



# BIS Working Papers

## No 1127

# The impact of green investors on stock prices

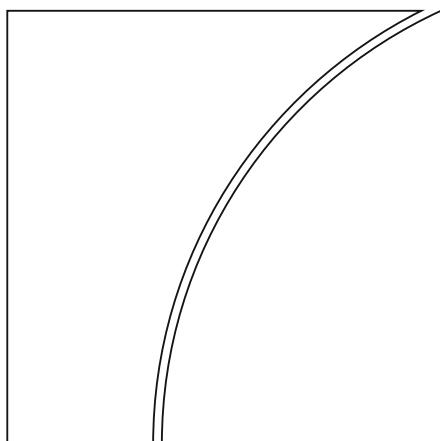
by Gong Cheng, Eric Jondeau, Benoit Mojon and Dimitri Vayanos

Monetary and Economic Department

September 2023 (revised March 2024)

JEL classification: G12, G23, Q54.

Keywords: Asset pricing, green investing, passive investing, portfolio rebalancing.



BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The papers are on subjects of topical interest and are technical in character. The views expressed in them are those of their authors and not necessarily the views of the BIS.

This publication is available on the BIS website ([www.bis.org](http://www.bis.org)).

© *Bank for International Settlements 2023. All rights reserved. Brief excerpts may be reproduced or translated provided the source is stated.*

ISSN 1020-0959 (print)  
ISSN 1682-7678 (online)

# The Impact of Green Investors on Stock Prices\*

Gong Cheng , Eric Jondeau<sup>1</sup>, Benoit Mojon<sup>2</sup>, and Dimitri Vayanos<sup>3</sup>

<sup>1</sup>HEC Lausanne and Swiss Finance Institute

<sup>2</sup>Bank for International Settlements

<sup>3</sup>London School of Economics, CEPR and NBER

Revised draft, March 12, 2024

## Abstract

We study the impact of green investors on stock prices in a dynamic equilibrium model where investors are green, passive or active. Green investors track an index that progressively excludes the stocks of the brownest firms, passive investors hold a value-weighted index of all stocks, and active investors hold a mean-variance efficient portfolio. Contrary to the literature, we find large drops in the stock prices of the brownest firms and moderate increases for greener firms. These effects occur primarily upon the announcement of the green index's formation and continue during the exclusion process. The announcement effects imply a first-mover advantage to early adopters of decarbonisation strategies.

**JEL:** G12, G23, Q54

**Keywords:** Asset pricing, green investing, passive investing, portfolio rebalancing

---

\*chengjacques@gmail.com, eric.jondeau@unil.ch, benoit.mojon@bis.org, d.vayanos@lse.ac.uk. We thank participants in the BIS Research Meeting and the NGFS Expert Network on Research webinar series for their constructive comments, and Jingtong Zhang for excellent research assistance. The project started when Gong Cheng was at the BIS. The views presented in this paper are those of the authors and do not necessarily reflect those of the BIS.

# 1 Introduction

In the fight against climate change, the role of financial asset owners and managers is widely debated. As large institutional investors are investing in financially diversified portfolios, they hold shares of firms with high greenhouse gas (GHG) emissions and thus contribute to global warming by financing polluting activities. An increasing number of corporate initiatives have sought to promote net zero investment in recent years. Central Banks, through the Network for Greening the Financial System (NGFS), have also been reflecting on greening their investment portfolios. Two broad approaches promoting green investment prevail. Investors can divest from the brownest firms (divestment) or influence the transition of those firms to greener operations through their votes at annual general meetings (engagement).

A key question that drives investors' consideration of divestment versus engagement concerns whether divestment effectively raises the cost of capital of brown firms versus green firms and thereby influences brown firms' future business development. In particular, shares of brown stocks sold by green investors will be bought by less climate-conscious investors. As a result, the impact of divestment on stock prices will depend on the willingness of those other investors to absorb additional shares of brown stocks. If there are few green investors relative to their counterparties, then the price impact will be small. This point is made by [Berk and van Binsbergen \(2022\)](#) in a thought-provoking paper. Within their calibration, which assumes a small community of green investors, they find that the change in the cost of capital resulting from divestment is insignificant.

Recent trends among financial investors suggest that the community of green investors might be far from small. Several initiatives have been launched by institutional investors to promote net zero investment, some of them under the umbrella or as partner of the United Nations Framework Convention on Climate Change.<sup>1</sup> These initiatives have committed to

---

<sup>1</sup>These groups include the Net Zero Asset Managers (NZAM) Initiative, the Net Zero Asset Owner (NZAO) Alliance, the Glasgow Financial Alliance for Net Zero (GFANZ), the Climate Action 100+, the Paris Aligned Asset Owners (PAAO), the Institutional Investors Group on Climate Change (IIGCC) among

transitioning investment portfolios to net zero emissions by 2050, which is consistent with a temperature rise of 1.5°C. Altogether, they represent a large share of the assets under management in the world, although not all of their assets are managed in the perspective of the net zero target. In such a situation, it might be that the counterparties of green investors require a significant price discount to buy brown stocks.

In this paper, we study within a dynamic equilibrium model the impact on stock prices of institutional investors reducing their exposure to the firms with the highest GHG emissions. We consider three categories of investors. Active investors hold a mean-variance efficient portfolio of all stocks. Passive investors hold a value-weighted index of all stocks and represent the large share of non-green passive institutional investors. Green investors hold a value-weighted index that progressively excludes brown firms. The brownest firms are excluded first from the green index, and every year an additional set of firms based on their carbon emissions is excluded. The green index replicates the strategy of a portfolio with a decreasing carbon footprint. Indices excluding brown stocks progressively are referred to as “net zero” or “Paris consistent” indices, and have been growing over time, as have the funds tracking them. As brown stocks are excluded progressively from the green index, active investors buy them at a discounted price that implies higher expected returns. Passive investors keep holding the market portfolio during this process and do not switch to an index that includes more brown stocks.

Given the investment demands of the three types of investors, we solve for equilibrium stock prices analytically following an approach similar to [Jiang, Vayanos and Zheng \(2022\)](#).<sup>2</sup> For a given calibration of the model and a decarbonisation trajectory of the green index, we determine the dynamics of equilibrium prices as well as those of investors’ demands. This approach allows us to assess the impact on expected and realised returns of different

---

many others.

<sup>2</sup>[Jiang, Vayanos and Zheng \(2022\)](#) study how passive investing affects equilibrium stock prices. They show that flows into passive funds raise disproportionately the stock prices of the economy’s largest firms. That result arises because of the re-pricing of systematic and large firms’ idiosyncratic risk.

scenarios regarding the proportion of green investors.

In our baseline calibration, the population of investors is split between 30% green investors, 50% passive investors and 20% active investors. These percentages broadly correspond to the current shares of the various investor types, as we argue in Section 4. The green index excludes progressively the 10% of brownest firms over ten years: the brownest 1% in the first year, the second brownest 1% in the second year, and so on. We find a substantial drop in the stock prices of the brown firms that are excluded and an increase in the stock prices of greener firms. These effects occur primarily when the exclusion strategy is announced, and continue during the transition process. By the end of the exclusion phase, after ten years, the price of the brownest firms declines by 5.6%, while the price of non-excluded firms rises by 0.7%. Overall, the cost of capital of the most polluting firms rises by 20 basis points (bp) compared to that of non-excluded firms. In addition, the large effects at the announcement date yield a first-mover advantage to green investors that adopt the decarbonisation (exclusion) strategy at an early stage. Additional calibrations in which the population of green investors grows over time yield even larger effects.

Two different mechanisms drive the profitability of green strategies in our model. Brown firms command a higher expected return after their exclusion, so that active investors are willing to increase their portfolio allocation to them. At the same time, brown firms experience a price drop at the announcement of the exclusion strategy, which results in a higher realised return for green investors compared to active and passive investors. The combined mechanism makes the decarbonisation strategy profitable during the exclusion phase (but not forever) even if brown firms earn higher expected return. This result is in line with empirical papers that have found negligible costs for green strategies.

Our paper is related to the literature on green investing and in particular on the impact that such investment choices might have on stock prices. [Bonnefon, Landier, Sastry and Thesmar \(2022\)](#) investigate investors' ethical preferences and argue that two types of

preferences drive responsible investors: in the first case, investors are reluctant to invest in companies that do not have a business model in line with their own moral values (value-alignment); in the second case, the primary drive of responsible investors is the societal outcomes of their investment choices (impact-seeking). This distinction often gives rise to the so-called divestment versus engagement (or exit versus voice) debate.

Theoretical models addressing the impact of divestment include [Pastor, Stambaugh and Taylor \(2021\)](#), [Pedersen, Fitzgibbons and Pomorski \(2021\)](#) and [Zerbib \(2022\)](#). The main mechanism is that brown assets should pay a higher expected return because green investors are reluctant to hold them and therefore active investors demand a premium to buy them. [Berk and van Binsbergen \(2022, BvB\)](#) quantify this mechanism in a calibration and find that the price effect of divestment is limited. This is because the depressing effect of green investors on brown assets is mitigated by a large number of active investors taking advantage of lower prices to purchase them. The effects that we uncover are larger than those in BvB by an order of magnitude. This is because we not only assume a larger fraction of green investors than in BvB, but we also drop the unrealistic assumption that all non-green investors are active. Because of the passive investors in our model, the elasticity of the demand function faced by green investors is smaller than in BvB by an order of magnitude and is closer to empirical estimates in the literature (e.g., [Gabaix and Koijen, 2021](#)).

The empirical literature presents a mixed panorama on the impact of divestment on stock prices. [Bolton and Kacperczyk \(2021\)](#) find that in the cross-section of U.S. stocks the level of carbon emissions affects stock returns significantly and gives rise to a carbon premium. A possible interpretation of this finding is that net-zero regulations target primarily activities with the highest emissions. [Bolton and Kacperczyk \(2022\)](#) find higher stock returns associated with higher levels and growth rates of carbon emissions in all sectors and most countries. [Hsu, Li and Tsou \(2023\)](#) study the interplay between industrial pollution and asset pricing, unveiling a “pollution premium” wherein firms with higher toxic emission in-

tensities yield superior average annual returns due to the exposure to a higher environmental policy uncertainty. [Becht, Pajuste and Toniolo \(2023\)](#) analyse another mechanism by which divestment is a form of disapproval, highlighting its significant influence on the stock prices of high carbon emitters, including firms not directly targeted by such actions. Their research underscores the transformative nature of divestment, evolving from a moral stance to a strategic exercise in risk management with far-reaching implications in the stock market.<sup>3</sup>

A few studies investigate the formulation of benchmark portfolios tailored for net-zero strategies. [Bolton, Kacperczyk and Samama \(2021\)](#) propose a tracking error relative to standard, business as usual benchmarks, albeit at the cost of extensive portfolio rebalancing.<sup>4</sup> Conversely, [Jondeau, Mojon and Pereira Da Silva \(2021\)](#) and [Cheng, Jondeau and Mojon \(2022\)](#) advocate an approach that maintains weights closely aligned with benchmarks, adjusting only for firms with extreme carbon emissions. The former paper adopts a forward-looking stance, assuming constant future carbon emissions. The latter paper adopts a backward-looking stance to construct a net zero benchmark over the recent period. In both methodologies, decarbonisation can be achieved at a relatively low cost in terms of financial performance and tracking error. Reasons for the low cost of decarbonisation are twofold. First, the demand pressure from green investors in favour of green assets results in higher prices of those assets, which partly compensate for the lower expected return, at least in the short run. Second, because of the extreme asymmetry in the distribution of carbon emissions, it is sufficient to divest from a small number of firms to achieve a large reduction

---

<sup>3</sup>Other papers explore the impact of green policies adopted by banks or capital-market investors on firms' financing avenues and their investment behaviour. [Green and Vallee \(2023\)](#) observe that in the coal industry, exit strategies by banks correlate with a reduction in debt issuance by firms engaged with those financial institutions, with limited substitution towards other banks or equity markets. [Kacperczyk and Peydro \(2022\)](#) similarly report that high-emissions firms experience a contraction in bank credit following their lending banks' commitment to green lending policies, yet note an absence of improvement in those firms' environmental performance.

<sup>4</sup>[Cenedese, Han and Kacperczyk \(2023\)](#) examine the construction of net zero portfolios and introduce the Distance-to-Exit (DTE) metric to evaluate the carbon-transition risk confronting companies. Their findings reveal that firms with elevated DTE values typically exhibit higher valuation ratios but lower expected returns, underscoring that DTE effectively captures carbon-transition risk.



in a portfolio's carbon footprint.

The rest of this paper is organised as follows. Section 2 presents the model. Section 3 solves for equilibrium prices and positions. Section 4 describes the calibration of the model and the various scenarios. Section 5 presents the results for the different scenarios. Section 6 discusses policy implications of our work and concludes. Appendix A provides proofs, technical details and additional results.

## 2 Model

Time  $t$  is continuous and goes from zero to infinity. The riskless rate is exogenous and equal to  $r > 0$ . There are  $K$  groups of  $N$  firms each. All firms in the same group have the same (unmodelled) level of GHG emissions. Firms in group  $K$ , with the highest indices  $n = (K - 1)N + 1, \dots, KN$ , have the highest emissions and are excluded from the index first. Firms in group  $K - 1$ , with the second highest indices  $n = (K - 2)N + 1, \dots, (K - 1)N$ , have the second highest emissions and are excluded second, and so on.

The stock of firm  $n = 1, \dots, KN$ , referred to as stock  $n$ , pays dividend flow  $D_{nt}$  per share and is in supply of  $\eta_n > 0$  shares. The dividend flow of stock  $n$  is

$$D_{nt} = \bar{D}_n + b_n^s D_t^s + b_n^c D_t^c + D_{nt}^i, \quad (2.1)$$

where  $\{\bar{D}_n, b_n^s, b_n^c\}_{n=1, \dots, KN}$  are constants and  $\{D_t^s, D_t^c, D_{nt}^i\}_{n=1, \dots, KN}$  are stochastic processes. We refer to  $\bar{D}_n$  as the constant component of the dividend flow,  $b_n^s D_t^s$  as the systematic component,  $b_n^c D_t^c$  as the climate component and  $D_{nt}^i$  as the idiosyncratic component. The systematic component is the product of a factor  $D_t^s$  times a factor loading  $b_n^s \geq 0$ . The factor  $D_t^s$  follows the square-root process

$$dD_t^s = \kappa^s (\bar{D}^s - D_t^s) dt + \sigma^s \sqrt{D_t^s} dB_t^s, \quad (2.2)$$

where  $\{\kappa^s, \bar{D}^s, \sigma^s\}$  are positive constants and  $B_t^s$  is a Brownian motion. The climate component is the product of a factor  $D_t^c$  times a factor loading  $b_n^c \geq 0$ . The factor  $D_t^c$  follows the square-root process

$$dD_t^c = \kappa^c (\bar{D}^c - D_t^c) dt + \sigma^c \sqrt{D_t^c} dB_t^c, \quad (2.3)$$

where  $\{\kappa^c, \bar{D}^c, \sigma^c\}$  are positive constants and  $B_t^c$  is a Brownian motion. The factors  $D_t^s$  and  $D_t^c$  are both systematic. We interpret the former as corresponding to the standard business-cycle risk and the latter as corresponding to climate transition risk. Climate transition risk refers to the uncertainty associated with the transition towards a low-carbon economy. It can arise from policies to mitigate climate change and achieve environmental sustainability goals, and the impact that those policies have on different firms. In Section 4, we directly relate the exposure to climate transition risk to the patterns in firms' carbon emissions. The idiosyncratic component follows the square-root process

$$dD_{nt}^i = \kappa_n^i (\bar{D}_n^i - D_{nt}^i) dt + \sigma_n^i \sqrt{D_{nt}^i} dB_{nt}^i, \quad (2.4)$$

where  $\{\kappa_n^i, \bar{D}_n^i, \sigma_n^i\}_{n=1, \dots, KN}$  are positive constants and  $\{B_{nt}^i\}_{n=1, \dots, KN}$  are Brownian motions. All Brownian motions are independent. By possibly redefining factor loadings, we set the long-run means  $\bar{D}^s$  and  $\bar{D}^c$  of the systematic factors to one. By possibly redefining the supply  $\eta_n$ , we set the long-run mean  $\bar{D}_n + b_n^s + b_n^c + \bar{D}_n^i$  of the dividend flow of stock  $n$  to one for all  $n$ . With these normalisations, we can write the dividend flow of stock  $n$  as

$$D_{nt} = 1 + b_s^n (D_t^s - 1) + b_n^c (D_t^c - 1) + (D_{nt}^i - \bar{D}_n^i). \quad (2.5)$$

The square-root specification (2.2)–(2.4) ensures that dividends and prices are always positive and the volatility of dividends per share increases with the level of dividends per share.<sup>5</sup>

---

<sup>5</sup>A geometric Brownian motion specification for dividends, which is commonly assumed in asset-pricing

Agents are competitive and form overlapping generations living over infinitesimal time intervals. Each generation includes active investors, passive investors and green investors. Active investors can invest in the riskless asset and in the stocks without constraints. Passive investors and green investors can invest in the riskless asset and in a stock portfolio that tracks an index. The index is a broad index for passive investors and a narrower one for green investors. Passive and green investors do not observe the values of the dividend flows (2.2)–(2.4) and make their investment decisions in expectation over these values.

The broad index includes all firms. The green index includes a set  $\mathcal{G}_t$  of firms that decreases with time  $t$ . At  $t = 0$ , all firms are included. At  $t = T$ , firms  $n = (K - 1)N + 1, \dots, KN$ , i.e., in group  $K$ , are dropped. At  $t = 2T$ , firms  $n = (K - 2)N + 1, \dots, (K - 1)N$ , i.e., in group  $K - 1$ , are also dropped. The process continues until  $t = K'T$  for  $K' < K$ , when firms  $n = (K - K')N + 1, \dots, (K - K' + 1)N$ , i.e., in group  $K'$ , are the last to be dropped. Times  $T, 2T, \dots, K'T$  correspond to rebalancing times for green investors.

The broad and the green indices are capitalisation-weighted, i.e., they weigh firms according to their market capitalisation. Therefore, the number of shares  $\eta_{Int}$  that the broad index includes of any firm  $n$  is proportional to the number of shares  $\eta_n$  issued by the firm. By possibly rescaling the broad index, we set  $\eta_{Int} = \eta_n$ . Likewise, the number of shares  $\eta_{Gnt}$  that the green index includes of any firm  $n \in \mathcal{G}_t$  is proportional to  $\eta_n$ . By possibly rescaling the green index, we set  $\eta_{Gnt} = \eta_n$  for  $n \in \mathcal{G}_t$ . Since  $\eta_{Gnt} = 0$  for  $n \notin \mathcal{G}_t$ , we can write  $\eta_{Gnt}$  for all  $n$  as  $1_{n \in \mathcal{G}_t} \eta_n$ .

We denote by  $W_{At}$ ,  $W_{It}$  and  $W_{Gt}$  the wealth of an active investor, a passive investor and a green investor, respectively, at time  $t$ , by  $z_{Ant}$ ,  $z_{Int}$  and  $z_{Gnt}$  the number of shares of firm  $n$  that these agents hold, and by  $\mu_{At}$ ,  $\mu_{It}$  and  $\mu_{Gt}$  the measure of these agents. A passive investor holds  $z_{Int} = \lambda_{It} \eta_n$  shares of firm  $n$ , and a green investor holds  $z_{Gnt} = \lambda_{Gt} \eta_{Gnt}$  shares of the firm, where  $\lambda_{It}$  and  $\lambda_{Gt}$  are proportionality coefficients that the agents choose

---

models, would also imply these properties. We adopt the square-root specification because it yields closed-form solutions.

optimally. The coefficients  $(\lambda_{It}, \lambda_{Gt})$  do not depend on the values of the dividend flows, which passive and green investors do not observe, but can depend on time. We assume that  $(\lambda_{It}, \lambda_{Gt})$  are constant in each of the intervals between rebalancing times  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$ , and denote their values by  $(\lambda_{Ik}, \lambda_{Gk})$  and  $(\lambda_{IK'}, \lambda_{GK'})$ , respectively. We likewise assume that  $(\mu_{At}, \mu_{It}, \mu_{Gt})$  are constant in each of these intervals, and denote their values by  $(\mu_{Ak}, \mu_{Ik}, \mu_{Gk})$  and  $(\mu_{AK'}, \mu_{IK'}, \mu_{GK'})$ , respectively.

The budget constraint of agent type  $i = A, I, G$  is

$$dW_{it} = \left( W_{it} - \sum_{n=1}^{KN} z_{int} S_{nt} \right) r dt + \sum_{n=1}^{KN} z_{int} (D_{nt} dt + dS_{nt}) = W_{it} r dt + \sum_{n=1}^{KN} z_{int} dR_{nt}^{sh}, \quad (2.6)$$

where  $dW_{it}$  is the infinitesimal change in wealth and  $dR_{nt}^{sh} \equiv D_{nt} dt + dS_{nt} - r S_{nt} dt$  is the share return of stock  $n$  in excess of the riskless rate. Agents have mean-variance preferences over  $dW_{it}$ . Active investors, who observe  $\{D_{nt}\}_{n=1, \dots, KN}$ , maximise the objective function

$$\mathbb{E}_t(dW_{At}) - \frac{\rho}{2} \text{Var}_t(dW_{At}) \quad (2.7)$$

over conditional mean and variance at time  $t$ . Passive and green investors, who do not observe  $\{D_{nt}\}_{n=1, \dots, KN}$ , maximise the objective function

$$\mathbb{E}_t^u(dW_{it}) - \frac{\rho}{2} \text{Var}_t^u(dW_{it}), \quad (2.8)$$

for  $i = I, G$ , over unconditional mean and variance at time  $t$ . The objective functions (2.7) and (2.8) can be derived from any Von Neumann–Morgenstern utility  $u$ , as shown in [Buffa, Vayanos and Woolley \(2022\)](#).

### 3 Equilibrium

We look for an equilibrium where the price  $S_{nt}$  of stock  $n$  is

$$S_{nt} = \bar{S}_{nt} + b_n^s S_t^s(D_t^s) + b_n^c S_t^c(D_t^c) + S_{nt}^i(D_{nt}^i), \quad (3.1)$$

where  $\bar{S}_{nt}$  is a deterministic function of  $t$ ,  $S_t^s(D_t^s)$  is a deterministic function of  $t$  and  $D_t^s$ ,  $S_t^c(D_t^c)$  is a deterministic function of  $t$  and  $D_t^c$ , and  $S_{nt}^i(D_{nt}^i)$  is a deterministic function of  $t$  and  $D_{nt}^i$ . The function  $\bar{S}_{nt}$  represents the present value of the constant component of dividends. The functions  $b_n^s S_t^s(D_t^s)$ ,  $b_n^c S_t^c(D_t^c)$ , and  $S_{nt}^i(D_{nt}^i)$  represent the present value of the systematic, climate and idiosyncratic components, respectively. Assuming that  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$ , and  $S_{nt}^i(D_{nt}^i)$  are twice continuously differentiable, we can write the share return  $dR_{nt}^{sh}$  of stock  $n$  as

$$\begin{aligned} dR_{nt}^{sh} &= (\bar{D}_n + b_n^s D_t^s + b_n^c D_t^c + D_{nt}^i)dt + (d\bar{S}_{nt} + b_n^s dS_t^s(D_t^s) + b_n^c dS_t^c(D_t^c) + dS_{nt}^i(D_{nt}^i)) \\ &\quad - r (\bar{S}_{nt} + b_n^s S_t^s(D_t^s) + b_n^c S_t^c(D_t^c) + S_{nt}^i(D_{nt}^i)) dt \\ &= \mu_{nt} dt + \sum_{j=s,c} b_n^j \sigma^j \sqrt{D_t^j} \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} dB_t^j + \sigma_n^i \sqrt{D_{nt}^i} \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} dB_{nt}^i, \end{aligned} \quad (3.2)$$

where

$$\begin{aligned} \mu_{nt} &\equiv \frac{\mathbb{E}_t(dR_{nt}^{sh})}{dt} = \bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r \bar{S}_{nt} \\ &\quad + \sum_{j=s,c} b_n^j \left[ D_t^j + \kappa^j (1 - D_t^j) \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} + \frac{1}{2} (\sigma^j)^2 D_t^j \frac{\partial^2 S_t^j(D_t^j)}{\partial (D_t^j)^2} + \frac{\partial S_t^j(D_t^j)}{\partial t} - r S_t^j(D_t^j) \right] \\ &\quad + D_{nt}^i + \kappa_n^i (\bar{D}_n^i - D_{nt}^i) \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} + \frac{1}{2} (\sigma_n^i)^2 D_{nt}^i \frac{\partial^2 S_{nt}^i(D_{nt}^i)}{\partial (D_{nt}^i)^2} + \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial t} - r S_{nt}^i(D_{nt}^i) \end{aligned} \quad (3.3)$$

is the instantaneous expected share return of stock  $n$ , and the second step in (3.2) follows from (2.2)–(2.4) and Ito's lemma.

Using (2.6) and (3.2), we can write the objective function (2.7) of active investors as

$$\sum_{n=1}^{KN} z_{Ant} \mu_{nt} - \frac{\rho}{2} \left[ \sum_{j=s,c} \left( \sum_{n=1}^{KN} z_{Ant} b_n^j \right)^2 (\sigma^j)^2 D_t^j \left[ \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} \right]^2 + \sum_{n=1}^{KN} z_{Ant}^2 (\sigma_n^i)^2 D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right]. \quad (3.4)$$

Active investors maximise (3.4) over positions  $\{z_{Ant}\}_{n=1,\dots,KN}$ . Their first-order condition is

$$\mu_{nt} = \rho \left[ \sum_{j=s,c} b_n^j \left( \sum_{m=1}^{KN} z_{Amt} b_m^j \right) (\sigma^j)^2 D_t^j \left[ \frac{\partial S_t^j(D_t^j)}{\partial D_t^j} \right]^2 + z_{Ant} (\sigma_n^i)^2 D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \quad (3.5)$$

and equates the instantaneous expected share return  $\mu_{nt}$  of stock  $n$  to the stock's contribution to instantaneous portfolio return variance times the risk-aversion coefficient  $\rho$ .

Market clearing requires that the aggregate demand of active investors, passive investors and green investors equals the supply coming from the issuing firm:

$$\mu_{At} z_{Ant} + \mu_{It} \lambda_{It} \eta_n + \mu_{Gt} \lambda_{Gt} 1_{n \in \mathcal{G}_t} \eta_n = \eta_n \Rightarrow z_{Ant} = \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} 1_{n \in \mathcal{G}_t}}{\mu_{At}} \eta_n. \quad (3.6)$$

Substituting  $z_{Ant}$  from (3.6) into (3.5) and conjecturing that the functions  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$  and  $S_{nt}^i(D_{nt}^i)$  are affine increasing in  $D_t^s$ ,  $D_t^c$  and  $D_{nt}^i$ , respectively, i.e.,

$$S_t^s(D_t^s) = a_{0t}^s + a_{1t}^s D_t^s, \quad (3.7)$$

$$S_t^c(D_t^c) = a_{0t}^c + a_{1t}^c D_t^c, \quad (3.8)$$

$$S_{nt}^i(D_{nt}^i) = a_{n0t}^i + a_{n1t}^i D_{nt}^i, \quad (3.9)$$

for  $(a_{0t}^s, a_{1t}^s, \{a_{n0t}^i, a_{n1t}^i\}_{n=1,\dots,KN})$  positive functions of  $t$ , we find

$$\bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r\bar{S}_{nt} + \sum_{j=s,c} b_n^j \left[ D_t^j + \kappa^j a_{1t}^j (1 - D_t^j) + \frac{da_{0t}^j}{dt} + \frac{da_{1t}^j}{dt} D_t^j - r(a_{0t}^j + a_{1t}^j D_t^j) \right]$$

$$\begin{aligned}
& + D_{nt}^i + \kappa_n^i a_{n1t}^i (\bar{D}_n^i - D_{nt}^i) + \frac{da_{n0t}^i}{dt} + \frac{da_{n1t}^i}{dt} D_{nt}^i - r(a_{n0t}^i + a_{n1t}^i D_{nt}^i) \\
& = \rho \left[ \sum_{j=s,c} b_n^j \left( \sum_{m=1}^{KN} \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} \mathbf{1}_{m \in \mathcal{G}_t}}{\mu_{At}} \eta_m b_m^j \right) (\sigma^j a_{1t}^j)^2 D_t^j \right. \\
& \quad \left. + \frac{1 - \mu_{It} \lambda_{It} - \mu_{Gt} \lambda_{Gt} \mathbf{1}_{n \in \mathcal{G}_t}}{\mu_{At}} \eta_n (\sigma_n^i a_{n1t}^i)^2 D_{nt}^i \right]. \tag{3.10}
\end{aligned}$$

Equation (3.10) is affine in  $(D_t^s, D_t^c, D_{nt}^i)$ . Identifying linear terms in  $D_t^j$  for  $j = s, c$  and recalling that  $(\lambda_{It}, \lambda_{Gt}, \mu_{At}, \mu_{It}, \mu_{Gt})$  are constant in each of the intervals  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$  yields a Riccati ordinary differential equation (ODE) in  $a_{1t}^j$  in each of these intervals. The solution in the interval  $[K'T, \infty)$  is constant. The solution in each interval  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  is time-varying. Identifying linear terms in  $D_{nt}^i$  yields an ODE of the same type in  $a_{n1t}^i$ . Identifying constant terms yields a linear ODE in each interval.

**Proposition 3.1.** *The equilibrium price function has the form (3.1) with  $S_t^s(D_t^s)$ ,  $S_t^c(D_t^c)$  and  $S_{nt}^i(D_{nt}^i)$  given by (3.7), (3.8) and (3.9), respectively. The function  $a_{1t}^j$  for  $j = s, c$  is given by  $a_{1t}^j = \bar{a}_{1K'}^j$  for  $t \in [K'T, \infty)$  and*

$$a_{1t}^j = \frac{\bar{a}_{1k}^j \left( g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j} \right) e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T-t]} - \frac{1}{\bar{a}_{1k}^j} \left( \bar{a}_{1k}^j - a_{1,(k+1)T}^j \right)}{\left( g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j} \right) e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T-t]} + g_k^j \left( \bar{a}_{1k}^j - a_{1,(k+1)T}^j \right)} \tag{3.11}$$

for  $[kT, (k+1)T)$  and  $k = 0, \dots, K' - 1$ , where

$$\begin{aligned}
\bar{a}_{1k}^j & \equiv \frac{2}{r + \kappa^j + \sqrt{(r + \kappa^j)^2 + 4g_k^j}}, \\
g_k^j & \equiv \rho \left( \sum_{m=1}^{KN} \frac{1 - \mu_{Ik} \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} \mathbf{1}_{\{m \leq (K-k)N\}}}{\mu_{Ak}} \eta_m b_m^j \right) (\sigma^j)^2.
\end{aligned}$$

The function  $a_{1nt}^i$  is given by  $a_{1nt}^i = \bar{a}_{n1K'}^i$  for  $t \in [K'T, \infty)$  and

$$a_{1nt}^i = \frac{\bar{a}_{n1k}^i \left( g_{nk}^i a_{n1,(k+1)T}^i + \frac{1}{\bar{a}_{n1k}^i} \right) e^{\left( g_{nk}^i \bar{a}_{n1k}^i + \frac{1}{\bar{a}_{n1k}^i} \right) [(k+1)T-t]} - \frac{1}{\bar{a}_{n1k}^i} \left( \bar{a}_{n1k}^i - a_{n1,(k+1)T}^i \right)}{\left( g_{nk}^i a_{n1,(k+1)T}^i + \frac{1}{\bar{a}_{n1k}^i} \right) e^{\left( g_{nk}^i \bar{a}_{n1k}^i + \frac{1}{\bar{a}_{n1k}^i} \right) [(k+1)T-t]} + g_{nk}^i \left( \bar{a}_{n1k}^i - a_{n1,(k+1)T}^i \right)}, \quad (3.12)$$

where

$$\begin{aligned} \bar{a}_{n1k}^i &\equiv \frac{2}{r + \kappa_n^i + \sqrt{(r + \kappa_n^i)^2 + 4g_{nk}^i}}, \\ g_{nk}^i &\equiv \rho \frac{1 - \mu_{Ik} \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} \mathbb{1}_{\{n \leq (K-k)N\}}}{\mu_{Ak}} \eta_n (\sigma_n^i)^2. \end{aligned}$$

The function  $\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i$  is given by

$$\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i = \frac{\bar{D}_n}{r} + \sum_{j=s,c} b_n^j \kappa_n^j \int_t^\infty a_{1t'}^j e^{-r(t'-t)} dt' + \kappa_n^i \bar{D}_n^i \int_t^\infty a_{n1t'}^i e^{-r(t'-t)} dt'. \quad (3.13)$$

The values of  $(\lambda_{Ik}, \lambda_{Gk})$  for  $k = 0, \dots, K'$  are determined from the first-order conditions of passive and green investors in Appendix A.

Figure 1 summarises the model set-up by illustrating the portfolio rebalancing between green and active investors. The figure assumes four groups of stocks, with the last two groups (most polluting firms) being excluded by green investors. At time 0, both active and green investors hold one quarter of their wealth in each of the four groups of stocks. At time 1, green investors sell a fraction of the shares of the most polluting firms (brown bars), which are bought by active investors. The rest of their portfolio is reallocated proportionately. The exclusion goes for a number of periods, as it will be defined in the model; at time  $t$ , the most polluting firms are completely excluded from the portfolio of green investors and other groups of stocks are held proportionately.

After all adjustments have been made by green investors, their portfolio consists of the



first two groups (least polluting firms). Active investors hold a large fraction of the last two groups (the share initially held by green investors and their own initial share) and much less of the first two groups (possibly nothing) bought by green investors.

## 4 Calibration and Scenarios

We consider 500 firms, categorised into 100 groups of 5 firms each ( $K = 100$  and  $N = 5$ ). Group 1 is the least polluting and Group 100 the most polluting. Firm characteristics are identical except for the loadings on the climate transition risk factor, which depend on firms' GHG emissions. The horizon of the decarbonisation strategy is ten years ( $K' = 10$ ). Firms in Group 100 are removed from the green index first, after one year. Firms in Group 91 are removed last, after ten years. All in all, 50 firms are excluded, which amount for 10% of all firms.

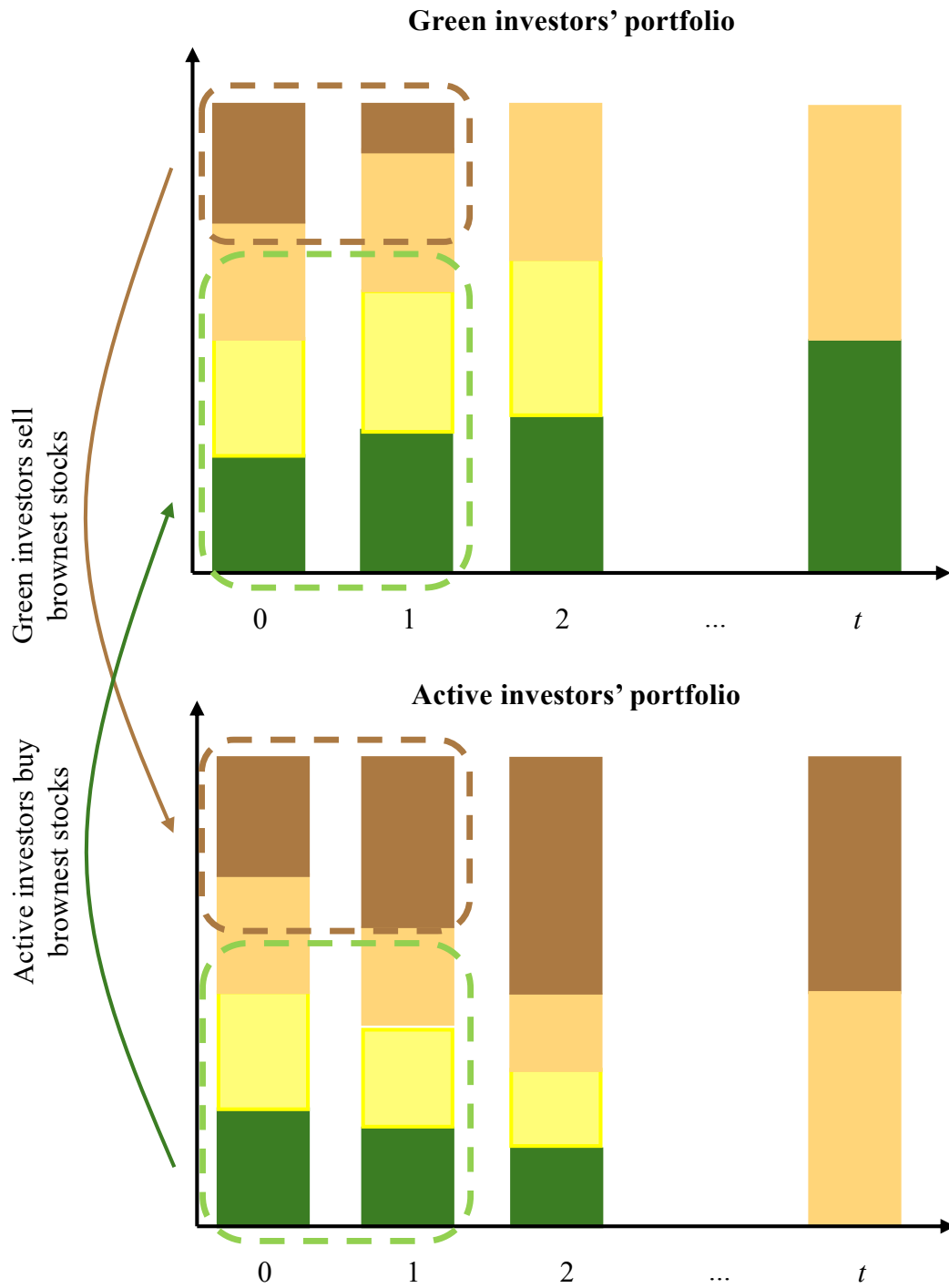
The calibration of the rate of exclusion aligns with recent empirical findings on the cross-sectional characteristics of carbon emissions and net zero investment strategies. Carbon emissions exhibit a Pareto distribution with a heavy right tail. [Jondeau et al. \(2021\)](#) find that eliminating the most polluting firms representing 1% of market capitalisation leads to an average reduction of 15% of the portfolio's carbon emissions.<sup>6</sup> In a net zero investment perspective, a cumulative reduction of portfolio emissions by 10% per year—corresponding to a 65% reduction over 10 years—would require gradually excluding the most polluting firms representing 1% of the market capitalisation every year. As a result, after ten years, the exclusion of the most polluting firms representing approximately 10% of the market capitalisation would reduce portfolio emissions by 65%.

The rationale behind the choice of a gradual exclusion of the most polluting firms is supported by operational considerations. Certain institutional investors, such as foundations

---

<sup>6</sup>According to a widely cited report by the Climate Disclosure Project or CDP published in 2017, 70.6% of global GHG emissions since 1988 are due to 100 companies. See <https://www.cdp.net/en/articles/media/new-report-shows-just-100-companies-are-source-of-over-70-of-emissions>.

Figure 1: Asset exclusion and exchange between green investors and active investors



or pension funds, aiming to decrease the carbon footprint of their portfolios, might be hesitant to implement rapid changes, given the obligation of maintaining a tracking error of their portfolio close to a standard benchmark. A gradual approach assists in spreading the impact on tracking error over multiple years while facilitating a swift reduction in emissions for the green portfolio. Nevertheless, our simulation results indicate that green investors embracing the decarbonisation strategy early gain a first-mover advantage over late adopters. This advantage stems from the decreasing trajectory of prices for brown stocks. Early movers can sell brown stocks at a higher price compared to later movers. It is noteworthy that the strategy adopted by green investors in our model closely aligns with the design of strategies for “Paris aligned” or “net zero” indices or funds.<sup>7</sup>

Importantly, throughout the decarbonisation process, green investors reinvest the funds generated from exclusions into less polluting firms, as depicted in Figure 1. Strategies with a more proactive approach, such as directing the exclusion proceeds toward the least polluting firms, would further strengthen the price impact documented in the paper.

The fractions of the three types of investors are key elements of our calibration. We consider three scenarios, as outlined in Table 1. In Scenario 1 we assume that the fractions are constant over time and equal to  $(\mu_A, \mu_I, \mu_G) = (20\%, 50\%, 30\%)$ . Setting the fraction  $\mu_I$  of passive investors to 50% aligns with evidence on the massive shift from active to passive strategies. Bloomberg report that index mutual funds and ETFs constituted 54% of U.S. equity mutual funds’ assets under management as of the end of 2020.<sup>8</sup>

The fraction of passive investors is even larger when adding to passive funds those active funds that track their benchmarks closely. [Chinco and Sammon \(2022\)](#) estimate that passive investors under this broader definition comprised 37.8% of all investors in the U.S. stock market in 2020. They arrive at their estimate using trading volumes around index additions

---

<sup>7</sup>MSCI and S&P have launched the MSCI Climate Paris Aligned Indexes family and the Paris Aligned & Climate Transition indexes family, respectively. Among others, Amundi, Lyxor, and iShares have launched ETFs or funds based on Paris aligned indices.

<sup>8</sup>See <https://www.bloomberg.com/professional/blog/passive-likely-overtakes-active>.

Table 1: Key parameters for the three baseline scenarios

	Green investors	Active investors	Passive investors	Climate transition risk factor
Scenario 1	$\mu_G = 30\%$	$\mu_A = 20\%$	$\mu_I = 50\%$	$b_n^c = 0$
Scenario 2	$\mu_G = 30\% \rightarrow 60\%$	$\mu_A = 20\%$	$\mu_I = 50\% \rightarrow 20\%$	$b_n^c = 0$
Scenario 3	$\mu_G = 30\% \rightarrow 60\%$	$\mu_A = 20\%$	$\mu_I = 50\% \rightarrow 20\%$	$b_n^c > 0$

and deletions.

If passive investors are defined even more broadly to include investors who do not respond to price changes, then their fraction can be even larger. Indeed, some long-term investors may not track a benchmark closely to be included in measures of passive investors but may still not respond aggressively to price changes of some stocks. [Koijen, Richmond and Yogo \(2020\)](#) find that the ownership share of passive and long-term investors exceeded 50% in 2019. [Van der Beck and Jaunin \(2021\)](#) estimate that the share of inelastic investors is close to 26%, while the share of purely passive investors is around 39%.

Setting the fraction  $\mu_G$  of green investors to 30% aligns with estimates from the Global Sustainable Investment Alliance, indicating approximately 36% of sustainable assets under management in 2020 ([GSIA, 2022](#)), although [OECD \(2023\)](#) contends that this may be an overestimate.<sup>9</sup> The fraction  $\mu_A$  of active investors is set to  $\mu_A = 1 - \mu_I - \mu_G = 20\%$ . Active investors are willing to buy brown stocks from green investors provided there is sufficient compensation for holding them.

In Scenario 2, we assume that the fraction  $\mu_G$  of green investors grows over time, starting at an initial 30% in year 1 and reaching 60% by year 10. We assume that the fraction  $\mu_A$  of active investors remains constant and equal to 20% and that the fraction  $\mu_I$  passive investors decreases from 50% in year 1 to 20% in year 10.

<sup>9</sup>Net-zero investors are a subset of green investors in the above estimates and their share is significantly smaller. According to the Phoenix Capital’s impact database, as of February 2023, 729 net zero aligned funds (from 325 organisations) have raised 289 billion euros of capital. 58% of these funds are open to investment. See <https://phenixcapitalgroup.com/download-impact-report-march-23-net-zero>.

In Scenario 3, we introduce climate transition risk while maintaining the fractions of investors as in Scenario 2. All firms exhibit positive exposure to climate transition risk ( $b_n^c > 0$  for all  $n$ ), with the firms with higher carbon emissions having higher exposure to that risk. The loading  $b_n^c$  on the climate transition risk factor is parameterised to align with the Pareto cross-section distribution of carbon emissions:  $b_n^c$  increases from 0.0027 for least polluting firms (Group 1) to 0.011 for the tenth most polluting firms (Group 91), 0.037 for the second most polluting firms (Group 99), and 0.1 for the most polluting firms (Group 100). The parameters of the distribution of  $b_n^c$  are chosen to reflect the extreme asymmetry in the distribution of carbon emissions across firms, with a few firms contributing massively to the global emissions.

Many of the remaining parameters are set following [Jiang, Vayanos and Zheng \(2022\)](#) and are chosen to match expected excess returns, CAPM beta and CAPM  $R^2$ . Key parameters are summarised in [Table 2](#).

Table 2: Calibrated parameters

	Volatility	Beta	Mean-reversion	Dividend
Systematic risk	$\sigma^s = 1.4$	$b_n^s = 0.82$	$\kappa^s = 0.04$	$\bar{D}^s = 1$
Climate transition risk	$\sigma^c = 1.4$		$\kappa^c = 0.04$	$\bar{D}^c = 1$
Idiosyncratic risk	$\sigma_n^i = \sigma^s \sqrt{\bar{D}_n^i}$		$\kappa_n^i = 0.04$	$\bar{D}_n^i = 0.18$
Riskless rate	$r = 3\%$			
Number of shares	$\eta = 0.001$			
Risk aversion	$\rho = 1$			
Number of groups	$K = 100$	$K' = 10$	$N = 5$	

The riskless rate is set to  $r = 3\%$ . The mean-reversion parameters ( $\kappa^s$ ,  $\kappa^c$ , and  $\kappa_n^i$ ) are set to 4%. The parameters  $b_n^s$  and  $\bar{D}_n^i$  are set to match the average CAPM  $\beta$  and  $R^2$  across stocks. The number of shares ( $\eta = 0.1\%$ ) and the risk aversion parameter ( $\rho = 1$ ) are chosen to generate expected excess returns across groups that lie between 4% and 6%. The diffusion

parameter  $\sigma^s = 1.4$  is chosen to maximise stocks' return variances. Returns are computed from share returns by dividing by the share price. The expected excess return of stock  $n$  is

$$dR_{nt} \equiv \frac{\mathbb{E}_t(dR_{nt}^{sh})}{S_{nt}} = \frac{D_{nt}dt + \mathbb{E}_t(dS_{nt})}{S_{nt}} - rdt \quad (4.14)$$

## 5 Simulation Results

### 5.1 Scenario 1: Modest Share of Green Investors

Our baseline case corresponds to a situation where the proportion of green investors is modest ( $\mu_G = 30\%$ ) and remains constant over the whole time horizon. Green investors reduce their exposure to brown stocks whereas active investors, who represent 20% of total investors, are ready to sell off green stocks and buy brown stocks. Figure 2 illustrates the evolution of prices and expected returns (or cost of capital) of three representative stocks, namely, stocks of the firms that are excluded in Year 1 (first period of exclusion), in Year 10 (last period of exclusion) and the firms that are not excluded from the green investors' portfolios. Table 3 provides summary statistics of the changes in prices, in the cost of capital as well as a few other characteristics (return volatility, and CAPM  $\beta$ ) of the representative stocks.

When green investors make their exclusion strategy (the 10-year exclusion horizon and the order of the groups of stocks to be excluded) public in year  $t = 0$ , prices and expected returns adjust instantaneously to the new equilibrium (Table 3,  $t = 0$  lines). Prices of the excluded stocks drop, with those of Group 1 firms being affected the most. The stock price of the firms excluded in Year 1 drops by 5.4%, while that of the firms to be excluded in Year 10 drops by 4%. In contrast, the stock price of the firms that will never be excluded during the 10-year exclusion period rises by 0.6%. The expected returns of the excluded firms increase to attract active investors. The instantaneous increase is the largest for the most polluting firms or the ones excluded first, by 13 basis points (bp). In contrast, the expected return of

non-excluded firms is reduced by 1 bp. After 10 years, at the end of the exclusion period when a new equilibrium is reached, the stock price of the excluded firms drops cumulatively by 5.6%, while the price of non-excluded firms increases by 0.7%. Over the same period, the cost of capital of the most polluting firms increases cumulatively by 18 bp, whereas that of non-excluded firms decreases by 2 bp, reflecting the higher cost of financing of brown firms relative to green firms.

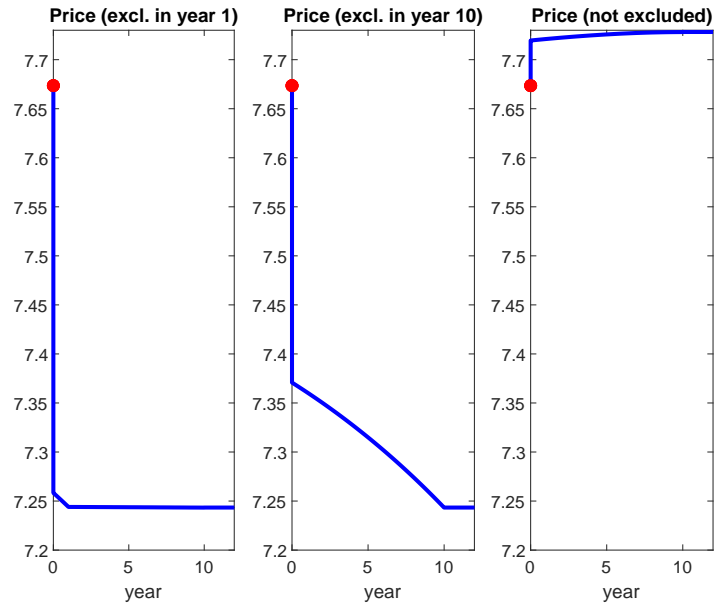
Table 3: Results - Scenario 1

	Firms excluded in Year 1	Firms excluded in Year 10	Non-excluded firms
$\Delta$ Price at $t = 0$	-5.41%	-3.95%	0.60%
$\Delta$ Price at $t = 10$	-5.61%	-5.61%	0.71%
$\Delta$ Cost of capital at $t = 0$	0.13%	0.11%	-0.01%
$\Delta$ Cost of capital at $t = 10$	0.18%	0.18%	-0.02%
Realised return over 10 years	4.38%	4.48%	4.80%
Return volatility at $t = 0$	20.16%	20.43%	20.11%
Return volatility at $t = 10$	20.11%	20.11%	20.14%
CAPM $\beta$ at $t = 0$	1.19	1.18	1.14
CAPM $\beta$ at $t = 10$	1.19	1.19	1.14

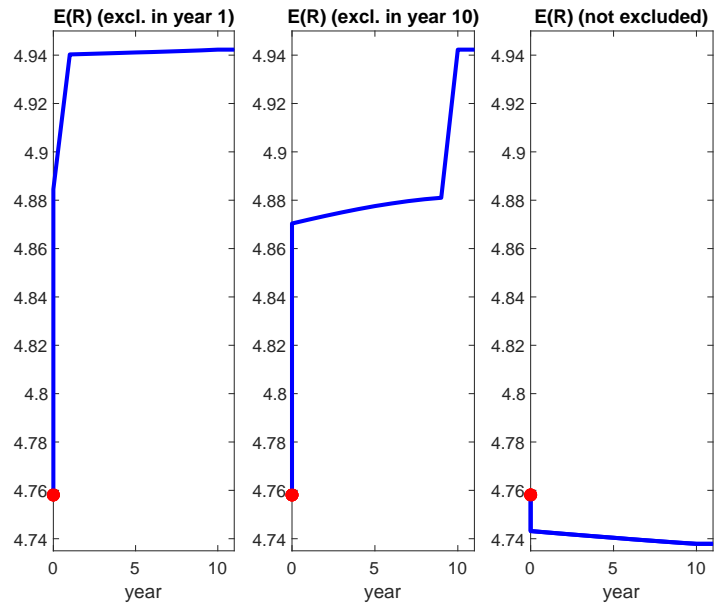
The effect of green investors' exclusion strategy on the cost of capital is large. In comparison, BvB find no detectable difference, around 0.44 bp, in the cost of capital between the firms that care about environmental and social costs and firms that do not. In our study, the difference reaches 20 bp, approximately 45 times bigger than what these authors estimate. Two main factors explain this difference. First, in BvB's baseline calibration, green investors only represent 2% of total wealth, whereas they represent 30% of wealth in our model. Even with a much higher fraction of green investors (33%), BvB find a small effect on the cost of capital, equal to 10.6 bp. To obtain a 20 bp effect in their framework, the fraction of green investors should equal 50% of total wealth. Second and more importantly, these authors

Figure 2: Impact of exclusion in Scenario 1

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the change in the price of a given Group of firms (in %) relative to the price before the announcement of the net zero strategy. The expected return impact is measured as the change in the expected return of a given Group of firms (in percentage point) relative to the expected return before the announcement of the net zero strategy.



assume that the remaining 98% of the market is composed of active investors, who can thus take over all brown assets that green investors want to sell, thus making polluting and less polluting assets close to perfect substitutes. In contrast, we assume a much smaller share of active investors in the market (20%) and the remaining 50% of investors are passive investors who cannot change the composition of their portfolio frequently, and must follow the development of market capitalisation. This set-up makes polluting and less polluting assets imperfectly substitutable.

Moreover, we highlight an interesting “first-mover advantage” of green investing. Our simulation results show that price and expected returns change at the announcement and over the 10-year transition period, between the first set and the last set of firms that are excluded. For the firms excluded in Year 1, the stock price instantaneously drops by 5.4% at the announcement of the exclusion strategy, very close to the final steady state price at the end of the 10-year transition period, i.e., 5.6%. In comparison, the initial price change in Year 1 for the firms to be excluded in the last round, i.e., in Year 10, is only about 4%, with the gap between the initial price and the final steady state price phased in gradually during the 10-year transition period (Table 3). The expected returns show similar patterns: the initial increase in expected returns reaches 13 bp for the first set of firms excluded in Year 1, against 11 bp for the last set of firms to be excluded. At the end of the transition, the expected returns increase by 18 bp for both groups. Therefore, given the perfect foresight of our model, green investment shows a “first-mover advantage”, as all brown assets that are expected to be excluded from the green investible space will take a price haircut at the very first moment when the green investing strategy is announced.

The benefit from the first-mover advantage can be measured by computing the realised return of the various firms from the investor’s perspective. The realised return over the 10 years is computed by adding the percentage price change in year  $t = 0$  with the cumulative expected return over the 10 years. We find that even if expected return of green firms is

lower after the announcement, their ex-post return is higher over the 10-year period. This observation is consistent with the paradox discussed by [Pastor, Stambaugh and Taylor \(2022\)](#) that brown firms should command a higher expected return but under-performed green firms over the recent period.

The table also reports the return volatility and the beta of the CAPM regression for the various groups of firms. As in Scenario 1, all firms have the same exposure to systematic risks, differences between groups are limited. Notably, excluded firms are more sensitive to market movements, with a slightly higher beta parameter.

We conduct a sensitivity analysis of the baseline results. Increasing the share of green investors ( $\mu_G$ ) does not change the price impact if the share of active investors ( $\mu_A$ ) increases in the same proportion. Varying the share of passive investors from  $\mu_G = 10\%$  to  $\mu_G = 90\%$  while keeping the same relative share of green and active investors ( $\mu_G/\mu_A = 2/3$ ) yields the same price impact .

Fixing the share of passive investors to  $\mu_G = 50\%$ , and varying the relative share of green and active investors from ( $\mu_G = 0.1$  and  $\mu_A = 0.4$ ) to ( $\mu_G = 0.4$  and  $\mu_A = 0.1$ ) changes the price impact significantly, as illustrated in [Figure 3](#) (Panel A). For instance, a shift from our baseline case ( $\mu_G = 0.3$  and  $\mu_A = 0.2$ ) to ( $\mu_G = 0.4$  and  $\mu_A = 0.1$ ) is sufficient to more than double the price impact. The final impact on the price of excluded firms increases from 5.6% to 12.2%, while the increase in the cost of capital rises from 18 bp to 43 bp (Panel B).

We finally investigate the effect of the net zero strategy on the realised return of the various groups of firms (Panel C). The initial price impact dominates the expected return impact over the 10-year period for all shares of green investors: from the investor's perspective, brown firms have a lower performance than green firms because of the initial price decrease.

Figure 3: Impact of changing the relative share of green and active investors

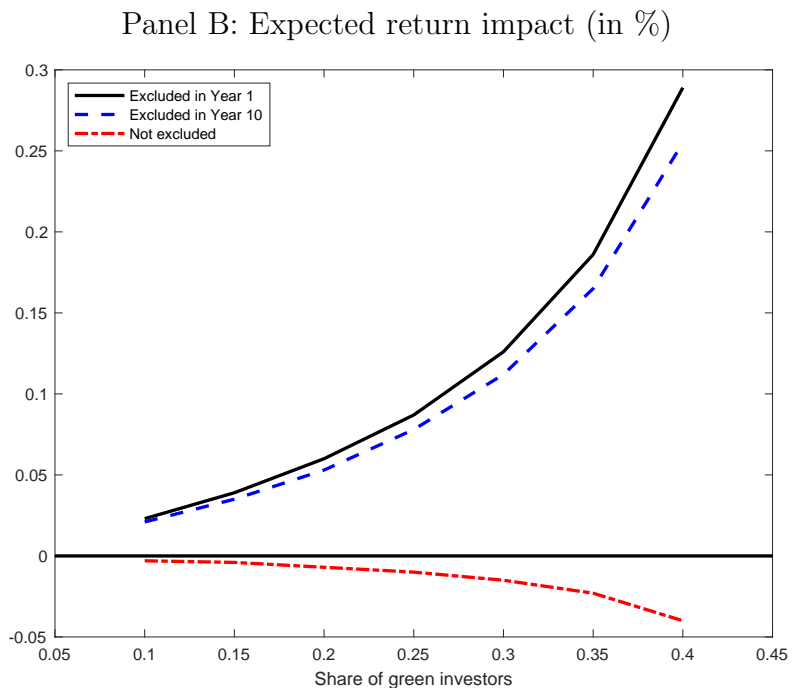
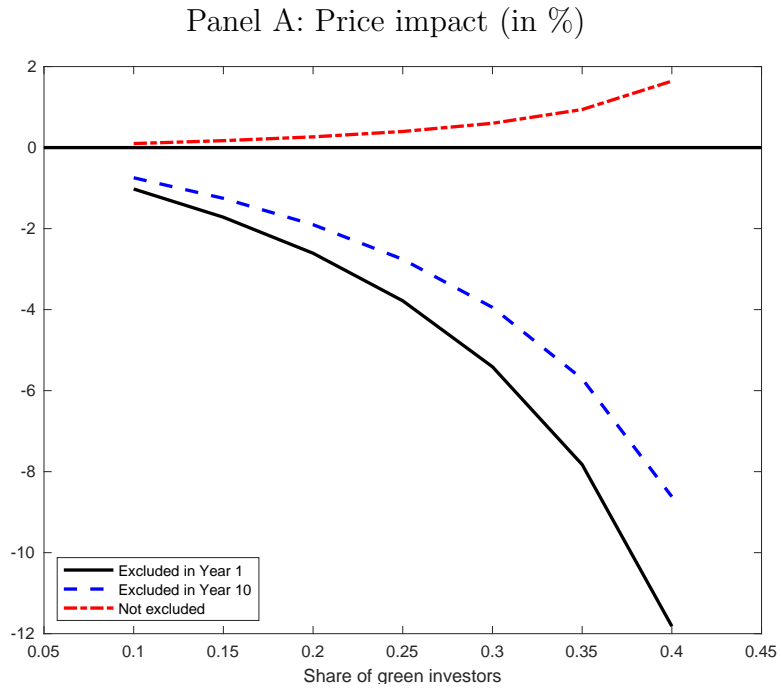
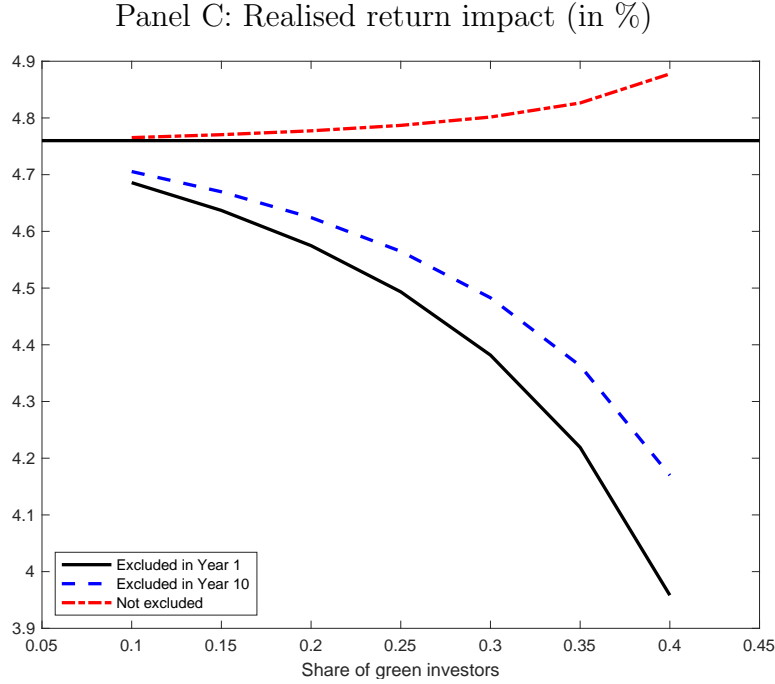


Figure 3 (Cont.): Impact of changing the relative share of green and active investors



Note: The price impact is measured as the change in the price of a given Group of firms (in %) relative to the price before the announcement of the net zero strategy (Panel A). The expected return impact is measured as the change in the expected return of a given Group of firms (in percentage point) relative to the expected return before the announcement of the net zero strategy (Panel B). The realised return impact is measured as the change in year  $t = 0$  in the price plus the average expected return over the 10 years of a given Group of firms (in percentage point) (Panel C).

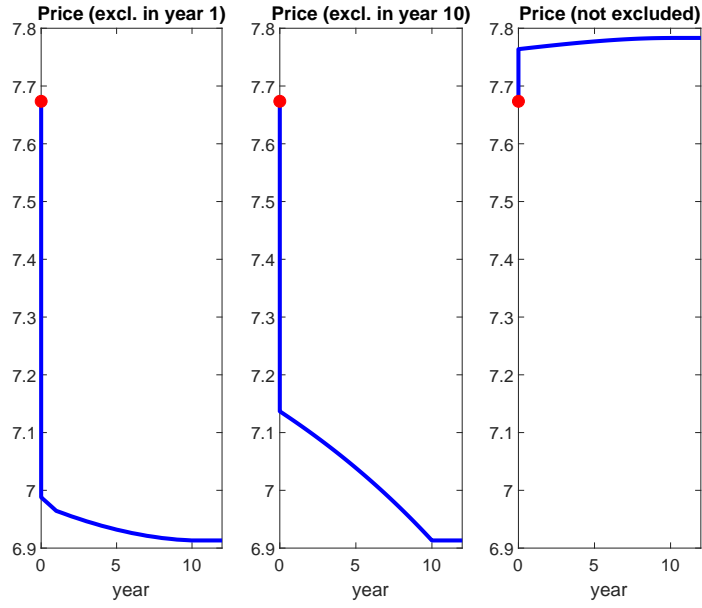
## 5.2 Scenario 2: Growing Proportion of Green Investors

To capture the impact of the size of green investors in driving our results, we now present Scenario 2, in which we allow the population of green investors to grow from 30% of the market ( $\mu_G = 30\%$ ) in Year 1 to  $\mu_G = 60\%$  in Year 10. We keep the proportion of active investors constant at  $\mu_A = 20\%$ , suggesting that active investors will be progressively less effective at compensating the willingness of green investors to divest from brown firms. The population of passive investors will reduce to cater for the increase in the proportion of green investors.

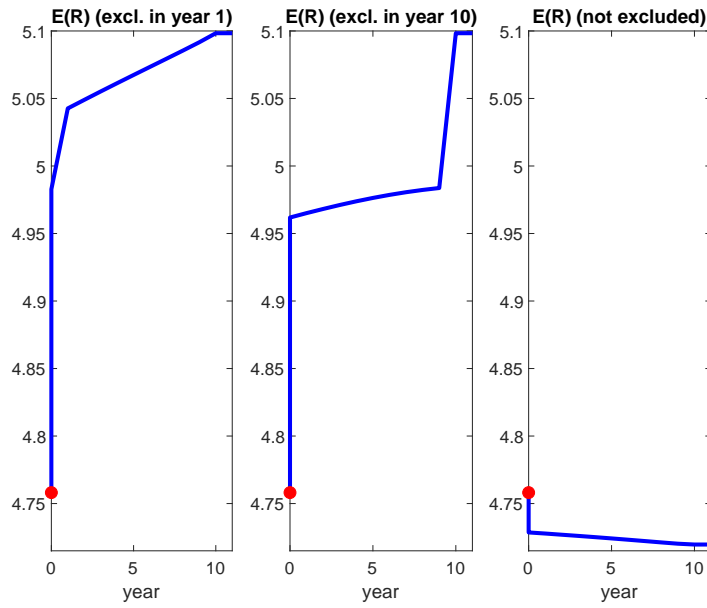
Figure 4 shows our simulation of stock prices and expected returns and Table 4 gives

Figure 4: Impact of exclusion in Scenario 2

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the change in the price of a given Group of firms (in %) relative to the price before the announcement of the net zero strategy. The expected return impact is measured as the change in the expected return of a given Group of firms (in percentage point) relative to the expected return before the announcement of the net zero strategy.

Table 4: Results - Scenario 2

	Firms excluded in Year 1	Firms excluded in Year 10	Non-excluded firms
$\Delta$ Price at $t = 0$	-8.94%	-7.00%	1.18%
$\Delta$ Price at $t = 10$	-9.91%	-9.91%	1.43%
$\Delta$ Cost of capital at $t = 0$	0.22%	0.20%	-0.03%
$\Delta$ Cost of capital at $t = 10$	0.34%	0.34%	-0.04%
Realized return over 10 years	4.15%	4.28%	4.84%
Return volatility at $t = 0$	20.32%	20.68%	20.09%
Return volatility at $t = 10$	20.09%	20.09%	20.14%
CAPM $\beta$ at $t = 0$	1.22	1.21	1.14
CAPM $\beta$ at $t = 10$	1.22	1.22	1.14

more detailed return moments. The impact of exclusion on prices and expected returns approximately doubles compared with Scenario 1, as the proportion of green investors increases while that of active investors remains constant, making brown and green assets strongly imperfect substitutes. First, the stock price of the excluded firms is massively reduced at the time of announcement, from 8.9% for the firms excluded in Year 1 to 7% for the firms to be excluded in Year 10. Not-excluded firms benefit from a 1.2% instantaneous increase in their stock prices. During the 10-year transition, prices continue to decrease for the firms listed for exclusion and to increase for the other firms.

As a consequence, differences in expected returns are exacerbated. For the firms excluded in Year 1, the expected return increases immediately by 22 bp at the time of announcement and by 34 bp after 10 years. The reduction in expected returns or the cost of capital for non-excluded firms remains moderate (3 bp in Year 1, 4 bp in Year 10), as they represent approximately 90% of the market portfolio. Despite the higher cost of capital of brown firms, the overall impact on the realised return is amplified in favour of green firms because the price impact dominates. On average over 10 years, the performance of green firms is equal

to 4.8%, while it is equal to 4.15% for the firms with the highest carbon emissions.

As risk exposures are not changed in this scenario, idiosyncratic and systematic risk measures are barely affected relative to Scenario 1.

### 5.3 Scenario 3: Climate Transition Risk and Growing Proportion of Green Investors

Finally, in Scenario 3, we introduce the climate transition risk factor on top of the growing share of green investors as in Scenario 2. The climate transition risk factor affects firm  $n$ 's per-share dividend flow according to Equation (2.1). Loadings on climate factor are calibrated to follow a power law across groups. Namely, 1% of most polluting stocks account for 15% of aggregate loading; 10% of most polluting stocks account for 40% of aggregate loading. These values are chosen based on the empirical findings of [Jondeau et al. \(2021\)](#).

In this scenario, climate transition risk is calibrated based on the differential between the cost of capital of brown stocks and that of green stocks in the absence of green investors. As Table 5 and Figure 5 demonstrate, the effects on stock prices and cost of capital are larger than in Scenario 2 for the stocks excluded in Year 1 and smaller for stocks excluded in later years or the non-excluded ones. At the announcement, firms in Group 100, namely to be excluded first, suffer from a 10.4% cut in stock prices, while the initial decrease for firms in Group 90, namely to be excluded last, is only about 5.3%. At the same time, stock prices of the non-excluded firms benefit from a 0.8% increase. After 10 years, the price drop is still strongest for firms in Group 100, reaching 12.1%, whereas prices fall by 7.5% for firms in Group 91.

The cost of capital is dramatically affected by the decarbonisation strategy. It increases by 21 bp for the firms excluded first and only by 10 bp for firms to be excluded last. Over time, at the end of the 10 years, the impact on the cost of capital more than doubles. The cost of capital for the not-excluded firms drops marginally by 2 bp at the announcement

date and at the end of the transition.

Table 5: Results - Scenario 3

	Firms excluded in Year 1	Firms excluded in Year 10	Non-excluded firms
$\Delta$ Price at $t = 0$	-11.80%	-5.74%	0.83%
$\Delta$ Price at $t = 10$	-14.03%	-8.06%	1.00%
$\Delta$ Cost of capital at $t = 0$	0.24%	0.10%	-0.01%
$\Delta$ Cost of capital at $t = 10$	0.59%	0.26%	-0.02%
Realized return over 10 years	3.29%	3.00%	3.37%
Return volatility at $t = 0$	19.53%	15.19%	14.64%
Return volatility at $t = 10$	19.07%	14.69%	14.68%
CAPM $\beta$ at $t = 0$	1.47	1.16	1.06
CAPM $\beta$ at $t = 10$	1.46	1.16	1.06

Because the cost of capital is substantially increased for brown firms, the difference in realised returns is reduced between brown and green firms. Eventually, in this scenario, the gap in the cost of capital is amplified because brown firms must compensate active investors for their additional exposure to climate transition risk. Consequently, the gap in the realised return is reduced.

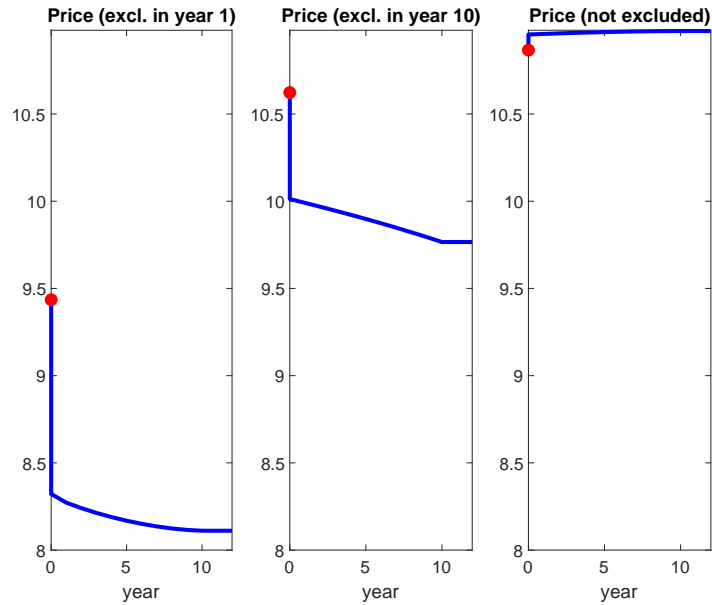
The intuition behind these results is that because of the climate transition risk factor, brown firms are more correlated with each other. As a consequence, their prices drop more when they are excluded from the index –because active investors are not willing to hold a marginal share of a brown firm knowing that it correlates heavily with the remainder of their portfolio of brown firms. The comparison of Table 4 and Table 5 shows that this effect is present for excluded firms, as their prices drop more in Scenario 3 than in Scenario 2. This accords with our intuition because the worse brown firms are heavily exposed to climate risk due to the power law assumption.

Regarding risk measures, the table also reveals that idiosyncratic volatility is substantially

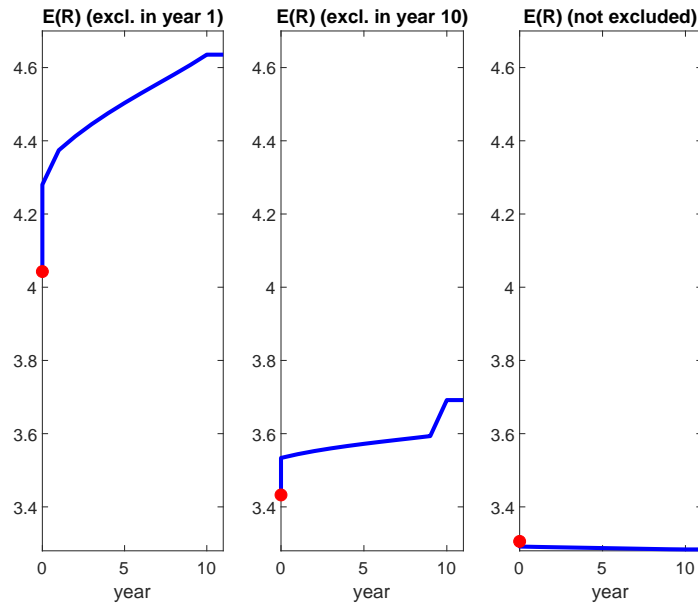


Figure 5: Impact of exclusion in Scenario 3

Panel A: Price impact



Panel B: Expected return impact



Note: The price impact is measured as the change in the price of a given Group of firms (in %) relative to the price before the announcement of the net zero strategy. The expected return impact is measured as the change in the expected return of a given Group of firms (in percentage point) relative to the expected return before the announcement of the net zero strategy.

increased for most polluting firms. In addition, these firms are more sensitive to the market dynamics. In particular, their beta parameter is increased from 1.2 in Scenarios 1 and 2 to 1.5 in Scenario 3. As both idiosyncratic and systematic risks increase for most polluting firms, the  $R^2$  of the CAPM regression barely changes.

## 6 Discussion

We study the impact of green investors on stock prices in a dynamic equilibrium asset-pricing model where three types of investors –green, passive and active– jointly determine stock prices and returns. Green investors aim to reduce their exposure to firms with the highest GHG emissions. Active investors maximise expected returns and can buy the stocks of brown firms whereas passive investors stick to an index of the entire market.

The decarbonisation strategy of green investors that we simulate in the model reflects what the academic literature and market practitioners would refer to as “Paris agreement” or “net zero” benchmark indices. The trajectory that we assume (the 1% most polluting firms are excluded every year in a cumulative way) corresponds to an annual carbon emission reduction rate of approximately 10% for the green portfolio, given the skewed distribution of carbon emissions. This is the necessary GHG reduction rate that green portfolios need to generate to stay roughly on a net zero trajectory by 2050.

We find a large fall in the stock prices of the high-emitting firms that are excluded by green investors and in turn an increase in the stock prices of greener firms when the exclusion strategy is announced and during the transition process. In the baseline scenario where there are 30% green investors, 20% active investors and 50% passive investors, the stock price of the firms excluded in Year 1 drops by 5.4%, while that of the firms to be excluded in Year 10 drops by 4%, immediately upon the announcement of the exclusion strategy. The changes in stock prices are reflected translated into variation in expected returns of the firms to be excluded and those that remain in the investable space. Over a 10-year transition period,

the cost of capital of the brown firms of the most polluting firms increases by 18 bp.

The large price effects we find complement the findings of earlier papers which generally find small price effects, especially when the community of green investors is small. The economically large effects of green investing on stock prices and expected returns stem from imperfect substitution across stocks. The assumption that the share of active investors is small implies that the market's demand function is significantly more price-inelastic. Moreover, the relative sizes of green and active investors matter for the magnitude of the price impact of green investing. For a given share of passive investors, the larger the share of green investors, the smaller the share of active investors, and the larger the price impact.

In addition, we assume perfect foresight regarding the green investing strategy, regarding the timing and the list of firms to be excluded. This assumption also contributes to the large impact on prices and expected returns. In practice, the process may not be perfectly predictable and this may attenuate the effect on impact. It is likely that the ultimate effect (on Year 10) would remain similar. It should be noted that the large initial impact could trigger a rush if investors want to hedge against the large fall in the price of brown stocks. As a consequence, we would expect a first-mover advantage for green investors to enter the decarbonisation strategy at an early stage.

In our baseline scenario, we assume the share of green investors to be modest and, more importantly, constant over time. An increase in the share of green investors would substantially raise the cost of capital of the most polluting firms: the stock price of excluded firms would eventually decrease by 10% and the cost of capital would increase by 34 bp (Scenario 2).

The assumptions behind our quantitative results are far from extreme. Only a small fraction of firms would be excluded in the process, some of them only after 10 years. Capital from green investors would flow from most polluting firms to less polluting firms. As the exclusion is based on the GHG emissions of individual firms and not on whether they belong

to a particular sector (no sector is a priori excluded), green investors could engage in a best-in-class approach and help the development of green technologies, including in the energy and electricity production industries.

Finally, our analysis focuses on the impact of green investors on stock prices and does not account for linkages between stock prices and corporate investment. The drop in the stock prices of the most polluting firms when they are excluded from the index could force them to cut down on investment, further accentuating the drop. Incentives by these firms to switch to less polluting technologies could thus be stronger than our analysis suggests. Extending our analysis to incorporate real investment and its two-way feedback with stock prices is a promising direction of future research.

## References

- Becht, M., Pajuste, A., Toniolo, A., 2023. Voice through divestment. European Governance Corporate Institute, Finance Working Paper N° 900/2023. URL: <https://ssrn.com/abstract=4386469>.
- Van der Beck, P., Jaunin, C., 2021. The equity market implications of the retail investment boom. Swiss Finance Institute Research Paper No. 21-12. URL: <https://ssrn.com/abstract=3776421>.
- Berk, J.B., van Binsbergen, J.H., 2022. The impact of impact investing. Stanford University Graduate School of Business Research Paper. URL: <https://ssrn.com/abstract=3909166>.
- Bolton, P., Kacperczyk, M., 2021. Do investors care about carbon risk? *Journal of Financial Economics* 142, 517–549. URL: <https://doi.org/10.1016/j.jfineco.2021.05.008>.
- Bolton, P., Kacperczyk, M., 2022. Global pricing of carbon-transition risk. *Journal of Finance* 78, 3677–3754. URL: <https://doi.org/10.1111/jofi.13272>.
- Bolton, P., Kacperczyk, M., Samama, F., 2021. Net-zero carbon portfolio alignment. Working Paper. URL: <https://ssrn.com/abstract=3922686>.
- Bonnefon, J.F., Landier, A., Sastry, P., Thesmar, D., 2022. The moral preferences of investors: Experimental evidence. HEC Paris Research Paper No. FIN-2019-1350. URL: <https://ssrn.com/abstract=3458447>.
- Buffa, A.M., Vayanos, D., Woolley, P., 2022. Asset management contracts and equilibrium prices. *Journal of Political Economy* 130, 3146–3201. URL: <https://ideas.repec.org/a/ucp/jpolec/doi10.1086-720515.html>.

- Cenedese, G., Han, S., Kacperczyk, M., 2023. Carbon-transition risk and net-zero portfolios. Working Paper. URL: <https://ssrn.com/abstract=4565220>.
- Cheng, G., Jondeau, E., Mojon, B., 2022. Building portfolios of sovereign securities with decreasing carbon footprints. Bank for International Settlements Working Papers 1038. URL: <https://ideas.repec.org/p/bis/biswps/1038.html>.
- Chinco, A., Sammon, M., 2022. The passive-ownership share is double what you think it is. Working Paper. URL: <https://ssrn.com/abstract=4188052>.
- Gabaix, X., Koijen, R., 2021. In Search of the Origins of Financial Fluctuations: The Inelastic Markets Hypothesis. NBER Working Papers 28967. National Bureau of Economic Research. URL: <https://EconPapers.repec.org/RePEc:nbr:nberwo:28967>.
- Green, D., Vallee, B., 2023. Can finance save the world? Measurement and effects of bank coal exit policies. Working Paper. URL: <https://ssrn.com/abstract=4090974>.
- GSIA, Global Sustainable Investment Alliance, 2022. Global sustainable investment review 2022. Technical Report. URL: <https://www.gsi-alliance.org/members-resources/gsir2022/>.
- Hsu, P.S., Li, K., Tsou, C.Y., 2023. The pollution premium. *Journal of Finance* 78(3), 1343–1392. URL: <https://doi.org/10.1111/jofi.13217>.
- Jiang, H., Vayanos, D., Zheng, L., 2022. Passive investing and the rise of mega-firms. Working Paper. URL: <https://ssrn.com/abstract=3749534>.
- Jondeau, E., Mojon, B., Pereira Da Silva, L.A., 2021. Building benchmark portfolios with decreasing carbon footprints. Swiss Finance Institute Research Paper No. 21-91. URL: <https://ssrn.com/abstract=3987186>.

- Kacperczyk, M.T., Peydro, J.L., 2022. Carbon emissions and the bank-lending channel. Working Paper. URL: <https://ssrn.com/abstract=3915486>.
- Koijen, R.S.J., Richmond, R.J., Yogo, M., 2020. Which investors matter for equity valuations and expected returns? NBER Working Papers 27402. National Bureau of Economic Research, Inc. URL: <https://ideas.repec.org/p/nbr/nberwo/27402.html>.
- OECD, Organisation for Economic Cooperation and Development, 2023. Towards orderly green transition: Investment requirements and managing risks to capital flows. Technical Report. URL: <https://www.oecd.org/investment/investment-policy/towards-orderly-greentransition.pdf>.
- Pastor, L., Stambaugh, R.F., Taylor, L.A., 2021. Sustainable investing in equilibrium. *Journal of Financial Economics* 142(2), 550–571. URL: <https://doi.org/10.1016/j.jfineco.2020.12.011>.
- Pastor, L., Stambaugh, R.F., Taylor, L.A., 2022. Dissecting green returns. *Journal of Financial Economics* 146, 403–424.
- Pedersen, L.H., Fitzgibbons, S., Pomorski, L., 2021. Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics* 142(2), 572–597. URL: <https://doi.org/10.1016/j.jfineco.2020.11.001>.
- Zerbib, O.D., 2022. A sustainable capital asset pricing model (S-CAPM): Evidence from environmental integration and sin stock exclusion. *Review of Finance* 26, 1345–1388. URL: <https://doi.org/10.1093/rof/rfac045>.

# Appendix

## A Proof of Proposition 3.1

We first derive the first-order conditions of passive and green investors. Using (2.6), (3.2) and  $z_{Int} = \lambda_I \eta_n$ , we can write the objective (2.8) of passive investors as

$$\begin{aligned} & \sum_{n=1}^{KN} \lambda_{It} \eta_n \mu_n - \frac{\rho}{2} \lambda_{It}^2 \left[ \left( \sum_{n=1}^{KN} \eta_n b_n^s \right)^2 (\sigma^s)^2 \mathbb{E}_t^u \left[ D_t^s \left[ \frac{\partial S_t^s(D_t^s)}{\partial D_t^s} \right]^2 \right] \right. \\ & \left. + \left( \sum_{n=1}^{KN} \eta_n b_n^c \right)^2 (\sigma^c)^2 \mathbb{E}_t^u \left[ D_t^c \left[ \frac{\partial S_t^c(D_t^c)}{\partial D_t^c} \right]^2 \right] + \sum_{n=1}^{KN} \eta_n^2 (\sigma_n^i)^2 \mathbb{E}_t^u \left[ D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \right] dt, \end{aligned} \quad (\text{A.1})$$

where the expectation is taken over  $(D_t^s, D_t^c, D_{nt}^i)$ . Noting that  $\lambda_{It}^2$  is assumed to be constant in each of the intervals  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$  and  $[K'T, \infty)$ , and using (3.7)–(3.9), we find the first-order condition

$$\begin{aligned} & \sum_{j=s,c} \left( \sum_{m=1}^{KN} \eta_m b_m^j \right) \left( \sum_{m=1}^{KN} [1 - (\mu_{AK'} + \mu_{IK'}) \lambda_{IK'} - \mu_{GK'} \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m b_m^j \right) (\sigma^j \bar{a}_{1K'}^j)^2 \\ & + \sum_{m=1}^{KN} [1 - (\mu_{AK'} + \mu_{IK'}) \lambda_{IK'} - \mu_{GK'} \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m^2 (\sigma_m^i \bar{a}_{m1K'}^i)^2 \bar{D}_m^i = 0 \quad (\text{A.2}) \end{aligned}$$

in  $[K'T, \infty)$ , and

$$\begin{aligned} & \sum_{j=s,c} \left( \sum_{m=1}^{KN} \eta_m b_m^j \right) \left( \sum_{m=1}^{KN} [1 - (\mu_{Ak} + \mu_{Ik}) \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m b_m^j \right) (\sigma^j)^2 \int_{kT}^{(k+1)T} (a_{1t}^j)^2 dt \\ & + \sum_{m=1}^{KN} [1 - (\mu_{Ak} + \mu_{Ik}) \lambda_{Ik} - \mu_{Gk} \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m^2 (\sigma_m^i)^2 \bar{D}_m^i \int_{kT}^{(k+1)T} (a_{m1t}^i)^2 dt = 0 \end{aligned} \quad (\text{A.3})$$



in  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$ . We can likewise write the objective (2.8) of green investors as

$$\begin{aligned}
& \sum_{n=1}^{KN} \lambda_{It} 1_{\{n \in \mathcal{G}_t\}} \eta_n \mu_n - \frac{\rho}{2} \lambda_{It}^2 \left[ \left( \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n b_n^s \right)^2 (\sigma^s)^2 \mathbb{E}_t^u \left[ D_t^s \left[ \frac{\partial S_t^s(D_t^s)}{\partial D_t^s} \right]^2 \right] \right. \\
& + \left. \left( \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n b_n^c \right)^2 (\sigma^c)^2 \mathbb{E}_t^u \left[ D_t^c \left[ \frac{\partial S_t^c(D_t^c)}{\partial D_t^c} \right]^2 \right] \right. \\
& + \left. \sum_{n=1}^{KN} 1_{\{n \in \mathcal{G}_t\}} \eta_n^2 (\sigma_n^i)^2 \mathbb{E}_t^u \left[ D_{nt}^i \left[ \frac{\partial S_{nt}^i(D_{nt}^i)}{\partial D_{nt}^i} \right]^2 \right] \right] dt, \tag{A.4}
\end{aligned}$$

where the expectation is taken over  $(D_t^s, D_t^c, D_{nt}^i)$ . The first-order condition is

$$\begin{aligned}
& \sum_{j=s,c} \left[ \left( \sum_{m=1}^{KN} 1_{\{m \leq (K-K')N\}} \eta_m b_m^j \right) \right. \\
& \times \left. \left( \sum_{m=1}^{KN} [1 - \mu_{IK'} \lambda_{IK'} - (\mu_{AK'} + \mu_{GK'}) \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m b_m^j \right) (\sigma^j \bar{a}_{1K'}^j)^2 \right] \\
& + \sum_{m=1}^{KN} [1 - \mu_{IK'} \lambda_{IK'} - (\mu_{AK'} + \mu_{GK'}) \lambda_{GK'} 1_{\{m \leq (K-K')N\}}] \eta_m^2 1_{\{m \leq (K-K')N\}} (\sigma_m^i \bar{a}_{m1K'}^i)^2 \bar{D}_m^i = 0 \tag{A.5}
\end{aligned}$$

in  $[K'T, \infty)$ , and

$$\begin{aligned}
& \sum_{j=s,c} \left[ \left( \sum_{m=1}^{KN} 1_{\{m \leq (K-k)N\}} \eta_m b_m^j \right) \right. \\
& \times \left. \left( \sum_{m=1}^{KN} [1 - \mu_{Ik} \lambda_{Ik} - (\mu_{Ak} + \mu_{Gk}) \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m b_m^j \right) (\sigma^j)^2 \int_{kT}^{(k+1)T} (a_{1t}^j)^2 dt \right] \\
& + \sum_{m=1}^{KN} [1 - \mu_{Ik} \lambda_{Ik} - (\mu_{Ak} + \mu_{Gk}) \lambda_{Gk} 1_{\{m \leq (K-k)N\}}] \eta_m^2 1_{\{m \leq (K-k)N\}} (\sigma_m^i)^2 \bar{D}_m^i \int_{kT}^{(k+1)T} (a_{m1t}^i)^2 dt = 0 \tag{A.6}
\end{aligned}$$

in  $[kT, (k+1)T)$  for  $k = 0, \dots, K' - 1$ .

We next determine  $a_{1t}^j$  for  $j = s, c$ . Identifying terms in  $D_t^j$  in (3.10) yields the ODE

$$1 - (r + \kappa^j)a_{1t}^j - g_k^j(a_{1t}^j)^2 + \frac{da_{1t}^j}{dt} = 0. \quad (\text{A.9})$$

When  $k = 0, \dots, K' - 1$ , (A.9) is defined over  $t \in [kT, (k+1)T)$ , and when  $k = K'$ , (A.9) is defined over  $t \in [K'T, \infty)$ . When  $k = K'$ , we look for a constant solution of (A.9), corresponding to the steady state. Such a solution  $\bar{a}_{1K'}^j$  must satisfy the quadratic equation

$$1 - (r + \kappa^j)\bar{a}_{1K'}^j - g_{K'}^j(\bar{a}_{1K'}^j)^2 = 0. \quad (\text{A.10})$$

Equation (A.10) has two solutions if

$$(r + \kappa^j)^2 + 4g_{K'}^j > 0,$$

which we assume. We focus on the smaller solution, which is the continuous extension of the unique solution when  $g_{K'}^j = 0$ , and is as in the proposition. When  $k = 0, \dots, K' - 1$ , we solve (A.9) recursively with terminal condition  $\lim_{t \rightarrow (k+1)T} a_{1t}^j = a_{1,(k+1)T}^j$ . We find

$$\begin{aligned} \frac{da_{1t}^j}{dt} &= g_k^j(a_{1t}^j)^2 + (r + \kappa^j)a_{1t}^j - 1 \\ \Rightarrow \frac{da_{1t}^j}{dt} &= (a_{1t}^j - \bar{a}_{1k}^j)(g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}) \\ \Rightarrow \frac{da_{1t}^j}{(a_{1t}^j - \bar{a}_{1k}^j)(g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j})} &= dt \\ \Rightarrow \frac{da_{1t}^j}{g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j}} \left( \frac{1}{a_{1t}^j - \bar{a}_{1k}^j} - \frac{g_k^j}{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}} \right) &= dt \\ \Rightarrow \log \left( \frac{a_{1,(k+1)T}^j - \bar{a}_{1k}^j}{g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j}} \right) - \log \left( \frac{a_{1t}^j - \bar{a}_{1k}^j}{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}} \right) &= \left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T - t] \\ \Rightarrow \frac{g_k^j a_{1t}^j + \frac{1}{\bar{a}_{1k}^j}}{a_{1t}^j - \bar{a}_{1k}^j} &= \frac{g_k^j a_{1,(k+1)T}^j + \frac{1}{\bar{a}_{1k}^j}}{a_{1,(k+1)T}^j - \bar{a}_{1k}^j} e^{\left( g_k^j \bar{a}_{1k}^j + \frac{1}{\bar{a}_{1k}^j} \right) [(k+1)T - t]}, \end{aligned}$$

which yields (3.11).

We next determine  $a_{1t}^n$ . Identifying terms in  $D_{nt}^i$  in (3.10) yields the ODE

$$1 - (r + \kappa_n^i) a_{n1t}^i - g_{nk}^i (a_{n1t}^i)^2 + \frac{da_{n1t}^i}{dt} = 0. \quad (\text{A.11})$$

When  $k = 0, \dots, K' - 1$ , (A.11) is defined over  $t \in [kT, (k+1)T)$ , and when  $k = K'$ , (A.11) is defined over  $t \in [K'T, \infty)$ . When  $k = K'$ , we look for a constant solution of (A.11). Proceeding as for  $a_{1t}^j$ , we find  $\bar{a}_{n1K'}^i$  in the proposition. When  $k = 0, \dots, K' - 1$ , we solve (A.11) recursively with terminal condition  $\lim_{t \rightarrow (k+1)T} a_{n1t}^i = a_{n1, (k+1)T}^i$ . Proceeding as for  $a_{1t}^j$ , we find (3.12).

Identifying the remaining terms yields the ODE

$$\bar{D}_n + \frac{d\bar{S}_{nt}}{dt} - r\bar{S}_{nt} + \sum_{j=s,c} b_n^j \left( \kappa^j a_{1t}^j + \frac{da_{0t}^j}{dt} - r a_{0t}^j \right) + \kappa_n^i a_{n1t}^i \bar{D}_n + \frac{da_{n0t}^i}{dt} - r a_{n0t}^i = 0 \quad (\text{A.12})$$

in the function  $\bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i$ . Its solution is

$$\begin{aligned} \bar{S}_{nt} + \sum_{j=s,c} b_n^j a_{0t}^j + a_{n0t}^i &= \int_t^\infty \left( \bar{D}_n + \sum_{j=s,c} b_n^j \kappa^j a_{1t'}^j + \kappa_n^i a_{n1t'}^i \bar{D}_n \right) e^{-r(t'-t)} dt' \\ &= \frac{\bar{D}_n}{r} + \sum_{j=s,c} b_n^j \kappa^j \int_t^\infty a_{1t'}^j e^{-r(t'-t)} dt' + \kappa_n^i \bar{D}_n \int_t^\infty a_{n1t'}^i e^{-r(t'-t)} dt'. \end{aligned} \quad (\text{A.13})$$

For  $t \in [K'T, \infty)$ , the solution is constant and equal to

$$\bar{S}_n + \sum_{j=s,c} b_n^j a_0^j + a_{n0}^i = \frac{\bar{D}_n + \sum_{j=s,c} b_n^j \kappa^j \bar{a}_{1K'}^j + \kappa_n^i \bar{D}_n \bar{a}_{n1K'}^i}{r}.$$

## Previous volumes in this series

1126 September 2023	CBDC and the operational framework of monetary policy	Jorge Abad, Galo Nuño and Carlos Thomas
1125 September 2023	Banks' credit loss forecasts: lessons from supervisory data	Martin Birn, Renzo Corrias, Christian Schmieder and Nikola Tarashev
1124 September 2023	The effect of monetary policy on inflation heterogeneity along the income distribution	Miguel Ampudia, Michael Ehrmann and Georg Strasser
1123 September 2023	Global supply chain interdependence and shock amplification – evidence from Covid lockdowns	Sally Chen, Eric Tsang and Leanne (Si Ying) Zhang
1122 September 2023	gingado: a machine learning library focused on economics and finance	Douglas K G Araujo
1121 September 2023	Margins, debt capacity, and systemic risk	Sirio Aramonte, Andreas Schrimpf and Hyun Song Shin
1120 September 2023	Energy Shocks as Keynesian Supply shocks: Implications for Fiscal Policy	Enisse Kharroubi and Frank Smets
1119 September 2023	Keep calm and bank on: panic-driven bank runs and the role of public communication	Olivier Coibion, Yuriy Gorodnichenko, Francesco Grigoli and Damiano Sandri
1118 August 2023	The origins of monetary policy disagreement: the role of supply and demand shocks	Carlos Madeira, João Madeira and Paulo Santos Monteiro
1117 August 2023	An impossibility theorem on truth-telling in fully decentralized systems	Rodney Garratt and Cyril Monnet
1116 August 2023	Absolute blockchain strength? Evidence from the ABS market in China	Jing Liu, Ilhyock Shim and Yanfeng Zheng
1115 August 2023	Sharks in the dark: quantifying HFT dark pool latency arbitrage	Matteo Aquilina, Sean Foley, Peter O'Neill and Thomas Ruf
1114 August 2023	The term structure of inflation forecasts disagreement and monetary policy transmission	Alessandro Barbera, Fan Dora Xia and Xingyu Sonya Zhu
1113 August 2023	To Lend or Not to Lend: the Bank of Japan's ETF purchase program and securities lending	Mitsuru Katagiri, Junnosuke Shino and Koji Takahashi

All volumes are available on our website [www.bis.org](http://www.bis.org).