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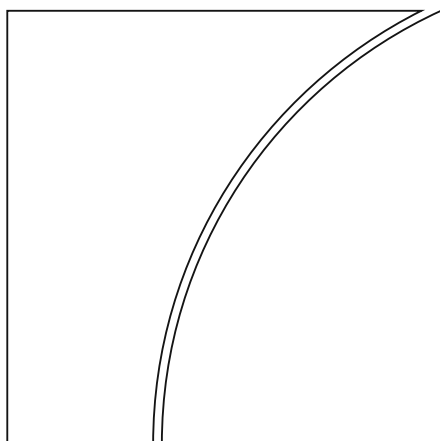
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Energy Shocks as Keynesian Supply shocks: Implications for Fiscal Policy*

Enisse Kharroubi[†] and Frank Smets[‡]

Abstract

This paper analyses the economic impact of and the optimal policy response to energy supply shocks in a flexible price model with heterogeneous households. We introduce energy as a consumption good on the demand side and as an input to production on the supply side. A distinguishing feature is that, in line with empirical evidence, we allow households' energy demand to be non-homothetic. The model provides three main insights. First, (negative) energy supply shocks act as a (negative) demand shock, or Keynesian supply shock, when three conditions are met: (i) household income heterogeneity is intermediate, neither too high nor too low; (ii) the fraction of poor and credit-constrained households is high and (iii) competition between firms is strong enough. Second, implementing the first-best allocation requires subsidising the poor and taxing the rich, and more so when the economy faces a negative energy shock. Last, issuing public debt can be part of the optimal policy response to a negative energy shock, if the shock is large and the economy's overall energy intensity is low.

Keywords: Energy shocks, non-homothetic demand, heterogeneous households, fiscal policy, public debt.

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*Corresponding author's email: enisse.kharroubi@bis.org. The views expressed here do not necessarily represent the views of the BIS or the ECB. The paper benefited from comments from participants at the BIS-SNB Research Workshop, the CEPR conference on Rethinking Macroeconomic Policy in Times of Turmoil and the ETH-University of Oxford Macro-Finance conference. The paper was written while Frank Smets was visiting the BIS, as a Lamfallussy Research Fellow.

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1 Introduction

The recent energy crisis has elicited much debate about its distributional and macroeconomic consequences and the appropriate fiscal (and monetary) policy response. One feature that has been highlighted is the negative impact of higher energy prices on the distribution of income and consumption. Poorer households typically spend a larger share of their income on energy (see Figure 1).¹ As a result, they experience a stronger decline in their real disposable income following negative energy shocks. Moreover, as these

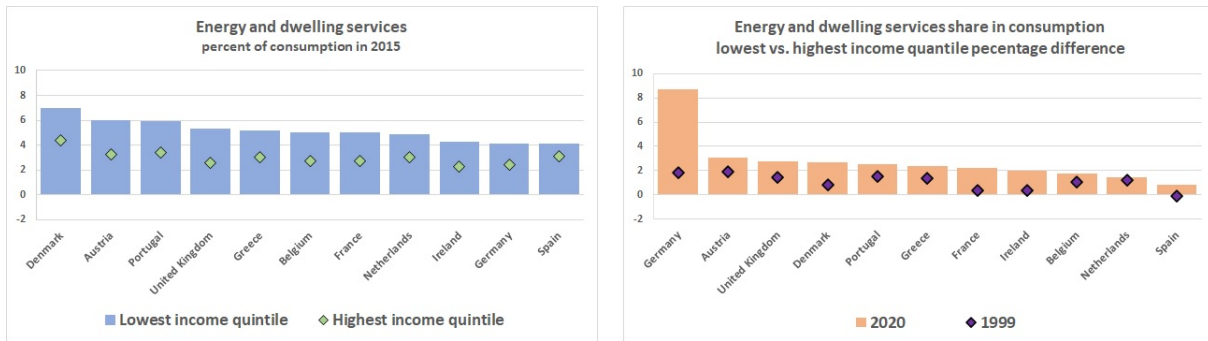


Figure 1: Poor households spend more on energy.

households often have low saving buffers and are typically credit constrained, they tend to adjust their non-energy consumption much more than richer households in response to an adverse energy shock. Accordingly, the fiscal policy debate has focused on the need to target income support (in the form of subsidies or tax rebates) to the most vulnerable households. This can alleviate the negative distributional consequences and support aggregate demand, while limiting the fiscal cost.²

In this paper we develop a flexible price model with heterogeneous households and non-homothetic demand for energy to investigate the economic impact of and the optimal fiscal policy response to energy supply shocks. Like previous papers in the literature, we introduce energy as a consumption good on the demand side and as an input to production on the supply side.³ In addition, our framework carries two distinguishing features. One

¹This evidence, based on Eurostat Household Budget Survey, also shows that the difference between the rich and the poor in the energy share in consumption has been growing over time.

²See, for instance, Ari et al. (2022) for a review of policy initiatives meant to alleviate the fallout of the energy crisis.

³see Kim and Loungani (1992), Hunt (2005), Bodenstein et al. (2008) or Dhawan and Jeske (2008) for a similar modelling.

is that households need to consume a minimum amount of energy.⁴ As a result, as income rises, the energy share in total expenditure falls. Second, households are heterogeneous in their income as well as in their ability to tap credit markets. As a consequence, the minimum energy consumption barely matters for high-income households. Conversely for low-income households, the minimum energy consumption may become binding, especially when negative income shocks hit.

In this environment, a negative energy supply shock typically has a larger impact on the ability of low-income, credit-constrained households to consume non-energy goods.⁵ We trace out the implications of this feature for the aggregate economy and the policy response and provide three main insights. First, in this environment, (negative) energy supply shocks can act as (negative) demand shocks, or Keynesian supply shocks, in particular when three conditions are met: the dispersion in household incomes is neither too high nor too low; the fraction of poor, credit-constrained households is large; the elasticity of substitution between non-energy goods is high. Second, implementing the first-best allocation requires subsidising the poor and taxing the rich, and the more so — relative to incomes— when the economy suffers a negative energy shock. Finally, issuing public debt can be part of the optimal policy response, particularly when the economy faces a large shock, and/or when the economy’s overall energy intensity is low.

The literature on the economic effects of energy shocks is large (see for instance Kilian (2008) for a survey). The distributional consequences and the optimal policy response are, however, less well explored. Our paper contributes to a recent literature focusing on this angle. First, several empirical papers document the distributional consequences of energy price shocks. Battistini et al. (2022), Gelman et al. (2023) and Känzig (2021) show that low-income and/or liquidity-constrained households adjust their non-energy expenditures to energy price shocks to a larger extent. Similarly, Peersman and Wauters (2022) show, using Belgian survey data, that non-energy consumption is more sensitive to energy price increases than to energy price decreases. Marginal propensities to consume

⁴We follow here Geary (1950) and Stone (1954) in assuming that households derive utility from energy consumption only when it exceeds a certain minimum amount.

⁵Motivated by the recent European experience, Gornemann et al. (2023) also consider energy shocks as reductions in the available quantity of energy, and how such shocks can create self-fulfilling fluctuations.

(MPCs) are also significantly larger for low-income and low-saving buffer households, while non-energy consumption declines more strongly for households who report higher uncertainty on their future financial situation. Our paper embeds these empirical findings in a two-agent macroeconomic model.

Second, a burgeoning literature uses TANK and HANK models to explore the distributional and macroeconomic implications of energy price shocks. Chan et al. (2022) develop a small, open-economy TANK model, where labor and energy complement each other. In this framework, higher energy prices typically reduce the labor share in total income, which depressed aggregate demand, because of borrowing constraints. In turn, price flexibility insures firm profits against adverse energy price shocks and further depresses labor income and demand. Pieroni (2023) and Auclert et al. (2023) develop a full-scale HANK model to address similar questions. Auclert et al. (2023) focuses on the open economy implications of alternative monetary and fiscal policy responses. They show that an increase in imported energy prices can cause a recession by pushing down real wages and consumer spending, provided the elasticity of substitution between energy and domestic goods is realistically low. They also analyse the cross-border impact of alternative monetary and fiscal policies. Our paper also emphasises the possible negative demand effects of an energy supply shock, but our mechanism does not rely on a limited substitutability on the supply side. Instead we emphasise the empirically relevant non-homothetic energy demand on the household side and the implications for MPCs of credit-constrained households. We also characterise optimal fiscal policy in this environment.

Specifically, we investigate three questions. First, what are the implications of a negative energy supply shock? Second, which distortions, if any, does the market allocation suffer from? Third, what tools can the social planner use to correct these distortions, and in particular, is there a role for public debt?

On the first question, a negative energy supply shock which makes energy more scarce, typically makes the economy as a whole poorer, as households and firms need energy for consumption and production. Aggregate output, aggregate consumption and real wages therefore all fall. Yet, households' demand for energy being non-homothetic, low-income, credit-constrained households have to cut significantly their demand for consumption

goods, as (some of) their demand for energy is sticky and does not respond to changes in relative prices. As a result, aggregate demand for consumption goods falls more than one-to-one, the relative price of consumption goods drops and high-income households end up better-off as the drop in the relative price of consumption goods dominates the drop in income due to the direct impact of the negative energy supply shock. As high-income households can freely borrow and lend, a temporary negative energy supply shock typically leads to a drop in the equilibrium rate of interest.

Negative energy shocks can act as Keynesian supply shocks when the pecuniary externality affecting the price of consumption goods, is large relative to the negative income effect due to adverse energy shocks. This happens when three conditions are met. First, household income heterogeneity needs to be neither too low nor too high. When income heterogeneity is low, consumption patterns are similar across households, and the price externality is weak, while with high income heterogeneity, low-income households do not purchase consumption goods in the first place. So, there can be no externality. Second, the demand for consumption goods needs to drop significantly following a negative energy shock, i.e. there needs to be enough low-income, credit-constrained households. Third, the price of consumption goods needs to be sufficiently responsive to the fall in demand, i.e. competition between firms producing consumption goods needs to be sufficiently large. Hence unlike other theories of Keynesian supply shocks looking at asymmetric shocks in the context of low elasticity of substitution between consumption goods (Guerrieri et al. (2022)), we highlight the role of inelastic demand for some goods like energy. Such inelastic demand can lead to a disproportionate reduction in the demand for other goods, paving the way for Keynesian supply shocks.

In our model, the decentralised equilibrium suffers two distortions. First, because of monopolistic competition, firms do not consume enough energy. Second, because households are heterogeneous, some consume too much (energy and consumption goods) while others consume too little. Putting these two distortions together, households, as a group, do not consume enough energy at the equilibrium when dispersion in household incomes is large. Conversely, firms do not consume enough energy at the equilibrium when dispersion in household incomes is low.

Looking into the normative analysis, the social planner can implement the first-best allocation, by subsidising low-income households—to ensure they can cover their inelastic demand for energy—and firms' energy consumption, to address the monopolistic competition distortion. Last but not least, the social planner needs to raise taxes on high-income households, to equalise MPCs across households and ensure a balanced budget.

The first-best allocation features two important properties. First, negative energy shocks always act as standard negative supply shocks and raise the equilibrium rate of interest, at the first-best. By equalising incomes across households, the social planner precludes any price externality resulting from some households disproportionately cutting consumption in response to negative energy shocks. Second, negative energy shocks require *proportionally* larger transfers to low-income households. This is because negative energy shocks typically hurt low-income households twice, as labour income falls *and* as a larger fraction of income is devoted to energy consumption. Implementing the first-best therefore requires to tax a larger fraction of rich households' income, the larger the negative energy shock.

In this context, there can be a role for public debt in implementing the social optimum. In principle, there should not be any. Under the first-best allocation, there is no shortage of demand, as negative energy supply shocks always raise the equilibrium rate. However, under balanced budget, implementing the first best allocation is possible, only insofar as tax rates can be set arbitrarily high. Yet if there is an upper bound on tax rates, as is likely in practise, large negative energy shocks may require taxing high-income households beyond what is feasible. Issuing public debt can then bypass this problem. In doing so, the social planner must still ensure that future tax revenues, which are themselves bounded by maximum feasible taxation, are enough to service the debt *and* implement the first-best allocation once the economy is back to the steady state. Summarising these constraints, we derive a "fiscal space" statistics, which computes the upper bound on the energy shock for which the social planner can implement the first-best allocation, using public debt, while ensuring it is sustainable. We then show that, provided the social planner can implement the first-best at the steady state, "fiscal space" is strictly positive. In addition, it is larger, in economies where energy-intensity is lower, i.e. where the share of energy in output

and consumption is lower, and/or also in economies where household inter-temporal elasticity of substitution is higher. Conversely "fiscal space" is smaller in economies where household income dispersion is larger, and/or in economies where public debt is higher to start with, these correlations being magnified in low energy-intensive economies.

The rest of the paper proceeds as follows. The next section lays out the main building blocks of the model. Section 3 derives the decentralised equilibrium and its main properties. The social optimum, its implementation and the implications for public debt issuance, are analysed in section 4. Finally conclusions are drawn in section 5.

2 The model

The model consists of households and firms. Households are heterogeneous, they supply labour to firms and consume goods that firms produce under monopolistic competition.⁶ Moreover, households consume energy (in addition to consumption goods) while firms use energy as an input to production (in addition to labour). Importantly, the demand for energy on the household side will be non-homothetic. Let us now describe the model's main assumptions more systematically, starting with households and following with firms.

2.1 The demand side

2.1.1 Household preferences

Households live indefinitely. Each period, they consume energy E and a composite consumption good C . In addition they are endowed each period with some quantity of energy and some quantity of labour—the latter is normalised to one for simplicity. Households' preferences U write as:

$$U = \sum_t \beta^t \frac{[u(E_t; C_t)]^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} \quad (1)$$

with

$$u(E_t; C_t) = \left[\delta_h^{\frac{1}{\sigma_h}} (E_t - e_h)^{1-\frac{1}{\sigma_h}} + (1 - \delta_h)^{\frac{1}{\sigma_h}} (C_t)^{1-\frac{1}{\sigma_h}} \right]^{\frac{1}{1-\frac{1}{\sigma_h}}} \text{ if } E_t \geq e_h \quad (2)$$

Here γ denotes households' inter-temporal elasticity of substitution, σ_h denotes households' (intra-temporal) elasticity of substitution between energy and consumption goods and β is the rate at which households discount the future. Following Geary (1950) and Stone (1954), we introduce a minimum (subsistence) consumption level—denoted e_h —for energy, which acts as a lower bound on households' demand for energy. Accordingly, energy consumption E_t carries an infinite marginal utility whenever $E_t \leq e_h$, making energy demand up to e_h inelastic to prices. Households therefore first devote their income to energy consumption up to $E_t = e_h$ and then split whatever income is left between energy and

⁶We restrict heterogeneity to the household side, while firms are assumed to be symmetric. Introducing heterogeneity on the firm side would add further mechanism in the model, not least in terms of reallocation across firms and sectors, that we leave out this paper.

consumption goods, the share of energy in household expenditures being then δ_h .⁷ The composite consumption good C , is in turn, a standard CES aggregation of consumption goods s produced across the different sectors in the economy:

$$C_t = \left[\int_0^1 [c_{st}]^{1-\frac{1}{\eta}} ds \right]^{\frac{1}{1-\frac{1}{\eta}}} \quad (3)$$

Here $\eta > 1$ is the elasticity of substitution between the different consumption goods. Denoting p_{et} the price of energy at time t and p_{st} the price of the consumption good s at time t , the price of the composite consumption good p_{ct} and the general price level P_t —assuming energy consumption E exceeds the minimal level e_h — respectively satisfy:

$$p_{ct}^{1-\eta} = \int_0^1 p_{st}^{1-\eta} ds \quad \text{and} \quad P_t^{1-\sigma_h} = \delta_h p_{et}^{1-\sigma_h} + (1 - \delta_h) p_{ct}^{1-\sigma_h} \quad (4)$$

2.1.2 Household demand for energy and consumption goods

Let us denote R_t , the nominal income net of the cost of minimum energy consumption, that a household devotes to purchases of energy and consumption goods. Household demands for energy and for the composite consumption good, respectively write as:

$$E_t = e_h + \frac{R_t^-}{p_{et}} + \delta_h \left[\frac{p_{et}}{P_t} \right]^{-\sigma_h} \frac{R_t^+}{P_t} \quad \text{and} \quad C_t = (1 - \delta_h) \left[\frac{p_{ct}}{P_t} \right]^{-\sigma_h} \frac{R_t^+}{P_t} \quad (5)$$

where $x^- = \min\{x; 0\}$ and $x^+ = \max\{x; 0\}$. When net income R_t is negative, i.e. gross income is not enough to cover for the cost of minimum energy consumption, households consume only energy and do not demand consumption goods. Both the average and the marginal propensity to consume energy (energy APC and energy MPC in Figure 2) are then equal to one. Conversely, when net income R_t is positive, i.e. gross income exceeds the cost of minimum energy consumption, the energy MPC drops from 1 to δ_h , while the non-energy MPC increases from 0 to $1 - \delta_h$. As a result the energy APC gradually falls with income, consistent with empirical evidence showing that energy accounts for a larger fraction of the consumption basket for lower-income households (see introduction). Last,

⁷Household demands for energy and consumption goods are formally derived in the next section.

the demand C_{st}^* for consumption good s writes as

$$C_{st}^* = \left[\frac{p_{st}}{p_{ct}} \right]^{-\eta} C_t^* = (1 - \delta_h) \left[\frac{p_{st}}{p_{ct}} \right]^{-\eta} \left[\frac{p_{ct}}{P_t} \right]^{-\sigma_h} \frac{R_t^+}{P_t} \quad (6)$$

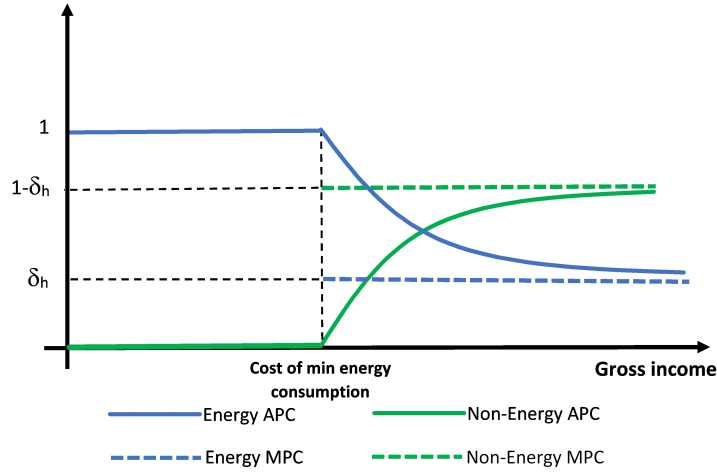


Figure 2: Households' average and marginal propensities to consume.

2.1.3 Household heterogeneity

There are two types of households. A first group of households—called *unconstrained* households—is endowed with a quantity of energy $(en)_u$ that exceeds the minimum energy consumption e_h : $(en)_u > e_h$; owns the firms producing consumption goods; and can freely lend and borrow. A second group of households—called *constrained* households—is endowed with a quantity of energy $(en)_c$ that falls short of the minimum energy consumption e_h : $(en)_c < e_h$; has no ownership rights over firms producing consumption goods; and cannot save nor borrow. A fraction ϕ of households are constrained.

Let e_c denote constrained households' energy needs, i.e. $e_c = e_h - (en)_c > 0$. Then constrained households' net income, when it is positive, is simply the difference between labour income w_t and the cost $p_{et}e_c$ of minimum energy consumption net of the energy endowment:

$$R_t^c \equiv [w_t - p_{et}e_c]^+ \quad (7)$$

Turning now to the case of unconstrained households, their net income writes as:

$$R_t^u \equiv w_t + p_{et}e_u + \frac{\pi_t}{1 - \phi} + (1 + i_{t-1})s_{t-1} - s_t \quad (8)$$

Here e_u denotes the energy endowment net of the minimum energy consumption, i.e. $e_u = (en)_u - e_h > 0$, s_t the savings at time t , π_t the firms' profits at time t , and i_t is the nominal interest rate on savings s_t .⁸ Last, unconstrained households set their net savings $[(1 + i_{t-1})s_{t-1} - s_t]$ at time t , consistent with the usual Euler equation:

$$\frac{R_t^u}{P_t} = \left[\frac{\beta(1 + i_t)}{P_{t+1}/P_t} \right]^{-\gamma} \frac{R_{t+1}^u}{P_{t+1}} \quad (9)$$

and welfare for constrained and unconstrained households respectively writes as

$$U^i = \sum \beta^t \frac{[R_t^i/P_t]^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} \text{ for } i = \{c; u\} \quad (10)$$

2.2 The supply side

2.2.1 Energy and good producing firms

On the supply side, there are firms producing energy and firms producing consumption goods. Energy is supplied by exogenous producers who sell energy to households and firms producing consumption goods. We do not model energy producers except for the assumption that supply of energy is infinitely elastic at the energy world price p_{et}^* . For now, let us simply equate the local and world prices of energy p_{et} and p_{et}^* . Later, we lay out the uncovered interest parity relationship linking the two prices.

Conversely firms producing consumption goods operate across the different sectors of the economy and are owned by *unconstrained* households —those who can borrow and lend and have a relatively large energy endowment. The next sections detail their technology and the choices they make on behalf of their shareholders.

⁸Given that e_u is positive, we can rule out the case where R_t^u would be negative.

2.2.2 Technology and demand for inputs

Firms produce consumption goods using labour which they rent from households, and energy which they purchase from unconstrained households and/or from the exogenous energy producers. Specifically, output y_{st} at time t for the firm operating in sector s , writes as:

$$y_{st} = \left[\delta_s^{\frac{1}{\sigma_s}} E_{st}^{1-\frac{1}{\sigma_s}} + (1 - \delta_s)^{\frac{1}{\sigma_s}} L_{st}^{1-\frac{1}{\sigma_s}} \right]^{\frac{1}{1-\frac{1}{\sigma_s}}} \quad (11)$$

Here E_{st} denotes the firm's energy consumption at time t and L_{st} the amount of labour the firm hires at time t from households. The elasticity of substitution between energy and labour is σ_s while δ_s measures the energy intensity of sector s .

Let mc_{st} denote the marginal cost to produce consumption good s at time t . The firm producing good s then sets its price p_{st} to maximise profits, given the marginal cost mc_{st} and household demand for its good:

$$\begin{aligned} & \max_{p_{st}} \quad \pi_{st} = [p_{st} - mc_{st}] y_{st} \\ \text{s.t.} \quad & \begin{cases} y_{st} = \left[\frac{p_{st}}{p_{ct}} \right]^{-\eta} C_t^* \quad \text{and} \quad mc_{st}^{1-\sigma_s} = \delta_s p_{ct}^{1-\sigma_s} + (1 - \delta_s) w_t^{1-\sigma_s} \\ C_t^* = (1 - \delta_h) \left[\frac{p_{ct}}{P_t} \right]^{-\sigma_h} \frac{R_t}{P_t} \quad \text{and} \quad R_t = (1 - \phi) R_t^u + \phi R_t^c \end{cases} \end{aligned} \quad (12)$$

Here the demand y_{st} for consumption good s depends on three terms. The first reflects the choice between different consumption goods, the second, the choice between energy and consumption goods and the last reflects the composition of aggregate demand R_t between constrained and unconstrained households' demands, as expressed in (7) and (8).

The optimal price p_{st} of consumption good s at time t simply writes as a constant markup μ over the sectoral marginal cost, $p_{st} = \mu mc_{st}$ with $\mu = \frac{\eta}{\eta-1}$ so that the price index for consumption goods writes as $p_{ct} = \mu mc_t$ where mc_t denotes the average marginal cost across sectors, i.e. $(mc_t)^{1-\eta} = \int_s (mc_{st})^{1-\eta} ds$. The equilibrium level of output in sector s is then

$$y_{st} = \frac{1 - \delta_h}{\mu} \left[\frac{mc_{st}}{mc_t} \right]^{1-\eta} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h} \frac{R_t}{mc_{st}} \quad (13)$$

As is visible from (13), sectoral output y_{st} has standard properties: it decreases in the sectoral marginal cost of production mc_{st} as well as in the average marginal cost of production

mc_t , while it increases with the elasticity of substitution between goods η , and with the share of income $1 - \delta_h$, households spend on consumption goods, in addition to increasing with households' income R_t . Then using (13), profits in sector s satisfy

$$\pi_{st} = \frac{\mu - 1}{\mu} \left[\frac{mc_t}{mc_{st}} \right]^{\eta-1} (1 - \delta_h) \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h} R_t \quad (14)$$

Under the assumption $\eta > \sigma_h \geq 1$, the comparative statics for profits π_{st} and output y_{st} are broadly similar: profits decrease in the sectoral marginal cost mc_{st} as well as in the average marginal cost mc_t while they increase with the share of income $1 - \delta_h$, households spent on consumption goods and with households' income R_t . There are still two main differences: one is the impact of competition, the other is the impact of energy prices. For competition, things are clear: higher competition, i.e. a higher parameter η , raises output but reduces profits. For energy prices, things are less clear-cut: Everything else equal, higher energy prices lead households to reduce their demand for energy and substitute with more demand for consumption goods, which would imply higher profits. But higher energy prices also raise the marginal cost to produce consumption goods so that energy-intensive sectors, i.e. those with higher δ_s , experience a reduction in their profits.

Last, firms' demands for energy and labour are simply the sum of firm-level energy and labour demands across firms. Using the expression in (13) for firm-level output, aggregate energy and labour demand by firms respectively writes as:

$$E_f = \int_0^1 \delta_s \left[\frac{p_{et}}{mc_{st}} \right]^{-\sigma_s} y_{st} ds = \frac{1 - \delta_h}{\mu} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h} \left[\int_0^1 \frac{\delta_s \left[\frac{mc_{st}}{mc_t} \right]^{1-\eta}}{p_{et}^{\sigma_s} [mc_{st}]^{1-\sigma_s}} ds \right] R_t \quad (15)$$

and

$$L_f = \int_0^1 (1 - \delta_s) \left[\frac{w_t}{mc_{st}} \right]^{-\sigma_s} y_{st} ds = \frac{1 - \delta_h}{\mu} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h} \left[\int_0^1 \frac{(1 - \delta_s) \left[\frac{mc_{st}}{mc_t} \right]^{1-\eta}}{w_t^{\sigma_s} [mc_{st}]^{1-\sigma_s}} ds \right] R_t \quad (16)$$

Having determined households' demand for energy and consumption goods as well as firms' demand for energy and labour, we are now equipped to solve for the general equilibrium of the economy. Moreover, to simplify the exposition, we will focus from now on, on the case where the elasticities of substitution σ_h and σ_f are both equal to one.

3 The decentralised equilibrium

In this economy, an equilibrium is a vector of prices and quantities such that:

- (i) Household demands for energy and consumption goods maximise intra-temporal utility; household borrowing and savings maximise inter-temporal utility.
- (ii) Firm' demands for energy and labour minimise total cost of output; consumption good prices maximises profits.
- (iii) The price of consumption goods balance households' demand and firms' supply; the wage rate balances households' labour supply and firms' labour demand; the interest rate balances the market for lending and borrowing.

To derive the decentralised equilibrium, we first need to determine the equilibrium of the labour market. Then we can close the model and go through the main take-aways. Considering a symmetric equilibrium, assuming sectoral heterogeneity away, i.e. $\delta_s = \delta_f$ and $\sigma_s = \sigma_f = 1$, and denoting $1 - \delta = (1 - \delta_f)(1 - \delta_h)$, we can derive the following result.

Proposition 1 *The equilibrium "real" wage rate $\omega = w_t/p_{et}$ satisfies*

$$(1 - \phi)(\omega + e_u) + \phi(\omega - e_c)^+ = \frac{\omega}{1 - \delta} [1 + \delta_h(\mu - 1)] \quad (17)$$

The equilibrium "real" price of consumption goods $p_c = p_{ct}/p_{et}$ writes as $p_c = \mu\omega^{1-\delta_f}$, and the equilibrium "real" interest rate satisfies $\beta(1 + r) = \frac{\beta(1+i_t)}{p_{et+1}/p_{et}} = 1$.

Proof 1 *cf. Appendix A.1.*

In this equilibrium, all prices move one-for-one with the price of energy: the wage rate deflated by the price of energy is constant and there is full pass-through of energy prices to consumption goods prices and to the general price level. Changes in the nominal interest rate also have no real effects, as energy prices would then move by the same amount, but in the opposite direction to leaving the real interest rate unchanged.⁹ For this reason, we abstract from now on, from the time subscript and denote p_e the price of energy.

⁹The price of energy being set abroad, a change, say a cut, in the domestic nominal interest rate would raise one to one the price of energy in local currency because of uncovered interest parity, so that the domestic real interest rate would typically be unaffected.

By contrast, fluctuations in the *quantity* of energy available, i.e. in the energy endowments $(e_u; e_c)$, do have real effects, and these effects depend on the shape of distribution of energy endowments. The following lemma looks at this question.

Lemma 2 Denoting the economy's net energy endowment $e = (1 - \phi)e_u - \phi e_c$, constrained households can participate in the demand for consumption goods if and only if

$$\frac{e_c}{e} \leq \frac{1 - \delta}{\delta + \delta_h(\mu - 1)} \quad (18)$$

In this case, households' aggregate real income is $R/P = \mu^{\delta_h} \left[\frac{\delta}{\delta + \delta_h(\mu - 1)} \right]^\delta \frac{e^\delta}{\delta^\delta (1 - \delta)^{1 - \delta}}$

Proof 2 cf. Appendix A.2.

When condition (18) holds, the energy need of constrained households e_c is small relative to the economy's net energy endowment e , implying that heterogeneity in households' energy endowments is relatively low. In this case, firms can use a large volume of energy for production and produce a large volume of output, without cutting households' energy consumption below the minimum level e_h .

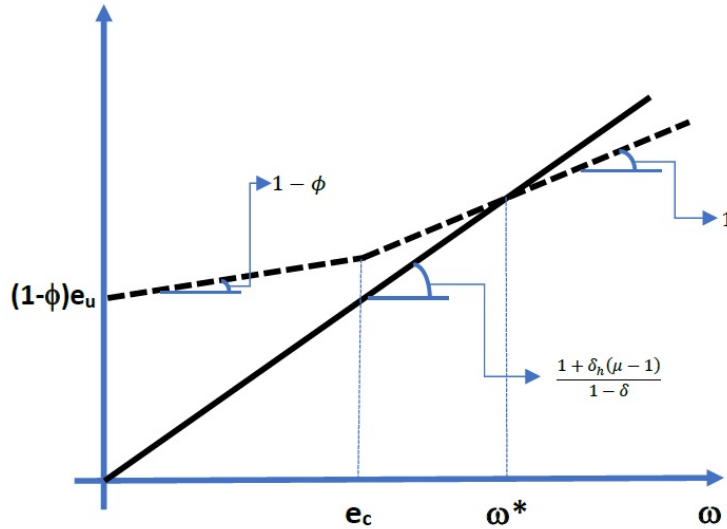


Figure 3: The complete participation equilibrium.

In addition, as firms produce large volumes of output, they also pay out relatively high wages, which further ensures that households, in particular constrained ones, are

able to cover their minimum energy consumption e_h and participate to the demand for consumption goods. We will refer to this equilibrium as "the complete participation equilibrium" (see Figure 3 above) in what follows.

Conversely, when condition (18) does not hold, the energy need of constrained households e_c is large relative to the economy's total net energy endowment e , i.e. heterogeneity in households' energy endowments is relatively large. As a result, constrained households spend most of their labour income on energy, not on consumption goods, and the demand for consumption goods is depressed. Demand for consumption goods being low, the equilibrium wage rate ω is also low, which further pushes constrained households against the minimum energy consumption constraint. In this case, constrained households are not able to cover their minimum energy consumption, nor to participate in the demand for consumption goods. We will refer to this situation in what follows as "incomplete participation" (see Figure 4 below).

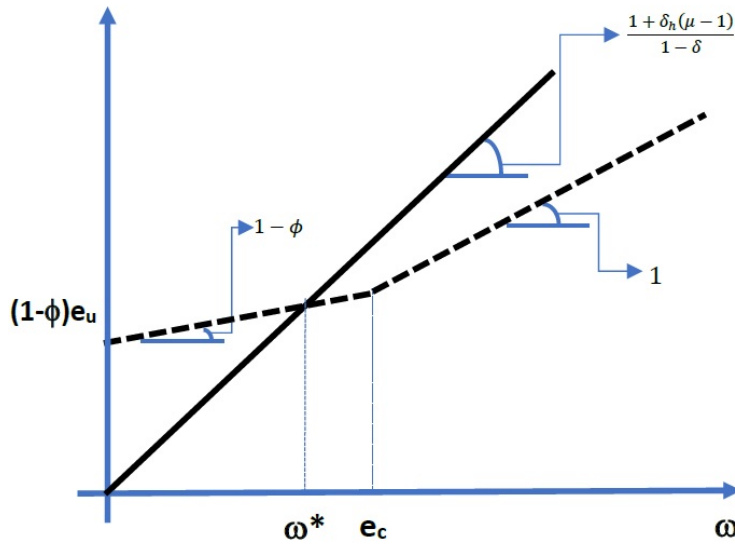


Figure 4: Incomplete participation.

3.1 Demand and supply effects of energy shocks

The two types of situations —complete and incomplete participation— described above have distinct properties, both at the aggregate and at the individual levels. Constrained households are trivially better-off in the equilibrium with complete participation. But

what about unconstrained households? And how do output and real incomes vary in response to shocks affecting the energy endowments e_u and e_c , in each case? The following proposition shed some lights on these questions.

Proposition 3 Denoting $\lambda = (1 + \delta_h(\mu - 1) - \frac{\delta\mu}{\phi}) / (1 - \delta)^2$, a shock that reduces the economy's energy endowment e , acts as a negative demand shock if and only:

$$[1 - \lambda] \frac{1 - \delta}{\delta + \delta_h(\mu - 1)} \leq \frac{e_c}{e} \leq \frac{1 - \delta}{\delta + \delta_h(\mu - 1)} \quad (19)$$

Proof 3 cf. Appendix A.3.

Following a negative energy supply shock, that cuts the energy endowment of constrained and unconstrained households, the equilibrium wage rate falls, which cuts both households incomes and the price of consumption goods. However, when participation is incomplete, only unconstrained households purchase consumption goods. As a consequence, the drop in the price of consumption goods is smaller than the drop in income, and households are worse-off. A negative energy supply shock therefore raises the equilibrium interest rate as households are temporarily poorer and therefore want to borrow to smooth out the shock.

Things can however be different in the complete participation equilibrium. In this equilibrium, both constrained and unconstrained households purchase consumption goods. As a result, when a negative energy supply shock hits, both types of households are poorer and hence reduce their demand for consumption goods. However, constrained households, as they get closer to minimum energy consumption, have to cut their purchases of consumption goods disproportionately, given that a large fraction of their demand for energy is sticky and cannot be compressed. Hence, the fall in the price of consumption goods can actually be larger than the fall in unconstrained households' income. Expression (20) isolates these two opposite forces.

$$\frac{R_t^u}{P_t} = \underbrace{\left[\frac{e_t^\delta}{\omega^{1-\delta}} \right]}_{(-)} \underbrace{\left[1 + \omega - \left[\delta_h + \frac{1 - \delta_h}{\mu} \right] \phi \left[1 - \omega \frac{e_{ct}}{e_t} \right] \right]}_{(+)} \quad (20)$$

On the one hand, a negative energy supply shock reduces income expressed in energy units. This income effect is the first term in (20). On the other hand, the disproportionate fall in the demand for consumption goods, cuts the relative price of consumption goods, expressed in energy units. This price effect is the second term of (20). The positive price effect then dominates the negative income effect and unconstrained households are better-off following a negative energy supply shock when the energy need e_c from unconstrained households is neither too large nor too low relative to the economy's total energy endowment e .

Figure 5 plots unconstrained households real income as a function of constrained households energy need e_c . It shows that a negative energy shock—that cuts households' energy endowment and hence raises constrained households energy need relative to the economy's overall energy endowment—first reduces the real income of unconstrained households. This (first green) region corresponds to the complete participation equilibrium where consumption patterns of constrained and unconstrained households are very similar, because constrained households' energy needs are relatively low. In this region, the relative price of consumption goods falls less quickly than income following a negative energy shock so that households' real income falls.

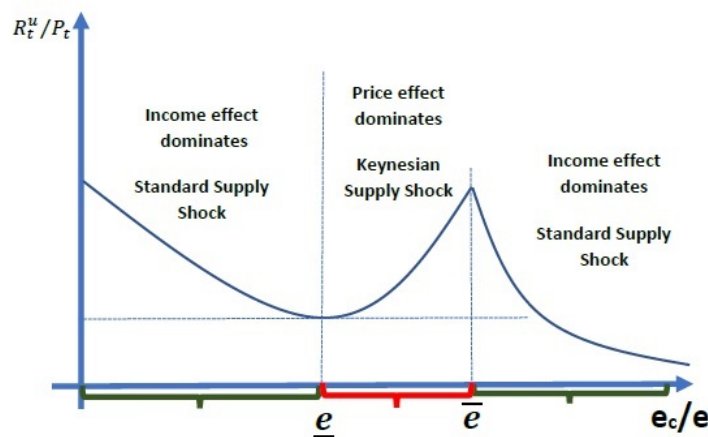


Figure 5: Negative energy supply shocks as negative demand shocks.

However, as constrained households' energy needs increase, the difference in consumption patterns between constrained and unconstrained households widens and a negative

energy shock triggers a significant fall in constrained household demand for consumption goods. The fall in the relative price of consumption goods then dominates the fall in household income and, unconstrained households' real income goes up. This is the second red region. Finally, when constrained households energy needs are large relative to the economy's overall energy endowment, i.e. when condition (18) holds, then the economy falls into the incomplete participation situation and negative energy shocks reduce incomes more than they cuts prices. As result, real incomes always fall. To wrap up, energy shocks work as Keynesian supply shocks when the dispersion in energy endowments between constrained and unconstrained households is intermediate, neither too low, nor too large.

Moreover, condition (19) in Proposition 2 shows that the parameter λ needs to be positive for negative energy shocks to operate as Keynesian supply shocks. This typically requires that the fraction ϕ of constrained households be sufficiently large and the markup μ charged by firms producing consumption goods be sufficiently low. When ϕ is high and there are many constrained households, a negative energy supply shock implies a larger negative shock to the demand for consumption goods, hence a larger fall, everything else equal, in the relative price of consumption goods. In addition, the drop in the relative price of consumption goods depends not only the drop in aggregate demand for consumption goods, but also on the elasticity of the inverse demand function, and hence on the markup μ . When the markup is low, the fall in the demand for consumption goods produces a large drop in the relative price of consumption goods, so that a negative energy supply shock is more likely to raise unconstrained households' real income.

To summarise, when three conditions are met, i.e (i) heterogeneity in energy endowments is intermediate, (ii) the fraction of constrained households is large (iii) the markup charged by firms is low, then negative energy supply shocks reduce the neutral real rate that is governed by the Euler equation (9), which is typical of a negative demand shock:

$$1 + r_t = \frac{1}{\beta} \left[\frac{R_{t+1}^u / P_{t+1}}{R_t^u / P_t} \right]^{\frac{1}{\gamma}} \Rightarrow \frac{\partial (R_t^u / P_t)}{\partial e} \leq 0 \Leftrightarrow \frac{\partial r_t}{\partial e} \geq 0$$

4 The social optimum

The social planner can improve on the decentralised equilibrium allocation in two ways. First there are welfare gains in redistributing income between constrained and unconstrained households as income differences imply differences in marginal utilities of consumption, and hence welfare losses. Second households and firms compete in the decentralised equilibrium for energy and there can be welfare gains in redistributing energy consumption between households and firms. Let us first focus on this last issue, derive the socially optimal allocation and compare it to that of the decentralised equilibrium.

4.1 The socially optimal allocation of energy.

Setting aside household heterogeneity, at the social optimum, the planner allocates energy to households and firms such that the allocation maximises households' welfare under the constraints that (i) households' consumption (of consumption goods) is equal to firms' output; (ii) firms use all of households' labour supply; and (iii) the sum of energy consumption by households and firms cannot exceed the total energy endowment in the economy. The problem for the social planner therefore writes as:

$$\begin{aligned} \max_{E_h; E_f} u(E_h; E_f) &= \left[\delta_h^{\frac{1}{\sigma_h}} (E_h - e_h)^{\frac{\sigma_h-1}{\sigma_h}} + (1 - \delta_h)^{\frac{1}{\sigma_h}} C^{\frac{\sigma_h-1}{\sigma_h}} \right]^{\frac{\sigma_h}{\sigma_h-1}} \\ &\left\{ \begin{array}{l} C = \left[\delta_f^{\frac{1}{\sigma}} E_f^{\frac{\sigma-1}{\sigma}} + (1 - \delta_f)^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ E_h + E_f = e \end{array} \right. \end{aligned} \quad (21)$$

We can then derive the following proposition.

Proposition 4 *Households consume too little energy (and firms too much) relative to the social optimum whenever:*

$$\frac{e_c}{e} \geq \frac{1 - \delta}{\delta} + \frac{(\mu - 1)\delta_h}{\delta\phi} \quad (22)$$

Proof 4 *cf. Appendix A.4.*

The decentralised equilibrium features two types of inefficiencies. The first is the standard distortion due monopolistic competition. The second is the distortion due to the

non-homotheticity in households' energy consumption. Because of monopolistic competition, firms produce too little output in the decentralised equilibrium. As a result, they use too little energy and the social planner corrects this inefficiency by allocating more energy to firms. However, when heterogeneity in households' energy endowments is large, constrained households are unable to cover their minimum energy consumption, at the equilibrium, and households, taken together, do not consume enough energy. The social planner then corrects for this inefficiency by allocating more energy to households (and less to firms) to ensure that constrained households can cover their minimum energy consumption and participate in the demand for consumption goods. This is why households as a group consume too little energy relative to the socially optimal allocation when the energy needs of constrained households e_c/e are relatively large.

In addition, in the decentralised equilibrium, constrained households' consumption of both energy and purchases of consumption goods is inefficiently low, as their marginal utility of consumption is always higher than that of unconstrained households. This is why a social planner who wants to implement the first best-allocation needs to address two issues: First it needs to neutralise the monopolistic competition distortion and provide firms with incentives to consume more energy. Second, it needs to equalise incomes across constrained and unconstrained households, which would eliminate welfare losses stemming from differences in marginal utilities of consumption, in addition to ensuring that constrained households are able to cover their minimum energy consumption.

4.2 Decentralising the socially optimal allocation

Consider now a social planner who aims at implementing the first best allocation, using taxes and subsidies. For this, let us assume the social planner grants a subsidy s_f to firms on energy consumption so that firms pay a price $(1 - s_f)p_e$ for energy instead of the market price p_e . Moreover, the social planner can raise a lump-sum tax T_u and extend a lump-sum subsidy S_c to households. Given this tax/subsidy scheme, denoting $\omega_t(s_f, S_c, T_u)$ the wage rate —expressed in energy units—, net income at time t of constrained households becomes $R_t^c/p_e = \omega_t(s_f, S_c, T_u) + S_c - e_c$, while net income of unconstrained households writes as $R_t^u/p_e = \omega_t(s_f, S_c, T_u) - T_u + e_u + \pi(s_f, S_c, T_u)/(1 - \phi)$. In this last expression,

$\pi(s_f, S_c, T_u)$ denotes firms' profits —expressed in energy units—, when the social planner extends a subsidy s_f to firms, a subsidy S_c to constrained households and imposes a tax T_u on unconstrained households.¹⁰ In this framework, the tax/subsidy policy implements the first-best allocation if and only if three conditions are satisfied:

- Firms and households should consume their first-best level of energy,
- Marginal utilities of consumption should be equalised across households,
- Tax revenues should cover for subsidy expenditures.

We can derive the following result.

Proposition 5 *The social planner can implement the first-best allocation, by paying a subsidy s_f to firms per unit of energy consumed, a subsidy S_c to constrained households, and raising a tax T_u on unconstrained households, such that:*

$$(1 - s_f)\mu = 1 \quad \text{and} \quad S_c = e_c + \frac{\mu - (1 - \delta)}{\mu\delta}e \quad \text{and} \quad (1 - \phi)T_u = \phi S_c + \frac{\mu - 1}{\mu} \frac{\delta - \delta_h}{\delta}e \quad (23)$$

In this case, the wage rate ω^o and households' real income R^o/P^o write as

$$\omega^o = \frac{1}{\mu} \frac{1 - \delta}{\delta}e \quad \text{and} \quad \frac{R^o}{P^o} = \frac{e^\delta}{(1 - \delta)^{1-\delta} \delta^\delta} \quad (24)$$

Proof 5 *cf. Appendix A.5.*

The tax and subsidy scheme that implements the first-best allocation has four important properties. First, the subsidy extended to firms s_f is strictly positive and independent of the economy's energy endowment e . When the elasticity of substitution between energy and labour σ_f is one, firms' total energy consumption moves one-for-one with the wage rate ω which itself moves one-for-one with the economy's energy endowment e . As a consequence, the monopolistic competition distortion can be corrected with a constant subsidy per unit of energy consumed. Second, the wage rate under the socially optimal allocation is lower than under the decentralised equilibrium, i.e. $\omega^o < \omega$. When firms get

¹⁰To simplify notations, the tax T_u and the subsidy S_c are expressed in energy units.

a subsidy for energy consumption, they increase their demand for energy, but they reduce their demand for labour and the more so, the higher the elasticity of substitution σ_f . Given that household labour supply is inelastic, the subsidy on energy consumption extended to firms typically reduces the equilibrium wage rate. The fall in the equilibrium wage rate in turn affects households' demand for consumption goods in two ways: through an income and through a substitution effect. On the one hand, cheaper energy and cheaper labour imply cheaper consumption goods, which leads households to increase their demand for consumption goods and reduce their demand for energy. On the other hand, a lower wage rate implies a lower income and hence a lower demand for both consumption goods and energy. When households' elasticity of substitution σ_h is one, these income and substitution effects typically cancel each other, leaving the demand for consumption goods unchanged. The demand for consumption goods being unaffected, the subsidy extended to firms ends up depressing the equilibrium wage rate. In addition, when the social planner levies a tax on unconstrained households and extends a subsidy to constrained households, this acts in practise as a transfer of resources from agents spending a large share of their income on consumption goods to agents spending a large share of their income on energy. As a result, the tax/subsidy scheme is depressing the demand for consumption goods (and increasing the demand for energy), which reinforces the drop in the demand for labour, and hence the fall in the market clearing wage rate. This is why the equilibrium wage rate under the first-best allocation is always lower than the equilibrium wage rate under the decentralised equilibrium.

Third, under the first-best allocation, households' real income R^o/P^o is typically increasing in the energy endowment e . As a result, a negative energy supply shock typically reduces unconstrained household's real income and therefore raise the equilibrium rate of interest. In other words, negative energy shocks always act as negative supply shocks under the first best allocation.

Fourth and last, the subsidy extended to constrained households and hence the tax levied on unconstrained households, are both increasing in the economy's energy endowment e , i.e. $\partial S_c/\partial e > 0$ and $\partial T_u/\partial e > 0$. Hence, following a negative energy supply shock, that cuts the economy's energy endowment e , the subsidy S_c extended to constrained

households, and the tax T_u imposed on unconstrained households both fall. However, the energy need e_c of constrained households being non-zero, the elasticity of taxes and subsidies to energy shocks is less than one:

$$\frac{\partial S_c}{\partial e} \frac{e}{S_c} < 1 \quad \text{and} \quad \frac{\partial T_u}{\partial e} \frac{e}{T_u} < 1 \quad (25)$$

Based on this property, we can then derive the following lemma.

Lemma 6 *Implementing the first best allocation requires taxing a larger fraction of unconstrained households' income when the economy faces a negative energy shock.*

Proof 6 *cf. Appendix A.6.*

A negative energy shock that cuts households' energy endowments therefore requires to impose a larger tax rate on unconstrained households. The reason is that, following negative energy shocks, constrained households' income takes a double hit, one from the fall in labour income ω and one from the increase in minimum energy consumption e_c . The marginal utility of consumption of constrained households therefore increases much more steeply than that of unconstrained households. Equalising incomes then requires transferring a larger share of total income to constrained households. In practise, this means that negative energy shocks, in spite of being uniformly distributed, act as a distributional shock, that hurt low-income, credit-constrained households, to a larger extent.

This is clearly visible in Figure 6 below, which plots the subsidy and tax rates needed to implement the first-best allocation. Following a negative energy shock, the subsidy to constrained households needs to increase for a given tax on unconstrained households (or alternatively a larger tax for a given subsidy) to ensure income equalisation (EQ and EQ' schedules), as the negative energy shock widens the income difference across households. Given that income equalisation has to take place under balanced budget (BB schedule), the social planner needs to impose a larger tax rate τ .

The comparative static results derived above have one obvious implication: Given that the social planner needs to impose a higher tax rate on unconstrained households in the

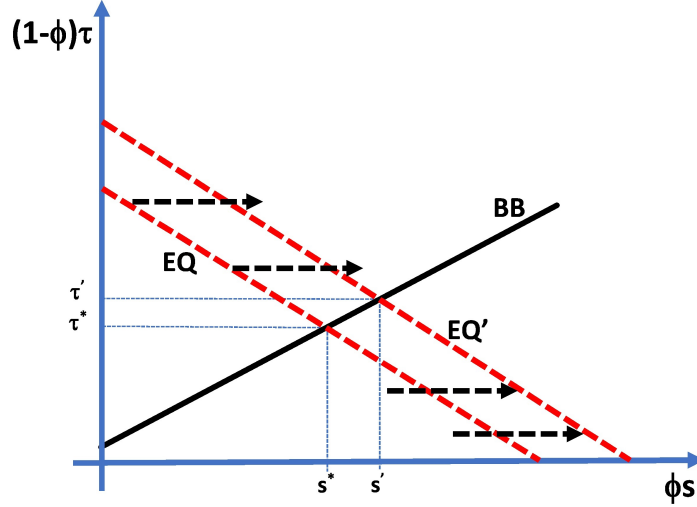


Figure 6: Negative energy shocks require larger rate of taxes and subsidies

face of negative energy shocks, i.e. at a time where incomes are lower, implementing the first-best allocation may become impossible if this implies imposing tax rates in excess of what is feasible in practise. The next proposition looks into this question.

Proposition 7 *When tax rates cannot exceed $\bar{\tau}$, implementing the first-best allocation, through subsidies to firm energy consumption and taxes/subsidies to households, is possible only if:*

$$\phi \frac{e_c}{e} \leq \frac{1}{1 - \bar{\tau}} \left[\frac{1 - \phi}{\delta} + \frac{(\mu - 1)(\delta - \delta_h)}{\delta \mu} \left[\frac{\mu - \phi(1 - \delta)}{\delta + (\mu - 1)\delta_h} - 1 \right] \right] - \frac{\mu - \phi(1 - \delta)}{\delta + (\mu - 1)\delta_h} \quad (26)$$

Proof 7 *To replicate the first-best allocation, the social planner needs to extend taxes and subsidies in line with (23). Then assuming tax rates cannot exceed $\bar{\tau}$, implementing the first-best allocation through a set of taxes and subsidies is not feasible if (26) holds.*

Expression (26) shows that implementing the first-best allocation by relying solely on taxes and subsidies is not possible whenever the maximum tax rate that the planner can impose is such that (26) does not hold even for $e_c = 0$. In this case, even if constrained households' energy endowment were to cover for their minimum energy consumption, tax revenues that can be levied from unconstrained households would fall short of covering the subsidy that needs to be extended to firms to correct for the monopolistic competition

distortion, *and* for the subsidy that needs to be extended to constrained households to equalise marginal utilities of consumption across households. Similarly the higher the fraction ϕ of constrained households, the more likely, tax revenues levied from unconstrained households will not be enough to cover for subsidies to firms and constrained households. In this case, the social planner may need additional tools. Issuing public debt could be one of them. We investigate this possibility in the next section.

4.3 Optimal public debt issuance.

Let us consider an economy where the energy endowment e —and its distribution across constrained and unconstrained households—are such that the social planner can implement the first-best allocation, by taxing unconstrained households' labour income and subsidising firms' energy consumption and constrained households' labour income. In other words, the net energy need of constrained households e_c/e is sufficiently low, so that condition (26) holds. Then, consider a negative energy shock that temporarily cuts energy endowments, so that condition (26) stops holding and the social planner cannot anymore implement the first best allocation, by solely relying on taxes and subsidies. We now ask if there is a role in this setting for public debt. More specifically, if the social planner cannot implement the first-best allocation because this would imply a tax rate that would exceed the maximum $\bar{\tau}$, could it be that issuing debt to raise the missing resources makes the economy as a whole better-off? And if so, under which conditions?

To answer these questions, let us first describe the social planner's problem in more details. Suppose until date $t - 1$, the economy's total energy endowment e is sufficiently large so that the social planner needs to impose a relatively low tax rate τ on unconstrained households to implement the first-best allocation, i.e. $\tau \leq \bar{\tau}$. In other words, the relative energy need of constrained households e_c/e is such that condition (26) holds. So until date $t - 1$, the social planner does not need to issue public debt, nor is there any public debt in the economy.

Then at date t , a one-off, temporary, shock hits the economy and cuts the energy endowment from e down to εe with $\varepsilon < 1$. Unconstrained households' energy endowment therefore drops from e_u to $e_u - (1 - \varepsilon)e$, constrained households' net energy need increases

from e_c to $e_c + (1 - \varepsilon)e$. Moreover, the negative energy shock is sufficiently large — ε is sufficiently low— so that condition (26) stops holding, i.e. the ε shock satisfies:

$$\varepsilon \leq \frac{(1 - \bar{\tau})\phi(1 + e_c/e)}{(1 - \bar{\tau})\left[\phi - \frac{\mu - \phi(1 - \delta)}{\delta + (\mu - 1)\delta_h}\right] + \frac{1 - \phi}{\delta} + \frac{(\mu - 1)(\delta - \delta_h)}{\delta\mu}\left[\frac{\mu - \phi(1 - \delta)}{\delta + (\mu - 1)\delta_h} - 1\right]} \quad (27)$$

Finally from date $t + 1$ onward, the energy shock disappears and the energy endowment is back to its initial steady-state level e so that the social planner can implement again, the first-best allocation by relying solely on taxes and subsidies.

While the social planner cannot implement the first-best allocation at date t , as this would imply setting the tax rate on unconstrained households above the maximum possible tax rate $\bar{\tau}$, it can still set the tax rate at its maximum level $\bar{\tau}$ and make up for the difference by issuing debt. The amount of debt per unconstrained household¹¹ then satisfies:

$$d_t = T_u(\varepsilon) - \bar{\tau}R^u(\varepsilon)/p_e = [\tau(\varepsilon) - \bar{\tau}]R^u(\varepsilon)/p_e \quad (28)$$

In this expression, $T_u(\varepsilon)$ represents the taxes, the social planner needs to raise to pay out the subsidies to constrained households and firms, when the economy is hit with a negative energy shock ε , $\bar{\tau}$ is the maximum tax rate that the social planner can impose, and $R^u(\varepsilon)/p_e$ is the income of unconstrained households in the decentralised equilibrium, when the economy is hit with a negative energy shock ε . Issuing debt, however, implies paying interest. How much interest the social planner has to pay at date $t + 1$ for the debt issued at date t depends on unconstrained households' Euler equation, which in turn depends on unconstrained households' real incomes at date t and date $t + 1$, respectively R_t^u/P_t and R_{t+1}^u/P_{t+1} . Given the expression for households' real income under the first-best allocation (24), the real interest rate at date t , i.e. at the time of the negative energy shock, satisfies

$$1 + r_t = \frac{1}{\beta} \left[\frac{R_{t+1}^u/P_{t+1}}{R_t^u/P_t} \right]^{\frac{1}{\gamma}} = \frac{1}{\beta\varepsilon^{\frac{\delta}{\gamma}}} \quad (29)$$

As noted above, under the first-best allocation, the real interest rate increases when a

¹¹it is convenient to express public debt as the amount held by each unconstrained household as only unconstrained households can borrow and save and hence buy financial instruments

negative energy shock hits, as unconstrained households who are temporarily poorer would like to borrow to smooth out the shock. Moreover, the increase in the real interest rate is larger when the energy intensity δ is higher. As would be expected, a negative energy shock is more costly in economies which are more energy intensive.

Then from date $t + 1$, when the economy is back to the steady-state, the social planner can simply implement the first-best allocation by simply levying taxes and paying out subsidies. It does however have to fund the debt issued to address the negative shock that hit at date t , which requires raising some additional taxes, to make sure the debt does not blow up out of proportion. Noting that at the first best, interest rates satisfy $\beta r_{t+n} = 1$ from date $t + n$ onwards (with $n \geq 1$), public debt at date $t + n$ writes as

$$d_{t+n} = \frac{R^o}{\beta^{n-1}} \left[(1 + r_t) \frac{\tau(\varepsilon) - \bar{\tau}}{1 - \tau(\varepsilon)} - \frac{1}{\beta} \sum_{k=0}^{n-1} \beta^{n-k} \tau_{t+n-k}^a \right] \quad (30)$$

where $\{\tau_{t+n}^a\}_{n \geq 1}$ denotes the additional taxes that the social planner raises to finance (part of) the debt coming due. Then considering the case where the social planner sets constant additional taxes, i.e. $\tau_{t+n}^a = \tau_a$ from date $t + n$, onwards (with $n \geq 1$), and given that additional and steady-state taxes cannot exceed maximal taxation, i.e. $\tau_a + \tau_{ss} \leq \bar{\tau}$, we can derive the following proposition.

Proposition 8 *The social planner can implement the first-best allocation on the back of a negative energy supply shock, by issuing public debt, raising taxes and paying subsidies, as long as the energy shock ε satisfies*

$$1 + \frac{\beta}{1 - \beta} [\bar{\tau} - \tau_{ss}] [\varepsilon]^{\frac{\delta}{\gamma}} \geq \frac{1 - \bar{\tau}}{1 - \tau(\varepsilon)} \quad (31)$$

Proof 8 *cf. Appendix A.7.*

Let us denote $\varepsilon(\bar{\tau})$ the largest negative energy shock for which the social planner can implement the first best without issuing any public debt; and ε_{\max} the shock for which (31) holds with equality. Then it is straightforward to note that as long as $\tau_{ss} \leq \bar{\tau}$, i.e. as long as the social planner can implement the first-best allocation at the steady state,

without having to issue public debt, we always have $\varepsilon_{\max} < \varepsilon(\bar{\tau})$. In other words, when the social planner can implement the first-best allocation at the steady state by setting a tax rate below $\bar{\tau}$, issuing public debt allows the social planner to accommodate a larger set of negative energy shock.

The intuition for this result is fairly simple. When the economy faces a large negative energy shock, implementing the first best allocation may imply taxing unconstrained households beyond what is practically possible. As a result, the social planner can issue public debt to make up for the missing resources and redistribute the proceeds to firms and constrained households. That said, the social planner has to commit to raise additional taxes in the future to back the current debt issuance. In practise, the face value of public debt cannot exceed the present value of all future additional tax revenues that the social planner can raise, once the shock has dissipated. Hence, for the very same reason that public debt can be useful, i.e. to bypass the social planner's limited taxation power, public debt cannot grow indefinitely, as it has to be backed by future tax revenues, which are themselves limited by the social planner's maximum taxation power.

Interestingly, issuing public debt, following a negative energy shock, can be useful for the social planner to implement the first-best allocation, even as the energy shock acts as a negative supply shock and raises the equilibrium rate of interest. There is hence a role for public debt in implementing the first best allocation, not simply when the negative energy shock acts as negative demand shock, but also when it acts as a negative supply shock. In other words, public debt is useful even in the absence of demand shortages.

Looking at comparative statics, the social planner can accommodate a wider set of negative shocks by issuing public debt when the inter-temporal elasticity of substitution γ is higher (salmon vs. yellow region in Figure 6). A high inter-temporal elasticity of substitution γ indeed means that unconstrained agents can more easily substitute between current and future consumption. As a result, the negative energy shock raises the equilibrium real rate, but to a lesser extent when γ is higher. The cost of issuing public debt when the economy faces a negative energy shock is then lower and so is the additional taxation needed to ensure public debt does not balloon out of proportion.

Similarly, the social planner can accommodate a larger set of negative energy shocks ε

by issuing public debt when the energy intensity δ is lower (salmon vs. yellow region in Figure 7). Hence when consumption and/or production are less energy-intensive, public debt is more useful, in the sense that the social planner has more “fiscal space” and can accommodate a wider set of negative energy shocks. Conversely, and as would be expected, when the social planner’s taxation power is more limited, i.e. when $\bar{\tau}$ is lower, the set of negative shocks ε under which the social planner can implement the first best allocation using public debt is also more limited. Simply put, the higher the public debt to start with, the larger the tax revenues needed to fund pre-existing public debt. As a result, the ability to raise additional taxes is lower and additional public debt can accommodate a smaller set of shocks. Moreover, this effect tends to be more pronounced when the energy-

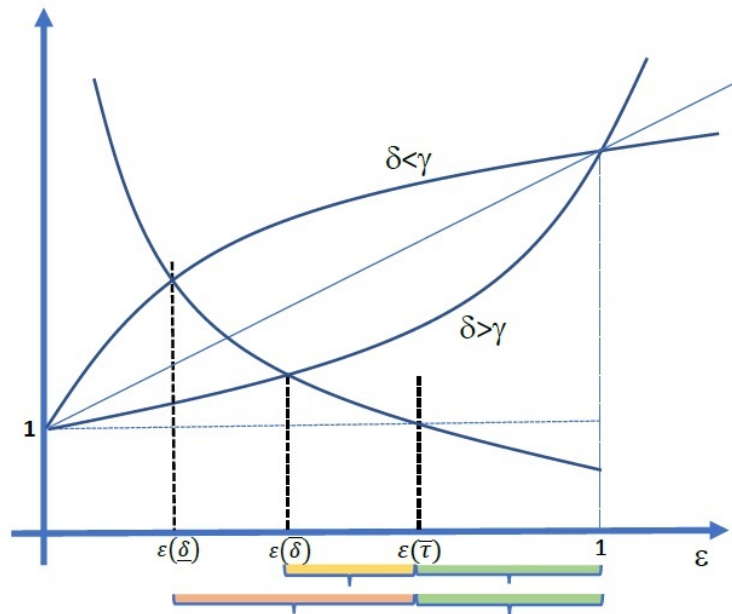


Figure 7: Negative energy shocks and public debt

intensity parameter δ is lower. Public debt can therefore help accommodate a wider set of negative energy supply shocks. But it is then also more sensitive to changes in the planner’s taxation power, i.e. to initial debt levels, as higher initial public debt shrinks the set of shocks that can be addressed using public debt to a larger extent when output and/or consumption are less energy intensive.

5 Conclusions

We investigated the distributional and economic impact of an energy supply shock in a two-agent model where households have a minimum need of energy and some households are poor and credit-constrained. We show that a negative energy supply shock may morph into a negative demand shock for consumption goods and put downward pressure on prices if the share of constrained households is large enough and prices of consumption goods are flexible enough. In that case, unconstrained households benefit from a negative energy supply shock as their real income increases. As they smooth overall consumption and the desired saving increases, the equilibrium real interest rate rises. We analyse optimal fiscal policy in this environment where aggregate output is too low due to monopolistic competition and consumption of energy and consumption goods is distorted due to credit constraints on poorer households. Optimal budget-neutral fiscal policy can replicate the first-best. It involves an energy subsidy to firms, a labour income subsidy to the poorer, credit-constrained households and an income tax on rich, unconstrained households. When energy supply temporarily falls the transfer from the rich to the poor is optimally increased. In this first-best world, output and consumption are higher, the equilibrium real wage is lower and negative energy supply shocks always act as negative supply shocks and raise the equilibrium real interest rate. Finally, if there is a relevant upper limit on the income tax rate that can be imposed on the rich households, public debt can help implement the first-best outcome by spreading out the budgetary cost of the subsidies to the poor provided there is enough fiscal space. The analysis was done in a model without nominal rigidities. A next step is to explore the impact of nominal frictions in wage setting and price setting for optimal fiscal and monetary policy.

References

ARI, A., S. BLACK, O. CELASUN, D. M. IAKOVA, A. MINESHIMA, V. MYLONAS, W. PARRY, IAN, I. TEODORU, AND K. ZHUNUSSOVA (2022): “Surging energy prices in Europe in the aftermath of the War: How to support the vulnerable and speed up the transition away

- from fossil fuels," *IMF Working Paper*, 152.
- AUCLERT, A., H. MONNERY, M. ROGNLIE, AND S. L. (2023): "Managing an energy shock: Fiscal and monetary policy," *mimeo*, Stanford University.
- BATTISTINI, N., V. DI NINO, M. DOSSCHE, AND A. KOLNDREKAJ (2022): "Energy prices and private consumption: what are the channels?" *Economic Bulletin*, European Central Bank.
- BODENSTEIN, M., C. J. ERCEG, AND L. GUERRIERI (2008): "Optimal monetary policy with distinct core and headline inflation rates," *Journal of Monetary Economics*, 55, S18–S33.
- CHAN, J., S. DIZ, AND D. KANNGIESSER (2022): "Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK," *MPRA Paper*, University Library of Munich.
- DHAWAN, R. AND K. JESKE (2008): "Energy Price Shocks and the Macroeconomy: The Role of Consumer Durables," *Journal of Money, Credit and Banking*, 40, 1357–1377.
- GEARY, R. C. (1950): "A Note on 'A Constant-Utility Index of the Cost of Living'," *Review of Economic Studies*, 18, 65–66.
- GELMAN, M., Y. GORODNICHENKO, S. KARIV, D. KOUSTAS, M. D. SHAPIRO, D. SILVERMAN, AND S. TADELIS (2023): "The response of consumer spending to changes in gasoline prices," *American Economic Journal: Macroeconomics*, 15, 129–60.
- GORNEMANN, N., S. HILDEBRAND, AND K. KUESTER (2023): "Limited Energy Supply, Sunspots, and Monetary Policy," *ECONtribute Discussion Papers Series 215*.
- GUERRIERI, V., G. LORENZONI, L. STRAUB, AND I. WERNING (2022): "Fiscal Policy in the Age of COVID: Does it 'Get in all of the Cracks?'" *American Economic Review*, 112, 1437–74.
- HUNT, B. (2005): "Oil Price Shocks: Can they Account for the Stagflation in the 1970's," *IMF Working Paper*, 215, 173–189.
- KILIAN, L. (2008): "The Economic Effects of Energy Price Shocks," *Journal of Economic Literature*, 46, 871–909.

KIM, I.-M. AND P. LOUNGANI (1992): “The role of energy in real business cycle models,” *Journal of Monetary Economics*, 29, 173–189.

KÄNZIG, D. R. (2021): “The macroeconomic effects of oil supply news: Evidence from OPEC announcements,” *American Economic Review*, 111, 1092–1125.

PEERSMAN, G. AND J. WAUTERS (2022): “Heterogeneous household responses to energy price shocks,” *NBB Working Paper 416*.

PIERONI, V. (2023): “Energy Shortages and Aggregate Demand: Output Loss and Unequal Burden from HANK,” *European Economic Review*, 154.

STONE, R. (1954): “Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand,” *Economic Journal*, 64, 511–527.

Appendix A.1: Proof of Proposition 1. With homogeneous sectors, given firms’ optimal pricing, the price of consumption goods in each sector simply writes as $p_{st}/p_{et} = \mu \left[\delta_f + (1 - \delta_f) \omega_t^{1-\sigma_f} \right]^{\frac{1}{1-\sigma_f}}$, where $\omega_t = w_t/p_{et}$. Applying this expression to the relative price of consumption goods p_c in the special case $\sigma_f = 1$ yields $p_c = p_{ct}/p_{et} = \mu \omega^{1-\delta_f}$. Then aggregate net saving being zero in equilibrium, aggregate income devoted to the purchase of consumption goods and energy, beyond and above the minimum energy consumption e_h , writes as $R_t/p_{et} = (1 - \phi)(\omega_t + e_u) + \phi(\omega_t - e_c)^+ + \pi_t/p_{et}$. Then using expression (14) for equilibrium profits π_t , household aggregate income at date t writes as:

$$\frac{R_t}{p_{et}} = \frac{(1 - \phi)(\omega_t + e_u) + \phi(\omega_t - e_c)^+}{1 - (1 - \delta_h) \frac{\mu-1}{\mu} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h}} \quad (32)$$

Solving for the equilibrium of the labour market in the special case where $\sigma_h = 1$, the wage rate ω_t that balances labour supply and demand satisfies (17). Finally, aggregate income for unconstrained households writes as $R_t^u = (1 - \phi)p_{et}(\omega_t + e_u) + \pi_t$. Again using expression (14) for equilibrium profits π_t , the expression for aggregate income R_t^u simplifies as

$$\frac{R_t^u}{p_{et}} = \frac{(\omega_t + e_u) + (1 - \delta_h) \frac{\mu-1}{\mu} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h} \frac{\phi}{1-\phi} (\omega_t - e_c)^+}{1 - (1 - \delta_h) \frac{\mu-1}{\mu} \left[\frac{p_{ct}}{P_t} \right]^{1-\sigma_h}} \quad (33)$$

Given previous results, aggregate income for unconstrained households R_t^u grows, at the steady state, at the same rate as energy prices, i.e. $\frac{R_{t+1}^u}{R_t^u} = \frac{p_{et+1}}{p_{et}}$. Applying this property to households' Euler equation yields the equilibrium "real" interest rate r .

Appendix A.2: Proof of Lemma 2. Given that parameters satisfy $\delta; \delta_h < 1 < \eta$, the economy has a unique and stable equilibrium, which can be of two sorts: When condition (18) holds, then the equilibrium wage ω^* given by (17), satisfies $\omega^* \geq e_c$, households are all able to cover their minimum energy needs and participate in the demand for consumption goods. Conversely, when condition (18) does not hold, then the equilibrium wage ω^* satisfies $\omega^* < e_c$; constrained households are not able to cover for their minimum energy consumption and therefore do not participate into the demand for consumption goods.

Appendix A.3: Proof of Proposition 3. When $\sigma_f = \sigma_h = 1$, then following expression (33) for unconstrained households' nominal income, unconstrained households' real income writes, up to a positive multiplicative constant, as

$$\frac{R_t^u}{P_t} = \frac{(1 - \phi)(\omega_t + e_u) + (1 - \delta_h)\frac{\mu-1}{\mu}\phi(\omega_t - e_c)^+}{\mu^{1-\delta_h}\omega_t^{1-\delta}} \quad (34)$$

Using condition (17) for the equilibrium wage rate ω_t and denoting $v = (1 + (\mu - 1)\delta_h)/(1 - \delta)$, and $\varphi = (1 - \delta_h)(1 - \phi)$, unconstrained households' real income writes, under incomplete participation, as

$$(1 - \phi)\frac{R_t^u}{P_t} = \frac{v\mu^{\delta_h}}{\varphi + (1 - \varphi)\mu} \left[\frac{(1 - \phi)e_u}{v - (1 - \phi)} \right]^\delta \quad (35)$$

A negative shock that cuts the economy's energy endowment e —and hence the net endowment e_u —therefore always reduces unconstrained households' real income. Conversely, in the complete participation equilibrium, unconstrained households' real income writes, up to a positive multiplicative constant, as:

$$(1 - \phi)\frac{R_t^u}{P_t} = \mu^{\delta_h} \left[\frac{\mu - \phi(1 - \delta)}{(1 - \delta)\mu} + \phi \frac{v - 1}{\mu} \frac{e_c}{e} \right] \left[\frac{e}{v - 1} \right]^\delta \quad (36)$$

A negative energy shock that equally cuts the net energy endowments e_u and $-e_c$ then raises unconstrained households' real income R_t^u/P_t if and only if

$$\frac{e_c}{e} \geq (1 - \lambda) \frac{1 - \delta}{\delta + \delta_h(\mu - 1)} \quad (37)$$

with $\lambda = (1 - \delta_h + \delta_h\mu - \delta\mu/\phi)/(1 - \delta)^2$. Given that the complete participation equilibrium requires condition (18) to hold, unconstrained households' real income cannot fall with the energy endowment e unless $\phi \geq \frac{\delta\mu}{1 + \delta_h(\mu - 1)}$. Given that $\phi \leq 1$, this requires $\delta_f\mu \leq 1$.

Appendix A.4: Proof of Proposition 4. Based on the first-order condition for the planner's problem (21), the socially optimal allocation of energy between households and firms is such that firms' energy consumption E_f^o satisfies

$$\left[1 + \frac{\delta_h}{1 - \delta_h} \left[\frac{1}{\delta_f} \right]^{\frac{\sigma_h}{\sigma_f}} \left[\delta_f^{\frac{1}{\sigma_f}} + (1 - \delta_f)^{\frac{1}{\sigma_f}} \left[E_f^o \right]^{\frac{1 - \sigma_f}{\sigma_f}} \right]^{\frac{\sigma_f - \sigma_h}{\sigma_f - 1}} \right] E_f^o = e \quad (38)$$

In the special case where the elasticities of substitution are both set to one, ie. $\sigma_h = \sigma_f = 1$, simplifying this expression, the socially optimal allocation of energy satisfies:

$$E_f^o = \left[1 - \frac{\delta_h}{\delta} \right] e \quad \text{and} \quad E_h^o = \frac{\delta_h}{\delta} e \quad (39)$$

Conversely in the decentralised equilibrium, following on expressions (15) for firms' demand for energy and (32) for households' total income net of minimum energy consumption, firms' energy use is $E_f = \frac{\delta_f}{1 - \delta_f} \omega^{\sigma_f}$ while the equilibrium wage rate ω satisfies (17).

Hence, in the complete participation equilibrium, i.e. when condition (18) holds, the amount of energy E_f^* that firms use in equilibrium, satisfies

$$\left[1 + \frac{\delta_h}{1 - \delta_h} \left[\frac{1}{\delta_f} \right]^{\frac{\sigma_h}{\sigma_f}} [\mu]^{\sigma_h} \left[\delta_f^{\frac{1}{\sigma_f}} + (1 - \delta_f)^{\frac{1}{\sigma_f}} \left[E_f^* \right]^{\frac{1 - \sigma_f}{\sigma_f}} \right]^{\frac{\sigma_f - \sigma_h}{\sigma_f - 1}} \right] E_f^* = e \quad (40)$$

Here again, in the particular case where the elasticities of substitution are both set to one,

ie. $\sigma_h = \sigma_f = 1$, the equilibrium allocation of energy simplifies as:

$$E_f^* = \left[1 - \frac{\mu\delta_h}{\delta + (\mu - 1)\delta_h} \right] e \quad \text{and} \quad E_h^o = \frac{\mu\delta_h}{\delta + (\mu - 1)\delta_h} e \quad (41)$$

Last, under incomplete participation, i.e. when condition (18) does not hold, then the amount of energy E_f^* that firms use in equilibrium, satisfies

$$\left[1 + \frac{\delta_h}{1 - \delta_h} \left[\frac{1}{\delta_f} \right]^{\frac{\sigma_h}{\sigma_f}} [\mu]^{\sigma_h} \left[\delta_f^{\frac{1}{\sigma_f}} + (1 - \delta_f)^{\frac{1}{\sigma_f}} \left[E_f^* \right]^{\frac{1 - \sigma_f}{\sigma_f}} \right]^{\frac{\sigma_f - \sigma_h}{\sigma_f - 1}} \right] E_f^* + \phi \left[\frac{1 - \delta_f}{\delta_f} E_f^* \right]^{\frac{1}{\sigma_f}} = (1 - \phi) e_u \quad (42)$$

When the elasticities of substitution are both set to one, ie. $\sigma_h = \sigma_f = 1$, the energy allocation simplifies as:

$$E_f^* = \left[1 - \frac{1 + \frac{\mu\delta_h}{\phi(1-\delta)}}{1 + \frac{\delta + (\mu - 1)\delta_h}{\phi(1-\delta)}} \right] (1 - \phi) e_u \quad \text{and} \quad E_h^o = \frac{1 + \frac{\mu\delta_h}{\phi(1-\delta)}}{1 + \frac{\delta + (\mu - 1)\delta_h}{\phi(1-\delta)}} (1 - \phi) e_u - \phi e_c \quad (43)$$

When the decentralised equilibrium features complete participation, expressions (39) and (41) show that firms consume too little energy in the decentralised equilibrium, i.e. $E_f^* < E_f^o$. Conversely, when there is incomplete participation, then expressions (39) and (43) show that the amount of energy E_f^* that goes to firms is independent of e_c , unlike in the social optimum, where it is decreasing in e_c . Hence there exists a threshold level for e_c such that when e_c is above this threshold, firms consume too much energy in the decentralised equilibrium relative to the social optimum. In the specific case where $\sigma_h = \sigma_f = 1$, the condition under which firms consume too much energy in the decentralised equilibrium relative to the social optimum simplifies as (22).

Appendix A.5: Proof of Proposition 5.

Suppose the social planner extends a subsidy s_f to firms for each unit of energy consumed. Then, when $\sigma_h = \sigma_f = 1$, firms' demand for energy writes as:

$$E_f = \frac{\delta_f}{1 - \delta_f} \frac{w_t}{(1 - s_f) p_e} = \frac{\delta_f}{1 - \delta_f} \frac{\omega}{1 - s_f} \quad (44)$$

Moreover, when the social planner extends a subsidy S_c to constrained households and levies a tax T_u on unconstrained households, the equilibrium of the labour market writes as:

$$(1 - \phi)(\omega + e_u - T_u) + \phi(\omega - e_c + S_c)^+ = \frac{\omega}{1 - \delta_f} \left[1 + \frac{\delta_h \mu}{1 - \delta_h} \right] \quad (45)$$

Re-writing expression (45), energy E_f consumed by firms at the equilibrium satisfies

$$\left[1 + \frac{\delta_h \mu}{\delta - \delta_h} (1 - s_f) \right] E_f + (1 - \phi) T_u = e + \phi S_c + s_f E_f \quad (46)$$

Moreover, at the social optimum, the planner should equalise incomes across constrained and unconstrained households. Denoting $\xi = (1 - \delta_h)(\mu - 1)/(1 + \delta_h(\mu - 1))$, the income equalisation condition writes as:

$$\omega_t + S_c - e_c = (1 + \xi)(\omega_t - T_u + e_u) + \xi \frac{\phi}{1 - \phi} (\omega_t + S_c - e_c) \quad (47)$$

Finally, tax revenues should cover for subsidy expenditures, i.e.

$$(1 - \phi) T_u = \phi S_c + s_f E_f \quad (48)$$

Using (46) and (48), one can easily check that firms consume the first-best level of energy E_f^o when the subsidy s_f satisfies $\mu(1 - s_f) = 1$. Then inverting firms' demand for energy (44), the equilibrium wage rate under the optimal policy writes as $\omega_t = \frac{1 - \delta}{\delta} \frac{e}{\mu}$. Finally, given the expressions for the subsidy s_f , the wage rate ω_t , and firms' energy consumption E_f , the income equalisation condition (47) together with the balanced budget condition (48) imply that the subsidy S_c to constrained households and the tax T_u on unconstrained households should satisfy (23).

Appendix A.6: Proof of Lemma 6.

The taxes T_t^u/p_{et} the social planner needs to raise on unconstrained households to

implement the first-best allocation write as:

$$(1 - \phi) T_t^u / p_{et} = \phi \left[e_c + \frac{\mu - (1 - \delta)}{\delta \mu} e \right] + \frac{\mu - 1}{\mu} \frac{\delta - \delta_h}{\delta} e \quad (49)$$

Moreover, in the decentralised equilibrium, i.e. absent any intervention from the social planner, unconstrained households' income R_t^u / p_{et} writes as:

$$(1 - \phi) R_t^u / p_{et} = (1 - \phi) (1 + \xi) (\omega_t + e_u) + \xi \phi (\omega_t - e_c) \quad (50)$$

Hence, the tax rate for unconstrained households, i.e. taxes T^u as a ratio of income R^u writes as

$$\frac{T_t^u}{R_t^u} = \frac{\delta \mu \phi (e_c / e) + [\mu - \phi (1 - \delta)] + [(\mu - 1)(\delta - \delta_h) - (1 - \phi) \mu]}{\delta \mu \phi (e_c / e) + \frac{\delta \mu}{\delta + (\mu - 1) \delta_h} [\mu - \phi (1 - \delta)]} \quad (51)$$

One can then easily check that the term on the RHS of (51) is increasing in the ratio e_c / e . As a consequence negative energy supply shocks that cut the energy endowment e and hence raise the ratio e_c / e typically require the social planner to levy a larger fraction of unconstrained households' income, to implement the first best allocation:

Appendix A.7: Proof of Proposition 8.

According to (30), to prevent public debt from spiralling up, the social planner must ensure that the present value of the additional taxes is greater than or equal to the date- $t+1$ value of the debt issued at date t .

$$\frac{1}{\beta} \sum_{k=0}^{n-1} \beta^{n-k} \tau_{t+n-k}^a \geq (1 + r_t) \frac{\tau(\varepsilon) - \bar{\tau}}{1 - \tau(\varepsilon)} \quad (52)$$

Given that the social planner cannot raise additional taxes τ_a in excess of the difference between maximum and steady state taxes, i.e. $\tau_a \leq \bar{\tau} - \tau_{ss}$, debt is sustainable in the long-run if and only if

$$\bar{\tau} - \tau_{ss} \geq \frac{1 - \beta}{\beta} [\varepsilon]^{-\frac{\delta}{\gamma}} \frac{\tau(\varepsilon) - \bar{\tau}}{1 - \tau(\varepsilon)} \quad (53)$$

Expression (53) can then be simplified, with simple algebra, as (31).

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