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Optimal dynamic hedging using futures under a borrowing constraint

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Abstract

Both financial and non-financial firms routinely implement hedging policies to mitigate their exposure to changes in asset prices. However, while these policies may perform satisfactorily in the limited sense of hedging the exposure under consideration, they might increase the overall likelihood of financial distress due to the liquidity risks that they create. This paper examines the case of hedging price risk using derivative contracts that are marked to market (such as futures contracts) and hence subject to margin calls. It is shown that liquidity risk, stemming from the need to meet margin calls on the futures position, can be a significant source of risk and can even lead to financial distress even though the firm remains "hedged". Such risks should therefore be taken into account in the formulation of an optimal hedging policy. This paper derives the dynamic hedging strategy of a firm that uses futures contracts to hedge a spot market exposure. The risk emanating from the margin requirement on futures contracts is incorporated into the hedging decision by restricting the borrowing capacity of the firm. It is shown that this leads to a substantial reduction in the firm's optimal hedge, especially if the hedging horizon is long. The results provide theoretical support for the low level of hedging observed empirically.

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1. Introduction

Active hedging of exposure to changes in asset and commodity prices has been an area of substantial academic research. Sophisticated policies have been recommended as tools for risk management which, in principle, have become much easier to implement due to the large variety of derivative contracts now available. However, these recommendations have not been widely adopted in practice. One of the reasons cited is that while these policies may perform satisfactorily in the limited sense of hedging the exposure under consideration, they often increase the likelihood of financial distress of the firm in a broader sense. The *Group of Thirty* Global Derivatives Study Group (1993) as well as the Committee on Payment and Settlement Systems (1998) identify liquidity risk, stemming from the possibility of temporary cash shortfalls that prevent a firm from making payment obligations, as one of the risks facing users of derivatives and other financial contracts. Such risks should be taken into account in the formulation of an optimal hedging policy. This paper derives the dynamic hedging strategy of a firm that uses futures contracts to hedge a spot market exposure. The risk emanating from the margin requirement on futures contracts is incorporated into the hedging decision by restricting the borrowing capacity of the firm. I find that this leads to a substantial alteration in the firm's optimal hedging strategy, especially if the hedging horizon is long.

Futures contracts are one of the most widely used derivative instruments in financial asset and commodity markets. Like a forward contract, a futures contract specifies the terms at which the buyer and seller can exchange a commodity or asset on a future date. Unlike forwards, however, futures contracts have highly uniform and well specified contract terms and are exchange-traded, leading to their high trading volume and low transactions costs. Each participant in the futures market is required to maintain a margin account with the clearing house of the exchange. Futures price changes are credited (or debited) to this account on a daily basis, an operation that is referred to as daily settlement or *marking-to-market*. The margin is a performance bond that minimises the risk of default, due to potentially adverse price moves, by either party involved in a contract. If the margin falls below a certain level, the losing party must deposit additional margin to remain in the market. Thus, a firm that assumes a futures position to lower the risk of its spot market exposure must make an appraisal of its ability to meet unexpectedly large margin calls. Failure to do so can result in substantial interim losses on the futures position and even lead to financial distress, as it did in the case of Metallgesellschaft.

In late 1993 and early 1994, MG Corporation, the US subsidiary of Germany's 14th largest industrial firm, Metallgesellschaft AG, reported staggering losses amounting to over $1.3 billion on its position in energy futures and swaps. The blame was pinned squarely on the firm's large position in energy derivatives, primarily oil futures, that it had undertaken to hedge long-term commitments, extending up to 10 years, to sell oil and oil products to its clients at a fixed price. By December 1993, the size of MG Corporation's energy derivatives position was equivalent to about 160 million barrels of oil. A sustained drop in the price of oil futures led to unexpectedly large margin calls on the firm's futures positions which it failed to meet due to lack of sufficient liquidity. This, in spite of the fact that the two largest banks in Germany, Deutsche Bank and Dresdner Bank, served on the supervisory board of Metallgesellschaft and together held 22% of the firm. Only a massive $1.9 billion rescue and restructuring operation coupled with a premature liquidation of its hedging position kept Metallgesellschaft from going to bankruptcy.[1]

A number of papers have looked at hedging problems in which interim profits and losses on the futures position are continuously marked-to-market in an interest-bearing margin account. None of these, however, consider the case where borrowing to meet margin calls may be costly or limited.[2] Given a predetermined cash position and fairly priced futures, Anderson and Danthine (1981) show that the static hedge ratio, the optimal number of futures to be held per unit of the spot asset, is given by the covariance of the futures price and the spot price scaled by the variance of the futures price.[3] In a subsequent paper (Anderson and Danthine (1983)), the result is extended to a multi-period setting with finite maturity of the cash position, obtaining the optimal dynamic hedge as the *discounted* value of the static hedge, the discount factor being the riskless interest rate and the discount period being the time remaining to maturity. Duffie and Jackson (1990) analyse the optimal futures hedging problem in several continuous-

[1] See Culp and Miller (1995), Culp and Hank(1994), Mello and Parsons (1994), and Edwards and Canter (1994) for a detailed discussion of the Metallgesellschaft case.

[2] The effects of borrowing constraints have been explored primarily in the context of the consumption-investment problem. See Merton (1971), Lehoczky, et. al. (1983), Karatzas, et. al. (1986), Sethi, et. al. (1992), and Vila and Zariphopoulou (1990). Grossman and Vila (1992) solve for optimal trading rules with a non-negativity constraint on wealth and a leverage constraint on investment in the risky asset.

[3] If the futures are not fairly priced, the optimal futures position consists of an additional term, called the speculative component of the hedge, which is proportional to the expected gain in the futures price. To focus on the hedging aspect of the problem, I assume martingale futures throughout the analysis.

time settings, solving explicitly for the optimal hedge. Papers by Karp (1988) and Ho (1984), among others, consider the problem where both price and output uncertainty are to be hedged. In all of these papers, the prescribed hedging policies implicitly require that firms have the ability to borrow arbitrarily large amounts to meet margin calls on their hedging positions. Such a requirement, simply to maintain a hedging position, seems unreasonable when the very purpose of hedging should be to avoid this in the first place.

In this paper I show that in the presence of borrowing constraints, firms hedge significantly less than they would if they had unlimited borrowing capacity. The optimal hedge is increasing in the level of the margin account and asymptotically approaches the unconstrained hedge. For a given level of margin wealth, the hedge comes closer to the unconstrained value as the contract advances towards maturity and the probability of the borrowing constraint becoming active declines. These results provide theoretical support for the relatively low level of corporate and agricultural hedging that has been documented in the literature. They also explain the high open interest in short maturity futures contracts relative to long ones.

The paper is organised as follows. Section 2 provides a simple example in discrete time to illustrate the role of the borrowing constraint on the margin account. The framework for the continuous time model and its assumptions is established in Section 3. Section 4 discusses the numerical procedure used to solve the partial differential equation obtained from the stochastic control problem. Section 5 presents the results and discusses their relevance in explaining various stylised facts about hedging behaviour and futures markets. Section 6 concludes the paper.

2. An Example in Discrete Time

I begin with a simple example in a discrete-time setting with three dates to illustrate the role that the borrowing constraint plays on the optimal dynamic hedging decision. The example also shows how this constraint leads to a "concavification" of the derived utility function relative to that of the unconstrained problem.

Consider a firm which owns one unit of an asset until date T, at which time it is sold at the prevailing random spot price. This asset has no carrying cost or convenience yield, and pays no dividends. The firm seeks to maximise its concave utility $U(\cdot)$ of date T wealth. Futures contracts based on this asset and having maturity T are available. Since the firm is risk averse, it takes a futures position to hedge the risk of variations in the spot price. The futures price on date t is denoted by F_t. The current date is $T-2$, at which time the futures price is $F_{T-2} = 16$. The price on the next date either doubles with probability 1/3, or becomes half of the current price with probability 2/3. The binomial tree of futures prices is shown in Figure 2.1. Note that the futures price process has zero drift so that the only motive for assuming a futures position is to hedge the spot price exposure.

The futures position can be revised on date $T-1$ after the realisation of the futures price for that date. Let θ_t denote the firm's futures position on date t. The firm maintains a margin account, whose level on date t is X_t, to which futures price changes are marked-to market. Hence an amount equal to $\theta_t(F_{t+1} - F_t)$ is credited (or debited) to the margin account on date $t+1$. The firm starts on date $T-2$ with wealth $X_{T-2}(\geq 0)$ in the margin account but *cannot* borrow on dates $T-1$ and T to fulfil margin account shortfalls. Hence it is required that $X_{T-1} \geq 0$ and $X_T \geq 0$. The margin account earns no interest, there are no transactions costs and the futures contracts are infinitely divisible.

The final period wealth consists of the spot price on the asset and the margin wealth, X_T. Since the futures contract has maturity T, the futures price is equal to the spot price on date T. Hence the final period wealth can be expressed as $F_T + X_T$. On date t, the firm solves:

$$\max_{\theta_t} \mathbb{E}_t[U(F_T + X_T)] \tag{2.1}$$

subject to the borrowing constraints:

$$X_s \geq 0, \qquad t \leq s \leq T \tag{2.2}$$

where $\mathbb{E}_t$ is the expectation operator conditional on the information on date t.

With no borrowing constraints on the margin account, the solution to this hedging problem is quite simple: $\theta_{T-2} = -1$ and $\theta_{T-1} = -1$, ie the firm hedges the spot price exposure completely. I will refer to this as a *total hedge*. This hedge eliminates all the spot price risk, allowing the firm to achieve final period wealth equal to $16 + X_{T-2}$ with probability one for any value of X_{T-2}. This is possible because in the absence of borrowing constraints, any shortfalls in the level of the margin account on dates $T-1$ and T due to a rise in futures prices can be replenished by borrowing freely.

However, in the presence of the borrowing constraints in equation (2.2), the firm might not be able to assume a fully hedged position for low levels of margin account wealth. Failure to maintain a positive balance in the margin account at any date will force the firm to liquidate all its futures positions from that date onwards until the final period. I solve the problem by dynamic programming where V_t denotes the value function on date t:

$$V_t \equiv \max_{\theta_t} \mathbb{E}_t[U(F_T + X_T)] \tag{2.3}$$

On date $T-1$, departure from the total hedge is driven entirely by the constraint $X_T = X_{T-1} + \theta_{T-1}(\overline{F}_T - F_{T-1}) \geq 0$ where $\overline{F}_T$ is the highest possible realisation of the futures price on date T conditional on the current price F_{T-1}. At the node $F_{T-1} = 32$, the maximum possible loss due to the (short) futures position is $32(= 64 - 32)$ per contract; hence if X_{T-1} is less than 32, only a fraction equal to $X_{T-1}/32$ of the contract can be purchased. A similar argument applies to the node $F_{T-1} = 8$. Therefore the optimal hedge on date $T-1$ is given by:

$$\theta_{T-1} = \begin{cases} \max(-1, -X_{T-1}/32) & \text{if } F_{T-1} = 32 \\ \max(-1, -X_{T-1}/8) & \text{if } F_{T-1} = 8 \end{cases} \tag{2.4}$$

The hedging decision on date $T-2$ is a little more complex. Of course, there is the next period constraint $X_{T-1} = X_{T-2} + \theta_{T-2}(\overline{F}_{T-1} - F_{T-2)} \geq 0$ which has to be satisfied. But $\theta_{T-2} = \max(-1, -X_{T-2}/16)$ is not the optimal hedge. Full or maximum possible hedging on date $T-2$ might constrain the hedging choice on date $T-1$ in the event that the futures price rises. Thus the firm has to make a trade-off between hedging fully on date $T-2$ and taking the risk of less flexibility in the date $T-1$ hedging decision, or remaining only partly hedged on date $T-2$ and having greater flexibility on date $T-1$. The optimal choice for θ_{T-2}, shown by the solid line in Figure 2.2, is given by:

$$\theta_{T-2} = \begin{cases} -X_{T-2}/16 & X_{T-2} \leq 16/3 \\ -1/4 - X_{T-2}/64 & 16/3 \leq X_{T-2} \leq 48 \\ -1 & X_1 \geq 48 \end{cases} \tag{2.5}$$

For high $X_{T-2}(\geq 48)$, the firm can hedge totally on both dates since the maximum possible loss can be covered by the margin account (16 on date $T-1$ and 32 on date T). For low $X_{T-2}(\leq 16/3)$, the constraint $X_{T-1} \geq 0$ is binding. However, in the intermediate range, the firm hedges less than it can without violating next period's margin account constraint so that it can put a “reasonable” hedge in place on date 2. The dotted lines in Figure 2.2 show the two conditional hedges on date $T-1$ as a function of margin wealth on that date.

Figure 2.3 takes a closer look at the hedging decision in this intermediate range and its implications for θ_{T-1} under the two possible outcomes for F_{T-1}. Consider, as an illustration, the hedging decision with $X_{T-2} = 40$. The myopic choice would be to hedge completely on date $T-2$. In that case, the hedge ratios along the row corresponding to $X_{T-2} = 40$ in Figure 2.3 would read (-1, -0.75, -1), denoting that the firm would be able to hedge only 75% of its exposure if the futures price rises to 32. Another option is to ensure that the firm is fully hedged on the final date, in which case the same row would read (-0.5, -1, -1), so that the firm hedges only partly on date $T-2$ but fully on date $T-1$. The optimal hedging decision, (-0.88, -0.81, -1), is in between these two limiting cases in which the firm hedges only partly on date $T-2$ and anticipates being able to hedge only partly on date $T-1$ as well if the futures price rises.

Finally, I look at the effect of the borrowing constraint on the value function (or the indirect utility function) on date $T-2$. Suppose the utility function is given by $U(F_T, X_T) = \log(F_T + X_T)$. The value function for the unconstrained problem is given by $\log(16 + X_{T-2})$ since the firm can sell the asset on date T at a price of 16 with probability one due to complete hedging at each date. For the constrained case, the value function is defined in equation (2.3) with the optimal values of the hedges obtained in equations (2.4) and (2.5). Both of these are shown in Figure 2.4. The value function for the constrained problem is below that for the unconstrained problem because the latter has a larger feasible set of hedging strategies. For margin wealth above 48, the value functions are the same since the firm can hedge the spot exposure completely as was seen in equation (2.5). More interestingly, the borrowing constraint makes the firm more risk averse in that its indirect utility is more concave than its direct utility for low levels of margin wealth. Surprisingly, the higher the induced risk aversion due to the borrowing constraint, the less the firm hedges. This seemingly counter-intuitive result arises because the constraint introduces the risk that the firm might deplete its margin wealth and hence be unable to hedge its final period payoff.

3. The Model and its Assumptions

The intuition gained through the simple example can now be formalised. In this section, the framework for the dynamic hedging problem is established in a continuous-time setting. The unconstrained problem is solved analytically followed by a discussion of the implicit assumptions that underlie an unconstrained hedging policy. Thereafter, a borrowing constraint is imposed to derive the partial differential equation and the boundary conditions for the constrained hedging problem.

3.1. Unconstrained Hedging

Consider a firm that is choosing a dynamic hedging strategy using futures contracts to maximise the expected utility of its terminal wealth.

Assumption 1: The firm owns π units of a single asset until a future date T at which time the asset is sold on the spot market. There is no uncertainty about the quantity. The spot price of this asset is S_t, which follows a geometric Brownian motion with drift equal to the constant riskless interest rate, $r(\geq 0)$:

$$\frac{dS_t}{S_t} = rdt + \sigma dZ_t \tag{3.1}$$

σ is the constant standard deviation of relative changes in the spot price and dZ_t is the increment of a standard Wiener process.

Assumption 2: Futures contracts with maturity T are available. The price of the futures contract, denoted by F_t, is that price which makes the futures contract valueless. Thus it is simply the price which, when discounted to the present, equals the value of the underlying asset (see Ross (1997)), giving the futures price dynamics:

$$\frac{dF_t}{F_t} = \sigma dZ_t \tag{3.2}$$

Assumption 3: A futures position is undertaken by buying or selling futures contracts in accordance with the process θ_t. All futures price changes are credited (or debited) to a margin account whose level is denoted by X_t and which earns interest at the constant riskless interest rate. Any shortfalls in the margin account can be replenished by borrowing without limit at the same interest rate. Hence the level of the margin account evolves according to the dynamics

$$dX_t = rX_t dt + \theta_t dF_t \tag{3.3}$$

There are no transactions costs and the futures contracts are infinitely divisible.

Assumption 4: The objective of the firm is to choose the number of futures contracts θ_t so as to maximise the expected utility of terminal wealth, which is composed of the spot portfolio, πS_T, and the margin account wealth, X_T. But since the futures contract matures at time T as well, the terminal wealth may also be written as $X_T + \pi F_T$. The firm has a utility function that exhibits constant relative risk aversion equal to $1 - \gamma$. Hence at any time $t (< T)$ the firm solves

$$\max_{\{\theta_t\}} \mathbb{E}_t[\frac{(X_T + \pi F_T)^\gamma}{\gamma}]; \qquad 0 < \gamma < 1 \tag{3.4}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on the information at time t.

Assumptions 1-4 lay out what can be called the *unconstrained* hedging problem since there are no constraints on the margin account. Let $V(x, f, t)$ denote the value function for this problem where $x = X_t$ and $f = F_t$. Then

$$V(x, f, t) \equiv \max_{\{\theta_t\}} \mathbb{E}_t[\frac{(X_T + \pi F_T)^\gamma}{\gamma}] \tag{3.5}$$

Proposition 1. In the absence of any borrowing constraints, the optimal number of futures contracts to be held is

$$\theta_t = -\pi \exp(-r(T - t)) \tag{3.6}$$

The value function for the unconstrained problem is given by:

$$V(x, f, t) = \frac{(xe^{r(T-t)} + \pi f)^\gamma}{\gamma} \tag{3.7}$$

Proof: See Appendix.

This is simply the continuous-time version of the solution derived by Anderson and Danthine (1983). If the interest rate on the margin account were zero, it is easy to see that a perfect hedge could be obtained by short-selling π (or buying $-\pi$) futures contracts. With a non-zero interest rate, the optimal hedge is just the present value of this quantity, thus obtaining the discount factor $\exp(-r(T - t))$. This solution for θ_t will be referred to as a *total hedge* since it provides a complete hedge for the spot price exposure. It may be noted that the total hedge does not depend on the level of the margin account. This is because any shortfalls in the margin account can be met by borrowing at the same riskless rate that the margin account earns when it has a positive balance. Moreover, it implicitly assumes that the firm can borrow without limit to fulfil margin calls.

What is the likelihood that a firm following a total hedging strategy might need to borrow at some time before the maturity of its spot position to fulfil margin calls? Figure 3.1 presents simulation results for some typical values. The results are shown for different levels of the margin account as a fraction of the total exposure at the current futures price. The interest rate on the margin account is 5% and the probabilities are calculated for two different values of volatility of the futures contract: 15% and 20% per annum. These results may also be interpreted as the probability that a firm following a total hedging strategy will be unhedged on the date of maturity of its spot position, given that it has no recourse to borrowing to fulfil margin calls. The initial margin on futures contracts typically varies between 1% and 10% of the size of the total exposure depending upon the volatility of the underlying asset. Suppose the initial margin requirement is 5%. With 15% volatility in futures prices, a firm which can only put up its initial margin would be unhedged with 88% probability a year hence when its spot position matures. Even a firm that faces a total exposure of $1 million and has $0.5 million available to fulfil margin account obligations will be forced to borrow with a 28% probability before maturity in a year's time to keep its hedge intact. The probabilities are even higher if the volatility is 20% per annum. Starting with as much wealth in the margin account as the total exposure still leaves a probability of 11% of having to borrow additional amounts to meet margin obligations before maturity a year later.

One way to meet margin account obligations is to generate sufficient liquidity within the firm. However, this is expensive because funds have to be allocated from elsewhere in the firm, thus incurring the opportunity cost of forgone lucrative projects[4]. Turning to external sources seems more likely but this can be difficult as well. This is because a creditor might not understand what the firm is doing. Also, external observers can fail to distinguish between a hedging position and a speculative one. It might be argued that cash losses on the futures position are matched by an increase in the value of the spot contract, so the firm is essentially

[4] See Froot et al (1993).

facing only a short-run liquidity crunch. Consequently the firm should be able to convince its creditors to extend the necessary support. But the panic set off by a string of losses on the futures position can often cause lenders to revoke liquidity support provisions at a time when they are most needed.[5] Thus external borrowing, even when available, is expensive and limited. Relying on large and frequent doses of additional credit either from within or outside the firm seems unrealistic. Moreover, it does seem unreasonable to ask a firm to have to make recurrent, expensive forays into credit markets simply to keep its hedging position intact when the whole objective behind hedging is to avoid this in the first place.

3.2. Constrained Hedging

The constrained dynamic hedging program incorporates the risk of unexpectedly large margin calls in a simplified manner by imposing the following constraint:

Assumption 5: The firm starts at a given level of the margin account X_0 and is required to maintain a balance in the margin account above a predetermined level $K(>0)$ till maturity with no recourse to additional borrowing. Hence, the maximisation in (3.4) is subject to the constraint

$$X_t \geq K \text{ for all } t \leq T \tag{3.8}$$

This assumption is not as restrictive as it might seem. It is clear that no firm would be willing to borrow infinite amounts simply to keep a hedging position in place. The extent to which the firm can borrow can be thought of as incorporated in the initial level of the margin account, X_0. It is given that $X_0 > K$.

Stochastic dynamic programming is used to solve the constrained problem. The state of the system at any time $t(\leq T)$ is completely denoted by the vector (x, f, t) where $x = X_t$ and $f = F_t$ at time t. Let $J(x, f, t)$ be the value function for the following problem:

$$J(x, f, t) \equiv \max_{\{\theta_t\}} \mathbb{E}_t[\frac{(X_T + \pi F_T)^\gamma}{\gamma}] \tag{3.9}$$

[5] Indeed, in the case of Metallgesellschaft, the initial reports were that the firm's subsidiary had been "betting" on the futures market. Culp and Hanke (1994) report: " ... four major European banks called in their outstanding loans to MGRM [the oil trading affiliate of Metallgesellschaft] when its problems became public in December 1993. Those loans, which the banks had previously rolled-over each month, denied MGRM much needed cash to finance its variation margin payments and exacerbated its liquidity problems."

subject to the constraint in equation (3.8) and the futures price process specified in equation (3.2). The dynamics for X_t is slightly modified to reflect that only balances exceeding K earn the interest at rate r:

$$dX_t = r(X_t - K)dt + \theta_t dF_t \tag{3.10}$$

The Hamilton-Jacobi-Bellman equation for this stochastic control problem is given by:

$$-\frac{\partial J}{\partial t} = \max_{\{\theta\}}[r(X-K)J_X + \frac{\sigma^2 F^2}{2}\{\theta^2 J_{XX} + 2\theta J_{XF} + J_{FF}\}] \tag{3.11}$$

where the subscripts denote partial derivatives. The necessary first-order condition of optimality is:

$$\theta = -\frac{J_{XF}}{J_{XX}} \tag{3.12}$$

which is also sufficient if $J(\cdot)$ is concave in (x, f).[6] Substituting this into equation (3.11) yields the partial differential equation:

$$0 = \frac{\partial J}{\partial t} + r(X-K)J_X + \frac{\sigma^2 F^2}{2}\{-\frac{J_{XF}^2}{J_{XX}} + J_{FF}\} \tag{3.13}$$

Proposition 2. If the margin account falls to K at any time before maturity, the firm is forced to liquidate its futures position and remains unhedged till maturity, ie

$$\text{If } X_s = K, \text{ then } \theta_t = 0 \text{ for } s \leq t < T.$$

Proof: From the dynamics specified in equation (3.10), once the margin account hits the level K, it earns no interest. Any non-zero futures position at this time can violate the constraint $X_t \geq K$ with positive probability. Hence it is necessary for the firm to liquidate its futures position at time s. Since this makes K an absorbing state for the X-process, θ_t remains at zero till maturity. □

The result in Proposition 2 provides one of the two boundary conditions for equation (3.13). Since the firm remains unhedged till maturity once the margin account falls to K,

$$J(K, f, t) = \mathbb{E}_t[\frac{(K + \pi F_T)^\gamma}{\gamma}] \tag{3.14}$$

[6] Once a candidate numerical solution has been found, it can be verified that this condition is indeed satisfied.

The second boundary condition is simply the value function at maturity which is equal to the utility function at T.

$$J(x, f, T; K) = \frac{(x + \pi f)^\gamma}{\gamma} \tag{3.15}$$

4. Solution of the Partial Differential Equation

The non-linear partial differential equation (PDE) in equation (3.13) is generally hard to solve. To the best of my knowledge, an analytical solution to this equation does not exist. I therefore use a numerical procedure based on the simultaneous Method of Lines. This section lays out certain salient features of the solution procedure.

A large number of problems in economics and finance do not have analytic solutions. For even simple problems in asset pricing, the PDE derived from the Hamilton-Jacobi-Bellman equation needs to be solved using numerical methods (for example, the American put option with dividends). Much progress has been made in solving them faster and to a greater degree of accuracy. In a large number of these asset pricing models, a linear pricing argument is invoked to preclude arbitrage, resulting in a PDE which is linear as well. In contrast, the optimal hedging problem is a stochastic control problem and the PDE obtained in equation (3.13) is non-linear due to the non-linearity of the optimal control θ_t in equation (3.12). Such PDEs would arise naturally in the derivation of optimal hedging or trading strategies (for example, portfolio insurance strategies). A second source of complexity for the problem is the imposition of the borrowing constraint on the margin account, which induces path dependencies on one of the two state variables.[7] Finally, while the numerical solution of the PDE will provide the value function, the main object of interest is the optimal control, which is only known as a function of the derivatives of the value function. In this model, the control (equation (3.12)) of the constrained hedging problem involves the ratio of two second-order derivatives of the value function. Obtaining a solution to the PDE that is smooth enough, so that the numerical derivatives obtained by the finite differencing of the value function are meaningful, is particularly challenging.

Three remaining boundary conditions for the PDE in equation (3.13) corresponding to $f = 0$, $f = \infty$ and $x = \infty$ are easily specified. The first uses the property that $f = 0$ is an

[7] A model with transactions costs is another example where path dependencies are present. Lo (1996) solves a PDE with dependencies to derive the optimal control of execution cost of buy or sell orders over a finite horizon.

absorbing state for the futures price process. The latter two use the analytic solution for the unconstrained problem obtained in equation (3.7).

The domain for x in the original problem is $[K, \infty]$. Setting $K = 0$, the infinite domain for x is mapped to $[1, 0]$ using the mapping $y = -\exp(-\lambda_1 x)$. Since the objective is to explore the hedging strategy for low levels of margin account wealth, the region of greatest interest is close to the boundary $x = 0$ (ie $y = -1$). Therefore the N grid points are equally spaced in the interval $[-1, 0]$ so that the exponential transform causes the step size in the x-domain to be progressively smaller as the boundary is approached. This is particularly useful because it is in this region that the borrowing constraint induces the most curvature on the value function. A similar transformation is performed for the other state variable f so that $z = -\exp(-\lambda_2 f)$ and the M grid points are equally spaced in $[-1, 0]$. The third state variable, t , enters the value function only in the form $(T - t)$, which is the remaining time to maturity of the spot position (and the futures contract). Let τ denote the time to maturity. The transformed PDE, now having the state vector (y, z, t), is

$$J_\tau = ry\log(-y)J_y + \frac{(\sigma\log(-z))^2}{2}[-\frac{y(zJ_{yz})^2}{yJ_{yy} + J_y} + z^2 J_{zz} + zJ_z] \tag{4.1}$$

where $J(\cdot)$ is defined over the finite domain $[-1, 0] \times [-1, 0] \times [0, T]$ and the boundary conditions are recast in terms of the new state vector. For a given value of τ, let any point in the (y, z) plane be denoted by (y_i, z_j) and let $G_{ij}(\tau) \equiv J(y_i, z_j, \tau)$. Replacing the partial derivatives with respect to y and z on the right hand side of equation (4.1) by their symmetric finite difference representations, the following reduced form equation in τ is obtained for each of the $(N - 2) \times (M - 2)$ points in the (y, z) plane

$$\frac{dG_{ij}(\tau)}{d\tau} = \Phi(y_l, z_{k,} G_{lk}(\tau); \{l\}_{i-1}^{i+1}; \{k\}_{j-1}^{j+1}) \quad i = 2, .., N-1; j = 2, .., M-1 \tag{4.2}$$

This is a non-linear ordinary differential equation (ODE) in τ with the boundary condition

$$G_{ij}(0) = \frac{(-\pi\log(-z_j)/\lambda_2 - \log(-y_i)/\lambda_1)^\gamma}{\gamma} \quad i = 2, .., N-1; j = 2, .., M-1 \tag{4.3}$$

The values of $G_{ij}(\cdot)$ along the edges of the (y, z) plane are already provided (also as functions of τ) by the boundary conditions corresponding to $y \in \{-1, 0\}$ and $z \in \{-1, 0\}$. Setting $\pi = 1$ avoids a singularity at the point $(0, 0)$, which corresponds to $(x = \infty, s = \infty)$, by ensuring that the limit in G_{NM} is approached at the same velocity from both the y and z directions.

Thus I now have a system of $(N-2) \times (M-2)$ simultaneous ODEs (and as many boundary conditions) in τ which can be solved using the wide variety of ODE solvers available. Two points require special attention in this final step. First, the symmetric finite difference approximation of the derivatives makes the ODE in equation (4.2) at (i, j) only a function of the G at that point and the eight surrounding points. This results in the Jacobian, for the system of ODEs, having a tri-diagonal banded structure of width three. This sparsity of the Jacobian is exploited to speed up the computations dramatically. Secondly, while equation (4.2) is an ODE, it is still non-linear. Most ODE solvers use linearly implicit formulas ie the computation of the value of solution at the next step requires the solution of a system of linear equations. However, the non-linearity of the ODE causes the coefficient of $dG_{ij}/d\tau$ to depend on τ in the linear solution scheme, making the system implicitly "stiff".[8] In view of this stiffness in the problem, an ODE solver provided by Shampine and Reichelt (1995) is used that can successfully tackle such situations.[9]

5. Results and Applications

The parameter values used for the main solution are as follows:

Number of spot market contracts owned by the firm, $\pi = 1$

Hedging horizon, $T = 52$ weeks (1 year)

Volatility of the futures prices = 15% per annum

Lower bound on margin wealth, $K = 0$

Coefficient of relative risk aversion, $1 - \gamma = 1/2$

Riskless interest rate, $r = 5\%$ per annum

Since the firm has a long position in the spot market ($\pi = 1$), the optimal hedge is a short position in the futures contract. However, for ease of exposition in the discussion that follows, θ_t refers to the magnitude of the futures position only. Further, all the results are reported as a function of $X_t/\pi F_t$, the ratio of the current level of the margin account to the value of the total exposure at the current futures price. Due to the scaling properties of power utility and geometric Brownian motion, the optimal control is only a function of this ratio.

[8] The general form of a stiff problem is $M(t)dy/dt = f(t, y)$.

[9] The essential feature of stiff solvers is that they avoid local extrapolation. Even one-step interpolation can be unsatisfactory if the problem is very stiff. The ODE solver used here makes a second-order local approximation for the interpolant. The solver may be found at the URL: http:\\www.mathworks.com

Figure 5.1 shows the optimal futures position as a function of the time to maturity in weeks and the ratio of margin wealth to total exposure. It is obvious that at (or an instant before) maturity, the optimal hedge ratio is one for all levels of the margin account since the firm can afford to hedge completely. Before maturity, no hedging is possible if the margin wealth is zero irrespective of the time that remains to maturity. This is in accordance with Proposition 2. As the margin wealth increases, the optimal hedge ratio approaches the total hedge (corresponding to the unconstrained problem) asymptotically. This is because a firm which is endowed with a very high (infinite) level of margin wealth faces (in the limit) negligible risk that the margin account will dwindle down to zero before maturity due to large margin calls. Hence it can adopt the hedging policy obtained for the unconstrained problem.

5.1. Margin Wealth

The solution is more interesting for intermediate levels of the margin wealth. Figure 5.2 shows the relation between the hedge ratio and the margin wealth-exposure ratio for three different hedging horizons. This corresponds to three different sections, parallel to the margin account/exposure axis, of the surface in Figure 5.1 for the three maturities. It is seen that departure from the total hedge is significant even when the margin level is substantial in comparison to the size of the futures position. Thus if a firm holds a million dollars' worth of futures contracts to hedge a spot position that would mature one year hence, and has half a million dollars to fulfil margin account obligations, it would hedge only 61% of its exposure. This is well below the unconstrained hedging level of 95% (Proposition 1) if it could borrow unlimited amounts at an interest rate of 5% per annum to fulfil any margin calls.[10] The intuition is that the firm faces a choice between either hedging less than the total exposure and bearing a fraction of the spot price risk at this time, or hedging out all of the spot price risk at this time but taking on the risk that it might be forced to liquidate its hedge at some time within the next year. The latter could force the firm to bear the risk of spot exposure in its totality at maturity. Even with a margin account level as large as the exposure, the hedge ratio is only 0.82.

These results are for an annual volatility of 15% in futures prices. The departure from

[10] It should be noted that for the case where the futures price is not perfectly correlated with the spot price, even the unconstrained hedge is below $\exp(-r(T-t))$ and the constrained hedge will be accordingly lower.

unconstrained hedging is even more if the volatility is 20%. Figure 5.3 shows the optimal hedge for three different values of futures price volatility. A volatility of 20% per annum makes the hedge almost half the exposure if the time to maturity is one year and the margin wealth is half the total exposure.

In a recent paper that provides empirical evidence on the determinants of corporate hedging policy, Mian (1996) finds that the only feature that distinguishes hedging firms from non-hedging firms is firm size. All the other factors that are usually put forth as determinants of hedging like costs of financial distress, contracting costs, capital market imperfections and taxes, seem to have no influence on a firm's decision to hedge or not to hedge. The results obtained above suggest an explanation for these findings. I have shown that the amount of liquid cash required to keep a hedging position is substantial relative to the size of the exposure. Firm size may be seen as a proxy for the extent to which a firm has access to sufficient liquidity to fulfil margin account obligations. Large firms can generate additional cash either internally or approach external capital markets with relatively greater ease and on better terms than small firms.

A similar argument explains the limited participation of farmers in agricultural futures markets. Surveys show that only about 2 to 13% of farmers hedge the exposure of their agricultural produce. Most farmers take on considerable debt at the beginning of the cropping season. Thus their ability to raise additional liquidity to fulfil potentially large margin calls from a futures hedging position is severely restricted.

5.2. Time to Maturity

Figure 5.4 shows the optimal hedge ratio as a function of the time to maturity of the spot position for different levels of the margin wealth-exposure ratio. The line marked '∞' corresponds to the unconstrained hedge as discussed in Proposition 1. For any given level of the margin wealth per unit exposure, the hedge ratio approaches the total hedge as the firm comes closer to maturity. This is because as the horizon shortens, the probability of running out of the margin wealth declines as well. Thus the spot price exposure can be hedged to a greater extent. Farther away from maturity, the optimal hedge declines sharply, especially if the level of margin wealth relative to the exposure is small. These results seem to suggest an explanation for the high open interest in short maturity futures compared to longer maturity futures contracts, a phenomenon prevalent across almost all financial and commodity futures markets.

6. Conclusion

I have shown that the risk emanating from unexpectedly large margin calls due to a hedging position in futures contracts is substantial. Hence I have considered a firm that is constrained in its ability to borrow for the purpose of fulfilling margin account obligations. It is found that the constrained optimal hedge is substantially lower than the unconstrained hedge. The optimal hedge approaches the unconstrained hedge as the contract advances towards maturity and the probability of the constraint becoming active in the future declines. These results provide theoretical support for the relatively low level of corporate and agricultural hedging that has been documented in the literature. They also suggest an explanation for the much higher open interest in short maturity futures contracts relative to longer maturity contracts. The model developed here could also be used to assess the incidence and effectiveness of the use of margin requirements to control the volume of trade in futures markets.

Though marking-to-market is an institutional feature of only exchange-traded derivative contracts, the use of collateral to mitigate counterparty and credit concerns is increasing in other transactions as well. This increased use of collateral has allowed the OTC market to expand beyond just market participants with high credit ratings. The International Swaps and Derivatives Association (1999) estimates that the total value of collateral in circulation across the privately negotiated derivatives industry in 1998 to be around \$175 to \$200 billion. The revaluation and remargining of collateralised positions along with the ability to raise collateral at short notice can be a significant source of liquidity pressure for counterparties in OTC transactions, as has been pointed out by the Committee on Global Financial System (2001).

Periodic marking-to market is an essential tool of risk management for users of all types of derivative contracts, exchange-traded or not. Indeed, a survey by the Group of Thirty (1994) showed that 99% of dealers and 91% of end-users of derivative contracts mark their derivative positions to market, most of them on a daily basis. However, mere marking-to-market does not lower the risk of large margin calls. The overall objective of risk management is fulfilled only if the users of these contracts can modify their derivative positions in response to this (internal or external) daily settlement procedure. To the extent that such positions have been assumed with a hedging objective, the methodology of this paper can be extended to other hedging instruments as well.

The main motivation behind hedging is a reduction in the volatility of future cash flows. However, this paper has shown that pursuit of the limited objective of hedging the spot exposure can often lead to more volatile cash flows. The hedging problem must be recast to include cash flows that result from the hedging position as well. Thus a hedger must optimise not only over the extent to which an exposure should be hedged but also over the choice of available hedging instruments, incorporating the inherent risks that their use entails. This paper has taken a first step in this direction by incorporating the daily settlement feature into the hedging decision. Further exploration of these issues constitutes an interesting area for future research.

Appendix

The value function for the unconstrained hedging problem is:

$$J(X_t, F_t, t) = \frac{(X_t + \pi F_t)^\gamma}{\gamma}$$

and the optimal number of futures to be held at time t is

$$\theta_t = -\pi \exp(-r(T-t))$$

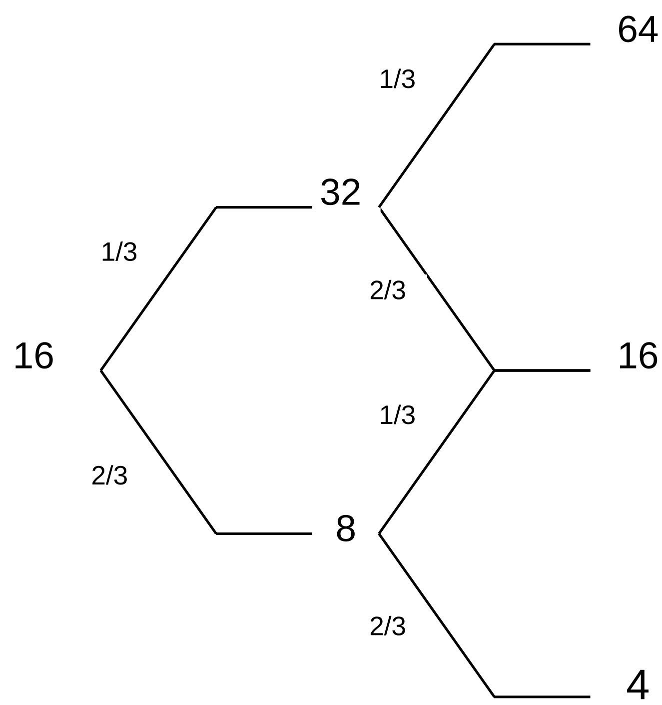
Proof: Since I already have a candidate solution, I can directly use the verification theorem of dynamic programming. It can be checked by direct substitution that the proposed solution satisfies the partial differential equation in equation (3.13) and the boundary condition

$$J(X_T, F_T, T) = \frac{(X_T + \pi F_T)^\gamma}{\gamma}$$

Hence it is the solution to the unconstrained problem. □

Figure 2.1

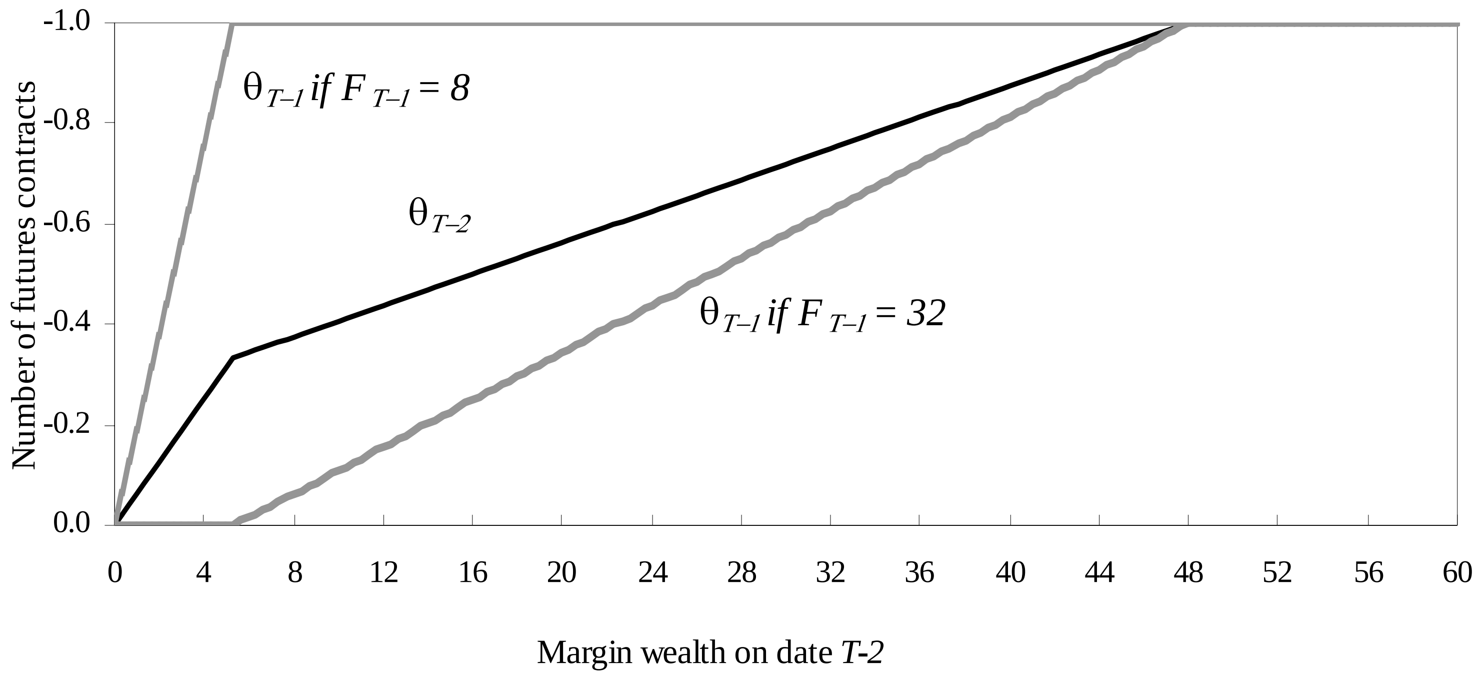
The futures price process



The binomial tree shows the futures price process. The price on date *T-2* is 16. The price on each successive date either doubles with probability 1/3 or becomes half the current price with probability 2/3. The process has zero drift and the riskless rate is zero.

Figure 2.2

Optimal hedging positions



The graph shows the optimal futures position to hedge one unit of the spot asset to be received on date T as a function of the margin wealth on date T-2. The solid black line is the hedge on date T-2 and the grey lines show the hedge on date T-1 conditional on the realisation of the futures price.

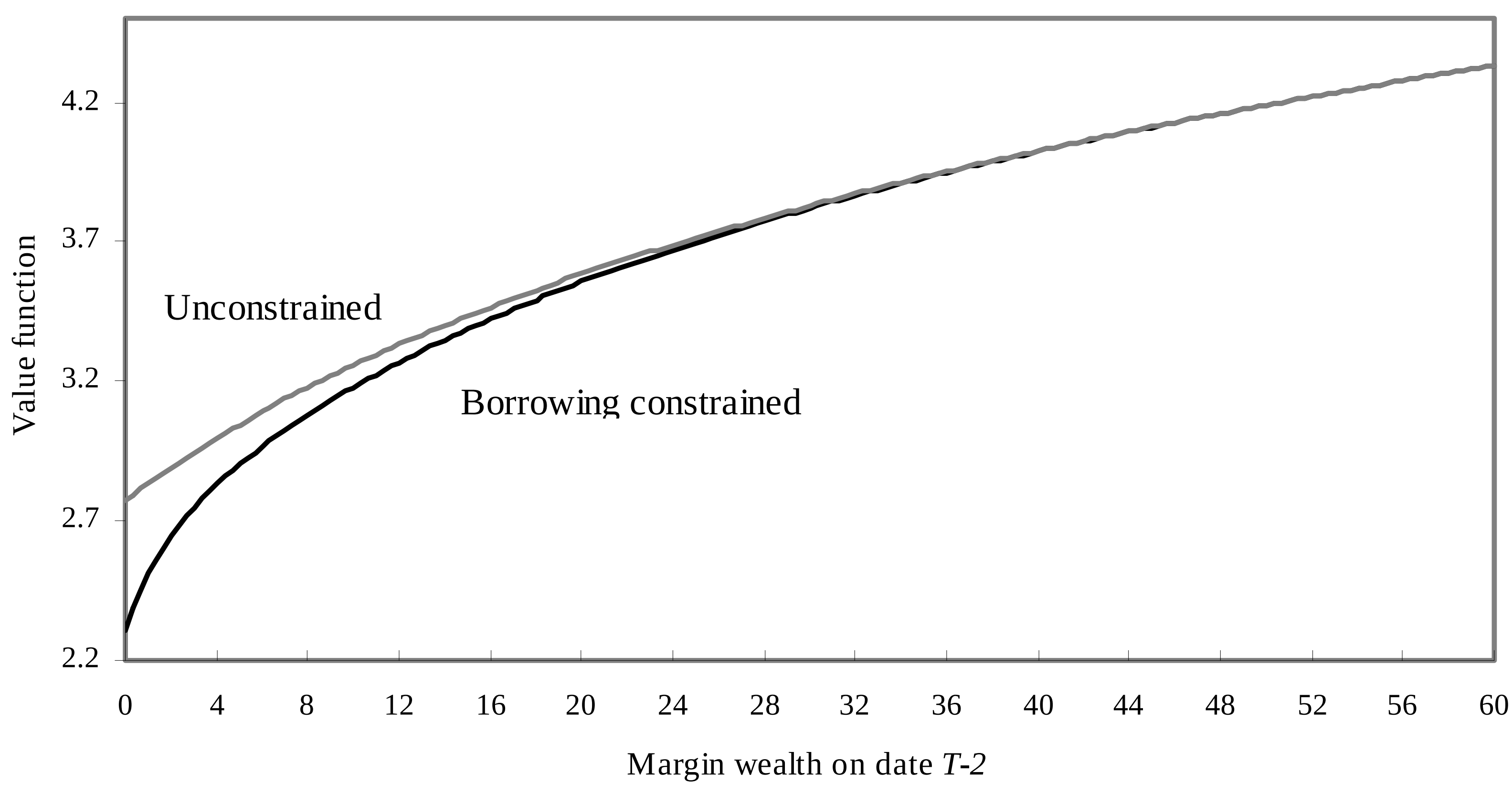
Figure 2.3

Conditional hedging positions

Margin wealth on date *t-2* (X_{T-2})	**Hedge ratio on date *t-2* (θ_{T-2})**	**Hedge ratio on date *t-1* (θ_{T-1})**	
		if $F_{T-1} = 32$	**if $F_{T-1} = 8$**
5	-0.31	0.00	-0.94
10	-0.41	-0.11	-1.00
15	-0.48	-0.23	-1.00
20	-0.56	-0.34	-1.00
25	-0.64	-0.46	-1.00
30	-0.72	-0.58	-1.00
35	-0.80	-0.70	-1.00
40	-0.88	-0.81	-1.00
45	-0.95	-0.93	-1.00
50	-1.00	-1.00	-1.00

This table shows the optimal futures position to hedge one unit of the spot asset to be received on date *T* as a function of the margin wealth on date *T-2*. The second column has the hedge ratios on date *T-2* and the third and fourth columns show the hedge on date *T-1* conditional on the realisation of the futures price.

Figure 2.4
Constraint-induced concavification of the value function



The graph shows the value function for the unconstrained (grey line) and the constrained (solid black line) hedging problems as a function of the wealth on date T-2. The borrowing constraint causes a "concavification" of the value function, ie the firm behaves in a more risk averse manner due to the constraint. The utility function is given by *U(w)* = log*(w)*.

Figure 3.1

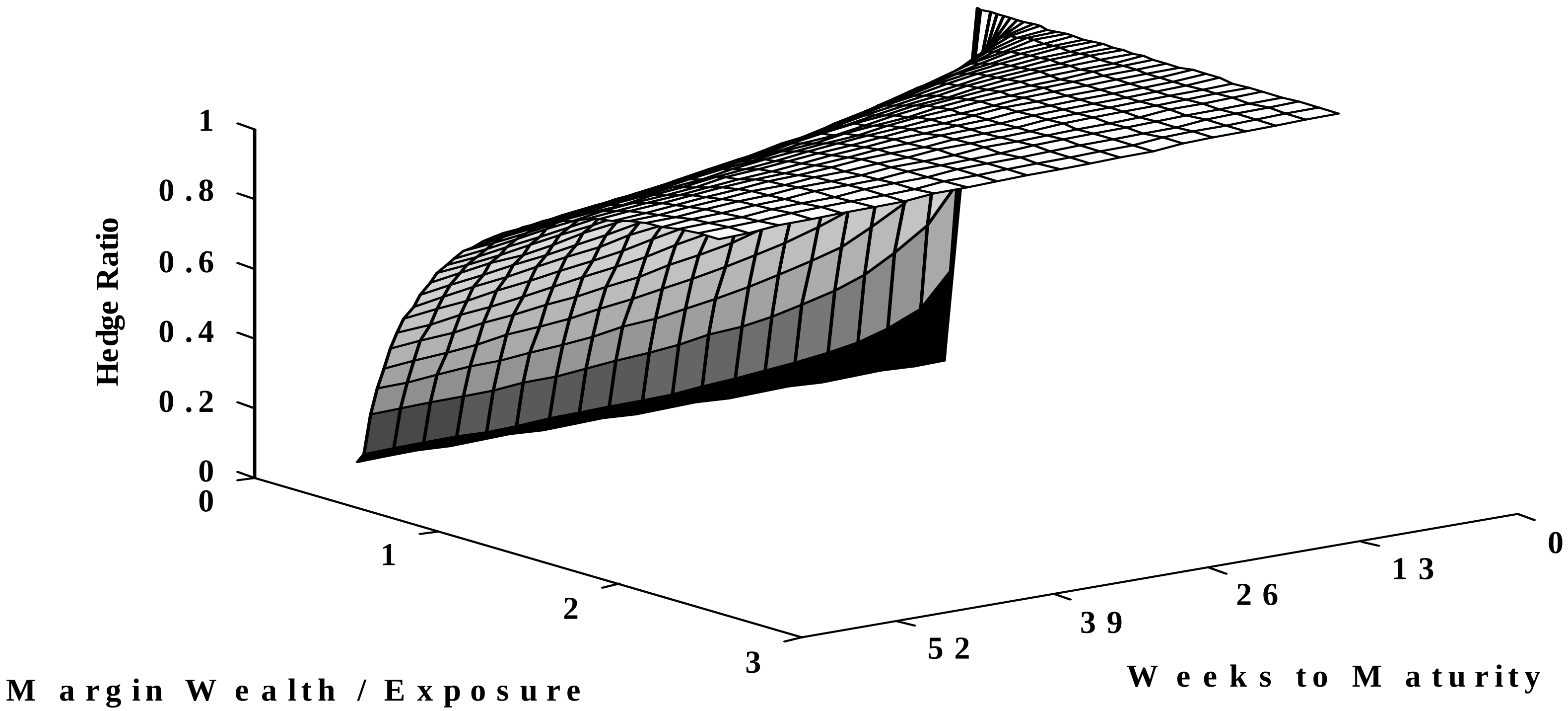
Probability of hitting the borrowing constraint before maturity

Volatility (per annum)	**Margin wealth / Exposure (*X* / *F*)**	**Probability [$X_t < 0$] before maturity (per cent)**		
		τ = 13 weeks	***τ = 26 weeks***	***τ = 52 weeks***
15 %	0.05	76	84	88
	0.10	59	71	79
	0.25	23	39	53
	0.50	3	12	28
	1.00	0	0	6
20 %	0.05	78	85	89
	0.10	63	73	81
	0.25	29	46	60
	0.50	6	18	34
	1.00	2	2	11

This table shows the probability that total hedging of the spot position will result in a depletion of the margin wealth before maturity. The margin wealth is expressed as a fraction of the current value of the futures position. τ is the time remaining to maturity. The riskless rate is 5% per annum. Probabilities estimated from 5,000 simulations.

Figure 5.1

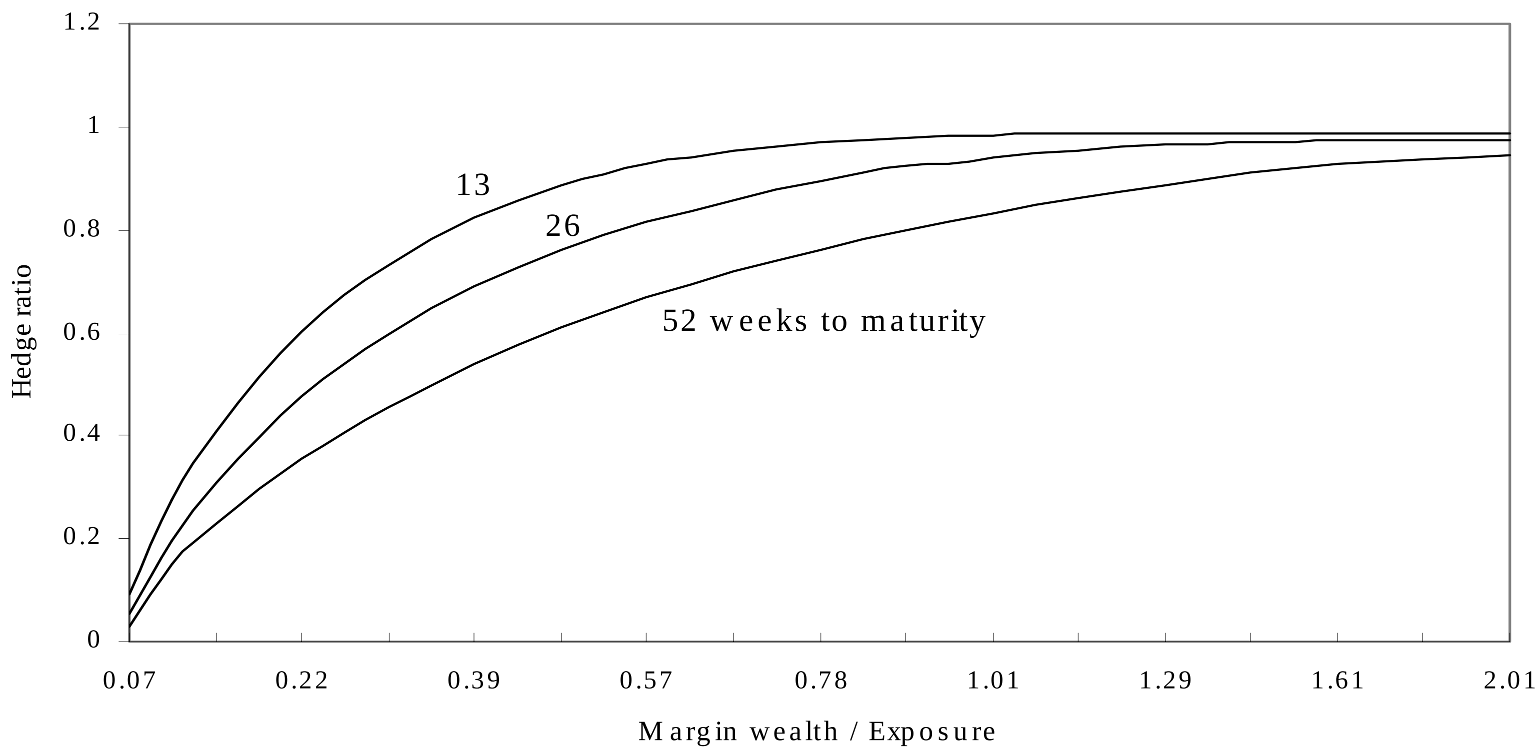
Optimal hedge ratios



This graph shows the optimal hedge for a firm that holds one unit of the spot asset. The optimal hedge is shown as a function of the margin wealth-exposure *(X/πF)* ratio for maturities extending up to one year. These values are calculated by solving the Bellman equation for the constrained problem numerically. Annual futures price volatility is 15%, the riskless rate is 5%, and the firm maximises CRRA utility of final period wealth with relative risk aversion of 0.5.

Figure 5.2

Effect of margin wealth



The graph shows how the the optimal hedge changes with the margin wealth-exposure (X/pF) ratio for three different times to maturity. The values are calculated for 15% annual volatility in futures prices and a riskless rate of 5% pa. The utility function is CRRA with relative risk aversion of 0.5.

Figure 5.3

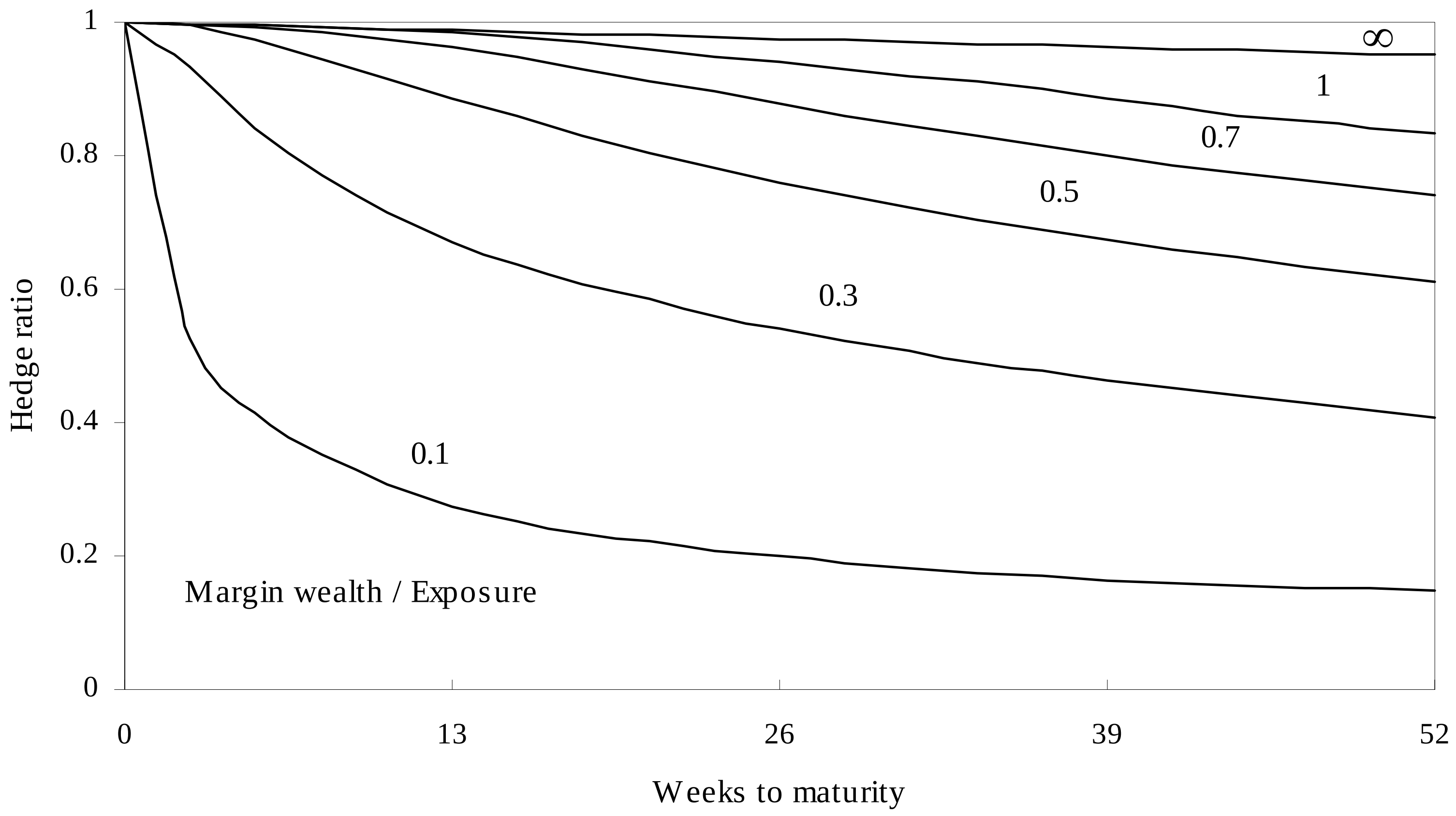
Effect of futures price volatility

Margin Wealth/ Exposure ($X / \pi F$)	Volatility (per annum)	Hedge Ratio (θ_t)		
		τ = *13 weeks*	τ = *26 weeks*	τ = *52 weeks*
1/2	10 %	0.93	0.80	0.69
	15 %	0.89	0.72	0.61
	20 %	0.84	0.67	0.53
1	10 %	0.99	0.95	0.89
	15 %	0.98	0.92	0.83
	20 %	0.98	0.88	0.79

This table shows the optimal hedge ratio for different volatilities as a function of the margin wealth-exposure $(X/\pi F)$ ratio and time to maturity of the spot position. The results are for CRRA utility with relative risk aversion of 0.5. The riskless rate is 5% per annum.

Figure 5.4

Effect of maturity



The graph shows how the optimal hedge changes with the time remaining to maturity of the spot position that is being hedged, for different values of the margin wealth-exposure *(X/πF)* ratio. The values are calculated for 15% annual volatility in futures prices and a riskless rate of 5% per annum. The utility function is CRRA with relative risk aversion of 0.5.

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