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Theory of supply chains: a working capital approach*

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Abstract

This paper presents a "time-to-build" theory of supply chains which implies a key role for the financing of working capital as a determinant of supply chain length. We apply our theory to global value chains (GVCs) and trade, where firms strike a balance between the productivity gain from longer GVCs against the greater financial cost due to longer supply chains. In equilibrium, there is a duality between real activity and financial conditions, where more accommodative financial conditions are associated with higher output.

JEL codes: F23, F36, G15, G21, L23
Keywords: global value chains, global liquidity, trade finance

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1 Introduction

Production takes time. Working capital in the form of inventories and receivables bridges the timing mismatch between incurring costs and realizing cash flow from sales. Inventories and receivables are assets of the firm and are financed by counterparts on the liabilities side of the balance sheet.

We propose a theory of supply chains based on the insight that the length of the supply chain formed by a group of firms determines the aggregate level of working capital needed to sustain the supply chain. Longer supply chains imply greater financing needs, but may bring real economic benefits such as higher productivity. Financing conditions then determine the terms of the tradeoff between real and financial considerations. As financial conditions fluctuate, the optimal investment in working capital will also fluctuate. Looser financing conditions lengthen optimal supply chains and increase the demand for working capital. Conversely, when financial conditions tighten, there is a corresponding contraction of working capital as firms choose a shorter horizon for their cashflows, entailing a contraction of working capital. In this sense, there is a duality between financing conditions and real economic outcomes. This duality introduces an important channel for financial conditions to affect real economic outcomes.

Figure 1 illustrates this relationship between working capital and financial conditions in a vivid way. The panels in Figure 1 show the median annual growth rate of accounts receivable for a balanced global panel of manufacturing firms from Capital IQ together with various measures of financial conditions.

The top left panel plots the median annual growth of accounts receivable with the annual change of the Goldman Sachs Financial Conditions Index (FCI). Higher FCI values indicate tighter financial conditions. The top right panel shows the scatter chart of this relationship. We see that tighter financial conditions are associated with a contraction in accounts receivable. The middle row of panels plots the co-movement of accounts receivable growth with the growth of dollar-denominated credit to non-banks outside the United States from the BIS global liquidity indicator (GLI) statistics. We see that

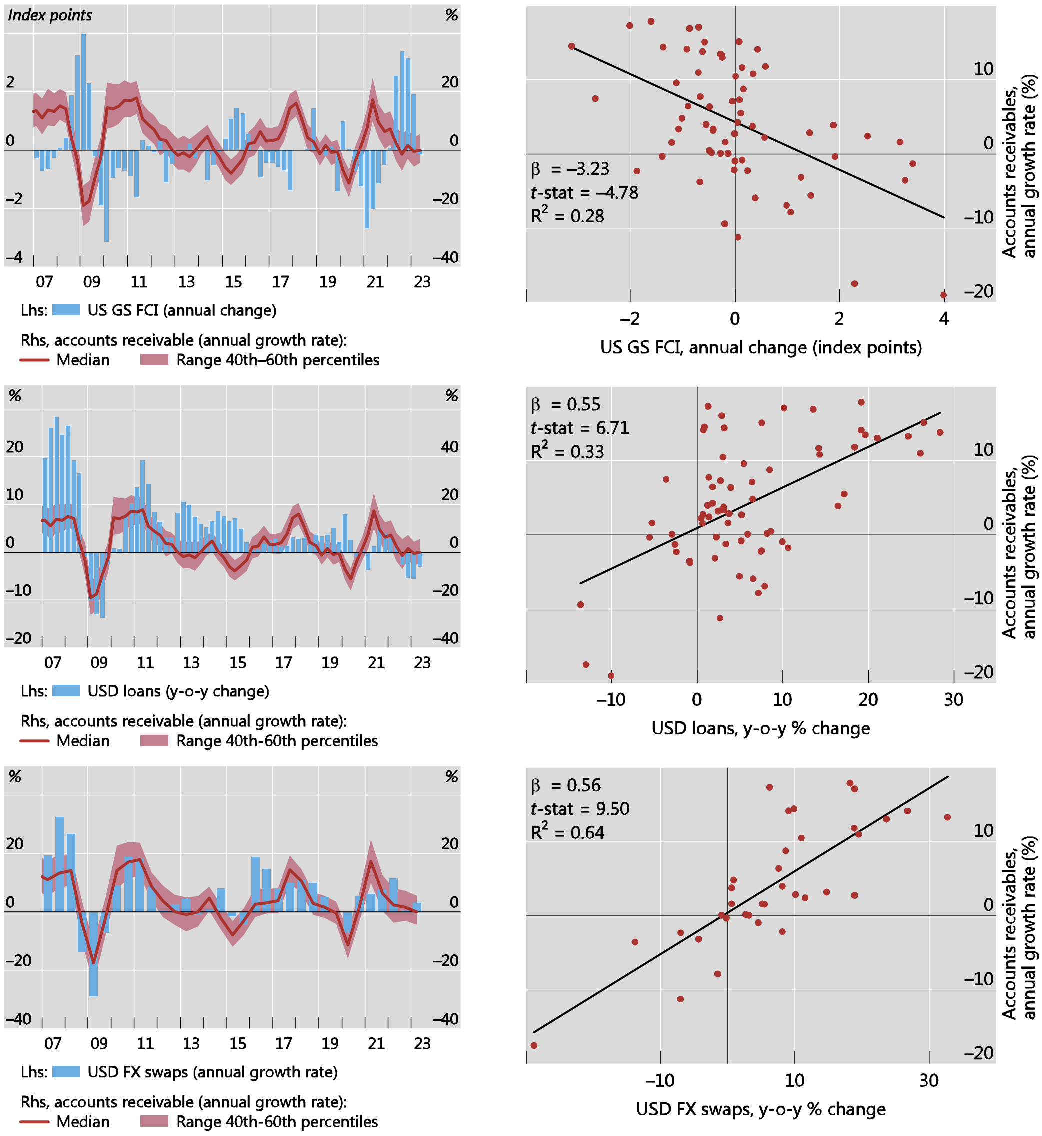

Figure 1. **Working capital and financial conditions.** These panels show the median annual growth rate of accounts receivable for a balanced global panel of manufacturing firms from Capital IQ together with measures of financial conditions. The top left panel plots the median annual growth of accounts receivable with the annual change of the Goldman Sachs Financial Conditions Index (FCI). Higher FCI values indicate tighter financial conditions. The top right panel shows the scatter chart of this relationship. The middle row plots the accounts receivable growth with the growth of dollar-denominated credit to non-banks outside the United States (from BIS GLI statistics). The bottom panels show accounts receivables growth with the annual growth of dollar FX swaps outstanding (from the BIS OTC derivatives statistics). In all three instances more accommodative financial conditions are associated with higher accounts receivable growth.

accounts receivable growth co-move with global dollar credit growth. Finally, the bottom two panels show the co-movement of accounts receivables growth with the annual growth of dollar FX swaps outstanding (from the BIS OTC derivatives statistics). In all three instances more accommodative financial conditions are consistently associated with more expansive accounts receivable.

Our theory of supply chains has a distinctively 'Austrian' flavor, in that longer supply chains are associated with higher productivity and output, so that the optimal investment in working capital considers the time dimension of production. The 'Austrian' label makes reference to the idea of "roundabout production" in the terminology of Böhm-Bawerk (1884) and the Austrian school of capital theory, where the output from one stage of production can be used as an input into production at a subsequent step. Antras (2022, 2023) has breathed new life into the Austrian approach in studying global value chains and trade.

We develop our theory in two stages. In the first stage, we derive the determinants of total working capital taking as given the financing cost facing firms in the global value chain. Our theory draws a close analogy between the investment in working capital and the investment in *physical* capital. Ramey (1989) first pioneered such an approach. The solution for the optimal working capital for any given financing cost can also be interpreted as the *demand for credit* as a function of the financing cost.

In the second stage, we close the model by combining the demand for credit with a credit supply function with a key role for a parameter for bank leverage that can capture the ebb and flow of financial conditions. Higher bank leverage is associated with higher credit supply and hence more permissive financial conditions. The resulting equilibrium outcome for working capital, supply chain length and total output is determined by the parameter for financing conditions, establishing the duality between real and financial variables in the economy.

By highlighting the analogy between physical capital and working capital, our theory connects with the classic theme of inventories as a driver of output fluctuations (Blinder (1986), Ramey (1989)). Rather than being a buffer stock, inventories reflect the active choice of working capital by firms. Indeed, Ramey (1989) has the title: "Inventories as a factor of production". The extent of roundabout production determines the investment in working capital, and firms strike the optimal balance between the productivity gains from working capital against the greater cost of that working capital. Kashyap, Lamont and Stein (1994) documented the sensitive nature of inventories to financial conditions, especially to shocks that reduced bank credit supply.

While the early literature on inventories and output fluctuations focused on the domestic context, our theory has greatest applicability in the international context given the central role of supply chains in international trade. Most closely related to our paper is the recent work of Antras (2022, 2023) who proposes an 'Austrian' theory of global value chains and international trade where firms engage in sequential production with a pre-determined number of stages, but in which the time length of each stage is a choice variable in the spirit of Findlay (1978). In such a setting, a lower interest rate is associated with longer production times at each stage, higher wages and higher final goods output.

Compared to Antras (2022), our theory focuses on the complementary notion of 'roundaboutness' in the *number of stages* in the supply chain, which proves useful in highlighting the inherently non-linear nature of working capital costs as supply chains lengthen. As such, our theory highlights the greater vulnerability of longer supply chains to a tightening of financial conditions. The non-linearity stems from earlier vintages of inventories reflecting more rounds of value-added. As a simple example, a firm with n production stages would be carrying n vintages of inventories. For simplicity, suppose the inventories (going from the newest to the oldest) are valued at $v, 2v, \cdots, nv$. Then steady state working capital is the sum: $v + 2v + \cdots + nv$, which increases at the rate of the *square* of the length of the production chain.

In this way, the working capital necessary to sustain the production chain becomes highly sensitive to the length of the chain, necessitating much greater incremental financing needs as production chains become longer. The upshot is that tighter financial conditions bite harder for longer supply chains.

One key contribution of our theory is to close the model and to derive the equilibrium interest rate as a function of credit supply. As with Antras (2022), a lower interest rate in our model is associated with higher output, higher productivity and wages. Antras (2022) introduces a capital market where the interest rate follows from the rate of time preference of agents. In our case, the model is closed by credit market clearing.

In the context of international trade, the length of supply chains has a direct bearing on the ratio of gross sales to value-added. We apply our framework to the optimal offshoring decision of a multinational firm. Even when the sequential production process is largely determined by the technology, a multinational firm may nevertheless have considerable leeway to choose its production time profile through the extent of offshoring. Offshoring can lower costs and raise productivity, but the financial cost of holding larger inventories introduces a countervailing element. The firm must finance inventories in transit as assets on the balance sheet, and the cost of financing will affect the net benefits of offshoring. We derive closed form solutions and show that the ratio of trade to value-added fluctuates with financial conditions. Easier credit conditions are associated with higher trade volumes, higher inventories and higher productivity.

Our focus on financial conditions as a determinant of trade volumes places our work in the literature on trade and finance. It is well understood that merchanise trade is dependent on trade finance for working capital (Amiti and Weinstein (2011), Niepmann and Schmidt-Eisenlohr (2017a), Bruno and Shin (2023)) and that global banks play the key intermediation role (BIS (2014), Niepmann and Schmidt-Eisenlohr (2017b), Claessens and Van Horen (2021), Goldberg (2023) and Matray et al. (2024)). Hardy and Saffie (2024) and Hardy, Saffie and Simonovska (2024) show that trade credit between non-financial firms also fluctuates in line with global conditions.

The study by Matray et al. (2024) is especially notable. They use the temporary shutdown of the Export-Import Bank of the United States (EXIM) as a natural experiment to estimate the real economy impact of trade financing and conclude that a 1 percent decrease in EXIM trade financing is associated with a 5 percent decrease in total exports. Minetti et al. (2019) find that credit conditions play a role in firms' decision to participate in supply chains. Our theory sheds light on the possible mechanisms involved.

Our theory has a point of contact with the literature on global factors in trade and macro fluctuations. Rey (2013), Miranda-Aggripina and Rey (2022) and Goldberg (2023) have highlighted the importance of global financial factors in the fluctuations of financial conditions across economies. There is now a growing literature that has noted that the dollar exchange rate itself is a global factor. The broad dollar index has attributes of a barometer of financial conditions, whereby a stronger dollar is associated with tighter credit conditions and a slower growth in trade (Avdjiev et al. (2019a), Avdjiev et al. (2019b), Bruno and Shin (2015, 2023), Bruno, Kim and Shin (2018), Caballero, Candelaria and Hale (2018), Cao and Dinger (2022), Lilley et al. (2022) and Obstfeld and Zhou (2022)).

In a micro study of export shipments, Bruno and Shin (2023) show that exporting firms that are more reliant for credit from banks that have a greater reliance on wholesale dollar funding suffer a sharper slowdown in exports due to the greater fluctuations in credit availability from such banks. They find that exports to the United States are subject to the same effects as exports to other destinations, even though a stronger dollar would entail an unambiguous improvement in trade competitiveness for the exporting firm. The evidence on exports to the United States allows the disentangling of the financial channel from the invoicing channel (Gopinath et al., 2020), as the invoicing currency (US dollars) coincides with the currency of the importer. Cook and Patel (2022) show in a model with dollar invoicing that a contractionary monetary shock reduces the ratio of gross to value-added exports, a pattern confirmed in the data.

Our paper has a point of contact with the large literature on global value chains (see Antras and Chor (2022) for a survey) and the propagation of shocks through interconnected sectors (Di Giovanni, Kalemli-Ozcan, Silva and Yildrim (2022)). Our focus on the role of financing conditions also provides a point of comparison with the literature on trade volumes at times of financial crises, especially during the Great Financial Crisis of 2008 (Amiti and Weinstein (2011), Chor and Manova (2012) and Manova (2013)).

The remaining sections of this paper proceed as follows. Section 2 presents the benchmark model. Sections 3 explores trade volumes determined by optimal offshoring and its financial determinants. Section 4 discusses banks' choice of credit supply. Section 5 concludes.

2 Benchmark 'Austrian' Model

We begin with an elementary model of supply chains that isolates the time dimension of production. There are no product or labor market frictions. The only friction is that production takes time, in the spirit of Böhm-Bawerk (1884).

The time dimension of production is best explained through a diagram. Consider Figure 2 which depicts the inventories of a firm with a three-stage production process. The firm undertakes the first production stage at date 1, sends the intermediate good to stage 2. At date 3, the firm has three vintages of inventories. The oldest inventory (3 periods old) has the highest value ($3v$) reflecting greater inputs in the past. The next oldest inventory (2 periods old) has the next highest value ($2v$), and so on.

In this setting, the total *stock* of inventories carried by the firm in steady state (given by $v + 2v + 3v$) can be represented by the area of the triangle in Figure 2. Since the area of the triangle is increasing at the rate of the *square* of the length of the production chain, the value of the stock of inventories is also increasing at the same rate. In this way, the working capital need increases non-linearly with the length of the chain, necessitating much greater incremental financing needs as production chains become longer.

2.1 Working capital and productivity

Consider production that takes place through chains of length n. There is a population of L workers. There are firms each owned by a penniless entrepreneur. In the elementary model, each firm is matched with one worker, so that there are L/n production chains in the economy. Our general model examines the joint decision to allocate labor and choose the length of the supply chain.

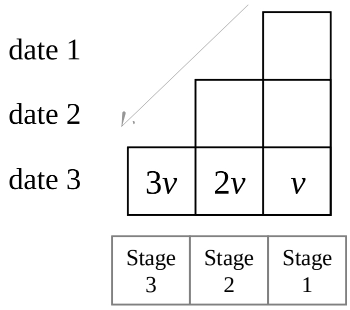

Figure 2. Inventories of a firm with a three-stage production process. At date 3, the firm has three vintages of inventories, and older vintages have higher value reflecting greater inputs in the past.

Within each production chain, there is a consumer-facing firm, labeled as firm n, which sells the final output. The other firms produce intermediate inputs in the production of the final good. Firm 1 supplies its output to firm 2, who in turn supplies output to firm 3, and so on.

There is a "time to build" element. It is assumed that firms face a fixed unit of time to successfully complete each stage. This assumption recognizes that the production process of any intermediate or final product may inherently require a minimum fixed duration. Each step of the production process takes one unit of time, where time is indexed by $t \in \{1, 2, \cdots\}$. A production chain of length n takes n units of time to produce the final output.

Although each step of the production process is identified with a firm, this is for narrative purposes only. In applied settings, our model may be better interpreted as a multi-plant firm with each stage corresponding to a plant. The model is silent on where the boundary of the firm lies along the chain. What matters is the *aggregate* financing need of the supply chain as a whole. For simplicity of exposition, we will say "firm" with the understanding that they can be units of a single multi-plant firm.

Wage costs cannot be deferred and must be paid immediately in the period when the production is carried out. Labor is provided inelastically, and total labor supply is fixed at L. There is no physical capital. The wage rate is w per period. A production chain hires one worker for each stage. The cashflow to the chain is given in the table below.

		Firms					cumulative
		n	$n-1$	$\cdots$	2	1	cashflow
	1					$-w$	$-w$
	2				$-w$	$-w$	$-3w$
date	$\vdots$			$\cdots$	$-w$	$-w$	$\vdots$
t	$n-1$		$-w$	$\cdots$	$-w$	$-w$	$-\frac{1}{2}(n-1)nw$
	n	$-w$	$-w$	$\cdots$	$-w$	$-w$	$-\frac{1}{2}n\,(n+1)\,w$
	$n+1$	$y\,(n)-w$	$-w$	$\cdots$	$-w$	$-w$	
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	

At date 1, firm 1 begins production by hiring a worker and paying wages. It sends its output to firm 2 at date 2. Firm 2 takes delivery and begins production at date 2, and sends the intermediate good to firm 3 at date 3, and so on. Meanwhile, at date 2, firm 1 starts another sequence of production by producing its output, which is sent to firm 2 at date 3.

The first positive cashflow to the chain comes at date $n+1$ when firm n sells the final output $y\,(n)$. The cash transfer upstream is instantaneous, so that all upstream firms are paid for their contribution to the output.

Firms face a borrowing rate of $r > 0$. For the analysis in this section, we take r as given. We will later endogenize r by introducing a banking sector and solve for r as the equilibrium interest rate that clears the credit market.

We assume for simplicity that firms face a financing cost of zero in the initial set-up phase until the first positive cashflow materializes from the sale of the final good. In Appendix A, we provide the solution for the general case where firms face positive interest cost from the outset, and show that the assumption of zero interest cost in the initial set-up phase is without loss of generality for our main results.

The working capital needed in the initial set-up phase is given by:

$$\tfrac{1}{2}n\,(n+1)\,w \tag{1}$$

reflecting the sum of all wages paid until the first cashflow from the sale of the final good. Firms start with no equity and all financing is done by borrowing. Note that the total borrowing is of the order of the *square* of the

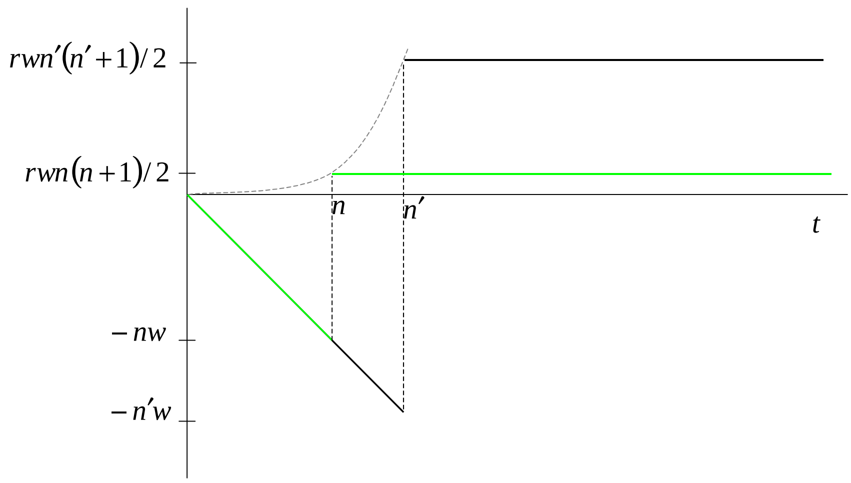

Figure 3. Profile of lenders' cash flow from lending to a production chain of length n (light line) and to a chain of length n' (dark line)

length of the production chain, corresponding to the area of the "triangle" in the cashflow diagram above.

There are L/n production chains, so that the aggregate working capital demand in the economy, denoted by K, is

$$\begin{aligned} K &= \tfrac{1}{2}n\,(n+1)\,w \times \frac{L}{n} \\ &= \frac{n+1}{2}wL \qquad (2) \end{aligned}$$

For firms, the choice of the length of the production chain takes account of the marginal increase in productivity from lengthening the chain against the increased cost of financing for working capital. From the lenders' perspective, the cash flow is negative until date n, but then they start receiving interest payment on the outstanding stock of loans. Figure 3 compares the profile of lenders' cash flows depending on the length of the supply chain. The green line gives the cash flow profile by lending to a supply chain of length n, while the black line gives the profile from lending to a chain of length $n' > n$.

The production chain consisting of n stages has final output $y\,(n)$, where

$$y\,(n) = A(n)l \qquad (3)$$

and l is total labor employed by the chain ($l = n$ when each stage employs one worker), and

$$A(n) = n^{\alpha}, \qquad (0 < \alpha < 1) \qquad (4)$$

so that productivity is an increasing and concave function of the length of production chain. Our assumption that $0 < \alpha < 1$ harks back to Böhm-Bawerk's (1884) discussion of "roundabout production" in which:

> "[t]he indirect method entails a sacrifice of time but gains the advantage of an increase in the quantity of the product. Successive prolongations of the roundabout method of production yield further quantitative increases though in diminishing proportion."[1]

The parameter α is the only "deep" technological parameter in our model, as the interest rate on working capital will be obtained by closing the model with credit supply.

2.2 Optimal length of supply chain

Supply chain length n maximizes the surplus of the chain as a whole, reflecting the interpretation of our model as the decision of a multi-plant firm. Since the borrowing rate is zero until date n and is r from date $n + 1$, the choice of n at date 0 maximizes the discounted sum of surpluses:

$$\begin{aligned} V &= \sum_{t=n+1}^{\infty} \frac{(n^\alpha zL - wzL - rzK)}{(1+r)^{t-n}} \\ &= (n^\alpha zL - wzL - rzK)\frac{1}{r} \end{aligned} \tag{5}$$

where z is the proportion of the labor force employed by the production chain. The above maximization problem boils down to the problem of maximizing the per period surplus:

$$\begin{aligned} \pi &= n^\alpha zL - wzL - rzK \\ &= \left[n^\alpha - w\left(1 + \tfrac{r(n+1)}{2}\right)\right] zL \end{aligned} \tag{6}$$

The first-order condition for n gives

$$n = \left(\frac{2\alpha}{wr}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

[1] Böhm-Bawerk (1884), p. 88 of 1959 English translation by G. Huncke, Libertarian Press.

In competitive markets firms bid away their surplus by competing for workers. The wage rate is determined by the zero profit condition:

$$n^{\alpha} = w\left(1 + \tfrac{r(n+1)}{2}\right) \tag{8}$$

The left-hand side of (8) is the marginal product of labor, while the the right-hand side is its marginal cost taking account of working capital costs.

Given this simple set-up, we can solve the model in closed form. The optimal chain length is

$$n = \frac{\alpha}{1-\alpha}\left(1 + \frac{2}{r}\right) \tag{9}$$

so that production chains are longer when the interest rate r is lower.[2]

Productivity, or output per worker is

$$A(n) = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha} \tag{10}$$

and total output of the economy Y is

$$Y = n^{\alpha}L = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha} L \tag{11}$$

so that productivity and output are decreasing in the interest rate r. The equilibrium wage w is also decreasing in r, since we have:

$$w = 2\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha}\left(\frac{1-\alpha}{2+r}\right) \tag{12}$$

Total working capital of all production chains in the economy is given by:

$$\begin{aligned} K &= \frac{n+1}{2}wL \\ &= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha}\left(\frac{\alpha}{r} + \frac{1-\alpha}{2+r}\right) L \end{aligned} \tag{13}$$

[2] Antras (2023) also derives the inverse relationship between the interest rate and roundaboutness, uderscoring the robustness of the conclusions. One distinction between the two models is our assumption that firms face a fixed unit of time to complete each stage, whereas Antras allows firms the flexibility to choose production time for each stage freely, without limitations. Antras's model suggests that without fixed production time, it can be the case that there is no association between the number of stages and the interest rate. Kim (2024) demonstrates that within Antras' model if both fixed and variable production time are introduced, the optimal number of stages may exhibit a negative correlation with the interest rate.

In our model, investment in working capital raises productivity and increases output. However, the increase in working capital comes at the cost of greater financing cost. Within the credit market, K is the *aggregate credit demand* in the economy. Equation (13) shows that credit demand is decreasing in the interest rate r. The credit to output ratio has the simple form as below, which also declines with the interest rate.

$$\frac{K}{Y} = \frac{\alpha}{r} + \frac{1-\alpha}{2+r} \tag{14}$$

Our model draws out the analogy between working capital and fixed capital. Indeed, Ramey's (1989) investigation of modeling inventories as a factor of production suggests that the analogy can be explored further. From (2) and (3), total output can be written as

$$\begin{aligned} Y(K,L) &= n^{\alpha} L \\ &= \left(\frac{2K}{wL} - 1\right)^{\alpha} L \\ &= \left(\frac{2}{w} - \frac{L}{K}\right)^{\alpha} K^{\alpha} L^{1-\alpha} \end{aligned} \tag{15}$$

where K here represents working capital.

Imposing a Cobb-Douglas functional form for output will result in a misspecified production function where the total factor productivity term $(2/w - L/K)^{\alpha}$ depends on endogenous variables. The measured TFP term is not well defined when $r = 0$ as the denominators of both expressions inside the brackets in (15) go to infinity. Figure 4 which plots TFP as a function of the borrowing rate r when $\alpha = 0.033$ suggests that the misspecified TFP term is decreasing in the borrowing rate. To outside observers who impose a Cobb-Douglas production function, they would observe that TFP undergoes shocks as financial conditions change. When financial conditions are tight and the risk premium in the borrowing rate increases, they will also observe that total factor productivity falls.

2.3 Sales versus value-added

Our model is well-suited to distinguishing total sales (gross output) from value-added (final good sale), which is equivalent to total output in our

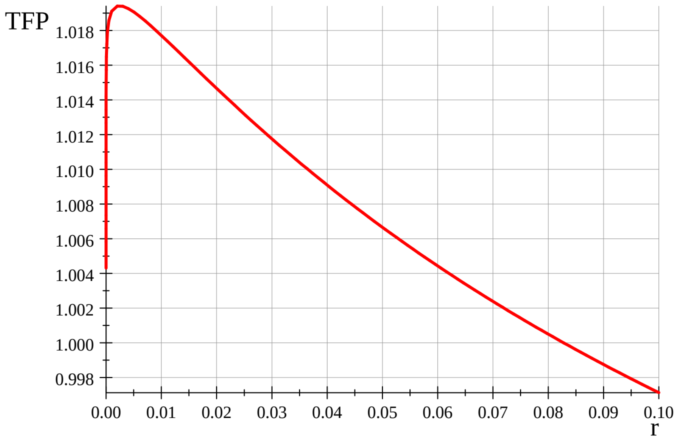

Figure 4. Total factor producitivity as a function of the market interest rate ($\alpha = 0.033$)

model.

Denote by p_i the price of the intermediate good produced by firm i. Given the zero profit condition, the intermediate good price is just sufficient to cover wages and the cost of intermediate inputs including the cost of working capital. Thus, in steady state, the prices of intermediate goods are given by:

$$\begin{aligned} p_1 &= w + rwn \\ p_2 &= w + rw(n-1) + p_1 \\ p_3 &= w + rw(n-2) + p_2 \\ &\vdots \\ p_n &= w + rw + p_{n-1} \end{aligned} \tag{16}$$

By recursive substitution, we have

$$\begin{aligned} p_1 &= w(1+rn) \\ p_2 &= 2w(1+rn) - wr \\ p_3 &= 3w(1+rn) - wr(1+2) \\ &\vdots \\ p_n &= nw(1+rn) - wr(1+2+\cdots+(n-1)) \end{aligned} \tag{17}$$

The economy has L/n of such production chains. Therefore, gross sales in steady state, denoted by S, can be written as:

$$S = \sum_{i=1}^{n} p_i \left(\frac{L}{n}\right) = \left[w\left(1+rn\right)\left(\sum_{i=1}^{n} i\right) - wr\sum_{i=1}^{n-1} i\left(n-i\right)\right]\left(\frac{L}{n}\right)$$

Using the algebraic identity $\sum_{i=1}^{n-1} i\left(n-i\right) = \frac{1}{6}n\left(n-1\right)\left(n+1\right)$, gross sales can be solved in closed form as:

$$\begin{aligned} S &= \left[\tfrac{1}{2}\left(1+rn\right)\left(n+1\right) - \tfrac{1}{6}r\left(n-1\right)\left(n+1\right)\right] wL \\ &= \tfrac{1}{6}\left(n+1\right)\left(r+2rn+3\right)wL \end{aligned} \tag{18}$$

Meanwhile, total value-added is the value of the final good (denoted by Y), which amounts to

$$Y = p_n\left(\frac{L}{n}\right) = \left[wn\left(1+rn\right) - wr\sum_{i=1}^{n-1} i\right]\left(\frac{L}{n}\right)$$

Therefore

$$Y = \left(\tfrac{1}{2}r\left(n+1\right)+1\right) wL \tag{19}$$

and the ratio of sales to value-added is:

$$\frac{S}{Y} = \frac{\left(n+1\right)\left(\frac{1}{3}r + \frac{2}{3}rn + 1\right)}{r(n+1)+2}$$

Substituting in the solution for n, we have

$$\begin{aligned} \frac{S}{Y} &= \frac{\left(r+2\alpha\right)\left(r+\alpha\left(1+r\right)+3\right)}{3r\left(1-\alpha\right)\left(r+2\right)} \\ &= \frac{1}{3\left(1-\alpha\right)}\left(1+\frac{2\alpha}{r}\right)\left(1+\alpha+\frac{1-\alpha}{2+r}\right) \end{aligned} \tag{20}$$

The sales to value-added ratio is decreasing in the interest rate r, reflecting the shorter production chains when financial conditions are tighter.

Gathering the findings in (9), (10), (11) and (20), we can summarize the main features of our model as follows.

Proposition 1 *A higher borrowing rate r is associated with (1) shorter supply chains, (2) lower productivity per worker, (3) lower output and (4) lower sales-to-value-added ratio.*

3 Model with general production function

Our benchmark model fixed labor allocation so as to focus on the time dimension of production. In this section, we present a more comprehensive Austrian model incorporating a CES production function. In this model, we present a more general model of the joint allocation of labor and working capital. We demonstrate that the key findings from the benchmark model are preserved even in the general case. To facilitate ease of computation, we employ a continuous-time version of the model.

3.1 Optimal labor allocation

The stage index, denoted as i, is a continuous variable defined over the interval $[0, n]$, where $i = 0$ represents the most upstream stage, and $i = n$ represents the most downstream stage. The production process commences at $t = 0$, and the final good is produced and sold starting from $t = n$. Consequently, the firm requires working capital to cover wage payments during the initial setup phase ($0 \leq t < n$).

The firm chooses the number of workers across different stages. When the firm opts to employ l_i workers for stage i, the working capital needed during the set-up phase is given by

$$\int_0^n (n - i)(wl_i)di \tag{21}$$

For simplicity, we continue to assume a zero interest rate on borrowing during the set-up phase.

The aggregate working capital demand in the economy is

$$K = \frac{\int_0^n (n - i)wl_i di}{\int_0^n l_i di} \times L \tag{22}$$

which simplifies to $\frac{n}{2}wL$, a continuous-time equivalent of eq. (2) in the benchmark model, when $l_i = 1$ for all i.

In steady state ($t \geq n$), the firm incurs production costs, which comprise the sum of wages in the current period and interest charged on the working capital accumulated during the setup periods:

$$\frac{\int_0^n wl_i di + r\int_0^n (n-i)wl_i di}{\int_0^n l_i di} l$$

where l represents the total labor employed by the chain.

We represent the production function of final goods using a constant elasticity of substitution (CES) form:

$$y = n^{\alpha+1}\left[\int_0^n \frac{1}{n}(l_i)^{-\rho} di\right]^{-\frac{1}{\rho}} \tag{23}$$

where ρ denotes the CES substitution parameter. Eq. (23) converges to a Leontief production function as $\rho \to \infty$ and to a Cobb-Douglas as $\rho \to 0$.

The labor productivity in this production chain is then expressed as:

$$A(n) = \frac{n^{\alpha+1}[\int_0^n \frac{1}{n}(l_i)^{-\rho} di]^{-\frac{1}{\rho}}}{\int_0^n l_i di}$$

which is independent of the number of workers hired by the chain. The final output y is expressed as

$$y = A(n)l = \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho} di]^{-\frac{1}{\rho}}}{\int_0^n l_i di} l$$

If l_i values are uniform across i, the final output becomes $y = n^\alpha l$ as in our benchmark model.

The production chain chooses l_i and n to maximize the per period surplus:

$$\begin{aligned} \pi(n) &= A(n)zL - wzL - rzK \\ &= \left[\frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho} di]^{-\frac{1}{\rho}} - \int_0^n (1+r(n-i))wl_i di}{\int_0^n l_i di}\right] zL \end{aligned}$$

where z is the proportion of the labor force employed by the production chain.

The first-order condition for labor employed in stage i (l_i) gives

$$\frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho} di]^{-\frac{1}{\rho}-1}(l_i)^{-\rho-1} - (1+r(n-i))w}{\int_0^n l_i di} - \frac{\pi(n)}{(\int_0^n l_i di)zL} = 0$$

In competitive markets firms bid away their surplus by competing for workers, which results in the zero profit condition:

$$n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}} - \int_0^n (1+r(n-i))wl_i di = 0 \tag{24}$$

From the first-order condition and the zero profit condition, we have the optimal labor allocation across stages

$$l_i = (1+r(n-i))^{-\frac{1}{1+\rho}} l_n \tag{25}$$

3.2 Optimal chain length

The first-order condition for chain length n, combined with the zero profit condition, is given by

$$\frac{d}{dn}\left[n^{\alpha+1+\frac{1}{\rho}}\left[\int_0^n (l_i)^{-\rho}di\right]^{-\frac{1}{\rho}}\right] = \frac{d}{dn}\int_0^n (1+r(n-i))wl_i di$$

Using eqs. (24) and (25), we can further simplify the first-order condition for n as (see Appendix C for additional details in this subsection):

$$(\alpha+1+\frac{1}{\rho})(\frac{1}{n})\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}}di - \frac{1}{\rho} = 1 + r\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}}di \tag{26}$$

The equation can be solved for n. This suggests that the equilibrium chain length n depends on the CES substitution parameter ρ, and the interest rate r.

Note that when $\rho \to \infty$ (Leontief production function), eq. (26) yields the optimal chain length as

$$n = \frac{\alpha}{1-\alpha}\left(\frac{2}{r}\right)$$

This is a continuous time equivalent of the optimal n in eq. (9) of our benchmark model, indicating the negative effect of the interest rate on the length of the production chain.

Now consider the case where $\rho < \infty$. By calculating two integrals in eq. (26) and applying Taylor's expansion for the interest rate r which is close to zero, we can express eq. (26) as

$$(\alpha+1+\frac{1}{\rho})(1+\frac{1}{2}(\frac{\rho}{1+\rho})rn) - \frac{1}{\rho} = 1 + rn - \frac{1}{2}(\frac{1}{1+\rho})r^2n^2$$

which yields a quadratic equation for n with finite ρ:

$$\frac{r^2}{1+\rho}n^2 + \frac{((\alpha-1)\rho-1)r}{1+\rho}n + 2\alpha = 0 \tag{27}$$

We can solve the equation (27) to determine the optimal value of n. If the parameter α, representing the impact of roundaboutness on productivity equals zero, the optimal soultion for n is zero ($n = 0$). Depending on the parameter values, the optimal solution may also be infinity.

To eliminate these unrealstic corner solutions, we assume that

$$0 < \alpha \ < \frac{1+\rho}{\rho} \ \text{ and } \ \left(\frac{(\alpha-1)\rho-1}{1+\rho}\right)^2 - \frac{8\alpha}{1+\rho} \geq 0$$

We also assume that for a technologically feasible upper bound on the chain length, denoted by η, the following inequality holds

$$\frac{r^2}{1+\rho}\eta^2 + \frac{((\alpha-1)\rho-1)r}{1+\rho}\eta + 2\alpha < 0$$

In this case, the quadratic equation yields two positive solutions for n, and the smaller of the two is identified as the optimal solution, satisfying the second-order condition. The optimal solution for n is expressed as a decreasing function of the interest rate:

$$n = \left[\frac{-(\frac{(\alpha-1)\rho-1}{1+\rho}) - [(\frac{(\alpha-1)\rho-1}{1+\rho})^2 - \frac{8\alpha}{1+\rho}]^{\frac{1}{2}}}{\frac{2}{1+\rho}}\right]\left(\frac{1}{r}\right) \tag{28}$$

which holds for any finite ρ.

3.3 Effects of the interest rate

The interest rate influences key macro variables, including productivity and output, through its impact on the length of the production chain. Utilizing Taylor's expansions, we can express productivity $A(n)$, output Y, the wage rate w and total working capital K as (refer to Appendix D for further details in this subsection)

$$A(n) = \frac{n^\alpha[1 + \frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}}{1 - \frac{1}{2}(\frac{1}{1+\rho})rn} \tag{29}$$

$$Y = \frac{n^{\alpha}[1 + \frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}}{1 - \frac{1}{2}(\frac{1}{1+\rho})rn}L \tag{30}$$

$$w = \frac{n^{\alpha}}{[1 + \frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}} \tag{31}$$

$$K = \frac{n^{\alpha+1}}{2[1 + \frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}(1 - \frac{1}{2}(\frac{1}{1+\rho})rn)} \times L \tag{32}$$

In the case of the Leontief function with ρ approaching infinity, closed-form solutions can be derived, revealing a clear negative association between the interest rate r and productivity $A(n)$, output Y, the wage rate w, and total working capital K.

In a more general environment where ρ is finite, it's important to note that the formulations above indicate that $A(n)$, Y, w, and K are unaffected by rn, which is solely dependent on α and ρ, not on r, as evident from (28). Thus, the interest rate r influences these key macro variables only through n^{α} or $n^{\alpha+1}$ in the equations, both of decrease with the interest rate. Therefore, for any values of the CES substitution parameter ρ, it can be established that

$$\frac{dA(n)}{dr} < 0, \ \ \frac{dY}{dr} < 0, \ \ \frac{dw}{dr} < 0, \text{ and } \frac{dK}{dr} < 0 \tag{33}$$

This indicates that an increase in the interest rate leads to a reduction in the length of the production chain, which in turn results in the fall of productivity, GDP and working capital. Importantly, this inverse relationship holds true for any value of ρ, signifying that it is independent of the CES parameter.

4 Application to international trade

We now turn to an application of our theory to offshoring and trade, and begin with a motivating example of offshoring for a multinational firm.

		Stages	
		2nd	1st
Date t	1		w
	2	w	w
	3	w	w
	⋮	⋮	⋮

		Stages		
		3rd	2nd	1st
Date t	1			c
	2		0	c
	3	w	0	c
	4	w	0	c
	⋮	⋮	⋮	⋮

Figure 5. **Costs of two-step production with and without offshoring.** A good is produced with two rounds of value-added. The left-hand diagram depicts production without offshoring. The right-hand diagram depicts the case when there is offshoring of the first stage of production. Without offshoring, each production stage takes one period and incurs cost of w. By offshoring the first stage, the firm reduces the first-stage cost to c but lengthens the time to produce the final good to three periods due to the transport stage.

4.1 Motivating example

Consider a two-stage production process without offshoring ($n = 2$), where a final good can be produced with two rounds of value-added. This case is depicted by the left-hand diagram in Figure 4. Each step in the production of the good takes one time period, and incurs a cost of w. At date 1, the firm that produces both stages completes the first production step at cost w and sends the intermediate good to the second step. At date 2, the firm goes through the second step of production incurring cost w. Meanwhile, the firm begins the first stage of the production of the next unit at cost w.

The firm begins to receive revenue of p from date 3 onwards, when it sells the good at price p. Before then, the firm finances the costs incurred during the initial phase (dates 1 and 2) by borrowing at interest rate r.

In steady state (from date 3 onwards), the firm's cashflow is

$$p - 2w - r\left(2w\left(1+r\right) + w\left(1+r\right)^2\right) \tag{34}$$

consisting of sales revenue p, per-period production cost $2w$ and the interest expense on the debt incurred during the initial phase of production (at the steady state interest rate $r > 0$).

Now, suppose that the firm can offshore the first stage of production abroad. The right-hand panel of Figure 4 depicts production with offshoring. By offshoring the first step of production, the firm enjoys a productivity gain and also save on the cost of the first step of production. But it has to lengthen the total production time to three periods to take account of the time taken to transport the intermediate good. The cost of the first step of production with offshoring (including transport cost) is c. At date 2, the intermediate good is transported, and the second step of production takes place at date 3. The firm receives revenue from the sale of the good from date 4 onwards.

In steady state (from date 4 onwards), the firm's cashflow is

$$\widetilde{p} - (c + w) - r \left((c + w)(1 + r) + c(1 + r)^2 + c(1 + r)^3 \right) \tag{35}$$

consisting of sales revenue $\widetilde{p}$ net of production cost $c + w$ and the cost of building up and carrying working capital. By offshoring the first step of production, the firm raises revenue to $\widetilde{p}$ and lowers the first stage cost to c, but incurs a higher working capital cost.

Denote by $\tilde{k}$ and k the working capital with and without offshoring, respectively. The firm chooses to offshore when the firm's steady-state cashflow with offshoring (35) is larger than without offshoring (34), or equivalently, when

$$(\widetilde{p} - p) + (w - c) > r(\tilde{k} - k) \tag{36}$$

where $\tilde{k} = (c + w)(1 + r) + c(1 + r)^2 + c(1 + r)^3$ and $k = 2w(1 + r) + w(1 + r)^2$. The firm can increase steady state profit through offshoring when the financing cost of offshoring is sufficiently small. However, higher r entails a higher hurdle for offshoring.

In what follows, we develop our model of offshoring by extending the benchmark model of general n-stage supply chains into a multi-country setting.

4.2 Model of offshoring and productivity

Consider a multinational firm with a presence in multiple locations. Each location has an absolute advantage in precisely one stage of the production process. The absolute advantage derives from the location, not the worker.

Specifically, there is a constant $b > 0$ such that the location with the absolute advantage in production stage i has productivity of $1+b$ compared to productivity of 1 in any other location for that task.

The output of the multinational firm ($y(n) = A(n)l$) then depends on the extent of offshoring, and productivity is given by:

$$A(n,s) = \left(\sum_{i=1}^{n} x_i \right)^{\alpha} \qquad (0 < \alpha < 1) \tag{37}$$

where $x_i = 1+b$ if the production of the ith stage takes place in the location with the absolute advantage in stage i while $x_i = 1$ if the production takes place elsewhere. Thus, if there are s stages where production takes place in the location with the absolute advantage, productivity is given by

$$A(n,s) = (n+bs)^{\alpha}$$

To highlight the choice of offshoring, we fix the length of the production chain at $n = \bar{n}$, and there are $\bar{n}$ locations. Then the productivity of a chain is

$$A(s) = (\bar{n}+bs)^{\alpha} \tag{38}$$

and the final output $y(s)$ is

$$y(s) = A(s)l = (\bar{n}+bs)^{\alpha} l \tag{39}$$

The firm's key decision is to choose s, the extent of offshoring.[3]

4.3 Inventories in transit

We assume that transport requires labor services just as production does. Offshoring incurs additional financing costs due to time needed to transport intermediate goods. As indicated by Amiti and Weinstein (2011), it is not unusual for the transportation of goods overseas to extend over a period of two months. We assume that if an intermediate good is transported to another location, transport takes one unit of time, which is the same as the time

[3]It is possible to allow for endogenous n by first deriving s and w as a function of n and then solve for n. The qualitative features of the model remain.

needed for production of an intermediate good. Within the same country, we assume that transport happens instantaneously and without labor cost.

If s stages are offshored, the time to production of the final good in this offshoring model rises from $\bar{n}$ to $\bar{n}+s$. With offshoring, a new type of inventory emerges - *inventory in transit.* Without offshoring, the multinational firm has $\bar{n}$ vintages of inventories at the steady state. With offshoring, multinational firms hold $\bar{n}+s$ vintages of inventories including s vintages of inventories in transit.

As in the benchmark model, wages cannot be deferred and firms that engage in intermediate good production or overseas transport need working capital to pay wages. Wages at each stage of production or transportation is paid in the period when the activity of the stage takes place. We assume that w is equal across locations and activities.[4] We maintain the assumption that firms finance working capital with debt from banks at zero interest rates during the initial periods ($t \leq \bar{n}+s$). The relaxation of the assumption of zero interest rates during the transition period does not alter the main results of the model (see Appendix B).

The multinational firm starts the production of the first stage intermediate good ($i=1$) at date 1. At date $\bar{n}+s$, it begins producing the final goods by inputing intermediate goods for which it has paid $\frac{1}{2}(\bar{n}+s)(\bar{n}+s-1)\,w$, and labor, for which it currently pays $(\bar{n}+s)\,w$. The steady state working capital is

$$\tfrac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w$$

which is equal to the sum of all the wages that have been paid during the initial set-up period. Note that offshoring raises the working capital from $\frac{1}{2}\bar{n}(\bar{n}+1)w$ to $\frac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w$ due to inventories in transit.

Total working capital for the global economy as a whole, denoted by K,

[4] We may introduce a model which allows for wage difference across countries, for example due to restrictions on international labor mobility. In this variant of the model, multi-national firms have incentive to offshore each stage of production chain to the country with the lowest wage in the stage. The solution of the variant model with wage difference for the optimal offshoring is similar to that of this section with productivity difference.

is then

$$
\begin{aligned}
K &= \tfrac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w \cdot \frac{L}{(\bar{n}+s)} \\
&= \frac{\bar{n}+s+1}{2} wL \qquad (40)
\end{aligned}
$$

where L is the population of workers. K also has the interpretation as the *total demand for credit* to finance working capital investment. Taking the borrowing rate as given for now, the per period borrowing cost is

$$
r \cdot \frac{\bar{n}+s+1}{2} wL
$$

The cost of financing for working capital increases with the number of offshored stages.

4.4 Optimal offshoring

The firm chooses s to maximize its value, given by the discounted sum of surpluses:

$$
V = \sum_{t=\bar{n}+s+1}^{\infty} \frac{(\bar{n}+bs)^{\alpha} zL - wzL - rzK}{(1+r)^{t-\bar{n}-s}} \qquad (41)
$$

where z is the proportion of the workforce employed by the firm. The maximization problem reduces to maximizing the per-period profit

$$
\begin{aligned}
\pi &= (\bar{n}+bs)^{\alpha} zL - wzL - rzK \\
&= \left[(\bar{n}+bs)^{\alpha} - w\left(1 + \frac{r\,(\bar{n}+s+1)}{2}\right)\right] zL
\end{aligned}
$$

The first-order condition for s yields

$$
\bar{n} + bs = \left(\frac{2b\alpha}{wr}\right)^{\frac{1}{1-\alpha}} \qquad (42)
$$

and the zero profit condition is

$$
(\bar{n}+bs)^{\alpha} = w\left(1 + \frac{r}{2}(1+\bar{n}) + \frac{r}{2}s\right) \qquad (43)
$$

Assume that $b > \frac{1}{\alpha}$. From eqs. (42) and (43) we can solve the model in closed form. Optimal extent of offshoring is

$$
s = \frac{\alpha}{1-\alpha}\left(1 + \bar{n} + \frac{2}{r}\right) - \frac{\bar{n}}{b(1-\alpha)} \qquad (44)
$$

which is positive since $b > \frac{1}{\alpha}$ and is decreasing in r. The offshoring ratio $s/\bar{n}$ captures what fraction of production is offshored, and is given by

$$\frac{s}{\bar{n}} = \frac{\alpha}{1-\alpha}\left(\left(1+\frac{2}{r}\right)\frac{1}{\bar{n}}+1\right) - \frac{1}{b(1-\alpha)}$$

Appendix C describes an accounting framework which can be used to approximate the offshoring ratio $s/\bar{n}$ using available data.

The productivity is given by

$$A(\bar{n}, r) = \left[\frac{(b-1)\alpha}{1-\alpha}\bar{n} + \frac{b\alpha}{1-\alpha}\left(1+\frac{2}{r}\right)\right]^{\alpha}$$

and the world output Y^{World} is

$$Y^{\text{World}} = (\bar{n}+bs)^{\alpha} L = \left[\frac{(b-1)\alpha}{1-\alpha}\bar{n} + \frac{b\alpha}{1-\alpha}\left(1+\frac{2}{r}\right)\right]^{\alpha} L$$

so that productivity and world output are declining in r.

By plugging (44) into (43), we derive the equilibrium wage as

$$w = \frac{2b\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left[(b(1+\bar{n})-\bar{n})r^{\frac{1}{1-\alpha}} + 2br^{\frac{\alpha}{1-\alpha}}\right]^{1-\alpha}} \tag{45}$$

which is also declining in r. The tightening of financial condition reduces offshoring and has a negative impact on the world productivity, wages and output.

Using eqs. (44) and (45), we can then solve for working capital K as $(\bar{n}+s+1)\,wL/2$, which is decreasing in r. Global demand for credit is therefore decreasing in r.

4.5 Ratio of trade to output

Our model's distinction between total sales and value-added allows us to track the ratio of trade to total output, or equivalently, the ratio of trade to value-added. From the zero profit condition, we can express the intermediate prices p_i as in eq. (16)

$$p_i = w + rw(n-i+1) + p_{i-1}$$

Total sales in steady state can be obtained as:

$$\sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s} \right) = \frac{1}{6} (\bar{n}+s+1)(r+2r(\bar{n}+s)+3) wL \tag{46}$$

We can obtain total trade per period (denoted by T^{World}) by multiplying total sales by the proportion of production stages that are offshored. Therefore,

$$\begin{aligned} T^{\text{World}} &= \frac{s}{\bar{n}+s} \sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s} \right) \\ &= \frac{s}{\bar{n}+s} \left(\frac{r+2r(\bar{n}+s)+3}{6} \right) (\bar{n}+s+1) wL \end{aligned} \tag{47}$$

which is increasing in s.[5]

Total output (Y^{World}) is given by the value of the final good, or equivalently, the total value-added:

$$Y^{\text{World}} = p_n \left(\frac{L}{\bar{n}+s} \right) = \left[\frac{1}{2} r(\bar{n}+s+1) + 1 \right] wL \tag{48}$$

Therefore, the ratio of total trade to output is

$$\begin{aligned} \frac{T^{\text{World}}}{Y^{\text{World}}} &= \frac{\frac{s}{\bar{n}+s} \sum_{i=1}^{\bar{n}+s} p_i (\frac{L}{\bar{n}+s})}{p_n(\frac{L}{\bar{n}+s})} \\ &= \frac{s}{\bar{n}+s} \left[\frac{(\bar{n}+s+1)\left(\frac{1}{3}r + \frac{2}{3}r(\bar{n}+s)+1\right)}{r(\bar{n}+s+1)+2} \right] \end{aligned} \tag{49}$$

Note that $\frac{s}{\bar{n}+s}$ increases with s, and the fraction inside the square bracket also tends to increase with s since the numerator is a convex function of s while the denominator is linear in s. Thus, a higher s is associated with a higher ratio of trade to output.

[5] In reality, overseas transport companies receive fees for shipment from exporters or importers rather than buy and sell goods with them. Considering this, total sales among the firms (S^{World}) can be approximated by

$$S^{World} = \sum_{i=1}^{\bar{n}+s} p_i (\frac{L}{\bar{n}+s}) - T^{World} = \frac{\bar{n}}{\bar{n}+s} \sum_{i=1}^{\bar{n}+s} p_i (\frac{L}{\bar{n}+s}).$$

Recall that the optimal offshoring s is decreasing in r (eq. (44)). Therefore, eq. (49) implies that the ratio of trade to output decreases with the interest rate r.[6]

Lastly, total inventories of intermediate goods is given by

$$\begin{aligned} I &= \sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s}\right) - p_n \left(\frac{L}{\bar{n}+s}\right) \\ &= \frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{2} wL \end{aligned} \tag{50}$$

We multiply eq. (50) by $\frac{s}{\bar{n}+s}$ to get total inventories in transit (I^{tr}):

$$\begin{aligned} I^{\text{tr}} &= (\frac{s}{\bar{n}+s})I \\ &= \frac{s}{\bar{n}+s}\left[\frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{2}\right] wL \end{aligned}$$

The ratio of inventory-in-transit to output is

$$\frac{I^{\text{tr}}}{Y^{\text{World}}} = \frac{s}{\bar{n}+s}\left[\frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{r(\bar{n}+s+1)+2}\right] \tag{51}$$

so that inventories-in-transit relative to output is increasing in s and hence decreasing in r.

We summarize our findings in terms of the following proposition.

Proposition 2 *A higher interest rate r is associated with (1) lower offshoring, (2) lower productivity per worker, (3) lower output, (4) lower trade-output ratio, and (6) lower inventories in transit as fraction of output.*

[6]To formally prove this, we can rewrite $\frac{T^{World}}{Y^{World}}$ as the product of three functions of the interest rate r as

$$\frac{T^{World}}{Y^{World}} = B(r)C(r)D(R)$$

where

$$\begin{aligned} B(r) &= (\bar{n}+s+1) \\ C(r) &= \frac{s}{r(\bar{n}+s+1)+2} r \\ D(r) &= \frac{\frac{1}{3}r+\frac{2}{3}r(\bar{n}+s)+1}{\bar{n}+s}\left(\frac{1}{r}\right) \end{aligned}$$

Given the assumption $b > \frac{1}{\alpha}(> 1)$, we can show that $\frac{dB(r)}{dr} < 0$, $\frac{dC(r)}{dr} < 0$ and $\frac{dD(r)}{dr} < 0$, and hence $\frac{d(\frac{T^{World}}{Y^{World}})}{dr} < 0$.

4.6 Offshoring with general production function

We now introduce a continuous-time version of the offshoring model that incorporates a CES production function. This model allows firms to hire a varying number of workers across different stages, similar to the model presented in Section 3.

As a generalization of the production function (39), we adopt a CES form:

$$y = (\bar{n} + bs)^{\alpha}(\bar{n} + s)[\int_0^{\bar{n}+s} \frac{1}{\bar{n}+s}(l_i)^{-\rho} di]^{-\frac{1}{\rho}} \tag{52}$$

This formulation is reduced to $y = (\bar{n} + bs)^{\alpha} l$ when l_i are equal across stages, as in our benchmark offshoring model.

The working capital required during the set-up phase ($0 \leq t < \bar{n} + s$) is given by

$$\int_0^{\bar{n}+s} (\bar{n} + s - i)(wl_i) di$$

This calculation is made under the assumption of a zero interest rate on borrowing during the set-up phase.

The production chain seeks to maximize the per period surplus:

$$\begin{aligned} \pi(s) &= \left[\frac{(\bar{n} + bs)^{\alpha}(\bar{n} + s)^{1+\frac{1}{\rho}}[\int_0^{\bar{n}+s}(l_i)^{-\rho} di]^{-\frac{1}{\rho}}}{\int_0^{\bar{n}+s} l_i di} \right] zl \\ &- \left[\frac{\int_0^{\bar{n}+s}(1 + r(\bar{n} + s - i))wl_i di}{\int_0^{\bar{n}+s} l_i di} \right] zL \end{aligned} \tag{53}$$

The first-order condition for labor, along with the zero profit condition, results in

$$l_i = (1 + r(\bar{n} + s - i))^{-\frac{1}{1+\rho}} l_{\bar{n}+s} \tag{54}$$

The first-order condition for the number of offshored stages s, combined with the zero profit condition, yields (refer to Appendix F for additional details in this subsection)

$$\begin{aligned} & [b\alpha(\bar{n} + bs)^{-1} + (1 + \frac{1}{\rho})(\bar{n} + s)^{-1}] \int_0^{\bar{n}+s} (1 + r(\bar{n} + s - i))^{\frac{\rho}{1+\rho}} di \\ = & \frac{1}{\rho} + 1 + r \int_0^{\bar{n}+s} (1 + r(\bar{n} + s - i))^{-\frac{1}{1+\rho}} di \end{aligned} \tag{55}$$

This equation can be solved for the equilibrium number of offshored stages s.

In the case of a Leontief production function ($\rho \to \infty$), the equilibrium s is given by

$$s = \frac{\alpha}{1-\alpha}\left(\bar{n} + \frac{2}{r}\right) - \frac{\bar{n}}{b(1-\alpha)}$$

This suggests that the optimal offshoring s is decreasing in the interest rate r.

In case of finite ρ, applying Taylor's expansion to (55) yields a quadratic equation for s

$$(\frac{b}{1+\rho})s^2 - g(r)s + h(r) = 0 \tag{56}$$

where $g(r) = \frac{b}{r}(1-\alpha(\frac{\rho}{1+\rho})) - (\frac{1+b}{1+\rho})\bar{n}$ and $h(r) = b\alpha\frac{2}{r^2} + (b\alpha(\frac{\rho}{1+\rho}) - 1)\frac{\bar{n}}{r} + (\frac{1}{1+\rho})\bar{n}^2$.

Depending on the parameter values, the equation may yield two corner solutions, specifically autarky ($s = 0$) and full offshoring ($s = \bar{n}$). Additionally, it provides an interior solution for s if the following conditions hold:

$$\frac{1}{b}(\frac{1+\rho}{\rho}) < \alpha < \frac{1+\rho}{\rho}, \qquad g(r)^2 - 4(\frac{b}{1+\rho})h(r) \geq 0 \tag{57}$$

and

$$(\frac{b}{1+\rho})\bar{n}^2 - g(r)\bar{n} + h(r) < 0$$

In such cases, the optimal solution is determined by

$$s = \frac{g(r) - [g(r)^2 - 4(\frac{b}{1+\rho})h(r)]^{\frac{1}{2}}}{\frac{b}{1+\rho}} \tag{58}$$

Note that $\frac{ds}{dg(r)} > 0$ and $\frac{ds}{dh(r)} > 0$. Considering the condition (57), we also observe $\frac{dg(r)}{dr} < 0$ and $\frac{dh(r)}{dr} < 0$. Consequently, we can conclude that

$$\frac{ds(r)}{dr} < 0 \tag{59}$$

This indicates that offshoring is diminishing with a decrease in the interest rate r irrespective of ρ. Furthermore, it suggests that productivity, GDP, the wage rate, and total working capital decline as the interest rate r decreases.

This inverse relationship remains consistent for any value of ρ, underscoring its independence from the CES parameter.

5 Closing the model with credit supply

So far we have treated the rate of interest r as given. We now close the model by introducing credit supply through a banking sector.

An advantage of closing the model with a banking sector is that we can address how short-term fluctuations in credit conditions that affect lending condition (such as through fluctuations in the leverage of the banking sector) can affect macro fluctuations through productivity and trade. In this way, our analysis opens up additional avenues for exploration in trade and finance. We begin our analysis by presenting a model with credit risk arising from possible failure of supply chains. We then introduce a banking sector whose total lending is determined through a contracting problem to overcome a moral hazard problem among banks, following Bruno and Shin (2015).

5.1 Supply chains with credit risk

We introduce risk of failure of supply chains. Starting at date $\bar{n}+s+1$ (when each supply chain starts receiving positive cash from sales), the supply chain associated with a multinational firm is subject to a hazard rate $\varepsilon > 0$ of failure with zero liquidation value. We assume that if firm fails, the constituent units can re-group costlessly under a new multinational firm who can borrow afresh.

The multinational firm's optimisation problem at date 0 is to choose s to maximize the expected firm value, V:

$$V = \sum_{t=\bar{n}+s+1}^{\infty} \frac{(1-\varepsilon)^{t-\bar{n}-s}\left((\bar{n}+bs)^{\alpha} zL - wzL - rzK\right)}{(1+r)^{t-\bar{n}-s}} \tag{60}$$

Note that V now also incorporates the hazard rate ε, as well as the interest rate r. The firm value (60) can be simplified to:

$$V = \left((\bar{n}+bs)^{\alpha} zL - wzL - rzK\right) \frac{1-\varepsilon}{r+\varepsilon} \tag{61}$$

Despite the inclusion of risk factor ε, therefore, the maximization problem is reduced to maximizing certain profits. Profits are expressed, for example, in the simple benchmark case as:

$$\pi = (\bar{n} + bs)^{\alpha} zL - wzL \left(1 + \frac{r(\bar{n} + s + 1)}{2}\right)$$

or eq. (53) in the case of CES production function.

The borrowing rate r here is an endogenous variable, reflecting risk premium, which is determined when the demand and supply of credit markets are equalized.

We have similar first-order condition for s and zero profit condition as in Section 4, which yields the optimal extent of offshoring as eq. (44)

$$s = \frac{\alpha}{1-\alpha}\left(1 + \bar{n} + \frac{2}{r}\right) - \frac{\bar{n}}{b(1-\alpha)}$$

or eq. (58) in the case of CES production function.

Then the global demand for credit (for working capital), denoted by K, is derived as a function of r. In the benchmark case, for example, K is determined by

$$K(r) = \frac{\bar{n} + s(r) + 1}{2} w(r) L \tag{62}$$

where $s(r)$ and $w(r)$ satisfy eqs. (44) and (45).

5.2 Credit supply by banks

Credit is supplied through banks which are subject to a moral hazard problem. The bank's equity e is fixed, with equity ownership evenly distributed among the investor population in the world. Bank credit is short-term, and rolled over every period.

Along the steady state, the bank lends k^S (for "credit") at date t at the lending rate of interest r, so that the bank is owed $(1+r)\,k^S$ at date $t+1$. The lending is financed from the combination of equity e and deposit funding d, which is raised from investors. The cost of debt financing (deposit) is f so that the bank owes $(1+f)\,d$ at date $t+1$ (its notional liabilities). We will show shortly that f is determined to be equal to the risk-free rate r^f, which is set at zero.

Each production chain is subject to a hazard rate $\varepsilon > 0$ of failure from date $\bar{n} + s + 1$ onwards, while the correlation in failure across chains follows the Vasicek (2002) model. More specifically, production chain j survives into the next period (so that the loan is repaid) when $z_j > 0$ along the steady state (from date $\bar{n} + s + 1$ on), where z_j is the random variable:

$$z_j = -\Phi^{-1}(\varepsilon) + \sqrt{\rho}H + \sqrt{1-\rho}h_j \tag{63}$$

Here $\Phi(.)$ is the cumulative distribution function of the standard normal, H and $\{h_j\}$ are independent standard normals, and ρ is a constant between zero and one. H has the interpretation of the economy-wide fundamental factor that affects all chains, while h_j is the idiosyncratic factor for chain j. The parameter ρ is the weight on the common factor. Note that the unconditional probability of default of each production chain is given by $\Pr(z_j < 0) = \Pr\left(\sqrt{\rho}H + \sqrt{1-\rho}h_j < \Phi^{-1}(\varepsilon)\right) = \Phi\left(\Phi^{-1}(\varepsilon)\right) = \varepsilon$, consistent with our assumption that each chain has a constant hazard rate of failure of ε. Given the economy-wide factor H, defaults of different chains may have positive correlation.

Banks are able to diversify their loan by lending to a large number of separate production chains. In this situation, banks' leverage is determined through the following contracting problem, which follows Bruno and Shin (2015).

Suppose that the banks face the choice between two alternative portfolios. The good portfolio consists of loans to production chains which have a probability of default ε and zero correlation of defaults across loans $\rho = 0$ (so that $z_j = -\Phi^{-1}(\varepsilon) + h_j$). The bad portfolio consists of loans to chains with a higher probability of default $\hat{\varepsilon} = \varepsilon + v$, for $v > 0$ and positive correlation of defaults across loans $\hat{\rho} > 0$ (hence $z_j = -\Phi^{-1}(\varepsilon + v) + \sqrt{\hat{\rho}}H + \sqrt{1-\hat{\rho}}h_j$). The bad portfolio generates greater dispersion in the outcome density for the loan portfolio. Since banks have limited liability, a greater probability of bank failure is associated with a higher option value of limited liability.

The notional value of the bank's total loan is $(1+r)k^S$. Conditional on H, defaults of individual loans are independent. By taking the limit where the bank diversifies its lending across a large number of firms, the realized value of the bank's loan portfolio can be written as a function of H by the law of large numbers.

Suppose that the bank chooses the bad portfolio of loans to production chains with $v > 0$ and $\hat{\rho} > 0$. Then the realized value of the bank's loan portfolio conditional on H, $a_B(H)$, is

$$\begin{aligned} a_B(H) &= (1+r)\,k^S \cdot \Pr\left(\sqrt{\hat{\rho}}H + \sqrt{1-\hat{\rho}}h_j \geq \Phi^{-1}(\varepsilon+v)\,|H\right) \\ &= (1+r)\,k^S \cdot \Phi\left(\tfrac{\sqrt{\hat{\rho}}H-\Phi^{-1}(\varepsilon+v)}{\sqrt{1-\hat{\rho}}}\right) \end{aligned} \tag{64}$$

If we normalize a_B by the face value of assets, the c.d.f. of normalized $\hat{a}_B$ is given by

$$\begin{aligned} F_B(u) &= \Pr(\hat{a}_B \leq u) \\ &= \Pr\left(H \leq \hat{a}_B^{-1}(u)\right) \\ &= \Phi\left(\hat{a}_B^{-1}(u)\right) \\ &= \Phi\left(\tfrac{\Phi^{-1}(\varepsilon+v)+\sqrt{1-\rho}\Phi^{-1}(u)}{\sqrt{\hat{\rho}}}\right) \end{aligned} \tag{65}$$

where $\hat{a}_B(H) \equiv a_B(H)/(1+r)\,k^S$.

Now suppose that the bank chooses the good portfolio consisting of loans to production chains with $v = 0$ and $\rho = 0$. Setting $v = 0$ and let $\rho \to 0$ in eq. (65), the good portfolio has the outcome distribution:

$$F_G(u) = \begin{cases} 0 & \text{if } u < 1-\varepsilon \\ 1 & \text{if } u \geq 1-\varepsilon \end{cases} \tag{66}$$

The good portfolio consists of i.i.d. loans, each of which has the default probability of ε, and the bank can fully diversify away credit risk. With a fully diversified loans, banks face the default of exactly ε fraction of borrowers. The realized value of the bank's portfolio is certain at $(1-\varepsilon)(1+r)\,k^S$.

The notional debt of the bank to depositors is $(1+f)\,d$. The debt of the bank normalized by the face value of assets, φ, is

$$\varphi \equiv (1+f)\,d/(1+r)\,k^S \tag{67}$$

which captures normalized leverage. We may consider φ as a measure of financial conditions, as higher φ corresponds to higher leverage and higher credit supply for given level of bank book equity.

More technically, φ can be seen as the strike price of the embedded option for the bank from limited liability. Let $\pi(\varphi)$ denote the value of the put option when the strike price is φ. Following Merton (1974), the bank's expected repayment to depositors is the repayment made in full in all states of the world (φ) minus the option value to default ($\pi(\varphi)$).

Then the expected payoffs of the bank is

$$E(\hat{a}) - [\varphi - \pi(\varphi)] \tag{68}$$

where $E(\hat{a})$ is the expected realization of the (normalized) loan portfolio.

The bank chooses d, k^S (equivalently, φ) and f so as to maximize its expected payoff (68) subject to the incentive compatibility constraint for the bank to choose the good portfolio, and the break-even constraint for depositors. If the expected payoff increases with leverage φ, the bank will increase leverage, but only until it hits the level that binds the incentive compatibility constraint.

The bank's incentive compatibility constraint to choose the good portfolio is

$$E_G(\hat{a}) - [\varphi - \pi_G(\varphi)] \geq E_B(\hat{a}) - [\varphi - \pi_B(\varphi)] \tag{69}$$

where the left-hand side is the expected payoff of the good portfolio and the right-hand side is that of the bad portfolio.

Denote the difference in option value to default by $\Delta\pi(\varphi) = \pi_B(\varphi) - \pi_G(\varphi)$, and note that $E_G(\hat{a}) - E_B(\hat{a}) = v$. Then eq. (69) can be written more simply as

$$\Delta\pi(\varphi) \leq v \tag{70}$$

The bank needs to keep leverage φ low enough that the higher option value to default of the bad portfolio does not exceed the greater expected payoff of the good portfolio.

From Breeden and Litzenberger (1978), the state price density is the second derivative of the option price with respect to its strike price. Using this, the difference in option value to default $\Delta\pi(\varphi)$ is given by

$$\Delta\pi(\varphi) = \begin{cases} \int_0^{\varphi} F_B(u)\, du & \text{if } \varphi < 1-\varepsilon \\ \\ \int_0^{1-\varepsilon} F_B(u)\, du - \int_{1-\varepsilon}^{\varphi} [1 - F_B(u)]\, du & \text{if } \varphi \geq 1-\varepsilon \end{cases} \tag{71}$$

Thus $\Delta\pi(\varphi)$ is single-peaked, reaching its maximum at $\varphi = 1-\varepsilon$. In addition, $\Delta\pi(\varphi)$ is increasing in leverage for $\varphi < 1-\varepsilon$, and $\Delta\pi(0) = 0$.

Note that

$$\begin{aligned} \Delta\pi(1) &= \int_0^1 [F_B(u) - F_G(u)]\, du \\ &= \int_0^1 [1 - F_G(u)]\, du - \int_0^1 [1 - F_B(u)]\, du \\ &= E_G(\hat{a}) - E_B(\hat{a}) = v \end{aligned} \tag{72}$$

that is, $\Delta\pi(\varphi)$ approaches v from above as $\varphi \to 1$. As $\varphi < 1$ for any bank with positive notional equity, there is a unique solution to $\Delta\pi(\varphi) = v$ in the range where $\Delta\pi(\varphi)$ is increasing. As $\Delta\pi(\varphi)$ is increasing in leverage for $\varphi < 1-\varepsilon$, the solution for φ^* satisfies $\varphi^* < 1-\varepsilon$. In sum, there is a unique level of (normalized) leverage φ^* that solves $\Delta\pi(\varphi) = v$, where $\varphi^* < 1-\varepsilon$. As such, the bank chooses the good portfolio and the leverage φ^* which is less than $1-\varepsilon$.

As a result of the bank's choice of good portfolio, the bank's probability of default becomes zero. Then the break-even constraint for depositors implies that the deposit rate of interest is equal to the risk-free rate, which is assumed to be zero: $f = r^f = 0$.

Using eq. (67) and the balance sheet identity $e + d = k^S$, we can solve for the bank's supply of credit, k^S, as

$$k^S = \frac{e}{1-(1+r)\varphi^*} \tag{73}$$

The total credit supply K^S across all banks is then given by:

$$K^S = \frac{me}{1-(1+r)\varphi^*} \tag{74}$$

where m is the number of banks in the world. This suggests that the global credit supply is increasing in r, e and φ^*. Especially, the credit supply increases with the bank lending rate r.

By combining the credit supply function given above (eq. (74)) with the credit demand function for financing working capital (eq. (62)), we can solve for the equilibrium borrowing rate r as the rate that clears the credit market. Any shock that reduces banking sector credit, such as credit losses that reduce bank equity e or an increase in hazard rate ε (which reduces leverage φ^*), will result in an upward shift of the credit supply curve, leading to an increase in the borrowing rate r. The increased borrowing rate will then kick in motion the combination of reduced productivity, reduced wages and lower offshoring activity described in Sections 2 and 3. We summarize our main result as follows.

Proposition 3 *A tightening of credit given by a decline in* φ *is associated with (1) an increase in the interest rate* r *(2) fall in productivity per worker, (3) fall in output (4) fall in equilibrium working capital, and (5) fall in the trade to output ratio.*

6 Concluding remarks

Fluctuations in financial conditions can have a substantial impact on macro and trade variables through their impact on the cost of working capital. Our results derive from the feature that production takes time and the operation of a production chain across national borders entails heavy demands on financing. One consequence of this feature is that long production chains and offshoring are sustainable only when credit is cheap, and chains that have become over-extended are vulnerable to financial shocks that raise the cost of borrowing.

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Appendix

A General financing cost in benchmark model

In the body of the text, we assumed for simplicity that the interest rate in the initial set-up phase is zero. In this appendix, we solve for the case with positive interest rates to demonstrate that our main results in the benchmark Austian model (Section 2) are unchanged in the general case.

Consider firm n which produces the most upstream stage good within a production chain that hires one worker for each stage. It borrows w to pay wages from a bank at date 1. The firm pays back $w(1+r_1)$ and borrows the same amount at date 2, where r_j is the interest rate at date j. It continues to roll over the principal and interest of the loan until date $n+1$. The value of the original loan of w becomes $w[\Pi_{j=1}^{n}(1+r_j)]$ at date $n+1$.

The firm engages in a second-round production and borrows w at date 2, and so on. As a result, total working capital financed by firm n at date $n+1$ is

$$\sum_{t=1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w \tag{75}$$

From date $n+1$ when the first final good of the production chain is sold, the firm receives the proceeds from its sales, with which it pays wages for the worker hired at the date and the interest for the working capital accumulated. Along the steady state, the firm rolls over the principal of the loan (75) but pays interest.

Firm i starts from date $n-i+1$. The working capital held by the firm at date $n+1$ is

$$\sum_{t=n-i+1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w$$

Then total working capital of the production chain that hires one worker for each stage at date $n+1$ is

$$\sum_{i=1}^{n}\sum_{t=n-i+1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w$$

In the steady state (that is, from date $n+1$ on), the production chain keeps rolling over the working capital.

There are L/n production chains of hiring one worker per stage, so that the aggregate demand for working capital along the steady state is given by

$$K = \left[\sum_{i=1}^{n} \sum_{t=n-i+1}^{n} (\Pi_{j=t}^{n}(1+r_j)) \right] w \cdot \frac{L}{n} \tag{76}$$

Suppose that the interest rate in the initial set-up phase is the same at $r_j = \hat{r}$ for all $j \leq n$. Then the expression inside the square bracket of eq. (76) can be rewritten as

$$\begin{aligned} \sum_{i=1}^{n} \sum_{t=n-i+1}^{n} [\Pi_{j=t}^{n}(1+r_j)] &= \sum_{i=1}^{n} (1+\hat{r}) \frac{(1+\hat{r})^i - 1}{\hat{r}} \\ &= \frac{(1+\hat{r})}{\hat{r}} \left(\frac{(1+\hat{r})^n - 1}{\hat{r}} (1+\hat{r}) - n \right) \end{aligned}$$

Using $(1+\hat{r})^n = 1 + \frac{n!}{1!(n-1)!}\hat{r} + \frac{n!}{2!(n-2)!}\hat{r}^2 + \frac{n!}{3!(n-3)!}\hat{r}^3 + ...\hat{r}^n$, we then have

$$\begin{aligned} K(n) &= \frac{(1+\hat{r})}{\hat{r}} \left(\frac{(1+\hat{r})^n - 1}{\hat{r}} (1+\hat{r}) - n \right) w \cdot \frac{L}{n} \\ &= \frac{(1+\hat{r})}{\hat{r}^2} Q(n) \cdot wL \end{aligned}$$

where $Q(n) = \left(\hat{r} + \frac{(n-1)}{2}\hat{r}^2 + \frac{(n-1)(n-2)}{6}\hat{r}^3 + ... + \hat{r}^{n-1} + \frac{1}{n}\hat{r}^n \right) (1+\hat{r}) - \hat{r}$.

Note that the interest rate $\hat{r}$ is so small that we may assume $\hat{r}^k$ for $k \geq 3$ goes to zero. Then we can approximate $Q(n)$ by $\frac{(n+1)}{2}\hat{r}^2$, which gives

$$K(n, \hat{r}) = (1+\hat{r}) \left(\frac{n+1}{2} \right) wL$$

This suggests that the aggregate working capital is a function of the length of production chain and the interest rate in the initial set-up period.

The firm seeks to maximize the profit along the steady state

$$\begin{aligned} \pi &= n^{\alpha} zL - wzL - rzK(n, \hat{r}) \\ &= \left[n^{\alpha} - w - r(1+\hat{r}) \left(\frac{n+1}{2} \right) w \right] zL \end{aligned}$$

where r is the interest rate in the steady state, which may differ or be equal to the interest rate in the initial set-up period $\hat{r}$.

The first-order condition for n gives

$$\alpha n^{\alpha-1} = r(\frac{1+\hat{r}}{2})w \tag{77}$$

The zero profit condition is

$$n^{\alpha} = \left[1 + r(1+\hat{r})\left(\frac{n+1}{2}\right)\right] w \tag{78}$$

From eqs. (77) and (78), the equilibrium chain length is

$$n = \frac{\alpha}{(1-\alpha)}\left(1 + \frac{2}{r(1+\hat{r})}\right)$$

which is reduced to $\frac{\alpha}{(1-\alpha)}\left(1+\frac{2}{r(1+r)}\right)$ in case with $\hat{r} = r$, and eq. (9) in the benchmark case with $\hat{r} = 0$.

This suggests that an increase in the interest rate, regardless of whether in the initial set-up period or the steady state, results in fall in the length of production chain. An increase in the interest rate in the set-up period leads to a shortening of production chain by increaing the steady state working capital $K(n, \hat{r})$, while that of the steady state interest rate does the same by raising the interest charged on the working capital r.

B Generalized offshoring model

In this appendix, we solve the model of offshoring in the general case where the interest rate in the initial set-up phase can be positive.

Consider firm i, which operates in the i-th from the most downstream among $\bar{n}+s+1$ stages of production/transportation of the global production chain. It begins production from date $\bar{n}+s-i+1$. The working capital that the firm holds at date $\bar{n}+s+1$ is

$$\sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} [\Pi_{j=t}^{\bar{n}+s}(1+r_j)]w$$

Total working capital of a global production chain that hires one worker for each stage at date $\bar{n}+s+1$ is given by

$$\sum_{i=1}^{\bar{n}+s} \sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} [\Pi_{j=t}^{\bar{n}+s}(1+r_j)]w$$

which the global production chain continues to roll over in the steady state.

The world's demand for working capital along the steady state is

$$K = \left[\sum_{i=1}^{\bar{n}+s} \sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} (\Pi_{j=t}^{\bar{n}+s}(1+r_j))\right] w \cdot \frac{L}{(\bar{n}+s)} \tag{79}$$

In an analogous way to the benchmark Austrian model case in Appendix A, we can show that with the assumption $r_j = \hat{r}$ for all $j \leq n$, the expression inside the square bracket of eq. (79) is simplified to

$$\frac{(1+\hat{r})}{\hat{r}}\left(\frac{(1+\hat{r})^{\bar{n}+s}-1}{\hat{r}}(1+\hat{r})-(\bar{n}+s)\right)$$

and we have

$$\begin{aligned} K(s) &= \frac{(1+\hat{r})}{\hat{r}}\left(\frac{(1+\hat{r})^{\bar{n}+s}-1}{\hat{r}}(1+\hat{r})-(\bar{n}+s)\right) w \cdot \frac{L}{(\bar{n}+s)} \\ &= \frac{(1+\hat{r})}{\hat{r}^2} Q(s) \cdot wL \end{aligned}$$

where $Q(s) = \left(\hat{r} + \frac{(\bar{n}+s-1)}{2}\hat{r}^2 + \frac{(\bar{n}+s-1)(\bar{n}+s-2)}{6}\hat{r}^3 + ... + \frac{1}{\bar{n}+s}\hat{r}^{\bar{n}+s}\right)(1+\hat{r}) - \hat{r}$.

Since $\hat{r}^k$ for $k \geq 3$ goes to zero, we can approximate $Q(s)$ by $\frac{(\bar{n}+s+1)}{2}\hat{r}^2$. Using this, we have

$$K(s,\hat{r}) = (1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right) wL$$

The global production chain chooses s to maximize the profit along the steady state

$$\begin{aligned} \pi &= (\bar{n}+bs)^{\alpha} zL - wzL - rzK(s,\hat{r}) \\ &= \left[(\bar{n}+bs)^{\alpha} - w - r(1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right) w\right] zL \end{aligned}$$

which yields the first-order condition for s

$$\alpha b(\bar{n}+bs)^{\alpha-1} = r(\frac{1+\hat{r}}{2})w \tag{80}$$

The zero profit condition gives

$$(\bar{n}+bs)^{\alpha} = \left[1 + r(1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right)\right] w \tag{81}$$

From eqs. (80) and (81), we derive the equilibrium extent of offshoring

$$s = \frac{\alpha}{(1-\alpha)}\left(1+\bar{n}+\frac{2}{r(1+\hat{r})}\right) - \frac{\bar{n}}{b(1-\alpha)}$$

which is expressed as $s = \frac{\alpha}{(1-\alpha)}\left(1+\bar{n}+\frac{2}{r(1+r)}\right) - \frac{\bar{n}}{b(1-\alpha)}$ in case where $\hat{r} = r$, and eq. (44) in the base case where $\hat{r} = 0$.

This tells us that an increase in the interest rate, regardless of whether it is in the set-up period or the steady state, results in fall in the extent of offshoring.

C Chain length with general production function

In this appendix, we derive the equilibrium condition to determine the optimal chain length in a generalized model with CES production function.

With the zero profit condition, the first-order condition for labor becomes

$$n^{(\alpha+1+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}-1}(l_i)^{-\rho-1} = (1+r(n-i))w \tag{82}$$

Substituting $i = n$ in eq. (82) yields

$$n^{(\alpha+1+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}-1}(l_n)^{-\rho-1} = w \tag{83}$$

From (82) and (83), we have the optimal labor allocation across stages

$$l_i = (1+r(n-i))^{-\frac{1}{1+\rho}} l_n \tag{84}$$

The first-order condition for chain length n is given by

$$\frac{\frac{d}{dn}[n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}} - \int_0^n (1+r(n-i))wl_i di]}{\int_0^n l_i di} - \frac{\pi(n)\frac{d}{dn}[\int_0^n l_i di]}{(\int_0^n l_i di)zL} = 0$$

Given the zero profit condition, the first-order condition for n simplifies to

$$\frac{d}{dn}[n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}}] = \frac{d}{dn}\int_0^n (1+r(n-i))wl_i di \tag{85}$$

Note that the left-hand side of (85) is

$$\begin{aligned}\frac{d}{dn}[n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}}] &= (\alpha+1+\frac{1}{\rho})n^{(\alpha+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}} \\ &\quad -(\frac{1}{\rho})n^{(\alpha+1+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}-1}(l_n)^{-\rho}\end{aligned} \tag{86}$$

Given (24), the first expression on the right-hand side of (86) is

$$(\alpha+1+\frac{1}{\rho})n^{(\alpha+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}} = (\alpha+1+\frac{1}{\rho})\frac{1}{n}\int_0^n (1+r(n-i))wl_i di$$

while (84) yields

$$\int_0^n (1+r(n-i))wl_i di = [\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}}di]wl_n$$

Hence, we have

$$(\alpha+1+\frac{1}{\rho})n^{(\alpha+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}} = (\alpha+1+\frac{1}{\rho})\frac{1}{n}[\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}}di]wl_n \tag{87}$$

With (83), the second expression on the right-hand side of (86) is given by

$$(\frac{1}{\rho})n^{(\alpha+1+\frac{1}{\rho})}[\int_0^n (l_i)^{-\rho}di]^{-\frac{1}{\rho}-1}(l_n)^{-\rho} = (\frac{1}{\rho})wl_n \tag{88}$$

Meanwhile, the right-hand side of (85) is

$$\begin{aligned}\frac{d}{dn}[w\int_0^n (l_i+rnl_i-ril_i)di] &= w(l_n+r\int_0^n l_i di + rnl_n - rnl_n) \\ &= wl_n + rw\int_0^n l_i di\end{aligned}$$

which, together with (84), results in

$$\frac{d}{dn}[w\int_0^n (l_i+rnl_i-ril_i)di] = [1+r\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}}di]wl_n \tag{89}$$

Using eqs. (87), (88) and (89), the first-order condition for n simplifies to

$$(\alpha+1+\frac{1}{\rho})(\frac{1}{n})\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}}di - \frac{1}{\rho} = 1+r\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}}di$$

which is eq. (26) in Section 5. This equation can be solved for n as a function of the CES substitution parameter ρ and the interest rate r.

Consider how n is determined in the case where $\rho \to \infty$ (Leontief production function). In this case, eq. (84) gives

$$l_i = l_n$$

which suggests that the number of workers employed are the same across stages. Hence we can interpret this case as our benchmark case in Section 2.

In this Leontief function case, eq. (26) becomes

$$(\alpha+1)(\frac{1}{n})\int_0^n (1+r(n-i))di = 1+r\int_0^n 1di$$

which, given that $\int_0^n(1+r(n-i))di = n+\frac{rn^2}{2}$, yields the optimal chain length as

$$n = \frac{\alpha}{1-\alpha}\left(\frac{2}{r}\right)$$

This is a continuous time equivalent of the optimal n in eq. (9) of our benchmark model, indicating the negative effect of the interest rate on the length of the production chain.

Now consider the case where $\rho < \infty$. By calculating two integrals in eq. (26) and applying Taylor's expansion for the interest rate r which is close to zero, we have

$$\int_0^n (1 + r(n-i))^{\frac{\rho}{1+\rho}} di = \frac{(1+rn)^{\frac{\rho}{1+\rho}+1} - 1}{r(\frac{\rho}{1+\rho}+1)} \simeq n + \frac{1}{2}(\frac{\rho}{1+\rho})rn^2$$

and

$$\int_0^n (1 + r(n-i))^{-\frac{1}{1+\rho}} di = \frac{(1+rn)^{-\frac{1}{1+\rho}+1} - 1}{r(-\frac{1}{1+\rho}+1)} \simeq n - \frac{1}{2}(\frac{1}{1+\rho})rn^2$$

Using the two equations, we can express eq. (26) as

$$(\alpha + 1 + \frac{1}{\rho})(1 + \frac{1}{2}(\frac{\rho}{1+\rho})rn) - \frac{1}{\rho} = 1 + rn - \frac{1}{2}(\frac{1}{1+\rho})r^2n^2$$

which yields a quadratic equation for n with finite ρ

$$\frac{r^2}{1+\rho}n^2 + \frac{((\alpha-1)\rho - 1)r}{1+\rho}n + 2\alpha = 0 \tag{90}$$

Eq. (90) can be solved to determine the equilibrium value of n. The equation may yield two unrealistice corner solutions for n, namely zero and infinity, depending on the parameter values. For instance, when the parameter α, representing the impact of roundaboutness on productivity equals zero, the optimal soultion for n is zero ($n = 0$). To ensure an interior solution, we impose the following conditions:

$$0 < \alpha \; < 1 + \frac{1}{\rho} \quad \text{and} \quad (\frac{(\alpha-1)\rho - 1}{1+\rho})^2 - \frac{8\alpha}{1+\rho} \geq 0$$

Additionally, we assume that for a technologically feasible upper bound on the chain length (η), the following inequality holds

$$\frac{r^2}{1+\rho}\eta^2 + \frac{((\alpha-1)\rho - 1)r}{1+\rho}\eta + 2\alpha < 0$$

Under the conditions, the coefficient of n is negative, while those of n^2 and the constant term are positive. Consequently, the quadratic equation yields two positive solutions for n, and the smaller of the two is identified as the optimal solution, satisfying the second-order condition. The optimal solution is given by

$$n = \left[\frac{-(\frac{(\alpha-1)\rho-1}{1+\rho}) - [(\frac{(\alpha-1)\rho-1}{1+\rho})^2 - \frac{8\alpha}{1+\rho}]^{\frac{1}{2}}}{\frac{2}{1+\rho}} \right] \left(\frac{1}{r} \right) \tag{91}$$

which holds for any finite ρ. For example, in Cobb-Douglas case of $\rho \to 0$, the optimal n is

$$n = \left[\frac{1 - (1 - 8\alpha)^{\frac{1}{2}}}{2} \right] \left(\frac{1}{r} \right)$$

This indicates that an increase in the interest rate leads to a fall in the length of the production chain for any value of ρ.

D Impact of interest rate in CES economy

We will now investigate the impact of the interest rate on equilibrium productivity, GDP, wage rate, and aggregate working capital. Utilizing equation (84), we can express the equilibrium productivity and GDP as:

$$\begin{aligned} A(n) &= \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho} di]^{-\frac{1}{\rho}}}{\int_0^n l_i di} \\ &= \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}} di]^{-\frac{1}{\rho}} l_n}{\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}} l_n di} \end{aligned} \tag{92}$$

and

$$Y = \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}} di]^{-\frac{1}{\rho}} l_n}{\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}} l_n di} L \tag{93}$$

The equibrium wage rate is determined by

$$\begin{aligned} w &= \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (l_i)^{-\rho} di]^{-\frac{1}{\rho}}}{\int_0^n (1+r(n-i)) l_i di} \\ &= \frac{n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}} di]^{-\frac{1}{\rho}}}{\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}} di} \\ &= n^{\alpha+1+\frac{1}{\rho}}[\int_0^n (1+r(n-i))^{\frac{\rho}{1+\rho}} di]^{-(\frac{1}{\rho}+1)} \end{aligned} \tag{94}$$

The working capital is expressed as

$$K = \frac{w\int_0^n (n-i)l_i di}{\int_0^n l_i di} \times L$$
$$= w\frac{\int_0^n (n-i)(1+r(n-i))^{-\frac{1}{1+\rho}}di}{\int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}}di} \times L \quad (95)$$

In the case where $\rho \to \infty$, the aforementioned formulations provide closed-form solutions for the equilibrium productivity, GDP, the wage rate, and the aggregate working capital:

$$A(n) = n^\alpha = \left(\frac{\alpha}{1-\alpha}\right)^\alpha \left(\frac{2}{r}\right)^\alpha$$

$$Y = n^\alpha L = \left(\frac{\alpha}{1-\alpha}\right)^\alpha \left(\frac{2}{r}\right)^\alpha L$$

$$w = \frac{n^\alpha}{1+\frac{rn}{2}} = (1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^\alpha \left(\frac{2}{r}\right)^\alpha$$

$$K = \frac{n^{\alpha+1}}{2+rn} \times L = \left(\frac{\alpha}{1-\alpha}\right)^\alpha \left(\frac{2}{r}\right)^\alpha \left(\frac{\alpha}{r}\right) L$$

These closed-form solutions suggest that productivity $A(n)$, GDP Y, the wage rate w and total working capital K all decrease with the interest rate r.

Now, let's consider a general case where $\rho < \infty$. Applying Taylor's expansion to (92), (93) and (94), we obtain

$$A(n) = \frac{n^\alpha[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}}{1-\frac{1}{2}(\frac{1}{1+\rho})rn}$$

$$Y = \frac{n^\alpha[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}}{1-\frac{1}{2}(\frac{1}{1+\rho})rn}L$$

$$w = \frac{n^\alpha}{[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}}$$

Note that

$$\begin{aligned}
& \int_0^n (n-i)(1+r(n-i))^{-\frac{1}{1+\rho}} di \\
= & \frac{1}{r}[\int_0^n (1+r(n-i))^{1-\frac{1}{1+\rho}} di - \int_0^n (1+r(n-i))^{-\frac{1}{1+\rho}} di] \\
= & \frac{1}{r}[(n+\frac{1}{2}(\frac{\rho}{1+\rho})rn^2) - (n-\frac{1}{2}(\frac{1}{1+\rho})rn^2)] \\
= & \frac{1}{2}n^2
\end{aligned}$$

Using this equation, we can simplify (95) to

$$\begin{aligned}
K &= \frac{n^\alpha}{[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}}[\frac{\frac{1}{2}n^2}{n-\frac{1}{2}(\frac{1}{1+\rho})rn^2}] \times L \\
&= \frac{n^{\alpha+1}}{2[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}(1-\frac{1}{2}(\frac{1}{1+\rho})rn)} \times L
\end{aligned}$$

Note that the fomulations suggest that $A(n)$, Y, w and K are not affected by rn, as evident from (91). Therefore, the interest rate affects these variables through n^α or $n^{\alpha+1}$, both of which decrease with the interest rate. Utilizing this, we can esablish that

$$\frac{dA(n)}{dr} = \alpha n^{\alpha-1}\frac{[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}}{1-\frac{1}{2}(\frac{1}{1+\rho})rn}(\frac{dn}{dr}) < 0$$

$$\frac{dY}{dr} = \alpha n^{\alpha-1}\frac{[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{-\frac{1}{\rho}}L}{1-\frac{1}{2}(\frac{1}{1+\rho})rn}(\frac{dn}{dr}) < 0$$

$$\frac{dw}{dr} = \frac{\alpha n^{\alpha-1}}{[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}}(\frac{dn}{dr}) < 0$$

$$\frac{dK}{dr} = \frac{(\alpha+1)n^\alpha L}{2[1+\frac{1}{2}(\frac{\rho}{1+\rho})rn]^{(\frac{1}{\rho}+1)}(1-\frac{1}{2}(\frac{1}{1+\rho})rn)}(\frac{dn}{dr}) < 0$$

This suggests that, for any values of CES substitution paprameter ρ, productivity, output, wage rate, and working capital all decrease with the interest rate r.

E Derivation of equations (55) and (56)

The first-order condition for the number of offshored stages s, combined with the zero profit condition, is given by

$$\begin{aligned} & \frac{d}{ds}[(\bar{n}+bs)^{\alpha}(\bar{n}+s)^{1+\frac{1}{\rho}}[\int_0^{\bar{n}+s}(l_i)^{-\rho}di]^{-\frac{1}{\rho}}] \\ = & \frac{d}{ds}\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))wl_idi \end{aligned} \tag{96}$$

The left-hand side of (96) is

$$\begin{aligned} & \frac{d}{ds}[(\bar{n}+bs)^{\alpha}(\bar{n}+s)^{1+\frac{1}{\rho}}[\int_0^{\bar{n}+s}(l_i)^{-\rho}di]^{-\frac{1}{\rho}}] \\ = & [b\alpha(\bar{n}+bs)^{-1}+(1+\frac{1}{\rho})(\bar{n}+s)^{-1}](\bar{n}+bs)^{\alpha}(\bar{n}+s)^{1+\frac{1}{\rho}}[\int_0^{\bar{n}+s}(l_i)^{-\rho}di]^{-\frac{1}{\rho}} \\ & -(\frac{1}{\rho})(\bar{n}+bs)^{\alpha}(\bar{n}+s)^{1+\frac{1}{\rho}}[\int_0^{\bar{n}+s}(l_i)^{-\rho}di]^{-\frac{1}{\rho}-1}(l_{\bar{n}+s})^{-\rho} \end{aligned}$$

which simplifies to

$$[b\alpha(\bar{n}+bs)^{-1}+(1+\frac{1}{\rho})(\bar{n}+s)^{-\frac{1}{\rho}}]^{-1}[\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{\frac{\rho}{1+\rho}}di]wl_{\bar{n}+s}-(\frac{1}{\rho})wl_{\bar{n}+s}$$

The right-hand side of (96) is

$$\begin{aligned} & \frac{d}{ds}\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))wl_idi \\ = & w(l_{\bar{n}+s}+r\int_0^{\bar{n}+s}l_idi+r(\bar{n}+s)l_{\bar{n}+s}-r(\bar{n}+s)l_{\bar{n}+s}) \\ = & [1+r\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{-\frac{1}{1+\rho}}di]wl_{\bar{n}+s} \end{aligned}$$

Consequently, the first-order condition for n (96) simplifies to

$$\begin{aligned} & [b\alpha(\bar{n}+bs)^{-1}+(1+\frac{1}{\rho})(\bar{n}+s)^{-1}]\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{\frac{\rho}{1+\rho}}di-\frac{1}{\rho} \\ = & 1+r\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{-\frac{1}{1+\rho}}di \end{aligned}$$

which is eq. (55).

We rearrange (55) as

$$\begin{aligned} & b\alpha(\bar{n}+bs)^{-1}\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{\frac{\rho}{1+\rho}}di \\ = & \frac{1}{\rho}+1+r\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{-\frac{1}{1+\rho}}di \\ & -(1+\frac{1}{\rho})(\bar{n}+s)^{-1}\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{\frac{\rho}{1+\rho}}di \end{aligned} \tag{97}$$

Applying Taylor's expansion, the left-hand side of eq. (97) can be expressed as

$$\begin{aligned} & b\alpha(\bar{n}+bs)^{-1}\int_0^{\bar{n}+s}(1+r(\bar{n}+s-i))^{\frac{\rho}{1+\rho}}di \\ = & b\alpha(\bar{n}+bs)^{-1}[(\bar{n}+s)+(\frac{\rho}{1+\rho})\frac{r(\bar{n}+s)^2}{2}] \\ = & [b\alpha+b\alpha(\frac{\rho}{1+\rho})\frac{r\bar{n}}{2}+b\alpha(\frac{\rho}{1+\rho})\frac{r}{2}s](\bar{n}+bs)^{-1}(\bar{n}+s) \end{aligned}$$

and the right-hand side of eq. (97) is

$$\begin{aligned} & 1+\frac{1}{\rho}+r[(\bar{n}+s)-\frac{1}{2}(\frac{1}{1+\rho})r(\bar{n}+s)^2]-(1+\frac{1}{\rho})[1+(\frac{\rho}{1+\rho})\frac{r(\bar{n}+s)}{2}] \\ = & r(\bar{n}+s)-\frac{1}{2}(\frac{1}{1+\rho})r^2(\bar{n}+s)^2-\frac{r(\bar{n}+s)}{2} \\ = & (\frac{r}{2}-\frac{1}{2}(\frac{1}{1+\rho})r^2(\bar{n}+s))(\bar{n}+s) \end{aligned}$$

Hence eq. (97) simplifies to

$$b\alpha+b\alpha(\frac{\rho}{1+\rho})\frac{r\bar{n}}{2}+b\alpha(\frac{\rho}{1+\rho})\frac{r}{2}s=r\frac{1}{2}(\bar{n}+bs)-\frac{1}{2}(\frac{1}{1+\rho})r^2(\bar{n}+s)(\bar{n}+bs)$$

This results in a quadratic equation for s

$$(\frac{b}{1+\rho})s^2+[b\alpha(\frac{\rho}{1+\rho})\frac{1}{r}+(\frac{1+b}{1+\rho})\bar{n}-\frac{1}{r}b]s+b\alpha\frac{2}{r^2}+b\alpha(\frac{\rho}{1+\rho})\frac{1}{r}\bar{n}-\frac{1}{r}\bar{n}+(\frac{1}{1+\rho})\bar{n}^2=0$$

This equation can be expressed in the form:

$$(\frac{b}{1+\rho})s^2-g(r)s+h(r)=0$$

where $g(r)$ and $h(r)$ are as defined in eq. (56).

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