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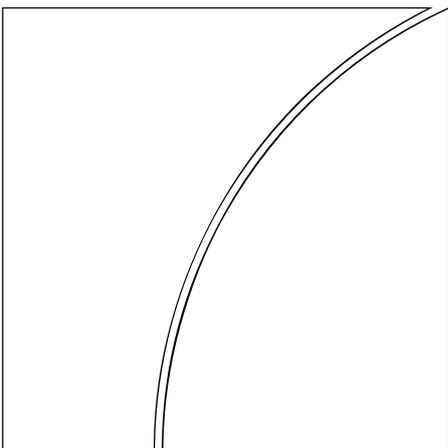
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Bank runs, welfare and policy implications

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Abstract

This paper proposes a model in which bank runs are closely related to the state of the business cycle. The benchmark model shows that, in a market economy, there are welfare losses due to the existence of bank runs. Extensions of the model explore the welfare effects of various government policies. The results suggest that an interest-cap deposit insurance scheme is an efficient policy to prevent bank runs, while other policies, including the suspension of convertibility, a penalty on short-term deposits and full-coverage deposit insurance schemes, will all have adverse side effects.

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1 Introduction¹

The banking sector is vulnerable to bank runs because, by nature, banks issue liquid liabilities but invest in illiquid assets. When a bank run occurs, agents rush to the banks and withdraw their funds as quickly as possible. Banks are driven into bankruptcy due to liquidity problems. The breakdown of the banking industry distorts capital allocation and in most situations adds downward pressure to the real economy.

Historically, bank runs occurred frequently in Europe in the 19th century, and plagued the United States until the reform of the Federal Reserve System after the crisis of 1933. Over the past two or three decades, the bank run phenomenon has hit most emerging countries (see Lindgren et al 1996). Recent work by Kaminsky and Reinhart (1999, 2000) suggests a new phenomenon since the 1980s in that bank runs have also played a very important role in the so-called “twin crises” episodes.

Given the frequent occurrence of bank runs and the associated destabilising costs, various policy instruments have been implemented to avoid the undesirable phenomena. In early time, policy-makers paid more attention on crisis resolution, or, how to stop a bank runs once it occurs. Such policy instruments include the suspension of convertibility of deposits and a penalty on short-term deposits (see Dwyer and Gilbert 1989). More recently, the policymakers have shifted their emphasis to crisis prevention, or, how to minimise the possibility of bank runs without hurting the banking sector’s role in providing liquidity. The proposed policies include holding appropriate provisions and capital reserves, strengthening banks’ self-regulation, and designing deposit insurance schemes (see FSF 2001). In this paper, I try to explore the welfare effects of these policy instruments. The question will be addressed in two parts. First, what are the effects of these policies in stopping bank runs? Second, and more importantly, what are the *ex ante* effects of these policies if their existence has been taken into account by the market?

To start the analysis, it is important to explain the microeconomic underpinnings of bank runs. There are two general views. One group of economists, including Diamond and Dybvig (1983),

¹This paper is part of my thesis at Duke University. I thank Craig Furfine, Philip Lowe, Enrique Mendoza, Pietro Peretto, Kostas Tsatsaronis, Diego Valderrama and Lin Zhou for helpful comments. I am also grateful for helpful suggestions from the seminar participants at Duke University, City University of Hong Kong, the BIS and the International Conference on Economic Globalisation. All errors remain mine.

Cooper and Ross (1998), Chang and Velasco (2000, 2001), Park (1997) and Jeitschko and Taylor (2001), consider bank runs as self-fulfilling prophecies, unrelated to the state of the real economy. There exist two equilibria in the banking sector. On the one hand, if no agent expects that a bank run will happen, the risk-sharing mechanism provided by the banking sector functions well and the economic resources are allocated in an efficient way. On the other hand, if all agents anticipate a bank run, then they all have the incentive to withdraw their deposits immediately and a bank run occurs as expected. Which of the two equilibria happens depends on the expectations of agents, which, unfortunately, are not addressed in their models.

The second view, as reflected in the empirical studies of Gorton (1988), Calomiris and Gorton (1991), and recent work by Allen and Gale (1998), Zhu (2001) and Goldstein and Pauzner (2000), considers bank runs as a phenomenon closely related to the state of the business cycle. Allen and Gale (1998) show that the business cycle plays an important role in generating banking crises. They also show that bank runs can be first-best efficient and central bank intervention may be undesirable in some situations. Zhu (2001) develops a two-stage model in which agents make withdrawal decisions sequentially. He shows that bank runs happen only when agents perceive a low return on bank assets, and banks may deliberately choose a bank-run contract over a run-proof alternative. Goldstein and Pauzner (2000) show that when agents receive slightly noisy signals regarding the fundamentals, the economy will feature a unique equilibrium in which the occurrence of bank runs is determined by the state of the business cycle.

This paper follows the business cycle origin model developed in Zhu (2001) for three major reasons. First, the model predicts that the occurrence of bank runs is related to economic fundamentals rather than a “sunspot phenomenon”. This prediction is consistent with recent empirical studies. Second, the model features a unique equilibrium and the probability of bank runs can be endogenously determined. This property eliminates the undesirable indeterminacy in the analysis. Third, and most importantly, the two-stage framework in the model allows us to study how the banks and agents will react to the government policies (or, the *ex ante* effects), which is absent from much of the existing literature.

The benchmark model illustrates that the banking sector provides a risk-sharing mechanism against the uncertainty in depositors’ liquidity needs. However, it can also be the source of insta-

bility because of the potential bank run problems. Hence, the equilibrium in a market economy is suboptimal.

Extensions of the model explore the welfare implications of various government policies. The main results are as follows. First, suspension of convertibility of deposits is both *ex post* and *ex ante* inefficient in preventing runs because it cannot distinguish between those with true liquidity needs and those who are running on the banks. Therefore, although bank runs are successfully stopped, it is very likely that some agents with true liquidity needs cannot withdraw their deposits in a timely manner, while other agents who do not have genuine liquidity needs will have their deposits repaid.

Second, taxation on short-term deposits, although it can affect both the quantity and composition of early withdrawals, introduces investment distortions to the economy and turns out to be inefficient as well.

Third, deposit insurance is an *ex post* efficient policy in preventing bank runs, but it is *ex ante* inefficient due to the “moral hazard” problems. Because the deposit insurance authority cannot monitor the banks’ investment behaviour perfectly, banks always have the incentive to behave aggressively by offering high interest rates. However, this paper proposes that substituting the full-coverage deposit insurance scheme with an interest-cap deposit insurance scheme can overcome this moral hazard problem and help the economy to achieve the socially optimal outcome.

Fourth, imposition of a capital requirement, or equivalently an capital/asset ratio requirement, is an efficient policy to prevent bank runs in the limit. As the capital requirement increases, the market equilibrium gradually converges to the social optimum. The problem is, however, that the capital requirement might be very high.

The remainder of this paper is organised as follows. Section 2 develops the benchmark model and defines the competitive equilibrium in the market. Section 3 analyses the welfare properties of the competitive equilibrium in comparison with two types of socially optimal allocations. Section 4 discusses the welfare effects of five different policies: suspension of convertibility of deposits, taxation on short-term deposits, full-coverage deposit insurance schemes, interest-cap deposit insurance schemes and capital requirements. Section 5 concludes.

2 Benchmark model

The benchmark model is based on the framework developed in Zhu (2001). There are three periods: $T = 0, 1, 2$. Two investment technologies are available in period 0: a storage technology and a risky technology. The storage technology is riskless: it yields a constant return of 1 in period 1 or 2. The risky asset yields a long-term return of \tilde{R} which has a support $[0, \infty]$. Besides, the risky asset is illiquid in that its liquidation value in period 1 is $(1 - \tau)\tilde{R}$.² The risky asset is more productive in the long run but less efficient in the short run on the assumption that $(1 - \tau)E(\tilde{R}) < 1 < E(\tilde{R})$.³

There are a large but finite number (N) of ex ante identical agents. Each agent is endowed with one unit of goods in period 0. Agents are subject to a preference shock in the interim period. A fraction (α) of these agents turn out to be impatient, implying that they derive utility from period 1 consumption only; the others ($1 - \alpha$) will be patient, who only care for period 2 consumption. Their utility functions are

$$u^1(c_1, c_2) = u(c_1) \tag{2.1}$$

$$u^2(c_1, c_2) = u(c_2) \tag{2.2}$$

respectively, where $u(\cdot)$ satisfies $u(0) = 0$, $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The banking sector is perfectly competitive. In period 0, banks compete with each other by offering demand-deposit contracts which specify a short-term interest rate (r_1) and a long-term interest rate (r_2). Individual agents then decide whether to deposit their endowments with the bank or not.⁴ After receiving the deposits, each bank chooses its optimal portfolio allocation between the safe asset ($1 - i$) and the risky asset (i).

In the interim period, the uncertainties in consumer types and asset returns are resolved. Each agent learns his own preference type, and he also receives a publicly observable signal (s) that

²The determination of liquidation value is exogenous in this paper. Some existing papers, such as Krugman (1998b) and Backus et al (1999), may shed light on future study in this direction. This paper employs the proportional liquidation value out of two considerations. First, due to a liquidity crunch, the assets are always sold at a lower price. Second, since the information is perfect in this model, the liquidation value of an asset should be associated with its true value.

³This is a necessary condition. A sufficient condition is $E[u[(1 - \tau)\tilde{R}]] < u(1) < E[u(\tilde{R})]$, where $u(\cdot)$ is the utility function for a representative agent.

⁴To simplify the algebra, I assume each agent has only two choices: either to deposit all his endowments or to deposit nothing. The main conclusions in this paper remain robust when agents are allowed to deposit a fraction of their endowments. Besides, if agents are indifferent between two contracts, they randomly pick up the deposit bank.

correctly reveals the return on the risky asset ($s = R$).⁵ Based on the public and private information, agents make withdrawal decisions sequentially. This sequential decision rule has a twofold meaning. First, at a certain time, only one agent is allowed to withdraw his deposit from the bank. Second, each agent can observe the actions of all agents ahead of him before he chooses his action.

Banks pay the short-term interest rate to the early consumers according to the “first come, first served” rule in period 1.⁶ In period 2, all late consumers share the remaining assets of the banks with a maximum payment of r_2 .

Lemma 1 (*Uniqueness of equilibrium outcome*) *Given the sequential decision rule, for a certain demand-deposit contract (r_1, r_2) and portfolio structure (i) , there is a unique equilibrium outcome. Equilibrium aggregate early withdrawal amount, L ,⁷ equals α when $R \geq \frac{r_1 - 1 + i}{i}$ and $L = 1$ otherwise.*

Proof: see Appendix A.

Lemma 1 states that, under a given contract, a bank run happens if and only if agents perceive a low return on bank assets. The intuition is as follows: if patient agents know that the economy is in a good state, they will have no incentive to run on the bank. The sequential decision rule provides a mechanism through which the patient agents can coordinate their actions. The first patient agent, knowing that the followers can observe his action, will choose to “wait” and send the signal “I am a patient agent and I am expecting the other patient agents not to run on the bank”. Observing this signal, the followers also choose to “wait”. Therefore, the sequential decision rule provides an equilibrium-selection mechanism in the Diamond-Dybvig framework and only the Pareto efficient outcome is chosen in equilibrium.

Lemma 1 also implies that the probability of bank runs can be endogenously determined by contract variables. Specifically, there will be no bank runs under contracts that feature $r_1 \leq 1 - i$. This is quite intuitive because the run-proof contracts penalise sufficiently early withdrawals and therefore never have the liquidity problem in the interim period.

In a two-stage game, banks are allowed to choose the interest rates (r_1 and r_2) and portfolio

⁵This paper only studies the case in which the information is perfect. When the information is imperfect, the banks are more vulnerable to runs, but the welfare analysis in the paper remains valid. See Zhu (2001).

⁶The sequential service constraint is not essential in the model. As shown in Zhu (2001), removing this sequential service constraint does not change the equilibrium properties.

⁷ L is defined as the proportion of early withdrawers with a bank.

structure (i) taking into account the withdrawal strategies of agents as specified in Lemma 1. In a competitive market, the competitive equilibrium can be defined by using the backward induction method.

Definition 1 *The competitive equilibrium contract (r_1^*, r_2^*, i^*) in the market economy should satisfy the following conditions:*

1. *Withdrawal decisions under a certain contract are as specified in Lemma 1.*
2. *i^* is chosen to maximise the bank's expected profit for a given (r_1, r_2) .*
3. *Agents deposit their endowments with the bank that offers the best contract (with the highest expected utility).*
4. *Interest rates (r_1^*, r_2^*) are chosen to maximise the bank's expected profit.*

In a market economy, the above problem is equivalent to “banks choose the best contract for a representative agent subject to the constraint that expected profit is zero”.⁸ The result is intuitive and can be easily shown by the Duality Theorem. A representative bank must make zero profit in the market equilibrium. If the profit is positive, at least one bank will deviate by offering a higher interest rate and it will win all deposits. This bid-up process will continue until all banks' profits are driven to zero.

This zero-profit property implies that, in a market economy, the effective long-term interest rate is determined by dividing the remaining resources among late consumers and therefore is state-contingent.⁹ In other words, financial intermediaries only provide a risk-sharing mechanism against the idiosyncratic risk but cannot diversify the aggregate risk in the economy.

Accordingly, the banks' optimisation problem is as follows:

Lemma 2 *The equilibrium contract in a market economy can be characterised by the following problem:*

$$\begin{aligned} \max_{r_1, i} \quad & \int_0^{R^*} \frac{1-i+iR(1-\tau)}{r_1} u(r_1) f(R) dR + \int_{R^*}^{\infty} [\alpha u(r_1) + (1-\alpha)u(r_2^e(R))] f(R) dR & (2.3) \\ \text{s.t.} \quad & 1-i \geq r_1 \alpha \end{aligned}$$

⁸Notice that the bank's expected profit is inversely related to a representative agent's expected utility.

⁹As discussed in Section 4, when the banks have their own stake (capital assets or deposit insurance premium), the specification of the long-term interest rate becomes much more important. Banks will make zero *expected* profit rather than make zero profit *in all states*.

$$R^* \equiv \frac{r_1 - 1 + i}{i}$$

$$r_2^e = \frac{1 - i + iR - r_1\alpha}{1 - \alpha}$$

The objective function consists of two parts. The first part refers to the case where the asset return is low and a complete bank run is unavoidable. Banks liquidate all their assets and pay the depositors according to the sequential service rule. Only a fraction $(\frac{1-i+iR(1-\tau)}{r_1})$ of agents are fully paid and the rest receive nothing. The second part corresponds to the case in which everyone knows that the economy is in a good state and no bank run occurs. Banks do not need to liquidate their productive assets in the interim period. Impatient agents receive the short-term interest rate and patient agents share the remaining assets in period 2.

The first constraint is the budget constraint in period 1, which requires that the minimum holding of safe assets be the expected short-term liabilities to impatient agents. This is quite intuitive because if the banks hold fewer safe assets, then they will always have to liquidate some risky assets in the interim period. Since the risky asset is less productive in the short run, this portfolio choice is never optimal. Another implication from the first constraint is that the banks might have an incentive to hold extra safe assets $(1 - i - r_1\alpha)$. Holding extra liquidity has two opposite effects. (1) It could be welfare-improving for two reasons. First, since the probability of default (bank runs) is decreasing in i , an extra liquidity holding will make the banking sector less vulnerable to runs. Second, if a bank run happens, holding more safe assets will reduce the liquidation costs. (2) However, holding extra liquidity is costly in that when no bank run occurs, the late consumers will receive a lower payment because the safe asset is less productive in the long run. How much extra liquidity the banks should hold depends on which effect plays a dominant role.

The second constraint defines the threshold return below which a bank run is inevitable as indicated by Lemma 1. The third constraint specifies the payoff for the late consumers by using the zero-profit property.

3 Socially optimal allocation

In this section, I introduce two types of socially optimal allocations and compare them with the equilibrium outcome in a market economy. Given the existence of two kinds of risks (the idiosyn-

cratic risk in preference type and the aggregate risk in asset return) in the economy, a social planner can provide a risk-sharing mechanism through setting up a national bank and allocating the resources according to the true liquidity needs. Depending on different types of risks being involved, I define two types of socially optimal allocations.

Definition 2 (*First-best allocation*) *A first-best allocation is defined in the following problem:*

$$\begin{aligned} \max_{r_1, i} \quad & \alpha u(r_1) + (1 - \alpha)u(r_2) \\ \text{s.t.} \quad & 1 - i \geq r_1 \alpha \\ & r_2 = \frac{1 - i + i \cdot E(\tilde{R}) - r_1 \alpha}{1 - \alpha} \end{aligned} \tag{3.1}$$

In the first-best allocation, the social planner pays each agent according to his true preference type. No costly liquidation occurs in the interim period. The social planner chooses the optimal demand-deposit contract and investment portfolio to maximise the representative agent's expected utility. The constraint equations refer to the budget constraints for the social planner in period 1 and period 2, respectively.

An important feature of the first-best allocation is that, in the period 2, the social planner actually provides a smoothing device across the state of the business cycle as the long-term interest rate is non-state-contingent. One possible explanation is that the social planner can subsidise the interest payments in bad states by using the profits in good states if it has enough reserve assets.¹⁰ As a result, both the idiosyncratic risk and the aggregate risk are diversified.

Lemma 3 *The first-best allocation is characterised by:*

$$u'(r_1^f) = E(\tilde{R}) \cdot u'(r_2^f) \tag{3.2}$$

$$\begin{aligned} i^f &= 1 - r_1^f \alpha \\ r_2^f &= \frac{i^f \cdot E(\tilde{R})}{1 - \alpha} \end{aligned} \tag{3.3}$$

Proof: see Appendix B.

¹⁰Another possible explanation is that each bank is confronted with individual investment risk. Therefore, there is no aggregate investment risk in the economy. A social planner can offer a risk-sharing scheme among all banks by using the profits from high-return banks to subsidise the low-return banks.

Equation (3.2) is the familiar Euler equation for an optimal contract, which balances the marginal cost and marginal benefit of changing interest rates in equilibrium. Equation (3.3) implies that the social planner should hold a minimum amount of safe assets for interim payment, and invest all the remaining deposits in the more productive technology. This is not surprising because there is no bank run in the socially optimal contract and therefore extra liquidity holding is undesirable.

If we now impose the restriction that the social planner cannot smooth the consumption across the state of the business cycle, the socially optimal allocation is defined as follows:

Definition 3 (*Second-best allocation*) *A second-best allocation¹¹ is defined in the following problem:*

$$\begin{aligned} \max_{r_1, i} \quad & E_R[\alpha u(r_1) + (1 - \alpha)u(r_2(R))] & (3.4) \\ \text{s.t.} \quad & 1 - i \geq r_1\alpha \\ & r_2(R) = \frac{1 - i + iR - r_1\alpha}{1 - \alpha} \end{aligned}$$

In the second-best allocation, all agents withdraw their deposits according to their true liquidity needs. The risk in idiosyncratic preference types is diversified through the contract. However, the aggregate risk still exists because the amount of total assets varies across the state of the business cycle. In contrast to the first-best contract, the long-term interest payment is state-contingent, and the social planner is required to break even in *every* state.

The second-best allocation should satisfy the following first-order condition:

$$u'(r_1^s) = E[u'(r_2^s(R)) \cdot R] \quad (3.5)$$

Proposition 1 *Comparing the above three allocations, a representative agent obtains the highest welfare in the first-best allocation and the lowest welfare in the competitive equilibrium contract. In other words, the equilibrium in the market economy is suboptimal.*

Proof: see Appendix C.

¹¹I am misusing the term “second-best” in this paper. It is different from the standard definition in welfare economics. Here I am using it to emphasise the fact that this allocation diversifies only the idiosyncratic risk in the economy.

This optimality sequence is not surprising. Figure 1 provides an intuitive explanation. While the first-best allocation provides an insurance against both the idiosyncratic risk and the aggregate risk, the second-best allocation only diversifies the idiosyncratic risk. In a market economy, the banking sector also insures against the idiosyncratic risk. This risk-sharing mechanism functions well when the economy is in a good state. However, when the asset return turns out to be low, all agents have the incentive to run on the banks and this risk-sharing mechanism breaks down. Due to the existence of the liquidation costs, the economy suffers large welfare losses. This destabilisation effect, which is related to the fragility of the banking sector, partially cancels out the risk-sharing benefit and makes the competitive equilibrium inferior to the second-best optimum.

For illustration, I provide a numerical example. Suppose that the proportion of impatient agents (α), the distribution of the asset return (\tilde{R}), the liquidation cost (τ), and the form of the utility function are as follows:

$$\alpha = 0.5, \quad \tilde{R} \sim \text{lognormal}(0.25, 0.5^2), \quad \tau = 0.5,$$

$$u(c) = \frac{(c+1)^{1-\beta}-1}{1-\beta}, \text{ where } \beta = 2$$

From Lemma 3, it is easy to define the first-best allocation. The optimal contract features $r_1^f = 1.0160$, $r_2^f = 1.4317$ and $i^f = 0.4920$. The expected utility for a representative agent is $E(U^f) = 0.5464$.

Similarly, the second-best allocation can be solved from problem (3.4). The contract is characterised by $r_1^s = 1.0335$ and $i^s = 0.4832$. The long-term interest rate is state-contingent and is determined by distributing the remaining assets among late consumers. A representative agent receives an expected utility of $E(U^s) = 0.5295$, which is lower than under the first-best allocation.

The equilibrium contract in a market economy, which is defined in problem (2.3), can be solved by using the grid-searching method. In the given example, it features $r_1^d = 0.83$,¹² $i^d = 0.38$ and $E(U^d) = 0.5164$. In equilibrium, banks choose a bank-run contract (bank runs occur when

¹²The short-term interest rate is lower than 1 in the competitive equilibrium. This is partly because the liquidation value is very low in this model. Banks are providing a risk-sharing mechanism to the agents by offering them a smoother consumption path.

$R < 0.5526$) instead of a run-proof contract. Besides, the banks are willing to hold extra safe assets ($1 - i - r_1\alpha > 0$) in equilibrium to mitigate the shocks caused by bank runs. Not surprisingly, the competitive equilibrium outcome is inferior to the second-best optimum.

4 Welfare analysis of various policies

In this section, I extend the benchmark model and discuss the welfare effects of five different government policies.

4.1 Suspension of convertibility of deposits

Suspension of convertibility of deposits allows the banks to suspend payment when the early withdrawals reach a certain level (αN) in the interim period. In the 19th and early 20th century, this policy was used very often during banking panics (see Dwyer and Gilbert 1989). Even in recent financial crises, some similar policies were adopted as temporary crisis management measures. For example, Malaysia decided to impose wide range capital controls¹³ soon after the occurrence of the 1997 East Asian crisis.

Diamond and Dybvig (1983), in their classic paper, claim that suspension of convertibility of deposits can be used to achieve optimal risk-sharing. They argue that the suspension policy puts a restriction on agents' expectations, eliminates possible bank runs and therefore achieves the social optimum. However, this conclusion is not valid in this model. In fact, the welfare effect of this policy solely depends on the decision sequence in period 1. In most situations, this policy turns out to be inefficient.

Proposition 2 *In a market economy, suspension of convertibility of deposits is both ex post and ex ante inefficient to prevent bank runs.*

Proof: according to the policy, the aggregate withdrawal in period 1 cannot exceed $\alpha \cdot N$. The payoff function for agent i , who observes an aggregate early withdrawal L_i ahead of him, is:

$$r_1(R, L_i) = \begin{cases} r_1 & \text{if } L_i < \alpha \\ 0 & \text{if } L_i \geq \alpha \end{cases} \quad \text{if } \textit{withdraw}$$

¹³The government announcement in September 1998 prohibited citizens from taking more than USD 100 out of Malaysia.

$$r_2 = \frac{1 - i - r_1\alpha + iR}{1 - \alpha} \quad \text{if wait}$$

A patient agent's strategy depends on the return on assets and his position in the queue. If $R \geq \frac{r_1 - 1 + i}{i}$, he always chooses to wait; if $R < \frac{r_1 - 1 + i}{i}$, he chooses to withdraw his deposit when fewer than $\alpha \cdot N$ agents have withdrawn their deposits and wait otherwise. Therefore, different decision sequences will lead to different policy effects.

Let us first consider the best scenario under which in period 1 agents make decisions according to the following sequence: all impatient agents make withdrawal decisions ahead of all patient agents. Given this decision sequence, suspending convertibility can achieve the second-best allocation. When $R \geq \frac{r_1 - 1 + i}{i}$, all patient agents choose to stay and no bank run occurs. When $R < \frac{r_1 - 1 + i}{i}$, only impatient agents are able to withdraw their deposits and the patient agents are forced to wait under the suspension policy. As a result, the destabilisation effect caused by bank runs is completely removed without any side effects. The new equilibrium insures completely against the idiosyncratic risk and achieves the second-best optimum (ex ante effect).

But this conclusion is no longer valid when the decision sequence is different. Consider another extreme case in which the decision sequence is the opposite to that above: all patient agents make withdrawal decisions ahead of impatient agents. In this case, the deposit suspension policy turns out to be inefficient. In particular, when $R < \frac{r_1 - 1 + i}{i}$, the policy causes large welfare losses for the impatient agents. Without the suspension policy, bank runs happen and the first $\frac{1 - i + iR(1 - \tau)}{r_1}$ agents receive the short-term interest rate, r_1 . With the suspension policy, however, only the patient agents are able to withdraw their deposits from the banks in period 1 (notice "withdraw" is the optimal strategy for each patient agent in this situation). Although the banks no longer suffer liquidation costs, in period 2 the remaining assets are useless to the impatient agents who have been forced to stay. The suspension policy leads to a severe misallocation of assets.

To summarise, the suspension policy has two effects. On the one hand, it protects the banks from runs and minimises the liquidation costs. On the other hand, it brings about the "misallocation effect" as some impatient agents may be prohibited from withdrawing in period 1. Conditional on the decision sequence specified in the model, the "misallocation effect" may become very serious and outweigh the benefit in most cases.

The underlying reason for the existence of the "misallocation effect" is that the suspension

policy can only control the quantity of aggregate early withdrawals but cannot distinguish the true liquidity needs of individual agents. Therefore, it cannot guarantee that the resources are allocated in the correct way. In most cases, this policy is not efficient in preventing bank panics.¹⁴

4.2 Taxation on short-term deposits

A second policy that has been used during banking crises is the imposition of taxes (or penalties) on early withdrawals. It is argued that this policy will indirectly prevent bank panics by increasing the cost of early consumption. In practice, Chile imposed a tax on short-term capital outflows in the early 1990s. The question is: is this policy an efficient way to prevent bank runs?

Suppose the government imposes a tax rate t on short-term deposits, and then returns the collected taxes to all agents as a lump sum transfer. Obviously, under this tax regime, it is the after-tax payoff that affects the patient agents' withdrawal decisions. As shown in Appendix D, under a certain contract (r_1, r_2, i) , when $R \geq R^* \equiv \frac{r_1(1-t)(1-\alpha)+r_1\alpha-1+i}{i}$, or equivalently $r_{1,after-tax} \leq r_{2,after-tax}$, all agents will report their types truthfully and no bank run happens. However, when $R < R^*$, "waiting" becomes a dominated strategy and a bank run occurs.

If no bank run happens, an early consumer receives the after-tax income of $r_1(1-t) + r_1 \cdot Lt$, in which $r_1(1-t)$ is the after-tax short-term interest rate, and $r_1 \cdot Lt$ is the government transfer. Instead, a late consumer would receive $r_2^e(R, L) + r_1 \cdot Lt$, which consists of the long-term interest payment and the government transfer. On the other hand, if a bank run does happen, a proportion ($p = \frac{1-i+iR(1-\tau)}{r_1}$) of the agents who receive the short-term interest payments pay the tax. After the government redistribution, the income distribution is as follows: a proportion p of agents receive $r_1(1-t) + p \cdot r_1 t$, and the rest of the agents receive the government transfer of $p \cdot r_1 t$.

The banks' optimisation problem is therefore as follows:

$$\begin{aligned} \max_{r_1, i} \quad & \int_0^{R^*} [p \cdot u(r_1(1-t) + p \cdot r_1 t) + (1-p)u(r_1 t)] f(R) dR \\ & + \int_{R^*}^{\infty} [\alpha u(r_1(1-t) + \alpha r_1 t) + (1-\alpha)u(\frac{1-i+iR-r_1\alpha}{1-\alpha} + \alpha r_1 t)] f(R) dR \end{aligned} \quad (4.1)$$

where p and R^* are defined as above.

This policy differs from the suspension of convertibility of deposits in that the suspension

¹⁴In the given example, under the worst scenario, the competitive equilibrium under the suspension policy features $r_1 = 0.80$, $i = 0.32$ and $E(U) = 0.5133$. A representative agent is even worse off under the new environment.

policy imposes a direct quantity control on early withdrawals, while the tax policy can control both the quantity and the composition of early withdrawals through its effect on the after-tax interest payment. Therefore, the tax policy will not bring about the “misallocation effect”. However, there are two disadvantages with the tax policy. First, it cannot eliminate the possibility of bank runs unless the tax rate is extremely high. Second, and more importantly, the early consumption tax will introduce distortions to banks’ investment decisions. In particular, banks now face a difficult trade-off. If they wish to maintain the after-tax short-term interest rate at the initial level, they have to invest less in the risky assets, therefore suffering the productivity loss. However, if they wish to maintain the investment in the more productive assets, the after-tax short-term interest rate has to be reduced, thereby damaging the risk-sharing benefit. This conflict will usually lead to underinvestment and involve welfare losses.

Proposition 3 *An early withdrawal tax is both ex post and ex ante inefficient and is always dominated by the second-best allocation.*

Proof: see Appendix E.

I use the given numerical example to illustrate the ex ante effects of the tax policy. Figure 2 reflects how the equilibrium after-tax short-term interest rate, the probability of bank runs, and expected utility change under different levels of taxes. The numerical results suggest:

(1) As the tax increases, the before-tax short-term interest rate (the dashed line) increases, but the after-tax short-term interest rate (the solid line) in equilibrium almost remains the same. However, the heavy burden in short-term liability forces the banks to invest more in the riskless assets and less in the more productive assets. The underinvestment phenomenon leads to welfare losses in the economy.

(2) The imposition of early withdrawal taxes does not improve the welfare of a representative agent. As Figure 2 shows, the welfare increases insignificantly when the tax is at a low or medium level, much inferior to the first-best and second-best outcomes. When the tax rate is very high, the equilibrium outcome is even worse than the benchmark result, suggesting that the distortion effect is much higher when the tax rate is higher.

(3) The tax policy is not very successful in preventing bank runs ex ante. In fact, the probability of bank runs in equilibrium even increases when the tax rate is not very high.

(4) In the extreme case $t = 1$, no bank run occurs in equilibrium. This is quite intuitive. Because if an agent chooses to withdraw early, he will lose everything and only receive the government transfer. Thus no patient agent will disguise himself as an impatient type. As a result, bank runs are completely eliminated. But the extremely high tax also brings severe investment distortion to the economy. On aggregate, the equilibrium outcome is much worse than the benchmark case.

4.3 Full-coverage deposit insurance (FCDI) scheme

The role of deposit insurance has been a very controversial topic. In some early work (Diamond and Dybvig 1983), deposit insurance is considered as an efficient policy to achieve the social optimum. However, continuing research suggests that we should be more cautious. Cooper and Ross (1998) point out that deposit insurance schemes eliminate the possibility of bank runs but at the same time reduce agents' incentive to monitor the banks. Krugman (1998a), after the 1997 East Asian crisis, argues that the implicit deposit insurance policy causes a severe moral hazard problem and leads to imprudent "overinvestment", which is the core element in the economic crash. A related debate is the role of the IMF. Some economists (Sachs 1998, Radelet and Sachs 1998) argue that a lender of last resort is an efficient way to prevent self-fulfilling financial panics; therefore the IMF should be expanded and a larger amount of funds should be provided more quickly when financial crises occur. At the other extreme, Schwartz (1998) and Calomiris (1998) criticise the IMF for acting as lender of last resort, arguing that such action causes moral hazard problems and in the long run increases the fragility of the world financial system. They suggest that IMF bailout schemes should be avoided.

This section explores the ex post and ex ante effects of an FCDI scheme, or a blanket guarantee scheme. Under an FCDI scheme, the central bank (or a public authority) guarantees depositors the promised interest rates when the banks are insolvent. And the funding source comes from the insurance premium payment by the banks.¹⁵

Under an FCDI scheme, the payoff function for each agent is:

$$r_1^e = r_1 \qquad r_2^e = r_2 \qquad (4.2)$$

¹⁵I assume the banks pay the premium out of their own funds rather than out of deposits. Obviously, any deposit insurance scheme without any insurance premium requirement on the banks is ex ante inefficient and unsustainable due to the moral hazard problems.

where r_1^e and r_2^e are the expected payoff an agent can receive if he chooses to withdraw early or to wait, respectively. Obviously, when $r_1 \leq r_2$, no patient agent has the incentive to misreport his preference type. As a result, no bank run happens in period 1.

Lemma 4 *A full-coverage deposit insurance plan is ex post efficient in that it can eliminate bank runs and avoid costly liquidation.*

A more interesting problem is whether the FCDI scheme is ex ante efficient. To put it another way, how will the banks respond to the FCDI scheme in choosing the deposit contract and portfolio structure? And how high an insurance premium should be charged in order to keep the plan sustainable?

I first study the behaviour of banks under a given insurance premium δ . The banks' problem is as follows:

$$\max_{r_1, r_2} \quad \alpha u(r_1) + (1 - \alpha)u(r_2) \quad (4.3)$$

$$\text{s.t.} \quad 1 - i \geq r_1 \alpha \quad (4.4)$$

$$R^* = \frac{r_1 \alpha + r_2 (1 - \alpha) - 1 + i}{i}$$

$$\int_{R^*}^{\infty} i(R - R^*) \cdot f(R) dR \geq \delta \quad (4.5)$$

$$i = \operatorname{argmax} \int_{R^*}^{\infty} i(R - R^*) \cdot f(R) dR$$

The objective function reflects the fact that there is no bank run in period 1 when an FCDI plan exists. All impatient agents get a consumption of r_1 and all patient agents receive r_2 . The first constraint is the usual budget constraint in period 1. The second constraint specifies the threshold return R^* below which a rescue package is needed. It comes from the condition that $\frac{1 - i + iR^* - r_1 \alpha}{1 - \alpha} = r_2$. When $R \geq R^*$, banks are able to earn positive profits. When $R < R^*$, banks are insolvent and a rescue package is implemented by the central bank. The third constraint is the incentive constraint for banks to join the insurance plan, which states that banks must be able to earn enough profits to cover the insurance premium payment. The last constraint determines the choice of portfolio structure, which maximises the banks' expected profits in equilibrium.

Lemma 5 *In problem (4.3), both restriction (4.4) and restriction (4.5) are binding.*

Proof: see Appendix F.

Lemma 5 states two important features of the equilibrium. First, banks choose to hold no excess liquidity and maximise their investments in the more productive technology. Second, in the competitive market, the net profit for a representative bank is zero.

Another feature of the equilibrium contract under the deposit insurance scheme is that the long-term payment for a patient agent is not state-contingent. Hence, the system provides a risk-sharing mechanism against both idiosyncratic risk and aggregate risk. The new question is: can this policy be used to achieve the first-best optimum? Unfortunately, the answer is no because of the moral hazard issues.

Based on Lemma 5, it is straightforward to define the equilibrium contract under a certain FCDI scheme.

Lemma 6 *The equilibrium contract (r_1, r_2) in problem (4.3) should satisfy the following conditions:*

$$u'(r_1) = u'(r_2) \cdot E[R|R \geq R^*] \quad (4.6)$$

$$\int_{R^*}^{\infty} (1 - r_1\alpha)(R - R^*) \cdot f(R)dR = \delta \quad (4.7)$$

where $R^* = \frac{r_2(1-\alpha)}{1-r_1\alpha}$.

Equation (4.6) is the familiar Euler equation. The intuition is as follows. If the short-term interest rate is reduced by an amount of Δr , the agent will suffer a loss of $u'(r_1) \cdot \Delta r$ in the short run. But the long run payment will be increased by $E[R|R \geq R^*] \cdot \Delta r$.¹⁶ In equilibrium, the marginal cost and marginal benefit should be equalised.

Notice the difference between the first-order conditions in the first-best allocation and the FCDI system. In the first-best environment, the marginal rate of transformation is determined by the unconditional mean of asset return. In the latter case, however, the banks do not care about the losses for the central bank. Therefore, the marginal rate of transformation is related to the conditional mean of asset return. This “extortion effect”, which refers to the fact that banks ignore the negative externality of higher bailout costs and offer too high interest rates, prevents the economy from achieving the first-best optimum.

¹⁶Consider the profit function for the banks. Banks lose nothing when $R < R^*$ (central bank will bail out) and gain the profits when $R \geq R^*$. The Δr assets will bring the banks an expected profit of $E[R|R \geq R^*] \cdot \Delta r$.

Lemma 7 *Under the full-coverage deposit insurance plan, the first-best allocation is feasible but not chosen in the market economy.*

Proof: I first show that the first-best allocation contract (r_1^f, r_2^f, i^f) is feasible when the central bank charges an insurance fee of $\delta^f = \int_{E(\tilde{R})}^{\infty} i^f [R - E(\tilde{R})] \cdot f(R) dR$. Under this FCDI scheme,

1. Banks can make enough profits to cover the insurance premium payment. From Lemma 3, $r_2^f(1 - \alpha) = i^f \cdot E(\tilde{R})$ and $i^f = 1 - r_1^f \alpha$, therefore $R^* = \frac{r_2^f(1 - \alpha)}{1 - r_1^f \alpha} = E(\tilde{R})$. From the definition of δ^f , the expected profit for the banks equals the insurance fee payment.

2. The bailout costs can be covered by the insurance premium payment because

$$\begin{aligned} & \int_0^{R^*} [r_1^f \alpha + (1 - \alpha)r_2^f - (1 - i^f + i^f R)] \cdot f(R) dR \\ &= \int_0^{E(\tilde{R})} i^f [E(\tilde{R}) - R] \cdot f(R) dR \\ &= \int_{E(\tilde{R})}^{\infty} i^f [R - E(\tilde{R})] \cdot f(R) dR \\ &= \delta^f \end{aligned}$$

Unfortunately, the first-best allocation will not be chosen in the market economy. From equation (4.6), it is obvious that banks will not choose the first-best contract. Instead, banks will offer the depositors higher interest rates and the expected bailout costs for the central bank are higher than δ^f .

To maintain a credible FCDI scheme, the central bank has to charge a higher insurance premium δ^* that is able to cover its bailout costs. In equilibrium,

$$\int_0^{R^*} i(R^* - R) \cdot f(R) dR = \delta^*$$

Combining with the equilibrium conditions (4.6) and (4.7), the self-sustainable market equilibrium $(r_1^*, r_2^*, i^*, \delta^*)$ under an FCDI scheme is characterised by:

$$\begin{cases} r_2^*(1 - \alpha) &= (1 - r_1^* \alpha) E(\tilde{R}) \\ u'(r_1^*) &= u'(r_2^*) \cdot E[R | R \geq E(\tilde{R})] \\ \delta^* &= \int_0^{E(\tilde{R})} i^* [E(R) - R] \cdot f(R) dR \\ i^* &= 1 - r_1^* \alpha \end{cases}$$

Proposition 4 *A full-coverage deposit insurance brings stability into the banking sector and improves the welfare of investors. However, it cannot achieve the first-best optimum due to the “moral hazard” problem.*

I still use the numerical example to illustrate the welfare effects of the FCDI scheme. Figure 3 shows that the first-best allocation is feasible but will not be chosen when $\delta = \delta^f$. The asterisk represents the first-best allocation, under which the deposit insurance authority's bailout costs can be fully covered by the insurance premium payment. But individual banks have the incentive to offer a higher average interest rate¹⁷ to investors and the FCDI scheme is no longer sustainable. Specifically, when $\delta = \delta^f = 0.1407$, the equilibrium contract will be $r_1 = 0.85$, $r_2 = 1.8044$, $i = 0.575$, $E(U) = 0.5514$ and the expected loss for the central bank is 0.0656. The agents are better off than under the first-best optimum, of course, at the cost of huge deficits to the central bank.

Figure 4 shows the contracts that the banks will offer under different insurance fees. When the insurance fee increases, the banks will offer a lower average interest rate, the central bank's balance sheet improves, and the welfare for a representative agent is lower. In the given example, the equilibrium contract in which the premium payments can fully cover the bailout costs is: $\delta^* = 0.1677$, $r_1^* = 0.8321$, $r_2^* = 1.6993$, $i^* = 0.584$ and $E(U)^* = 0.5419$. It is inferior to the first-best allocation due to the existence of moral hazard problems.

4.4 Interest-cap deposit insurance (ICDI) scheme

Due to the existence of moral hazard problems under the FCDI scheme, policymakers have been looking for different variants of deposit insurance plans to mitigate or to remove this adverse effect. Two major variants were proposed in the recent report by the Financial Stability Forum (2001): a limited-coverage deposit insurance scheme and coinsurance. The limited-coverage deposit insurance scheme protects the principal and interest of each depositor up to a certain limit.¹⁸ The coinsurance system specifies that only a proportion of deposits (including interest) are protected. In this subsection, I propose that a similar variant of deposit insurance scheme, which I refer to as the "interest-cap deposit insurance" (ICDI) scheme, can remove the moral hazard problems and achieve the first-best optimum.

Under an ICDI scheme, the maximum basic protection each depositor can receive is his principal

¹⁷Defined as $\alpha r_1 + (1 - \alpha)r_2$.

¹⁸For example, the maximum protection for each depositor is USD 100,000 in the United States and CAD 60,000 in Canada.

and a certain amount of interest that does not exceed a predetermined cap ($\bar{r} - 1$). In other words, the maximum payment a depositor can receive upon bank default is \bar{r} .¹⁹ As shown below, a well-designed ICDI can overcome the conflict of interests between the central bank and deposit banks, and achieve the first-best social optimum. By setting the interest cap on protection, the deposit insurance authority indirectly imposes a cap on the interest rate that a deposit bank will offer to agents.

Proposition 5 *An interest-cap insurance scheme is efficient in preventing bank runs and can achieve the first-best social optimum.*

In particular, an ICDI scheme with $\bar{r} = r_2^f$ and deposit insurance premium $\delta^f = \int_{E(\tilde{R})}^{\infty} i^f [R - E(\tilde{R})] \cdot f(R) dR$ is able to achieve the first-best social optimum. Appendix G shows that, under this ICDI scheme, banks will choose the first-best contract ($r_1 = r_1^f$, $r_2 = r_2^f$). On the one hand, the banks will not choose a lower r_2 because they always have the incentive to maximise the utilisation of deposit insurance. More importantly, on the other hand, the banks have no incentive to increase the interest rate offer as under the FCDI scheme. Under the FCDI scheme, the banks choose to increase the long-term interest rate and decrease the short-term interest rate (to maintain expected zero profit). The marginal cost of the lower short-term interest rate will be compensated by the fact that the long-term interest rate is higher *in all states*. However, under the ICDI scheme, this incentive no longer exists because a representative agent cannot earn a higher long-term interest rate when banks are insolvent due to the existence of a coverage limit. Therefore, the initial moral hazard problem, in which banks increase the central bank's bailout costs through offering higher interest rates, no longer exists under the specific ICDI scheme.²⁰

One important implication from Appendix G is that the ICDI scheme should cover both the principal and part of (or all) interest rate payments. This is not surprising. The maximum protection must be greater than the short-term interest rate to induce the agents not to run on the banks.²¹

¹⁹In this model, this ICDI scheme is actually the same as the limited-coverage deposit insurance scheme. They differ when agents are heterogeneous.

²⁰However, we should be cautious about this result in practice. This conclusion is based on the assumption that all banks are faced with the same productivity shock. In reality, considering the fact that banks are also confronted with idiosyncratic productivity shocks, a uniform interest cap is not able to catch this heterogeneity.

²¹In practice, to reduce the bailout costs of deposit insurance schemes, or to increase the large agents' incentive to

4.5 Capital requirement

Another widely used tool in bank regulation is the imposition of a capital requirement. In general terms, a capital requirement specifies how much equity a bank should hold for each unit of deposits. This equity can be invested in either technology and can be used to repay the depositors when asset returns are low. Throughout this paper, I use κ to represent the capital requirement for each unit of deposit.²² From the definition, the capital requirement relates to the banks' own funds.

Suppose individual banks invest $1-i+\kappa$ in the safe assets and i in the risky assets. The existence of capital reduces the probability of bank runs because it increases the banks' solvency ability. Following Lemma 1, equilibrium aggregate early withdrawal, L , equals α when $R \geq \frac{r_1-(1-i+k)}{i}$ and $L = 1$ otherwise. There are three possible outcomes (see Figure 5):

$$(1) R < R_1, \text{ where } R_1 \equiv \frac{r_1-(1-i+k)}{i}$$

When the return on risky assets is very low, the bank default is unavoidable. All agents rush to banks in the hope of retrieving part of (or all) their deposits. A bank run happens and banks lose their entire capital.

$$(2) R_1 \leq R < R_2, \text{ where } R_2 \equiv \frac{r_1\alpha+r_2(1-\alpha)-(1-i+k)}{i}$$

When the return is within the intermediate level, the banks are not in immediate danger of default and patient agents are willing to wait in the interim period and no bank run happens. But banks have to use all their capital to repay the demand-deposit contracts. Patient agents receive a payment of $r_2^e = \frac{1-i+k+iR-r_1\alpha}{1-\alpha}$, which is higher than r_1 but less than the promised long-term interest rate, r_2 .

$$(3) R \geq R_2$$

When the return on risky assets is high, all late consumers receive the promised interest rate, r_2 , in the long run. Bank runs never occur. The payoff function for banks is subtler. Define $R_3 \equiv \frac{r_1\alpha+r_2(1-\alpha)-(1-i)}{i}$. When $R_2 \leq R \leq R_3$, banks have to use part of their capital collateral to pay the late consumers. When $R \geq R_3$, the capital collateral is untouched and the banks earn positive profits.

Combining the above analysis, under a certain capital requirement (κ), the optimisation problem

monitor the banks, the deposit insurance authority may choose a lower interest cap or the coinsurance scheme.

²²Obviously, a capital requirement of κ is equivalent to a capital ratio of $\frac{\kappa}{1+\kappa}$ because assets = liabilities = deposits + capital.

for a representative bank is:

$$\max_{r_1, r_2} \int_0^{R_1} \frac{1 - i + \kappa + iR(1 - \tau)}{r_1} u(r_1) \cdot f(R) dR \quad (4.8)$$

$$+ \int_{R_1}^{R_2} [\alpha u(r_1) + (1 - \alpha)u(r_2^e(R))] \cdot f(R) dR$$

$$+ \int_{R_2}^{\infty} [\alpha u(r_1) + (1 - \alpha)u(r_2)] \cdot f(R) dR$$

$$\text{s.t.} \quad i \text{ maximises } \int_0^{R_2} -\kappa \cdot f(R) dR + \int_{R_2}^{\infty} i(R - R_3) \cdot f(R) dR \quad (4.9)$$

$$\int_0^{R_2} -\kappa \cdot f(R) dR + \int_{R_2}^{\infty} i(R - R_3) \cdot f(R) dR = \kappa[E(\tilde{R}) - 1] \quad (4.10)$$

$$1 - i + \kappa \geq r_1 \alpha \quad (4.11)$$

The objective function, which specifies the expected utility for a representative agent, consists of three parts that correspond to three possible outcomes. Equation (4.9) states that the choice of portfolio structure should maximise the banks' expected profits. Equation (4.10) is the incentive constraint for individual banks, which suggests that the expected profits for banks should cover the opportunity costs of the collateral assets in equilibrium. Since the risk-neutral bankers can always invest their equity assets in the more productive technology and obtain an expected return of $E(\tilde{R})$, the banking sector should assure them the same payoff. Equation (4.11) specifies the minimum holding of riskless assets.

The welfare effects of this policy, accordingly, are different in different states of the economy:

- When $R \geq R_2$, both idiosyncratic risk and aggregate risk are eliminated. Agents receive constant payments based on their true liquidity needs. The banking sector functions well and no default occurs.

- When $R_1 \leq R < R_2$, impatient agents receive a constant payoff but the patient agents receive a state-contingent payoff. Only the idiosyncratic risk is diversified. The banking sector defaults in period 2 but no bank run happens in period 1.

- When $R < R_1$, bank runs happen. Banks go into bankruptcy and the risk-sharing mechanism breaks down. The destabilisation effect leads to costly liquidation and welfare losses.

In summary, imposition of a capital requirement gives the banks a partial defense against bank runs without causing new distortions. It cannot eliminate the occurrence of bank runs, though. However, as the capital requirement increases, the threshold returns R_1 and R_2 decrease,

suggesting that the probability of default and the probability of bank runs are smaller. Therefore the risk-sharing benefit dominates and the market equilibrium gradually converges to the first-best optimum.

Proposition 6 *As κ increases, the equilibrium contract in a market economy converges gradually to the first-best optimum.*

Proof: see Appendix H.

Figure 6 provides the results from numerical simulations. The horizontal axis represents the capital ratio ($\frac{\kappa}{1+\kappa}$). As the capital ratio increases, the equilibrium contract in the market economy, including the interest rate structure, and investment portfolio, converges to the first-best optimum. Besides, the banking sector becomes more stable as a higher capital requirement is imposed.

However, a potential problem with the capital requirement policy is the speed of convergence. Theoretically, the market equilibrium converges to the social optimum only as the capital ratio approaches 100%. In the given numerical example, the welfare in the market equilibrium reaches the second-best optimum when the capital ratio is about 13%, and is equivalent to the outcome under the FCDI scheme ($\delta^* = 16.77\%$) when the capital ratio is 32.43%. This requirement is obviously very high.

5 Conclusion

This paper proposes a model in which bank runs are closely related to the state of the business cycle. Extensions of the model study the welfare effects of five different policies: suspension of convertibility of deposits; taxation on short-term deposits; full-coverage deposit insurance schemes; interest-cap deposit insurance schemes and capital requirements.

The results suggest that, in a competitive market, an interest-cap deposit insurance scheme can be used to achieve the first-best social optimum. The limited guarantee removes the banks' incentive to offer interest rates that are too high, therefore avoiding the moral hazard issues confronted by a full-coverage deposit insurance scheme and restoring the economy to the first-best optimum.

My study also suggests that the other policies designed to prevent bank runs will have adverse side effects. First, suspension of convertibility of deposits can only control the quantity of early

withdrawal but cannot distinguish the true liquidity needs of depositors. Although such a policy can successfully stop a bank run, the resources might be allocated in an inefficient way because, when the asset return is low, some depositors with true liquidity needs will not be able to withdraw their funds while other depositors who do not have genuine liquidity needs will obtain access to their funds. Second, taxation on early withdrawals introduces investment distortions into the economy. The taxation policy reduces the depositors' incentive to run on the banks, but meanwhile the banks have to reduce their investment in the more productive assets. Third, the full-coverage deposit insurance scheme is ex post efficient but ex ante inefficient in preventing bank runs. On the one hand, the blanket guarantee promises the late consumers a long-term payment that is at least as good as the short-term payment, thereby successfully protecting the banks from runs. On the other hand, this scheme will cause moral hazard, which arises from the conflict of interests between the deposit insurance authority and individual banks. Finally, a capital requirement provides a partial cushion against bank runs without introducing any new distortion to the economy. However, the market equilibrium converges to the social optimum only as the capital ratio approaches 100%.

A possible extension of this paper is to relax the assumption of a representative bank setting. Banks in this paper are homogeneous and face the same aggregate uncertainty. By allowing for both a heterogeneous banking industry and the existence of an interbank credit market, we can study how a run on an individual bank spreads across the whole banking industry, and how the liquidation value of bank assets is endogenously determined. This line of research is potentially very promising.

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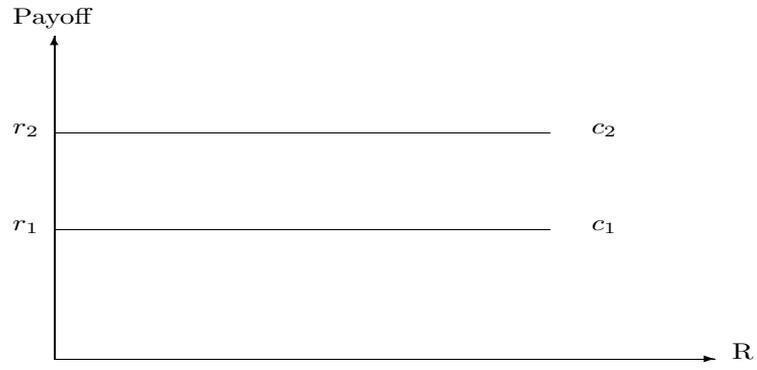
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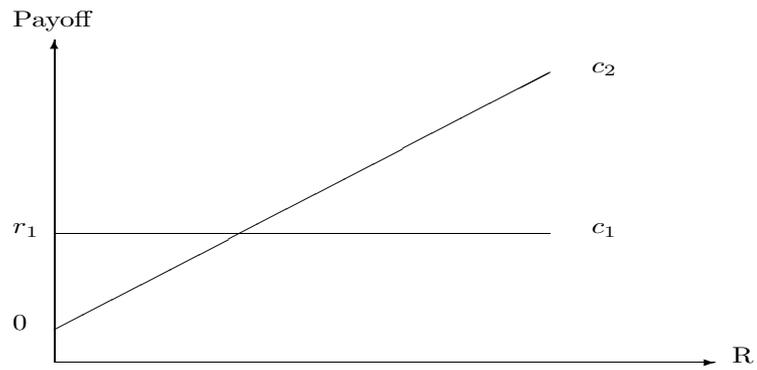
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Figure 1

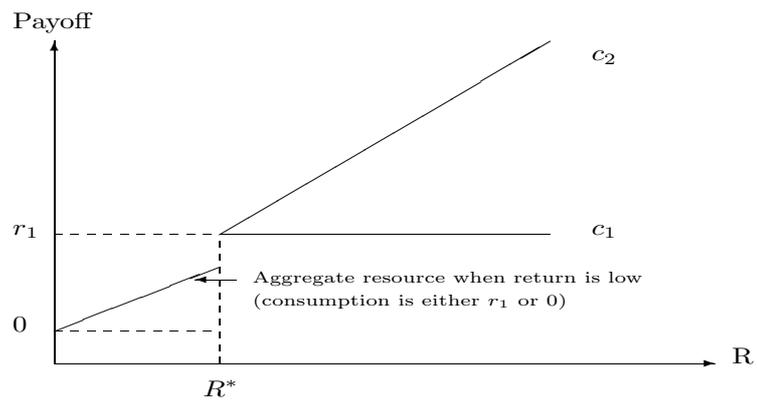
Comparison of three different allocations



First-best allocation



Second-best allocation



Equilibrium allocation in a market economy

Figure 2

Market equilibria under different early withdrawal taxes

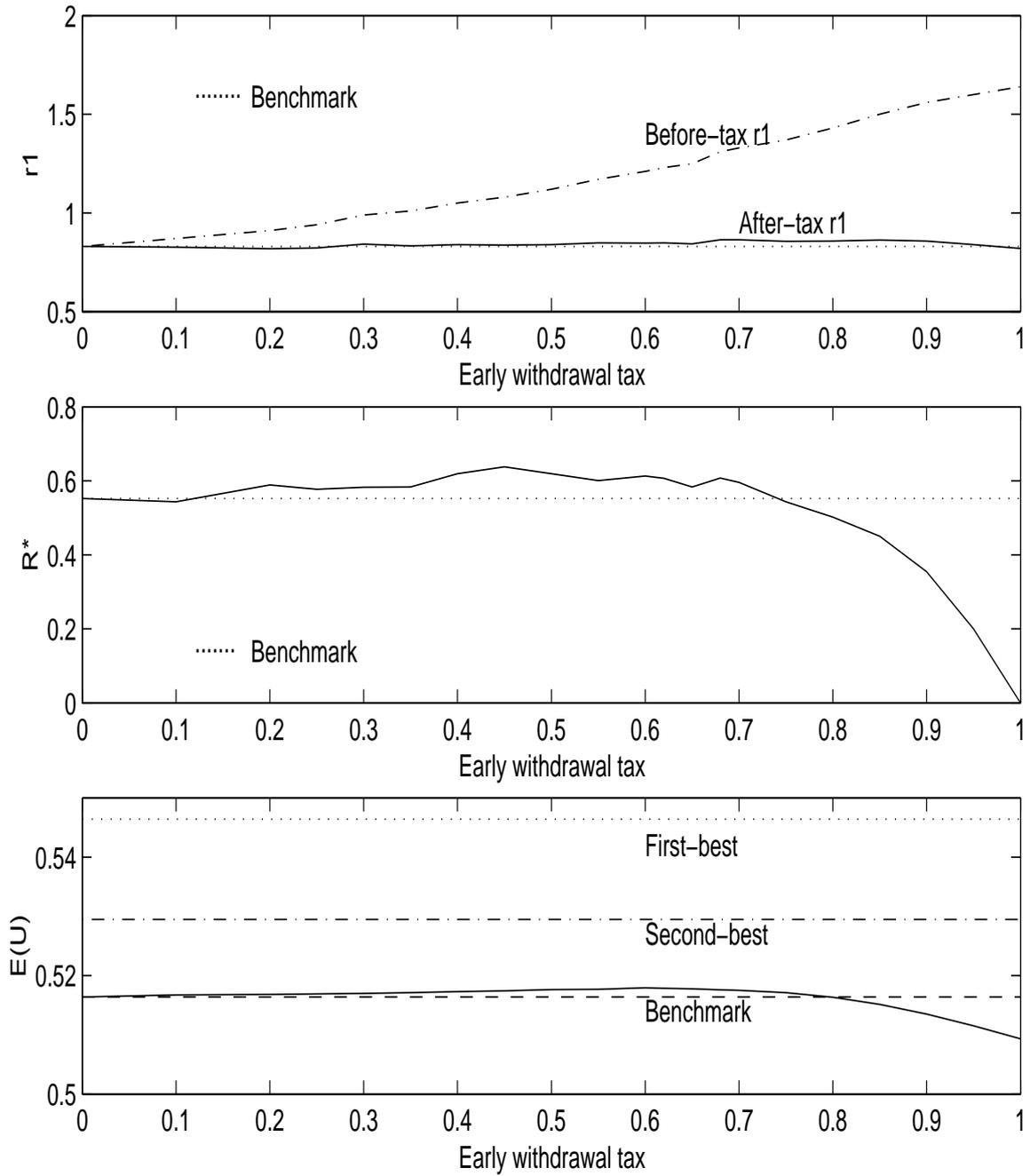


Figure 3

First-best allocation is feasible but not chosen under the FCDI scheme
 (insurance premium $\delta^f = \int_{E(\tilde{R})}^{\infty} i^f [R - E(\tilde{R})] \cdot f(R) dR = 0.1407$)

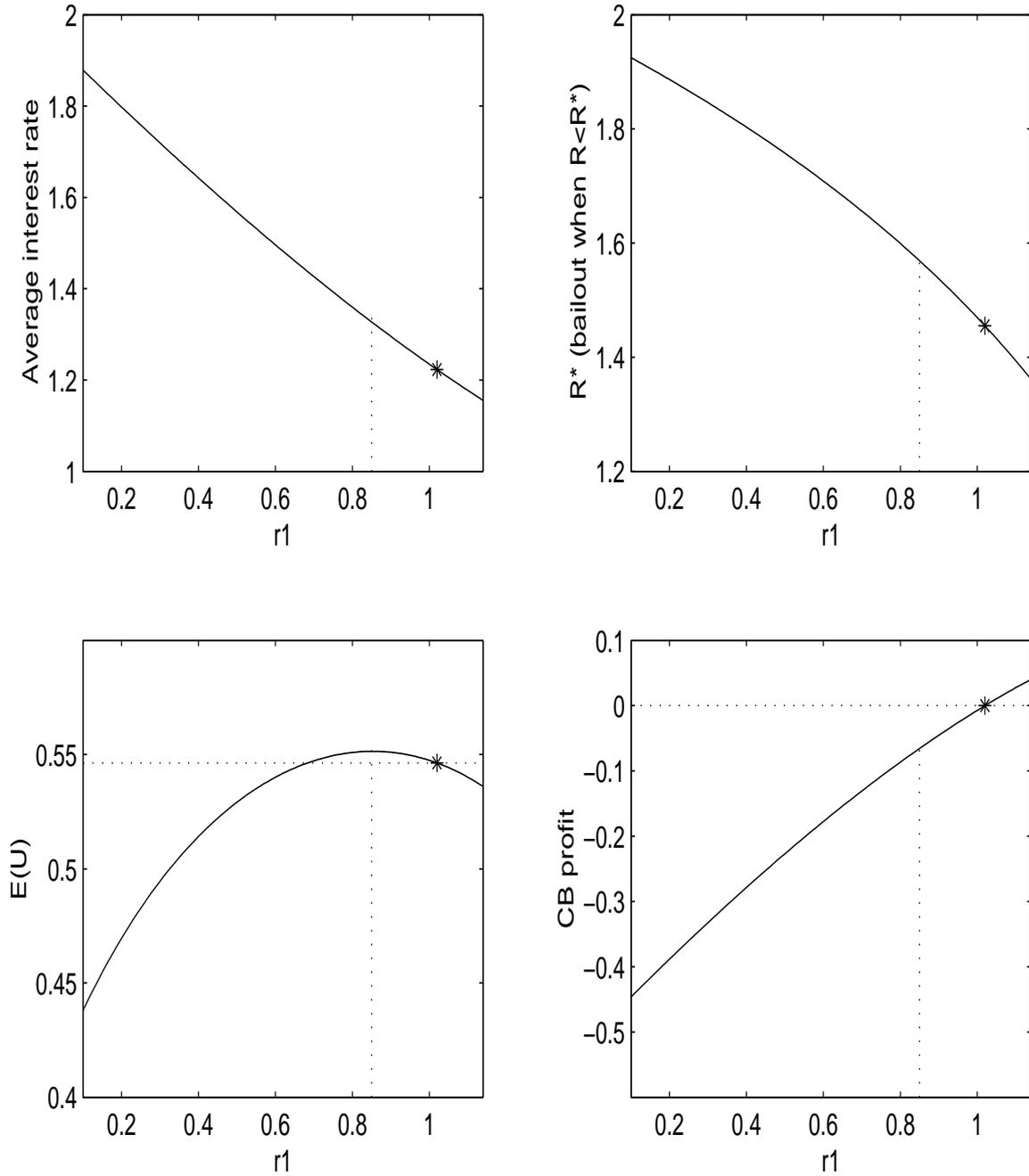


Figure 4
Optimal contracts under FCDI schemes

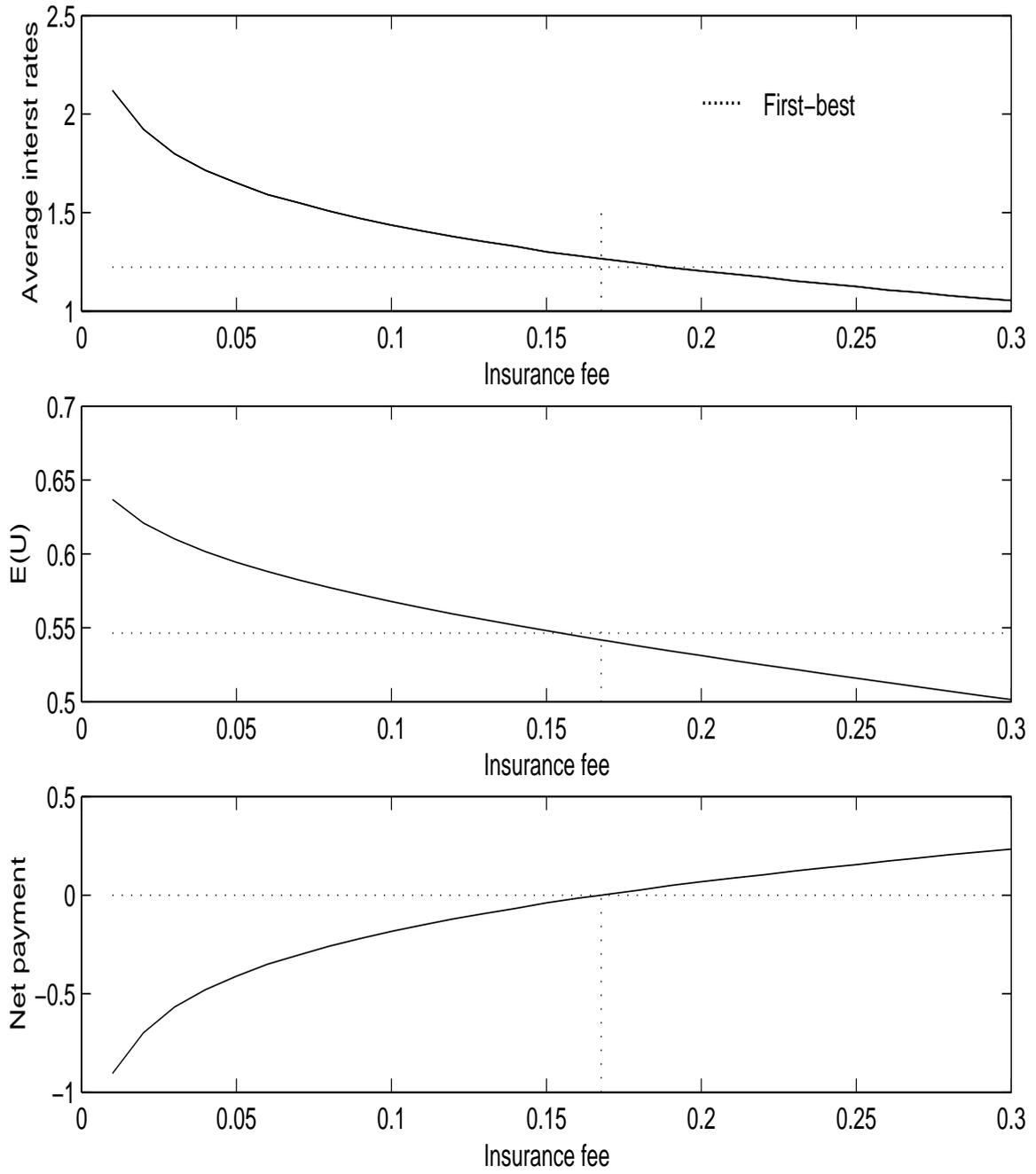


Figure 5
 Payoff functions under capital requirements

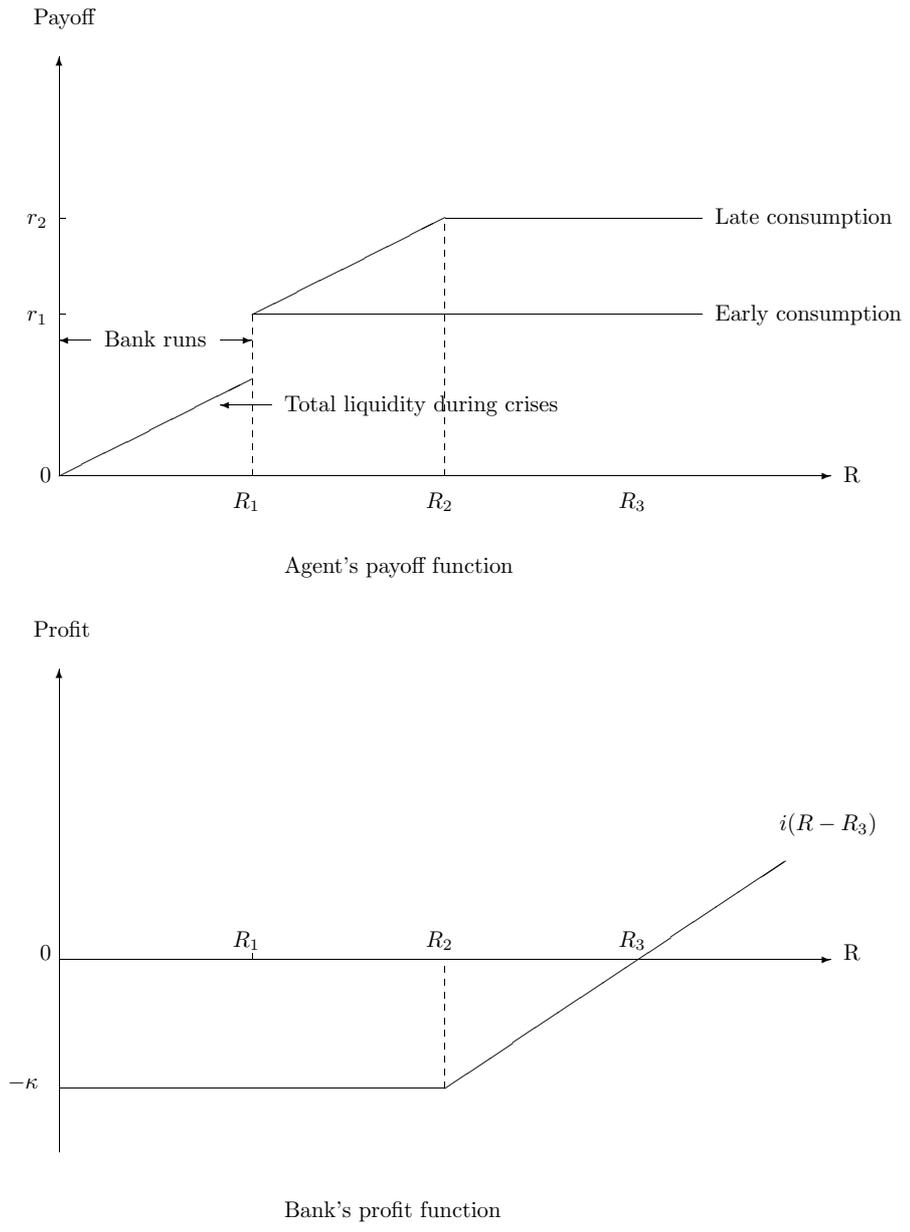
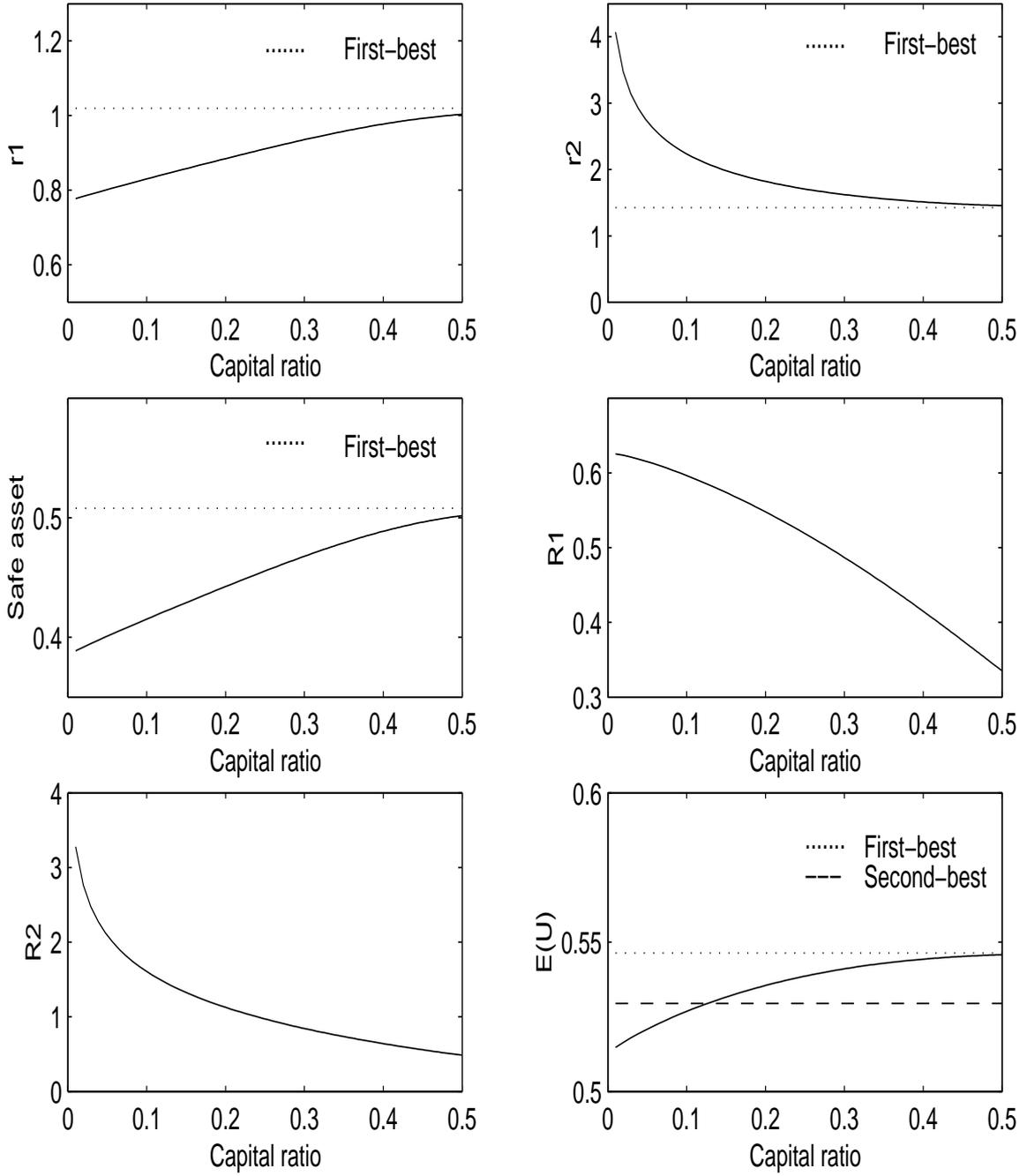


Figure 6

Optimal contracts under capital requirements



Appendix

A Proof of Lemma 1

The proof follows Zhu (2001). Impatient agents always choose to withdraw early. A patient agent, if he chooses to withdraw early, will receive r_1 ; If he chooses to wait, the expected payoff is:

$$r_2^e(R, L) = \begin{cases} \min\left(\frac{1-i+iR-r_1L}{1-L}, r_2\right) & \text{when } r_1L \leq 1-i \\ \min\left(\frac{1-i-r_1L+i(1-\tau)R}{(1-L)(1-\tau)}, r_2\right) & \text{when } r_1L > 1-i \end{cases} \quad (\text{A.1})$$

where L is the aggregate early withdrawal.

Simple algebra shows that when $R < \frac{r_1-1+i}{i}$, the payoff function $r_2^e(R, L) < r_1$ for all L . When $R \geq \frac{r_1-1+i}{i}$, it is true that $r_2^e(R, \alpha) \geq r_1$.

Using the conclusion in Zhu (2001), a bank run happens if and only if $R < \frac{r_1-1+i}{i}$.

B Proof of Lemma 3

Rewrite problem (3.1) as

$$\max_{r_1} \max_{i \leq 1-r_1\alpha} \alpha u(r_1) + (1-\alpha)u\left(\frac{1-i+i \cdot E(R) - r_1\alpha}{1-\alpha}\right)$$

For a given r_1 , banks will choose to maximise their holdings of risky assets. This is because:

$$\frac{dU^f}{di} = u'(r_2) \cdot [E(\tilde{R}) - 1] > 0$$

Therefore $i = 1 - r_1\alpha$ and problem (3.1) is equivalent to:

$$\begin{aligned} \max_{r_1} \quad & \alpha u(r_1) + (1-\alpha)u(r_2) \\ \text{s.t.} \quad & r_2 = \frac{(1-r_1\alpha) \cdot E(\tilde{R})}{1-\alpha} \end{aligned} \quad (\text{B.1})$$

The first-order condition is

$$u'(r_1) = E(\tilde{R}) \cdot u'(r_2) \quad (\text{B.2})$$

C Proof of Proposition 1

I prove the proposition in two steps.

1. The first-best allocation is superior to the second-best allocation.

Suppose the second-best allocation is characterised by r_1^s and i^s . By using the same r_1 and i , a representative agent obtains an expected utility of $\alpha u(r_1^s) + (1 - \alpha)u[E(r_2^s)]$ under a first-best contract. Due to concavity of the utility function, a representative agent is better off in the first-best contract environment. And from the definition of first-best allocation, $\alpha u(r_1^f) + (1 - \alpha)u(r_2^f) \geq \alpha u(r_1^s) + (1 - \alpha)u[E(r_2^s)]$. Therefore, the first-best optimum is superior to the second-best optimum.

2. The second-best allocation is superior to the equilibrium allocation in the market economy.

Suppose that the equilibrium contract in the market economy is characterised by r_1 and i (r_2 is state-contingent). The expected utility for a representative agent is

$$U^d = \int_0^{R^*} \frac{1 - i + iR(1 - \tau)}{r_1} u(r_1) f(R) dR + \int_{R^*}^{\infty} [\alpha u(r_1) + (1 - \alpha)u(r_2^e(R))] f(R) dR$$

where $r_2^e = \frac{1 - i + iR - r_1\alpha}{1 - \alpha}$.

Now consider that the social planner adopts the same contract in the second-best contract environment. The expected utility for a representative agent is

$$U^s = \int_0^{\infty} [\alpha u(r_1) + (1 - \alpha)u(r_2^e(R))] f(R) dR$$

Therefore,

$$\begin{aligned} U^s - U^d &= \int_0^{R^*} [\alpha u(r_1) + (1 - \alpha)u(r_2^e(R)) - \frac{1 - i + iR(1 - \tau)}{r_1} u(r_1)] f(R) dR \\ &> \int_0^{R^*} [\alpha u(r_1) + (1 - \alpha)u(\frac{1 - i + iR - r_1\alpha}{1 - \alpha}) - \frac{1 - i + iR}{r_1} u(r_1)] f(R) dR \\ &= \int_0^{R^*} [(1 - \alpha)u(\frac{1 - i + iR - r_1\alpha}{1 - \alpha}) - \frac{1 - i + iR - r_1\alpha}{r_1} \cdot u(r_1)] f(R) dR \\ &> 0 \end{aligned}$$

where the last inequality comes from the concavity property for the $u(\cdot)$ function and $u(0) = 0$.

Therefore, the second-best optimum must be superior to the market equilibrium.

D Withdrawal decisions under taxation on short-term deposits

Given (r_1, r_2, i) , the after-tax consumption from “wait” and “withdraw” is:

$$\begin{aligned} c_1 &= r_1(1-t) + r_1Lt \\ c_2 &= \frac{1-i+iR-r_1L}{1-L} + r_1Lt \quad \text{when } r_1L < 1-i \\ c_2 &= \frac{1-i+i(1-\tau)R-r_1L}{1-L} + r_1Lt \quad \text{when } r_1L \geq 1-i \end{aligned}$$

(1) When $R > R^* \equiv \frac{r_1(1-t)(1-\alpha)+r_1\alpha-1+i}{i}$.

$$\begin{aligned} c_2(L=\alpha) &= \frac{1-i+iR-r_1\alpha}{1-\alpha} + r_1\alpha t \\ &> \frac{r_1(1-t)(1-\alpha)}{1-\alpha} + r_1\alpha t \\ &= r_1(1-t) + r_1\alpha t = c_1 \end{aligned}$$

therefore patient agents choose to wait when $R > R^*$.

(2) When $R < R^*$.

Similarly, it can be shown that $c_2(L=\alpha) < c_1$, and $\frac{\partial c_2(L)}{\partial L} < 0$ for $L \geq \alpha$. Thus $c_2(L) < c_2(\alpha) < c_1$ for all $L \geq \alpha$. As a result, a bank run happens.

E Proof of Proposition 3

Suppose the equilibrium contract under a certain early withdrawal tax, (r_1, i) , has been defined by solving problem (4.1). Define $R^* = \frac{r_1(1-t)(1-\alpha)+r_1\alpha-1+i}{i}$ and $p = \frac{1-i+iR(1-\tau)}{r_1}$. In equilibrium, bank runs happen if and only if $R < R^*$.

The equilibrium welfare is:

$$\int_0^{R^*} [pu(c_{12}) + (1-p)u(c_{22})]f(R)dR + \int_{R^*}^{\infty} [\alpha u(c_{11}) + (1-\alpha)u(c_{21})]f(R)dR$$

where $c_{11} = r_1(1-t) + \alpha r_1 t$, $c_{21} = \frac{1-i+iR-r_1\alpha}{1-\alpha} + \alpha r_1 t$, $c_{12} = r_1(1-t) + pr_1 t$, and $c_{22} = pr_1 t$.

Now consider a contract in the second-best allocation setting. Let $r_1^* = r_1(1-t) + \alpha r_1 t$, $i^* = i$; the long-term payment is therefore

$$\begin{aligned} r_2^*(R) &= \frac{1-i+iR-r_1^*\alpha}{1-\alpha} \\ &= \frac{1-i+iR-r_1\alpha}{1-\alpha} + \alpha r_1 t \\ &= c_{21} \end{aligned}$$

These two contracts lead to the same allocation of resources when $R > R^*$. However, when $R < R^*$, the contract in the second-best environment is better because $\alpha u(r_1^*) + (1 - \alpha)u(r_2^*) > pu(c_{12}) + (1 - p)u(c_{22})$ for all $R < R^*$. There are two reasons.

First, the total resources under the second-best allocation are higher because there is no liquidation cost. Specifically, $\alpha r_1^* + (1 - \alpha)r_2^* = 1 - i + iR$ and $p \cdot c_{12} + (1 - p)c_{22} = 1 - i + iR(1 - \tau)$.

Second, the second-best allocation divides the resources more evenly. Notice that $r_1^* > r_2^*$ and $c_{12} > c_{22}$ when $R < R^*$. From $1 - i \geq r_1\alpha \Rightarrow p > \frac{r_1\alpha + iR(1-\tau)}{r_1} > \alpha$, we have

$$r_1^* - c_{12} = (\alpha - p)r_1t < 0$$

Thus $r_1^* < c_{12}$ and $r_2^* > c_{22}$. From the concavity of the utility function, the contract in the second-best environment must be better.²³

Combining the above results suggests that, for any contract under the tax policy, there always exists a better contract in a second-best environment. The early withdrawal tax is ex ante inefficient.

F Proof of Lemma 5

(1) Denote $E(\Pi) = \int_{R^*}^{\infty} i(R - R^*) \cdot f(R)dR$,

$$\begin{aligned} \frac{d[E(\Pi)]}{di} &= \int_{R^*}^{\infty} (R - 1)f(R)dR - i(R^* - R^*)\frac{\partial R^*}{\partial i}f(R^*) \\ &= \int_{R^*}^{\infty} (R - 1)f(R)dR \\ &> 0 \end{aligned}$$

As a result, i must be chosen at its maximum value, $1 - r_1\alpha$, in the equilibrium contract.

(2) The constraint (4.5) should be binding, too. Otherwise the banks can offer a better contract to depositors and attract more deposits by bidding up the interest rates. This process continues until the net profit is driven down to zero.

G Proof of Proposition 5

When $\bar{r} = r_2^f$ and $\delta^f = \int_{E(\bar{R})}^{\infty} i^f[R - E(\bar{R})] \cdot f(R)dR$, the banks can choose from two types of contracts.

²³Concavity of $u(\cdot)$ implies: if $x_1 > y_1 > y_2 > x_2$, then $\alpha u(x_1) + (1 - \alpha)u(x_2) < \beta u(y_1) + (1 - \beta)u(y_2)$ for any α, β so long as $\beta y_1 + (1 - \beta)y_2 = \alpha x_1 + (1 - \alpha)x_2$.

(1) $r_2 \leq r_2^f = \bar{r}$, in which the ICDI is actually an FCDI scheme.

$$\begin{aligned}
& \max_{r_1, r_2} && \alpha u(r_1) + (1 - \alpha)u(r_2) \\
& \text{s.t.} && 1 - i \geq r_1 \alpha \\
& && R^* = \frac{r_1 \alpha + r_2 (1 - \alpha) - 1 + i}{i} \\
& && \int_{R^*}^{\infty} i(R - R^*) \cdot f(R) dR \geq \delta \\
& && i = \operatorname{argmax} \int_{R^*}^{\infty} i(R - R^*) \cdot f(R) dR
\end{aligned}$$

Following Appendix F, both restrictions are binding. Therefore $i = 1 - r_1 \alpha$ and $\int_{R^*}^{\infty} (1 - r_1 \alpha)(R - R^*) \cdot f(R) dR = \delta$. This suggests that both i and r_1 can be determined once r_2 is chosen, because

$$\begin{aligned}
\frac{dU}{dr_2} &= \frac{\partial U}{\partial r_2} + \frac{\partial U}{\partial r_1} \cdot \frac{dr_1}{dr_2} \\
&= (1 - \alpha)u'(r_2) + \alpha u'(r_1) \frac{-\int_{R^*}^{\infty} (1 - \alpha)f(R) dR}{\int_{R^*}^{\infty} \alpha R f(R) dR} \\
&\geq [u'(r_2^f) \cdot E(R|R \geq R^*) - u'(r_1^f)] \cdot \frac{1 - \alpha}{E(R|R \geq R^*)} \\
&= u'(r_2^f)[E(R|R \geq R^*) - E(R)] \cdot \frac{1 - \alpha}{E(R|R \geq R^*)} \\
&> 0
\end{aligned}$$

The first inequality comes from the fact that $r_2 \leq r_2^f$ and $r_1 \geq r_1^f$. As a result, banks will choose the first-best contract $r_1 = r_1^f$, $r_2 = r_2^f$ among contracts of this type.

(2) $r_2 \geq r_2^f = \bar{r}$.

The optimisation problem for the banks is:

$$\begin{aligned}
& \max_{r_2} && \int_0^{R^*} [\alpha u(r_1) + (1 - \alpha)u(\bar{r})] f(R) dR \\
& && + \int_{R_1^*}^{R^*} [\alpha u(r_1) + (1 - \alpha)u(\frac{1 - i + iR - r_1 \alpha}{1 - \alpha})] f(R) dR \\
& && + \int_{R^*}^{\infty} [\alpha u(r_1) + (1 - \alpha)u(r_2)] f(R) dR \\
& \text{s.t.} && 1 - i = r_1 \alpha \\
& && R_1^* = \frac{r_1 \alpha + \bar{r}(1 - \alpha) - 1 + i}{i} \\
& && R^* = \frac{r_1 \alpha + r_2(1 - \alpha) - 1 + i}{i} \\
& && \int_{R^*}^{\infty} i(R - R^*) \cdot f(R) dR = \delta
\end{aligned}$$

The objective function reflects the fact that the maximum amount of payment a depositor can receive is \bar{r} when the bank is insolvent. And similarly, both the budget constraint and the incentive constraint are binding; therefore the only choice variable is r_2 . After some algebra, it is shown that

$$\begin{aligned}
\frac{dU}{dr_2} &= \frac{\partial U}{\partial r_2} + \frac{\partial U}{\partial r_1} \cdot \frac{dr_1}{dr_2} \\
&= \frac{\int_{R^*}^{\infty} (1-\alpha)f(R)dR}{\int_{R^*}^{\infty} \alpha R f(R)dR} \cdot [u'(r_2) \int_{R^*}^{\infty} \alpha R f(R)dR \\
&\quad + \int_{R_1^*}^{R^*} \alpha R u'(\frac{(1-r_1\alpha)R}{1-\alpha})f(R)dR - \alpha u'(r_1)] \\
&< \frac{1-\alpha}{E(R|R \geq R^*)} \cdot [u'(\bar{r}) \int_{R_1^*}^{\infty} R f(R)dR - u'(r_1)] \\
&< \frac{1-\alpha}{E(R|R \geq R^*)} \cdot [u'(r_2^f)E(R) - u'(r_1^f)] \\
&= 0
\end{aligned}$$

where the first inequality comes from the fact that $r_2 \geq \bar{r}$ and $\frac{(1-r_1\alpha)R}{1-\alpha} > \bar{r}$ for $R \in [R_1^*, R^*]$, and the second inequality uses the fact that $r_1 \leq r_1^f$ when $r_2 \geq r_2^f$. Therefore, the first-best contract is also chosen.

Combining the results, under the ICDC with $\bar{r} = r_2^f$ and $\delta^f = \int_{E(\tilde{R})}^{\infty} i^f [R - E(\tilde{R})] \cdot f(R)dR$, the banks will choose the first-best contract and the economy achieves the first-best social optimum.

H Proof of Proposition 6

The proof is divided into the following steps.

– Step 1: $i = 1 + \kappa - r_1\alpha$.

Define $E(\Pi)$ as the expected profit for the banks as specified in equation (4.9). It is easy to show that

$$\frac{\partial E(\Pi)}{\partial i} = \int_{R_2}^{\infty} (R-1)f(R)dR > 0$$

Therefore, equation (4.11) is binding. Accordingly, the incentive constraint (equation 4.10) is rewritten as:

$$\int_{R_2}^{\infty} (1 + \kappa - r_1\alpha)(R - R_2)f(R)dR - \kappa \cdot E(R) = 0 \tag{H.1}$$

The Lagrange equation for the problem is:

$$\begin{aligned}
\text{MAXU}(\kappa) &= \max_{r_1, r_2} \int_0^{R_1} \frac{r_1 \alpha + (1 + \kappa - r_1 \alpha) R (1 - \tau)}{r_1} u(r_1) \cdot f(R) dR \\
&+ \int_{R_1}^{R_2} [\alpha u(r_1) + (1 - \alpha) u(\frac{(1 + \kappa - r_1 \alpha) R}{1 - \alpha})] \cdot f(R) dR \\
&+ \int_{R_2}^{\infty} [\alpha u(r_1) + (1 - \alpha) u(r_2)] \cdot f(R) dR \\
&+ \lambda \{ \int_{R_2}^{\infty} (1 + \kappa - r_1 \alpha) (R - R_2) f(R) dR - \kappa \cdot E(R) \}
\end{aligned}$$

where $R_1 = \frac{r_1(1-\alpha)}{1+\kappa-r_1\alpha}$, $R_2 = \frac{r_2(1-\alpha)}{1+\kappa-r_1\alpha}$, and λ is the Lagrange multiplier. $\text{MAXU}(\kappa)$ is the indirect utility function.

– Step 2: The first-order conditions for the above problem are:

$$\begin{aligned}
\frac{\partial \text{MAXU}(\kappa)}{\partial r_1} &= \int_0^{R_1} \left\{ \frac{(1 + \kappa - r_1 \alpha) R (1 - \tau)}{r_1} u'(r_1) - \frac{(1 + \kappa)(1 - \tau) R u(r_1)}{r_1^2} \right\} f(R) dR \\
&+ \alpha u'(r_1) - \int_{R_1}^{R_2} \alpha R u' \left[\frac{(1 + \kappa - r_1 \alpha) R}{1 - \alpha} \right] f(R) dR \\
&- (1 - \alpha) \tau u(r_1) f(R_1) \frac{(1 - \alpha)(1 + \kappa)}{(1 + \kappa - r_1 \alpha)^2} - \lambda \alpha \int_{R_2}^{\infty} R f(R) dR \\
&= 0
\end{aligned} \tag{H.2}$$

$$\begin{aligned}
\frac{\partial \text{MAXU}(\kappa)}{\partial r_2} &= \int_{R_2}^{\infty} (1 - \alpha) [u'(r_2) - \lambda] f(R) dR \\
&= 0 \\
\Rightarrow u'(r_2) &= \lambda
\end{aligned} \tag{H.3}$$

– Step 3: As $\kappa \rightarrow \infty$, the equilibrium converges to the first-best allocation.

When $\kappa \rightarrow \infty$, by using $R_1 \rightarrow 0$, $R_2 \rightarrow 0$, and $R_3 \rightarrow 1$, equation (H.2) can be written as

$$\begin{aligned}
\alpha u'(r_1) - \lambda \alpha E[\tilde{R}] &= 0 \\
\Rightarrow u'(r_1) &= \lambda E[\tilde{R}]
\end{aligned}$$

Combined with equation (H.3), we obtain the familiar Euler equation $u'(r_1) = E[\tilde{R}] \cdot u'(r_2)$.

Furthermore, I show that $r_2 = \frac{(1-r_1\alpha)E[\tilde{R}]}{1-\alpha}$ satisfies the incentive constraint (equation H.1) as $\kappa \rightarrow \infty$. As $i = 1 + \kappa - r_1\alpha$, $r_2 = \frac{(1-r_1\alpha) \cdot E(R)}{1-\alpha}$, when $\kappa \rightarrow \infty$,

$$\int_{R_2}^{\infty} (1 + \kappa - r_1 \alpha) (R - R_2) f(R) dR - \kappa E(R)$$

$$\begin{aligned} &\rightarrow \int_0^\infty (1 + \kappa - r_1\alpha)Rf(R)dR - r_2(1 - \alpha) - \kappa E(R) \\ &= (1 - r_1\alpha)E(R) - r_2(1 - \alpha) = 0 \end{aligned}$$

From Lemma 3, it is safe to conclude that the market equilibrium converges to the first-best allocation as the capital requirement increases.

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