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Forecasting swap rate volatility with information from swaptions*

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Abstract

We examine the predictability of the model-free implied volatility from swaptions on future realized volatility of the underlying swap rates. The model-free implied volatility demonstrates significant predictability on future realized volatility of swap rates along a wide cross-section of tenors. The predictive power of the model-free implied volatility is superior to the predictability of lagged realized volatility and GARCH-type conditional volatility. The superior predictive power of the model-free implied volatility also holds out of sample, in different market states and with longer forecasting horizons.

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1. Introduction

Volatility forecasting is important for financial investment and risk management. On the one hand, volatility is crucial for portfolio management as it is an important consideration for investors when making investment decisions and creating portfolios. A good forecast for volatility over the investment holding period is essential for assessing investment risks. On the other hand, volatility forecasting is helpful for risk management. One is likely to obtain proper estimates of value at risk (VaR) or expected shortfall given a good forecast of future volatility.

Interest rate swap is one of the largest segments in the over-the-counter derivatives market. The trading volume of interest rate derivatives in the over-the-counter market more than doubled from 2016 to 2022 (McGuire (2022)). The total outstanding notional value is more than 460 trillion dollars by the end of 2020.[1] Swap rates are essential in the global financial system as they reflect funding costs for major financial institutions. In this paper, we exploit over-the-counter swaption[2] data to construct the model-free implied volatility and examine its predictive ability on future realized volatility of interest rate swap rates across various tenors.

With the inherent ex-ante forward-looking feature, option-implied volatility contains useful information about future realized volatility. There is a stream of literature that examines the information content of option-implied volatility in the equity market. In earlier work, implied volatility used is either simply the implied volatility of the at-the-money option or the weighted average of implied volatilities of close to at-the-money options (Christensen and Prabhala (1998)). Later, a model-free implied volatility measure is introduced by Demeterfi, Derman, Kamal, and Zou (1999), Carr and Madan (2001), Britten-Jones and Neuberger (2000) and Jiang and Tian (2005). The predictability of the model-free implied volatility on future realized volatility has been extensively investigated since then.[3] These studies show that option-implied volatility provides more useful

[1]Bank for International Sentiment: OTC derivatives statistics.(https://stats.bis.org/statx/srs/table/d7)

[2]A swaption refers to an option to enter into an interest rate swap or some other types of swap.

[3]Since the introduction of the VIX by the CBOE Options Exchange, most exchanges worldwide have launched their own volatility indexes by accommodating the calculation methodology of the VIX. Kourtis, Markellos, and Symeonidis (2016) study the information content of option-implied volatility indexes across various countries/regions. Wayne, Lui, and Wang (2010) compare the information of options traded OTC

information in forecasting future realized volatility in the equity market.

By contrast, the swaption market has been less studied. Trolle and Schwartz (2014) investigate the dynamics of the swap rate moments implied from swaption data and document the negative variance risk premium for swap rates. Grishchenko, Song, and Zhou (2017) find that the swap rate variance risk premium has predictive power for future Treasury bond returns. Our study complements the literature by examining the predictive power of the model-free implied volatility estimated from swaptions on future realized volatility for swap rates across different tenors.

We obtain a rich USD swaption cube data set from JP Morgan, one of the largest interdealer brokers in the interest rate derivatives market. The swaption data is at daily frequency, with 19 different underlying swap tenors, ranging from 1-month to 30-year, and 25 different expirations, ranging from 1-week to 30-year. For each expiration and each underlying interest rate swap, there are 7 different strike rates. Relying on such a rich set of over-the-counter swaption cube data, we calculate the model-free implied volatility with different times to maturity for swap rates with various tenors.

We focus on investigating the predictive power of the model-free implied volatility on future realized volatility of the underlying swap rates. Used alone, the model-free implied volatility shows both statistically and economically significant predictability on 1-month ahead realized volatility for swap rates across various tenors, complementing the findings from the equity and foreign exchange markets (e.g. Jorion (1995)). Moreover, the model-free implied volatility from swaptions is an upward biased predictor of future realized volatility.

Then, we compare the predictability of the model-free implied volatility to two other predictors: the lagged realized volatility and the conditional volatility estimated from a set of GARCH models. Although these two alternative predictors have strong predictive power for future realized volatility, the performance of the model-free implied volatility is superior to all these measures. In particular, the in-sample adjusted R-squared is much higher when predicting with the model-free implied volatility than when predicting either with the lagged realized volatility or with the GARCH-type conditional volatility. In addition, with the model-free implied volatility, adding the lagged realized volatility or the

and that traded on exchanges in Hong Kong SAR and Japan.

GARCH-type conditional volatility neither weakens the effect of the model-free implied volatility, nor improves the forecasting performance.

Besides the various in-sample tests and model comparisons, we examine the out-of-sample performance by comparing the root mean squared errors of the forecasts from different forecasting models. The superior forecasting performance of the model-free implied volatility also holds out-of-sample. The root mean squared errors generated from the linear forecasting model in which the model-free implied volatility is used as the predictor are not only significantly lower than those from the benchmark model in which the forecast is simply the historical average, but also significantly lower than those from alternative forecasting models in which either lagged realized volatility or the GARCH-type conditional volatility is used as the predictor. With the model-free implied volatility, adding alternative predictors does not further reduce the root mean squared errors significantly.

The superior predictability of the model-free implied volatility is robust across different market states. When the market is in business cycle recessions or when the monetary policy uncertainty is high, the model-free implied volatility demonstrates superior forecasting performance. Finally, both of the in-sample and out-of-sample superior performances of the model-free implied volatility persist when we extend the forecasting horizons from 1 month to 3 months, 6 months and 1 year. If anything, the degree of superior performance decreases as the forecast horizon becomes longer.

Our study contributes to the literature in two aspects. First, this paper contributes to the literature on the information content of the interest rate derivatives market. The interest rate derivatives market is very large. Aside from the theoretical derivative pricing, the information content of derivative prices for the underlying instruments is less studied. This is perhaps because of data limitations. Reliable quotes from principal interdealer brokers are necessary. The data we use spans a long sample period from January 1997 to June 2022. This data is rich with a wide variety of times to maturity and a wide cross-section of swap rate tenors.

Second, this paper contributes to the literature on volatility forecasting. The superior information content of the option-implied volatility has been studied on the equity market and foreign exchange market (Jorion (1995)). We complement the literature by focusing on another large market: the interest rate swap market. The swaption-implied information, swap rate model-free implied volatility (SRMFIV), shows strong and significant predictive power on future realized volatility for swap rates with various tenors. We confirm the superior forecasting performance of SRMFIV not only in sample but also out of sample. Both the financial industry and academia pay continuing attention to the VIX, the fear index in the equity market. Our study helps draw attention to the fear index in the interest rate swap market.

The remainder of the paper is organized as follows. Section 2 describes the swaption data, the methodology used to calculate the swap rate model-free implied volatility and the measurement of the realized volatility. Section 3 examines the in-sample predictive power of the swap rate model-free implied volatility on future realized volatility for swap rates with various tenors. The predictability of the swap rate model-free implied volatility is compared with that of other commonly used predictors in Section 4. Section 5 examines the out-of-sample predictive performance. Section 6 provides robustness analysis and examines the predictive power at different market states and with longer forecasting horizons. Section 7 concludes the paper.

2. Data

In this section, we describe the swaption cube data, the methodology used to calculate the model-free implied volatility from swaptions and the way we measure realized volatility. Statistical properties of the model-free implied volatility and realized volatility are also discussed.

2.1. Swaption cube

Broadly speaking, a swaption is an option granting its owner the right, not the obligation, to enter into an underlying swap. Although the underlying could be a variety of swaps, the term “swaption” typically refers to options on interest rate swaps. A receiver swaption gives the owner of the swaption the right to enter into a swap in which they receive the fixed leg and pay the floating leg. A payer swaption gives the owners of the swaption the right to enter into a swap where they pay the fixed leg and receive the floating leg. The participants in the swaption market are dominated by large corporations, banks, financial institutions and hedge funds. The swaption market is primarily over-the-counter (OTC). Interdealer brokers serve as the main intermediaries providing liquidity.

We obtain USD swaption data from JP Morgan, one of the largest interdealer brokers in the interest rate derivatives market. Since the underlying interest rate swaps vary with tenors, the swaption price quotation data is usually called a swaption cube which varies along three dimensions: the tenor of the underlying interest rate swap, the time-to-expiration of the option and the strike swap rate. The swaptions are customarily quoted in terms of Black or basis point implied volatilities. Black volatilities can be easily converted into payer and/or receiver premiums using the Black (1976) formula, which will be described in detail later. The data we obtained is in daily frequency and the sample period is from May 01, 1992 to June 30, 2022. The swaption data are rich with 19 different underlying swap tenors (1-month, 3-month, 6-month, 1-year, 1.5-year, 2-year, 3-year, 4-year, 5-year, 6-year, 7-year, 8-year, 9-year, 10-year, 12-year, 15-year, 20-year, 25-year and 30-year) and with 25 different times to expiration (1-week, 2-week, 3-week, 1-month, 2-month, 3-month, 6-month, 9-month, 1-year, 1.5-year, 2-year, 2.25-year, 2.5-year, 3-year, 4-year, 5-year, 6-year, 7-year, 8-year, 9-year, 10-year, 12-year, 15-year, 20-year and 30-year). For each time-to-expiration and each underlying interest rate swap, there are 7 different strike rates quoted as: at-the-money forward swap rate (ATMF)−300bp, ATMF−100bp, ATMF−50bp, ATMF, ATMF+50bp, ATMF+100bp, and ATMF+300bp. Since some of the quotes are missing in the early sample period, our analysis focuses on the sample period from January 02, 1997 to June 30, 2022. Given that not all the interest rate swaps are popular and have enough liquidity, we focus on swaps with the following eight tenors: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year. Balancing the length of the final sample and non-overlapping sampling frequency, we mainly focus on swaptions with 1-month time-to-expiration and construct the sample with monthly frequency.

2.2. Swap rate model-free implied volatility

Swap rate model-free implied volatility (SRMFIV) is a forward-looking estimate of swap rate volatility, similar to the VIX which is estimated from S&P 500 index options. In practice, there are two versions of swap rate model-free implied volatility: log-normal model-free implied volatility and basis point (normal) model-free implied volatility. Different from stock index levels and individual stocks prices, absolute (level) changes describe risks more effectively than relative (percentage) changes in the context of yields and spreads. Market participants think of volatility in terms of absolute changes. Therefore, following Mele and Obayashi (2015), Mele, Obayashi, and Shalen (2015) and Trolle and Schwartz (2014), we estimate the basis point swap rate model-free implied volatility.

We briefly describe the methodology used to calculate the swap rate model-free implied volatility. The detailed and rigorous proofs could be found in Mele and Obayashi (2014), Mele and Obayashi (2015) and Trolle and Schwartz (2014). When we consider a fixed versus floating interest rate swap for the period T_m to T_n with a fixed rate of K, the SRMFIV is calculated by:

$$SRMFIV_{t,m,n} = 10000 \times \sqrt{\frac{1}{T_m - t} Var_t^{\mathbb{A}}\left(S_{m,n}(T_m)\right)}, \tag{1}$$

$$Var_t^{\mathbb{A}}\left(S_{m,n}(T_m)\right) = \frac{2}{A_{m,n}(t)} \left(\int_{S_{m,n}(t)}^{\infty} P_{m,n}(t,K)dK + \int_0^{S_{m,n}(t)} R_{m,n}(t,K)dK \right), \tag{2}$$

where $P_{m,n}(t,K)$ is the payer swaption premium and $R_{m,n}(t,K)$ is the receiver swaption premium at time t with the strike swap rate K. $T_m - t$ is the remaining time-to-expiration of the swaption and $T_n - T_m$ is the tenor of the underlying swap rate. $S_{m,n}(t)$ is the at-the-money strike swap rate (i.e. forward swap rate) and $A_{m,n}(t)$ is the annuity factor. With multiplying by 10000, the units of the estimated SRMFIV is in basis point.

As mentioned in the previous section, the swaption data we obtained from JP Morgan is quoted in Black volatility, so that we need to transform the Black volatility to payer premium or receiver premium in order to estimate Eq. (1) and Eq. (2). The swaption pricing formula by Black (1976) are:

$$P_{m,n}(t,K) = A_{m,n}(t) \left(S_{m,n}(t)\Phi(d_1) - K\Phi(d_2) \right), \tag{3}$$

$$R_{m,n}(t,K) = A_{m,n}(t) \left(-S_{m,n}(t)\Phi(-d_1) + K\Phi(-d_2) \right), \tag{4}$$

where $d_1 = \frac{ln(S_{m,n}(t)/K) + \frac{1}{2}\sigma^2(T_m - t)}{\sigma\sqrt{T_m - t}}$ and $d_2 = d_1 - \sigma\sqrt{T_m - t}$. $\Phi(\cdot)$ is the cumulative normal distribution function and σ is the Black volatility. Note that the annuity factor will be eliminated in the final calculation of SRMFIV.

To empirically estimate the integration in Eq. (2), numerical integration methods are necessary. Such methods have the following limitations: (i) the limited range of strike rates: only ±300bps from the forward swap rate; and (ii) the sparse set of the discrete strike rates: only 7 different strike rates. To empirically approximate Eq. (2), we take the following steps.[4] We first expand the range of the strike swap rate to $[0.1 \times ATMF, 2 \times ATMF]$ and equally space the range to 1000 points. Second, we linearly interpolate the Black volatility between the available strike swap rates and extrapolate with constant Black volatility outside of the available strike swap rates. For strike swap rates smaller than the lowest available strike swap rate, we use the Black volatility at the lowest strike swap rate. For strike swap rates larger than the available highest strike swap rate, we use the Black volatility at the highest strike swap rate. Third, the Black volatility is transformed to payer premium or receiver premium according to Eq. (3) or Eq. (4). Finally, numerical approximation of Eq. (2) is obtained.[5]

2.3. Realized volatility

The true volatility is unobservable and is usually proxied by realized volatility. Realized variance is calculated as the sum of squared returns sampled at some intervals. Quadratic variation theory implies that realized variance asymptotically converges to the actual unobserved variance as the sampling frequency increases to infinity (Barndorff-Nielsen and Shephard (2002)). The literature that focuses on the equity market usually measures the realized volatility with intra-day, such as 5-minute returns (Jiang and Tian (2005), Kourtis et al. (2016)). However, since the interest rate swap market and swaption market are OTC, we do not have high quality intra-day data for the underlying swap rates. Consistent with common market practice, we use absolute changes of swap rates at the daily frequency to measure realized volatility, following Trolle and Schwartz (2014). Specifically, with daily

[4]The empirical steps closely follow Jiang and Tian (2005). Jiang and Tian (2005) carefully examine several implementation issues such as truncation error, discretization error and limited availability of strike prices, and conclude that curve-fitting and extrapolation could provide accurate estimation of the integral in the equation.

[5]We tried different strike swap rate ranges, different numbers of equally-spaced points and different filtration criteria, and the resulting SRMFIV series do not change much.

absolute changes, the ex-post realized volatility could be calculated as:

$$RV_t = 10000 \times \sqrt{\frac{252}{N_t} \sum_{i=1}^{N_t} (S_i - S_{i-1})^2}, \quad (5)$$

where S_i is the swap rate on day i in month t and N_t is the number of available days in month t. The realized volatility is annualized and expressed in basis point.[6]

2.4. *Summary statistics*

As described in the previous section, we obtain a set of SRMFIV series for swap rates with various tenors and various times to expiration. As a first step to justify the quality of the data we use and our numerical approximation methodology, we compare our SRMFIV series for 1-year time-to-expiration and the underlying swap rate tenor of 10 years to the SRVIX index [7] (CBOE (2018)) and the VIX index which are calculated and disseminated by Chicago Board Options Exchange (CBOE). The dynamics of the time series of our SRMFIV (blue line), the SRVIX (red line) from CBOE and the VIX (purple line) from CBOE are plotted in Figure 1.

[Figure 1 about here]

The sample period for CBOE SRVIX is from June 18, 2012 to February 11 2022. As seen from Figure 1, our SRMFIV tracks the SRVIX closely. For the common sample period, the correlation between our SRMFIV and SRVIX is about 0.9617.

The dynamics of SRMFIV is different from that of VIX, the implied volatility of equity market. SRMFIV and VIX are positively correlated with a correlation coefficient of only 0.4112. There are pretty much notable deviations between these two fear indices. During our sample period, VIX exhibits a larger volatility and more frequent fluctuations than SRMFIV does. Moreover, SRMFIV and VIX react differently to events from the debt

[6]There are alternative ways to form a proxy for the latent volatility. For example, a range-based estimator by Garman and Klass (1980) is popular. However, we do not have reliable data on daily high or daily low for the swap rates from the JP Morgan data. Our method to calculate the realized volatility exactly follows Trolle and Schwartz (2014). Although there are concerns related to the estimation of the realized volatility, we believe that the method we used is acceptable, taking into consideration of the data availability and market practice.

[7]SRVIX is 1-year implied volatility of the swap rate with the tenor of 10 years (https://www.cboe.com/us/indices/dashboard/SRVIX/).

and equity markets (Mele et al. (2015)). For example, SRMFIV suddenly hiked by 35bps from about 77bps to 112bps around late 2013 when US Treasury yields surged after the Federal Reserve announced a tapering of quantitative easing (known as Taper Tantrum), while VIX did not experience significant spikes during the same period. Also, during the business cycle recession in 2001, SRMFIV kept climbing up by around 25bps, while VIX dipped and showed a V-shaped fluctuation. Along with the market expectations on the Federal Reserve's policy rate normalization, both the SRMFIV and VIX increased since the beginning of 2022. Therefore, the SRMFIV capture the market concerns in a different manner from the VIX. In the following baseline analysis, we focus on swaptions with expiration of one month. The dynamics of the model-free implied volatility are shown in Figure 2.

[Figure 2 about here]

The basic statistical properties of the 1-month SRMFIV and realized volatility are summarized in Table 1. Panel A shows the summary statistics for the annualized 1-month realized volatility. For the swap rate with tenor of 3 months, the realized volatility has the mean of 23.99 basis points and the standard deviation of 33.89 basis points. Its distribution is right skewed with heavy tails. The results from the Jarque-Bera test suggest that the distribution is not normal. The realized volatility demonstrates strong persistence with the autocorrelations at lags 1, 2 and 3 being 0.55, 0.35 and 0.31, respectively. The mean realized volatility generally increases as the tenor increases, which is as expected when we estimate the volatility of the absolute changes for the swap rates. The 10-year swap rate has the highest average realized volatility of 86.16 basis points. The degree of right skewness of the distribution of the realized volatility decreases as the swap tenor increases.

[Table 1 about here]

Panel B reports the summary statistics for the swap rate model-free implied volatility we estimated. The averages of all the SRMFIV series are larger than those of the realized volatility in Panel A. For the swap rate with the tenor of 3 months, the 1-month SRMFIV is 46.15 basis points, which is larger than the average realized volatility of 23.99 basis points. For the swap rate with the tenor of 1 year, the 1-month SRMFIV is 66.01 basis points, which is again larger than the average realized volatility of 48.79 basis points. For the swap rate with the tenor of 10 years, the 1-month SRMFIV is 100.97 basis points, which is larger than the average realized volatility of 86.16 basis points. These imply negative variance risk premium for the swap rate. Moreover, most of the SRMFIV series have lower standard deviations and larger autocorrelations than those of the realized volatility. This is consistent with the finding that implied volatility is a smoothed expectation of realized volatility, in which case it should be less variable and more persistent than realized volatility (Christensen and Prabhala (1998)). The correlations between the realized volatility and SRMFIVs are reported in Table 11 in the Appendix.

3. Information content of SRMFIV

In this section, we investigate the information content of the SRMFIV implied from swaptions for forecasting swap rate realized volatility over the remaining life of the swaptions. In the main analysis, we focus on the non-overlapping sample. Specifically, the sampling frequency is one month. The remaining time-to-expiration of the swaptions and the time horizon for measuring the realized volatility are also one month. Following the literature which examines the information content of the model-free implied volatility on the equity market, such as Christensen and Prabhala (1998), Mayhew and Stivers (2003), Koopman, Jungbacker, and Hol (2005), Jiang and Tian (2005) and Wayne et al. (2010), we examine the predictability of SRMFIV on future swap rate realized volatility by estimating a time-series predictive regression of the following form:

$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \varepsilon_{t+1}, \tag{6}$$

where RV_{t+1} is the swap rate realized volatility in month $t+1$ and $SRMFIV_t$ is the swap rate model-free implied volatility at the end of month t. The standard errors are calculated taking into account the possible heteroscedastic and autocorrelated structure in the errors (Newey and West (1987)).

Following Christensen and Prabhala (1998) and Jiang and Tian (2005), three hypotheses could be tested: (1) if SRMFIV contains information about future realized volatility, β_1 should be non-zero; (2) if SRMFIV is an unbiased forecast of future realized volatility, we should obtain $\beta_0 = 0$ and $\beta_1 = 1$ and (3) if SRMFIV is efficient for forecasting future realized volatility, the residuals, ε_{t+1}, in the above regression should be white noise and uncorrelated with any variables in the market's information set.

[Table 2 about here]

The univariate predictive regression estimation results are reported in Table 2 Panel A. Each column reports the estimates for the swap rate with the following eight different tenors: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year. The estimates of β_1 are all positive, ranging from 0.78 to 0.85 and with strong statistical significance. Thus, SRMFIV contains significant information about future realized volatility for swap rates across a wide range of tenors. The last row reports the F statistic for the joint hypothesis: $\beta_0 = 0$ and $\beta_1 = 1$. Large F statistics, ranging from 41.16 to 169.92, show that SRMFIV is a biased forecast of future realized volatility for all swap rates. The estimated adjusted R^2s are large and range from 0.43 to 0.71. The Durbin-Watson statistics (Durbin and Watson (1950)) are around two, indicating that the residuals from the univariate regression are not autocorrelated.

In addition, we equally separate the whole sample period into two parts: from January 1997 to September 2009 and from October 2009 to June 2022, and re-estimate the predictive regression of Eq.(6). The results are reported in Panel B and Panel C of Table 2. The estimates show that the significant information content of the SRMFIV implied from swaptions on future realized volatility of the underlying swap rates is present in the two subsample periods.

In summary, by estimating an univariate predictive regression as in Eq.(6), we show that the swap rate model-free implied volatility, inferred from the swaptions across different strike rates, contains significant information for future realized volatility of swap rates across a wide range of tenors. The results are comparable to the literature which focuses on other asset classes, such as stock, currency and commodity. The information content of SRMFIV is strong, albeit the swaption market is OTC. How is the information content of such swap rate model-free implied volatility compared to that of other commonly used predictors? We address this question in the next section.

4. Comparison with other models

In the previous section, we have shown that the swap rate model-free implied volatility alone demonstrates significant predictability for future realized volatility of swap rates across various tenors. In this section, we compare the predictability of SRMFIV on future realized volatility to some commonly used alternative predictors: lagged realized volatility and conditional volatility estimated from GARCH-type models.

4.1. Comparison with lagged realized volatility

Lagged realized volatility is one of the commonly used measures to predict future realized volatility. We compare the predictability of the swap rate model-free implied volatility to lagged realized volatility with a set of regressions of the following form:

$$RV_{t+1} = \beta_0 + \beta_2 RV_t + \varepsilon_{t+1}, \tag{7}$$

$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \beta_2 RV_t + \varepsilon_{t+1}, \tag{8}$$

where RV_t is the realized volatility in month t and other variables are defined in the same way as those used in previous sections. The results of the regressions are presented in Table 3.

[Table 3 about here]

Panel A of Table 3 reports the estimation results from the univariate regression in Eq.(7), in which the lagged realized volatility is used alone to predict the 1-month ahead future realized volatility. Panel A shows that, used alone, the lagged realized volatility demonstrates strong predictability of future realized volatility. The estimates of the coefficient on RV_t range from 0.56 to 0.78 and are all statistically significant. The estimates of the adjusted R-squared range from 0.3 to 0.6. When we compare the results from Panel A with those from Table 2, the magnitudes of both the slope estimate and the adjusted R-squared are larger in Table 2, indicating that the predictability of SRMFIV is stronger than that of the simple lagged realized volatility.

Panel B reports the estimates of the multivariate regression of Eq. (8), in which both the lagged realized volatility and SRMFIV are included in the predictive regression. The

SRMFIV still shows significant predictability of the future realized volatility. However, the predictive power of the lagged realized volatility either weakens or disappears in most cases. Moreover, the magnitude of the coefficients on SRMFIV from the multivariate regressions in Panel B does not change much compared to that in Table 2.

These results show that the swap rate model-free implied volatility contains significant information content for future realized volatility and the coefficient on SRMFIV remains significant when we do a horse race between the SRMFIV and the lagged realized volatility..

4.2. Comparison with GARCH-type model

Other commonly used alternative models for swap rates are the time series models (Poon and Granger (2003)).[8] Specifically, we choose the $GARCH(1,1)$ model for the daily absolute change of swap rates:

$$
\begin{aligned}
\Delta S_t &= \mu + \varepsilon_t \\
\varepsilon_t &= \sigma_t z_t \\
\sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \gamma \sigma_{t-1}^2.
\end{aligned}
$$

At the end of month t, we use the swap rate's daily changes in the most recent two years to fit a $GARCH(1,1)$ model. Then, following Polkovnichenko and Zhao (2013) and Rosenberg and Engle (2002), with the estimated $GARCH(1,1)$ model parameters and the filtered innovation terms, we simulate[9] the model 22 steps ahead, which is about one month forward,[10] and over 1000 sample paths. Finally, the monthly swap rate change is obtained by aggregating each sample path, and the standard deviation across the 1000 simulated points is estimated as a predictor for future realized volatility. One month later, everything repeats until the sample period ends. With a rolling window of two years, we try to balance between estimation accuracy which needs as many observations as possible and potential structural change which prefers a short and most recent historical sample.

[8]The NYU V-Lab models important market series with various time series models. Among them are the interest rate swaps (https://vlab.stern.nyu.edu/analysis/VOL.IRSWAP10%3AGOVT-R.GARCH)

[9]A more traditional method to generate forecasts with the GARCH model is to generate the forecasts with the estimated model parameters. However, we employ the simulation method here by taking into account the empirical fact that the standardized filtered innovations show heavier tails than the standard normal density assumption. Simulation with the estimated model parameters and the empirical innovation density thus captures the swap rate dynamics more closely.

[10]For a forecasting horizon of one month, 22 trading days is a commonly used assumption. The results are robust with other assumptions such as 20 days.

We denote the predictor estimated in such a way by $VGARCH$. As in the previous section, we investigate the predictability of $VGARCH$ and compare it with $SRMFIV$ by the following set of regressions:

$$RV_{t+1} = \beta_0 + \beta_2 VGARCH_t + \epsilon_{t+1}, \tag{9}$$

$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \beta_2 VGARCH_t + \epsilon_{t+1}, \tag{10}$$

where the variables are defined in the same way as those used in the previous sections. The estimates are reported in Table 4.

[Table 4 about here]

Panel A shows that the volatility predictor estimated from the $GARCH(1,1)$ model demonstrates statistically significant predictability for the future realized volatility, with the slope coefficients ranging from 0.45 to 0.81. However, the magnitude of the adjusted R-squared is much smaller than that in Table 2. Moreover, the predictive power of the conditional volatility estimated from the GARCH model is even weaker than that of the lagged realized volatility in Table 3. Panel B shows the estimates when both $VGARCH$ and $SRMFIV$ are included in the predictive regression. With $SRMFIV$ included in the predictive regression, the predictive power of $VGARCH$ all disappears.

In summary, all these results show that the model-free implied volatility inferred from swaptions is superior to the conditional volatility estimated from the GARCH-type model in terms of predicting future realized volatility for swap rates with different tenors.

4.3. Multivariate regression with all alternative predictors

Finally, we examine the predictive power of the swap rate model-free implied volatility while controlling all the other alternative predictors. We run a regression of the following form:

$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \beta_2 RV_t + \beta_3 VGARCH_t + \epsilon_{t+1}, \tag{11}$$

where all the variables are defined in the same way as those from the previous sections. The estimates are reported in Table 5.

[Table 5 about here]

All the columns of Table 5 show that the swap rate model-free implied volatility possesses superior predictive ability for future realized volatility. All the alternative predictors, the lagged realized volatility and the conditional volatility estimated with the GARCH model, do not have any marginal predictability when the swap rate model-free implied volatility is included in the predictive regression.

In summary, by various comparisons, we show that the swap rate model-free implied volatility, estimated from swaptions which are traded OTC, is a superior predictor for the future realized volatility of swap rates across various tenors. With the swap rate model-free implied volatility included, alternative predictors do not provide much additional predictability. These results are similar to the findings from the exchange-traded equity index options market (Christensen and Prabhala (1998) and Jiang and Tian (2005)). The forward-looking feature of the model-free implied volatility contributes to such superiority.

5. Out-of-sample performance

In the previous sections, the superior performance of the swap rate model-free implied volatility on future realized volatility is examined in-sample. However, despite the strong in-sample predictive performance, whether the superior predictive performance also holds out-of-sample is not clear (Poon and Granger (2003)). In this section, we examine the predictability of the swap rate model-free implied volatility and other alternative models and compare these models from the perspective of out-of-sample predictive performance.

As an out-of-sample performance measure, we choose the root mean squared error (RMSE) which is commonly used in the forecasting literature (Poon and Granger (2003)). For each model, we calculate the RMSE in the following way:

$$RMSE = \sqrt{\frac{1}{T-T_0}\sum_{t=T_0+1}^{T}\left(RV_t - \widehat{RV}_t\right)^2}, \tag{12}$$

where RV_t is the realized volatility for month t and $\widehat{RV}_t$ is the predicted volatility for month t and estimated from each of the various forecasting models at the end of month $t-1$. The first T_0 months of the sample are used for the initial estimation. The sample used for estimation is expanded as time goes on.

As the benchmark model, we choose the historical mean method where the forecast of the volatility for month t is simply the average of the historical volatilities up to month $t-1$. Then we consider three models. "Model 1" uses the swap rate model-free implied volatility as the predictor in a univariate predictive regression as in Eq. (6). "Model 2" uses lagged realized volatility as the predictor in the univariate predictive regression as in Eq. (7). Finally, "Model 3" uses the GARCH-type conditional volatility as the predictor in the univariate predictive regression as in Eq. (9).

To investigate the performance of the predictive regressions with different predictors, we compare the RMSEs of Model 1, Model 2, and Model 3 to the RMSE of the benchmark model. The benchmark model can also be regarded as a restricted model from each of the three models, in which the slope coefficient of the predictor is restricted to be zero. The difference between RMSEs from nested models is tested with the method by Clark and West (2007). In addition, to show the superior predictability of the swap rate model-free implied volatility to alternative predictors, we compare the RMSE of Model 1 to those of Model 2 and Model 3. The difference between RMSEs from different models (such as Model 1 v.s. Model 2 or Model 1 v.s. Model 3) is tested based on the method by Diebold and Mariano (2002). The empirical results are reported in Table 6.

[Table 6 about here]

The first 60 monthly observations are used for the initial estimation. Panel A reports the RMSEs for the benchmark model for swap rates with various tenors. The RMSEs vary between 32 and 43 basis points. Panel B reports the RMSEs for the three univariate predictive regression models. All of the three models generate lower RMSEs than the benchmark model reported in Panel A. The difference is examined based on the Clark-West test (Clark and West (2007)). All of the CW-statistics are positive and large in magnitude, for all of the three univariate predictive regression models and for swap rates with various tenors. These results suggest the superior out-of-sample performance of the various univariate forecasting models.

Moreover, Panel B demonstrates that Model 1 in which the swap rate model-free implied volatility is used as a single predictor is associated with the lowest RMSEs across all the tenors. For the swap rate with the tenor of 1 year, the RMSE generated by Model 1 is only 19.41 basis points, while the RMSEs generated by Model 2 and Model 3 are

23.80 and 30.60 basis points, respectively. For the swap rate with the tenor of 10 years, the RMSE generated by Model 1 is 22.58 basis points, while the RMSEs generated by Model 2 and Model 3 are 25.61 and 34.41 basis points, respectively. We conduct the formal test of comparing Model 1 to Model 2 or Model 3 based on the Diebold-Mariano test (Diebold and Mariano (2002)) and report the results in Panel C. The positive and high DM-statistics show that both Model 2 in which lagged realized volatility is used as a single predictor and Model 3 in which GARCH volatility is used as a single predictor generate higher RMSEs than Model 1. Moreover, the superior out-of-sample performance of Model 1 is robust along various tenors.

The in-sample examinations reported in Table 3, Table 4, and Table 5 show that the information content of the swap rate model-free implied volatility is superior to that of other commonly used predictors. Adding lagged realized volatility or conditional volatility from GARCH model neither weakens the contribution of the $SRMFIV$ nor improves the overall predictive performance. We further investigate this from the perspective of out-of-sample performance. We calculate the RMSEs for the three multivariate predictive regression models: "Model 4" is the model in which both swap rate model-free implied volatility and lagged realized volatility are included in the predictive regression as in Eq. (8). "Model 5" is the model in which both the swap rate model-free implied volatility and GARCH volatility are included in the predictive regression as in Eq. (10). "Model 6" is the model in which the swap rate model-free implied volatility, lagged realized volatility, and conditional volatility estimated from the GARCH model are all included in the predictive regression as in Eq. (11).

The empirical results are reported in Panel D of Table 6. Panel D shows that the RMSEs of Model 4, Model 5, and Model 6 are similar to those generated by Model 1 as shown in Panel B. The RMSEs range from 19 to 25 basis points. Since Model 1 is a nested version of Model 4, Model 5, and Model 6, we formally test our observations by the Clark-West test. All the CW-statistics are close to zero, indicating that none of Model 4, Model 5, or Model 6 is better than Model 1 in terms of out-of-sample predictive performance. Therefore, with the inclusion of the swap rate model-free implied volatility in the linear predictive model, neither the lagged realized volatility nor the conditional volatility estimated from the GARCH model provides any marginal contribution when predicting one month ahead future realized volatility for swap rates of various tenors.

In summary, we show that the superior predictability of the swap rate model-free implied volatility on future realized volatility for swap rates of various tenors is not only significant not only in-sample, but also out-of-sample.

6. Additional analyses

6.1. Robustness

In this section, we do a set of robustness examinations to further confirm our in-sample and out-of-sample findings documented in the previous sections.

6.1.1. Comparison with the GJR-GARCH model

In the previous sections, we rely on the GARCH(1,1) model to generate alternative volatility predictors. In this section, we further employ the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model (Glosten, Jagannathan, and Runkle (1993)) as a robustness test. Specifically, we choose the GJR-GARCH(1,1) model, which captures the leverage effect and the potential conditional heteroskedasticity properties of the daily changes of swap rates. The results are reported in Table 12 in the appendix. The results are similar to those shown in Table 4 and the conclusion remains robust. $VGJR$ (the conditional volatility estimated with a GJR-GARCH(1,1) model) alone demonstrates some predictive ability for future realized volatility of swap rates across various tenors. However, with $SRMFIV$ included in the predictive regression, the predictive power of $VGJR$ disappears.

6.1.2. Out-of-sample performance with rolling window

In section 5, we examine the out-of-sample performance with an expanding window scheme. As a robustness check, we further examine the out-of-sample performance of various models with a rolling window scheme, with the window length of 60 months. The results are reported in Table 13 in the appendix. The results are similar to those reported in Table 6. The swap rate model-free implied volatility shows superior out-of-sample predictive ability for future realized volatility with the rolling window scheme.

6.2. Predictability at different states of the market

In the previous sections, we have shown that the swap rate model-free implied volatility is a superior predictor for future realized volatility of swap rates with various tenors. The superior performance of the $SRMFIV$ results from the feature that it contains important forward-looking information. Forward-looking information should be more important for prediction when the dynamics of the variable of interest is volatile or when there is a potentially regime switching. Therefore, we examine this possibility by examining the predictive power of the model-free implied volatility at different states of the market.

First, we are interested in the relative strength of predictability during National Bureau of Economic Research (NBER)-dated business-cycle recessions and expansions. Following Neely, Rapach, Tu, and Zhou (2014), we tabulate the in-sample R^2 statistics for sample periods of recessions and expansions, separately. Specifically, we calculate the following version of in-sample R^2 [11]:

$$R^2_{IS,c} = 1 - \frac{\sum_{t=1}^{T} \mathbf{I}_t^c \hat{\varepsilon}_t^2}{\sum_{t=1}^{T} \mathbf{I}_t^c (RV_t - \overline{RV})^2} \quad \text{for } c \in \{REC, EXP\}, \tag{13}$$

where $\mathbf{I}_t^{REC}$ ($\mathbf{I}_t^{EXP}$) is an indicator variable which equals one when month t is a recession (expansion) and zero otherwise, $\hat{\varepsilon}_t$ is the fitted residual from the predictive regressions of Eqs.(6), (7) or (9), $\overline{RV}$ is the full sample average of RV. Similarly, to evaluate the out-of-sample performance, we calculate the following version of out-of-sample R^2:

$$R^2_{OOS,c} = 1 - \frac{\sum_{t=1}^{T} \mathbf{I}_t^c (RV_t - \widehat{RV}_t)^2}{\sum_{t=1}^{T} \mathbf{I}_t^c (RV_t - \overline{RV}_t)^2} \quad \text{for } c \in \{REC, EXP\}, \tag{14}$$

where $\widehat{RV}_t$ is the out-of-sample predicted one-month ahead future realized volatility based on models defined by predictive regressions of Eq. (6), (7) or (9) and $\overline{RV}_t$ is the out-of-sample predicted one-month ahead future realized volatility based on the benchmark model which is simply the historical average. The empirical results are reported in Table 7.

[Table 7 about here]

Panel A presents the results for in-sample performance and Panel B those for out-of-sample performance. In each panel, rows 1 to 2 are for Model 1 in which the swap

[11] Following Neely et al. (2014), there is no clear way of decomposition of the full sample R^2. The decomposition used is a natural way and by definition R^2_{REC} or R^2_{EXP} could be negative.

rate model-free implied volatility is used as the predictor in the predictive regression as in Eq.(6), rows 3 to 4 are for Model 2 in which lagged realized volatility is used as the predictor in the predictive regression as in Eq. (7), and rows 5 to 6 are for Model 3 in which GARCH-type conditional volatility is used as the predictor in the predictive regression as in Eq. (9).

For each model, the in-sample performance in recession periods is better than that in expansion periods, with $R^2_{IS,REC}$ larger than $R^2_{IS,EXP}$. However, in terms of out-of-sample performance, $R^2_{OOS,REC}$ is smaller than $R^2_{OOS,EXP}$. This is not surprising as the in-sample performance is estimated using the full sample and out-of-sample performance is evaluated with the expanding sample. In the sample period of our study, there are not many observations during recessions in the early training samples.

We find that the superior performance of Model 1 persists in both market states, whether measured in-sample or out-of-sample. Both $R^2_{IS,REC}$ and $R^2_{IS,EXP}$ ($R^2_{OOS,REC}$ and $R^2_{OOS,EXP}$) from Model 1 are larger than those from Model 2 or Model 3. Moreover, conditional on economic recession, the performance improvement of Model 1 is more remarkable. For swap rates with the tenor of 1 year, estimates of $R^2_{IS,REC}$ ($R^2_{OOS,REC}$) for Models 1, 2 and 3 are 66.58%, 51.15% and 26.13% (52.74%, 36.42% and -2.33%), while 68.49%, 60.45% and 38.23% (82.91%, 71.71% and 50.04%) for $R^2_{IS,EXP}$ ($R^2_{OOS,EXP}$), respectively. Alternatively, for swap rates with the tenor of 10 years, estimates of $R^2_{IS,REC}$ ($R^2_{OOS,REC}$) for Models 1, 2 and 3 are 64.53%, 48.37% and 1.32% (60.05%, 44.21% and -3.86%), respectively, while 48.88%, 41.61% and 11.88% (60.72%, 51.12% and 14.14%) for $R^2_{IS,EXP}$ ($R^2_{OOS,EXP}$), respectively. To further illustrate the differences in performance of Model 1 to other models in recessions and expansions, we take the average of $R^2_{IS,REC}$, $R^2_{IS,EXP}$, $R^2_{OOS,REC}$ and $R^2_{OOS,EXP}$ across all the tenors, respectively, and plot the results in Figure 3. From Panel B, we find that the difference between $R^2_{OOS,REC}$ and $R^2_{OOS,EXP}$ of Model 1 is smaller than those from Model 2 or Model 3, implying the superior performance of SRMFIV in predicting future realized volatility in recessions.

We also try to measure the market condition by monetary policy uncertainty (MPU) (Baker, Bloom, and Davis (2016))[12]. Their monetary policy uncertainty measure is a

[12]The data for monetary policy uncertainty (MPU) is directly downloaded from the website: https://www.policyuncertainty.com/bbd_monetary.html

newspaper-based index and is constructed as scaled frequency counts of newspaper articles that discuss monetary policy uncertainty. Monetary policy uncertainty is a natural measure of market condition for the interest rates market. When the monetary policy uncertainty is high, sophisticated market participants demonstrate better ability to extract information about interest rates, including the second-order moments. As is well known, the participants in the swaption market are dominated by large corporations, banks, financial institutions and hedge funds, all of which are sophisticated investors. Therefore, we hypothesize that the information content of the swap rate model-free implied volatility for future realized volatility is superior when the monetary policy uncertainty is high. Specifically, month t is classified as "HIGH" if the MPU is above the sample median and as "LOW" otherwise. We measure the forecasting performance following Eq. (13) and Eq. (14), with $c \in \{HIGH, LOW\}$. The empirical results are reported in Table 8.

[Table 8 about here]

Table 8 shows that the superior performance of the swap rate model-free implied volatility is achieved in both the "HIGH" and "LOW" market states and from the perspective of both in-sample and out-of-sample. When the MPU is "HIGH", the performance improvement of Model 1 is more remarkable, especially for swap rates with longer tenors. For swap rates with the tenor of 10 years, estimates of $R^2_{IS,HIGH}$ ($R^2_{OOS,HIGH}$) for Models 1, 2 and 3 are 55.74%, 41.07% and 1.39% (57.62%, 44.16% and -0.11%), respectively, while 54.44%, 49.48% and 17.29% (63.38%, 53.37% and 18.73%) for $R^2_{IS,LOW}$ ($R^2_{OOS,LOW}$), respectively. Moreover, we take the average of $R^2_{IS,HIGH}$, $R^2_{IS,LOW}$, $R^2_{OOS,HIGH}$ and $R^2_{OOS,LOW}$ across all the tenors, respectively, and plot them in Figure 3. Panel D shows that the difference between $R^2_{OOS,HIGH}$ and $R^2_{OOS,LOW}$ is much smaller for Model 1 than those from Model 2 or Model 3, implying the superior performance of SRMFIV on predicting future realized volatility when monetary policy uncertainty is relatively high.

[Figure 3 about here]

All the above findings show the superiority of the predictive performance of the model-free implied volatility in recessions and when monetary policy uncertainty is relatively high.

This confirms the forward-looking nature of the volatility information implied from swaptions. Realized volatility and GARCH-type conditional volatility are more about historical information. Although, to some extent, historical information is useful for predicting the future, the forward-looking information is more timely when the market is more volatile and more likely to switch regimes.

6.3. Longer forecasting horizon

In the previous sections, we study the predictability of the swap rate model-free implied volatility for the time horizon of one month. In this section, we extend the examination to the following longer forecasting horizons: 3 months, 6 months and 1 year. We focus on swap rates with the tenors of 1 year, 5 years and 10 years. Similar to the previous sections, we investigate the in-sample predictability by estimating a regression of the following form:

$$RV_{t+1,t+h} = \beta_0 + \beta_1 SRMFIV_{t,t+h} + \beta_2 RV_{t-h+1,t} + \beta_3 VGARCH_{t,t+h} + \epsilon_{t+1,t+h}, \quad (15)$$

where $RV_{t+1,t+h}$ is the realized volatility from month $t+1$ to $t+h$, $SRMFIV_{t,t+h}$ is the model-free implied volatility, estimated at the end of month t, from swaptions with time-to- expiration of h months, $RV_{t-h+1,t}$ is the lagged realized volatility from month $t-h+1$ to month t and $VGARCH_{t,t+h}$ is the conditional volatility, estimated at the end of month t from a GARCH model, for months from $t+1$ to $t+h$. With the sampling frequency being monthly, the observations are overlapping. The standard errors are corrected with lags being the length of the overlapping window, h. The empirical results are reported in Table 9.

[Table 9 about here]

Panel A shows the estimates for the univariate regression when SRMFIV is used alone as the predictor. First, with longer forecasting horizons, SRMFIV is also an important predictor for future realized volatility. The estimates of the slope coefficients on SRMFIV range from 0.61 to 0.83 and the adjusted R-squared ranges from 0.38 to 0.68. The F statistics are large, ranging from 4.53 to 73.70, indicating that the model-free implied volatility is a biased forecast of future realized volatility of swap rates. Second, the predictive power decreases as the forecasting horizon increases. With longer horizons, both the slope coefficient and the adjusted R-squared decrease: for the swap rate with the tenor of 1 year, the slope estimates decrease from 0.83 to 0.61 and the adjusted R-squared decreases from 0.68 to 0.45; for the swap rate with the tenor of 5 years, the slope estimates decrease from 0.81 to 0.74 and the adjusted R-squared decreases from 0.61 to 0.46 and for the swap rate with the tenor of 10 years, the slope estimates decrease from 0.77 to 0.70 and the adjusted R-squared decreases from 0.50 to 0.38. Panel B reports the results when the lagged realized volatility and conditional volatility estimated from the GARCH model are included in the regression. The swap rate model-free implied volatility is still a strong predictor of future realized volatility. Except for the case for the swap rate with the tenor of 1 year and forecasting horizon of 3 months and cases with forecasting horizon of 6 months, where the lagged realized volatility shows some weak predictive power, neither lagged realized volatility nor conditional volatility estimated from a GARCH model has much additional predictive power.

Besides the in-sample investigation, we also examine the out-of-sample performance. The structure of the empirical setting is similar to those in the previous sections. The results are reported in Table 10.

[Table 10 about here]

Panel B shows that the RMSEs of Model 1, ranging from 19.50 to 28.14 basis points, are much smaller than those, ranging from 29.99 to 37.07 basis points, of the benchmark model shown in Panel A. The superior performance is further confirmed by the positive and large CW-statistics. Although both lagged realized volatility and conditional volatility estimated from a GARCH model help reduce the RMSEs compared to the benchmark model, the decrease in RMSEs is largest for Model 1 in which SRMFIV is used as the predictor. The DM-statistics from Panel C are positive and large in magnitude. Panel D examines whether there are additional contributions by the lagged realized volatility or conditional volatility estimated from a GARCH model. Similar to the previous sections, we find that adding either the lagged realized volatility or conditional volatility estimated from a GARCH model to the SRMFIV does not help improve the out-of-sample performance much. Finally, the superior out-of-sample performance of Model 1 decreases as the forecasting horizon becomes longer. Although the SRMFIV stands out by its forward-looking feature, the forward-looking information fades as forecasting horizon becomes longer.

Overall, we find that the superior performance of the swap rate model-free implied volatility on predicting future realized volatility persists with longer forecasting horizons. The superior performance is confirmed both in sample and out of sample.

7. Conclusion

We calculate the swap rate model-free implied volatility from over-the-counter swaption quotes for swap rates with a wide cross-section of tenors and with a variety of times to expiration. We examine the predictive power of the swap rate model-free implied volatility on future realized volatility of swap rates. We find that the swap rate model-free implied volatility shows strong and significant predictability for future realized volatility. The predictive power of the swap rate model-free implied volatility is stronger than either the lagged realized volatility or the conditional volatility estimated from a GARCH model. The information content of the swap rate model-free implied volatility is is superior to other commonly used predictors. Adding other predictors either does not weaken the effects of the swap rate model-free implied volatility or does not improve the forecasting performance further. The superior predictive performance is further confirmed by investigating the out-of-sample performance and also persists conditional on different market conditions and with longer forecasting horizons.

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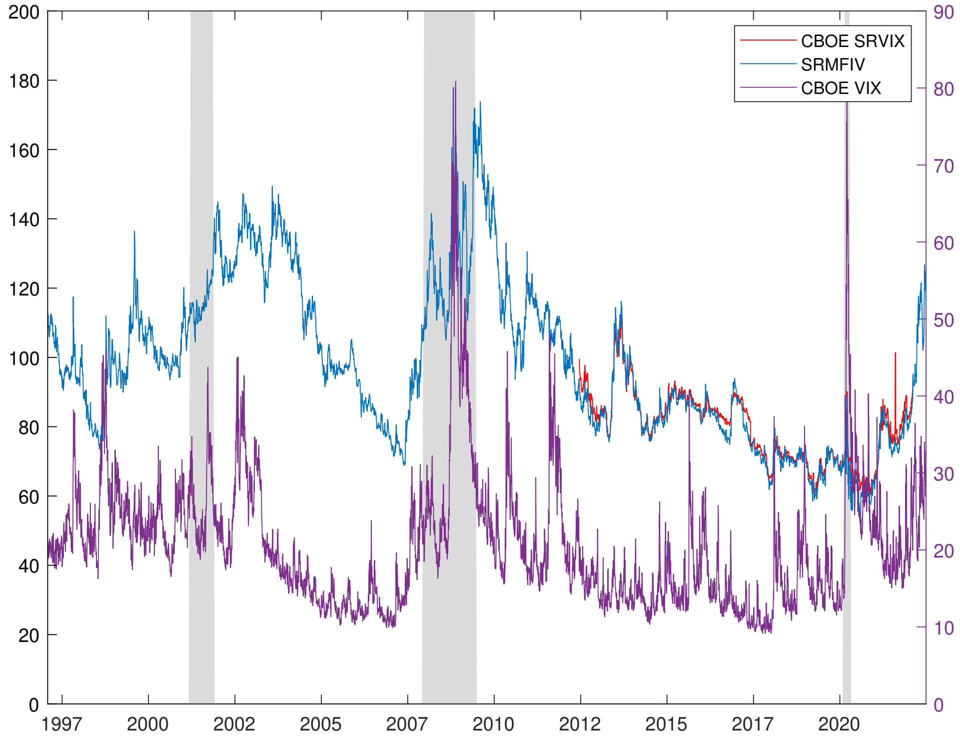

Figure 1
Time series of SRMFIV, SRVIX, and VIX

This figure plots the time series dynamics of the SRMFIV for the swap rate with times to expiration of 1 year and the tenor of 10 years. The sample period is from January 1997 to June 2022. The sample period for CBOE SRVIX is from June 18, 2012 to February 11, 2022. For the common sample period, the correlation between SRMFIV and SRVIX is about 0.9617. The correlation between SRMFIV and VIX is 0.4112. NBER defined recessions are indicated with gray areas.

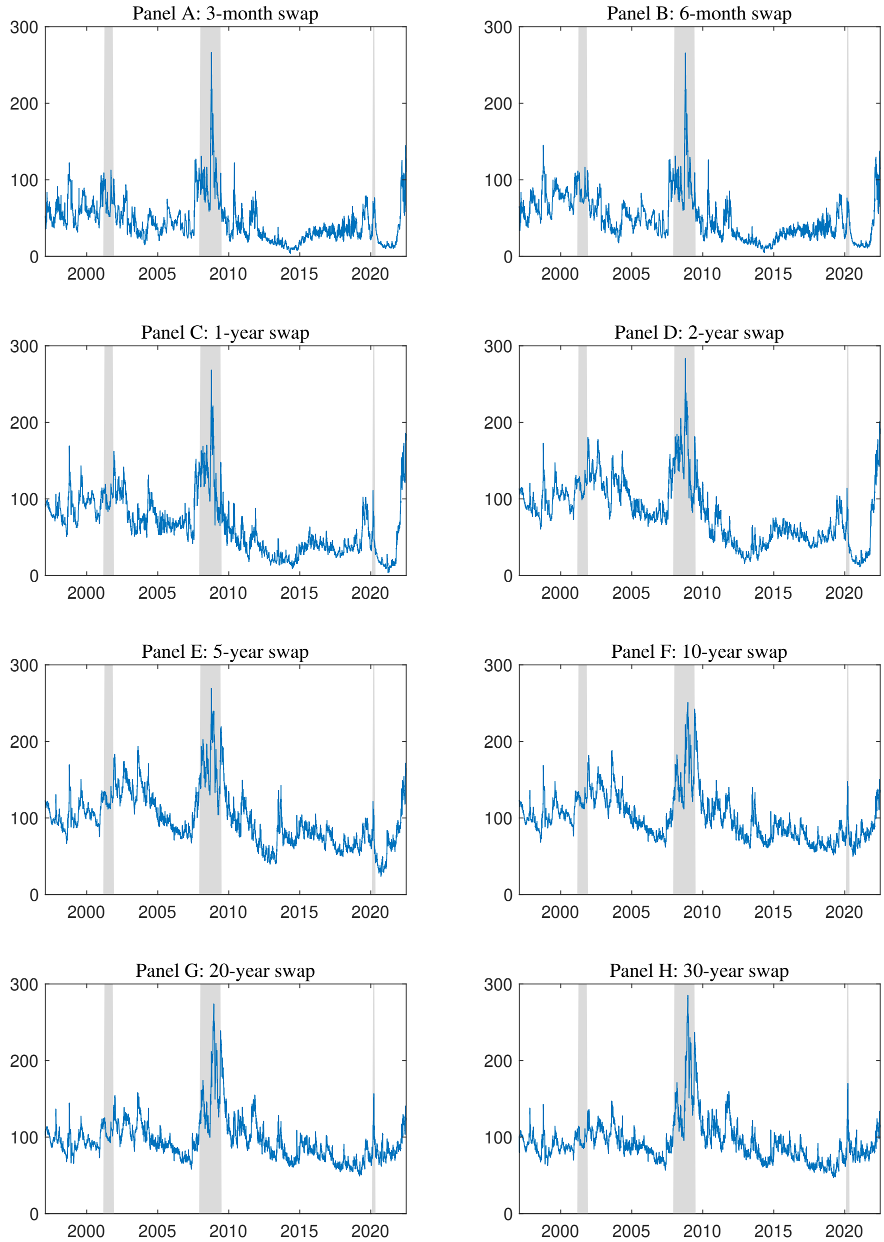

Figure 2
Time series of SRMFIV for the swap rate with various tenors

This figure plots the time series dynamics of the SRMFIV for the swap rate with the following eight tenors: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year. The sample period is from January 1997 to June 2022. NBER defined recessions are indicated with gray areas.

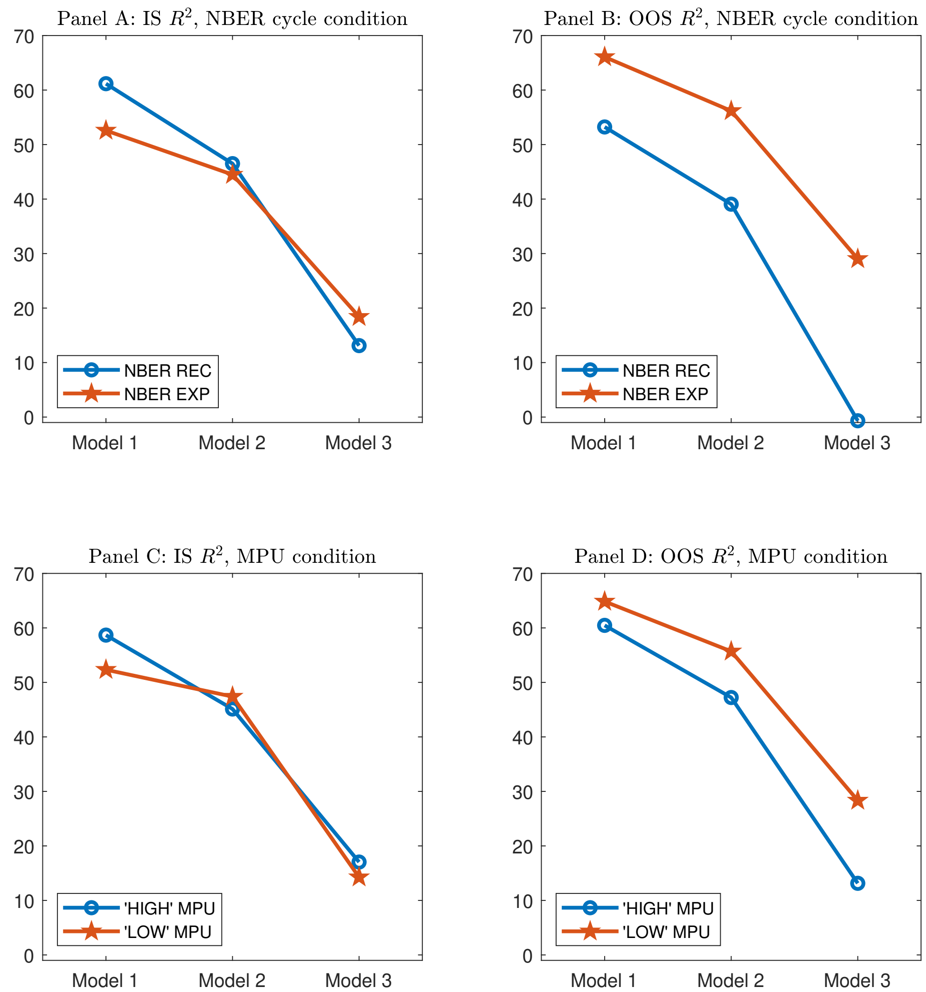

Figure 3
In-sample and out-of-sample performance with different market conditions

This figure plots the in-sample and out-of-sample forecasting performance with different market conditions of different models: "Model 1" uses model-free implied volatility as the predictor; "Model 2" uses lagged realized volatility as the predictor; and "Model 3" uses a GARCH model estimated conditional volatility as the predictor. Panel A and B shows the in-sample and out-of-sample forecasting performance with NBER defined business cycles (recession v.s. expansion). Panel C and D shows the in-sample and out-of-sample forecasting performance with monetary policy uncertainty states (high v.s. low).

Table 1
Summary statistics for realized volatility and swap rate model-free implied volatility (SRMFIV)

This table presents the summary statistics for the realized volatility (RV) and swap rate model-free implied volatility (SRMFIV) for the swap rates with the following eight tenors: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year. The summary statistics include the number of monthly observations (N), mean (Mean), median (Median), standard deviation (St.d.), skewness (Skew.), Kurtosis (Kurt.), autocorrelation with lags 1, 2, and 3 (AC(1), AC(2), and AC(3)), the p-value from Jarque-Bera test (Jarque and Bera (1987)) of normality (JB-test) and the p-value from KPSS test (Kwiatkowski, Phillips, Schmidt, and Shin (1992)) of stationarity (KPSS-test). Summary statistics for realized volatility are reported in Panel A and those for SRMFIV are reported in Panel B. The sample period is from January 1997 to June 2022.

Tenor	N	Mean	Median	St.d.	Skew.	Kurt.	AC(1)	AC(2)	AC(3)	JB-test	KPSS-test
Panel A: Realized volatility (RV)											
3-month	306	23.99	13.80	33.89	3.76	20.52	0.55	0.35	0.31	0.001	0.010
6-month	306	31.08	21.48	31.70	2.75	13.35	0.67	0.52	0.46	0.001	0.010
1-year	306	48.79	40.90	36.41	1.52	6.36	0.75	0.69	0.64	0.001	0.010
2-year	306	66.95	59.47	40.62	1.17	5.09	0.78	0.73	0.69	0.001	0.010
5-year	306	84.26	77.82	37.90	1.19	4.82	0.71	0.62	0.57	0.001	0.010
10-year	306	86.16	78.02	34.52	1.41	5.64	0.67	0.53	0.47	0.001	0.010
20-year	306	82.62	75.92	31.99	1.70	7.29	0.63	0.49	0.43	0.001	0.010
30-year	306	81.24	74.51	32.15	1.96	8.69	0.64	0.49	0.44	0.001	0.010
Panel B: Swap rate model-free implied volatility (SRMFIV)											
3-month	306	46.15	41.75	26.76	1.58	7.58	0.82	0.72	0.61	0.001	0.010
6-month	306	50.01	45.18	28.26	1.20	5.62	0.84	0.76	0.66	0.001	0.010
1-year	306	66.01	63.26	37.15	0.81	3.71	0.88	0.83	0.78	0.001	0.010
2-year	306	82.45	80.17	40.52	0.58	3.07	0.91	0.86	0.82	0.003	0.010
5-year	306	98.84	92.83	35.97	0.82	3.84	0.89	0.82	0.76	0.001	0.010
10-year	306	100.97	94.07	31.40	1.31	5.49	0.88	0.79	0.71	0.001	0.010
20-year	306	96.46	91.99	29.02	2.01	9.47	0.86	0.76	0.67	0.001	0.010
30-year	306	93.97	89.78	28.81	2.39	11.77	0.85	0.76	0.66	0.001	0.010

Table 2
Forecasting future swap rate realized volatility with swap rate model-free implied volatility

This table reports the estimated results from the univariate predictive regression of the following form:

$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \varepsilon_{t+1},$$

where RV_{t+1} is the realized volatility in month $t+1$ and $SRMFIV_t$ is the swap rate model-free implied volatility estimated at the end of month t. Heteroscedasticity and autocorrelation consistent t-statistics with 4 lags (as in Newey and West (1987)) are reported in parentheses. DW is the Durbin-Watson statistic of the autocorrelation test for the residuals. F is the statistic for the joint hypothesis: $\beta_0 = 0$ and $\beta_1 = 1$. The sample period is from January 1997 to Jun 2022. Panel A reports the estimates for the full sample period, Panel B for the subsample period from January 1997 to September 2009 and Panel C for the subsample period from October 2009 to June 2022.

Dependent	RV_{t+1}							
Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Panel A: Full sample period								
Intercept	-14.68	-10.46	-5.08	-3.22	1.51	3.79	7.68	7.81
	(-3.75)	(-4.67)	(-3.25)	(-1.60)	(0.43)	(0.87)	(1.42)	(1.36)
$SRMFIV_t$	0.84	0.83	0.82	0.85	0.84	0.82	0.78	0.78
	(7.99)	(12.94)	(24.45)	(26.58)	(20.54)	(16.80)	(12.81)	(11.89)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.43	0.54	0.68	0.71	0.63	0.55	0.49	0.49
DW	1.67	1.69	2.11	2.10	1.90	1.78	1.82	1.80
F	135.99	169.92	120.84	84.85	55.86	54.32	49.82	41.16
Panel B: Sub-sample: January 1997 to September 2009								
Intercept	-27.41	-15.76	-4.53	-3.38	-5.18	-0.52	8.52	8.41
	(-4.38)	(-3.70)	(-0.90)	(-0.47)	(-0.67)	(-0.07)	(1.07)	(1.07)
$SRMFIV_t$	1.03	0.90	0.82	0.85	0.88	0.84	0.76	0.77
	(7.88)	(10.75)	(13.80)	(12.84)	(13.22)	(13.13)	(9.75)	(9.31)
N	153	153	153	153	153	153	153	153
$\bar{R}^2$	0.41	0.44	0.43	0.46	0.53	0.51	0.49	0.51
DW	1.94	1.80	2.13	2.11	1.93	1.78	1.79	1.74
F	111.11	67.82	47.04	38.20	39.45	36.93	31.40	25.78
Panel C: Sub-sample: October 2009 to June 2022								
Intercept	-5.35	-7.80	-4.45	-3.75	1.89	0.54	2.42	3.15
	(-1.56)	(-2.80)	(-2.07)	(-1.24)	(0.30)	(0.06)	(0.29)	(0.39)
$SRMFIV_t$	0.61	0.78	0.80	0.87	0.85	0.87	0.85	0.84
	(3.98)	(6.58)	(11.71)	(12.57)	(9.11)	(7.74)	(7.81)	(8.02)
N	152	152	152	152	152	152	152	152
$\bar{R}^2$	0.32	0.48	0.67	0.70	0.52	0.43	0.40	0.39
DW	0.98	1.43	1.97	2.06	1.88	1.85	1.85	1.86
F	101.81	164.28	92.15	64.21	22.04	23.00	21.72	17.23

Table 3

Comparison with lagged realized volatility

This table reports the estimated results for the predictive regressions of the following form:

$$\begin{aligned} RV_{t+1} &= \beta_0 + \beta_2 RV_t + \varepsilon_{t+1}, \\ RV_{t+1} &= \beta_0 + \beta_1 SRMFIV_t + \beta_2 RV_t + \varepsilon_{t+1}, \end{aligned}$$

where RV_{t+1} is the realized volatility in month $t+1$, RV_t is the realized volatility in month t and $SRMFIV_t$ is the swap rate model-free implied volatility at the end of month t. Panel A and B report estimates for the univariate regression and multivariate regression, respectively. Heteroscedasticity and autocorrelation consistent t statistics with 4 lags (Newey and West (1987)) are reported in parentheses. The sample period is from January 1997 to June 2022.

Panel A: Univariate regression

Dependent	RV_{t+1}							
Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	10.81	10.30	12.00	14.64	24.22	28.51	30.43	29.57
	(5.17)	(6.11)	(6.49)	(5.59)	(7.28)	(6.81)	(6.55)	(6.16)
RV_t	0.56	0.68	0.76	0.78	0.71	0.67	0.63	0.64
	(7.15)	(17.36)	(26.00)	(22.73)	(18.34)	(12.98)	(10.55)	(10.01)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.30	0.44	0.56	0.60	0.50	0.44	0.40	0.40

Panel B: Multivariate regression

Dependent	RV_{t+1}							
Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	-11.67	-7.75	-4.50	-3.34	1.42	4.61	8.54	8.96
	(-2.91)	(-2.98)	(-2.35)	(-1.41)	(0.38)	(1.05)	(1.63)	(1.69)
$SRMFIV_t$	0.70	0.67	0.77	0.87	0.85	0.74	0.68	0.65
	(4.99)	(5.85)	(8.21)	(8.27)	(8.55)	(8.23)	(6.94)	(6.54)
RV_t	0.16	0.18	0.06	-0.01	-0.01	0.07	0.10	0.13
	(1.63)	(2.16)	(0.72)	(-0.12)	(-0.11)	(0.99)	(1.33)	(1.68)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.44	0.55	0.68	0.71	0.62	0.55	0.50	0.49

Table 4
Comparison with conditional volatility estimated from GARCH model

This table reports the estimated result for the predictive regressions of the following form:

$$\begin{aligned} RV_{t+1} &= \beta_0 + \beta_2 VGARCH_t + \varepsilon_{t+1}, \\ RV_{t+1} &= \beta_0 + \beta_1 SRMFIV_t + \beta_2 VGARCH_t + \varepsilon_{t+1}, \end{aligned}$$

where RV_{t+1} is the realized volatility in month $t+1$, $VGARCH_t$ is the conditional volatility for month $t+1$, estimated with a $GARCH(1,1)$ model at the end of month t and $SRMFIV_t$ is the swap rate model-free implied volatility at the end of month t. Panel A and B report estimates for the univariate regression and multivariate regression, respectively. Heteroscedasticity and autocorrelation consistent t statistics with 4 lags are reported in parentheses. The sample period is from January 1997 to June 2022.

Panel A: Univariate regression

Dependent	RV_{t+1}							
	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	10.15	12.56	15.54	30.78	51.68	56.35	50.50	45.10
	(3.43)	(4.86)	(4.59)	(5.88)	(6.78)	(5.82)	(4.67)	(4.22)
$VGARCH_t$	0.50	0.62	0.81	0.71	0.52	0.45	0.48	0.55
	(3.96)	(5.54)	(8.60)	(7.98)	(4.84)	(3.14)	(2.96)	(3.32)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.17	0.25	0.33	0.27	0.13	0.07	0.07	0.08

Panel B: Multivariate regression

Dependent	RV_{t+1}							
	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	-14.63	-10.64	-4.90	-2.68	3.63	9.45	10.50	7.53
	(-3.96)	(-5.04)	(-3.36)	(-1.46)	(1.02)	(1.83)	(1.69)	(1.20)
$SRMFIV_t$	0.79	0.78	0.83	0.88	0.87	0.87	0.80	0.78
	(7.17)	(9.83)	(15.63)	(17.48)	(17.13)	(15.94)	(11.02)	(10.41)
$VGARCH_t$	0.08	0.09	-0.02	-0.05	-0.09	-0.16	-0.07	0.01
	(0.89)	(1.07)	(-0.34)	(-0.72)	(-1.43)	(-2.15)	(-0.75)	(0.07)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.43	0.54	0.68	0.71	0.63	0.55	0.49	0.49

Table 5
Comparison with all the alternative predictors

This table reports the estimated result for the predictive regressions of the following form:

$$RV_{t+1} \quad = \quad \beta_0 + \beta_1 SRMFIV_t + \beta_2 RV_t + \beta_3 VGARCH_t + \epsilon_{t+1},$$

where RV_{t+1} is the realized volatility in month $t+1$, $SRMFIV_t$ is the swap rate model-free implied volatility at the end of month t, $VGARCH_t$ is the conditional volatility for month $t+1$, estimated with a $GARCH(1,1)$ model at the end of month t and RV_t is the realized volatility in month t. Heteroscedasticity and autocorrelation consistent t statistics with 4 lags (Newey and West (1987)) are reported in parentheses. The sample period is from January 1997 to June 2022.

Dependent	RV_{t+1}							
	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	-11.85	-8.06	-4.33	-2.79	3.54	9.96	11.36	8.83
	(-3.03)	(-3.24)	(-2.43)	(-1.29)	(0.94)	(1.95)	(1.90)	(1.51)
$SRMFIV_t$	0.68	0.64	0.78	0.89	0.88	0.81	0.70	0.65
	(4.59)	(5.09)	(7.12)	(7.50)	(8.03)	(8.35)	(6.40)	(6.00)
RV_t	0.15	0.17	0.06	-0.01	-0.01	0.06	0.10	0.13
	(1.63)	(2.11)	(0.72)	(-0.12)	(-0.11)	(0.83)	(1.33)	(1.69)
$VGARCH_t$	0.04	0.07	-0.02	-0.05	-0.09	-0.16	-0.07	0.00
	(0.52)	(0.94)	(-0.34)	(-0.72)	(-1.42)	(-2.10)	(-0.76)	(0.03)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.44	0.55	0.68	0.71	0.63	0.55	0.50	0.49

Table 6

Out-of-sample predictive performance

This table reports the out-of-sample predictive performance of various forecasting models. Root mean squared errors (RMSE) are used to evaluate the out-of-sample predictive performance. Panel A reports the RMSEs for the benchmark model where the forecast is simply the historical average. Panel B reports the RMSEs for three models: "Model 1" uses swap rate model-free implied volatility as the predictor; "Model 2" uses lagged realized volatility as the predictor; and "Model 3" uses GARCH model estimated conditional volatility as the predictor. Each of the three models is compared to the benchmark model and the CW-statistic (Clark and West (2007)) is reported. Panel C compares different univariate predictive regression models and reports the DM-statistic (Diebold and Mariano (2002)). Panel D presents the RMSEs of various multivariate regression models: "Model 4" is the model in which both SRMFIV and lagged realized volatility is included in the predictive regression as in Eq.(8). "Model 5" is the model in which both SRMFIV and conditional volatility estimated from the GARCH model are included in the predictive regression as in Eq.(10). "Model 6" is the model in which SRMFIV, lagged realized volatility and conditional volatility estimated from the GARCH model all are included in the predictive regression as in Eq. (11). We further compare each of them to "Model 1" and the CW-statistics (Clark and West (2007)) are reported in parentheses. The first 60 months are used for the initial estimation and the subsequent estimation is done with expanding sample window. The out-of-sample evaluation period is from January 2002 to June 2022.

Variable	*RMSE*							
Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Panel A: Benchmark model								
Benchmark model	33.15	32.41	38.14	43.05	39.52	35.74	33.59	34.13
Panel B: Models with a single predictor								
Model 1: $SRMFIV$	23.95	20.63	19.41	21.15	22.17	22.58	23.07	23.93
Model 2: Lagged RV	26.52	23.24	23.80	25.46	26.27	25.61	25.60	26.13
Model 3: $VGARCH$	29.90	27.69	30.60	36.28	36.61	34.41	32.41	32.87
CW: Model 1 v.s. Benchmark	(4.47)	(7.29)	(13.38)	(14.53)	(11.86)	(8.93)	(6.44)	(5.51)
CW: Model 2 v.s. Benchmark	(3.89)	(5.94)	(11.30)	(12.03)	(9.62)	(7.65)	(6.60)	(5.87)
CW: Model 3 v.s. Benchmark	(4.58)	(6.08)	(9.74)	(9.68)	(6.83)	(4.60)	(4.66)	(4.61)
Panel C: Model comparison								
DM: Model 1 v.s.Model 2	(1.29)	(3.38)	(4.81)	(4.53)	(3.84)	(3.34)	(2.47)	(1.95)
DM: Model 1 v.s.Model 3	(1.91)	(3.63)	(5.51)	(8.20)	(6.98)	(6.08)	(4.22)	(3.64)
Panel D: Models with multiple predictors								
Model 4: $SRMFIV$ and Lagged RV	23.94	20.45	19.60	21.28	22.30	22.63	23.19	24.04
CW: Model 4 v.s. Model 1	(0.52)	(2.05)	(-1.56)	(-1.83)	(-1.06)	(-0.05)	(0.17)	(0.66)
Model 5: $SRMFIV$ and $VGARCH$	24.31	20.84	19.99	21.48	22.46	22.63	23.19	24.16
CW: Model 5 v.s. Model 1	(-0.35)	(-2.00)	(0.09)	(-1.11)	(-0.21)	(0.64)	(-1.06)	(-0.13)
Model 6: $SRMFIV$, Lagged RV and $VGARCH$	24.30	20.64	20.13	21.61	22.58	22.72	23.33	24.26
CW: Model 6 v.s. Model 1	(-0.01)	(0.56)	(-0.19)	(-1.71)	(-0.47)	(0.56)	(-0.36)	(0.39)

Table 7

Forecasting performance conditional on NBER-dated business cycle

This table reports the in-sample and out-of-sample R^2s conditional on market states: NBER-dated business cycle. The sample period is classified as recessions and expansions according to the NBER recession indicator. We calculate the in-sample $R^2_{IS,REC}$ and $R^2_{IS,EXP}$ according to Eq. (13) and we calculate the out-of–sample $R^2_{OOS,REC}$ and $R^2_{OOS,EXP}$ according to Eq. (14). Panel A reports the in-sample estimates and Panel B for the out-of-sample estimates.

	Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Panel A: In-sample performance									
Model 1: $SRMFIV$	$R^2_{IS,REC}(\%)$	48.59	57.19	66.58	66.76	68.25	64.53	59.16	58.29
	$R^2_{IS,EXP}(\%)$	34.46	52.28	68.49	73.13	60.01	48.88	42.41	41.01
Model 2: Lagged RV	$R^2_{IS,REC}(\%)$	37.67	43.68	51.15	52.96	51.80	48.37	42.89	43.53
	$R^2_{IS,EXP}(\%)$	19.32	45.86	60.45	64.89	49.36	41.61	37.06	37.36
Model 3: $VGARCH$	$R^2_{IS,REC}(\%)$	27.48	31.97	26.13	7.87	0.58	1.32	3.93	5.57
	$R^2_{IS,EXP}(\%)$	1.15	12.72	38.23	39.17	22.05	11.88	10.60	11.40
Panel B: Out-of-sample performance									
Model 1: $SRMFIV$	$R^2_{OOS,REC}(\%)$	43.52	46.11	52.74	52.25	62.52	60.05	54.74	53.90
	$R^2_{OOS,EXP}(\%)$	55.58	74.38	82.91	83.34	71.83	60.72	51.60	48.21
Model 2: Lagged RV	$R^2_{OOS,REC}(\%)$	30.50	36.21	36.42	39.24	47.75	44.21	38.44	39.71
	$R^2_{OOS,EXP}(\%)$	45.68	61.34	71.71	73.48	59.37	51.12	44.14	42.63
Model 3: $VGARCH$	$R^2_{OOS,REC}(\%)$	9.64	10.96	-2.33	-6.34	-7.96	-3.86	-3.84	-1.74
	$R^2_{OOS,EXP}(\%)$	32.52	40.13	50.04	40.55	24.05	14.14	14.99	15.69

Table 8

Forecasting performance conditional on monetary policy uncertainty

This table reports the in-sample and out-of-sample R^2s conditional on market states: monetary policy uncertainty. The sample period is classified as "HIGH" when monetary policy uncertainty is above the sample median and "LOW" otherwise. We calculate the in-sample $R^2_{IS,HIGH}$ and $R^2_{IS,LOW}$ according to Eq. (13) and we calculate the out-of–sample $R^2_{OOS,HIGH}$ and $R^2_{OOS,LOW}$ according to Eq. (14), with $c \in \{HIGH, LOW\}$. Panel A reports the in-sample estimates and Panel B for the out-of-sample estimates.

	Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Panel A: In-sample performance									
Model 1: $SRMFIV$	$R^2_{IS,HIGH}(\%)$	45.46	55.57	69.09	75.45	66.99	55.74	51.04	50.10
	$R^2_{IS,LOW}(\%)$	34.73	53.29	64.01	61.12	56.13	54.44	47.48	47.46
Model 2: Lagged RV	$R^2_{IS,HIGH}(\%)$	32.45	40.77	55.85	63.17	51.06	41.07	37.53	39.06
	$R^2_{IS,LOW}(\%)$	23.60	59.56	57.69	54.81	48.88	49.48	42.99	42.23
Model 3: $VGARCH$	$R^2_{IS,HIGH}(\%)$	24.36	30.26	35.68	27.90	10.82	1.39	1.98	3.93
	$R^2_{IS,LOW}(\%)$	-7.56	-1.39	25.94	26.57	19.61	17.29	17.10	16.55
Panel B: Out-of-sample performance									
Model 1: $SRMFIV$	$R^2_{OOS,HIGH}(\%)$	45.28	53.13	71.19	78.09	71.63	57.62	53.90	52.89
	$R^2_{OOS,LOW}(\%)$	60.36	80.24	79.55	71.33	64.07	63.38	51.57	48.30
Model 2: Lagged RV	$R^2_{OOS,HIGH}(\%)$	31.65	42.16	58.10	66.98	57.46	44.16	38.24	39.12
	$R^2_{OOS,LOW}(\%)$	53.63	67.35	67.47	61.38	52.90	53.37	45.89	43.66
Model 3: $VGARCH$	$R^2_{OOS,HIGH}(\%)$	13.65	21.71	30.69	26.70	9.78	-0.11	1.34	1.35
	$R^2_{OOS,LOW}(\%)$	35.24	36.95	43.25	34.58	23.95	18.73	16.08	17.82

Table 9
Forecasting swap rate volatility with the swap rate model-free implied volatility: longer forecasting horizons

This table reports the estimated results for the predictive regressions of the following form:

$$RV_{t+1,t+h} = \beta_0 + \beta_1 SRMFIV_{t,t+h} + \beta_2 RV_{t-h+1,t} + \beta_3 VGARCH_{t,t+h} + \epsilon_{t+1,t+h},$$

where $RV_{t+1,t+h}$ is the realized volatility from month $t+1$ to $t+h$, $SRMFIV_{t,t+h}$ is the model-free implied volatility estimated at the end of month t with time to maturity of h months, $RV_{t-h+1,t}$ is the lagged realized volatility from month $t-h+1$ to month t and $VGARCH_{t,t+h}$ is the conditional volatility for months from $t+1$ to $t+h$ and estimated at the end of month t from a GARCH model. We choose h being 3 months, 6 months and 1 year, respectively and we focus on swap rate with tenors of 1-year, 5-year and 10-year. The sample period is from January 1997 to June 2022. Panel A and B report estimates for the univariate regression and multivariate regression, respectively. Heteroscedasticity and autocorrelation consistent t statistics with $h+1$ lags are reported in parentheses.

Panel A: Univariate regression

Dependent variable	$RV_{t+1,t+h}$								
Tenor	1-year			5-year			10-year		
Time-to-expiration	3-month	6-month	1-year	3-month	6-month	1-year	3-month	6-month	1-year
Intercept	-6.46	-5.00	-0.30	6.53	9.89	14.11	11.31	15.39	20.33
	(-2.28)	(-0.96)	(-0.04)	(1.31)	(1.29)	(1.41)	(1.83)	(1.59)	(1.72)
$SRMFIV_t$	0.83	0.75	0.61	0.81	0.78	0.74	0.77	0.74	0.70
	(15.15)	(8.56)	(5.28)	(14.09)	(8.86)	(6.40)	(11.34)	(7.06)	(5.45)
N	303	300	294	303	300	294	303	300	294
$\bar{R}^2$	0.68	0.61	0.45	0.61	0.56	0.46	0.50	0.45	0.38
DW	0.70	0.29	0.08	0.67	0.29	0.11	0.61	0.28	0.10
F	73.70	48.92	32.21	19.80	9.61	4.81	18.58	8.87	4.53

Panel B: Multivariate regression

Dependent variable	$RV_{t+1,t+h}$								
Tenor	1-year			5-year			10-year		
Time-to-expiration	3-month	6-month	1-year	3-month	6-month	1-year	3-month	6-month	1-year
Intercept	-3.46	-0.73	1.29	10.17	14.48	15.46	15.38	19.29	20.76
	(-1.49)	(-0.17)	(0.18)	(2.19)	(2.17)	(1.75)	(2.47)	(2.18)	(1.91)
$SRMFIV_t$	0.69	0.57	0.45	0.80	0.58	0.78	0.85	0.51	0.67
	(6.95)	(4.72)	(3.79)	(6.76)	(3.81)	(4.27)	(6.33)	(2.94)	(3.46)
RV_t	0.25	0.52	0.31	0.10	0.40	0.17	0.01	0.38	0.22
	(2.17)	(2.89)	(1.57)	(0.77)	(2.63)	(0.74)	(0.08)	(1.88)	(0.71)
$VGARCH_t$	-0.14	-0.34	-0.08	-0.16	-0.25	-0.24	-0.17	-0.19	-0.21
	(-1.44)	(-2.48)	(-0.69)	(-1.76)	(-1.85)	(-0.96)	(-1.81)	(-1.29)	(-0.66)
N	303	300	294	303	300	294	303	300	294
$\bar{R}^2$	0.69	0.67	0.47	0.62	0.60	0.47	0.50	0.49	0.39

Table 10

Out-of-sample performance: longer forecasting horizons

This table reports the out-of-sample performance of various forecasting models with longer forecasting time horizons: 3 months, 6 months and 1 year. We focus on the swap rate with the tenors of 1 year, 5 years and 10 years. Root mean squared error (RMSE) is used to evaluate the performance. Descriptions of different models are the same as those from Table 6.

Variable	*RMSE*								
Tenor	1-year			5-year			10-year		
Time-to-expiration	3-month	6-month	1-year	3-month	6-month	1-year	3-month	6-month	1-year
Panel A: Benchmark model									
Benchmark model	36.71	35.99	35.48	37.07	35.76	34.71	32.87	31.24	29.99
Panel B: Models with a single predictor									
Model 1: $SRMFIV$	19.50	22.70	28.14	22.26	23.91	26.83	22.54	23.20	24.64
Model 2: Lagged RV	22.46	25.07	30.99	24.96	24.74	30.09	25.15	23.71	26.59
Model 3: $VGARCH$	29.43	29.84	31.98	34.32	33.17	32.74	31.07	28.81	28.03
CW: Model 1 v.s. Benchmark	(14.65)	(13.27)	(10.50)	(14.00)	(14.18)	(13.19)	(11.12)	(12.07)	(12.95)
CW: Model 2 v.s. Benchmark	(13.16)	(12.42)	(8.50)	(12.39)	(12.21)	(10.97)	(9.31)	(8.83)	(9.78)
CW: Model 3 v.s. Benchmark	(10.84)	(9.88)	(8.14)	(7.19)	(7.39)	(6.72)	(5.98)	(7.36)	(6.48)
Panel C: Model comparison									
DM: Model 1 v.s.Model 2	(3.23)	(1.83)	(3.95)	(3.25)	(1.01)	(5.57)	(3.53)	(0.58)	(3.72)
DM: Model 1 v.s.Model 3	(7.64)	(8.59)	(4.11)	(7.81)	(7.17)	(7.23)	(7.49)	(6.26)	(5.28)
Panel D: Models with multiple predictors									
Model 4: $SRMFIV$ and Lagged RV	19.36	23.33	29.33	22.51	23.98	29.56	22.83	23.39	26.54
CW: Model 4 v.s. Model 1	(2.11)	(3.42)	(0.64)	(-0.71)	(2.15)	(-3.23)	(-0.62)	(1.44)	(-3.79)
Model 5: $SRMFIV$ and $VGARCH$	20.08	23.55	29.08	22.48	24.69	27.90	22.63	23.78	25.64
CW: Model 5 v.s. Model 1	(0.53)	(-1.43)	(0.25)	(0.06)	(-0.77)	(-2.01)	(0.40)	(-0.75)	(-2.92)
Model 6: ALL	19.84	22.94	29.87	22.74	24.41	31.17	22.92	23.84	28.13
CW: Model 6 v.s. Model 1	(1.89)	(3.03)	(0.93)	(-0.58)	(1.62)	(-3.34)	(-0.32)	(1.17)	(-3.65)

Appendix

Table 11

Correlation among realized volatility and SRMFIV

This table presents the correlations for the realized volatility and the swap rate model-free implied volatility for swap rates with the following eight tenors: 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year.The results for realized volatility are reported in Panel A and those for the swap rate model-free implied volatility in Panel B. The sample period is from January 1997 to June 2022.

Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
				Panel A: Realized volatility (RV)				
3-month	1.00							
6-month	0.92	1.00						
1-year	0.77	0.93	1.00					
2-year	0.63	0.81	0.95	1.00				
5-year	0.56	0.71	0.81	0.91	1.00			
10-year	0.53	0.64	0.70	0.77	0.93	1.00		
20-year	0.50	0.59	0.61	0.65	0.84	0.97	1.00	
30-year	0.48	0.55	0.55	0.58	0.78	0.93	0.99	1.00
				Panel B: Swap rate model-free implied volatility (SRMFIV)				
3-month	1.00							
6-month	0.97	1.00						
1-year	0.89	0.91	1.00					
2-year	0.80	0.82	0.96	1.00				
5-year	0.69	0.70	0.82	0.92	1.00			
10-year	0.63	0.63	0.71	0.79	0.94	1.00		
20-year	0.59	0.56	0.60	0.65	0.82	0.95	1.00	
30-year	0.55	0.52	0.53	0.57	0.75	0.90	0.99	1.00

Table 12
Comparison with conditional volatility estimated from the GJR-GARCH model

This table reports the estimated result for the predictive regressions of the following form:

$$RV_{t+1} = \beta_0 + \beta_2 VGJR_t + \varepsilon_{t+1},$$
$$RV_{t+1} = \beta_0 + \beta_1 SRMFIV_t + \beta_2 VGJR_t + \varepsilon_{t+1},$$

where RV_{t+1} is the realized volatility in month $t+1$, $VGJR_t$ is the conditional volatility for month $t+1$, estimated with a $GJR-GARCH(1,1)$ model at the end of month t, and $SRMFIV_t$ is the swap rate model-free implied volatility at the end of month t. Panels A and B report estimates from the univariate regression and multivariate regression, respectively. Heteroscedasticity and autocorrelation consistent t-statistics with 4 lags are reported in parentheses. The sample period is from January 1997 to June 2022.

Panel A: Univariate regression

Dependent	RV_{t+1}							
	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	9.60	12.88	17.53	32.24	51.66	54.95	51.91	49.59
	(3.37)	(5.02)	(4.62)	(5.82)	(6.54)	(5.67)	(4.44)	(4.21)
$VGJR_t$	0.54	0.61	0.78	0.68	0.51	0.46	0.46	0.48
	(4.32)	(5.49)	(8.22)	(7.57)	(4.80)	(3.27)	(2.64)	(2.68)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.19	0.24	0.29	0.25	0.13	0.07	0.06	0.07

Panel B: Multivariate regression

Dependent	RV_{t+1}							
	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Intercept	-14.57	-10.67	-4.61	-2.46	3.48	9.69	10.93	8.30
	(-4.03)	(-5.03)	(-3.14)	(-1.30)	(0.95)	(1.85)	(1.67)	(1.24)
$SRMFIV_t$	0.78	0.79	0.84	0.88	0.87	0.87	0.80	0.78
	(6.97)	(9.86)	(16.81)	(18.11)	(17.42)	(15.83)	(11.18)	(10.56)
$VGJR_t$	0.11	0.09	-0.05	-0.06	-0.08	-0.17	-0.08	-0.01
	(1.13)	(1.07)	(-0.71)	(-0.93)	(-1.23)	(-2.09)	(-0.76)	(-0.11)
N	305	305	305	305	305	305	305	305
$\bar{R}^2$	0.43	0.54	0.68	0.71	0.63	0.55	0.49	0.49

Table 13

Out-of-sample performance: with the rolling window of 60 months

This table reports the out-of-sample performance of various forecasting models. Root mean squared error (RMSE) is used to evaluate the out-of-sample performance. Panel A reports the RMSE for the benchmark model in which the forecast is simply the historical average. Panel B reports the RMSEs for three models: "Model 1" adopts the swap rate model-free implied volatility as the predictor; "Model 2" adopts the lagged realized volatility as the predictor and "Model 3" adopts the GARCH model-estimated conditional volatility as the predictor. Each of the three models is compared to the benchmark model and the CW-statistic (Clark and West (2007)) is reported. Panel C compares different univariate predictive regression models and reports the DM-statistic (Diebold and Mariano (2002)). Panel D presents the RMSEs of various multivariate regression models: "Model 4" is the model in which both SRMFIV and lagged realized volatility are included in the predictive regression as in Eq.(8). "Model 5" is the model in which both SRMFIV and conditional volatility estimated from the GARCH model are included in the predictive regression as in Eq.(10). "Model 6" is the model in which SRMFIV, the lagged realized volatility and the conditional volatility estimated from the GARCH model all are included in the predictive regression as in Eq.(11). We further compare each of them to Model 1 and the CW-statistics (Clark and West (2007)) are reported in parentheses. The first 60 months are used for the initial estimation and the remaining estimation is done with a rolling window of 60 months. The out-of-sample evaluation period is from January 2002 to June 2022.

Variable	*RMSE*							
Tenor	3-month	6-month	1-year	2-year	5-year	10-year	20-year	30-year
Panel A: Benchmark model								
Benchmark model	33.78	32.36	35.74	40.27	39.26	36.89	35.01	35.45
Panel B: Models with a single predictor								
Model 1: $SRMFIV$	24.18	21.11	21.07	23.28	24.70	25.30	25.55	26.22
Model 2: Lagged RV	28.37	24.49	24.70	27.07	28.51	28.09	27.98	28.39
Model 3: $VGARCH$	28.58	27.57	31.96	37.85	38.56	35.73	34.16	33.94
CW: Model 1 v.s. Benchmark	(4.03)	(5.25)	(8.50)	(9.68)	(9.06)	(7.13)	(5.91)	(5.32)
CW: Model 2 v.s. Benchmark	(3.49)	(5.12)	(7.44)	(7.82)	(6.94)	(6.00)	(5.64)	(5.25)
CW: Model 3 v.s. Benchmark	(2.05)	(2.55)	(4.28)	(4.05)	(3.99)	(4.39)	(4.61)	(4.12)
Panel C: Model comparison								
DM: Model 1 v.s.Model 2	(1.79)	(2.39)	(3.41)	(3.70)	(3.99)	(3.65)	(2.98)	(2.53)
DM: Model 1 v.s.Model 3	(1.96)	(4.28)	(5.74)	(7.44)	(6.54)	(4.94)	(3.49)	(2.86)
Panel D: Models with multiple predictors								
Model 4: $SRMFIV$ and Lagged RV	24.89	21.61	21.46	23.74	25.16	25.75	26.27	26.75
CW: Model 4 v.s. Model 1	(1.32)	(1.45)	(-0.63)	(-1.10)	(-1.62)	(-0.85)	(-0.65)	(-0.61)
Model 5: $SRMFIV$ and $VGARCH$	25.17	20.82	21.49	23.79	25.11	25.80	25.85	26.42
CW: Model 5 v.s. Model 1	(0.85)	(1.09)	(0.86)	(-1.44)	(-1.33)	(-0.80)	(-0.39)	(0.81)
Model 6: $SRMFIV$, Lagged RV and $VGARCH$	25.11	20.99	21.98	24.23	25.60	26.26	26.57	26.95
CW: Model 6 v.s. Model 1	(1.27)	(1.13)	(0.37)	(-1.63)	(-1.87)	(-1.13)	(-0.70)	(0.03)

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