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Bank runs without self-fulfilling prophecies

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Abstract

This paper proposes that bank runs are unique equilibrium outcomes instead of self-fulfilling prophecies. By assuming that depositors make their withdrawal decisions sequentially, the model provides an equilibrium-selection mechanism in the economy. A bank run would occur if and only if depositors perceive a low return on bank assets. Furthermore, a panic situation arises only when the market information is imperfect. A two-stage variant of the model shows that banks would deliberately offer a demand-deposit contract that is susceptive to bank runs.

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1 Introduction[1]

Bank runs are a matter of great public concern. In the 1980s and 1990s, 73% of the IMF's member countries suffered some form of banking crisis (see Lindgren et al 1996), and many of these had some form of bank run. These runs have played an important role not only in causing financial instability, but also in generating balance of payments (BOP) crises in a range of emerging markets, including Southeast Asia, Russia, Turkey and Ecuador (see Kaminsky and Reinhart 1999, 2000). Therefore, a good understanding of the mechanism of bank runs is not only crucial for bank supervision, but also important for understanding the recent crisis episodes. In this paper, I will try to answer the following three relevant questions. What are the underlying driving forces of bank runs? Are bank runs unavoidable in the banking industry? What is the relationship between the occurrence of bank runs and the quality of market information?

Banks have always been vulnerable to systemic crises, panics or runs because, by their nature, they issue liquid liabilities but invest in illiquid assets. Throughout this paper, I define a "bank run" as a situation in which all depositors decide to withdraw their funds before the bank's assets mature. However, it is important to distinguish between two types of bank runs. A type-I run fits the conventional view of bank runs as a panic situation in which there are no fundamental problems. Depositors rush to withdraw their money for "non-economic" reasons, such as pessimistic expectations or herd behaviour. On the other hand, a type-II bank run happens when the economy is in a bad state.

This paper provides a framework that can explain the occurrence of both types of bank runs. As in most of the existing literature, the starting point is the framework developed in the classic paper by Diamond and Dybvig (1983). They present a benchmark model in which bank runs are self-fulfilling prophecies. Demand-deposit contracts offered by banks provide a risk-sharing mechanism among risk-averse agents. Diamond and Dybvig show that there exist multiple equilibria due to the

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illiquidity problem of banks. On the one hand, if all investors withdraw their deposits according to their true liquidity needs, the economy is able to achieve the social optimum and the risk is successfully diversified. On the other hand, if all depositors anticipate a bank run, then everyone has the incentive to withdraw immediately and a bank run occurs as a self-fulfilling phenomenon. Which of these two equilibria occurs is determined exogenously, say, by a "sunspot".

The Diamond-Dybvig (D-D) model is very attractive in that it can explain the existence of bank runs within a simple framework.[2] However, further research has highlighted several important drawbacks. First, the D-D model does not give any hints as to when a bank run will occur. The shift in agents' expectations, which underpins the switch from one equilibrium to the other, is left unexplained. Second, the D-D framework is incomplete in that it does not consider the impact of possible bank runs on the behaviour of banks. As suggested by Cooper and Ross (1998), it is unclear why banks are willing to offer the demand-deposit contract from the beginning if bank runs are possible. Thus, it is important to consider how banks choose the demand-deposit contract in response to the possibility of bank runs. Finally, the property that all banking crises are self-fulfilling is controversial. Gorton (1988), Calomiris and Gorton (1991), Corsetti et al (1999) and Shen (2000) conduct a broad range of empirical studies and conclude that the data do not support the "sunspot" view that banking crises are random events. Instead, the empirical evidence suggests that bank runs are closely related to the state of the business cycle.

These issues suggest that the multiple-equilibrium models might overstate the indeterminacy of agents' beliefs. In fact, there are several papers in the existing literature that aim to unveil the "fundamentals" that affect agents' expectations and trigger bank runs. Chari and Jagannathan (1988), for example, provide a signal-extracting story. In their model, liquidity needs are uncertain and agents have asymmetric information on asset returns. Uninformed agents observe total early withdrawals and try to figure out what information the informed agents have. Chari and Jagannathan show that the equilibrium is unique and a bank run can happen even when no agent has adverse information because uninformed agents will incorrectly infer that the future return is low when liquidity-based withdrawals turn out to be unusually high. Jeitschko and Taylor (2001)

[2] This self-fulfilling story has been widely cited to explain bank panics (eg Alonso 1996, Cooper 1999, Cooper and Ross 1998). It has also played a key role in the recent so-called "third-generation" models, which consider the currency crises as the by-product of self-fulfilling bank crises (see Chang and Velasco 2000, 2001).

study a repeated-game version of the D-D model and show that the coordination failure always occurs as the "fear" that some agents may be discouraged will trigger a complete bank run at some point. Allen and Gale (1998) also develop a model that is consistent with the business cycle view of the origins of banking panics. In their model, they use the Pareto efficiency criteria to refine the equilibria. They show that a bank run is an inevitable result of the standard deposit contract and can be first-best efficient in a world with aggregate uncertainty. In more recent work by Goldstein and Pauzner (2000), Frankel et al (2000) and Morris and Shin (2000), it is shown that there is a unique equilibrium when the actions of agents rely on slightly noisy signals, and the probability of bank runs can be endogenously determined.

This paper extends the existing literature by proposing another possible "equilibrium-selection" mechanism. It builds on the basic D-D model but features significant differences:

(1) The traditional D-D model assumes that an agent cannot observe the actions of others when he makes his withdrawal decision. The validity of this assumption is doubtful, however. In the real world, agents usually do not make decisions simultaneously, and the followers are able to observe partial or complete information about those that make decisions before them. For example, an investor in the stock market can observe the trading history before he gives his order. Recent work by Green and Lin (2000) and Peck and Shell (1999) also highlights the importance of sequentiality of withdrawals in banking theory. Hence, this paper assumes that agents are scheduled to make their decisions sequentially and the followers can observe the actions of those in front of them.

(2) This paper allows for aggregate uncertainty in the economy: in particular, the long-term asset return is random.[3] The introduction of aggregate uncertainty allows me to study the relationship between the occurrence of bank runs and the state of the business cycle. This paper also introduces a publicly observable signal of asset returns.[4]

(3) I develop the D-D model into a two-stage game. In the first stage, banks compete with each other by offering demand-deposit contracts and agents choose their deposit banks. In the second

[3]See Chari and Jagannathan (1988) and Allen and Gale (1998).

[4]The signal can be any information from the TV, newspaper, internet, company reports, and so on. One important feature, which distinguishes this paper from both the information-based models (see Chari and Jagannathan 1988, Jacklin and Bhattacharya 1988, Goldstein and Pauzner 2000) and herd behaviour stories (see Banerjee 1992, Chari and Kehoe 1999, Calvo 1999) is that the information on asset return is the same for all agents. Hence, this paper focuses on how the sequential decision rule enables the agents to exchange information on their true liquidity needs instead of information on economic fundamentals.

stage (post-deposit stage), each agent observes his private liquidity need and then decides whether to withdraw the funds from the banks sequentially. As pointed out earlier, the two-stage analysis completes the model by explaining how the possibility of bank runs will affect the behaviour of the banking sector.

(4) A crucial assumption in the Diamond-Dybvig framework is the "first come, first served" constraint. This model does not impose this restriction.

The post-deposit analysis shows that there is a unique Perfect Bayesian Equilibrium (PBE) in the interim period and the probability of bank runs can be endogenously determined by contract variables. Because the front agents know that their actions can be observed by the followers, they are able to send the latecomers a signal that reflects their preference types. This "signalling effect" provides a coordination scheme among agents and yields a unique equilibrium outcome. I show that a bank run happens if and only if agents perceive a low return on bank assets. Furthermore, the quality of the public signal is very important in determining the characteristics of bank runs. When information becomes noisy, the banking sector is more vulnerable to runs and the probability of type-I bank runs increases.

Since the probability of bank runs can be endogenously determined, the two-stage model is able to study how banks will choose the optimal demand-deposit contract in response to the possibility of bank runs. I show that, in equilibrium, banks always choose a bank-run contract over a run-proof alternative, which suggests that the risk-sharing benefit outweighs the destabilising cost of bank runs.

The remainder of this paper is organised as follows. Section 2 describes the setup of the model. Section 3 analyses the equilibrium withdrawal decisions for agents under a given demand-deposit contract. Section 4 studies how banks respond to the possibility of bank runs when they are allowed to choose the demand-deposit contracts. Section 5 concludes.

2 Model setup

The model follows the setup of the standard Diamond-Dybvig framework. It has three periods ($T = 0, 1, 2$), one investment technology, and two sets of players: private agents and banks.

2.1 Investment technology

There exists an efficient (in the long run) but illiquid (in the short run) technology in the economy, which produces a random return $\tilde{R}$[5] in period 2 but only yields a constant return of 1 in period 1.[6] For simplicity, I assume that $\tilde{R}$ is uniformly distributed within $[R_L, R_H]$.

2.2 Agents

There are N agents in the economy. Each agent is endowed with one unit of goods. As in the D-D framework, there are potentially two types of agents: impatient agents and patient agents. Their utility functions are given by

$$u^1(c_1, c_2) = u(c_1) \tag{2.1}$$

$$u^2(c_1, c_2) = u(c_2) \tag{2.2}$$

respectively, where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma > 1$. That is, all agents are risk-averse, and impatient agents derive utility only from period 1 consumption while patient agents only care about period 2 consumption.

The type of each agent is unknown in period 0. In period 1, the idiosyncratic preference shock is realised. Each agent then knows only his own type but not that of others. For simplicity, I also assume that M out of N agents (or, a proportion $\alpha \equiv \frac{M}{N}$ of agents) are impatient. In other words, there is no aggregate uncertainty about the true liquidity needs in period 1.[7]

2.3 Banks

There are K banks in the economy.[8] Banks compete with each other by choosing the demand-deposit contracts in period 0.[9] A demand-deposit contract specifies a short-term interest rate, r_1,

[5]The technology is more efficient in the long run on the assumption that $E(u(\tilde{R})) > u(1)$, in which $u(\cdot)$ is the utility function.

[6]The determination of liquidation value is exogenous in this paper. Some existing papers may shed light on future study along these lines. Krugman (1998) provides a "fire-sale" story in which the firms are forced to liquidate their assets early at a price far below the real values when a self-fulfilling crisis happens. In a more recent paper, Backus et al (1999) propose a model with a liquidity crunch and imperfect information. They show that an idiosyncratic bank run might be contagious and banks' assets are liquidated at a very high cost because of the shortage of liquidity supply in the asset market.

[7]This assumption can be justified by the law of large numbers if N is large enough.

[8]The number of banks, K, is assumed to be much smaller than the number of agents (N, M).

[9]Green and Lin (2000) point out that the demand-deposit contract might not be efficient. They suggest that the optimal contract should involve making different payoffs to early consumers according to their sequence. In this

and a long-term interest rate, r_2.[10]

In periods 1 and 2, banks pay agents according to the terms specified in the contract. Whenever a bank goes bankrupt, it divides all resources among the remaining customers.

2.4 Information

In period 1, all agents receive a public signal s, which is an unbiased estimator of future asset return. In particular,

$$s = R + \epsilon \tag{2.3}$$

where ϵ is uniformly distributed within $[-e, e]$.[11] Thus it is straightforward to show that when an agent observes a signal s, he would expect the real return R to be uniformly distributed between $[s - e, s + e]$. In other words,

$$f(R|s) = \begin{cases} \frac{1}{2e} & \text{if } s - e \leq R \leq s + e \\ 0 & \text{otherwise} \end{cases}$$

2.5 Timing

In period 0, K banks compete for deposits by announcing the demand-deposit contracts, which specify a non-state-contingent short-term interest rate (r_1). Individual agents then choose the deposit banks. When agents are indifferent between two contract offers, they randomly pick the deposit bank.

In period 1, the type of each agent is realised and the public signal s is revealed. The decision each agent makes is simple: either to withdraw immediately or to wait (withdraw in period 2). Essential to this model is that agents make withdrawal decisions sequentially. In addition, each agent can observe the actions of all agents ahead of him before he makes his decision.

In period 2, the return on the risky asset is realised, and banks divide the remaining resources among all late consumers.

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paper, however, I restrict the study to demand-deposit contracts.

[10]In most of the existing literature, including this paper, the demand-deposit contract only needs to specify the short-term interest rate, and the long-term payoff is determined by evenly distributing the remaining resources among the late consumers. This assumption is innocuous because, in a competitive market, banks should make zero profit in all states. However, the specification of the long-term interest rate becomes very important when a capital requirement is introduced. See Zhu (2001) for more discussion.

[11]I assume that e is much smaller than the range of R. Obviously, the signal is perfect when $e = 0$.

3 Optimal decisions under a given contract

This section studies how the depositors make their withdrawal decisions in period 1. Since the withdrawal decisions in different banks are independent, we can start by analysing the equilibrium outcome in an individual bank.

Suppose a bank k offers an interest rate r_1 in period 0, and obtains total deposits of N_k, of which M_k are from potentially impatient agents. Depositors realise their liquidity needs and then make withdrawal decisions one after another according to a randomly determined sequence. I will show that there is a unique Perfect Bayesian Equilibrium (PBE) in this post-deposit subgame. Agents will run on the bank if and only if they perceive a low return on bank assets.

3.1 Definitions

In the post-deposit game, each agent i observes his own preference type, p_i. He also observes the previous withdrawal history. Denote a_j as the withdrawal decision made by agent j, where $a_j = 1$ indicates that agent j chooses to withdraw and $a_j = 0$ indicates that he chooses to wait. The withdrawal history observed by agent i then is $h_i = \{a_1, a_2, \cdots, a_{i-1}\}$. The aggregate withdrawal observed by agent i is $L_i = \sum_{j=1}^{i-1} a_j$.

Agent i makes his withdrawal decision based on his information set $I_i = \{p_i, h_i, F\}$, in which F includes the common knowledge information, such as the public signal on asset return, contract terms (r_1), proportion of impatient agents ($\alpha_k \equiv \frac{M_k}{N_k}$), and structure of the game.

An impatient agent's strategy is trivial. Because the long-term consumption is worthless to him, he always chooses to withdraw his deposit early. In other words, $a_i(p_i = \text{“}impatient\text{”}) = 1$.

3.2 Payoff function

A patient agent's strategy is more complicated. I start the analysis by looking at the payoff function for a patient agent i. He will obtain a constant amount of consumption (r_1) if he chooses to withdraw early and hold until period 2. However, if he chooses to wait, he will obtain a random amount of consumption in the long run, which depends on the expected aggregate early withdrawal from the bank, L.

$$r_2(R, L) = \frac{(N_k - r_1 L)R}{N_k - L} \tag{3.1}$$

Equation (3.1) means that the bank pays the early withdrawal ($r_1 L$) in period 1, and divides the remaining assets ($(N_k - r_1 L)R$) among the remaining depositors ($N_k - L$) in period 2. Obviously, a patient agent will choose to stay if and only if $E[u(r_2(R, L))|I_i] \geq u(r_1)$.

Notice that $r_2(R, L)$ is decreasing in L when $r_1 > 1$ and increasing in L when $r_1 < 1$.[12] This is quite intuitive. When $r_1 > 1$, an early consumer consumes more than the liquidation value of his deposits (liquidation value is 1). The more agents that decide to withdraw, the fewer assets are available for each late consumer. On the other hand, if $r_1 < 1$,[13] an early consumer consumes less than the liquidation value of his deposits, and the rest of his deposits are divided among late consumers. Therefore, the decision of a patient agent, which depends on both the contract variable (r_1) and his expectation of aggregate early withdrawal (L), is shown as in Table 1.

Table 1: Decisions of patient agents

	$L > L^*$	$L < L^*$
$r_1 > 1$	withdraw	wait
$r_1 < 1$	wait	withdraw

The threshold amount of withdrawal, L^*, at which a patient agent is indifferent between "withdraw" and "wait" is defined by $E[u(r_2(R, L^*))|I_i] = u(r_1)$. When the information is perfect ($e = 0$), the definition can be simplified to $L^* = \frac{N_k(R-r_1)}{r_1(R-1)}$.

Unfortunately, a patient agent cannot directly use the above decision rule to determine his strategy because the aggregate early withdrawal, L, is unobservable. Therefore, the central problem for each patient agent is how to form a reasonable expectation of L based on his private information set I_i.[14]

3.3 Equilibrium

Two important features of this post-deposit game are worth noting. First, because the impatient agents always choose to withdraw, a patient agent can reveal his true preference type by not

[12] $\frac{\partial r_2}{\partial L} = \frac{N_k(1-r_1)R}{(N_k-L)^2}$.

[13] When $r_1 < 1$, a representative agent accepts a negative short-term interest rate in the hope of obtaining a higher long-term interest rate when he turns out to be patient.

[14] In the D-D framework, there is no restriction on how agents form their expectations. As a result, multiple equilibria arise naturally. The efficient equilibrium, which occurs when no agents expect a bank run, is replaced by a bank-run equilibrium when the expectations shift in the opposite direction.

withdrawing. Second, a patient agent's decision depends not only on the withdrawal history he observes, but also on how (he believes) the followers will respond to his action.

It is natural to use the concept of Perfect Bayesian Equilibrium (PBE) developed by Fudenberg and Tirole (1991) to study the equilibrium outcome. The equilibrium should satisfy the following conditions: (i) based on individual information sets, agent i forms a belief on the preference types of front agents: $b_i = \{p_1, p_2, \cdots, p_{i-1}\}$; (ii) the strategy for each depositor is sequentially rational given the belief system $\{b_i\}$; (iii) the belief vector is consistent with the equilibrium strategy at all reachable information sets; (iv) agents' behaviour is rationalised on the off-equilibrium path.

The sequential decision rule provides an important channel through which patient agents are able to coordinate their actions. The front agent, by choosing "wait", is sending the signal "I am a patient agent, *and* I am expecting to receive a higher payment in period 2" to the followers. This "signalling effect" gives the front agent the power in initiating the coordination among agents and can yield a unique equilibrium outcome. Since the properties of the payoff function are different under two types of contracts (Table 1), the equilibrium strategies and equilibrium outcomes are also different. Lemma 1 and Lemma 2 specify the unique PBEs under the two types of contracts, respectively.

Lemma 1 *Given a contract $r_1 > 1$, there is a unique PBE in the post-deposit game. The equilibrium outcome is as follows: when $L^* \geq M_k$, every agent reports his preference type truthfully and the banking sector functions efficiently; when $L^* < M_k$, a bank run is inevitable.*

Proof: see Appendix A.

When $r_1 > 1$, the demand-deposit contract has a potential liquidity problem as the short-term interest rate is higher than the liquidation value. Patient agents decide whether it is desirable to coordinate with each other based on the information they receive. Lemma 1 implies that the patient agents will choose to coordinate their actions so long as this can yield a higher long-term payment. The key parameter is the threshold amount of withdrawal (L^*) at which a patient agent is indifferent between "withdraw" and "wait", which is endogenously determined by the public signal and contract variables. When L^* is greater than the number of impatient agents (M_k), the long-term return is higher so long as all patient agents do not withdraw their deposits. Lemma 1 shows that, in this situation, the front patient agent will initiate a coordination proposal by choosing to wait. The follower patient agents, knowing that it is beneficial to all of them, will also choose to wait. This coordination turns out to be stable and is the unique equilibrium due to the existence of the sequential decision rule. On the other hand, when $L^* < M_k$, the coordination is undesirable and a bank run happens.

Lemma 1 also implies that whether a bank run happens or not solely depends on the market's perception on asset return. From the definition of L^*, when $r_1 > 1$, $L^* \geq M_k$ is equivalent to $s \geq s^*$, in which s^* is defined by $u(r_1) = E[u(\frac{(N_k - r_1 M_k)\cdot R}{N_k - M_k})|s^*]$, or $u(r_1) = E[u(\frac{(1-r_1\alpha_k)\cdot R}{1-\alpha_k})|s^*]$, where α_k is the proportion of impatient agents among the depositors with bank k. In other words, Lemma 1 actually suggests that a bank run happens if and only if agents perceive a bad return on bank assets.

Lemma 2 *Given a contract $r_1 < 1$, there is a unique PBE in the post-deposit game. The equilibrium outcome is as follows: when $L^* \leq M_k$, all agents report their preference types truthfully; when $L^* > M_k$, a partial bank run happens and the aggregate early withdrawal is L^*.*[15]

Proof: see Appendix B.

Lemma 2 states that agents will choose to coordinate when $L^* \leq M_k$. There are two points worth mentioning. First, when $r_1 < 1$, $L^* \leq M_k$ is equivalent to $s \geq s^*$. Therefore the lemma suggests that a bank run does not occur when agents perceive a high return on bank assets. Second, a complete bank run never occurs under the contract that features $r_1 < 1$. This is because banks will never have liquidity problems in the interim period. Even when the asset return is low, only some of the patient agents have the incentive to withdraw early and this early withdrawal behaviour stops at a certain point. Why? According to Table 1, as more agents choose to withdraw their deposits in period 1, the expected long-term payment is higher. When it comes to the point where patient agents are indifferent between "withdraw" and "wait", the run phenomenon immediately disappears.

Since no complete bank run occurs when $r_1 < 1$, throughout this paper I refer to this type of contract as a "run-proof" contract.

[15] The probability of a partial bank run occurring is usually very small. For example, when we impose the assumption that the minimum long-term return is the same as the liquidation value ($R_L = 1$), it is easy to show that $s^* < 1 < R$ and therefore a partial bank run never occurs.

Combining Lemma 1 and Lemma 2, it is straightforward to derive the unique PBE in the post-deposit game.

Proposition 1 *In the post-deposit stage, there is a unique equilibrium outcome:*

(1) when $r_1 > 1$, a bank run happens if and only $s < s^*$;

(2) when $r_1 < 1$, a partial bank run happens if and only if $s < s^*$; and when a partial bank run happens, the patient agents are indifferent between "wait" and "withdraw".

Proposition 1 implies that, in the post-deposit stage, the equilibrium outcome solely depends on the market's perception of the state of the economy. A bank run happens if and only if agents perceive a low return on bank assets. Furthermore, the probability of bank runs can be endogenously determined by calculating the probability of the market observing a signal worse than s^*.

3.4 Equilibrium properties

To close the discussion of the post-deposit equilibrium, it is crucial to study the properties of s^*, the critical value below which a bank run is unavoidable. As stated above, s^* is defined by the equation $u(r_1) = E[u(\frac{(1-r_1\alpha_k)\cdot R}{1-\alpha_k})|s^*]$. The properties of s^* can be derived as follows:

Lemma 3 *s^* has the following properties: (i) when $e = 0$, $s^* = R^* \equiv \frac{(1-\alpha_k)r_1}{1-r_1\alpha_k}$; (ii) when $r_1 > 1$, $R^* > r_1 > 1$; (iii) when $e > 0$, $R^* < s^* < R^* + e$; (iv) s^* increases in r_1 and e.*

Proof: see Appendix C.

Lemma 3 states several important facts (when $r_1 > 1$):

(1) The level of s^* is endogenously determined by contract variables r_1 and the quality of the market signal. As a result, the probability of bank runs, or the probability of bank default, is endogenously determined.

(2) When the information is perfect ($e = 0$), only a type-II bank run is possible. Notice that R^* is a threshold return above which the occurrence of bank runs is not consistent with economic fundamentals. Under perfect information, bank runs would never occur when the asset return is higher than R^*. In other words, there is no type-I bank run.

(3) When the information is imperfect ($e > 0$), both types of bank runs are possible. Because $s^* > R^*$, the type-I bank run, in which agents choose to run on the bank when $R > R^*$, becomes

a realistic possibility. There are two underlying reasons. First, agents might incorrectly infer that the economy is in a bad state when they receive an undervalued signal. Second, since agents are risk-averse, they would require a certain amount of risk premium to compensate for the uncertainty in future payoff. This consideration induces the patient agents to take more precautionary actions (withdraw early and obtain a constant amount of consumption).

(4) In both cases, the occurrence of bank runs is related to the fundamentals. When the economy is in very good shape, no bank run occurs. When the economy is in a bad state, a bank run is unavoidable. When the economy is in some intermediate state and the information is imperfect, whether a bank run happens or not is ambiguous and depends upon the market's perception of the fundamentals.

(5) Banks are more vulnerable to runs when the short-term interest payment is higher. This is quite intuitive. When the short-term interest rate is increased, the short-term liabilities of the banks increase, forcing the banks to liquidate more assets. The long-term interest rate decreases and it is less worthwhile for a patient agent to choose to wait. On the other hand, the patient agents will demand a higher long-term payment for not running on the bank. Both effects increase the fragility of the banking sector.

(6) Banks are more vulnerable to runs when the public signal becomes noisier. Since s^* is increasing in e, the probabilities of both types of bank runs are higher when there is more noise in the market information. Therefore, the occurrence of bank runs under different market information structures differs not only qualitatively, but also quantitatively. This highlights the importance of the quality of the public signal (or transparency of market information) in preventing bank runs.

(7) Imperfect information leads to welfare losses in the economy. Figure 1 compares the welfare under two different information systems. When the information is perfect, a bank run happens if and only if $R < R^*$. On the other hand, when the information is imperfect, a bank run happens if and only if $s < s^*$. In both case I ($R < R^*$ and $s < s^*$) and case II ($R > R^*$ and $s > s^*$), the equilibrium outcomes under the two systems are the same. However, the two systems have different equilibrium outcomes in the following two situations:

– Case III ($R > R^*$ and $s < s^*$): in this case, an unnecessary bank run happens under the imperfect information system. Bank assets are liquidated and every agent receives the liquidation value of his deposit. This leads to a welfare loss of $\alpha_k u(r_1) + (1-\alpha_k)u(r_2) - u(1)$ for a representative depositor, where $r_2 = \frac{(1-\alpha_k r_1)R}{1-\alpha_k} > r_1$.

– Case IV ($R < R^*$ and $s > s^*$): in this case, because the market observes too optimistic a signal, no bank runs happen although it is incentive-compatible for each agent to make a run. The welfare loss is $u(1) - [\alpha_k u(r_1) + (1-\alpha_k)u(r_2)]$.[16]

To summarise, the occurrence of bank runs is related to the state of the business cycle. Agents run on the banks if and only if they perceive a low return on bank assets. Moreover, a panic-situation bank run, or the type-I run as defined above, occurs only when the market signal is imperfect. As the market signal becomes noisier, the banking sector becomes more fragile and the probabilities of both types of bank runs are higher. In the extreme case where the public signal reveals nothing about the economic fundamentals (or, equivalently, where there is no signal at all), the coordination mechanism among agents breaks down and a multiple equilibria phenomenon arises as in the traditional Diamond-Dybvig model. An implication of this is that improving the quality of market information can make a contribution to the stability of financial systems.

4 Equilibrium in the two-stage game

Until now, I have defined the equilibrium outcome under a certain demand-deposit contract. A bank run happens if and only if the market perceives a low return on bank assets. The probability of bank runs, which depends on the contract variables, can be endogenously determined. Based on these results, this section studies the type of demand-deposit contract that banks will offer taking into account the possibility of bank runs.

Because the banking sector is assumed to be perfectly competitive, a representative bank tries to maximise its expected profits, knowing that agents will deposit their endowments with the bank that offers the best contract. From principles of welfare economics, it is easy to redefine the problem. First, a representative bank must earn zero profit in equilibrium. This is quite intuitive: if the profit is positive, at least one bank will deviate by offering a higher interest rate and it will win all deposits. This bid-up process will continue until all banks' profits are driven to zero. Second, each bank is actually fighting for more deposits by offering the best demand-deposit contract that

[16] It might be negative, though, which means that preventing bank runs is desirable by avoiding the liquidation costs.

maximises the expected utility of a representative agent. According to the assumption, agents will randomly choose the deposit bank from those which offer the best contracts. The proportion of impatient agents with each deposit bank is the same as that of the whole population[17] ($\alpha_k = \alpha$).

Under the perfect information regime, the banks solve the following maximisation problem.

$$max_{r_1} E(U)|_{perfect} =$$

$$\begin{cases} \int_{R_L}^{R^*} u(1) f(R) dR + \int_{R^*}^{R_H} [\alpha u(r_1) + (1-\alpha) u(r_2(R))] \cdot f(R) dR & \text{if } r_1 \geq 1 \\ \int_{R_L}^{R^*} u(r_1) f(R) dR + \int_{R^*}^{R_H} [\alpha u(r_1) + (1-\alpha) u(r_2(R))] \cdot f(R) dR & \text{if } r_1 < 1 \end{cases}$$

The banks understand that if a bank-run contract ($r_1 > 1$) is chosen, a complete bank run will happen if and only if $R < R^*$; and if a run-proof contract ($r_1 < 1$) is chosen, a partial bank run occurs when $R < R^*$ and patient agents are indifferent between "wait" and "withdraw" when a partial bank run occurs.

Alternatively, when the information is imperfect, according to the welfare comparison in Section 3.4, the banks choose the optimal contract by solving the following problem:

$$max_{r_1} E(U)|_{imperfect} =$$

$$E(U)|_{perfect} - \int_{R^*}^{s^*+e} [\alpha u(r_1) + (1-\alpha) u(r_2) - u(1)] f(R) \cdot Pr(s < s^*|R) dR$$
$$- \int_{s^*-e}^{R^*} [u(1) - \alpha u(r_1) - (1-\alpha) u(r_2)] f(R) \cdot Pr(s \geq s^*|R) dR$$

The second term on the right-hand side represents the welfare losses when an unnecessary bank run happens under the imperfect information system. The third term represents the welfare losses when patient agents choose not to run on the banks while banks actually have fundamental problems. Since both R and ϵ are uniformly distributed, the conditional probabilities are given as follows.

$$Pr(s < s^*|R) = \frac{s^* + e - R}{2e} \qquad\qquad Pr(s \geq s^*|R) = \frac{R - (s^* - e)}{2e}$$

One important question here is whether banks are still willing to choose a bank-run contract when they take into consideration the possibility of bank runs. There are two opposite effects for a bank-run contract. On the one hand, it offers the impatient agents higher payments than the

[17]The key element that determines the equilibrium outcome in the post-deposit game, s^*, only depends on the proportion of impatient agents and has nothing to do with the number of agents.

liquidation value of their assets, therefore providing a smoother consumption path for a representative agent. This risk-sharing effect insures individual agents against the idiosyncratic risk and is welfare-improving. On the other hand, this contract makes the banking sector more vulnerable to runs and brings instability into the economy. The choice of banks depends on the relative magnitude of the risk-sharing effect and the destabilising effect. Proposition 2 gives a positive answer: the risk-sharing effect plays a dominant role and banks will always choose a bank-run contract in equilibrium.

Proposition 2 *In the two-stage analysis, $r_1^* > 1$ if the coefficient of risk aversion is greater that 1. In other words, banks always choose a bank-run demand-deposit contract over a run-proof alternative in equilibrium.*

Proof: see Appendix D.

Proposition 2 shows that a bank-run contract would improve the welfare even though it also brings instability to the economy. I now analyse how the optimal r_1^* is determined. For simplicity, I consider the case where information is perfect. The first-order condition is

$$\int_{R^*}^{R_H} [\alpha u'(r_1) - \alpha u'(r_2)R] f(R)dR - [u(r_1) - u(1)]f(R^*) \cdot \frac{\partial R^*}{\partial r_1} = 0$$
$$\Rightarrow u'(r_1) = \int_{R^*}^{R_H} u'(r_2)R \cdot f(R)dR + \frac{[u(r_1) - u(1)]f(R^*)}{\alpha \cdot \int_{R^*}^{R_H} f(R)dR} \cdot \frac{\partial R^*}{\partial r_1} \tag{4.1}$$

In the familiar Euler equation, the left-hand side is the marginal gain from increasing the short-term interest rate. The first term on the right-hand side is the marginal cost that results from transferring consumption from patient agents to impatient agents when no bank run happens. The second term on the right-hand side is the marginal cost that results from a more fragile banking system. In equilibrium, the banks should balance the marginal benefit with the marginal cost of changing r_1.

5 Conclusions, policy implications and extensions

This paper analyses a two-stage model of bank runs in which agents make their withdrawal decisions sequentially and banks offer demand-deposit contracts taking into account the possibility of bank runs. The model features a unique equilibrium, in which a bank run happens if and only if the market perceives a low return on bank assets. Moreover, banks will deliberately choose a contract that is susceptive to runs over a run-proof alternative in equilibrium. Although such a contract implies a more fragile banking sector, the risk-sharing effect plays a dominant role and the representative agent's welfare is still improved.

The study also highlights the importance of the transparency of the information system. Imperfect information is the only reason that causes a type-I bank run (a panic situation) in this model, in contrast to the results of the standard D-D model. A noisier signal makes the banking sector more vulnerable to runs and therefore brings welfare losses. Thus, this paper suggests that increasing the transparency and quality of market information is a key element in establishing a healthy banking system.

The unique-equilibrium explanation of the bank run phenomenon also has different policy implications from the traditional Diamond-Dybvig framework. The two policies that have been advocated to prevent bank runs, suspension of convertibility of deposits and deposit insurance, turn out to be inefficient. Although both policies can prevent a bank run in the interim period, they both introduce new distortions to the economy and therefore cannot achieve the social optimum. First, suspension of convertibility of deposits only controls the quantity of aggregate early withdrawal but cannot distinguish the true liquidity needs of agents. For example, when the market observes a bad signal on the asset return (this signal might be wrong), the dominant strategy for both impatient agents and patient agents is "withdraw the deposit in period 1 *if allowed*". Although the suspension policy successfully stops a bank run, its effect depends on who has the right to withdraw first. When all impatient agents are able to withdraw from the bank first, the run will be successfully prevented and the economy will achieve the social optimum. However, in a general situation, at least some patient agents can take advantage of their positions in the sequence and withdraw their deposits before the impatient agents. As a result, some impatient agents are unable to withdraw in period 1. Although a bank run does not occur, the resources are not allocated efficiently because long-term consumption is useless to impatient agents. This misallocation effect brings a new distortion into the economy and causes welfare losses.

Second, the deposit insurance policy cannot achieve the social optimum either. On the one hand, the patient agents will have no incentive to run on the bank because the deposit insurance scheme guarantees them the minimum consumption. However, if we consider the ex ante effect within the two-stage game, it is obvious that the deposit insurance scheme will cause moral hazard problems. Banks will behave more aggressively by offering a higher interest rate to attract more deposits. And knowing that their deposits are guaranteed by the safety net, individual agents will always choose the contract with the highest interest payment. This moral hazard issue will increase the bailout costs of the deposit insurance authority and in the long run the deposit insurance system is not sustainable.[18]

We see the basis for further work in several directions. An obvious one is to relax the assumption on the decision sequence. In this paper, the decision sequence is randomly and exogenously determined, and each agent knows his position in the sequence. The exogeneity of the decision sequence, however, seems innocuous. Gul and Lundholm (1995) and Zhang (1997) study a decision pattern in which all agents are allowed to choose the timing of their actions. Their research suggests that endogenous timing may not change the properties of equilibrium outcome.[19]

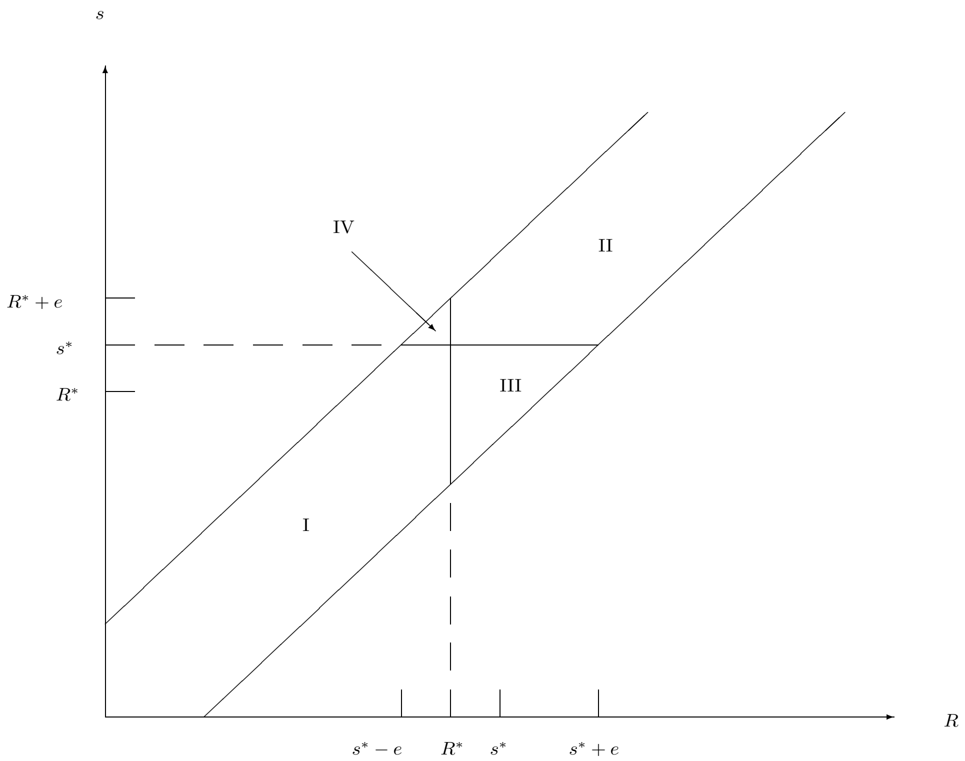
A more interesting direction is to extend the static model into a dynamic setting and study how the banks finance the long-term project by rolling over short-term debts. Financial crises in Thailand, Korea and other countries suggest that the inability to roll over short-term debts can create significant financial stress. A dynamic model of the debt restructuring issue might provide insightful understanding of this type of financial crisis.

[18]See Zhu (2001) for more detailed discussion.

[19]As suggested by Curtis Taylor, one possible solution is to suppose that agents learn their types before observing the public signal. In this case, the M impatient agents will be first in the sequence since they do not need to wait for the signal.

Figure 1

Welfare under perfect information vs imperfect information



Case I: $R < R^*$ and $s < s^*$

Case II: $R > R^*$ and $s > s^*$

Case III: $R > R^*$ but $s < s^*$

Case IV: $R < R^*$ but $s > s^*$

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Appendix

A Proof of Lemma 1

Proof: in the post-deposit game, the threshold withdrawal amount, L^*, is the public information. Suppose $L^* \in [\underline{L}, \underline{L}+1)$, where $\underline{L}$ is an integer. There are three possible situations:

(1) $\underline{L} < M_k$

In this case the patient agent knows that it is always better to withdraw immediately. The reason is simple: because the minimum aggregate early withdrawal, M_k, is larger than L^*, the long-term interest payment must be lower than the short-term interest rate. Everyone has the incentive to withdraw their deposit as quickly as possible. As a result, a bank run is inevitable.

(2) $\underline{L} \geq N_k - 1$ [20]

In this case a patient agent always chooses to wait because it will guarantee him a better payment in the long run no matter what strategies the other patient agents are adopting.

(3) $N - 1 > \underline{L} \geq M_k$

I use the induction principle to show that there is a unique equilibrium outcome in which all patient agents choose to wait (which I refer to as the "truth-telling" equilibrium) in this situation.

Step (i): in any case where $N_k - 2 = \underline{L} \geq M_k$, there is a unique "truth-telling" equilibrium.

$N_k = \underline{L} + 2$ means that it is better for a patient agent to wait so long as there are at least two agents who choose to wait in the interim period. In other words, a patient agent always chooses to wait whenever he observes that at least one agent has chosen to wait. Knowing this, if the first agent is patient, then he has a dominant strategy – "wait" – because he knows that the other patient agents will choose to wait, too. As a result, if patient agent 2 observes an aggregate withdrawal $L_2 = 1$, he concludes that the first agent must be impatient and he is the first patient agent in the line. Similarly, he will choose to wait to induce the other patient agents to wait. Following this argument iteratively, all patient agents who observe $L_i \leq \underline{L}$ will choose to wait. The unique equilibrium outcome is the "truth-telling" equilibrium.

Step (ii): in any case where $N_k - 3 = \underline{L} \geq M_k$, there is a unique PBE in which all agents report truthfully.

[20] This situation is very unlikely to arise. For example, in the perfect information case, it can be shown that L^* must be less than N_k, and $\underline{L} \geq N_k - 1$ is possible only when r_1 is very close to 1.

First, we determine the strategy for the first agent if he is patient. He knows that if he chooses to wait, his action will send the signal "agent 1 is patient" to all other agents. The other $N_k - 1$ agents will face a subgame with $N_k^{'} = N_k - 1 = \underline{L} + 2$, $M^{'} = M$ and $L^{*'} = L^*$. As discussed in step (i), this subgame features a unique equilibrium in which all agents report their true preference types. Thus patient agent 1 should choose to wait since his action will induce all other patient agents to wait.

Taking this into account, when patient agent 2 observes $a_1 = 1$, he (together with other agents) concludes that the first patient agent must be impatient. Therefore the $N_k - 1$ agents are confronted with a new subgame with $N_k^{'} = N_k - 1 = \underline{L}^{'} + 3$, $M_k^{'} = M_k - 1$, $L^{*'} = L^* - 1$ and $\underline{L}^{'} = \underline{L} - 1$. Following the same argument, the first patient agent in the new subgame (agent 2, who is patient and observes $L_2 = 1$ in the whole game) will choose to wait because it is a dominant strategy. Continue this process iteratively, and all patient agents who observe $L_i \leq \underline{L}$ choose to wait in equilibrium.

Step (iii): if it is true that there is a unique "truth-telling" equilibrium in a game that features $N_k = \underline{L} + d$ ($d \geq 2$) and $\underline{L} \geq M_k$, then it is also true that there is a unique "truth-telling" equilibrium in the new game that features $N_k^{'} = N_k + 1 = \underline{L} + d + 1$, $L^{*'} = L^*$ and $M^{'} = M_k$.

Step (ii) reveals two important features of the game. First, when the first impatient agent makes his decision, he only needs to consider the reactions of other agents if he chooses to wait because his payoff from choosing "withdraw" is independent of other agents' actions. Second, the game has a very attractive recursive structure. The first patient agent is able to reveal his preference type to the others through his action in equilibrium. The other agents, after observing the action of the first agent, are factually facing a new subgame with a similar structure.

Therefore, the proof is carried out in a similar way to step (ii). First, the first agent, if he is patient, must choose to wait because he knows that this action is able to induce the other agents to choose the "truth-telling" equilibrium in the new subgame that features $\{N_k, L^*, M_k\}$ (from the given condition). Second, given that $a_1 = 0$ is the dominant strategy for patient agent 1, if $a_1 = 1$, all agents are able to infer that the first agent is impatient and therefore they are confronted with a new subgame that features $\{N_k, L^* - 1, M_k - 1\}$, which belongs to the same type as the whole game. Using the same argument, the first patient agent in the new subgame has a dominant strategy –

"wait". Continue this analysis iteratively, and there is a unique "truth-telling" equilibrium in the game that features $\{N+1, L^*, M\}$.

Combining steps (i), (ii) and (iii), it is straightforward that the uniqueness of the "truth-telling" equilibrium is valid for all $N_k - 1 > \underline{L} > M_k$. QED.

B Proof of Lemma 2

Proof: when $r_1 < 1$, there also exists a critical L^*. However, a larger early withdrawal will increase the long-term interest payment and make late consumption more attractive. As a result, the strategy for a patient agent is different from when $r_1 > 1$ (see Figure 2).

(1) $L^* \leq M_k$ or $L^* \geq N_k - 1$

In both cases, the strategies for patient agents are just the opposite of when $r_1 > 1$. Since the long-term payoff is increasing in the aggregate early withdrawal, when $L^* \leq M_k$, the long-term interest rate is higher than r_1 and the patient agents will always choose to wait. If $L^* \geq N_k - 1$, however, they will choose to withdraw their deposits immediately.

(2) $L^* \in (M_k, N_k - 1)$

When $L^* \in (M_k, N_k - 1)$, the patient agents' strategies are as follows. First, if a patient agent observes $L_i > L^*$ (region I in case 3), he will choose to wait because he is sure to receive a higher payoff in the long run. Second, if a patient agent observes $i - L_i > N_k - L^*$ (region II in case 3), he will choose to withdraw because otherwise the aggregate early withdrawal must be less than L^* (since $L_i + (N_k - i) < L^*$) and all late consumers will receive a lower payment. Taking this into consideration, it can be shown iteratively that the patient agents in category III ($i - L_i \leq N_k - L^*$ and $L_i \leq L^*$) will always choose to wait, expecting that the followers will choose to withdraw until the aggregate early withdrawal equals L^*. In equilibrium, the patient agents are indifferent between "withdraw" and "wait".[21]

Therefore, when $r_1 < 1$, the optimal strategy for a patient agent is to withdraw when $i - L_i > N_k - L^*$ and to wait otherwise. The equilibrium outcome is therefore as stated in Lemma 2. QED.

[21]Strictly speaking, the aggregate early withdrawal equals the ceiling integer of L^* and the agents who choose to stay are slightly better off. When N_k is large enough, this difference goes to zero.

C Proof of Lemma 3

(1) When $e = 0$, s^* is defined by $u[\frac{(N_k - r_1 M_k)R}{N_k - M_k}] = u(r_1)$. The solution is $s^* = R^*$.

(2) Since $r_1 > 1$, $N_k - M_k > N_k - M_k r_1$ and therefore $R^* > r_1$.

(3) When $e > 0$, s^* is defined by $\int_{s^*-e}^{s^*+e} u[r_2(R)] \cdot f(R|s^*)dR = u(r_1)$, where $r_2(R) = \frac{(N_k - r_1 M_k)R}{N_k - M_k}$ and $f(R|s^*) = \frac{1}{2e}$ for $R \in [s^* - e, s^* + e]$. After some algebra, we have

$$\frac{\partial s^*}{\partial e} = -\frac{B}{A} > 0$$

where $B \equiv \frac{1}{e} \cdot [\frac{u[r_2(s^*+e)] + u[r_2(s^*-e)]}{2} - \int_{s^*-e}^{s^*+e} \frac{u[r_2(R)]}{2e} dR]$ is negative due to the concavity of the utility function, and $A \equiv \frac{1}{2e}\{u[r_2(s^* + e)] - u[r_2(s^* - e)]\}$ is positive. Besides, $-\frac{B}{A} < 1$ because it is equivalent to

$$\int_{s^*-e}^{s^*+e} \frac{u[r_2(R)]}{2e} dR < u[r_2(s^* + e)]$$

which is always true for an increasing utility function. Therefore, $R^* < s^* < R^* + e$.

(4) $\frac{\partial s^*}{\partial r_1} = \frac{2e \cdot u'(r_1) + \int_{s^*-e}^{s^*+e} u'[r_2(R)] \cdot \frac{MR}{N-M} dR}{u[r_2(s^*+e)] - u[r_2(s^*-e)]} > 0$

D Proof of Proposition 2

I first show that $r_1^* > 1$ is true under the perfect information, then show it is also valid under the imperfect information.

(1) Perfect information

First, the ex ante expected utility is increasing in r_1 for a run-proof contract because

$$\frac{\partial E(U)}{\partial r_1}|_{r_1<1} = \int_{R_L}^{R^*} u'(r_1)f(R)dR + \int_{R^*}^{R_H} [\alpha u'(r_1) - \alpha u'(r_2)R]f(R)dR$$

When the coefficient of risk aversion is greater than 1 ($\frac{-u''(c) \cdot c}{u'(c)} > 1$), it is straightforward to show that $u'(r) \cdot r$ is decreasing in r. The second term on the right-hand side must be positive because when $r_1 < 1$:

$$\begin{aligned} & \int_{R^*}^{R_H} [\alpha u'(r_1) - \alpha u'(r_2)R]f(R)dR \\ > & \int_{R^*}^{R_H} [\alpha u'(r_1) - \alpha u'(r_1)R^*]f(R)dR \\ > & \int_{R^*}^{R_H} \alpha u'(r_1) \cdot \frac{(1-r_1)}{1-\alpha r_1} \cdot f(R)dR > 0 \end{aligned}$$

Therefore, $E(U)$ is increasing in r_1 when $r_1 < 1$.

Second, $r_1 = 1$ cannot be the optimal contract because

$$\frac{\partial E(U)}{\partial r_1}|_{r_1 \geq 1} = \int_{R^*}^{R_H} [\alpha u'(r_1) - \alpha u'(r_2)R]f(R)dR - [u(r_1) - u(1)]f(R^*) \cdot \frac{\partial R^*}{\partial r_1}$$

When $r_1 = 1$, the corresponding $R^* = 1$ and $r_2 = R$. The first-order derivative at $r_1 = 1$ is therefore

$$\begin{aligned} \frac{\partial E(U)}{\partial r_1}|_{r_1=1} &= \int_{1}^{R_H} \alpha[u'(1) - u'(R)R]f(R)dR \\ &> 0 \end{aligned}$$

Therefore, $r^* > 1$ in equilibrium.

However, it is uncertain whether r_1^* is larger or smaller than the social optimal solution. Under the social optimum, the FOC is

$$u'(r_1) = \int_{R_L}^{R_H} u'(r_2) \cdot R \cdot f(R)dR \tag{D.1}$$

When r_1 is increased, the marginal cost under the social optimum is the decrease in long-term payoff in all states. Under the market economy (equation 4.1), the marginal cost is the decrease in long-term payoff when no bank run happens ($R > R^*$), which is less than under the social optimum. However, there is another cost in that the higher short-term interest rate makes the banks more vulnerable to runs (R^* increases). Combining these two effects, it is uncertain whether the marginal cost is larger or smaller in the market economy. As a result, r_1^* might be larger than, or less than, or equal to the short-term interest rate under the social optimum.

(2) Imperfect information

When the information is imperfect, the expected utility of a represented agent can be derived from the benchmark situation in which the information is perfect.

$$max_{r_1} E(U)|_{imperfect} =$$

$$\begin{aligned} E(U)|_{perfect} &- \int_{R^*}^{s^*+e} [\alpha u(r_1) + (1-\alpha)u(r_2) - u(1)]f(R)\frac{s^* + e - R}{2e}dR \\ &- \int_{s^*-e}^{R^*} [u(1) - \alpha u(r_1) - (1-\alpha)u(r_2)]f(R)\frac{R - (s^* - e)}{2e}dR \end{aligned}$$

Because $\frac{\partial s^*}{\partial r_1} > 0$ and $\frac{\partial R^*}{\partial r_1} > 0$, after some tedious algebra, it can be shown that

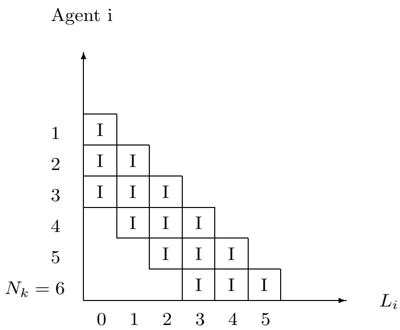
$$\begin{aligned} \frac{\partial E(U)|_{imperfect}}{\partial r_1}|_{r_1=1} &= \int_1^{R_H} \alpha[u'(1) - u'(R)R]f(R)dR \\ &- \int_1^{s^*+e} \alpha[u'(1) - u'(R)R]f(R)dR \\ &+ \int_{s^*-e}^{1} \alpha[u'(1) - u'(R)R]f(R)dR \end{aligned} \tag{D.2}$$

which is positive so long as $e <<$ range of R.

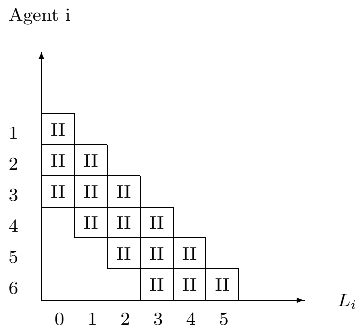
Therefore, $r_1^* > 1$ is also true under the imperfect information.

Figure 2

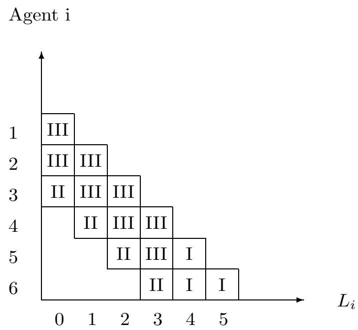
Example ($N_k = 6,\ M_k = 3$)



Case 1: $L^* \leq M_k$



Case 2: $L^* \geq N_k - 1$



Case 3: $L^* \in (3, 4)$

Note: In the example, a patient agent chooses “wait” in region I, and chooses “withdraw” in region II. By using backward induction iteratively, a patient agent chooses “wait” in region III.

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