BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers
No 1046

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by Rodney Garratt, Jiaheng Yu and Haoxiang Zhu

Monetary and Economic Department

November 2022

JEL classification: E42, G21, G28, L11, L15.

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This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

The Case for Convenience: How CBDC Design Choices Impact Monetary Policy Pass-Through*

Rodney Garratt[†] Jiaheng Yu[‡] Haoxiang Zhu[§]

Abstract

Banks of different sizes respond differently to interest on reserves (IOR) policy. For low IOR rates, large banks are non-responsive to IOR rate changes, leading to weak pass-through of IOR rate changes to deposit rates. In these circumstances, a central bank digital currency (CBDC) may be used to provide competitive pressure to drive up deposit rates and improve monetary policy transmission. We explore the implications of two design features: interest rate and convenience value. Increasing the CBDC interest rate past a point where it becomes a binding floor, increases deposit rates but leads to greater inequality of market shares in both deposit and lending markets and can reduce the responsiveness of deposit rates to changes in the IOR rate. In contrast, increasing convenience, from sufficiently high levels, increases deposit rates, causes market shares to converge and can increase the responsiveness of deposit rates to changes in the IOR rate.

Keywords: central bank digital currency, interest on reserves, payment convenience, deposit rates, bank lending
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*The views expressed are those of the authors and do not necessarily reflect those of the BIS. We thank Todd Keister for multiple discussions in the early stages of this work. We also thank Yu An, David Andolfatto, Ben Bernanke, Darrell Duffie, Zhiguo He, Michael Kumhof, Jiaqi Li, Debbie Lucas, Monika Piazzesi, Alessandro Rebucci, Daniel Sanches, Antoinette Schoar, Harald Uhlig, Yu Zhu, and Feng Zhu, as well as seminar participants at the Bank of Canada, Luohan Academy, MIT Sloan, Nova SBE, Microstructure Online Seminars Asia Pacific, the University of Hong Kong, the Canadian Economics Association, the JHU Carey Finance Conference, the Philadelphia Workshop on the Economics of Digital Currencies, the CB & DC Online Seminar, Boston University, the Federal Reserve Bank of New York, Shanghai Advanced Institute of Finance, MIT Computer Science and Artificial Intelligence Lab, the 2021 ECB Money Market Conference, AFA 2022 and the Future(s) of Money Conference, Paris 2022.

[†]Bank for International Settlements. Email: rodney.garratt@bis.org

[‡]MIT Sloan School of Management. Email: yujh@mit.edu.

[§]MIT Sloan School of Management and NBER. Email: zhuh@mit.edu. Haoxiang Zhu's work on this paper was completed prior to December 10, 2021, when he joined the U.S. Securities and Exchange Commission.

"If all a CBDC did was to substitute for cash – if it bore no interest and came without any of the extra services we get with bank accounts – people would probably still want to keep most of their money in commercial banks."
—Ben Broadbent, Deputy Governor of the Bank of England, in a 2016 speech

1 Introduction

A central bank digital currency (CBDC) "is a digital payment instrument, denominated in the national unit of account, that is a direct liability of the central bank" (BIS, 2020). Over the last few years, interest in CBDC has grown to the point where at present 90 percent of central banks are investigating options for introducing CBDCs (BIS, 2021). As indicated in Broadbent's remarks (in the epigraph), policymakers initially contemplated a CBDC that duplicated features of cash, without adding design characteristics that would make it more likely to compete with money issued by commercial banks – the so called disintermediation problem. However, more recently central banks have taken a broader view, and have been more open to the possibility that CBDCs can help them to fulfill their mandates, either in the present or the future. Central banks are increasingly viewing CBDCs as a way to improve the payment system, promote financial inclusion, enhance monetary policy transmission, and reduce systemic risk (BIS, 2020).

The likelihood that a CBDC will achieve any of the desired central bank objectives depends upon its design features and how they interact. CBDCs can offer both pecuniary benefits, in the form of interest payments, and nonpecuniary benefits. These can include a host of features that enhance the performance of CBDC as a medium of exchange. Examples include the quality of the user interface, processing speed, privacy and access to markets.[1]

[1] A publicly provided CBDC could be less expensive to use and more widely accessible than existing, privately offered payment methods, and it could offer access to new platforms and services. See, for example, the Bank of England discussion paper on CBDC (Bank of England, 2020) which describes the potential for third-party payment interface providers to provide overlay services on top of CBDC balances. A CBDC could also provide privacy. Policy makers have argued that the central bank is specially positioned to provide privacy in payments because the central bank does not have a profit motive to exploit consumer

We lump these possibilities together under the heading "payment convenience."

In this paper, we explore the implications of introducing a CBDC that is interest-bearing and offers payment convenience. All else equal, consumers will prefer to hold a currency that pays higher interest. However, if a currency is easier to use, or accepted at more places, then these non-pecuniary benefits may offset interest payments. Hence both interest rate *and* payment convenience are important choice parameters for a CBDC, as both will determine consumer demand for CBDC and hence its ultimate impact on monetary policy objectives.

We seek to evaluate design choices for CBDC in a model that is descriptive of the current US financial system, which is characterized by large excess reserves and in which the main monetary policy variable is interest on reserves (IOR).[2] Crucially, we also want our analysis to capture heterogeneity in bank size. Bank size matters for two key reasons. First, bank size impacts the cost basis of loans, as it determines the likelihood of retained reserves. Loan issuance involves the creation of deposits. When these deposits are spent by the borrower there is a chance that the recipient will belong to the same bank, in which case there is no associated transfer of reserves to another bank. This likelihood is not negligible for the largest banks in the US economy and hence the impact of retained reserves should not be ignored.[3] Second, large bank deposits may offer a higher convenience value than small bank deposits. For example, a large bank could have a more expansive network of branches and

payment data (Lagarde, 2018).

[2]In the United States, IOR has been paid since October 2008. Since the financial crisis of 2008-09, interest on excess reserves (IOER) has become the Federal Reserve's main policy tool to adjust interest rates. In July 2021, the Federal Reserve renamed IOER to interest on reserve balances (IORB), as required reserves are currently zero. For simplicity, we use the acronym IOR.

[3]During 2010-2020, based on Call Reports data, the top four largest banks captures 35% of US deposit market. Their deposit market shares are large *and* stable over the last decade, with the averages listed in the following: Bank of America (11%), Chase (10%), Wells Fargo (10%), and Citi (4%). Given the stationarity of deposit shares, and the fact that deposits are created through commercial banks making loans (Bank of England Quarterly Bulletin, 2014), the fraction of a lent dollar that ultimately flows to each bank should approximate the deposit shares, regardless of the number of transactions or transfers. Hence the high deposit market shares of the largest banks have non-negligible impact on their opportunity costs of making loans. In local deposit markets, concentration is also salient. Drechsler, Savov, and Schnabl (2017) measure this concentration by Herfindahl index (HHI), calculated by summing up the squared deposit-market shares of all banks that operate in a given county. Their calculation indicates that around 50% of US counties have an HHI that is higher than 0.3 (at least one bank's deposit share is greater than 30%), and 25% of US counties have an HHI that is higher than 0.5 (at least one bank's deposit share is greater than 50%).

ATMs, a better mobile App, or a wider range of other (unmodeled) services. The higher convenience value of its deposit may allow the large bank to offer a lower deposit rate than the small bank and yet still maintain a larger market share.

In our model, the CBDC is offered through commercial banks. While CBDC balances are the direct liability of the central bank, we envision that commercial banks will act as the central bank's agents to conduct KYC (Know Your Customer) and AML (Anti-Money Laundering). This "tiered" design of CBDC is consistent with the recent pilot of E-Krona in Sweden, the CBDC experiment in China, and the Banking for All Act in the United States.[4]

The heterogeneous-bank model we develop is new. The first part of our analysis seeks to validate the model by demonstrating that it explains aspects of US deposit markets that are not explained by existing, homogeneous-bank models of the US economy with large reserves. In particular, we are able to explain the observed lack of interest-rate pass-through in US deposit markets. Interest rate pass-through in the US economy is far from complete. Drechsler, Savov, and Schnabl (2017) find that "[f]or every 100 bps increase in the Fed funds rate, the spread between the Fed funds rate and the deposit rate increases by 54 bps." Duffie and Krishnamurthy (2016) document a sizable dispersion of a broad range of money market interest rates, which widened as the Fed raised its interest on reserves. Our analysis shows that the low correlation between movements in policy rates that determine the fed funds rate and movements in deposit rates can be partly attributed to the differential impact these changes have on banks of different sizes.

The intuition behind this result is as follows. The large bank's ability to offer a lower rate than the small bank may place them at a "corner solution" introduced by the fact that deposit rates cannot go below zero. The large bank will set a deposit rate of zero, that is non-responsive to changes in the policy rate, until that rate rises to a level where the zero

[4]The Act argues that Digital Dollar Wallets should provide a number of auxiliary services including debit cards, online account access, automatic bill-pay and mobile banking. These features (in particular mobile banking which could give access to a variety of platforms that customers of a particular bank might otherwise not have access to) could result in a CBDC with its own convenience value. Similar provisions are outlined in the ECB's digital euro report (2020).

lower bound on deposit rates is no longer binding. We will show that the zero lower bound on deposit rates binds only when IOR rate is low. Hence for low levels of IOR we expect deposit rates set by a large share of the banking sector to be non-responsive to changes in IOR. For high levels of IOR, the lower bound is not binding and we expect all banks to adjust deposit rates in response to changes in IOR (or the federal funds rate if this becomes the relevant opportunity cost of lending). We demonstrate that both of these model predictions seem to be true empirically.

In light of these observations regarding weak pass-through for low IOR rates, we examine how outcomes in both the deposit market (deposit shares at the different banks and deposit rates) and the lending market (loan volumes and rates) may be impacted by the choice of design characteristics of a CBDC. First, we vary the CBDC interest rate. Increasing the CBDC interest rate while holding the IOR rate and the CBDC convenience value fixed raises the deposit rates of both banks, thus bringing their weighted average closer to the IOR rate. Setting the CBDC interest rate equal to the interest rate on reserves would result in full monetary policy pass-through. However, by forcing both banks to raise interest rates, a higher CBDC interest rate makes it more difficult for the small bank to compete with the large bank by offering higher deposit rate. Thus, a higher CBDC interest rate reduces the market share of the small bank in deposit and lending markets, further widening the large bank-small bank gap.

Second, we vary the CBDC convenience value, holding the IOR rate and CBDC interest rate fixed. Making the CBDC more convenient weakens the market power of the large bank by narrowing the convenience gap between the two banks. For example, by hosting a convenient CBDC, a small community bank partially "catches up" with large global banks in offering payment functionalities. The most immediate implication is that a convenient CBDC results in a lower deposit rate at the small bank, because the small bank does not have to compensate depositors as much for forgoing the large bank's convenience. The deposit rate at the large bank initially remains unchanged, and hence the average deposit rate for the market falls as convenience is increased from zero. However, as convenience rises, a point is eventually reached where the large bank is no longer constrained by the zero lower bound and starts to raise its deposit rate to compete with the small bank. The result is an increase in the average deposit rate. The implication is that for any given level of IOR, pass-through of IOR to deposit rates is reduced for low levels of convenience and increased for high levels of convenience.

Finally, we address the issue of how a given CBDC design impacts the sensitivity of deposit rates to changes in the IOR rate. In the equilibrium where the lower bound on deposit rates is not binding, pass-through is complete regardless of the levels of the CBDC interest rate or convenience value. In equilibria where the lower bound is binding, we establish two results that hold under some distributional assumptions. First, within the constrained equilibrium, where the large bank's deposit rate is non-responsive to changes in the IOR rate, the response of the small bank's deposit rate to increases in the IOR rate decreases as the CBDC interest rate increases and increases as convenience increases. Second, higher levels of the CBDC interest rate increase the range of IOR rates for which the large bank's deposit rate is non-responsive and higher levels of convenience decrease the range. Thus, a positive interest on CBDC necessarily weakens monetary policy transmission from IOR to deposit rates when the IOR rate is low, while increasing convenience necessarily increases monetary policy transmission.

The paper is organized as follows. Section 2 provides a literature review. Section 3 introduces the model and characterizes the constrained and unconstrained equilibrium. There is a critical level of IOR rate at which the economy transitions from the the constrained to unconstrained equilibrium. Hence, this analysis provides an explanation for weak pass-through of IOR rate to deposit rates at low levels of IOR and strong pass-through at high levels. Section 4 separately evaluates the direct impact on deposit rates of increasing the CBDC interest rate s and convenience level v on deposit rates. Section 5 examines the sensitivity of deposit rates to changes in the IOR rate under different CBDC designs. Section 6 concludes.

2 Literature

Our work builds on previous literature that has modelled deposit and lending markets in the current regime of large excess reserves. In Martin, McAndrews, and Skeie (2016), a loan is made if its return exceeds the marginal opportunity cost of reserves, which can be either the federal funds rate or the IOR rate, depending on the regime. Our model differs in that we have multiple banks and hence lent money may return to the same bank as new deposits. Hence, our opportunity cost of lending is lower. Nevertheless, we share the conclusion that the aggregate level of bank reserves does not determine the level of bank lending.

There is now a growing literature that seeks to examine the impact of CBDC on deposit and lending markets. The conclusions vary and depend upon the level of competition, the interest rate on the CBDC, and other features (e.g., liquidity properties of CBDC and reserve requirements). Keister and Sanches (2021) consider a competitive banking environment in which deposit rates are determined jointly by the transactions demand for deposits and the supply of investment projects. If the CBDC serves as a substitute for bank deposits, then its introduction causes deposit rates to rise, and the levels of deposits and bank lending to fall.

In contrast, if banks have market power in the deposit market, the introduction of a CBDC does not disintermediate banks, as banks can prevent consumers from holding the CBDC by matching its interest rate. This lowers their profit margin, but does not lower the level of deposits, and may even increase it. This is true in the model proposed by Andolfatto (2021), where the bank is a monopolist. In that paper, an interest bearing CBDC causes deposit rates to rise and the level of deposits to increase. Likewise, in that paper, banks have monopoly power in the lending market, and, as in Martin, McAndrews, and Skeie (2016), lending is not tied directly to the level of deposits, Hence, a CBDC does not impact the interest rate on bank lending or the level of investment.

Chiu, Davoodalhosseini, Jiang, and Zhu (2019) also consider banks with market power and show that an interest-bearing CBDC can lead to more, fewer or no change in deposits, depending on the level of the CBDC interest rate. In an intermediate range of rates, the CBDC impacts the deposit market in a manner similar to Andolfatto (2021) in that banks offer higher deposit rates and increase deposits. Since, similar to Keister and Sanches (2021), lending is tied to the level of deposits, adding the CBDC results in increased lending.

Our work is closest to Andolfatto (2021). We do not specify the overlapping generations framework that he uses to make money essential. However, like Andolfatto (2021), in our model, reserves are abundant, lending is determined by a performance threshold, and banks have monopoly power in lending market. Hence, lending is determined not by deposit levels, but instead by the opportunity cost of funds. In our model, this opportunity cost is lower than the IOR rate, since we allow for the realistic feature that reserves come back to the lending bank with a probability that depends on the deposit market share. Unlike Andolfatto (2021), and the other works mentioned above, we incorporate two key design aspects of CBDC, interest rate and convenience value, and we examine the combined impact these features have on market outcomes in an environment with heterogeneous banks.

The impact of adding a CBDC can be richer in the presence of other frictions. In a model with real goods and competitive banks, Piazzesi and Schneider (2020) find that the introduction of CBDC is beneficial if all payments are made through deposits and the central bank has a lower cost in offering deposits. However, they also find that the CBDC can be harmful if the payer prefers to use a commercial bank credit line, but the receiver prefers central bank money. Parlour, Rajan, and Walden (2022) argue that a wholesale CBDC that enhances the efficiency of interbank settlement system could exacerbate the asymmetry between banks if the CBDC does not distinguish net-paying and net-receiving banks. Agur, Ari, and Dell'Ariccia (2022) consider an environment where households suffer disutility from using a payment instrument that is not commonly used. They examine trade-offs faced by the central bank in preserving variety in payment instruments and show that the adverse effects of CBDC on financial intermediation are harder to overcome with a non-interest-bearing CBDC.

Fernández-Villaverde, Sanches, Schilling, and Uhlig (2021) extend the analysis of CBDC to a Diamond and Dybvig (1983) environment in which banks are prone to bank runs. In this setting, the fact that the central bank may offer more rigid deposit contracts allows it to prevent runs. Since commercial banks cannot commit to the same contract, the central bank becomes a deposit monopolist. Provided that the central bank does not exploit this monopoly power, the first-best amount of maturity transformation in the economy is still achieved.

Brunnermeier and Niepelt (2019) and Fernández-Villaverde, Sanches, Schilling, and Uhlig (2021) derive conditions under which the addition of a CBDC does not affect equilibrium outcomes. Key to their result is the central bank's active role in providing funding to commercial banks in order to neutralize the CBDC's impact on their deposits.

3 Model and Equilibrium

3.1 Setup

The economy has a large bank (L) and a small bank (S).[5] There are $X = X_S + X_L$ reserves in the banking system, where X_S denotes the reserve holding of the small bank and X_L denotes the reserve holding of the large bank.[6] For simplicity, the banks start off holding reserves as their only asset, balanced by exactly the same amount of deposits. Following Martin, McAndrews, and Skeie (2016), we assume that the level of reserves X is exogenously determined by the central bank and is assumed to be large. The central bank pays the two commercial banks an exogenously determined interest rate f on their reserve holdings, which

[5] The assumption of two banks is, of course, a simplification. However, the situation may accurately describe the retail depositors' decision making process. Using survey data, Honka, Hortaçsu, and Vitorino (2017) find that US consumers were, on average, aware of only 6.8 banks and considered 2.5 banks when shopping for a new bank account. More than 80% considered fewer than 3 banks when shopping for a new bank account.

[6] We normalize the size of an individual loan to be \$1, so reserves are in units of the standard loan size. For example, if a loan size is \$1 million and the actual reserve is \$1 trillion, then in our model, X is interpreted as $\$1\, trillion/\$1\, million = 10^6$.

is called interest on reserves (IOR). The large and small banks pay depositors endogenously determined deposit rates r_L and r_S, respectively. Thus, if nothing else happens, bank j's total profit would be $X_j(f - r_j)$.

Commercial bank deposits are valuable not just for the interest they pay, but also for the payment services they provide, the benefit of which we refer to as convenience value. The convenience value of deposits in the small bank is normalized to be zero. The convenience value of deposits in the large bank is a random variable $\delta \geq 0$ that has the twice differentiable, cumulative distribution function G. We make the following assumption on G that we impose throughout the paper:

Assumption 1. *The function G satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$.*

This condition ensures the second order condition of a bank's optimization problem is satisfied. The condition is also used in the comparative statics analysis. Bounding the curvature of G bounds the masses of agents who value large bank's deposits highly and lowly, and ensures that both banks will compete to win additional depositors by raising their deposit rates when f and other parameters, which we introduce in the following paragraphs, change. Each depositor in the economy draws their large-bank convenience value δ independently from the distribution G. This process reflects the idea that enhanced payment services are not valued the same by all depositors.

The central bank offers a "retail" CBDC that is universally available. The CBDC has two features: it pays an interest rate of s to depositors who use it and it provides a per-dollar convenience value $v \geq 0$ to users that is the same across all depositors.[7] The convenience value can be interpreted as a benefit that depositors receive from transacting using central bank money. This benefit can include access to platforms on which CBDC can be spent,

[7]The uniform nature of CBDC convenience value reflects the idea that CBDC should ideally create no discrimination. This is also without loss of generality. If v varies across people, let's say $v = \bar{v} + \tilde{v}$, where $\bar{v}$ is the average convenience value that can be adjusted by the central bank, and $\tilde{v}$ represents the individual deviation from the average, we would only need to let G describe the distribution of $\delta - \tilde{v}$.

aspects of the mobile user interface (app features) or any other account services that are associated with central bank accounts.

We assume that the CBDC is offered via commercial banks, and that money can be transferred seamlessly between a depositor's deposit account at a commercial bank and their CBDC account offered via the commercial bank.[8] Because a depositor can transfer money between her deposit account and her CBDC account at no cost, she can obtain a convenience value in payments that is equal to the maximum of the two options. A depositor at the large bank receives convenience value $\max(\delta, v)$ and a depositor at the small bank receives convenience value $\max(0, v) = v$. The convenience value v acts as a lower bound on the payment convenience obtained by all depositors and thus narrows the gap between payment convenience levels that depositors receive across banks of different sizes.

There is a unit mass of agents, and each potentially plays three roles: entrepreneur (borrower), worker, and depositor. The main heterogeneity among the agents is their convenience value for large bank deposits.

The model has four periods. At $t = 0$, the commercial banks set the deposit rates r_L and r_S. The central bank sets the interest on reserves rate f, the CBDC interest rate s, and the CBDC convenience value v. In the model, f, s, and v are exogenous, and r_L and r_S are endogenous. At the start of the model, a fraction m_L of agents have existing deposits at the large bank and a fraction $m_S = 1 - m_L$ of agents have existing deposits at the small bank. The amount of deposits per capita across agents is identical. This means $m_L = X_L/X$ and $m_S = X_S/X$. Because users can seamlessly transfer money between the CBDC account and the deposit account, a CBDC account offered via a commercial bank effectively provides the same convenience value as the deposit account of that commercial bank. For this reason, the CBDC interest rate s is a lower bound on banks' deposit rates, i.e., $r_L \geq s$ and $r_S \geq s$.

[8]The Chinese CBDC experiment pivots around the e-CNY wallet mobile phone app. Embeded in the app are interfaces connecting to deposit accounts at eight authorized commercial banks, as well as AliPay and TenPay. Users can transfer money seamlessly between the deposit accounts and the e-CNY wallet, with just a click, and make payments from the e-CNY wallet. The transfer incurs no fee. The CBDC launched in Nigeria through the eNaira wallet app has the same characteristics. Chiu et al. (2019) also assume that CBDC and deposits are perfect substitutes in terms of payment functions.

At $t = 1$, the agents act as entrepreneurs and workers. Each agent is endowed with a project, and each project requires \$1 of investment and pays $A > 1$ with probability q_i and zero with probability of $1 - q_i$, where q_i has the distribution function Q and A is a commonly known constant. The expected payoff per dollar invested is thus $q_i A$. Each agent can only borrow from the bank where she keeps her deposit (the "relationship" bank). The bank prices the loan as a monopolist. If the loan is granted, the entrepreneur pays \$1 to a randomly selected agent from the same population. The selected agent plays the role as a worker and completes the project. The main point of introducing workers is to generate some money flow in the economy.

At $t = 2$, agents play the role as depositors. Workers who receive wages choose where to deposit the wage. The depositor can pick either the large bank or the small bank, and within a bank, the depositor can pick either the bank's own deposit account or the CBDC account. These choices are made after considering the depositor's own convenience value for large bank deposits, the convenience value of the CBDC, and all relevant interest rates.

At $t = 3$, the projects succeed or fail. The banks earn interest on reserves and pays depositors according to their deposit holdings and the deposit rates.

3.2 Bank deposit creation

For the purpose of illustration it is convenient to illustrate the deposit creation process by considering a discrete set-up, in which we characterize the bank's decision to make a single loan. The condition on bank lending that we derive will be applicable to the continuum model in which borrowers (i.e., the entrepreneurs) are infinitesimal.

The tables below show the sequence of changes in the large bank's balance sheet in the loan process. The changes in the small bank's balance sheet in the loan process are entirely analogous.

1. Before lending, the large bank starts with X_L reserves. Its balance sheet looks like:

Asset	Liability
Reserves X_L	Deposits X_L

2. If the large bank makes a loan of \$1, it immediately creates deposit of \$1 in the name of the entrepreneur. The balance sheet of the bank becomes:

Asset	Liability
Reserves X_L	Deposits X_L
Loans 1	New Deposits 1

3. Eventually, the entrepreneur will spend her money to pay a worker. The large bank anticipates that, in expectation, a fraction α_S of the \$1 new deposit will be transferred to the small bank, leading to a reduction of reserves by the same amount. The fraction α_L remains in the bank because the worker has an account with the same bank. The bank's balance sheet becomes:

Asset	Liability
Reserves $X_L - \alpha_S$	Deposits X_L
Loans 1	New Deposits α_L

If the large bank makes the \$1 loan to entrepreneur i, and charges interest rate R_i, its total expected profit, by counting all items in the balance sheet, will be

$$\underbrace{(X_L - \alpha_S)f}_{\text{Interest on reserves}} + \underbrace{[q_i(1 + R_i) - 1]}_{\text{Gross profit on the loan}} - \underbrace{(X_L + \alpha_L)r_L}_{\text{Cost of deposits}}. \tag{1}$$

If the large bank does not make the loan, then its total profit will be

$$X_L(f - r_L). \tag{2}$$

The large bank's marginal profit from making the loan, compared to not making it, is

$$\pi_i = \underbrace{q_i(1+R_i) - (1+f)}_{\text{Net profit on the loan}} + \underbrace{\alpha_L(f - r_L).}_{\text{Profit on deposit}} \tag{3}$$

In the expression of π_i, the net profit on the loan reflects the true opportunity cost of capital. Besides the usual profit on the loan, the large bank makes an additional profit equal to $\alpha_L(f - r_L)$. This is because each \$1 lent out stays with the large bank with probability α_L and earns the bank the IOR-deposit spread of $f - r_L$. The corresponding term for the small bank's marginal profit of lending is $\alpha_S(f - r_S)$. In the equilibrium we characterize, it will be the case that $\alpha_L(f - r_L) > \alpha_S(f - r_S)$, i.e., the large bank's convenience value of deposits translates into an advantage in the lending market. Such a feature would not be present if banks were homogeneous.

3.3 Equilibrium

We solve the model backward in time.

Deposit market at $t = 2$. A depositor with a large-bank convenience value of δ faces four choices:

	Large bank		Small bank	
	Deposit	CBDC	Deposit	CBDC
Convenience value	$\max(\delta, v)$	$\max(\delta, v)$	v	v
Interest rate	r_L	s	r_S	s

Obviously, the small bank attracts no depositors if $r_S < r_L$. So $r_S \geq r_L$ in equilibrium. For technical simplicity, whenever a depositor is indifferent between two choices, their preference is the small bank, the large bank, and finally the CBDC, in this order.[9]

[9] The tie-breaking rule between commercial banks and the CBDC is without loss of generality because a commercial bank can always offer ϵ above s so that depositors strictly prefer commercial bank deposits to the CBDC. The tie-breaking rule also preserves continuity in the fractions of depositors as parameters change to make depositors indifferent.

We will characterize parameter conditions under which $r_S > r_L$. This implies that a depositor with convenience value δ chooses the large bank if and only if

$$\delta > v \text{ and } r_L + \delta > r_S + v \Rightarrow \delta > r_S - r_L + v. \tag{4}$$

Therefore, the eventual market shares of the banks in the newly created deposits are

$$\alpha_L = 1 - G(r_S - r_L + v) \tag{5}$$

$$\alpha_S = G(r_S - r_L + v). \tag{6}$$

Loan market at $t = 1$. In the previous section we derived the marginal profit of a bank from making a loan. While the entrepreneur is infinitesimal here, expression (3) still applies.

The monopolist position of each bank in the lending market implies that a bank can make a take-it-or-leave-it offer to the entrepreneur. The bank's optimal interest rate quote would be $R_i = A - 1$ (or just tiny amount below), and the entrepreneur, who has no alternative source of funds, would accept. The lending bank takes the full surplus.

Hence, the large bank makes the loan if and only if

$$q_i A - (1 + f) + \alpha_L (f - r_L) > 0, \tag{7}$$

or

$$q_i > q_L^* = \frac{1 + f - \alpha_L (f - r_L)}{A}. \tag{8}$$

Exactly the same calculation for the small bank yields the comparable investment threshold

$$q_S^* = \frac{1 + f - \alpha_S (f - r_S)}{A}. \tag{9}$$

Choice of deposit rates at $t = 0$. Again, we start with the large bank. The large bank makes profits in two ways. Because the large bank is a monopolist when lending to its customers, its first source of profit is on the loans, $m_L \int_{q_l^*}(qA - 1 - f)dQ(q)$. The second source of the large bank's profit is on the interest rate spread. The existing deposit in the banking system is $X = X_L + X_S$. As discussed above, the lending process also creates new deposits. The amount of new deposit created by the large bank is $m_L(1 - Q(q_L^*))$, by the normalization that each loan is of \$1. Likewise, the small bank creates new deposit $m_S(1 - Q(q_S^*))$. When the two banks compete for depositors by setting the deposit rates r_L and r_S, we already show above that a fraction $\alpha_L = 1 - G(r_S - r_L + v)$ of total deposits end up with the large bank, enabling the large bank to collect a spread of $f - r_L$ per unit of deposit held.

Adding up the two components, we can write the large bank's total profit as

$$\begin{aligned}\Pi_L &= m_L \int_{q_L^*}^1 (qA - 1 - f)dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L) \\ &= m_L \int_{q_L^*}^1 [qA - (1 + f) + \alpha_L(f - r_L)]dQ(q) + [X_L + X_S + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L).\end{aligned} \tag{10}$$

Likewise, the small bank's total profit is

$$\Pi_S = m_S \int_{q_S^*}^1 [qA - (1 + f) + \alpha_S(f - r_S)]dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*))]\alpha_S(f - r_S). \tag{11}$$

As discussed before, the CBDC interest rate puts a lower bound on commercial banks' deposit rates, i.e., $r_L \geq s$, and $r_S \geq s$. There are two cases. The first is that $r_L > s$, so that the CBDC interest rate does not constrain the commercial banks' deposit rates. We call the first case the *unconstrained equilibrium.* The second case is that $r_L = s$, i.e., the CBDC interest rate binds the large bank's deposit rate. We call the second case the *constrained equilibrium.*

Unconstrained equilibrium. Assuming that Π_L is strictly quasi-concave in r_L, the sufficient condition for a unique maximum of the function Π_L with respect to r_L is

$$
\begin{aligned}
\frac{d\Pi_L}{dr_L} &= m_L(1-Q(q_L^*))\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_L \underbrace{[q_L^* A-(1+f)+\alpha_L(f-r_L)]}_{=0}\frac{dq_L^*}{dr_L} \\
&\quad + [X_L+X_S+m_S(1-Q(q_S^*))]\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{dq_S^*}{dr_L} \\
&= [X_L+X_S+m_L(1-Q(q_L^*))+m_S(1-Q(q_S^*))]\cdot[(f-r_L)G'(r_S-r_L+v)-1+G(r_S-r_L+v)] \\
&\quad - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{(f-r_S)G'(r_S-r_L+v)}{A}. \qquad (12)
\end{aligned}
$$

Likewise, the first-order condition of the small bank is

$$
\begin{aligned}
\frac{d\Pi_S}{dr_S} &= [X_L+X_S+m_L(1-Q(q_L^*))+m_S(1-Q(q_S^*))]\cdot[(f-r_S)G'(r_S-r_L+v)-G(r_S-r_L+v)] \\
&\quad - m_L\alpha_S(f-r_S)Q'(q_L^*)\frac{(f-r_L)G'(r_S-r_L+v)}{A}. \qquad (13)
\end{aligned}
$$

For simplicity, let $Q(\cdot)$ be the uniform distribution on $[0,1]$. And further impose a stationarity condition that the market shares of deposits $\{\alpha_j\}$ are identical to the starting market shares $\{m_j\}$. The first-order conditions simplify to

$$
\begin{aligned}
0 = \frac{d\Pi_L}{dr_L} &= [X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)]\cdot[(f-r_L)G'(r_S-r_L+v)-1+G(r_S-r_L+v)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f-r_L)(f-r_S)G'(r_S-r_L+v), \qquad (14) \\
0 = \frac{d\Pi_S}{dr_S} &= [X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)]\cdot[(f-r_S)G'(r_S-r_L+v)-G(r_S-r_L+v)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L+v). \qquad (15)
\end{aligned}
$$

From the above conditions we derive

$$(r_S-r_L)G'(r_S-r_L+v)+2G(r_S-r_L+v)=1. \qquad (16)$$

Proposition 1. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$ and that $G(v) < 0.5$. Let r_L and r_S solve equations (14)–(15). If $r_L > s$ and $r_S > s$, then it is an unconstrained equilibrium that the banks set r_L and r_S as their deposit rates. In this equilibrium:*

1. *The large bank sets a lower deposit rate ($r_L < r_S < f$) and has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

Proofs are in Appendix A.

The condition $G(v) < 0.5$ ensures that the CBDC does not increase the market share of the small bank so much that it fully eliminates the large bank's convenience value advantage in its deposits. Consequently, the small bank still needs to compete by offering a higher deposit rate than the large bank.

Further intuition of the equilibrium may be gained by considering an example. Suppose that $G(\delta) = \delta/\Delta$, where $\delta \in [0, \Delta]$ for a sufficiently large Δ. Then $G'(\cdot) = 1/\Delta$. The two first-order conditions reduce to

$$\frac{f - r_L}{\Delta} = 1 - \frac{r_S - r_L + v}{\Delta} + B \tag{17}$$

$$\frac{f - r_S}{\Delta} = \frac{r_S - r_L + v}{\Delta} + B \tag{18}$$

where

$$B \equiv \frac{\frac{1}{A\Delta}\alpha_L \alpha_S (f - r_L)(f - r_S)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{19}$$

As the total reserve X becomes large, B becomes close to zero. So the equilibrium deposit rates of the two banks become approximately $r_L \approx f - \frac{2}{3}\Delta + \frac{1}{3}v$ and $r_S \approx f - \frac{1}{3}\Delta - \frac{1}{3}v$. This shows directly how an increase in convenience reduces the spread between deposit rates.

Constrained equilibrium. The second case of the equilibrium is that the CBDC interest rate s becomes binding for the large bank. Recall the tie-breaking rule that at $r_L = s$, depositors use the large bank.

The small bank's profit function and first-order condition are as before:

$$0 = \frac{d\Pi_S}{dr_S} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)] - \frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v). \tag{20}$$

By contrast, the large bank's first order condition takes an inequality because the conjectured optimal solution is at the left corner:

$$0 > \frac{d\Pi_L}{dr_L}\Big|_{r_L \downarrow s} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v)] - \frac{1}{A}\alpha_S\alpha_L(f - s)(f - r_S)G'(r_S - s + v). \tag{21}$$

Proposition 2. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$. Suppose that $v < \bar{v}$ for some $\bar{v}$ that may depend on f and s. Let r_S solve equation (20). If, at r_S, equation (21) also holds, then it is a constrained equilibrium that the large bank sets s and the small bank sets r_S as their deposit rates. In this equilibrium:*

1. *The large bank sets a lower deposit rate ($s < r_S$) and has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

The condition that v cannot be too high guarantees that the small bank still wishes to compete by offering a higher deposit rate. This is analogous to the restriction on $G(v)$ in Proposition 1.

While the model described in this section is stylized, it potentially explains an important fact about the U.S. deposit market: deposit rates are below and only partially responsive to the key policy rate (Federal Funds rate and IOR) set by the central bank. We discuss in Appendix B how our model, under a certain parameterization and without the laborious calibration to the data, already generates predicted deposit rates that are largely similar to actual U.S. deposit rates from 1986 to 2021. We contend that this conformance provides essential support for the validity of the model's predictions regarding the introduction of a CBDC.

Parlour, Rajan, and Walden (2022) also analyze asymmetries in the banking sector and their consequences. In their model, a bank that is a net payer incurs an additional settlement cost and hence reduces lending, compared to net-receiving bank. In this sense, the net payer bank in their model looks like the small bank in ours. Despite similar predictions on lending, the two models are driven by different mechanisms. In our model, there is no exogenous cost associated with interbank settlement; rather, the main advantage of the large bank in lending is a higher likelihood that a lent dollar stays with the large bank and earns interest on reserves from the central bank. Moreover, the size of the large bank's advantage depends on the interest rate paid on reserves and the CBDC design, including its interest rate and convenience value, as we see in the next section.

Using confidential FedWire transaction data, Li and Li (2021) calculate the volatility of daily net payments as a fraction of daily gross payments for various banks. They find that banks with higher payment volatility pay a higher deposit rate and have lower loan volume growth, controlling for a set of observables. While our model does not have payment volatility, our predictions are consistent with the negative cross-sectional correlation they compute between the deposit rate and lending.

4 Impact of CBDC Interest Rate and Convenience Value

In this section, we discuss the consequences of varying the CBDC interest rate s or convenience value v. The results are summarized in Propositions 3 and 4.

4.1 Impact of CBDC interest rate s

When the federal reserve introduced the overnight reverse repo program (ONRRP) as a temporary facility to support its IOR policy, it began by testing the facility by varying the ONRRP rate between 1 basis point and 10 basis points, while holding the IOR rate fixed at 25 basis points. Here we examine how market outcomes change as s varies from a rate of 0 to f, while holding f fixed.

We focus on the case where, given a fixed value of v, f is sufficiently low that the constrained equilibrium applies. This case is most relevant to the current economic environment in the United States. In the unconstrained equilibrium, market outcomes are invariant to the CBDC interest rate s by definition.

Before we provide a formal statement of the comparative statics, it is useful to illustrate the impact of CBDC interest rate changes in an example. The top row of Figure 1 plots the behavior in the deposit markets as the CBDC interest rate rises from 0 to $f = 2\%$. The charts are computed numerically using a uniform distribution for G and a zero CBDC convenience value ($v = 0$). As we see in the top left plot, raising the CBDC interest rate increases the deposit rates of both banks as well as the weighted average deposit rates. The top right plot shows the corresponding changes in deposit market shares α_j, $j = L, S$, which are easily computed from (5) and (6). Since the large bank's deposit rate rises faster than the small bank's, the large bank gains market share from the small bank. Intuitively, the small bank competes with the large bank primarily by offering a higher deposit rate. As s increases, the maximum spread $f - s$ shrinks, limiting the small bank's ability to compete with its interest rate choice. Once the deposit rates are equal at f, the large bank obtains the entire market share of depositors, given the higher convenience value of its deposits.

The bottom row of Figure 1 illustrates the impact raising the CBDC interest rate has on the lending market. Raising the CBDC interest rate changes the incentives to make loans via the expected profit on the interest rate spread, $\alpha_j(f - r_j)$. Because, as shown above, both α_S and $f - r_S$ decrease in s, so does $\alpha_S(f - r_S)$. Thus, the small bank's loan quality threshold, $q_S^* = \frac{1+f-\alpha_S(f-r_S)}{A}$, increases in s, and its loan volume, $\alpha_S(1-q_S^*)$, decreases in s. In this example, the large bank's loan quality threshold, $q_L^* = \frac{1+f-\alpha_L(f-r_L)}{A}$, increases in s, and its loan volume $\alpha_L(1-q_L^*)$, also increases due to its larger market share. In this example, the total loan volume declines in s.

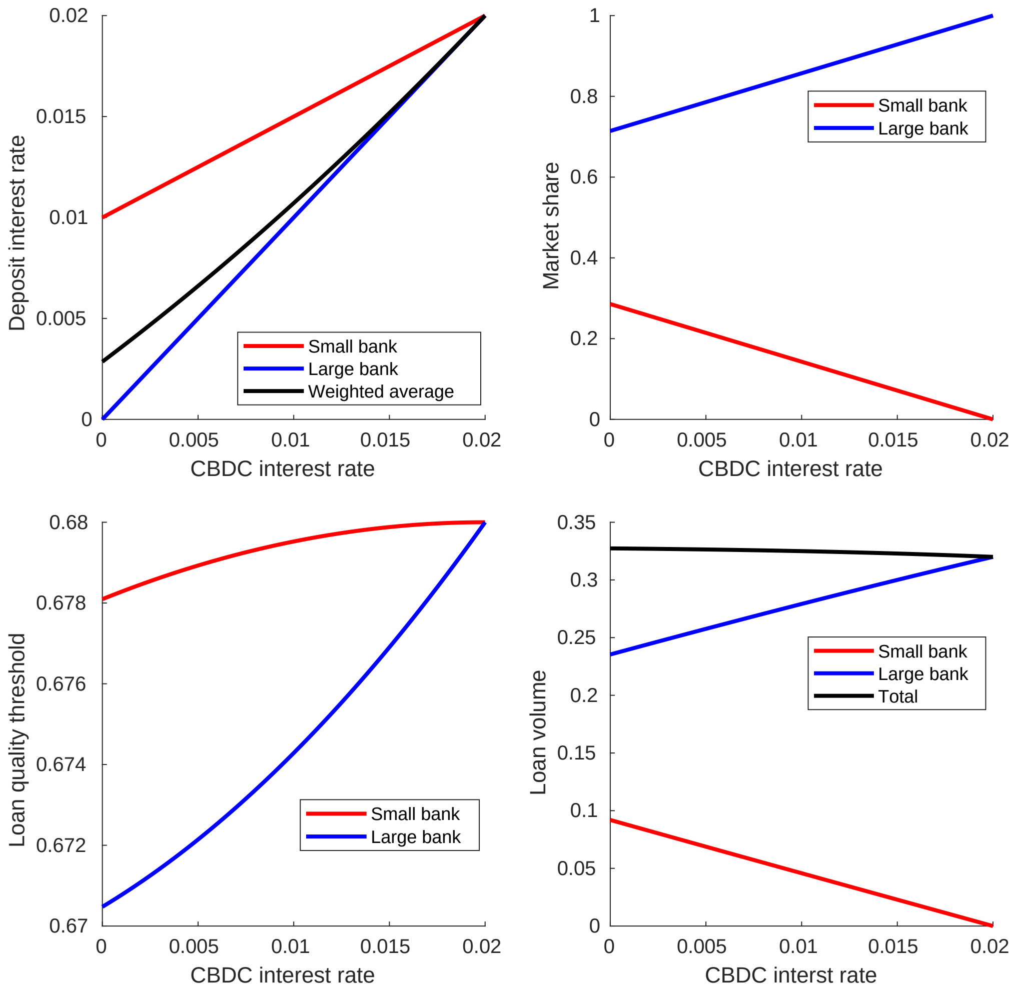

Figure 1: Impact of CBDC interest rate on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, v = 0$.

The following proposition characterizes the impact of CBDC interest rate s on the deposit and lending markets in more general cases.

Proposition 3. *For a sufficiently large X, increasing the CBDC interest rate in a constrained equilibrium has the following impact on the deposit and lending markets:*

	As s increases	
	Large	*Small*
Deposit rates r_L *and* r_S	$\uparrow$	$\uparrow$
Deposit market shares α_L *and* α_S	$\uparrow$	$\downarrow$
Weighted average deposit rate	$\uparrow$	
Loan quality thresholds q_L^* *and* q_S^*	$\uparrow$	$\uparrow$
Loan volume $\alpha_L(1-Q(q_L^*))$ *and* $\alpha_S(1-Q(q_S^*))$	$\uparrow$ *or* $\downarrow$	$\downarrow$
Total loan volume, i.e., total deposit created	*?* ($\downarrow$ *if* $G'' \leq 0$)	

Most of the qualitative aspects illustrated in Figure 1 are true generally, and are analytically proven in Proposition 3. The exceptions are that, in general, the large bank's loan volume may go up or down in s and when $G'' > 0$, we do not know what will happen to total loan volume.[10]

4.2 Impact of CBDC convenience value v

A convenient CBDC reduces the large bank's convenience advantage and hence has an impact even if its interest rate is zero. We illustrate the impact of a convenient CBDC by considering this polar case in Figure 2. The top row shows the outcomes in the deposit market. As v rises, the inconvenience disadvantage of the small bank shrinks. As long as the large bank's deposit rate remains at the floor rate, the small bank can afford to lower its interest rate and still capture a growing market share. Once v gets large enough, the large bank responds by raising its interest rate; however, the small bank can still afford to continue lowering its deposit rate for the same reason that the convenience gap between the two banks continues to shrink. Throughout this process the large bank loses market share and the small bank gains market share, albeit at a slower rate once the large bank is no longer constrained. The

[10] An example illustrating the ambiguity in large bank loans volumes is seen by setting $G(\delta) = \delta/0.035, A = 1.05, X = 10, f = 0.02, v = 0$. Then, in the constrained equilibrium, the large bank's loan volume first increases and then decreases with s.

overall impact of increasing the CBDC convenience value is the convergence of the deposit rates and market shares for the two banks.[11]

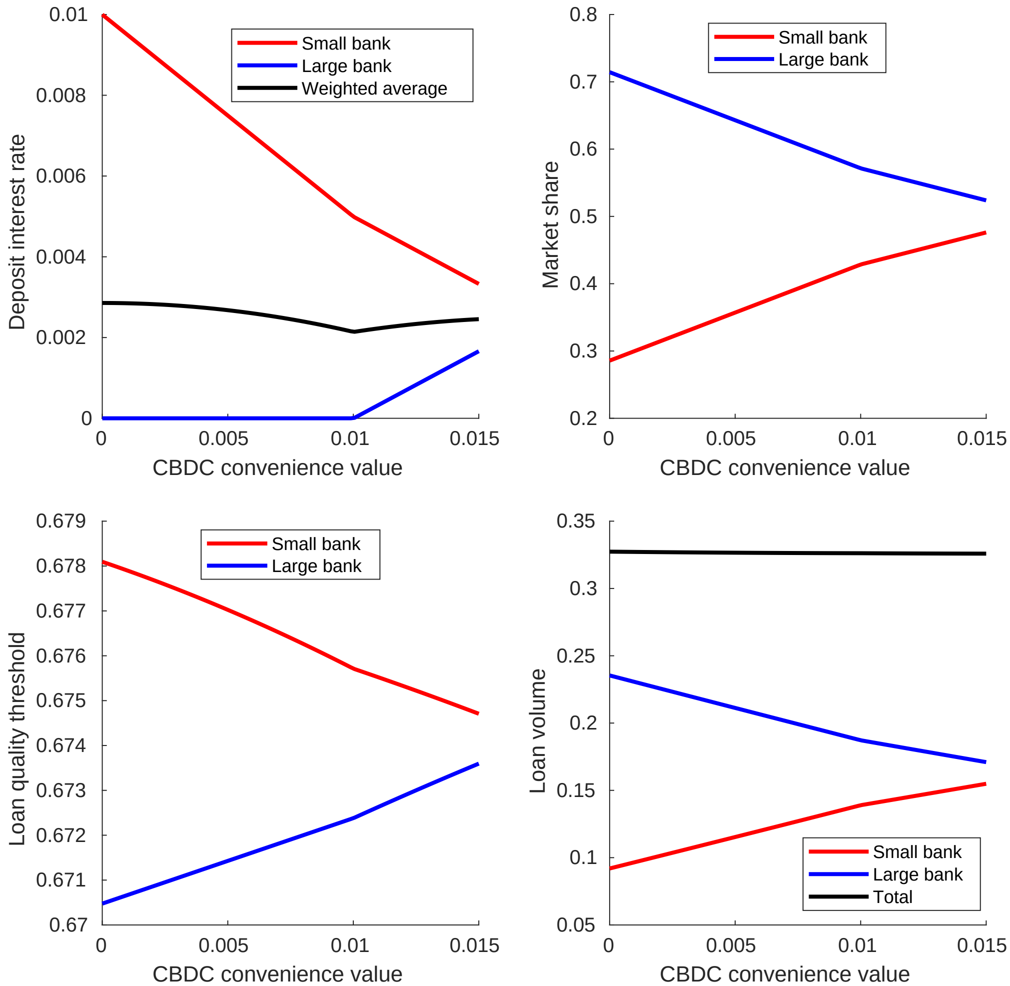

Figure 2: Impact of CBDC convenience value on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, s = 0$. The equilibrium transitions from constrained to unconstrained at $v = 0.01$.

The CBDC convenience value has a nuanced impact on the weighted average deposit rates. In a constrained equilibrium, a higher v results in a lower weighted average deposit rate when the large bank's deposit rate is at the lower bound. That is, a convenient CBDC weakens the transmission of monetary policy to the deposit market through IOR. Once the

[11] In fact, a modest CBDC convenience value may be enough to fully level the playing field. Under the uniform distribution of large-bank preference δ, when v rises to the point $v = \Delta/2$, depositors with $\delta > \Delta/2$ strictly prefer the large bank, and depositors with $\delta < \Delta/2$ strictly prefer the small bank. That is, the deposit market shares become equal and so do the deposit rates, loan quality thresholds, and loan volume.

economy transitions to an unconstrained equilibrium with a sufficiently high v, however, a higher CBDC convenience value increases the average deposit rate, increasing the transmission of monetary policy.

The bottom row of Figure 2 shows the outcomes in the lending market. Because the two deposit rates and the deposit market shares get closer to each other as v rises, it is unsurprising that the loan quality thresholds and loan volume of the two banks are also getting closer to each other. In this example, the total loan volume is almost invariant to v, and the most salient effect is the reallocation of loans from the large bank to the small one.

Figure 3 below further demonstrates the potential impact of CBDC convenience value on loan volume, using a different parametrization of $f = 3\%$ and $s = 1.25\%$. These parameters lead to a constrained equilibrium, with $r_L = s$. In this example, the total lending volume (left axis) is first decreasing in v and then increasing in v. The magnitude of the axes suggests that the more salient action is, again, the shift of lending from the large bank to the small one.

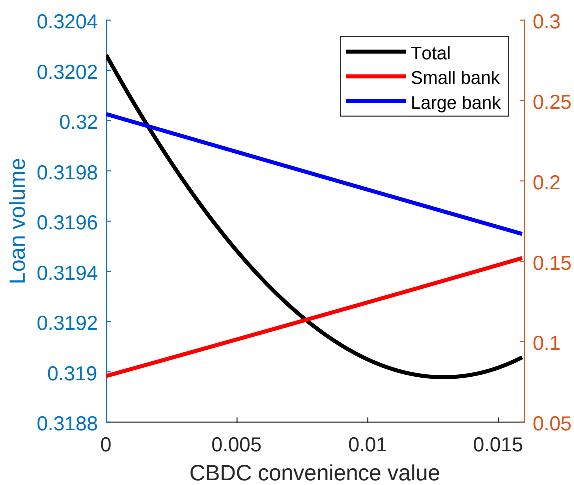

Figure 3: CBDC convenience value and loan volume. Total loan volume in on left axis. Loan volume of the two banks are on right axis. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.03, s = 0.0125$. All values of v in the depicted range correspond to a constrained equilibrium.

The next proposition summarizes the comparative statics with respect to v.

Proposition 4. *For a sufficiently large X, the impact of increasing v is given in the following table:*

As v increases	*Constrained*		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Deposit rates r_L and r_S	*Flat(=s)*	$\downarrow$	$\uparrow$	$\downarrow$
Deposit market shares α_L and α_S	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Weighted average deposit rate	$\uparrow$ *or* $\downarrow$ ($\downarrow$ *if* $G'' \leq 0$)		$\uparrow$ *or* $\downarrow$ ($\uparrow$ *if* $G'' \geq 0$)	
Loan quality thresholds q_L^* and q_S^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Loan volume $\alpha_L(1-q_L^*)$ and $\alpha_S(1-q_S^*)$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Total loan volume, i.e., total deposit created	$\uparrow$ *or* $\downarrow$		$\uparrow$ *or* $\downarrow$ ($\downarrow$ *if* $G'' \geq 0$)	

As in the previous subsection, most of the comparative static results that apply for uniform G are true more generally and are stated in Proposition 4. There are a few exceptions. The impact on the weighted average interest rate, $\alpha_S r_S + \alpha_L r_L$, is ambiguous. An increase in v shifts market share to the small bank and reduces the small bank's deposit rate, but the small bank has a higher deposit rate to start with, so the overall effect can go in either direction. In the constrained equilibrium, a concave G means that relatively more depositors have a weak (but still positive) preference for large bank's deposits, so a higher v quickly eliminates the large bank's advantage. As a result, the small bank can afford to reduce its deposit rate quickly, leading to a lower weighted average $\alpha_S r_S + \alpha_L r_L$.[12] In the unconstrained equilibrium, r_L increases in v. A convex G means that relatively more depositors have a strong (but still positive) preference for large bank's deposits, so the large bank raises r_L aggressively compared to the reduction in r_S, leading to a higher $\alpha_S r_S + \alpha_L r_L$.[13] The ambiguity in total loan volume for the constrained case is illustrated by the numerical example in Figure 3 where G is uniform and total loan volume first decreases and then increases in

[12]When $A = 1.5, X = 10, f = 0.02, s = 0, G = Gamma(5, 150)$ with mean $1/30$, then $0 < G''(r_S - r_L + v) < G'(r_S - r_L + v)/f$, yet weighted average deposit rate first increases then decreases in v, in the constrained equilibrium.

[13]When $A = 1.5, X = 10, f = 0.02, s = 0, G = Gamma(3, 200)$ with mean $3/200$, then $-G'(r_S - r_L + v)/f < G''(r_S - r_L + v) < 0$, yet weighted average deposit rate first increases then decreases in v, total loan volume first decreases then increases in v, in the unconstrained equilibrium.

v. Total loan volume unambiguously decreases in v in the unconstrained equilibrium if G is weakly convex. Intuitively, the weighted average deposit rate increases in v, so the IOR-deposit rate spread is compressed, which discourages lending. When G is strictly concave, however, the change in total loan volume resulting from an increase in v can be in either direction.

5 Sensitivity

We now address the issue of how a given CBDC design impacts the sensitivity of deposit rates to changes in the IOR rate f. In the unconstrained equilibrium, deposit rates of both the large and the small bank move one-for-one with the IOR rate f. This means the pass-through of f is perfect when the large bank is not constrained, that is, when the large bank competes with the small bank on the deposit rate margin. In the constrained equilibrium, the large bank's deposit rate is capped at CBDC interest rate s, and only the small bank's deposit rate reacts to changes in f, hence the pass-through of f to the average deposit rate is much weaker.

When the IOR rate f is low, deposit and lending markets are characterized by the constrained equilibrium and when f is high they enter into the unconstrained equilibrium. Let f^* denote the threshold value of the economy transitions from the constrained equilibrium to the unconstrained equilibrium. Since the pass-through of f is vastly different between the constrained equilibrium and the unconstrained equilibrium, it is important to understand how the CBDC interest rate s and the convenience value v affect the cut-off value of IOR,

f^*, that separates the two equilibria. We solve for f^*, from the following FOCs.

$$\begin{aligned}
0 &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f^* - r_S)G'(r_S - s + v) - G(r_S - s + v)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f^* - s)(f^* - r_S)G'(r_S - s + v). \\
0 &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f^* - s)G'(r_S - s + v) - 1 + G(r_S - s + v)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f^* - s)(f^* - r_S)G'(r_S - s + v). \qquad (22)
\end{aligned}$$

The following proposition summarizes the results.

Proposition 5. *For a sufficiently large X:*

1. *In the unconstrained equilibrium, r_L and r_S move one-for-one with f;*

2. *In the constrained equilibrium, $\frac{dr_S}{df}$ decreases with s and increases with v if $\frac{G''(\delta)}{G'(\delta)}$ is increasing in δ; and*

3. *f^* increases with s. f^* decreases with v if $G''(\delta) \geq 0$.*

With abundant reserves, the spread between IOR and the deposit rate is the main factor that determines profits for the banks. In the unconstrained equilibrium, the two banks compete primarily for deposit market shares, and the market shares do not vary with f. As a result, r_L and r_S move one-for-one with f.

Part 2 of Proposition 5 requires that $G''(\delta)/G'(\delta)$ is an increasing function as a sufficient condition.[14] Such distributions have decreasing probability density functions and large mass at small values. Under these assumptions, in the constrained equilibrium, higher s or lower v decreases the sensitivity of the small bank's deposit rate r_S to changes in f. This occurs under the stated convexity condition because when s is higher, or v is lower, the convenience value for the large bank of the marginal consumer that is indifferent between choosing the small bank and the large bank takes a lower value where the convexity of G is lower. This

[14] This condition is satisfied by Gamma distributions with a shape parameter less than 1. The proposition is also true for exponential distributions, where $G''(\delta)/G'(\delta)$ is a non-zero constant.

is where r_S needs to react less to offset the change introduced by f. Since r_S becomes less sensitive to f as s increases or v decreases, pass-through is decreased.

A higher s increases the cutoff value f^*, which means the Fed needs to set a higher IOR to enter into the high pass-through region. Intuitively, there needs to be a large spread between IOR and s in order to induce the large bank to compete with the small banks via its deposit rate policy. A higher s necessarily increases f^*. A higher v reduces the large bank's competitive advantage, and forces it to compete with the small bank sooner; that is, a higher v decreases the cutoff value f^*. The sufficient condition for this is that G is convex. Convex G means relatively more depositors have a strong preference for the large bank's deposits, so the large bank competes sooner on the deposit rate margin to compensate for the reduction in the convenience advantage.

6 Concluding Remarks

Payment convenience is a crucial aspect of CBDC design that may be more desirable than interest rate policy. A highly convenient CBDC produces sufficient competitive pressure in deposit markets to raise deposit rates for any given level of IOR and increases the responsiveness of deposit rates to IOR rate changes. Convenience also has favorable effects on market composition by leveling the playing field. Interest rate policy is less desirable in the sense that it may weaken the responsiveness of deposit rates to IOR rate changes and it increases the inequality of market shares.

An interesting aspect of our analysis is that the provision of CBDC impacts equilibrium outcomes even though the currency is not held in equilibrium. Hence there is no disintermediation. This is also true in Chiu et al. (2019) and Garratt and Lee (2021), where the option to use CBDC changes the equilibrium outcome even it is not exercised. An exception is Keister and Sanches (2021), where the CBDC has specific liquidity benefits that leads to its use. The idea that a central bank introduces a program to influence market rates by increasing the bargaining power of lenders is not new. Early descriptions of the overnight reverse repurchase agreement facility that the Federal Reserve Bank of New York began testing in September 2013 indicated that "the option to invest in ON RRPs [Overnight Reverse Repurchase Agreement Facility] also would provide bargaining power to investors in their negotiations with borrowers in money markets, so even if actual ON RRP take-up is not very large, such a facility would help provide a floor on short-term interest rates..." (Frost et al., 2015).

The results of our paper could be extended in multiple directions. One possible extension is to add short-term investment vehicles such as money market mutual funds and repurchase agreements that typically pay higher interest rates than bank deposits but cannot be easily used for processing payments. If the CBDC pays a sufficiently high interest rate, it is possible that money would flow out of these short-term investment vehicles into the CBDC, i.e., investors would earn returns from the Fed rather than short-term Treasury Bills. This additional channel is unlikely to affect lending because money market investors do not make loans. Another possibility is to consider heterogeneous CBDC interest rates paid to banks of different sizes, which adds yet another degree of freedom in the central bank's toolkit. In particular, the central bank could use heterogenous CBDC interest rates to fine-tune the competitive positions of large and small banks. These extensions are left for future research.

Appendix A: Proofs

Proof of Proposition 1

Taking the difference of the two FOCs, we have $(r_S - r_L)G'(r_S - r_L + v) = 1 - 2G(r_S - r_L + v)$. If $r_S \leq r_L$, then the left-hand side is non-positive but the right-hand side is $1 - 2G(r_S - r_L + v) \geq 1 - 2G(v) > 0$, a contradiction. So $r_S > r_L$. This implies that $G(r_S - r_L + v) < 0.5$ in equilibrium, i.e., $\alpha_L > \alpha_S$.

Let

$$B \equiv \frac{\frac{1}{A}\alpha_L \alpha_S (f - r_L)(f - r_S) G'(r_S - r_L + v)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{23}$$

The two FOCs are separately written as

$$(f - r_L) G'(r_S - r_L + v) = \alpha_L + B, \tag{24}$$

$$(f - r_S) G'(r_S - r_L + v) = \alpha_S + B. \tag{25}$$

So both r_L and r_S are below f. Take the ratio:

$$\frac{f - r_L}{f - r_S} = \frac{\alpha_L + B}{\alpha_S + B} > 1 > \frac{\alpha_S}{\alpha_L}. \tag{26}$$

Hence, $(f - r_L)\alpha_L > (f - r_S)\alpha_S$, and $q_L^* < q_S^*$.

Proof of Proposition 2

By the assumption that the function Π_S is well behaved to admit a unique global maximum, the derivative $d\Pi_S/dr_S$ should be strictly decreasing in r_S. To show that $r_S > s$, it is sufficient that the right-hand side of (20) is positive at $r_S = s$, i.e.,

$$[X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)][(f - s)G'(v) - G(v)] - \frac{1}{A} G(v)(1 - G(v))(f - s)^2 G'(v) > 0. \tag{27}$$

Clearly, the above equation holds at $v = 0$. By continuity, it also holds if v is below a cutoff, say $\bar{v}$. If $v \in [0, \bar{v})$, we have $r_S > s = r_L$.

Let $B \equiv \frac{\frac{1}{A}\alpha_L \alpha_S (f - s)(f - r_S) G'(r_S - s + v)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0$. The two FOCs are separately written as

$$(f - s) G'(r_S - s + v) < \alpha_L + B, \tag{28}$$

$$(f - r_S) G'(r_S - s + v) = \alpha_S + B. \tag{29}$$

Since $B > 0$, $G'(r_S - s + v) > 0$, we know $r_S < f$. Take the difference, we have $0 <$

$(r_S - s)G'(r_S - s + v) < \alpha_L - \alpha_S$. That is, $\alpha_L > \alpha_S$. It follows that $(f - s)\alpha_L > (f - r_S)\alpha_S$ and $q_L^* < q_S^*$.

Proof of Proposition 3

Since the large bank is constrained by the lower bound, its deposit rate rises in step with the CBDC interest rate s. Meanwhile, the small bank adjusts its equilibrium deposit rate at a slower pace, continuing to balance its ability to maintain depositors while its profit margin shrinks. To see how r_S is affected by s, let $\Gamma_S = d\Pi_S/dr_S$, $\Gamma_L = d\Pi_L/dr_L$, and start with the expression

$$0 = \frac{\partial \Gamma_S}{\partial s} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{ds}. \tag{30}$$

Because $\partial\Gamma_S/\partial r_S < 0$, a sufficient condition for $dr_S/ds > 0$ is $\partial\Gamma_S/\partial s > 0$. Writing total loan volume as $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$, we have

$$\begin{aligned}\frac{\partial \Gamma_S}{\partial s} = &\left(-\alpha_L\frac{\alpha_L}{A}\right)[(f - r_S)G'(r_S - s + v) - G(r_S - s + v)]\\ &+ (X + V)[-(f - r_S)G''(r_S - s + v) + G'(r_S - s + v)]\\ &+ \frac{1}{A}(f - r_S)\frac{\partial}{\partial s}[\alpha_L\alpha_S(f - s)G'(r_S - s + v)].\end{aligned} \tag{31}$$

On any closed region of f and s, the first and third term are bounded, by G being twice-differentiable. So if X is sufficiently large, the second term dominates. Under the assumption that $G''(\delta) < G'(\delta)/f$ for $\delta \in [0, f - s + v]$, we have $-(f - r_S)G''(r_S - s + v) + G'(r_S - s + v) > 0$, so a sufficiently large X would imply that $\partial\Gamma_S/\partial s > 0$, and so is dr_S/ds.

Next, we show that $r_S - s$ decreases in s. We have

$$\begin{aligned}\frac{\partial \Gamma_S}{\partial r_S} = &X\frac{\partial}{\partial r_S}[(f - r_S)G'(r_S - s + v) - G(r_S - s + v)]\\ &+ \frac{\partial}{\partial r_S}\{[\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)]\}\\ &- \frac{\partial}{\partial r_S}\left[\frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v)\right].\end{aligned} \tag{32}$$

The second and the third term are bounded on any closed region of r_S. The first term equals $X[(f-r_S)G''(r_S-s+v)-2G(r_S-s+v)]$. Hence,

$$\frac{dr_S}{ds}=-\frac{\partial\Gamma_S/\partial s}{\partial\Gamma_S/\partial r_S}\to\frac{(f-r_S)G''(r_S-s+v)-G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}, \tag{33}$$

as X becomes sufficiently large. We also have $d(r_S-s)/ds=dr_S/ds-1=\frac{G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}$, whose denomintor is negative under the condition that $G''(\delta)<G'(\delta)/f$ Hence, $d(r_S-s)/ds<0$. This implies that $\alpha_S=G(r_S-s+v)$ is decreasing in s, and α_L is increasing in s.

The weighted average interest rate is $\alpha_S r_S+\alpha_L s$. Its derivative with respect to s is

$$\frac{d(\alpha_S r_S+\alpha_L s)}{ds}=\frac{d\alpha_S}{ds}r_S+\alpha_S\frac{dr_S}{ds}+\frac{d\alpha_L}{ds}s+\alpha_L=[(r_S-s)G'(r_S-s+v)+\alpha_S]\left(\frac{dr_S}{ds}-1\right)+1. \tag{34}$$

By the calculation earlier, as X becomes large, $\frac{dr_S}{ds}-1\to\frac{G'(r_S-s)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}>-\frac{f}{f+r_S}$, where the inequality follows from $G''(\delta)<G'(\delta)/f$ for any $\delta\in[0,f-s+v]$. So, as X becomes large,

$$\frac{d(\alpha_S r_S+\alpha_L s)}{ds}>1-\frac{f}{f+r_S}[(r_S-s)G'(r_S-s+v)+\alpha_S]\geq 1-\frac{f}{f+r_S}>0, \tag{35}$$

where the second last inequality follows from the large bank's FOC that, $\lim_{X\to\infty}(f-s)G'(r_S-s+v)+G(r_S-s+v)\leq 1$.

Now we turn to loan market outcomes. Since α_S decreases in s and r_S increases in s, $\alpha_S(f-r_S)$ is decreasing in s and q_S^* is increasing in s. The small bank's loan volume, $\alpha_S(1-q_S^*)$, is then decreasing in s.

For the large bank's loan quality q_L^*, we have

$$\frac{dq_L^*}{ds}=-\frac{1}{A}\Big[G'(r_S-s+v)(1-\frac{dr_S}{ds})(f-s)-1+G(r_S-s+v)\Big]. \tag{36}$$

For the first term in the brackets, we know that $G'(r_S-s+v)(1-\frac{dr_S}{ds})(f-s)<(f-$

$s)G'(r_S - s + v)$, since $dr_S/ds > 0$. Also, from the large bank's optimality condition, as X is sufficiently large, we know that $(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v) \leq 0$. That means $dq_L^*/ds > 0$ and q_L^* is increasing in s. However, the impact of s on the large bank's loan volume $\alpha_L(1 - q_L^*)$ is ambiguous.

The total loan volume is $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$. Its derivative with respect to s is

$$\frac{1}{A}[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]G'(r_S - s + v)\left(\frac{dr_S}{ds} - 1\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds}. \tag{37}$$

While the first term is positive, the last two terms are negative. It is, however, possible to show that this derivative is negative if $G''(\delta) \leq 0$ and X is sufficiently large. As X becomes large, the two first-order conditions imply that

$$\begin{aligned} \lim_{X\to\infty}(f - s)G'(r_S - s + v) - \underbrace{(1 - G(r_S - s + v))}_{\alpha_L} &\leq 0, \\ \lim_{X\to\infty}(f - r_S)G'(r_S - s + v) - \underbrace{G(r_S - s + v)}_{\alpha_S} &= 0. \end{aligned} \tag{38}$$

Multiplying α_L to the first equation and α_S to the second equation, we have

$$\begin{aligned} \lim_{X\to\infty}\alpha_L(f - s)G'(r_S - s + v) - \alpha_L^2 &\leq 0, \\ \lim_{X\to\infty}\alpha_S(f - r_S)G'(r_S - s + v) - \alpha_S^2 &= 0. \end{aligned} \tag{39}$$

Plugging these in Equation (37), we have, as X becomes large,

$$\begin{aligned} \lim_{X\to\infty}\frac{d}{ds}(\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)) &\leq \frac{1}{A}(2\alpha_L^2 - 2\alpha_S^2)\left(1 - \frac{dr_S}{ds}\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds} \\ &= \frac{1}{A}\left[\left(1 - 2\frac{dr_S}{ds}\right)\alpha_L^2 + \left(\frac{dr_S}{ds} - 2\right)\alpha_S^2\right], \end{aligned} \tag{40}$$

Because $\frac{dr_S}{ds} < 1$, $(\frac{dr_S}{ds} - 2)\alpha_S^2 < 0$. If $G''(\delta) \leq 0$ and X is sufficiently large, we know from the expression of $\frac{dr_S}{ds}$ above that $\frac{dr_S}{ds} \geq \frac{1}{2}$. That means $(1 - 2\frac{dr_S}{ds})\alpha_L^2 \leq 0$ as well. So the total loan is decreasing in s in the limit. Because the limit is strictly negative, it is also negative for finite but large enough X.

Proof of Proposition 4

First we consider the unconstrained equilibrium and then the constrained one.

The unconstrained equilibrium

We know that $r_L < r_S < f$, and $\alpha_S < \frac{1}{2} < \alpha_L$. Let $\Gamma_S = d\Pi_S/dr_S$, $\Gamma_L = d\Pi_L/dr_L$. To calculate how r_L and r_S are affected by v, we take derivative of Γ_L and Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_L}{\partial v} + \frac{\partial \Gamma_L}{\partial r_L}\frac{dr_L}{dv} + \frac{\partial \Gamma_L}{\partial r_S}\frac{dr_S}{dv}, \quad (41)$$

$$0 = \frac{\partial \Gamma_S}{\partial v} + \frac{\partial \Gamma_S}{\partial r_L}\frac{dr_L}{dv} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \quad (42)$$

We solve for $\frac{dr_L}{dv}$ and $\frac{dr_S}{dv}$ from above equations. Denote $A_v = \frac{\partial \Gamma_L}{\partial v}$, $A_L = \frac{\partial \Gamma_L}{\partial r_L}$, $A_S = \frac{\partial \Gamma_L}{\partial r_S}$, $B_v = \frac{\partial \Gamma_S}{\partial v}$, $B_L = \frac{\partial \Gamma_S}{\partial r_L}$, and $B_S = \frac{\partial \Gamma_S}{\partial r_S}$. Then we have,

$$\frac{dr_L}{dv} = \frac{A_S B_v - B_S A_v}{A_L B_S - B_L A_S} \quad (43)$$

$$\frac{dr_S}{dv} = \frac{B_L A_v - A_L B_v}{A_L B_S - B_L A_S} \quad (44)$$

When X is sufficiently large, A_v is dominated by $X[(f - r_L)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$, so $A_v \approx X[(f - r_L)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$. Similarly, $B_v \approx X[(f - r_S)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$, $A_L \approx X[-(f - r_L)G''(r_S - r_L + v) - 2G'(r_S - r_L + v)]$, $A_S \approx X[(f - r_L)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$, $B_L \approx X[-(f - r_S)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$, and $B_S \approx X[(f - r_S)G''(r_S - r_L + v) - 2G'(r_S - r_L + v)]$. Hence, $\frac{dr_L}{dv} \to \frac{(f-r_L)G''(r_S-r_L+v)+G'(r_S-r_L+v)}{(r_S-r_L)G''(r_S-r_L+v)+3G'(r_S-r_L+v)}$, and $\frac{dr_S}{dv} \to \frac{(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)}{(r_S-r_L)G''(r_S-r_L+v)+3G'(r_S-r_L+v)}$. Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, $(r_S - r_L)G''(r_S - r_L + v) + 3G'(r_S - r_L + v)$ is positive, and $(f - r_L)G''(r_S -$

$r_L+v)+G'(r_S-r_L+v)$ is positive, so $\frac{dr_L}{dv}>0$. Also, $(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)$ is negative, so $\frac{dr_S}{dv}<0$. So r_L is increasing and r_S is decreasing in v.

For deposit market share $\alpha_S = G(r_S - r_L + v)$, we take the difference of the two FOCs, and have

$$(r_S - r_L)G'(r_S - r_L + v) + 2G(r_S - r_L + v) = 1. \tag{45}$$

Write $y = r_S - r_L + v$, and take derivative of the above equation with respect to v, then we have

$$[3G'(y) + (r_S - r_L)G''(y)]\frac{dy}{dv} - G'(y) = 0 \tag{46}$$

Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, we know that $3G'(y)+(r_S-r_L)G''(y) > 0$, hence $\frac{dy}{dv} > 0$. So α_S is increasing in v, and α_L is decreasing in v.

The weighted average deposit rate is $\alpha_S r_S + \alpha_L r_L = \alpha_S(r_S - r_L) + r_L$. Its derivative with respect to v is

$$\begin{aligned}\frac{d(\alpha_S r_S + \alpha_L r_L)}{dv} &= \frac{d\alpha_S}{dv}(r_S - r_L) + \alpha_S\frac{d(r_S - r_L)}{dv} + \frac{dr_L}{dv} \\ &> \frac{d\alpha_S}{dv}(r_S - r_L) + \frac{1}{2}\frac{d(r_S - r_L)}{dv} + \frac{dr_L}{dv} = \underbrace{\frac{d\alpha_S}{dv}}_{>0}(r_S - r_L) + \frac{1}{2}\frac{d(r_L + r_S)}{dv},\end{aligned} \tag{47}$$

where the inequality follows from $\alpha_S < \frac{1}{2}$ and $r_S - r_L$ decreasing in v. As X becomes large,

$$\frac{d(r_S + r_L)}{dv} \to \frac{(2f - r_L - r_S)G''(r_S - r_L + v)}{(r_S - r_L)G''(r_S - r_L + v) + 3G'(r_S - r_L + v)} \tag{48}$$

The denominator is positive as $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$. As $r_L + r_S < 2f$, when $G''(\delta) \geq 0$, we have $\frac{d(r_L+r_S)}{dv} \geq 0$, and hence $\alpha_S r_S + \alpha_L r_L$ increases in v.

For loan quality thresholds, since α_L is decreasing in v and r_L is increasing in v, q_L^* is increasing in v. Since α_S is increasing in v and r_S is decreasing in v, q_S^* decreasing in v.

For loan volumes, $\alpha_L(1-q_L^*)$ is decreasing in v, since α_L is decreasing and q_L^* is increasing. Similarly, $\alpha_S(1-q_S^*)$ is increasing in v.

Total loan volume equals $\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*) = 1 - \frac{1+f}{A} + \frac{\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)}{A}$. Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f-r_S)-2\alpha_L(f-r_L)]\frac{d\alpha_S}{dv} - \alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv}\right\}. \tag{49}$$

where $2\alpha_S(f-r_S)-2\alpha_L(f-r_L)<0$, $\frac{d\alpha_S}{dv}>0$, $\frac{dr_L}{dv}>0$, and $\frac{dr_S}{dv}<0$. We know $-\alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv} < -\alpha_S^2\frac{d(r_L+r_S)}{dv}$. If $G''(\delta)\geq 0$, we know from above that $\frac{d(r_L+r_S)}{dv}\geq 0$, so $-\alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv}\leq 0$, and so Equation (49) is negative. If $G''(\delta)<0$, however, the sign of the equation is ambiguous.

The constrained equilibrium

To calculate how r_S is affected by v, we take derivative of Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_S}{\partial v} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \tag{50}$$

When X is sufficiently large, the term $X[(f-s)G''(r_S-s+v) - G'(r_S-s+v)]$ dominates $\frac{\partial \Gamma_S}{\partial v}$. Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, we know that $\frac{\partial \Gamma_S}{\partial v}<0$. The second-order condition implies that $\partial\Gamma_S/\partial r_S<0$. Hence $\frac{dr_S}{dv}<0$, i.e., r_S is decreasing in v.

For deposit market share $\alpha_S = G(r_S-s+v)$, when X becomes sufficiently large, we have

$$\frac{dr_S}{dv} = -\frac{\partial\Gamma_S}{\partial v}\Big/\frac{\partial \Gamma_S}{\partial r_S} \to -\frac{(f-r_S)G''(r_S-s+v)-G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}. \tag{51}$$

Hence,

$$\frac{d(r_S - s + v)}{dv} = \frac{dr_S}{dv} + 1 = \frac{-G'(r_S - s + v)}{(f - r_S)G''(r_S - s + v) - 2G'(r_s - s + v)}, \tag{52}$$

where the numerator and the denominator are both negative. So $\frac{d(r_S - s+v)}{dv} > 0$, i.e., α_S is increasing in v and α_L is decreasing in v.

For weighted average deposit rate, take derivative with respect to v:

$$\begin{aligned}\frac{d}{dv}(\alpha_L s + \alpha_S r_S) &= \frac{d\alpha_S}{dv}(r_S - s) + \alpha_S \frac{dr_S}{dv} \\ &= \frac{(f - s)dr_S/dv + r_S - s}{f - r_S}\alpha_S\end{aligned} \tag{53}$$

where the second equality uses $d\alpha_S/dv \to \frac{\alpha_S}{(f - r_S)}(dr_S/dv + 1)$, implied by the small bank's FOC when X is sufficiently large. The derivative is negative if and only if $(f-s)dr_S/dv + r_S - s < 0$. Write $y = r_S - r_L + v$. Plugging in $\frac{dr_S}{dv} = -\frac{(f-r_S)G''(y) - G'(y)}{(f-r_S)G''(y) - 2G'(y)}$, the derivative is negative if and only if $G''(r_S - s + v) \leq \frac{f+s-2r_S}{(f-r_S)^2}G'(r_S - s + v)$. We now show that $f + s - 2r_S \geq 0$. We know that r_S is decreasing in v. So we only need to show, given s, $f + s - 2r_S \geq 0$ when $v = 0$. Let $l(x) = (f-x)G'(x-s) - G(x-s)$, then $l(r_S) = 0$, and $\frac{dl(x)}{dx} = (f-x)G''(x-s) - 2G'(x-s) < 0$ under the condition that $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$. To show that $f + s - 2r_S \geq 0$, we only need $l(\frac{1}{2}(f + s)) \leq 0$. That is, $\frac{f-s}{2}G'(\frac{f-s}{2}) - G(\frac{f-s}{2}) \leq 0$. This is true because if we let $m(x) = xG'(x) - G(x)$, then $\frac{dm(x)}{dx} = xG''(x) \leq 0$. And since $m(0) = 0$, we have $m(\frac{f-s}{2}) \leq 0$.

For loan quality thresholds and individual banks' loan volumes, the same proofs for the unconstrained equilibrium apply and are omitted.

Total loan volume equals $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) = 1 - \dfrac{1+f}{A} + \dfrac{\alpha_L^2(f-s) + \alpha_S^2(f - r_S)}{A}$. Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]\frac{d\alpha_S}{dv} - \alpha_S^2\frac{dr_S}{dv}\right\}. \tag{54}$$

Its sign is ambiguous because while the first term in the brackets is negative, the second term is positive.

Proof of Proposition 5

In the unconstrained equilibrium, the difference of the two banks' FOCs leads to Equation (16) . So $r_S - r_L$ does not vary with f. Further, let $B \equiv \frac{\frac{1}{A}\alpha_L \alpha_S (f-r_L)(f-r_S)G'(r_S-r_L+v)}{X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)}$, then the two FOCs are separately written as

$$(f - r_L)G'(r_S - r_L + v) = 1 - G(r_S - r_L + v) + B \tag{55}$$

$$(f - r_S)G'(r_S - r_L + v) = G(r_S - r_L + v) + B \tag{56}$$

When X is sufficiently large, $B \to 0$. Hence as f changes, r_L and r_S move one-for-one with f.

In the constrained equilibrium, let $\Gamma_S = d\Pi_S/dr_S$. To study the sensitivity of r_S to f, we take derivative of Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_S}{\partial f} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{df} \tag{57}$$

When X becomes sufficiently large, we have

$$\frac{dr_S}{df} = -\frac{\partial \Gamma_S}{\partial f}/\frac{\partial \Gamma_S}{\partial r_S} \to \frac{1}{-(f-r_S)\frac{G''(r_S-s+v)}{G'(r_S-s+v)} + 2} \tag{58}$$

We know that $(f - r_S)$ is decreasing in s and increasing in v, and that $r_S - s + v$ is decreasing in s and increasing in v. If G satisfies that $\frac{G''(\delta)}{G'(\delta)}$ is increasing in δ for any $\delta \in [0, f - s + v]$, then the denominator is increasing in s and decreasing in v . Also, since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$, the denominator is positive. So $\frac{dr_S}{df}$ is decreasing in s and increasing in v.

Given v and s, we solve for the cut-off value f^* that separates the constrained equilibrium and the unconstrained equilibrium. Let r_S^* be the equilibrium value of r_S when $f = f^*$. When X is sufficiently large, the small bank's FOC and the large bank's FOC are

$$(f^* - r_S^*)G'(r_S^* - s + v) - G(r_S^* - s + v) = 0 \tag{59}$$

$$(f^* - s)G'(r_S^* - s + v) - 1 + G(r_S^* - s + v) = 0 \tag{60}$$

Taking the difference of the two equations, we have

$$\Delta = (r_S^* - s)G'(r_S^* - s + v) - 1 + 2G(r_S^* - s + v) = 0 \tag{61}$$

So $r_S^* - s = m(v)$ is a function of v. By the small bank's FOC, we have

$$f^* = s + \frac{G(m(v) + v)}{G'(m(v) + v)} + m(v) \tag{62}$$

Hence f^* increases one-for-one with s. To show that f^* is decreasing in v, we take derivative of Δ to solve for $\frac{dm(v)}{dv}$

$$0 = \frac{\partial \Delta}{\partial v} + \frac{\partial \Delta}{\partial m(v)} \frac{dm(v)}{dv} \tag{63}$$

Hence, $\frac{dm(v)}{dv} = -\frac{\partial \Delta}{\partial v} / \frac{\partial \Delta}{\partial m(v)} = -\frac{m(v)G''(m(v)+v)+2G'(m(v)+v)}{m(v)G''(m(v)+v)+3G'(m(v)+v)}$. Denoting $y = m(v)+v = r_S^* - s + v$, we have

$$\begin{aligned} \frac{df^*}{dv} &= \frac{d}{dv}\left\{\frac{G(y)}{G'(y)}\right\} + \frac{dm(v)}{dv} \\ &= \frac{G'(y)^2[-m(v)G''(y) - G'(y)] - G''(y)G'(y)G(y)}{G'(y)^2[m(v)G''(y) + 3G'(y)]} \end{aligned} \tag{64}$$

where $r_S^* - s = m(v) > 0$. For G that satisfies $0 \leq G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$, we know the denominator is positive, and the numerator is negative, so $\frac{df^*}{dv} < 0$.

Appendix B: Fitting U.S. deposit rates

In this appendix, we illustrate the predictive performance of our model using U.S. data on deposit rates from 1986 to 2021. While our model is highly stylized and features only two banks, it captures qualitative features of deposit rates: they are lower than and somewhat non-responsive to the interest rate on reserves.

We use the model to predict deposit rates. The opportunity cost of funds for banks is determined in part by either the IOR rate or the federal funds rate, whichever is larger. In the period before the 2008-09 crisis the relevant rate was the federal funds rate. In the period after the crisis the relevant rate was (generally) the IOR rate. We use the higher of the two rates in each period as f, and apply the model under a specific parameterization. Specifically, we assume the convenience value for deposit at the large bank, δ, is distributed uniformly from 0 to 3.5%. The resulting predicted data series seems to fit the actual data reasonably well. Figure 4 shows the actual and predicted U.S. deposit rates from 1986Q1 to 2008Q2 relative to the federal fund rate. Figure 5 shows the actual and predicted U.S. deposit rates from May 2009 to February 2021 relative to the IOR rate.

In Figure 4, predicted deposit rates match the levels of actual deposit rates quite well and move almost one-for-one with the federal funds rate. The main deviation from the actual data is that predicted rates are too sensitive to changes in the federal funds rate. This is not surprising, as in our model, deposit rates of both the large and small bank move one-for-one with the federal funds rate in the unconstrained equilibrium. A noticeable deviation occurs during the pre-crisis period from 2001Q3 to 2004Q3. This is when the large bank's predicted deposit rate is constrained at the zero lower bound. The transition to low deposit rates associated with the constrained equilibrium is immediate in our model, but not in the data. Our model does not build in any "stickiness" into the deposit rate that would be necessary to match the data more closely.

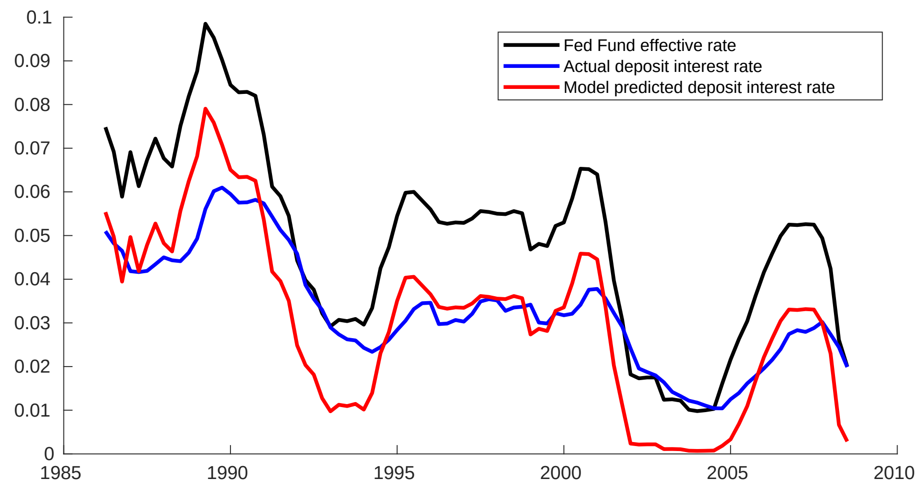

Figure 4: Actual and predicted U.S. deposit rates from 1986Q1 to 2008Q2. Domestic deposit rates are quarterly, calculated from call reports, as total interest expense on domestic deposits divided by total domestic deposits, multiplied by 4. The model-implied interest rate is the weighted average of the large bank's and the small bank's deposit rates, weighted by their market shares. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

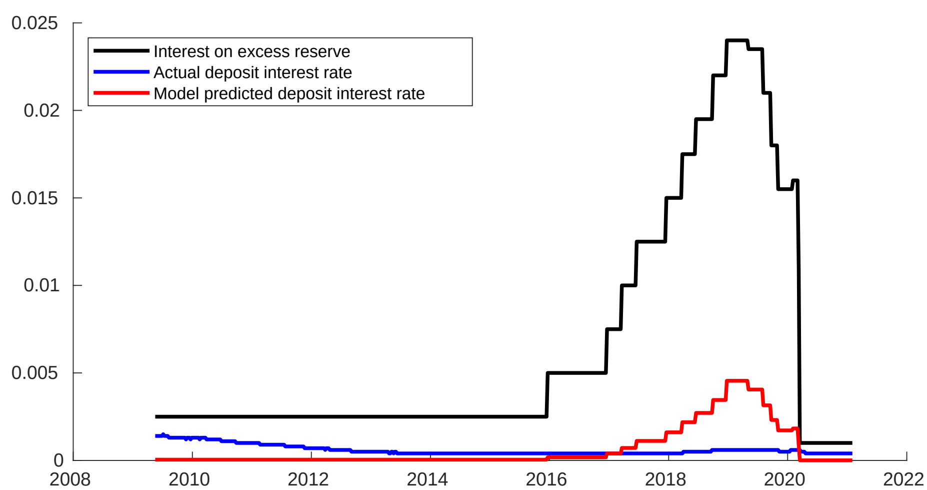

Figure 5: Actual and predicted U.S. deposit rates from May 18, 2009 to February 1, 2021. Weekly deposit rates for amounts less than \$100,000 are obtained from the FDIC through FRED. The model-implied interest rate is the weighted average of the large bank's and the small bank's deposit rates, weighted by their market shares. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

The time period in Figure 5 is characterized by a long stretch of near zero rates in the Federal Funds market and an IOR rate of 25 basis points. As the IOR rate was typically higher than the federal funds rate, the IOR rate is the relevant variable for predicting deposit rates. Deposit rates fell slowly during this period toward zero until the Fed began to raise the IOR rate in December 2015. The Fed raised the IOR rate multiple time reaching a peak of 2.40% from December 2018 to April 2019, but deposit rates reacted very slowly. Our model's predicted deposit rate captures this non-responsiveness. It is still too sensitive to changes in IOR compared to the data, but the deviation is not large. The low deposit rates that are predicted by our model occur because at the low IOR rates that existed during most of this period the zero lower bound is binding in our model and hence average market rates are determined largely by large bank deposit rates which are constrained at zero.

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