BIS Working Papers
No 1034
Debt sustainability and monetary policy: the case of ECB asset purchases
by Enrique Alberola, Gong Cheng, Andrea Consiglio and Stavros A Zenios
Monetary and Economic Department
July 2022

JEL classification: E52, H63, H68.
Keywords: Debt sustainability analysis, risk management, unconventional monetary policy, monetary-fiscal mix, PEPP, CVaR optimisation.
BIS Working Papers are written by members of the Monetary and Economic Department of the Bank for International Settlements, and from time to time by other economists, and are published by the Bank. The papers are on subjects of topical interest and are technical in character. The views expressed in them are those of their authors and not necessarily the views of the BIS.

This publication is available on the BIS website (www.bis.org).

© Bank for International Settlements 2022. All rights reserved. Brief excerpts may be reproduced or translated provided the source is stated.

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)
Debt sustainability and monetary policy:
The case of ECB asset purchases

Enrique Alberola  Gong Cheng  Andrea Consiglio  Stavros A. Zenios\textsuperscript{1}


Abstract

We incorporate monetary policy into a model of stochastic debt sustainability analysis and evaluate the impact of unconventional policies on sovereign debt dynamics. The model optimizes debt financing to trade off financing cost with refinancing risk. We show that the ECB pandemic emergency-purchase programme (PEPP) substantially improves debt sustainability for euro area sovereigns with a high debt stock. Without PEPP, debt would be on an increasing (unsustainable) trajectory with high probability, while, with asset purchases, it is sustainable and the debt ratio is expected to return to pre-pandemic levels by about 2030. The improvement in debt dynamics extends beyond the PEPP and is larger for more gradual unwinding of the Central Bank balance sheet. Optimal financing under PEPP induces an extension of maturities reducing the risk without increasing costs. The analysis also shows that inflation surprises have relatively little impact on debt dynamics, with the direction and magnitude of the effect depending on the monetary policy response.

Keywords: Debt sustainability analysis; risk management; unconventional monetary policy; monetary-fiscal mix; PEPP; CVaR optimization.

\textsuperscript{1}The authors are with the Bank for International Settlements (BIS) (Enrique.Alberola@bis.org at the BIS, Basel, Switzerland, Gong.Cheng@bis.org at the BIS, Hong Kong SAR, China), University of Palermo, Palermo, Italy (andrea.consiglio@unipa.it), and University of Cyprus, Nicosia, CY, Cyprus Academy of Sciences, Letters, and Arts, Non-Resident Fellow Bruegel, Brussels (zenioss@ucy.ac.cy). We gratefully acknowledge valuable comments from Matthias Drehmann, Boris Hofmann, Athanasios Orphanides, Ilhyock Shim, discussant Junko Koeda at the 29th NBER-TCER-CEPR Conference, conference participants at the 5th Sovereign Debt Research and Management Conference and the Bruegel Senior Economists Meeting, and seminar participants at the BIS and the University of Cyprus Economics Department. We also thank Jimmy Shek for his excellent research assistance.

Disclaimer: The views presented in this paper are those of the authors and do not necessarily represent those of the Bank for International Settlements.
1 Introduction

We integrate monetary policy into stochastic debt sustainability analysis (DSA) and study the impact of unconventional monetary policymaking on sovereign debt. The sovereign debt-monetary policy nexus has been of increasing importance since the Great Financial Crisis when Central Banks in advanced economies implemented large asset purchase programs\footnote{In 2010, the ECB holdings of euro area government debt accounted for only 2% of total outstanding government debt, but surged to 20.5% by end 2020. The share of US government debt held by the Federal Reserves increased from 7.8% to 18.3% during the same period. Data from Arslanalp and Tsuda (2014), with the euro area data excluding Latvia and Lithuania.}. The Covid-19 crisis reinforced this nexus, raising concerns going forward (BIS 2020). We introduce monetary policy through two channels: a macro-monetary model for conventional monetary policy and a model of the effect of unconventional policies through public debt purchases on debt financing spreads. We use the extended DSA model to assess the impact of ECB’s Pandemic Emergency Purchase programme (PEPP) on eurozone sovereign debt sustainability\footnote{PEPP was launched at the onset of Covid-19, see www.ecb.europa.eu/mopo/implement/pepp/html/index.en.html.}

Current DSA models consider the following variables governing debt dynamics: legacy debt level, projected output growth, interest rate on debt, and fiscal balance (Bouabdallah et al., 2017; European Commission, 2020; IMF, 2021; Zenios et al., 2021). Monetary policy and inflation are assumed as given and considered as neutral in the long run. We introduce the macro-monetary block with standard IS and Phillips curves determining the output gap and inflation dynamics, and a Taylor rule determining the policy rate and conventional monetary policymaking.

Importantly, given the intensified use of asset purchase programmes, we consider unconventional monetary policy in the DSA. The withdrawal of public debt from the markets to be held at the balance sheets of central banks contributes to reducing risk premia across the yield curve (Beckmann et al., 2020; Hondroyiannis and Papaoikonomou, 2021). This is the channel that we incorporate into DSA. Quantitative tightening (QT) works in the opposite direction as the central bank unwinds purchases as they mature, and we assess the impact from both cumulation (purchases) and depletion (unwinding) of central bank’s assets.

Our model enriches the methodological framework of Zenios et al. (2021) with the two dimensions of monetary policy. The model accounts for stochastic correlated deviations from the steady-state using a discrete time- and state-space scenario tree, calibrated to match expectations from institutional forecasts of the key variables, and historically observed volatilities and correlations. In line with current practices (Dias, Richmond, and Wright, 2014; Irwin, 2015), we model both debt flow and stock. The model optimizes debt financing trading off financing costs and refinancing risks. We employ a measure of tail risk (Conditional Value-at-Risk of Artzner, Delbaen, Eber, and Heath (1999))) of the debt stochastic variables to constrain the optimization problem of the public debt management office (PDMO). Specifically, if debt flow (gross financing needs) exceed the threshold that markets can finance, the sovereign can face a liquidity crisis. The optimization allows to draw inferences about sustainability. We consider that debt is sustainable when it is on a decreasing trajectory in the long term (10 years) with a high degree of probability (75%). An extension of the model estimates the fiscal effort required
to stabilise debt or reduce it to a desired level. Our approach allows for granular debt maturities, preserving the realism of PDMO decisions and allowing us to look at the effect of PEPP on debt financing maturities. Optimizing debt financing in a cost-risk tradeoff makes our model unique among the high-realism DSA models of the international institutions.

The stochastic DSA model with monetary policy is our methodological contribution. Our empirical contribution is to assess the implications of PEPP on debt sustainability, and to identify optimal debt financing responses to asset purchase programs. We also use the model to analyse the impact on debt dynamics of central bank reaction to inflation surprises.

The PEPP was launched in March 2020 to inject liquidity into the financial system. The initial programme envelope was €750bn, expanded to €1850bn by December 2020. The size of the programme relative to sovereign financing needs has been high. In 2020, when issuance spiked, PEPP financed more than 1.5 times the Greek borrowing needs, all of the Netherlands, and over half of the needs of thirteen countries, including Portugal, Spain, and Germany.

Figure 1 – The ECB Pandemic Emergency Purchase Programme

(A) Sovereign 10-year spreads

(B) Counterfactual for high debt countries

To what extent did the PEPP provide relief to sovereign borrowers? A first step in answering this question is to estimate the impact of PEPP on sovereign financing costs. Figure panel A shows eurozone sovereign 10-year bond spreads of several euro area countries relative to the German Bund. Credit spreads spiked in the wake of the pandemic, raising debt sustainability concerns in fiscally vulnerable countries. Spreads dropped with the announcement of the PEPP and tightened to below pre-pandemic levels with programme implementation. From November 2021, when speculation about ending the PEPP started, some countries saw their financing costs rise. In panel B we display the changes in sovereign spreads after the PEPP announcement, together with the increase in sovereign debt-to-GDP ratio for high debt countries (Cyprus, Greece, Italy, Portugal, and Spain). The actual cumulative changes in spreads (light blue bars) are significantly lower than the counterfactual (dark blue) that would have resulted from the rapid rise in debt levels (line). (The counterfactual is estimated from a panel regression of spreads on debt levels, as obtained in section 3.3.1). The spread compression for the high debt

3 See those of the IMF, EC, or ECB. Among stylized models, the single-period ones consider a binary long-short maturity structure ([Arellano and Ramanarayanan 2012 Conesa and Kehoe 2017], whereas multi-period models assume debt structures that are analytically tractable, such as consols with exponentially decaying coupons ([Angeletos 2002 Bocola and Dovis 2019 Hatchondo and Martinez 2009]).
countries is more than 100 basis points, highlighting the relevance of PEPP in the sovereign debt markets.

Estimating the suppression of credit risk spreads as a function of cumulative bond purchases we assess the expected effect of future PEPP unwinding and show the deeper impact of PEPP on long-term debt sustainability. We calibrate the extended DSA model to a representative eurozone high-debt country and integrate the estimated spread suppression function, to estimate the impact of the ECB unconventional policy response to the pandemic on government financing costs, debt dynamics, and debt management, under different unwinding assumptions. We find as follows:

1. The PEPP substantially improves debt dynamics and the effects persist well beyond the end of the programme. Without the PEPP, the debt level would be unsustainable. A persistent primary balance of 1.3% of GDP per annum (p.a.) over eighteen years would be required to stabilise debt dynamics at the end of the simulation horizon. In contrast, with the PEPP public debt is sustainable, returning to below pre-pandemic levels by 2030.

2. Unwinding the central bank balance sheet as public debt holdings mature (passive quantitative tightening) puts upward pressure on spreads, which is more intense the earlier the QT.

3. An inflation surprise does not have a strong impact on debt dynamics, with the magnitude depending on the monetary policy reaction. If the central bank views the shock as transitory and sees through it, debt dynamics slightly improve. If monetary policy tightens following a persistent inflation shock, debt dynamics slightly worsen. An indirect unfavourable impact, also of relatively small magnitude, would arise if the inflation shock accelerates the PEPP unwinding.

4. The PEPP induces a substantial Pareto improvement as regards the tradeoff between financing cost and refinancing risk. Maturities can be lengthened to reduce the refinancing risks without increasing financing cost.

Whereas we find overall positive impact of PEPP, it should not be viewed as a free lunch. Our model focuses on debt dynamics and abstracts from some elements that represent risks, that are the more relevant the larger the size of the balance sheet and when there is a change in the monetary policy environment.

Our study is related to an extensive literature on the determinants of sustainable sovereign debt levels. We follow Zenios et al. (2021) where the

---

4See “Helicopter money: The illusion of a free lunch”, https://voxeu.org/article/helicopter-money-illusion-free-lunch. Asset purchase has as a counterpart the increase of short-term bank reserves in the central bank balance sheet and from a consolidated perspective, public sector debt is not reduced. Given the maturity swap, the overall maturity of public sector debt decreases, entailing also a maturity mismatch in the central bank balance sheet that can generate losses to the central bank in a situation of rising interest rates with a concomitant fiscal cost as central bank transfers to the government evaporate. Other risks arise from political economy considerations, as unconventional policies blur the separation between monetary and fiscal policy. Governments favoured by lower financing cost, may resist monetary policy reversals, generating tensions with the central banks that could jeopardise central bank independence.

5See Angelesco (2002); Arellano and Ramanarayanan (2012); Bocola and Dovis (2019); Chatterjee and Eyangor (2012); Conesa and Kehoe (2017); Fernández and Martín (2014); Hatchondo and Martinez (2009); Missale (2012); Niepelt (2014), among others.
fiscal agent actively manages debt financing to minimize interest payments subject to constraints on risks of potential default. This reference and the earlier works, look for sovereign debt financing in the context of debt crises. Instead, we ask the question of sustainable debt financing in the context of active monetary policymaking.

The introduction of a macroeconomic framework with monetary policymaking in DSA is largely inspired by the blossoming literature on the interactions between monetary and fiscal policies. [Zampolli (2012)] is among the first to look at the role of sovereign debt management as a tool of monetary policy in the aftermath of the great financial crisis. [Debrun et al. (2021)] revisit the policy mix in the euro area during the pandemic from a central bank’s perspective, focusing on the ramifications for price stability and central bank credibility. [Hofmann et al. (2021)] present a macro model to explore the implications on fiscal sustainability of monetary policy at the zero lower bound. [Arslan, Drehmann, and Hofmann (2020)] analyse empirically the impact of central bank bond purchases on government financing conditions during the pandemic. These papers look at the monetary-fiscal nexus from the monetary perspective, whereas we examine the other side of the same coin, namely how monetary policy contributes to fiscal sustainability. We rely on these works for the parametrization of our model.

Our paper also endorses the empirical findings of [Plessen-Mátyás, Kaufmann, and von Landesberger (2021)] who identify the lengthening of maturities by the eurozone PDMO in response to asset purchase programs. We show that this lengthening is a Pareto improvement that optimally reduces the refinancing risk.

The paper is organised as follows: Section 2 presents the stochastic DSA model enriched with the macroeconomic framework and conventional and unconventional monetary policymaking. Section 3 describes the data, calibrates the DSA model to a representative eurozone high debt country, and estimates the impact of PEPP on sovereign spreads. Section 4 puts the model to work under different PEPP unwinding paths, and studies the effects of central bank responses to an inflation surprise. In section 5 we take a step further to look at the effect of the PEPP on debt financing and maturities. We conclude in Section 6.

2 Debt sustainability analysis with monetary policy

We first describe debt dynamics, introduce the scenario tree of stochastic debt variables, define the risk measure, and develop an optimization model for public debt financing. We then introduce the macroeconomic framework with monetary policymaking.

2.1 Debt dynamics and optimal public debt management

2.1.1 Debt dynamics

We consider a sovereign that at period $t$ is endowed with nominal output $Y_t$, owes debt stock $D_{t-1}$, and runs a primary balance $PB_t$, with the legacy debt $D_0$ as an initial condition. The sovereign’s gross financing needs are given by the flow variable $GFN_t$ as follows:

$$GFN_t = i_{t-1}D_{t-1} + A_t - PB_t,$$  \hspace{1cm} (1)
where \( i_{t-1} \) is the effective nominal interest rate on debt, and \( A_t \) denotes the part of the debt stock \( D_{t-1} \) which comes due. The debt stock is given by

\[
D_t = (1 + i_{t-1})D_{t-1} - PB_t. \tag{2}
\]

The sovereign finances its needs by issuing debt instruments of different maturities, denoted by \( j = 1, 2, \ldots, J \), with financing decisions \( X_t(j) \) denoting the nominal amount of debt issued at \( t \). The debt financing equation satisfies

\[
\sum_{j=1}^{J} X_t(j) = GFN_t. \tag{3}
\]

The nominal interest rate on the issued debt is determined by the risk-free rate \( (i_{ft}) \) plus premia idiosyncratic to the sovereign, which depend especially on the debt level (Bassanetti, Cottarelli, and Presbitero 2018; Paesani, Strauch, and Kremer 2006). A significant feedback effect, especially for high-debt countries, is between a sovereign’s debt and the level and slope of the yield curve. We model this feedback loop using risk and term premia. Following the literature we model premia as a function of the debt-to-GDP ratio, \( d_t = D_t/Y_t \), and of the maturity of the issued instrument. The interest rate for instrument \( j \) issued at \( t \) is given by

\[
r_t(j) = i_{ft} + \rho(d_t, j), \tag{4}
\]

where \( \rho(d, j) \) captures risk premium and term premia for the \( j \)th instrument maturity.

Thus, the effective interest rate in (2) is a function of the debt financing decisions

\[
i_t = \frac{i_{t-1}(D_{t-1} - A_t) + \sum_{j=1}^{J} r_t(j)X_t(j)}{D_t}. \tag{5}
\]

The financing decisions determine debt dynamics which in turn condition the evolution of risk and term premia and ultimately influence the financing maturities. Thus stock and flow are linked not only through quantities but also through prices in a feedback loop.

2.1.2 Uncertainty and a risk measure

The debt variables—economic output, primary balance, inflation, and debt financing rates—are uncertain, and we represent them by a discrete time- and state-space scenario tree, as shown in Figure 2 panel A. Time is indexed by \( t = 0, 1, 2, \ldots, T \), where \( T \) is the risk horizon of our simulation, and \( n \in N_t \) denotes the state at \( t \). The number of states at \( t \) is \( N_t \), with total number of states \( N \). Not all states at \( t \) can be reached from every state at \( t - 1 \), and \( P(n) \) denotes the set of states on the unique path from the root state 0 to \( n \). Each path leading to a terminal state \( n \in N_T \) is a scenario, with probability \( \text{Prob}^{(n)} \), which we set without loss of generality to \( 1/N \). For each state \( n \), all information at states \( m \) on the path \( P(n) \) is known since \( m \) precedes \( n \). Problem data and model variables are indexed by the state to which they belong, and we can model the dynamics of debt stock and flow on the states of the tree.
Panel B illustrates the distribution of debt-to-GDP ratio across time. In this example we observe a shift of the debt distribution towards lower values, with increasing variability for longer horizons. The high debt quantiles are declining and we can infer with high confidence that debt is sustainable. On the tree we also have a distribution of gross financing needs. The distributions of flow and stock dynamics provide rich information on sustainability. We can assess, for instance, if refinancing needs are likely to be large, with a given probability, leading to a liquidity crisis, or if the debt ratio is on a non-increasing trajectory and hence sustainable.

To quantify the information from these distributions we introduce a risk measure. We use the **Conditional Value-at-Risk** (CVaR) of the ratio of gross financing needs to GDP, defined as the expected value of financing needs above the right $\alpha$ quantile, i.e., the expected value of the right tail of the distribution of flow. Using CVaR as our risk measure we arrive at a model with constraints on the tail risk of gross financing needs at a confidence level $\alpha$. CVaR is widely used in banking and insurance, and is well suited for modeling extreme events for DSA. As CVaR increases so do the debt financing risks. Using $gfn$ to denote the stochastic variable over all
time periods, we define the CVaR risk function $\Psi(\cdot)$ for flow by

$$
\Psi(gfn) = E(gfn \mid gfn \geq gfn^\diamond).
$$

(6)

$gfn^\diamond$ is the right $\alpha$-percentile of the gross financing needs, i.e., the lowest value of $gfn$ such that the probability of gross financing needs less or equal to $gfn^\diamond$ is greater or equal to $\alpha$. $gfn^\diamond$ is the Value-at-Risk of debt flow. $\Psi(gfn)$ measures the refinancing risk, with higher values making it more difficult to refinance debt.

2.1.3 Optimal debt management with risk constraints

We determine optimal debt financing decisions to minimize the expected interest payment (IP) on the tree subject to acceptable levels of refinancing risk, in line with typical PDMO mandates.

Interest payments on state $n$ of the tree consist of interest on legacy debt $I^n_t$ plus service payments on debt created by the financing decisions. To track service payments on a path leading to $n$ we exploit the tree structure. Let $CF^t_n(j,m)$ denote the nominal amount of interest payment due at state $n$ of period $t$, per unit of debt $X^m_{\tau(m)}(j)$ issued at state $m$ of an earlier period $\tau(m)$ on path $\mathcal{P}(n)$. This amount is computed from scenarios of the term structure of interest rates, the term of the issued debt, and the premia eqn. (4). The state-dependent interest payment is given by

$$
IP^n_t = I^n_t + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^J X^m_{\tau(m)}(j) CF^t_n(j,m).
$$

(7)

Interest payments is what the PDMO controls through the financing decisions $X$.

The model minimizes the expected cost of debt subject to a flow risk constraint:

$$
\begin{align*}
\text{Minimize} & \quad \sum_{n \in \mathcal{N}_t} \text{Prob}^{(n)} IP^n_t \\
\text{s.t.} & \quad \Psi(gfn) \leq \omega.
\end{align*}
$$

(8)

(9)

This is a parametric model with an embedded tradeoff between debt financing cost and debt refinancing risk. Constraint (9) bounds the refinancing risk, i.e., the tail of gross financing needs, by a parameter $\omega$ as a percentage of GDP. If it is below some threshold that the markets can finance, then we can ascertain, with high confidence (e.g., 0.75), that debt financing needs can be met. For very high values the sovereign may face a liquidity crisis.

Varying this parameter we trace the efficient frontier of debt cost vs refinancing risk illustrated in Figure 3.

---


7 The threshold is taken to be 15% of GDP for emerging and 20% for developed economies by international institutions.
Lower refinancing risk, typically through issuance of long-maturity debt, increases the cost of debt financing and the dynamics of debt-to-GDP ratio shift upwards. If the debt stock dynamics follow a stable or decreasing trajectory over a 10 year horizon with a high probability (0.75), then the sovereign debt is sustainable. On the contrary, increasing debt dynamics indicate that the sovereign is facing a debt trap that can lead to insolvency and debt is unsustainable. This tension between stock and flow can be controlled through a constraint on stock dynamics

\[ \frac{\partial d}{\partial t} \leq \delta. \] (10)

If a solution can be obtained with \( \delta \leq 0 \) with a high probability, then debt is sustainable. Since \( d \) is a random variable taking scenario values \( d^n_t \), the constraint is also implemented using the risk measure [Zenios et al. 2021, Online Appendix A.2].

What would happen if a solution with stable debt is not possible? A model extension [Zenios et al., 2021, section 6] optimizes the timing and magnitude of stabilising fiscal effort that renders debt stock dynamics sustainable with high probability. Whether the resulting fiscal effort is politically tenable or excessively ambitious is not for the model to say, but [Eichengreen and Panizza (2016)] provide empirical guidance. We use this model feature in our tests.

Figure 3 – Tradeoff of debt financing cost with refinancing risk

---

2.2 Macroeconomic framework and monetary policymaking

We now describe the macroeconomic framework and introduce monetary policymaking.

2.2.1 Macroeconomic framework

Inflation \( (\pi_t) \) is driven by the [Phillips, 1962] curve augmented with inflation expectations

\[ \pi_t = \beta\pi_{t-1} + (1 - \beta\pi)E\pi_{t+1} + \gamma \hat{y}_t + \epsilon_{\pi,t}, \] (11)
where $E_t \pi_{t+1}$ is the expectation at time $t$ of inflation at $t+1$, $\hat{y}_t$ is the output gap, $\beta_\pi$ and $\gamma$ are parameters, with $\beta_\pi$ denoting persistence, and $\epsilon_{\pi,t}$ is a centred error term.

For the output gap ($\hat{y}_t$) we use the Mundell-Fleming IS curve

$$\hat{y}_t = \delta_y \hat{y}_{t-1} + (1 - \delta_y) E\hat{y}_{t+1} - \alpha_\pi (i_{ft} - r^*) + \epsilon_{\hat{y},t},$$

(12)

where the gap depends on the expectation $E\hat{y}_{t+1}$ and the difference of the policy interest rates $i_{ft}$ from the natural rate $r^*$. $\alpha_\pi$ and $\delta_y$ are parameters, with the latter denoting persistence, and $\epsilon_{\hat{y},t}$ is a centred error term.$^8$

Solving (11)–(12) we obtain equilibrium inflation and output gap. Assuming constant potential growth, the dynamics of real GDP growth ($g_t^*$) are determined by output gap changes

$$g_{t+1}^* = g_t^* + \Delta \hat{y}_t.$$  

(13)

DSA works with nominal economic output, since debt is in nominal values, given by $Y_t = Y_{t-1}(1 + g_t)$ where $g_t = g_t^* + \pi_t$.

The policy interest rate follows a Taylor (1993) rule

$$i_{ft} = \theta_i \; i_{f(t-1)} + (1 - \theta_i)[\pi_{t-1} + r^* + \alpha_\pi (\pi_{t-1} - \pi_t^*)] + \alpha_i \hat{y}_{t-1} + \epsilon_{i,t},$$

(14)

where $\alpha_\pi, \alpha_i$ are parameters, $\theta_i$ denotes persistence, and $\epsilon_{i,t}$ is a centred error term.

The interest rate for debt financing includes credit risk premia on top of the policy rate that are idiosyncratic to the $i$th sovereign (eqn. 4). The unconventional monetary policy is introduced through the impact of central bank purchases of sovereign securities on the spreads, and we account for both conventional and unconventional policies by modeling the spread over the risk free rate $i_{ft}$ without and with PEPP.

The spread $\rho(d_{i,t}, p_{i,t})$ is a function of the debt level of the country $d_{i,t}$, the net asset purchase $p_{i,t}$, and the maturity of the issued debt $j$ (term premium). We express spread as the product of a function of the debt in the market $\rho_C(d_{i,t}, p_{i,t}, j)$ times $[1 - \text{spread suppression function } \rho_U(p_{i,t}, j)]$$^9$ Spread suppression takes the value of zero before the PEPP is launched and is greater than zero as long as the central bank holds public debt. We have

$$\rho(d_{i,t}, p_{i,t}, j) = \rho_C(d_{i,t}, p_{i,t}, j) \times (1 - \rho_U(p_{i,t}))$$

(15)

$$\rho_U(p_{i,t}) \in [0, 1].$$

The risk premia $\rho_C(\cdot)$ are a function of the total debt in the market, which, in the case of central bank asset purchases, are less than the total sovereign debt. Unconventional monetary policy enters DSA through its impact on credit risk premia, and, hence, debt financing cost.

---

$^8$A more appropriate specification would take into account the full term structure including risk premia $\hat{y}_t = \delta_y \hat{y}_{t-1} + (1 - \delta_y) E\hat{y}_{t+1} - \alpha_\pi \left( \frac{1}{2} \sum_j r_t(j) - r^* \right)$. To limit the number of state variables in our DSA model we approximate the average of the term structure by the baseline interest rate in (12). Our debt estimates are best case and they would worsen if spreads significantly suppress growth, through the effect on the denominator of the debt-to-GDP ratio.

$^9$The nominal value of asset purchases is expressed as a percentage of country GDP in $\rho_C(\cdot)$, and as a percentage of the country’s PEPP envelope in $\rho_U(\cdot)$. With a slight abuse of notation we use $p_{i,t}$ in both functions.
Switching from conventional to unconventional policies in the context of DSA is straightforward, given the known timing of the PEPP and its calibrated effects. Reverting to the conventional regime requires assumptions on the timing of quantitative tightening by unwinding the purchased debt.

2.2.2 Risk and term premia under conventional monetary policy

The debt refinancing rates under conventional monetary policy are obtained from (4) and (15) with \( \rho_{U_i}(p_{i,t}) = 0 \). Following Zenios et al. (2021) we express the spread as

\[
\rho_C(d, j) = a_j + (1 + b_j)\hat{\rho}_C(d),
\]

where \( a_j \) and \( b_j \) are term premia, and \( \hat{\rho}_C(d) \) is the effect on interest rates (risk premium) of the debt stock in the market

\[
\hat{\rho}_C(d) = \hat{\rho} \left[ \frac{d_{\text{max}} - d}{1 + \exp(d_{\text{max}} - d)} - \frac{d_{\text{min}} - d}{1 + \exp(d_{\text{min}} - d)} \right].
\]

Eqns. (16)–(17) generate yield curves that shift and twist with debt changes. When \( b_j \) is higher (lower) for long- than short-term debt, a debt increase causes curve steepening (flattening) around the benchmark maturity rate. Eqn. (17) is a smooth sigmoid approximation of a piecewise linear function, with lower bound zero for debt-to-GDP ratio up to \( d_{\text{min}} \) and increasing linearly for higher debt ratios until it hits a ceiling at \( d_{\text{max}} \) when the sovereign loses market access and needs to solicit financing from multilateral institutions.

2.2.3 Unconventional monetary policy

The spread suppression function under unconventional monetary policy \( \rho_{U_i}(p_{i,t}) \) in (15) is a function diminishing in central bank purchases, reaching a maximum value when purchases exceed \( \bar{p} \). For each country we estimate the piecewise quadratic-linear relation

\[
\rho_{U_i}(p_{i,t}) = \begin{cases} 
\beta_0 + \beta_1 p_{i,t} + \beta_2 p_{i,t}^2 & 0 \leq p_{i,t} \leq \bar{p}_i \\
\bar{p}_{i,t} & p_{i,t} > \bar{p}_i.
\end{cases}
\]

We assume that the spread suppression is symmetric when ECB purchases assets as when it reduces its balance sheet.

3 Model calibration

We now calibrate the standard DSA model to a representative high-debt country over a 30-year period from 2021 to 2050, estimate the PEPP effect on spreads, and explain how the macroeconomic framework with monetary policy (subsection 2.2) is integrated into public debt management (subsection 2.1). Thus, we bring the pieces together to put the model to work.
3.1 Data

We construct the representative country by averaging the sovereign debt of Italy, Spain, and Portugal, equally weighted. The data are given in online Appendix A.1. The starting debt ratio is 135% GDP with a risk premium of about 250bp. For reference, the pre-pandemic debt ratio was about 115%. For primary balance, we average the countries’ projections from the 2021 IMF World Economic Outlook, starting at -5% in 2020 and converging to 0.05% from 2025 onwards.

For macroeconomic variables, we obtain eurozone equilibrium values using the parameters for the Phillips curve, IS curve, and Taylor rule from existing literature (Hofmann et al., 2021), with $r^*$ set to zero; see online Appendix A.2. The eurozone output gap starts at -3.1% and converges to 0% by 2030, with inflation starting at 1.8% and converging to 2% by 2027. From (12)–(13) we obtain eurozone GDP growth, starting at 5.0% in 2021 and converging to 1.4% by 2026.

To calibrate a scenario tree of the stochastic variables, we estimate volatilities and correlations of real GDP growth, output gap, primary balance, and inflation, from yearly data spanning 1998–2019; see online Appendix A.3.

3.2 Calibration of the DSA model

We calibrate the standard DSA model for the representative country. For real GDP growth, output gap, primary balance, and inflation scenarios, we use the values from the data section, and build a scenario tree using the moment matching method of Consiglio, Carollo, and Zenios (2016). We match the first moments of the state variables from the scenario tree to the equilibrium values, and the second moments and correlations to the historical estimates above. Thus, by construction, the expected values of the correlated stochastic fiscal variables and inflation on the scenario tree represent equilibrium values. The policy rate is derived from the inflation and output gap state variables using the Taylor rule (14). In online Appendix A.2, we give the fan charts from the calibrated tree.

For the risk and term premia parameters, we calibrate (16)–(17) using panel data from twenty-three EU countries over the period 1995–2020. We estimate the effects of debt stock on sovereign spreads over the German Bund $\hat{\rho}_C(d)$ from a panel regression for the linear segment of (17) to obtain the coefficient $\hat{\rho}$. We take the spreads of the benchmark 10-year bonds of a sample comprised of our high debt countries (Italy, Portugal, Spain) spanning 2015Q1–2020Q1. We start the calibration from 2015 to minimise any effects from the euro debt crisis and end prior to the launching of PEPP. We obtain an estimate $\hat{\rho} = 3.2$, i.e., each percentage point of debt to GDP adds 3.2 basis points to the spreads, in line with estimates in the literature (Zenios et al., 2021). Online Appendix A.5 reports the calibrated premia.

3.3 Calibration of the PEPP effects on spreads

We calibrate the unconventional monetary policy effects on spreads in two steps. First, we obtain an estimate of the effects of the PEPP on sovereign spreads by computing the counterfactual. We assume this rate from eurozone recent past data, which is much lower than the 2021 inflation, and simulate an inflationary surprise later.
Second, we calibrate PEPP-induced spread suppression as a function of cumulative sovereign debt holdings by the ECB\textsuperscript{11}.

### 3.3.1 Counterfactual spreads

We use $\rho(d_{i,t}, p_{i,t})$ without PEPP, i.e., $p_{i,t} = 0$, and using the calibrated (16)–(17) above we obtain out-of-sample predictions of spreads with the increase of debt from 2020Q2 onwards. This is the counterfactual should there be no PEPP illustrated in Figure 4 with the average of the eurozone high-debt countries shown in Figure 1 panel B.

**Figure 4 – Actual vs fitted sovereign spreads**

(A) Italy
(B) Portugal

(C) Spain

### 3.3.2 PEPP-induced spread suppression

We estimate the unconventional policy effect $\rho_{U_i}(p_{i,t})$ by examining the relationship between actual benchmark maturity spreads of the $i$th country, $\rho_{i,t}$, the out-of-sample counterfactual spread estimated above, and the net asset purchases $p_{i,t}$ as a proportion of the country’s PEPP envelope. We estimate the spread suppression rate as the proportional reduction of actual

\textsuperscript{11}The model can be adapted to include term premia, which play a central role in asset purchase programs such as the Federal Reserve Large-Scale Asset Purchases during the great financial crisis (Bowdler and Esteves, 2013; Gagnon et al., 2011).
spreads relative to the counterfactual

\[
\hat{\rho}_U(p_{i,t}) = \left[1 - \frac{\rho_{i,t}}{\hat{\rho}_C(d_t)}\right].
\]  

(19)

Using data over the period spanning the start of ECB purchases in April 2020 to January 2021 (see footnote [2]), we compute the suppression rates for increasing ECB sovereign debt holdings for each country; see online Appendix Figure [B.1] panel A. ECB purchases strongly suppress spreads for all high debt countries in our sample, albeit at a decreasing rate.

The data are clustered by country and given the small number of observations we fit a mixed-effect model over the whole data-set accounting for the correlation among the data due to grouped observations. That is, we fit equation (18) with random intercepts and slopes.

We fit different specifications assuming the same polynomial for all countries. The best fit is given by a country-specific intercept and linear slope with random effects, see online Appendix [B]. We display the fit for each country in Figure 5 together with the OLS fit over all countries. We use only the rising part, with the spread suppression reaching a plateau as the holdings approach the country’s envelope as is visible for Portugal but not yet for Italy and Spain. From the roots of (18) we estimate the maximum spread suppression rate \( \bar{\rho}_i \) and the amount \( p_i \) at which the maximum is reached, to complete the empirical calibration; details can be found in online Appendix [B]. For our tests we use the OLS estimates.

**Figure 5** – PEPP-induced spread suppression

![Figure 5](image)

### 4 Model at work

We perform DSA for the representative country under the counterfactual and under three PEPP unwinding alternatives, namely (i) purchases rolled over until 2024 with unwinding from 2025 lasting for five years, referred to as early quantitative tightening (EarlyQT), (ii) late quantitative tightening (LateQT) with unwinding commencing in 2030, and (iii) debt rolled over forever
(QEternity). We also consider an inflation surprise with and without monetary policy reaction. Our baseline tests are with the intermediate debt financing strategy (point B in Figure 3).

4.1 Debt sustainability without PEPP

We run the model in the absence of PEPP and show in Figure 6 the scenarios of risk premia and debt stock ratios. The corresponding expected cost of debt financing is 4.4% of GDP. Panel A shows large and growing risk premia, reminiscent of the eurozone crisis, and the debt stock ratio in panel B remains above the pre-pandemic level and is increasing with a very high probability at the 10 year horizon. Thus, without PEPP, debt is unsustainable according to our criterion. Additional results at the minimum cost and minimum risk points and for debt flow are in online Appendix C.

Figure 6 – Debt sustainability analysis without PEPP

Dashed line denotes the mean value and the shaded areas denote 0.25, 0.75, and 0.90 quantiles.

(A) Risk premia

(B) Debt stock

If we impose the debt sustainability constraint with \( \delta = 0 \) the model becomes infeasible, and we cannot identify a debt financing strategy to stabilise debt. Using the model extension from (Zenios et al., 2021, section 6) we compute the stabilising fiscal effort that renders debt stock dynamics sustainable with probability at least 0.75 by the end of the simulation horizon. We find that a fiscal effort averaging 1.28% GDP p.a. over eighteen years will stabilise debt. Such level of fiscal adjustment are excessively ambitious (Eichengreen and Panizza, 2016) and hard to sustain.

4.2 Debt sustainability with PEPP

The PEPP reduces spreads according to the spread suppression function. When the program unwinds we assume that the effect reverts symmetrically back to the origin, so that the effect of unwinding is the same as the effect of purchases.\(^{13}\)

\(^{12}\)We set \( \alpha = 0.95 \) to solve the model with high precision, but we report results and draw inferences at the more realistic, for policy analysis, confidence level 0.75. For instance, the 0.95 quantile in Figure 2 Panel B, is slightly increasing, but the 0.75 quantile is declining and for all practical purposes we consider this example to represent sustainable debt dynamics.

\(^{13}\)PEPP was launched in the context of spooky markets. If the unwinding takes place under normal market conditions, we could expect an asymmetric effect, as the reverse impact on spreads could be more muted.
We run the model with the PEPP and the different unwinding strategies, and report the results in Figure 7. The risk premia (panels A-C) are significantly reduced during the programme implementation, as expected. Once the PEPP is fully implemented, but before unwinding commences, the spreads are stable with a high probability of declining due to GDP growth (denominator effect). Upon unwinding, the spreads increase sharply, but even with an early quantitative tightening the average spreads return to levels slightly below the counterfactual mean levels shown by the dashed blue line. The expected cost of debt financing is reduced from 4.4% of GDP in the counterfactual to 3.0%, 2.4%, and 1.6% under the three unwinding strategies, respectively.

The corresponding average debt ratios (panels D-F) are below the pre-pandemic level, in contrast to the no PEPP case. With late unwinding the spreads remain much below the counterfactual, because of the denominator effect and of low refinancing costs during the PEPP contributing to lower debt servicing costs. In all cases, debt is on a declining trajectory at the 10 year horizon with a probability 0.75, that is, debt is sustainable under all unwinding strategies. In panel G we show the mean and the interquartile range of the debt ratio at the end of the simulation horizon, and for all PEPP schedules, the debt ratio converges to an average value below the starting point.

Panels D-F show that at the end of the simulation period debt trends upwards. We go a step further and calculate the stabilising fiscal effort to put debt dynamics on a non-increasing trajectory by the end of the horizon with probability 0.75 and show the results in Figure 8. The bars (left y-axis) show the fiscal effort as percentage of GDP p.a., averaged over the number of years during which this effort must be sustained (solid line, right y-axis). The fiscal effort is lower should the PEPP be kept longer. Without the PEPP, as noted above, we need a fiscal effort averaging 1.28% GDP p.a. over eighteen years. With EarlyQT the effort is about 0.90% p.a. over ten years and it is further reduced to 0.24% p.a. over 5 years under LateQT. With QEternity the spreads are very low and debt can be stabilised even if the country runs a deficit, corroborating the arguments of Blanchard (2019) that in a low interest rate environment public debt may be rolled over without incurring future fiscal cost. Early unwinding of PEPP implies higher stabilising primary balance over longer periods.

In conclusion, the PEPP substantially lowers debt stock with its effects going beyond the end of the programme, and public debt is restored to the pre-pandemic level by 2030. Unwinding the central bank balance sheet, puts upward pressure on spreads. The expected value of debt stock is stabilised or declining in the 10-year horizon. A primary balance of 0.2-0.9% of GDP over five to ten years, depending on the speed of unwinding, can stabilise debt dynamics with a high probability by the end of the simulation. This dynamic epitomises the significant interaction between fiscal and unconventional monetary policies.

There are two assumptions of our model that moderate the impact of the PEPP and its unwinding on the observed debt trajectories. First, only the policy risk free rate enters in the IS curve, as including spreads as functions of debt would increase the computational complexity (footnote 8). Increases in spreads are expected to depress output; if this effect were captured, the favourable impact of PEPP on the debt ratios would be bigger. Second, if the unwinding is done in tranquil times, as opposed to the turbulent period in which the PEPP was launched and
Figure 7 – Debt sustainability analysis with PEPP

This figure displays results of debt sustainability analysis under the unconventional PEPP intervention. The x-axis denotes years and the y-axes denote spreads in bp (Panels A-C) and debt-to-GDP ratios in % (Panels D-F). The dashed black line denotes the mean value and the shaded areas denote 0.25, 0.75, and 0.90 quantiles. The panels also show (blue line) the mean values with no PEPP and the pre-pandemic values, for comparison. Panel G illustrates the mean and inter-quartile range of the debt ratio at the end of the horizon, together with the initial and the pre-pandemic debt ratio.

(A) Risk premia, EarlyQT  
(B) Risk premia, LateQT  
(C) Risk premia, QEternity

(D) Debt stock, EarlyQT  
(E) Debt stock, LateQT  
(F) Debt stock, QEternity

(G) Mean and interquartile range of debt stock at end of simulation horizon

we used for the calibration, we would expect that the upward pressure from PEPP unwinding on spreads would be less than the downward push at the program inception. Then the effect would be asymmetric, instead of symmetric as assumed by our suppression function (footnote 13), reducing the negative impact of unwinding on debt dynamics. Therefore, our exercises may
Figure 8 – Stabilising the debt dynamics with fiscal effort

This figure displays the fiscal effort required to put debt dynamics on a non-increasing trajectory by the end of the horizon, with probability at least 0.75. The bars (left y-axis) show the fiscal effort as a % of GDP p.a., averaged over the number of years during which this effort must be sustained, shown by the line (right y-axis).

<table>
<thead>
<tr>
<th>PEPP exit strategy</th>
<th>Fiscal effort (% GDP)</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>NoPEPP</td>
<td>0.0</td>
<td>-5</td>
</tr>
<tr>
<td>EarlyQT</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>LateQT</td>
<td>1.0</td>
<td>5</td>
</tr>
<tr>
<td>QEternity</td>
<td>-0.5</td>
<td>10</td>
</tr>
</tbody>
</table>

underestimate the positive effect of launching the PEPP programme and its unwinding on debt sustainability.

4.3 Inflation surprise

We consider an inflation shock of 4% with little persistence (online Appendix Figure A.3 panel A) with the central bank responding with a Taylor rule interest rate hike (panel B). Alternatively, we design another exercise in which the central bank views the shock as transitory and does not react to it (panel C), such as was the stance of the ECB until the end of 2021.14

We show the effects of inflation on debt dynamics in Figure 9. The fan charts display the risk premium (panel A) and debt stock (panel B) of the no-PEPP counterfactual without inflation shock, with the dashed black lines showing the mean values. The blue lines show the mean values in the presence of an inflation shock when the central bank responds by raising interest rates, and the green lines show the means without central bank response. The results are intuitive in the direction of change, and the magnitude of the inflation effect is small, in line with Hilscher, Raviv, and Reis (2021). Debt dynamics worsen if the central bank responds to inflation, but the increase of debt ratio is small since the increase in financing costs (i.e., numerator) is partly offset by the increase of nominal GDP (i.e., denominator). If the central bank does not increase the rates, debt dynamics improve slightly due to the inflation effect. Overall, a short lived inflation shock and conventional monetary policy response have small effects on debt, supporting the view that monetary policy and inflation are neutral in the long run. The debt of the representative country remains on an increasing trajectory.

Would the PEPP improve debt dynamics in the presence of an inflation shock? In Figure 9, panel C, we show the mean debt ratio and the inter-quartile range at the end of the horizon, together with the starting debt ratio as a reference. The first set of lines displays the ratios without the PEPP and for the two different scenarios of how the central bank reacts to the inflation shock. The other three sets show the outcome with the PEPP unwinding strategies in the presence of an inflation shock. In all cases we observe the same slight deterioration of debt dynamics when the central bank responds to inflation by increasing interest rates, but the strong positive effect of the PEPP persists.

An inflation spike may have an indirect effect on debt by accelerating the PEPP unwinding, with EarlyQT being more likely than LateQT as the ECB has suggested recently.\textsuperscript{15} Comparing these two cases we observe that more rapid unwinding worsens debt dynamics. This comparison uncovers another channel of unconventional monetary policy effects on sovereign debt.

5 Optimal debt financing and monetary policy

We use the optimal debt financing feature of the model to explore the cost-risk tradeoff and the optimal debt maturity issuance under different PEPP schemes.

5.1 The cost-risk tradeoff

We vary the risk parameter $\omega$ to trace the efficient frontiers of expected interest payments vs gross financing needs, with different unwinding schedules. We show the results in Figure 10 together with the counterfactual. We note a frontier shift down and to the left, with significant reduction of both financing cost and refinancing risk. The improvements are larger for longer duration PEPP. EarlyQT produces cost savings up to 1.5% GDP over no PEPP, with a simultaneous significant reduction of tail risk of gross financing needs.

We illustrate on this figure possible Pareto moves of a PDMO that stays within its mandate to finance government borrowing at the lowest cost against acceptable risks. For instance, from the intermediate point B without PEPP a move along the vertical arrow reduces financing cost at constant risk. This move brings us closer to the minimum cost point C suggesting issuing debt of shorter maturities. Alternatively, a move along the diagonal reduces both cost and risk, but we are now closer to the minimum risk point A suggesting maturity lengthening.

5.2 Optimal debt maturities

How do these moves affect the issued debt maturities? We look at the debt financing strategy that reduces simultaneously cost and risk by moving along the diagonal arrow in Figure 10 and plot the results in Figure 11. The bars in panel A are the optimal weighted average maturity at issuance (WAMI, left y-axis), and the black line shows the refinancing risk (right y-axis). The expected cost of debt financing (not shown) is reduced from 4.4% of GDP without PEPP to 3.2%, 2.5%, and 2.0% as PEPP unwinding takes longer. We observe a lengthening of maturities with a lowering of risks. For instance, EarlyQT increases WAMI by about 12 months, corresponding

\textsuperscript{15}Financial Times, ECB scales back stimulus plan as Ukraine war drives up inflation expectations, March 10, 2022.
Figure 9 – Effects of an inflation surprise

The figure displays the fan charts for risk premium (panel A) and debt stock (panel B) under different conventional monetary policy responses to an inflation shock. The coral shaded fan chart is the baseline without PEPP or inflation surprise, with the dashed black lines showing the mean values. The dashed black line denotes the mean value and the shaded areas denote 0.25, 0.75, and 0.90 quantiles. The dashed blue lines are the mean values with an inflation shock and a conventional monetary policy response using the Taylor rule, and the dashed green lines are the mean values when the central bank does not react to the inflation spike. Panels C illustrates the mean and inter-quartile range of the debt ratio at the end of the horizon without PEPP and for the three unwinding strategies, with the dashed line indicating the initial debt ratio. Lines marked A are without inflation shock, B are with inflation shock and increase in interest rates, and C are when the central bank does not react.

(A) Risk premia

(B) Debt stock

(C) Debt ratio at the end of the horizon
to a lengthening of about 10 months per percentage point of financing cost reduction. Panel B shows the same information for a move down the vertical arrow. WAMI is reduced from 5.7 to 4.1 years with the expected cost (not shown) reduced from 4.4% to 1.5% of GDP. Under PEPP we can achieve significantly lower cost at the same or lower risk. Maturities lengthen or shorten depending on PDMO preferences.

We compare the optimal financing strategies with the empirical findings of Plessen-Matyás et al. (2021) who document a link between the Public Securities Purchase Programme and longer-dated public debt in the eurozone. They show that a one p.p. decrease in government
bond yields led to a lengthening of issuance maturities by seven months on average. This is in line with the maturity extension of ten months per p.p. of yield reduction obtained from our model under a simultaneous reduction of cost and risk. The model optimal recommendations are in the direction of the eurozone PDMO behaviour, and quantitatively they are remarkably close.

In conclusion, the model suggests an optimal adjustment of PDMO debt financing strategy in response to the PEPP. The policy implication is that funding maturities should be treated as endogenous in impact assessment of central bank purchase programmes.

6 Conclusions

We embed a macroeconomic framework with monetary policy in stochastic debt sustainability analysis to study the effects of monetary policy on debt sustainability. Monetary policy is introduced into DSA through a macro monetary model that incorporates conventional monetary policymaking and the effect of central bank’s debt purchases on debt financing costs.

We calibrate the extended DSA model to a representative high-debt eurozone country, including an estimation of the impact of the PEPP on sovereign spreads, and use it to assess the impact of the programme on debt sustainability. Our analysis highlights the relevance of PEPP in the sovereign debt markets. We find that (i) the program has substantial favourable effects on debt dynamics that persist after the programme is finished, rendering debt sustainable; without it a very large fiscal effort would be required to stabilise debt, (ii) reducing the size of central bank balance sheet as holdings of public debt mature puts upward pressure on spreads; while the rise in spreads adversely impact the debt dynamics (the larger with earlier unwinding), debt remains sustainable with a high probability (iii) an inflation surprise does not impact strongly debt dynamics and the magnitude of the impact depends on the monetary policy reaction. Yet, higher inflation could accelerate PEPP unwinding, with a negative impact on debt dynamics.

These findings point to sizeable favourable effect of the PEPP on debt sustainability which are only partly reversed as the programme is unwound. Although passive quantitative tightening impacts negatively on debt dynamics it does not jeopardise fiscal sustainability. This finding suggests that the risk of fiscal dominance going forward is limited.

Our analysis has implications for public debt management. The PEPP induces a Pareto improvement in the tradeoff between financing cost and refinancing risk. We find that it is optimal for public debt issuers to lengthen maturities to reduce the risk without increasing cost, and the model recommendation is in line with the empirically observed behaviour of eurozone public debt management offices during the Public Securities Purchase Programme. This has a policy implication that funding maturities should be treated as endogenous in impact assessment of asset purchase programmes.

A direction for future work is to refine the model. We could account for the effect of spreads on growth through the IS curve (footnote 8). Perturbations of the scenario tree to account for the spreads, given the debt level could, refine the estimates. A second refinement would be to allow for asymmetric effects of the PEPP unwinding in the spread suppression function (footnote 13) by muting the effect of the unwinding if done under normal market conditions.
Note that these two refinements, if anything, would strengthen our finding on the positive impact of the PEPP on debt sustainability.

Our methodological innovation is relevant beyond the study of asset purchase programs. Introducing a central bank allows us to perform DSA for long-horizon problems such as those arising from the fiscal pressures of an ageing population (Dieppe and Guarda, 2015; Kamiguchi and Tamai, 2019) or from climate change (Boone, Fels, Jorda, Schularick, and Taylor, 2021; Zenios, 2021). For these long-horizon problems, projections about monetary conditions from historical data are of limited value, whereas a central bank module can help to deliver theoretically sound forward-looking projections.
References


A Data Appendix

A.1 Representative country debt

The legacy debt of the representative high-debt country is shown in Figure A.1.

Figure A.1 – Legacy debt of eurozone representative high-debt country

A.2 Conventional monetary policy

In Table A.1 we give the parameters for the Phillips curve eqn. (11), output gap eqn. (12), and Taylor rule eqn. (14), drawing from existing literature (Hofmann et al., 2021). \( r^* \) is set to zero.

Table A.1: Parameters of the calibrated macroeconomy and monetary policy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_\pi )</td>
<td>0.3</td>
<td>Persistence of past inflation</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.2</td>
<td>Coefficient of output gap in Phillips curve</td>
</tr>
<tr>
<td>( \delta_{\dot{y}} )</td>
<td>0.2</td>
<td>Persistence of past output gap</td>
</tr>
<tr>
<td>( \alpha_{\dot{y}} )</td>
<td>0.5</td>
<td>Coefficient of interest rate in output gap</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>0.2</td>
<td>Persistence of past interest rate in Taylor rule</td>
</tr>
<tr>
<td>( \alpha_\pi )</td>
<td>0.4</td>
<td>Coefficient of inflation in Taylor rule</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>0.25</td>
<td>Coefficient of output gap in Taylor rule</td>
</tr>
<tr>
<td>( \pi^* )</td>
<td>2</td>
<td>Target inflation rate</td>
</tr>
</tbody>
</table>

A.3 Scenario tree moments

We estimate standard deviations and correlations of the macroeconomic variables from historical yearly data from the WEO over the period 1998 to 2019, and show the results in Table A.2.
For the real growth and primary balance time series we average the data of Italy, Spain, and Portugal, equally weighted. The inflation rate and output gap time are for the Eurozone.

Table A.2: Moments for the calibration of a scenario tree

<table>
<thead>
<tr>
<th>Factors</th>
<th>Standard deviation</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real growth</td>
<td>1.91</td>
<td>1 0.51 0.42 0.30</td>
</tr>
<tr>
<td>Output gap</td>
<td>1.62</td>
<td>1 0.37 0.62</td>
</tr>
<tr>
<td>Primary balance</td>
<td>1.33</td>
<td>1 0.10</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.82</td>
<td>1</td>
</tr>
</tbody>
</table>

A.4 Scenario tree

We calibrate the scenario tree for the representative country in the eurozone. The tree is centred on market expectations, and matches the standard deviations and correlations of Table A.2 using the moment matching method of Consiglio et al. (2016). For computational tractability we calibrate a tree of 256 scenarios over five years and after that we assume that the variables converge to their long-term trends. The scenario tree is calibrated for 2021–2050 and we display in Figure A.2 the fan charts of the four state variables, real GDP growth, output gap, primary balance, and inflation. The dashed lines show the equilibrium input data. We also show in the figure the nominal interest rate obtained by applying the Taylor rule on states of the tree.
Figure A.2 – Fan charts from the calibrated scenario tree

(A) Real GDP growth  
(B) Output gap

(C) Primary balance  
(D) Inflation

(E) Nominal interest rates
A.5 Risk and term premia

For the term-premia parameters in (16) and the other parameters of (17), we follow Zenios et al. (2021) and calibrate using panel data from 23 EU countries over the period 1995–2020. We obtain an estimate $\hat{\rho} = 3.2$, with $a_j$ equal to -25, 0, and 50, for the 3-, 5- and 10-year bonds. $b_j$ has marginal effects on our results and we set it equal to zero, $d_{\text{min}} = 60$ and $d_{\text{max}} = 220$. The maximum spread with this calibration is about 500bp, as observed during the eurozone debt crisis before the crisis countries were cut off the markets and debt financing cost was stabilised by official sector support.

A.6 Inflation surprise

The inflation shock is shown in Figure A.3 panel A. Panel B shows the monetary policy response to this shock, with interest rates set according to our calibration of the Taylor rule with $\theta_i = 0.2$ from Table A.1. Panel C shows the case where the central bank views the inflation shock as transitory and does not adjust the interest rate with the baseline Taylor rule but instead persists in its pre-inflation trajectory with $\theta_i = 0.7$.

Figure A.3 – Inflation surprise and conventional monetary policy responses

(A) Inflation shock

(B) Nominal interest rates response

(C) Persistent nominal interest rates
Fitting the PEPP spread suppression models

Using data over the period spanning the start of purchases in April 2020 to January 2021, we display in Figure B.1 the realized suppression rates versus the ECB holdings of sovereign debt for each country in our sample. We first fit a second order polynomial using OLS on the panel data as our baseline Model-0. We then test the following country-specific mixed-effect models to fit eqn. (18):

Model-1. \( \beta_{0i} = \gamma_{00} + u_{0i} \), where the intercept varies by country with the random effect \( u_{0i} \).

Model-2. \( \beta_{0i} = \gamma_{00} + u_{0i} \) and \( \beta_{1i} = \gamma_{10} + u_{1i} \), where the intercept and the first order coefficient are country dependent with random effect terms \( u_{0i} \) and \( u_{1i} \).

Model-3. \( \beta_{0i} = \gamma_{00} + u_{0i} \), \( \beta_{1i} = \gamma_{10} + u_{1i} \) and \( \beta_{2i} = \gamma_{20} + u_{2i} \), where all coefficients vary across countries, with \( u_{0i} \), \( u_{1i} \), and \( u_{2i} \) describing the grouping effect.

The mixed-effect models are fitted using the R package \texttt{lme4}, and we report the Akaike and Bayesian information criteria in Table B.1. We observe that Model-2 is the best performing, and the estimated parameters are displayed in Table B.2.

![Figure B.1 – Spread suppression rates vs ECB holdings](image)

<table>
<thead>
<tr>
<th>Model</th>
<th>Akaike</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-0</td>
<td>120.3</td>
<td>123.4</td>
</tr>
<tr>
<td>Model-1</td>
<td>109.8</td>
<td>113.7</td>
</tr>
<tr>
<td>Model-2</td>
<td>104.8</td>
<td>109.4</td>
</tr>
<tr>
<td>Model-3</td>
<td>114.3</td>
<td>122.1</td>
</tr>
</tbody>
</table>

In Figure B.2 we plot the residuals versus predictive values of Model 2, with the QQ and index plot, and the histogram of Pearson residuals with the fitted normal. The residual plot...
Table B.2: Estimates of the best fitting PEPP spread suppression Model-2

<table>
<thead>
<tr>
<th>Country</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$p_t$ (at Jan. 2021)</th>
<th>$\bar{p}$</th>
<th>$\bar{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>-26.70</td>
<td>3.17</td>
<td>-0.03</td>
<td>49.90</td>
<td>55.79</td>
<td>61.71</td>
</tr>
<tr>
<td>Portugal</td>
<td>-1.28</td>
<td>2.92</td>
<td>-0.03</td>
<td>46.48</td>
<td>51.31</td>
<td>73.51</td>
</tr>
<tr>
<td>Spain</td>
<td>-21.12</td>
<td>3.12</td>
<td>-0.03</td>
<td>47.25</td>
<td>54.97</td>
<td>64.73</td>
</tr>
</tbody>
</table>

shows no specific pattern around the horizontal dashed line (as also the index plot), suggesting that the chosen model is reasonable. Moreover, residuals scatter uniformly around the 0-line, implying that the error term is homoscedastic and Gaussian distributed, also seen in the Q-Q plot and the histogram.

Figure B.2 – Diagnostic tests of a mixed-effects PEPP model of spreads
C Additional results

We show here results for the risk premium, debt stock, and debt flow at the minimum risk and the minimum cost financing strategies, together with the intermediate strategy used in the main paper. Debt violates the 20% threshold with very high probability starting at 2027. The results shown in Figure C.1 without PEPP and Figure C.2 corroborate the findings we report in the main paper. The minimum cost strategy lowers the pace at which debt dynamics increase without PEPP, but they still do, whereas PEPP has significant positive impact and renders debt sustainable even at the low-risk/high-cost strategy. Our main findings are robust to the debt financing strategy used.

Figure C.1 – Debt sustainability analysis without PEPP

This figure displays the counterfactual of debt sustainability variables without the unconventional PEPP intervention at three different points of the efficient frontier of Figure 10: minimum risk point A, minimum cost point C, and intermediate tradeoff point B.
Figure C.2 – Debt sustainability analysis with PEPP

(A) Risk premia, EarlyQT

(B) Risk premia, LateQT

(C) Risk premia, QEternity

(D) Debt stock, EarlyQT

(E) Debt stock, LateQT

(F) Debt stock, QEternity

(G) Debt flow, EarlyQT

(H) Debt flow, LateQT

(I) Debt flow, QEternity
## Previous volumes in this series

<table>
<thead>
<tr>
<th>Volume</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1033</td>
<td>The Holt-Winters filter and the one-sided HP filter: A close correspondence</td>
<td>Rodrigo Alfaro and Mathias Drehmann</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1032</td>
<td>Capital flows and monetary policy trade-offs in emerging market economies</td>
<td>Paolo Cavallino and Boris Hofmann</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1031</td>
<td>Risk capacity, portfolio choice and exchange rates</td>
<td>Boris Hofmann, Ilhyock Shim and Hyun Song Shin</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1030</td>
<td>Mis-allocation within firms: internal finance and international trade</td>
<td>Sebastian Doerr, Dalia Marin, Davide Suverato and Thierry Verdier</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1029</td>
<td>Bank of Japan’s ETF purchase program and equity risk premium: a CAPM interpretation</td>
<td>Mitsuru Katagiri, Junnosuke Shino and Koji Takahashi</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1028</td>
<td>Fiscal deficits and inflation risks: the role of fiscal and monetary regimes</td>
<td>Ryan Banerjee, Valerie Doctor, Aaron Mehrotra and Fabrizio Zampolli</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1027</td>
<td>What drives repo haircuts? Evidence from the UK market</td>
<td>Christian Julliard, Gabor Pinter, Karamfil Todorov and Kathy Yuan</td>
</tr>
<tr>
<td>July 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1026</td>
<td>Monetary policy announcements and expectations: the case of Mexico</td>
<td>Ana Aguilar, Carlo Alcaraz Pribaz, Victoria Nuguer and Jessica Roldán-Peña</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1025</td>
<td>Communication, monetary policy and financial markets in Mexico</td>
<td>Ana Aguilar and Fernando Pérez-Cervantes</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1024</td>
<td>Forward guidance and expectation formation: A narrative approach</td>
<td>Christopher S Sutherland</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1023</td>
<td>Monetary policy press releases: an international comparison</td>
<td>Mario Gonzalez and Raul Cruz Tadle</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1022</td>
<td>Effects of Banco de la Republica’s communication on the yield curve</td>
<td>Luis Fernando Melo-Velandia and Juan J Ospina-Tejeiro</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1021</td>
<td>Seeing the forest for the trees: Using hLDA models to evaluate communication in Banco Central do Brasil</td>
<td>Angelo M Fasolo, Flávia M Graminho and Saulo B Bastos</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1020</td>
<td>Alternative monetary-policy instruments and limited credibility: an exploration</td>
<td>Javier García-Cicco</td>
</tr>
<tr>
<td>June 2022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1019</td>
<td>Unconventional credit policy in an economy under zero lower bound</td>
<td>Jorge Pozo and Youel Rojas</td>
</tr>
<tr>
<td>May 2022</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All volumes are available on our website www.bis.org.