

## What drives investor risk aversion? Daily evidence from the German equity market<sup>1</sup>

Stock prices move as corporate earnings prospects change but they also move as investors change their aversion to risk. Aversion to risk gives rise to a risk premium, which consists of an expected extra return that investors require to be compensated for the risk of holding stocks. Option prices are a unique source of information for the estimation of risk premia. The way strike prices in option contracts distinguish between outcomes that are relatively favourable to investors and those that are relatively unfavourable allows an estimate of risk aversion to be extracted from observed option prices. This is done by comparing what is implied in option prices with the probabilities of various outcomes from a purely statistical point of view.

The purpose of this special feature is to explain daily movements in the risk aversion of investors in the German stock market as reflected in option prices.<sup>2</sup> We focus on the main German index, the Dax, which summarises the stock prices of 30 major German companies. Our data on Dax option prices consist of daily observations from December 1995 to May 2002. To explain movements in our measure of risk aversion, we examine indicators of expectations about economic growth, market volatility, credit risk premia and negative news events. We find that investors in the German equity market seem to have become increasingly risk-averse since 1998. In addition, we note that movements in US stock prices have a strong impact on this risk aversion.

We complement the study of Tarashev et al (also in this *Quarterly Review*) in three respects. First, we analyse risk aversion at a higher frequency: we examine daily movements, while they examine monthly movements. Second, we measure risk aversion in a slightly different way – particularly in estimating statistical probabilities – thus allowing a comparison of two measures and potentially providing a sense of the robustness of option-based measures. Finally, we go a step further by attempting to identify factors that would explain the changes in risk aversion from one day to the next.

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<sup>1</sup> This work was largely carried out while the author was visiting the BIS. The views expressed in this article are those of the author and do not necessarily reflect those of the BIS or the Austrian National Bank. I am indebted to Ernst Glatzer for expert statistical help.

<sup>2</sup> Beber and Brandt (2003), Rosenberg and Engle (2002), Aït-Sahalia et al (2001) and Jackwerth (2000) also explore measures of risk aversion.

## Estimating two distributions

### Measuring risk aversion

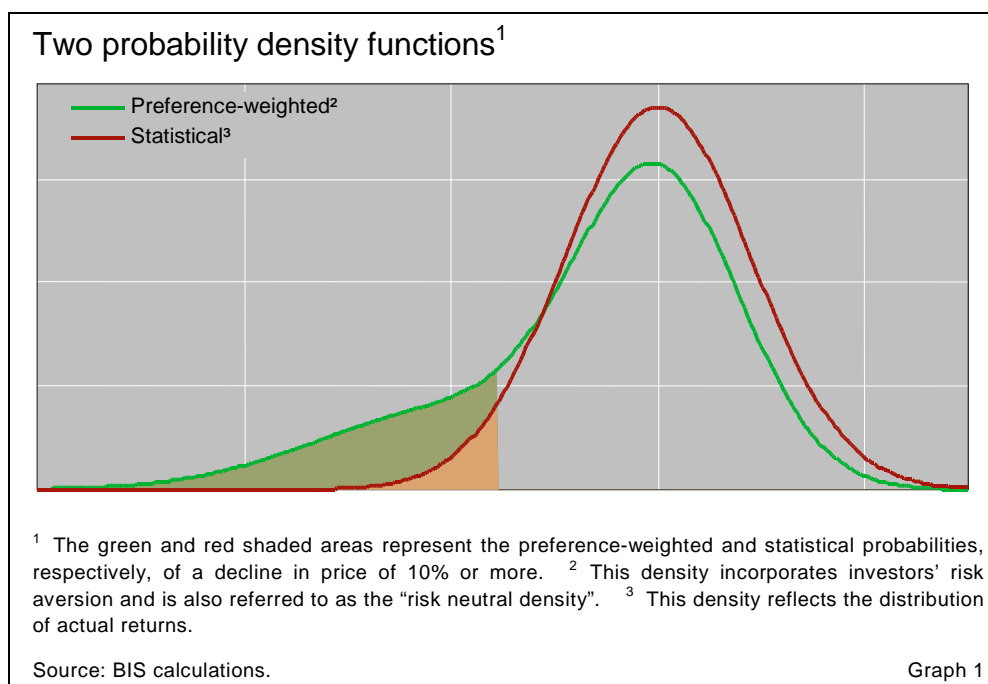
We measure risk aversion by comparing two estimates of the probability density function (PDF) for future stock prices. One estimate is extracted from option prices, while the other is estimated from realised movements in stock prices. Risk aversion can be viewed as accounting for the difference between those two estimates. The components of our methodology are illustrated in Graph 1. The green line represents the probability density extracted from option prices and the red line the density estimated from actual stock price movements. The estimation details are outlined in the box on the next page.

When traders price options, they are in effect applying preference-weighted probabilities of different possible asset price outcomes for the period until the derivative expires. In other words, the observed option price incorporates the traders' perceptions of the future movement of the asset price together with their degree of risk aversion, which may change over time.<sup>3</sup> From the time series of actual returns we estimate the statistical PDF, represented by the red line in Graph 1. This is a purely statistical model and therefore contains no information on risk aversion. In our comparison of the two PDFs, we concentrate on the probability of declines in stock prices. From both PDF estimates, we compute the probability of a Dax decline of 10% or more relative to the forward value of the index over 49 days. These probabilities correspond to the two shaded areas in the left tails.

If traders pay more attention to events with negative consequences for their wealth, the preference-weighted probability of future declines differs from the statistical probability by a distance reflecting the risk aversion. If traders are

Risk aversion drives a wedge between statistical probabilities ...

... and probabilities that reflect investors' preferences



<sup>3</sup> Jackwerth (1999) offers a survey on the estimation of the PDF from option prices.

## Estimation methods

### *Specification of the preference-weighted PDF*

The estimation of the preference-weighted PDF is based on the daily prices of put and call options and futures on the Dax index from December 1995 to May 2002.<sup>①</sup> Options are traded on the Eurex exchange by means of an electronic system. The original maturities of the options in our sample are one and two months.

In order to eliminate time to maturity effects from our estimates, we need to estimate a preference-weighted PDF with a constant horizon. If the effect of the downward trend in the maturity of the options were neglected, the parameters might change solely due to the fact that the expiry date was approaching. In particular, volatility decreases with each time increment, as uncertainty about the asset price on the day of maturity declines. To construct preference-weighted PDFs that are free of these spurious effects, we use an interpolation procedure on the implied volatility.<sup>②</sup> We interpolate between the contracts with one-month and two-month horizons to obtain a constant horizon of 49 calendar days.<sup>③</sup> Our proxy for the risk-free rate is the set of interbank interest rates.

Our parameterisation of the option-implied PDF is the mixture of log-normals model.<sup>④</sup> This flexible specification can generate a variety of shapes for the preference-weighted PDF. The mixture of two log-normal distributions is based on two regimes. In each regime, the stock price is log-normally distributed with a different mean and dispersion. The estimation of the parameters relies on non-linear least squares.<sup>⑤</sup>

### *Specification of the statistical PDF*

We specify the statistical PDF of the returns as an asymmetric GARCH model with a constant mean and a conditional normal distribution. This specification is frequently applied in the literature. The GARCH model is motivated by the stylised fact that the volatility of stock returns moves over time in a predictable manner. In our model, today's conditional volatility depends on yesterday's volatility, on a term representing the asymmetric relation between volatility and stock returns and on the squared return observed yesterday.

Our database for the statistical PDF consists of the daily closing values of the Dax index for the period between December 1993 and May 2002. The literature offers two methods for constructing the sample, namely an expanding or a rolling window of stock returns. While Tarashev et al (also in this *Quarterly Review*) use the former, we perform rolling GARCH estimations with a moving window of 500 observations. After each estimation, we compute volatility forecasts for the next 49 days. Then we move the sample forward by one observation and restart the estimation and forecasting algorithm. Given that we consider a rather long horizon, namely almost two months, we assume that the 49-day distribution can be approximated by a conditional normal PDF. Therefore, the estimation of the statistical PDF only requires the volatility forecast as an input. Once the 49-day conditional distribution is obtained, we compute the probability of a decline of 10% or more over seven weeks relative to the forward value of the index from the tail of the statistical PDF.

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<sup>①</sup> Dax options are specified as European options, therefore there is no need to take account of the early exercise of contracts. <sup>②</sup> Details of our estimation and filtering procedure are given in Glatzer and Scheicher (2003). <sup>③</sup> For the purpose of comparison, we have also estimated PDFs with a horizon of 42 and 56 days. The results remain unchanged. <sup>④</sup> This specification of the option-implied PDF was introduced by Melick and Thomas (1997). <sup>⑤</sup> Engle (2001) gives a concise introduction to GARCH models.

risk neutral, they do not charge a risk premium and the statistical and preference-weighted probabilities are equal. In reality, we observe that market participants assign higher weights to economic outcomes where wealth is low, such as in periods of economic downturn. This type of investor behaviour is called risk aversion and leads to significant premia for bearing risk. In consequence, we would expect traders to behave as if the probability of a future stock market decline were higher than the historical data would deem

necessary. This additional probability, illustrated by the difference between the shaded areas in the left tails of the two hypothetical PDFs in Graph 1, is exactly the difference between the preference-weighted PDF and the statistical PDF, which we attempt to capture empirically.

In the academic literature, the concept of the preference-weighted PDF is equivalent to that of the risk neutral PDF. Under risk neutrality, the preference-weighted probabilities incorporate risk aversion in such a way that the preference-weighted probabilities assigned to undesirable outcomes exceed the corresponding statistical probabilities. The preference-weighted probabilities are constructed such that if one uses them to take expected values, one will obtain the observed market price of the asset in question.

### *Estimation results*

For our last estimation date, in May of last year, a comparison of the two PDF estimates shows evidence of risk aversion. The likelihood of a Dax decline of at least 10% relative to the mean amounted to 10.7% in the preference-weighted PDF and 7.7% in the statistical PDF. This difference of 3% between the preference-weighted and the statistical probability is mainly caused by the longer tail of the option-implied PDF. Hence, the traders in the options market behaved as if the probability of large future declines of the Dax index were higher than historical experience would suggest. For instance, a fall from 4,800 index points to a level of 3,500 has a statistical probability close to zero, but the preference-weighted assessment of this possibility is still significant.

Risk aversion explains a 3% difference in probabilities

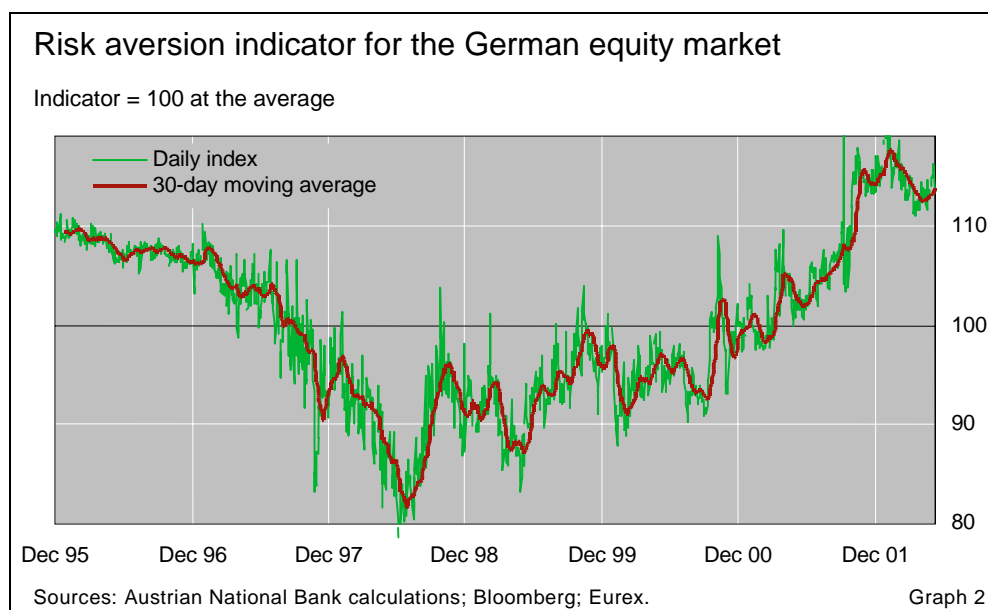
The time series of our measure of risk aversion in Graph 2 shows an upward trend since summer 1998. Inspection of Graph 2 also reveals that the downward movement on the German stock market since the first half of 2000 coincided with an increase in our estimated risk aversion. Note that there is a difference between our measure and that of Tarashev et al, especially for late 1997, when their measure shows a sharp rise in risk aversion. The difference indicates that the estimation of risk aversion measures is sensitive to the choice of the empirical procedure. In particular, the way the sample for the estimation of the statistical PDF is constructed affects the resulting risk aversion.<sup>4</sup>

Risk aversion has risen since summer 1998

The variability of the measure indicates substantial changes in the day-to-day behaviour of risk aversion. Overall, the measure starts above its mean in January 1996, moves to a minimum in June 1998 and then increases up to the end of our sample. Our measure of risk aversion is clearly related to the movement of the stock price in general. Linking the sharp upward moves in Graph 2 to the specific dates, we find a close correspondence between market developments and our measure of risk aversion. The first major spike is on 30 September 1998, when the financial crisis in Russia and the collapse of LTCM caused widespread turbulence in the global capital markets. The last visible jump occurred in the aftermath of the events of 11 September 2001.

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<sup>4</sup> In the present paper, we use a rolling sample of 500 observations, whereas Tarashev et al use an expanding sample, starting in January 1988 and increasing with each day by one observation.



A caveat in our analysis is that the difference between the preference-weighted PDF and the statistical PDF is affected not only by risk aversion, but also by structural differences between the stock and options markets. Our analysis of the determinants of risk aversion in the next section therefore relies on the changes in risk aversion. Hence, a constant bias between the two PDFs that is not related to risk aversion does not affect our results.

## Determinants of changes in risk aversion

### *Which factors drive the movement of risk aversion?*

Risk aversion may be affected by the business climate ...

Determinants of investors' risk aversion identified in the asset pricing literature are economic growth prospects, measures of equity and credit market risk, fluctuations in the exchange rate and negative news events in other equity markets.<sup>5</sup> A poorer outlook for the economy may raise risk aversion, because investors react to the increased likelihood of lower-wealth situations by reducing their willingness to bear risk. The slope of the term structure reflects the pessimism of market participants about the economic climate, because of the linkage of the term structure to investors' portfolio decisions. If investors expect the business climate to improve, they will shift some of their assets from short-maturity instruments into long-term bonds. This change in the portfolio composition will increase the short rate relative to the long rate, leading to a flatter slope of the term structure. Rising risk in the equity and credit markets makes it likely that future wealth will be lower and hence may lead to higher risk aversion. Our measures of equity market and credit risk are US implied equity volatility and the credit spread in the US capital markets.<sup>6</sup>

<sup>5</sup> See, for example, De Santis and Gerard (1997).

<sup>6</sup> The slope of the term structure is defined as the yield on 10-year benchmark German government bonds minus the three-month money market rate. To measure credit and equity

Declines in major equity markets may also explain the movements in risk aversion. In particular, the US stock market is a key source of negative news for German equity investors as Tarashev et al find that risk aversion indicators in the US, German and UK equity markets co-move significantly over time. We measure negative news from the US stock market by means of an interaction dummy set to unity for negative S&P 500 returns.

... by price developments in credit and stock markets ...

Changes in the exchange rate can reduce the profit flows of companies trading internationally. Many companies in the Dax index generate a substantial portion of their cash flows abroad, which is why the movement of exchange rates also influences their earnings. Depreciation of the US dollar relative to the euro, or the Deutsche mark,<sup>7</sup> leads to a fall in the relative competitiveness of German exports and hence may affect the risk aversion of traders by increasing uncertainty about future profits. Again, we represent negative news as an interaction dummy variable, which is set to unity for negative changes in the exchange rate.

... or by movements in exchange rates

We evaluate the effects of these five factors by means of a standard regression approach.<sup>8</sup> As mentioned, we analyse the first differences of the risk aversion. Thus, in our regression model we evaluate how a change in, for example, the slope of the term structure changes the risk aversion indicator. The estimation in first differences helps us mitigate any bias caused by systematic differences between the statistical and the preference-weighted PDF that are not related to risk aversion.

### *Empirical results*

In the simultaneous estimation with five explanatory variables, we find that three factors are significant.<sup>9</sup> The regression results in Table 1 demonstrate that risk aversion is strongly linked to changes in US volatility and to negative news with regard to the exchange rate. We observe a weaker linkage of risk aversion with the slope of the term structure and no relation to the downturn in the S&P 500 Index or the credit risk indicator. By means of the five variables, we achieve an explanatory value of 9% for the daily variability of risk aversion.

German risk aversion is indeed affected by exchange rates ...

The signs of the three significant factors are in accordance with the argument outlined above. The significant positive sign of US volatility indicates how the transmission of US stock market developments into risk aversion takes place. Risk aversion in the German equity market rises due to higher US volatility and not because of negative returns on the S&P 500. The negative linkage with the exchange rate confirms our interpretation in the framework

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market risk, we use the VIX series of implied volatility for the S&P 100 Index and the spread between the yields on BBB- and AAA-rated US industrials.

<sup>7</sup> We use the daily US dollar rate relative to the euro, because the trade-weighted (effective) exchange rate is only available at a monthly frequency.

<sup>8</sup> We use the ordinary least squares approach with White standard errors.

<sup>9</sup> To investigate robustness, we have repeated the regressions with the PDF difference for the probability of a fall of 20% or more in 49 days. The results remain unchanged.

Estimates of factor effects on changes in risk aversion		
Sample period: December 1995–May 2002		
Variable	Coefficient	t-statistic
German yield curve	-0.0204	<b>-1.9897</b>
Negative US stock return	-0.0257	-0.2470
US volatility	0.0027	<b>6.6364</b>
Depreciation in US dollar	-0.4665	<b>-3.2695</b>
Credit spread	-0.0063	-0.6492

Note: All variables enter the regression in first differences. Coefficients significant at the 5% level are highlighted in bold. Adjusted R<sup>2</sup> = 0.09.

Sources: Austrian National Bank calculations; Datastream. Table 1

of the relationship between exports and exchange rates. The importance of foreign trade for German companies implies that their earnings forecasts will deteriorate given an appreciation of the euro, or the Deutsche mark before 1999. These concerns about reduced profitability do indeed seem to raise the risk aversion measure. The slope of the German term structure has a negative effect on risk aversion in the German stock market. Our results show that a poorer outlook for the overall business climate raises the risk aversion of holders of German equities. Hence, investors react to poorer growth prospects by increasing the weight on economic outcomes where the payoffs are low.

Overall, we observe that the primary determinant of risk aversion is the uncertainty of traders in US stock markets, as represented by the implied volatility of US stock prices. Ranking the five explanatory variables according to their explanatory power, we find that US equity volatility significantly exceeds the other four variables.<sup>10</sup>

## Conclusion

In this special feature we have studied a daily measure of risk aversion for investors in the German stock market. We have explored linkages of the measure with developments in the US stock market, the term structure of German interest rates and the exchange rate against the US dollar. Our first finding is that risk aversion seems to have risen since summer 1998. We also record a volatility spillover from the US equity market into risk aversion in the German equity market. In this context, it must be mentioned that our estimate of US volatility is an option-implied measure. Hence, part of the linkage between German risk aversion and US volatility may reflect a linkage between risk aversions in the two countries. Our findings thus parallel the results of Tarashev et al. Despite differences in the estimation results due to the way the samples are constructed, we both document the influence of international information on movements in risk aversion.

<sup>10</sup> In the bivariate regression, the VIX change achieves an R<sup>2</sup> of 8%, whereas values for the other four variables are below 1%.

... and especially by US stock market volatility

A link between US and German risk aversion

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