The size of hedge adjustments of derivatives dealers' US dollar interest rate options

by

John Kambhu*

Federal Reserve Bank of New York

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Abstract

The potential for the dynamic hedging of written options to lead to positive feedback in asset price dynamics has received repeated attention in the literature on financial derivatives. Using data on OTC interest rate options from a recent survey of global derivatives markets, this paper addresses the question whether that potential for positive feedback is likely to be realised. With the possible exception of the medium term segment of the term structure, transaction volume in available hedging instruments is sufficiently large to absorb the demands resulting from the dynamic hedging of US dollar interest rate options even in response to large interest rate shocks.

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The size of hedge adjustments of derivatives dealers' US dollar interest rate options

Dealers' hedging transactions in underlying fixed income markets required for the management of the price risks of their options' business raises two questions. First, might dealers' hedging demands be so large as to disrupt the markets in the available hedging products? Second, is the dynamic hedging of dealers' residual exposures sufficiently large to justify a concern about positive feedback in price dynamics in the fixed income market?

The potential for dynamic hedging of written options positions to introduce positive feedback in asset price dynamics has received repeated attention in the literature on financial derivatives. A short and incomplete list would include, Grossman (1988), Gennotte and Leland (1990), Fernald, Keane and Mosser (1994), Bank for International Settlements (1986, 1995), and Pritsker (1997). Using data on OTC US dollar interest rate options from a survey of global derivatives markets, this paper assesses the likelihood of such positive feedback caused by dynamic hedging of options. The OTC interest rate options market is an interesting place to explore the positive feedback issue because dealers are net writers of these options (see Annex Table A2).

The estimates in this paper suggest that, with the possible exception of the medium term segment of the term structure, transaction volume in available hedging instruments is sufficiently large to absorb the demands resulting from the dynamic hedging of US dollar interest rate options. While a definitive answer to the positive feedback question would require data on investors' demand for interest rate products in addition to dealers' hedging demand arising from dynamic hedging of options (see Pritsker 1997), comparing potential hedging demand with transaction volume in typical hedging instruments might give a provisional assessment of the likelihood of positive feedback.

1. Introduction

The data in this paper are global market data for US Dollar OTC interest rate options from the April 1995 Central Bank Survey of Derivatives Markets (Bank for International Settlements, 1996). Using data on notional amounts and market values, strike prices were estimated such that when applied to the notional amounts, the strike prices generate the observed market values of the options. In particular, given maturity data (from the Survey) and market growth data (from ISDA[1]), estimates were generated of the notional amount of options by maturity and origination date (going back 10 years). Strike prices, based on historical interest rate data, were then assigned to the options originated at each point in time, such that the strike prices produced option values equal to those observed in the survey.

[1] International Swaps and Derivatives Association.

With the estimated strike prices and a postulated interest rate shock, we ask what would be the change in dealers' hedge positions that would restore the net delta of a (hedged) option portfolio to its initial level? This estimated hedge adjustment is the incremental net demand of dealers for hedge instruments, given the assumed interest rate shock. The estimated demand for hedge instruments might give some indication of the potential for positive feedback effects attributable to derivatives dealers' hedging of their OTC options portfolios.

2. Price sensitivity of the global dealer portfolio

Figure 1 shows the estimated price sensitivity of the global dealers' portfolio. The value at the prevailing forward rates is the amount reported in the Survey, and the values at the indicated changes in interest rates are estimated values. While dealers have sold more options than they have purchased, at the prevailing forward rates the bought options had higher market values and the net value of the global portfolio was positive (see Annex Tables A1 and A2). This relationship between the notional amounts and market values of bought and sold options implies that the options sold to customers had a lower degree of "moneyness" than options purchased from customers. The estimated strike prices are consistent with this relationship, as relative to swap rates at origination, sold options were found to be out-of-the money while options purchased from customers were estimated to be in-the-money.

Since dealers were net sellers of options, large interest rate shocks that drive the sold options into-the-money will cause the value of the sold options to dominate the portfolio value. Hence, the aggregate dealers' portfolio value becomes negative at interest rate shocks of more than 100 basis points. Figure 1 shows, however, that if the portfolio is hedged (but the hedge not dynamically adjusted) the value of the hedged portfolio would turn negative only after an extremely large interest rate shock. A rise of interest rates of almost 200 basis points would be required before the hedged portfolio value turns negative. Dynamically adjusting the hedge position as interest rates change would make such an adverse outcome even less likely.

The curverture of the option value function implies that the hedge position must be adjusted after an interest rate shock because the option values decrease at an increasing rate as interest rates rise. Without the hedge adjustment, the gain in value of the initial hedge position would no longer be sufficient to compensate for the declining option values. This need to dynamically adjust the hedge position as interest rates change introduces a potential for positive feedback. Since the required hedge is a short position in fixed income securities, the hedge adjustment would introduce additional sales into the market on top of the initial selling pressure that accompanied the initial interest rate shock.

Another feature of the aggregate dealer position is its exposure to rising interest rates: the negative slope of the option value curve at the prevailing forward rates in Figure 1. The conventional view of financial institutions' interest rate risk profile holds that these firms have a structural long position in the fixed income market. Namely, exposure to rising rates. Thus Figure 1 implies that, in the aggregate, dealers as a group can not hedge their net option exposures with offsetting structural exposures from other business lines. While some dealers may have offsetting exposures elsewhere in their firms that hedge their options position, Figure 1 suggests that not all dealers can fully hedge internally.

3. Dynamic hedging estimates

Dealers' options positions, especially of longer maturities, are most likely hedged with a variety of interest rate instruments. The market for US dollar interest rate products is sufficiently large and diverse that options dealers can choose from a wide range of hedging instruments, such as futures contracts, FRAs, interest rate swaps, Treasury securities, and interbank loans. While these instruments are not perfect substitutes because of differences in credit risks, transactions costs, and liquidity, economies of scale and diversification help dealers manage and intermediate these risks. If dealers have sufficient time to hedge a position or replace a hedge with a cheaper alternative, they are unlikely to encounter difficulty meeting their hedging needs. For immediate hedge adjustments in large volume, however, their alternatives may be more limited. Across the range of maturities that need to be hedged, the most liquid instruments available are Eurodollar futures, Treasury securities, and Treasury futures.

Eurodollar futures

The Eurodollar futures market appears to have transaction volume sufficiently large to accommodate the estimated hedge adjustments for small interest rate shocks. At shorter maturities, the Eurodollar futures market is more than large enough to accommodate dealers' hedging demands, even for large interest rate shocks. For hedging of longer maturity exposures, however, the Eurodollar futures market appears to be able to accommodate only the hedging of residual exposures (after the use of other hedging instruments) and marginal adjustments to hedge positions.

The largest daily turnover volume of Eurodollar futures contracts exceeds the estimated hedge adjustments: out to 10 year maturities, for a 10 basis point change in forward rates; out to 4 to 5 years, and also between 8 and 10 year maturities for a 25 basis point change in forward rates (Table 1); and, out to only 2 year maturities, for a 75 bp change in forward rates (Table 2). To put these figures in perspective, a 25 basis point change is slightly less than the largest daily change, and a 75 basis point change is slightly less than the largest two-week change, in forward rates in the 4 to 7 year segment of the yield curve (during the period 1991 to 1995).

The estimated hedge adjustments are smaller than the stock of outstanding futures contracts at all maturities. Even in the case of hedge adjustments to a 75 basis point change in forward rates, except for contracts between 7 and 8 years maturity, the estimated hedge adjustment in most cases is much less than half of outstanding futures contracts (Table 3.)

With respect to the estimated hedge position, rather than adjustments to the hedge position, for longer maturity exposures the Eurodollar futures market is not large enough to accommodate the entire hedge demands that would be generated by a fully delta neutral hedging strategy, especially for exposures beyond 4 or 5 years (Table 3.)

Treasury securities

To hedge exposures to forward rates between 5 and 10 years maturity, a possible hedge position in Treasury securities consists of a short position (sale of a borrowed security) in the 10 year note, and a long position in the 5 year note.

For adjustments to hedge positions, the on-the-run security turnover volume exceeds estimated dealers' dynamic hedging demands (Table 4, Panel A). For an extremely large shock to forward interest rates, however, such as a 75 basis point shock to forward rates beyond 5 years out, the estimated hedge adjustment in the 5 and 10 year note would be approximately half of average daily turnover.

With regard to the hedge position, the on-the-run issue volume appears to be too small to accommodate hedging demand if a fully delta neutral hedging strategy were attempted exclusively in the cash market in Treasury securities. For example, if dealers fully hedged their exposures beyond 5 years with 5 and 10 year on-the-run issues, the required hedge position would be approximately equal to the outstanding amount of the on-the-run 5 and 10 year notes (Table 4, Panel A).

Two means by which the Treasury market may accommodate this hedging demand exist. *First*, the existence of a large repo (collateralized security lending) market in Treasury securities allows a fixed stock of on-the-run Treasury securities to meet trading demands that exceed the size of the on-the-run issue. Through the repo market, a trader that establishes a short position enables another trader to establish a long position in the security. Hence, the size of market participants' long position in the security can be larger than the outstanding stock of the security. *Second*, off-the-run issues when available can also be used, further enlarging the pool of available hedging instruments.

Futures on Treasury securities

In addition to the cash market in Treasury securities, dealers can also hedge with futures contracts on Treasuries. As seen in Panel B of Table 4, open interest and turnover volume in the Treasury futures market exceeds estimated dealers' hedging demand.

While outstandings and turnover volume in the cash and futures markets in Treasury securities exceeds estimated dealers' hedging demands, that demand could be significant relative to the size of the market. For example, the estimated hedge adjustment to a 75 basis point shock could be large as 25% of the combined average daily turnover in both markets, while the estimated hedge position could be as large as a third of total outstanding in both markets (see Table 4, Panels A and B).

Interest rate term structure models

If dealers are willing to accept model risk (correlation risk), they could also hedge exposures beyond 5 years by spreading their hedging demands across a wider maturity range of securities than only the 5 and 10 year notes. For example, with the use of a two (or more) factor interest rate term structure model, a dealer could construct a hedge of exposures between 5 and 10 years using a position in one year bills and 30 year bonds that replicate the exposure to the term structure factors that drive forward rates between 5 and 10 years. Such hedges, however, would be vulnerable to atypical price shocks that the term structure model does not account for.

Conclusions

The estimated size of dealers' hedge positions of longer maturity exposures, suggests that dealers' hedges, especially of exposures beyond 4 years maturity, are distributed over a range of fixed income instruments. While outstanding Eurodollar futures contract volume is smaller than the estimated size of the hedge position beyond 5 years, the large size of the US dollar fixed income market suggests that the hedge positions can still be absorbed by the markets in other fixed income instruments. With regard to an immediate dynamic hedge adjustments to an interest rate shock, however, the ideal hedging instrument is one that is liquid and has low transactions costs, such as Eurodollar futures, on-the-run Treasury securities, or Treasury futures.

Impact on transaction volume

The Eurodollar futures, on-the-run Treasury securities, and Treasury futures markets together can easily absorb hedge adjustments to shocks to the forward curve as large as 25 basis points along the entire term structure (Tables 1 and 4). For example, the estimated hedge adjustment for 5 to 10 year exposures to a 25 basis point shock is approximately 10% of the combined turnover in the Treasury on-the-run cash and futures markets.

For an extremely large interest rate shock, however, such as a 75 basis point shock to forward rates, dealers' dynamic hedge adjustments would generate significant demand relative to turnover and outstanding in these hedging instruments (see Tables 2 and 4). In this case, by bearing the price risk of a partially hedged position and spreading the hedge adjustment over more than one day, the hedge adjustment could be broken into smaller pieces that would be small relative to daily turnover. The terms of this trade-off between price risk and the cost of immediacy or liquidity of course would depend on the volatility of interest rates, and volatility may rise at the same time that liquidity is most impaired.

These results suggest that dealers' inter mediation of price risks through market making in interest rate options is supported by liquidity in underlying markets that allow them to manage their residual price risks. Transaction volume in the standard hedging instruments appear to be large enough to accommodate dealers' hedge adjustments in all but the most extreme periods of interest rate volatility.

Price impact

With regard to the price impact of dynamic hedging our results are less clear. For a definite answer an analysis of demands of other market participants would be required (see Pritsker, 1997). For example, investors whose demands are driven by "fundamentals" could be expected to undertake transactions in the opposite direction of dealer's dynamic hedging flows if those transactions drove interest rates to levels that appeared unreasonable to the "fundamentals" investors." If these investors constitute a sufficiently large part of the market, then their transactions would stabilise prices and keep positive feedback dynamics in check. However, such stabilising investors are not the only other market participants. Other participants include traders who follow short term market trends either because of "technical trading" strategies or because they interpret short term changes to be driven by transactions of better informed "fundamentals" investors. The trades by these investors could amplify the price impact of dealers dynamic hedging. Thus, the ultimate impact of dealers' dynamic hedging would depend on the relative sizes of these types of market participants, as described in Pritsker (1997).

At shorter maturities, transaction volume and open interest of the most liquid trading instruments are so much larger than dealers' dynamic hedging flows that positive feedback driven by dealers' dynamic hedging seems unlikely, even with very large interest rate shocks. However, at longer maturities around 5 to 10 years, dynamic hedging in response to an extremely large interest rate shock could be of significant volume relative to total transaction volume and open interest in the most liquid trading instruments. Hence, at this segment of the yield curve, the positive feedback hypotheses in the case of a very large interest rate shock can not be dismissed. The dynamic hedging volume in response to an unusually large interest rate shock could be large enough to have a significant impact on order flows in the medium term segment of the yield curve-maturities between 5 and 10 years. Such order flows might have a transitory impact on this segment of the yield curve.

4. The data and estimation

Option characteristics

Option type

All options were assumed to be caps and floors on a 6-month interest rate. A cap payoff at period t is,

$$y_t = \max\left[f_t - x, 0\right]\frac{0.5}{1+0.5\,f_t}n, \quad t < M$$

where f_t is the interest rate at period *t, x* is the strike rate, n is the notional amount, and *M* is the maturity of the cap. The payoff on the 6-month rate between periods *t* and *t*+1 is paid at the beginning of period *t*.

Counterparty type

The Survey data has three counterparty types, and options are either inter dealer options, options bought from customers, or options sold to customers. Dealers are net writers of options, as they have sold significantly more options to customers than they have bought (see Annex Table A2).

Maturities

Options are assumed to have maturities up to 10 years, in 6 month increments. The first caplet in any cap has a maturity of 3 months (mid-point of the first 6-month maturity band): a 3-month option on the 6-month rate that applies between 3 months and 9 months. The last caplet in any cap has a maturity 6 months shorter than the maturity of the cap: an option on the 6-month rate that applies for the last 6 months of the cap's term.

Origination dates

Options are assumed to have been originated up to 10 years earlier.

Strike prices

Strike prices are derived from historical term structure data. For example, a 5 year cap originated at period *p* will have a strike proportional to the 5 year swap rate at period *p*. Thus, two caps originated at the same time may have different strikes if their maturities differ. The distinction between bought and sold options also implies that two caps with the same remaining maturity and origination date may have different strikes if one is a sold option and the other is a bought option - given that the options are not inter-dealer.

Maturity distribution

The maturity distribution of options originated at any date is assumed to be described by a quadratic function. The notional amount of options with *t* periods remaining maturity, originated *p* periods in the past is

$$n(t,p) = \left(\coprod_{j=0}^{p} g_j\right)\left(a + b(t+p) + c(t+p)^2\right) \tag{1}$$

and,

$$n(t,p) = 0, \; for\; t + p < 1 year,$$

where t is remaining maturity, $t < 10$ years; p is the origination date (periods earlier), $p < 10$ years; $t+p$ is the original maturity, $t+p < 10$ years; g_j is the market growth term at period j, where $g = \frac{1}{1+r}$, and r is the growth rate from period j-1 to period j. The growth rates r are growth rates of notional amounts outstanding of US dollar interest rate options obtained from ISDA's surveys. The restriction in (1) forces caps and floors to have maturities of at least one-year when originated. (Regardless of this restriction, the first caplet (option) in any cap or floor has a maturity of 3 months (the midpoint of the first 6-month time band). Estimates without this restriction are shown in Section 5.

The maturity distribution is found by solving for the parameters (a,b,c) of the quadratic function in:

$$\sum_{t \le 1yr} n(t,p) = N_1 \tag{2a}$$

$$\sum_{1yr < t \le 5yrs} n(t,p) = N_5 \tag{2b}$$

$$\sum_{5yrs < t \le 10yrs} n(t,p) = N_{10} \tag{2c}$$

where N_m are notional amounts in the survey's three maturity categories (see Annex Table A3), and the function *n(.)* is as defined in equation (1).

Separate maturity distributions were estimated for interdealer options, options purchased from customers, and options sold to customers. The maturity data, however, were available only for all sold options and all bought options, where interdealer options were included in the maturity data of both bought and sold options. The maturity distribution of interdealer options was assumed to be the average of the bought and sold options' maturity distribution. Most outstanding contracts were of less than five years remaining maturity and were estimated to have been originated within three years of the survey date.

Option price function

The options are valued using Black's forward interest rate option model (see Hull 1993). The value of the period t payoff of a cap (floor) with strike rate x and notional amount n is

$$C(n,t,x) \quad = \quad e^{-r_t t}\left[f_t \, N\big(D1(t,x)\big) - x\, N\big(D2(t,x)\big)\right] \frac{\lambda}{1+\lambda f_t} n$$

$$F(n,t,x) \quad = \quad e^{-r_t t}\left[x\, N\big(-D2(t,x)\big) - f_t \, N\big(-D1(t,x)\big)\right] \frac{\lambda}{1+\lambda f_t} n$$

where, $D1(t,x) = \dfrac{1n\left(\dfrac{f_t}{x}\right) + \dfrac{\sigma_t^2 t}{2}}{\sigma_t \sqrt{t}}$, $D2(t,x) = D1(t,\ x) - \sigma_t \sqrt{t}$,

and λ is the length of the period for which the reference interest rate applies (6-months), f_t is the period t interest rate, σ_t is its volatility, and $N(.)$ is the standard normal distribution function. The value of a cap (floor) with maturity m is,

$$V^c(n,m,x) = \sum_{t<m} C(n,t,x),$$

$$V^f(n,m,x) = \sum_{t<m} F(n,t,x),$$

The valuation used the term structure of forward rates and the term structure of implied volatilities at end-of-March 1995 (Derivatives Week, 1996)[2].The section at the end of the paper presents estimates using alternative implied volatility structures.

Strike prices

Strike prices were derived from historical yield curves. Because separate market values were not available for caps and floors, a relationship between the strikes of caps and floors was required in the estimation. The structure was chosen on the assumption that buyers (sellers) of caps and floors had similar preferences regarding their options' moneyness. Thus, if buyers of caps desired out-of-the money options because of their cheaper premia, then buyers of floors would also. This structure regarding the options' moneyness was implemented in three different ways. These implementations gave similar results as shown in Table 5.

First, a proportional displacement of the strike price from the swap rate. The strikes of caps and floors are,

$$X^{cap}(t,p,A) = h(t+p,p)A \tag{3.1a}$$

$$X^{fir}(t,p,A) = \frac{h(t+p,p)}{A} \tag{3.1b}$$

where, t is the remaining maturity of the cap, p is the origination period (periods earlier), $t+p$ is the cap's original maturity, $h(m,p)$ is the historical swap rate of p periods earlier for a m period maturity swap, and A is a scaling factor.[3]

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2 The Derivatives Week forward rates and implied volatility data are consistent with those implied by Eurodollar futures prices and Eurodollar future options prices.

3 A complete 10 year time series for swap rates could not be found (data were available only from 1988). To complete the time series the missing values were assumed to equal the corresponding Treasury rate plus the last available swap spread.

Second, a cap and floor are assumed to have equal premia at origination,

$$v^{cap}\left(n,t,h(t,p)A^{cap}\right) = v^{flr}\left(n,t,h(t,p)A^{flr}\right), \tag{3.2a}$$

where the option values are evaluated at the term structures prevailing at origination, the strikes are defined as,

$$x^{cap}(t,p,A) = h(t,p)\,A^{cap}, and,\ x^{flr}(t,p,A) = h(t,p)A^{flr}, \tag{3.2b}$$

and A^{cap} and A^{flr} are separate scaling factors for caps and floors.

Third, caps and floors are assumed to have equal deltas at origination,

$$\Delta v^{cap}\left(n,t,h(t,p)A^{cap}\right) = \left|\Delta\ v^{flr}\left(n,t,h(t,p)A^{flr}\right)\right|, \tag{3.3}$$

where $\Delta v^{cap}$and Δv^{flr} are the deltas of a cap and floor (evaluated at the term structures prevailing at origination), and the strikes are defined as in (3.2b).

The scaling factors *(A)* are chosen so that the option values at the resulting strike prices equal the observed market values in the Survey. In each of the above specifications, the restrictions are applied to bought and sold options separately, with different scaling factors (*A*) for bought and sold options. In these strike price specifications, a cap will be out-of-the-money when a floor is out-of-the-money. Alternative strike price specifications are presented in Section 5.

Estimated strike prices and option values

Given the strike prices defined in (3), total values for bought and sold customer options, and interdealer options can be defined as functions of the scaling factors (A),

$$V_b\left(A^b\right)=\sum_t\sum_p v^c\left(B^c(t,p),t,x^c\left(t,p,A^b\right)\right)+\sum_t\sum_p v^f\left(B^f(t,p),t,x^f\left(t,p,A^b\right)\right) \tag{4a}$$

$$V_s\left(A^s\right)=\sum_t\sum_p v^c\left(S^c(t,p),t,x^c\left(t,p,A^s\right)\right)+\sum_t\sum_p v^f\left(S^f(t,p),t,x^f\left(t,p,A^s\right)\right) \tag{4b}$$

$$V_D\left(A^D\right)=\sum_t\sum_p v^c\left(D^c(t,p)t,x^c\left(t,p,A^D\right)\right)+\sum_t\sum_p v^f\left(D^f(t,p),t,x^f\left(t,p,A^D\right)\right) \tag{4c}$$

where *B* and *S* are notional amounts for bought and sold customer options, *D* is notional amount of interdealer options; and $v(n,t,x)$ is the value of a cap (floor) with notional amount *n*, maturity *t*, and strike price *x*. The index *t* represents remaining maturity, the index *p* is the origination date, where $t+p < 10$ years, and the superscripts *c* and *f* denote caps and floors.

On the basis of ISDA data we assume that caps amount to 73% of the options with the remainder being floors. The notional amounts of bought and sold options are derived from equations (1) and (2), and assigned to the caps and floors using the 73% ratio from the ISDA data. A small proportion of interest rate options are swaptions (19% at year-end 1994 in the ISDA data). However, for simplicity, we treat all options as either caps or floors.[4]

The value of each group of options in (4) is determined by the scaling factors in the strike rates - the parameter A in the strike price equations (3) and the value equations (4). The estimation is to find values of A^b, A^s, and A^D, such that:

$$V_b\left(A^b\right)+V_D\left(A^D\right)=V_b \tag{5a}$$

$$V_s\left(A^s\right)+V_D\left(A^D\right)=V_s \tag{5b}$$

subject to the restriction in (3.1, 3.2, or 3.3), where v_b (and v_s) is the observed market value of all US dollar options bought (and sold) by dealers including interdealer options.

Given the value of interdealer options (see below), in the case of the strike price structure (3.1), the estimation for bought options consists of solving for the single parameter A^b in equation (5a). In the strike price structure (3.2), however, the estimation for bought options consists of solving for the two parameters A^{cap}, A^{flr} in the two equations (3.2a) and (5a).

Interdealer options

Separate market values of interdealer US dollar options are unavailable. (The interdealer market values is available only in aggregate across all currencies, see Annex Table A1). For that reason, the problem in (5) is solved using four alternative assumptions: (1) inter-dealer options have strikes equal to the reference rate, $A^D=1$, in (3.1), (at-the-money strikes, relative to the swap term structure); (2) inter-dealer options have the same strikes as options bought from customers, $A^D=A^b$; (3) inter-dealer options have the same strikes as options sold to customers, $A^D=A^s$, and; (4) estimate the value of US Dollar interdealer options from the data in Annex Tables A1 and A2. The last estimation method (4) distributes the market value of interdealer options in Annex Table A1 between US dollar and other currencies so as to minimise the error in the ratios of market value to notional amounts relative to the margin ratios of the totals in Annex Tables A1 and A2.

The first and last alternatives produce comparable values for interdealer options. The at-the-money assumption (1) produces a value of interdealer options of \$11.3 billion, while the

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[4] This assumption is not likely to alter the paper's conclusions. For example, if a one year option on a five year swap were reported as a one year option, then the swaptions would appear as shorter maturity options in the data. Hence, the true exposures of shorter maturity would be less than assumed in the estimation, with the result that hedging demand for shorter maturity instruments would be smaller than estimated. This effect would only strengthen the conclusion that shorter maturity hedging volumes are small relative to transaction volume in Eurodollar futures. On the other hand, however, the swaptions would add to the estimated hedging demand at longer maturities Nevertheless, since swaptions are only 19% of the market, the net increment to estimated hedging demand would not significantly change the conclusions. The effect would be to strengthen the conclusions that longer maturity hedging demand could be significant relative to order flows in longer maturity hedge instruments but not so much larger as to overwhelm the market.

estimation in (4) results in a value of interdealer options of $10.9 billion. Table 7 shows the comparability of the hedge estimates with assumptions (1) and (4). Results using the other assumptions (2 and 3) were also similar to those in (1) and (4). The results reported in Sections 2 and 3 were derived using assumption (4).

An implication of the comparability of methods (1) and (4) is that inter-dealer options have strikes closer to at-the-money than customer options. This result is plausible, since dealers who use the interdealer market to hedge their net short volatility (negative gamma) position would obtain more hedging benefit from at-the-money options since such options have larger gamma.

Options sold to customers

In the strike rate equation (3), the value of the scaling factor that solves the sold options value equation (5b) is $A^s = 1.18$. Thus, for caps sold to customers, strike prices consistent with the observed market values are 18% higher than swap rates of comparable maturity at origination. These estimated strikes for sold options are predominantly deep out-of-the money (relative to swap rates of comparable maturity) at origination.

Options bought from customers

A solution to the bought options value equation (5a) requires a scaling factor in the strike rate equation (3) of, $A^b = 0.94$. For caps bought from customers, strike prices consistent with the observed market values are 6% smaller than swap rates of comparable maturity at origination. Thus, bought options are predominantly in-the-money (relative to swap rates of comparable maturity) at origination. In addition, strike rates for floors in this solution are higher than strike rates for caps. This relationship is the opposite of the relationship found for options sold to customers.

While this result might appear counterintuitive and could point to a problem in the estimation, it is consistent with market commentary in the early 1990s. An implication of this result is that customers looking for "yield-enhancement" during the low-interest rate regime of the early '90s, acquired "higher" yield by selling interest rate caps to dealers that were in-the-money relative to the swap term structure. While this "higher yield" is the market price or compensation for the expected pay out of the option, investors speculating on the path of interest rates would obtain higher investment returns (or losses) by selling in-the-money options. In addition, investors who believed that the forward curve was an overestimate of the future path of spot rates would sell options that were in-the-money relative to the forward curve. In retrospect, for positions that were not leveraged, the risks appear to have been moderate.

Assumptions regarding hedging

The analysis of dealers' hedging behaviour relies on the following assumptions.

(a1) Customers do not hedge their options positions.

Customers who have sold or bought options are assumed not to hedge, because doing so would negate what ever hedging or investment objective the options were used for. Customers who have sold options to dealers presumably did so for speculative "yield enhancement" or intertemporal income shifting. In which case, the costs of delta hedging the options would negate that investment objective. On the other hand, customers who have bought options from dealers for hedging purposes would not hedge the option since doing so would expose the underlying position the option was hedging.

If customers were to hedge their options, perhaps due to a reassessment of risks, then the market impact of dealers hedge adjustments would be smaller because they would be offset by customers' hedging. Since the predominance of our results support the claim that the market impact of dealers' hedging is small relative to the size of the market, dropping assumption (a1) would only strengthen the results.

(a2) Dealers restore the net delta of their position after an interest rate shock to its initial level.

Regardless of whatever hedge ratio they had initially, subsequent to an interest rate shock dealers are assumed to adjust their hedge position to bring the net delta of the portfolio back to its initial level. Dealers may or may not fully hedge the initial delta of the options book, and whatever hedging is initially done may be accomplished either internally with offsetting positions in the firm or with external hedging transactions. These initial offsetting positions, either internal or external, are assumed to have small gamma so that a change in the options' delta requires additional hedging transactions to return the portfolio's net delta to its original level.

(a3) An option exposure to a period t interest rate is hedged with an instrument that also has exposure to the period t interest rate - no basis risk in hedged positions.

With this assumption, a separate hedge ratio was calculated for each maturity's exposure.

Estimated hedge

The delta and the change in delta of the global dealers' portfolio was calculated given the notional amounts (from equation (2)) and estimated strike prices (from equation (5)). The estimated delta is the net hedge position of all dealers' (if they fully hedged) and the change in delta given an assumed interest rate shock is the change in the dealers' net hedge position. In response to an interest rate shock, if dealers are assumed to restore the net delta of their portfolios to their initial levels, then the change in delta of the global portfolio is the net dealer demand for hedge instruments. If hedging is executed with futures contracts, the estimated hedge adjustments are shown in Tables 1 and 2, and the hedge position (assuming complete hedging) is shown in Table 3. Table 4 shows the hedge adjustment and hedge position, if hedging of 5 to 10 year exposures is done with Treasury securities and futures on treasuries. These results are described in Section 3.

5. How robust are the results?

The results shown in Tables 1 to 4 are the results with the basic assumptions described above with the strike price restriction (3.1). To explore whether these results were sensitive to the assumptions, estimates were also performed using a variety of assumptions regarding the structure of strike prices, implied volatility, and other restrictions. The estimated hedge position and its change due to interest rate shocks were comparable across these different specifications and do not alter the conclusions. The results with these alternative assumptions are shows in Tables 5 through 8. The first column in these tables is the result under the basic assumptions, and the other columns are the results with the alternative assumptions.

Strike price variations

Distribution of strike prices

Instead of assuming that all options of a given maturity and origination date had the same strike rate, these options were distributed over two different strike prices, with the larger strike 22% higher than the smaller (10% above and below the reference rate for that option). Instead of equation (3), the strike prices were estimated using the following restrictions,

$$x^{cap}(t,p,A)_{high} = (1+\alpha)\, h\,(t+p,p)A \tag{6a.i}$$

$$x^{cap}(t,p,A)_{low} = (1-\alpha)\, h\,(t+p,p)A \tag{6a.ii}$$

$$x^{fir}\,(t,p,A)_{high} = (1+\alpha)\,\frac{h(t+p,p)}{A} \tag{6b.i}$$

$$x^{fir}\,(t,p,A)_{low} = (1-\alpha)\,\frac{h(t+p,p)}{A} \tag{6b.ii}$$

where $\alpha = 0.1$, and $h(m,p)$ is the historical swap rate of p periods earlier for a m period maturity swap. (The size of ∀ was chosen from inspection of the range of strike prices over which the bulk of Eurodollar futures options were distributed.)

Maturity variation in strike prices

For options bought from customers, instead of the strike price restriction in equation (3), the options' "moneyness" was assumed to vary with original maturity. In the first variation, the deviation of the strike from the swap reference rate decreased with maturity, and in the second the deviation increased with maturity.

Identical strike prices for caps and floors

Instead of the strike price structure in (3) for bought options, caps and floors were assumed to have identical strikes. This alternative specification produced in-the-money caps and out-of-the money floors. Applying a similar restriction for sold options was not meaningful, as it produced option values that exceeded the observed values. This result supports the use of equation (3) for sold options.

Implied volatility variations

Cap and floor implied volatilities

Instead of using a common implied volatility for both caps and floors, different implied volatilities were used. Caps were estimated using the Derivatives Week implied volatility data as in the basic assumptions, but implied volatilities for floors were adjustment upwards to conform with the difference between cap and floor implied volatility in DRI data. (The DRI implied volatility data are available only from January, 1996; while the Derivatives Week implied volatility data are derived from caps only).

Volatility smile

As an alternative to a common implied volatility across all degrees of "moneyness," results were also estimated using a volatility smile. A volatility smile consistent with Eurodollar futures options prices was constructed, and extrapolated across all maturities using the base volatility term structure as the at-the-money volatility.

Other variations

Options on 3-month interest rates

Instead of assuming that all options were on the 6-month interest rate, results were also derived on the assumption that the options were 3-month interest rate options. This variation doubles the number of individual options in a cap (floor).

Growth rate assumption in maturity distribution

The ISDA market size data for interest rate options contained a number of anomalous growth rates between certain dates. On the possibility that these growth rates were due to survey problems at those dates, alternative smoothed growth rates were derived by ignoring the market volumes at the anomalous dates. The notional amounts from the Central Bank Survey were then distributed across maturities and origination dates using these alternative growth rates in equations (1) and (2).

Unrestricted maturity distribution

As an alternative to the assumption that all caps (floors) have a maturity of at least one year when originated, the distribution of notional amounts across maturities and origination dates in (1) and (2) was estimated without the restriction in the maturity distribution (1).

Simultaneous volatility and interest rate shock

The results in Section 2 were estimated under the assumption that the volatility of interest rates remained constant while interest rates changed. In contrast, the hedge adjustments in Table 8 were estimated assuming simultaneous volatility and interest rate shocks. Interest rate volatility was assumed to increase by 25% relative to initial volatility levels, while the forward curve was assumed to increase by 75 basis points. While the estimated hedge adjustment is larger, the difference does not appreciably change the conclusions.

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Annex: Data

Table A1
Market values of OTC interest rate options
Billions of US dollars

	Bought			Sold		
	USD	Other	Total	USD	Other	Total
Dealer			22.4			21.6
Customer			15.2			14.6
Total	20.9	16.7	37.6	19.4	16.8	36.2

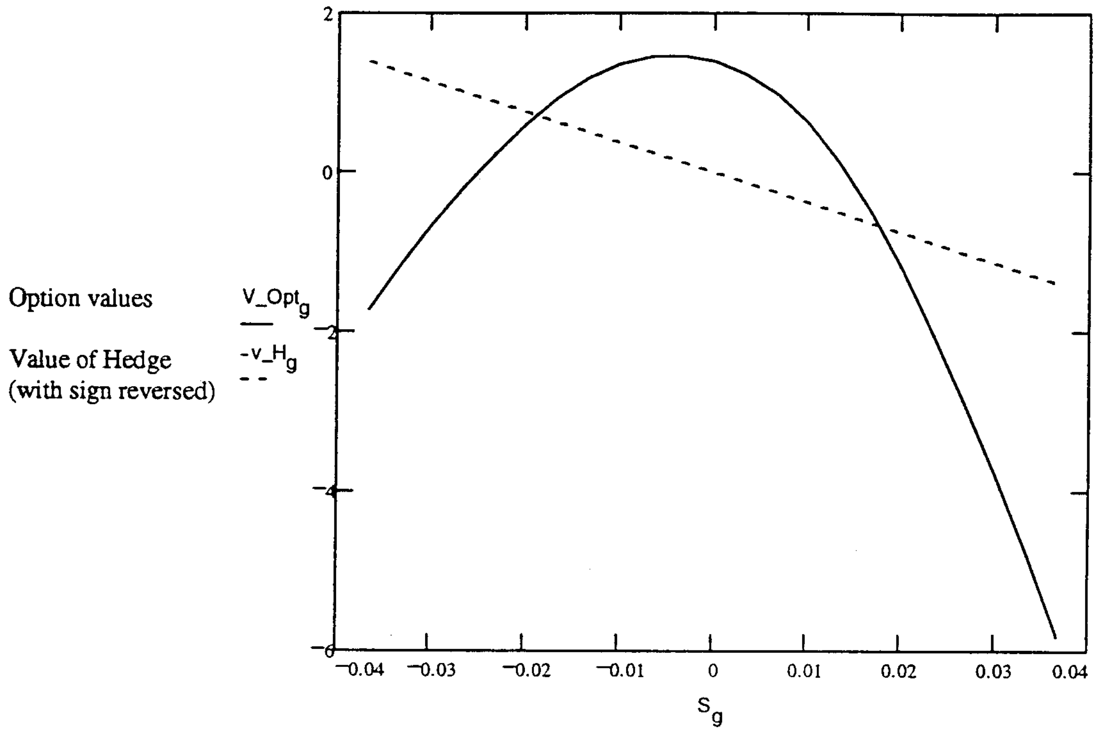
Table A2
National amounts of OTC interest rate options
Billions of US dollars

	Bought			Sold		
	USD	Other	Total	USD	Other	Total
Dealer	529.4	726.5	1255.9	576.1	681.9	1258.1
Customer	432.7	340.6	772.2	690.4	398.1	1088.4
Total	961.1	1067.1	2028.1	1266.5	1080.0	2346.5

Table A3
Maturity distribution of US dollars interest rate options

	Bought options	Sold options
Up to one year	30%	28%
Over one and up to five years	58%	56%
Over five years	12%	15%

Figure 1
Net Options and Hedge Values (-)



Notes:

(1) Vertical axis is market value in billions, and horizontal axis is interest rate change in percentage points (0.01 is 100 bp).

(2) The solid curve is the options value, and the dotted line is the mirror image of a hedge portfolio that delta hedges the options at the initial interest rate. The hedged portfolio has positive value when the solid curve (the options value) is above the dotted line (the hedge).

Table 1

Change in required hedge position compared to daily volume of Eurodollar futures 25 BP change in forward curve

Maturity (years)	Change in hedge position	Largest volume: Volume of 1st contract	Largest volume: Volume of 2nd contract	Average volume: Volume of 1st contract	Average volume: Volume of 2nd contract
0.5	- 6.3	**374.0**	**334.1**	**115.73**	**148.36**
1.0	- 9.2	**260.9**	**135.2**	**92.05**	**35.81**
1.5	- 7.7	**55.1**	**39.7**	**19.99**	**14.00**
2.0	- 5.7	**26.9**	**18.9**	**9.40**	**5.96**
2.5	- 4.6	**9.2**	**7.5**	4.02	3.26
3.0	- 3.7	**7.3**	**4.5**	2.69	1.94
3.5	- 3.1	**3.9**	2.6	1.52	1.32
4.0	- 2.6	**2.7**	**3.3**	1.20	1.09
4.5	- 2.1	**2.4**	**2.3**	0.89	0.79
5.0	- 1.9	**2.0**	1.4	0.75	0.46
5.5	- 1.6	1.3	**2.4**	0.20	0.23
6.0	- 1.4	1.3	1.3	0.22	0.20
6.5	- 1.2	1.0	1.2	0.17	0.15
7.0	- 1.0	**3.3**	0.7	0.20	0.12
7.5	- 0.9	0.6	**1.2**	0.07	0.09
8.0	- 0.6	**0.8**	**3.7**	0.07	0.11
8.5	- 0.4	**1.2**	**1.2**	0.11	0.09
9.0	- 0.3	**1.2**	**1.7**	0.08	0.08
9.5	- 0.1	**1.0**	**0.7**	0.07	0.06
10.0		1.2	1.0	0.06	0.04

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) The second column is the change in hedged position by maturity exposure. (3) The middle columns are the largest daily volume of futures contracts (by maturity of contract) in the first half of 1995. (4) The right most columns are the average daily volume (by maturity) in the first half of 1995. (5) The first and second futures contracts in the futures volume columns represent the two back to back contracts on 3-month interest rates required to hedge a six month exposure. (6) Bold indicates contract volume in excess of change in hedge position. (7) Negative values indicate an increase in a short position.

Table 2

Change in required hedge position compared to daily volume of Eurodollar futures 75 BP change in forward curve

Maturity (years)	Change in hedge position	Largest volume		Average volume	
		Volume of 1st contract	Volume of 2nd contract	Volume of 1st contract	Volume of 2nd contract
0.5	- 31.9	**374.0**	**334.1**	**115.73**	**148.36**
1.0	- 31.2	**260.9**	**135.2**	**92.05**	**35.81**
1.5	- 23.7	**55.1**	**39.7**	19.99	14.00
2.0	- 17.2	**26.9**	**18.9**	9.40	5.96
2.5	- 13.6	9.2	7.5	4.02	3.26
3.0	- 11.0	7.3	4.5	2.69	1.94
3.5	- 9.0	3.9	2.6	1.52	1.32
4.0	- 7.6	2.7	3.3	1.20	1.09
4.5	- 6.2	2.4	2.3	0.89	0.79
5.0	- 5.5	2.0	1.4	0.75	0.46
5.5	- 4.7	1.3	2.4	0.20	0.23
6.0	- 4.1	1.3	1.3	0.22	0.20
6.5	- 3.5	1.0	1.2	0.17	0.15
7.0	- 3.0	**3.3**	0.7	0.20	0.12
7.5	- 2.4	0.6	1.2	0.07	0.09
8.0	- 1.9	0.8	**3.7**	0.07	0.11
8.5	- 1.3	1.2	1.2	0.11	0.09
9.0	- 0.7	**1.2**	**1.7**	0.08	0.08
9.5	- 0.3	**1.0**	**0.7**	0.07	0.06
10.0		1.2	1.0	0.06	0.04

See notes to Table 1.

Table 3

Required hedge position in Eurodollar futures contracts compared to contracts outstanding

Maturity (years)	Hedge position	Open interest 1st contract	Open interest 2nd contract	Change in hedge position (75 BP Chg)
0.5	38.3	**561.9**	**366.4**	- 31.9
1.0	23.9	**279.7**	**222.0**	- 31.2
1.5	2.8	**174.0**	**145.4**	- 23.7
2.0	- 4.0	**114.2**	**96.3**	- 17.2
2.5	- 9.8	**84.9**	**68.6**	- 13.6
3.0	- 13.4	**60.3**	**54.8**	- 11.0
3.5	- 16.4	**49.5**	**38.8**	- 9.0
4.0	- 17.9	**34.4**	**27.2**	- 7.6
4.5	- 20.2	**22.6**	14.5	- 6.2
5.0	- 18.9	12.9	9.5	- 5.5
5.5	- 18.8	7.5	7.7	- 4.7
6.0	- 18.4	6.2	5.9	- 4.1
6.5	- 17.5	6.7	6.8	- 3.5
7.0	- 15.1	6.8	4.5	- 3.0
7.5	- 12.6	3.8	2.5	- 2.4
8.0	- 9.6	1.6	2.2	- 1.9
8.5	- 6.2	1.8	1.8	- 1.3
9.0	- 3.4	1.7	2.0	- 0.7
9.5	- 1.4	0.8	0.9	- 0.3
10.0		0.8	0.0	

Notes: (1) Billions of USD. Hedge estimates and open interest at end of March 1995. (2) The second column is the hedge position by maturity of exposure. (3) The middle columns are the outstanding volume of futures contracts at end of March 1995. (4) The first and second futures contracts in the futures volume columns represent the two back to back contracts on 3-month interest rates required to hedge a six month exposure. (5) Bold indicates contract volume in excess of hedge position. (6) Negative values indicate a short position or an increase in a short position.

Table 4

Hedge position in bonds using 5 and 10 year securities

Panel A: Treasury securities							
	Hedge position	**Chg hedge (10 BP)**	**Chg hedge (25 BP)**	**Chg hedge (75 BP)**	**On-the-run treasury**		
					Outstanding	**Daily volume**	
5 year	13.0	0.4	1.0	2.9	13.2	6.0	
10 year	- 13.0	- 0.4	- 1.1	- 3.3	13.8	4.0	
Panel B: Treasury futures							
	Hedge position	**Chg hedge (10 BP)**	**Chg hedge (25 BP)**	**Chg hedge (75 BP)**	**Treasury futures**		
					Open interest	**Large daily volume**	**Av. daily volume**
5 year	13.0	0.4	1.0	2.9	19.7	12.3	5.1
10 year	- 13.0	- 0.4	- 1.1	- 3.3	25.8	24.4	9.2

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Treasuries outstanding at end of March 1995; daily volume is from GovPx only (Fleming, 1997). (3) Treasury futures are the 5 and 10 year note contracts. Open interest as of end of March 1995, and volume is over first half of 1995. (4) Negative values indicate a short position or an increase in a short position.

Table 5

Strike price variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	**Base**	**Equal premia**	**Equal delta**	**Strike distr.**	**Maturity vrtn. 1**	**Maturity vrtn. 2**	**Identical caps/floors**
Change in futures hedge							
0.5	- 31.9	- 38.3	- 33.3	- 34.9	- 38.5	- 24.8	- 55.8
1.0	- 31.2	- 32.9	- 30.5	- 27.1	- 33.3	- 29.6	- 42.2
1.5	- 23.7	- 24.2	- 22.8	- 21.3	- 24.3	- 23.4	- 29.3
2.0	- 17.2	- 17.3	- 16.6	- 15.9	- 17.4	- 17.2	- 20.1
2.5	- 13.6	- 13.6	- 13.2	- 12.7	- 13.6	- 13.7	- 15.4
3.0	- 11.0	- 10.9	- 10.7	- 10.4	- 10.9	- 11.1	- 12.1
3.5	- 9.0	- 8.9	- 8.8	- 8.6	- 8.9	- 9.2	- 9.8
4.0	- 7.6	- 7.5	- 7.4	- 7.2	- 7.5	- 7.7	- 8.1
4.5	- 6.2	- 6.2	- 6.1	- 6.0	- 6.2	- 6.4	- 6.6
Change in bond hedge							
5 year	2.9	2.9	2.9	2.8	2.9	3.0	3.1
10 year	- 3.3	- 3.2	- 3.2	- 3.2	- 3.2	- 3.3	- 3.4

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Column headings indicate the assumption as described in the text. (3) Negative values indicate an increase in a short position.

Table 6

Volatility variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	Base	Cap/floor volatility	Volatility smile	Cap/floor and smile
Change in futures hedge				
0.5	- 31.9	- 31.5	- 27.2	- 26.8
1.0	- 31.2	- 31.2	- 27.8	- 27.7
1.5	- 23.7	- 23.7	- 21.1	- 21.0
2.0	- 17.2	- 17.2	- 14.6	- 14.5
2.5	- 13.6	- 13.6	- 11.4	- 11.3
3.0	- 11.0	- 10.9	- 9.1	- 9.0
3.5	- 9.0	- 9.0	- 7.4	- 7.4
4.0	- 7.6	- 7.5	- 6.2	- 6.2
4.5	- 6.2	- 6.2	- 5.3	- 5.3
Change in bond hedge				
5 year	2.9	2.9	2.6	2.6
10 year	- 3.3	- 3.3	- 2.9	- 2.9

See notes to Table 5.

Table 7

Other variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	Base	Dir. option at-the-m	Options on 3-month rate	Growth rate	Unrestr. mtry dstr.
Change in futures hedge					
0.5	- 31.9	- 25.2	- 32.5	- 38.6	- 35.9
1.0	- 31.2	- 28.6	- 30.4	- 32.2	- 30.9
1.5	- 23.7	- 22.6	- 23.4	- 25.5	- 24.4
2.0	- 17.2	- 16.6	- 17.0	- 19.2	- 18.1
2.5	- 13.6	- 13.2	- 13.5	- 15.4	- 14.5
3.0	- 11.0	- 10.7	- 10.9	- 12.5	- 11.7
3.5	- 9.0	- 8.9	- 9.0	- 10.2	- 9.6
4.0	- 7.6	- 7.5	- 7.5	- 8.3	- 7.9
4.5	- 6.2	- 6.2	- 6.2	- 6.6	- 6.4
Change in bond hedge					
5 year	2.9	2.9	2.9	2.4	2.8
10 year	- 3.3	- 3.3	- 3.3	- 2.7	- 3.1

See notes to Table 5.

Table 8

Change in required hedge position due to simultaneous volatility and forward rate shocks

Maturity (years)	I.R. shock only	Volt. shock only	I.R. and volt. shock
Change in futures hedge			
0.5	- 31.9	- 6.0	- 40.7
1.0	- 31.2	- 9.7	- 38.7
1.5	- 23.7	- 8.7	- 29.4
2.0	- 17.2	- 7.7	- 22.6
2.5	- 13.6	- 6.2	- 17.9
3.0	- 11.0	- 4.9	- 14.4
3.5	- 9.0	- 3.9	- 11.6
4.0	- 7.6	- 3.0	- 9.5
4.5	- 6.2	- 2.2	- 7.5
Change in bond hedge			
5 year	2.9	0.8	3.4
10 year	- 3.3	- 0.8	- 3.8

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Forward rates increase by 75 basis points, and volatility increases by 25% relative to initial volatility levels at short maturities, and by 8% at 10 years. (3) Negative values indicate an increase in a short position.