Dynamic macro stress exercise including feedback effect

by

Tokiko Shimizu*

Institute for Monetary and Economic Studies

Bank of Japan

Abstract

The goal of this study is to illustrate a viable way to explore macro risk in markets, not only from a static viewpoint but also from a dynamic one. In this paper, I focus mainly on the feedback effect caused by a market stress and try to present a possible analytical framework to incorporate the effect into a macro stress exercise. I discuss how to take into account feedback effects employing two approaches to the estimation of market participants' behaviors in response to a stress. One approach assumes typical portfolio rebalancing of each agent based on the available information, including the agent's trading strategy and its loss cutting rules, etc. The other approach involves the extraction of a pattern of portfolio rebalancing of each agent based on the historical data on its portfolio profile, such as sensitivity to risk factors, by utilizing a neural network. A dynamic stress exercise taking into account any feedback effect will provide us with more useful and vivid information on the macro market risk profile under stress and enable us to prepare for stress in a more efficient and effective way.

* I am indebted to Mr. Shigeru Yoshifuji of the Institute for Monetary and Economic Studies of the Bank of Japan for his providing trading data for the simulations. The views expressed in the paper are those of the author and do not necessarily represent those of the Bank of Japan, the Institute for Monetary and Economic Studies or the Euro-Currency Standing Comittee. Although the paper benefited from comments by the staff of the Institute, the author is solely responsible for any remaining errors.

1. Introduction

1.1 Framework of Macro Stress Exercise

The inputs for the Macro Stress Exercise which will be a possible tool for market perticipants to comprehend the macro market risk profile: a stress scenario provided by central banks and portfolio sensitivity data for reporting institutions (Figure 1). The output is an aggregate risk measure covering the institutions. There could be two approaches to aggregating micro risk figures into a macro risk measure. One can be called as the "Revaluation Approach," in which firms are expected to report expected loss amounts in a given stress scenario. The other can be called as the "Sensitivity Approach," in which firms are expected to report summary data on their sensitivity to market risk factors.

My research goal is to illustrate a viable way to explore macro risk in markets, not only from a static viewpoint but also from a dynamic one. In this paper, I focus mainly on the feedback effect caused by a market stress and try to present a possible analytical framework to incorporate the effect into the exercise. I discuss how to take into account feedback effects basically employing the Sensitivity Approach, which provides us with more flexibility in aggregating risk. The Revaluation Approach, however, could be dealt with by constructing the actual stress scenario obtained by employing an initial stress scenario via the feedback effect.

1.2 Issues to be discussed under Dynamic Stress Exercise

There are various factors to be considered when aggregating micro risk figures into a macro risk measure. Especially when conducting a macro stress exercise, we have to take into account not only static risk profiles in individual institutions but also the dynamic effects caused in them by those institutions' reactions against an initial shock. Feedback effect and liquidity effect are the key issues to be discussed in this context.

In a static world, if Bank-i's expected loss amount is R_i under a stress scenario, $\sum R_i$ could be an aggregate risk measure. However, once initial stress occurs in a market, traders in each bank begin to rebalance their portfolios or hedge their positions to minimize future losses. Along the way, the initial stress scenario could be either exacerbated or alleviated by such reactions. Feedback effect is defined as the impact on market price caused by traders' trading behaviors towards market price movements, which are realized as a result of traders' trading strategies and traders' needs for portfolio rebalancing. Liquidity effect including market impact caused by position liquidation behaviors is another important issue to be explored in order to capture a dynamic picture of macro risk under a stress. In the following chapters, I mainly focus on feedback effect, by presenting a viable analytical framework and discussing the implications of a dynamic macro stress exercise which takes into account the feedback effect.

2. Analytical Framework

2.1. Expanding a Framework of Static Stress Exercise

We need to expand the static framework in two dimensions in order to take the feedback effect into consideration. The first dimension is time horizon. I employ a sequential framework to conduct a dynamic stress exercise. My purpose in this study is not to aggregate micro risk figures at a static point, but to construct a model through which we can comprehend the dynamics of a macro risk profile that is affected by institutions' reactions at intervals (a multiperiod model). There are many impressive studies employing a multiperiod model in market microstructure. The Glosten and Milgrom model (GM model) [1985] employs multiperiod models to analyze the dynamic features of market impacts.

Secondly, we need to expand our model to correspond with a variety of agents. Each agent's behavior varies, depending on its portfolio mix and trading strategy. Glosten and Milgrom assume that there are three types of agents in a market, namely, informed trader, uninformed trader, and market maker. They analyze the decision-making process of a market maker by modeling price mechanisms in a market. The Gennottee and Leland model (GL model) [1990] is another example which takes the variety of agents' behaviors into consideration. The GL model takes into account the factor of supply from hedge traders who employ a portfolio insurance strategy. The model allows them to review a market meltdown mechanism caused by the hedging behaviors of a significant proportion of market participants, as was the case on Black Monday in 1987.

In the following chapters, I examine two approaches to the estimation of various behaviors of agents. One approach assumes that each agent employs a typical trading strategy and that we know it a priori. In the other approach, I derive the trading pattern of each agent from historical data on market movements and individual portfolio profiles.

2.2 Multiperiod Model

I develop a multiperiod model in order to take the feedback effect into account when I conduct the stress exercise described in this chapter. Bank-i's expected loss under a provided stress scenario is R_i at time t+1. Each agent reacts against the shock at t+2, and we finally obtain an aggregate macro risk measure after considering the feedback effect at t+3 (Figure 2). When we employ the Sensitivity Approach, we can estimate the aggregate risk measure at t+3 by assuming agents' reactions and calculating the risk amount based on sensitivity. Using finally realized prices to construct an actual stress scenario, we can take the feedback effect into account, even if we adopt the Revaluation Approach, by applying this scenario to reporting institutions.

2.2.1 *Framework of Multiperiod Model*

The portfolio value of the i-th agent at time t is expressed by F_{it}. F_{it} is a function of the portfolio mix f_{it} and risk factor prices x_t. Here, we need to consider sequential movements not only in risk factor prices but also in the portfolio mix.

$$F_{it} = f_{it}(x_t)$$

When an initial stress scenario (ISS, S_0) is provided, the portfolio value at t+1 can be shown as below. I assume that there is only one risk factor x_t and that f_{it} is consistent from t to t+1.

$$S_0 = dx$$

$$F_{it+1} = f_{it+1}(x_{t+1}) = f_{it}(x_t + dx)$$

We can obtain static aggregate risk R_s

$$R_s = \sum R_i = \sum dF_i = \sum \frac{\partial f_i}{\partial x} dx$$

Then, I take into account the i-th agent's portfolio rebalancing ($f_{it+1} \rightarrow f_{it+2}$). Portfolio value at t+2 is

$$F_{it+2} = f_{it+2}(x_t + dx)$$

Assume that there are n banks in our model and that they react to the ISS individually. Kawahara [1996] argues that the market impact on the risk factor price (dx') caused by an agent's trading can be expressed as a function of the macro trade imbalance, i.e., net supply in the market.

$$dx' = G\left(\sum_{i=1}^{n} \frac{dF_i}{x}\right)$$

The actual stress scenario (ASS, S_1) is provided as follows.

$$S_1 = dx + dx' = dX$$

Dynamic aggregate risk including feedback effect R_d is

$$
\begin{aligned}
R_d &= \sum \left(F_{it+2} - F_{it} \right) \\
&= \sum \left(f_{it+2}\left(x_t + dX \right) - f_{it}\left(x_t \right) \right) \\
&= \sum \left(\frac{\partial f_i}{\partial x}\left(dx + dx' \right) + \frac{\partial f_i}{\partial t} dt \right)
\end{aligned}
$$

2.3 Considering the variety of agents' behaviors

The multiperiod model described in Chapter 2.2 includes agents' portfolio rebalancing behaviors towards ISS, which depend on each agent's portfolio mix at time t and its trading strategy. We can take the variety of their behaviors into consideration by providing various types of $\frac{\partial f_i}{\partial t} dt$ in equation (1) corresponding to agents' types. In this chapter, I examine two alternatives:

1) assuming typical portfolio rebalancing of each agent based on the available information including its trading strategy and loss cutting rules, etc.
2) extracting a pattern of portfolio rebalancing of each agent based on the historical data on its portfolio profile - sensitivity to risk factors.

2.3.1 Assuming Typical Portfolio Rebalancing

I present a simple example to show how we obtain R_d, taking into account the feedback effect . Assume that there are three agents in a market where only one tradable risk factor, x, exists and that each agent has the portfolio mix described as follows:

Agent 1 : holding a_t units of asset x at t. $a_t = a$ constant

Agent 2 : holding b_t units of asset x at t. $db_t = b / x_t$ b is constant.

Agent 3 : holding c_t units of asset x at t. $dc_t = c \times dx_t$ c is constant.

Each agent has the trading strategy described below:

Agent 1 never trades

Agent 2 buys a constant amount (b dollars) of x every period. This strategy is the so-called "dollar-cost-average strategy".

Agent 3 buys x after x has risen or sells after it has fallen. The trading amount depends on the magnitude of the change in x in the previous period. If an agent employs a portfolio insurance strategy involving dynamic hedging, we can observe trading behavior which is the same as that of Agent 3.

I can explore dynamic aggregate risk based on the multiperiod model including the three agents described above. Each agent has 100 units of x at time t.

$$x_t = 100$$

$$S_0 = -6.5$$

$$x_{t+1} = 93.5$$

$$F_{it} = f_{it}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$F_{it+1} = f_{it+1}(x_{t+1}) = 100 \times x_{t+1} = 100 \times 93.5 = 9350$$

Static aggregate risk R_s can be calculated as

$$R_s = \sum R_i = \sum dF_i = 1950$$

Then, I consider agents' reactions against ISS.

$$F_{2t} = f_{2t}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$b = 2000$$

$$F_{2t+1} = f_{2t+1}(x_{t+1}) = \left(100 + b / x_{t+1}\right) \times x_{t+1} = \left(100 + 2000 / 93.5\right) \times 93.5 = 11350$$

Here, Agent 2 is assumed to buy $\frac{dF_2}{x_{t+1}} = +21$ units of x.

$$F_{3t} = f_{3t}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$c = 5$$

$$F_{3t+1} = f_{3t+1}(x_{t+1}) = \left(100 + c \times dx\right) \times x_{t+1} = \{100 + 5 \times (-6.5)\} \times 93.5 = 6311$$

Here, Agent 3 is assumed to sell $\frac{dF_3}{x_{t+1}} = -32.5$ units of x.

Assume that the market impact caused by three agents' reactions can be obtained as a linear function of the trade imbalance. I can get ASS;

$$
\begin{aligned}
dx' &= G\left(\sum_{i=1}^{n}\frac{dF_i}{x}\right)\\
&= k\cdot(21-32.5)\\
&= -11.5k
\end{aligned}
$$

If we assume k=0.2

$$S_1 = dx + dx' = -6.5 - 2.3 = -8.8$$

Dynamic aggregate risk R_d is calculated as follows;

$$
\begin{aligned}
R_d &= \sum\left(F_{it+2} - F_{it}\right)\\
&\cong \sum F_{it+2}\left(1-\frac{x_{it}}{x_{it+2}}\right)\\
&= \left|-880-1065-544\right|\\
&= 2539
\end{aligned}
$$

Dynamic aggregate risk is 1.3 times static aggregate risk. We also observe that ISS will cause a further price decline in x. This information that we obtain via a dynamic stress exercise is more useful than that from a static exercise. However, we should be careful about the probability and accuracy of the behavioral assumptions that I employ in the model and further study is necessary.

2.3.2 Extracting a trading pattern of portfolio rebalancing of each agent

I present another approach assuming more realistic trading behavior in this chapter. Utilizing a neural network system, it would be possible to extract trading patterns of agents from historical risk factor price data and corresponding changes in their portfolio profiles (see Appendix).

First of all, we have to decide on a set of data to be used as inputs and outputs to and from the neural network. Since my purpose is to estimate the probable reaction of an agent against risk factor price movements, inputs must be factors which affect a trader's decisions and outputs are some indicators of its trading behavior. Candidates for inputs would be;

1) the agent's portfolio mix at time t,

2) business circumstances surrounding the agents, such as profit/loss conditions and adequacy of risk capital, and

3) risk factor price movements.

The portfolio rebalancing behaviors that I am trying to estimate can be expressed as movements in agents' portfolio mixes, in other words, agents' positions. Since it does not seem possible for us to gather historical data on agents' portfolio mixes, I have to regard changes in portfolio sensitivity to major risk factors or actual profit and loss figures as proxies for portfolio rebalancing behaviors and select them as outputs of neural network analysis.

Inputs to a neural network can vary because of the flexibility of the neural network system. At this stage I use data on risk factor price movements and news as inputs for the analysis. Other economic or financial measurements could be candidates for analytical inputs, and I will continue to explore the selection of suitable and effective inputs and outputs.

I conduct a set of simulations using historical market data and trading data during a certain period. Trading data is obtained by letting one of the staff of the Institute, who has nine years experience as a bond trader in a bank, simulate daily trading based on market data during a set historical period. Details of simulations are described below.

Risk factors

Yen interest rates are selected as risk factors in our simulation. Three points on a yield curve, 3-year, 5-year, and 10-year swap rates, are regarded as factors of yen interest rate risk.

Simulation period

It is essential to provide learning data which include stress periods for neural networks in order to make networks capable of estimating agents' behaviors under stress. I chose the period from October 1993 to March 1994 as a learning period which includes market stress. As shown in Figure 3, we experienced several sudden rises of yen interest rates at the beginning of 1994. After the strong downward trend in interest rates toward the end of 1993, both futures and cash JGB markets faced significant short trading triggered by the MOF's operation of selling JGB of 14th January 1994. I have also conducted out-of-sample simulations using market and trading data from April to September 1994.

Characteristics of agents

There are three agents in the simulation. They trade JGB in three maturities: 3-year, 5-year and 10-year. They decide their trading volume and direction based on their own trading strategies, and their portfolio sensitivity to the three risk factors and actual P/L are assumed to be available for us. Each agent has its own typical trading pattern shown in Table 1. Then, we observe daily interest rate data, news of economic and financial events, their realized P/L, and sensitivity to the risk factors. Reported data on sensitivity are converted to the equivalent positions of 10-year JGB. Each agent has its own target level of profit, position limit, and loss cutting rules. If the level of accumulated loss reaches the targeted profit level, the agent has to close his/her position at once.

Inputs for learning

I adjust the data mix of learning inputs in the following sequence.

First I provide a set of daily returns for four risk factors and interest rate volatility to a network as inputs for learning. In this case, the level of estimation accuracy of network which has learned agent 1's trading pattern is only 71.67% (see Figure 4-1).

Second, I include realized P/L data among the inputs, since each agent's appetite for trading is constrained by its accumulated P/L condition. The estimation power thus increases to 76.67%. from 71.67% (see Figure 4-2). However it still cannot follow the movements in actual data during the periods particularly when it changes in an accelerating fashion, such as around the 20th, 45th, and 103rd sample data.

Finally, I include three days recent market movements and news of financial and economic events among the inputs in order to improve the estimation power of networks (see Figure 4-3, Figure 5). Among the market movements and news, I put more weight on the more recent information. This adjustment improves the network's estimation level to 85.42%. Regarding Agent 3, an effective way of improving the network's estimation power further is to add input data on a market trend over a longer horizon, since he/she has a view of longer horizon than the other agents (See Figure 6). Table 2 shows the process of improving the estimation power regarding each agent by adjusting the inputs.

I conduct out-of-sample simulations using the networks which have learned the trading patterns of the agents. According to the simulation results, it seems relatively easy to follow the trading patterns of Agent 1 and Agent 2 (see Figure 7-1 and Figure 7-2). However, the network cannot follow Agent 3 very well (see Figure 7-3). The reason why the network fails in the case of Agent 3 is the difference in trading patterns between a dealer-type of agent and an investor-type of agent. During the former sub-period, Agent 3 has almost fixed its position to the long side because of market trend shows the interest rate falling. Since the market trend has drastically changed in the latter sub-period, it is very difficult for the network, which has only learned the agent's trading pattern in the period when the market trend was only moving in one direction, to predict position changes from short to long during a period which includes fluctuating market trends.

Stress scenario

I pick up the largest daily change in both upward and downward directions in each risk factor during the period and construct stress scenarios by combining these figures. In Figure 3, we can see which days are picked up as a stress on each risk factor. Regarding 3-year and 10-year swap rates, the largest changes in both directions occurred in March 1994. The period from the end of November 1993 to the middle of January 1994 was selected as a stress period for 5-year swap rates.

The magnitude of these market changes are shown in Figure 8-1 to Figure 8-3 and Table 3. A pair of parallel lines in each figure show 2 standard deviations during the period.

By combining these figures, I provide 4 types of stress scenario: parallel shift of the yield curve; stress at the shorter end of maturity; the middle zone of the yield curve; and the longer end of maturity. Each type of scenario has two directions, upward and downward. We therefore obtain altogether eight stress scenarios (see Table 4). In each scenario, volatility level is also set as the level equivalent to its largest change during the period.

Feedback effect

According to the simulation results (see Table 5), the downward stress in the yield curve cause larger changes in the agents' positions (scenario 2, 4, and 8) than the upward one. Directions in total demand/supply differ between a stress at the shorter end and a stress in the longer end. Downward stress at the shorter end of maturity causes a fair amount of supply to the JGB market, which can produce negative feedback, offsetting the initial stress. On the other hand, a downward stress at the longer end of maturity causes a fair amount of demand to the market, and it would produce positive feedback to the initial stress.

In order to determine the magnitude of the feedback effect, I regard total demand/supply, a figure summed up all agents' delta changes as a measurement. The average of daily total demand/supply during the period is - 0.2 billion yen, and its standard deviation is 12 billion yen (see Figure 9).

Parallel shift scenario (scenarios 1 and 2)

Regarding the directions of the feedback effects under the scenarios, both of them would cause negative feedback and offset the initial stress. Upward parallel shift pushes Agent 2 into buying a relatively large amount of its position, 26 billion yen. Total supply to the market, however, is only 12 billion, which is equivalent to the standard deviation of total demand/supply changes during the simulation period, and the direction of the feedback effect would be negative. Although downward parallel shift would cause a fair amount of selling of JGB, it also produces negative feedback to the stress under this stress scenario. These results could be interpreted to mean that a parallel-shift type of yield curve stress is less harmful from the view point of the macro feedback effect.

Stress at the shorter end of maturity (scenarios 3 and 4)

Simulation results show that upward stress at the shorter end of maturity causes a fair amount of demand for JGBs. On the other hand, downward stress causes a significant supply of such bonds. As same as the results in the parallel shift scenarios, stress at the shorter end of maturity would also cause negative feedback effect to the yield curve.

Stress in the middle zone of the yield curve (scenarios 5 and 6)

Stress in the middle zone causes a smaller magnitude of change in total demand/supply than the other stress scenarios. The direction of the feedback effect cause by upward stress in the zone, however, would be positive. It means that if we face upward stress in the middle zone, the longer maturity interest rate (10-year JGB price) is expected to increase (fall) further as a consequence of stress.

Stress at the longer end of maturity (scenarios 7 and 8)

Downward stress causes a more significant effect than does a downward one. Increase in the JGB price causes a fair amount of demand for JGB. It means that there could be positive feedback effect under this scenario.

Risk profiles of the agents

Assume that the magnitude of the feedback effect on JGB prices depends on the total demand/supply volume which is realized by agents' response to a stress. If market price changes can be described as a linear function of total demand/supply volume as follows, we can estimate the level of loss each agent will face because of the feedback effect. Estimated risk profiles of each agent are shown in Table 6. Figures in Table 6 show agents' loss amounts determined by the additional market price change as a result of feedback effect and delta position after the feedback trading. Further discussion on the definition of estimated loss caused by feedback effect will be necessary, particularly whether implicit profit/loss of feedback trading is taken into account.

$$\Delta P = \alpha \sigma_p \cdot \beta$$

ΔP : JGB price change

α : (total D/S) / (standard deviation of total D/S during the period)

σ_p : standard deviation of JGB price changes during the period

β : ratio of price change caused by market demand and supply conditions (in this simulation, $\beta = 1$)

Among the eight scenarios, scenario 2, the downward parallel-shift type of stress, has the potential to cause the most severe feedback effect in relation to the level of the macro risk. According to the simulation result, I can say that all of these scenarios would have low potential to produce a severe feedback effect, because each agent would not face a loss which exceeds VaR at time t, just before stress. I can explore which kind of stress scenarios can produce severe feedback effect and which type of agents play a critical role in stress by analyzing the results of a simulation, as we have shown in this chapter.

I am improving the estimation power of the network by umbundling the output, delta, into direction and volume in its changes. The prediction on whether an agent increses or decreases its delta is essential to estimate the direction of feedback effect. The predicton on tradeing volume is necessary to capture the magnitude of feedback effcet. The unbundlled outputs will provide us with more accurate approximations of the feedback effect. The tentative results of the simulation with new outputs give better estimation of both agent's trade direction and its volume. Figure 10-1 shows the estimation results of trade direction, where if agent increases/decreases its delta, the parameter is set +0.5/-0.5. The network fails to predict agent's trade direction only twice out of 120 data. Figure 10-2 shows the estimation results of trading volume. Comparing to Figure 10-3, which shows the results obtained via the simulation with the former output, i.e. delta itself, the estimation error is reduced to 44 from 78.

I also attempt to explore the effect of the variation in stress scenario. As I show in this chapter, the stress scenario provided here do not produce severe impact on macro risk profiles. I am now conducting a stress simulation, which assumes a stress not only in the yield curve but also with news. The tentative results of the simulation show that if agents face a stress with news which exacerbate the stress, the feedback effect becomes greater than that in a stress without any news.

3. Implication for implementing Dynamic Stress Exercise

3.1 Necessary data for Macro Stress Exercise

Under the Revaluation Approach, each institution reports an expected loss amount under a provided stress scenario. The Sensitivity Approach requires institutions to report summary data on their sensitivity to market risk factors.

If we try to conduct a dynamic macro stress exercise as described in this note, no matter which approach we employ, risk profile data on sensitivity will be necessary. Even under the Revaluation Approach, data which show reactions against an initial shock will be required to construct an ASS.

As I pointed out in 2.3.2, historical data on institutions' risk profiles need to be provided to networks so that they can learn those institutions' portfolio rebalancing patterns. Further study on the candidates for inputs will be necessary in order to estimate their trading behavior more effectively. Historical P/L data would be another candidate as an output of network simulation, since P/L data seems to be more available than data on sensitivity.

It is true that the availability of these data, such as daily sensitivity and P/L, is not full enough for us to be able to ask that they be reported at this moment. However, since those data constitute fundamental information for internal risk management in financial institutions, banks which actively conduct trading business will in the near future come to use these kind of data more frequently as a tool for daily risk management.

3.2 How can exercise results be utilised?

Information we obtain via a stress exercise will vary according to the choice of approach employed at every stage, such as the Revaluation Approach or the Sensitivity Approach, and according to whether a neural network is used or whether assumptions on trading behavior are made. The information which obtained from static and dynamic exercises are listed as follows:

1) Static Stress Exercise : ISS and static aggregate risk
2) Dynamic Stress Exercise : ISS, static aggregate risk, institutions' reaction against ISS, ASS, and dynamic aggregate risk

The difference between the Revaluation Approach involving a stress scenario, and the Sensitivity Approach is the flexibility of scenario used for calculating the aggregate risk measure. The choice of assumptions concerning institutions' reactions also affects the informational content of exercise results. If we compare the two alternatives for estimating traders' behavior examined in this note, it is safe to point out that the approach which uses an AI system for learning the trade pattern can provide the exercise results with a more realistic shape.

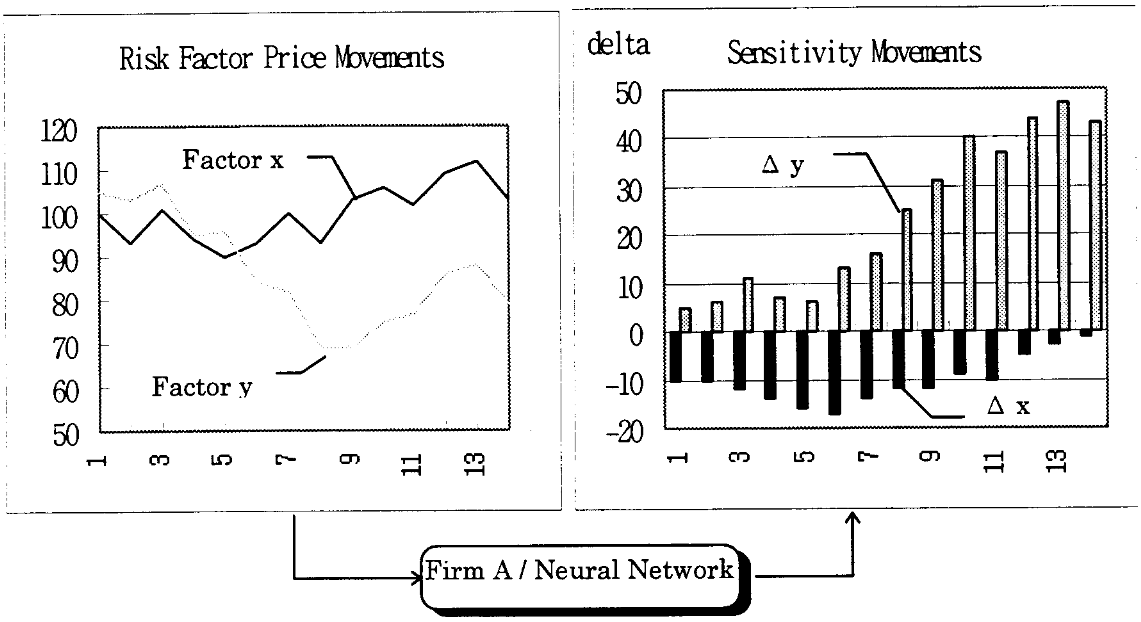
Great attention must be paid to the way that stress exercise results are used. A dynamic exercise taking into account any feedback effect will provide us with more useful and vivid information on the macro market risk profile under stress. If we obtain the information listed above via a dynamic stress exercise, we can prepare for stress in a more efficient and effective way. The information that most institutions will begin to sell the asset in reaction to the initial crash makes us more secure than in the case when we don't have any idea what their reaction might be. For example, as the simulation results in this note show, if we know that the upward parallel shift of the yen yield curve has a higher possibility of causing a positive feedback effect which could trigger systemic risk than the other stress scenarios, we should pay more attention to yield curve movements in this direction in our daily market monitoring. On the other hand, the prediction of a negative feedback effect under a stress at the shorter end of maturity affords us more room for making the political decision to conduct a necessary operation, such as supplying liquidity to a market, when we face a sudden crash of the short term interest rate.

When we expand the time horizon of the exercise, its result will have useful implications for the framework of financial or trading systems. For example, we can discuss in which situations a circuit breaker system works well or not from the point of view of systemic risk. My model can be a tool to simulate systemic meltdown in markets by expanding its time horizon and examining cases where price equilibrium disappears.

APPENDIX

Macro dynamic simulation using neural networks

1. Inducing information on a firm's behavioural pattern



$X_t = (x_t, y_t)$

Firm's portfolio value at t: $f_t(X_t) \rightarrow$ Sensitivity data at t: $\frac{\partial f_t(X_t)}{\partial x}, \frac{\partial f_t(X_t)}{\partial y}, \quad . \ . \ .$

Firm's portfolio value at $t+1$: $f_{t+1}(X_{t+1}) \rightarrow \frac{\partial f_{t+1}(X_{t+1})}{\partial x}, \frac{\partial f_{t+1}(X_{t+1})}{\partial y}, \quad . \ . \ .$

If we have daily data on a firm's sensitivity and risk factor price movements, we could estimate how each firm rebalances its portfolio in response to market movements. Daily change in a firm's sensitivity data is caused by

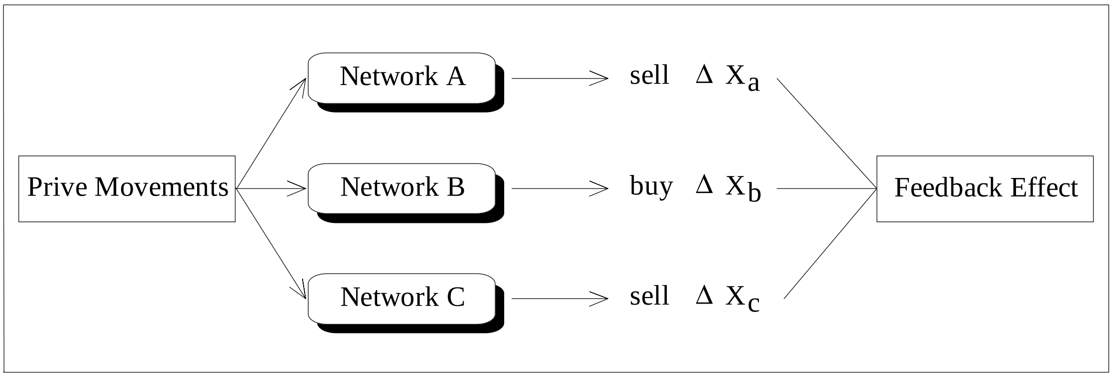
(a) risk factor price movements $\frac{\partial f_t(X_t)}{\partial x} \rightarrow \frac{\partial f_t(X_{t+1})}{\partial x}$, and

(b) portfolio rebalancing $\frac{\partial f_t(X_{t+1})}{\partial x} \rightarrow \frac{\partial f_{t+1}(X_{t+1})}{\partial x}$.

When we exclude (a) the effect of risk factor price movements from changes in sensitivity, we could obtain information on (b), the effect of the firm's portfolio rebalancing, $f_t \rightarrow f_{t+1}$.

Using neural networks, which can learn changing patterns of non-linear functions, we could make quasi firms by providing each network with the corresponding firm's portfolio rebalancing information based on the data set of daily changes in the firm's sensitivity and risk factor price movements.

2. Simulating a feedback effect of the rebalancing of portfolios by firms



Each network which learns a corresponding firm's portfolio rebalancing pattern functions as a quasi firm in our simulation. When we provide an initial price movement of asset X to networks, they decide whether and how much to buy/sell X based on the trading pattern they have learned. Their trade orders are aggregated and a new equilibrium found in the quasi market. If the price at the new equilibrium is significantly below/above the initial price, we can say that negative/positive feedback could be caused by the price movement. Furthermore, using this quasi market model, we could simulate a market price movement taking into account a feedback effect without incurring any reporting burden on firms.

Figure 1

Stress Exercise Structure

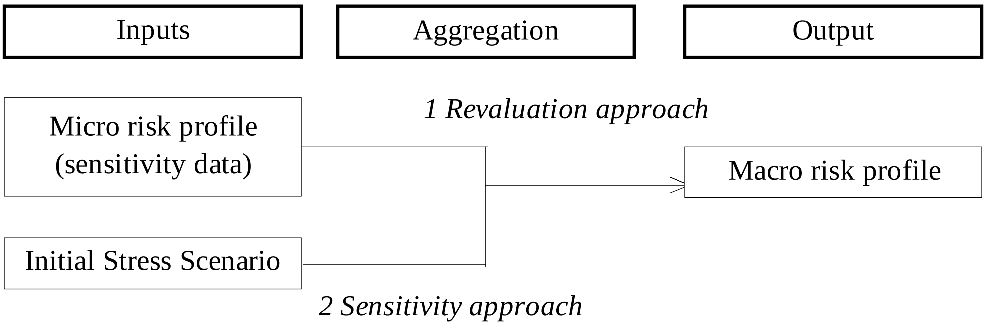


Figure 2

Multiperiod Model

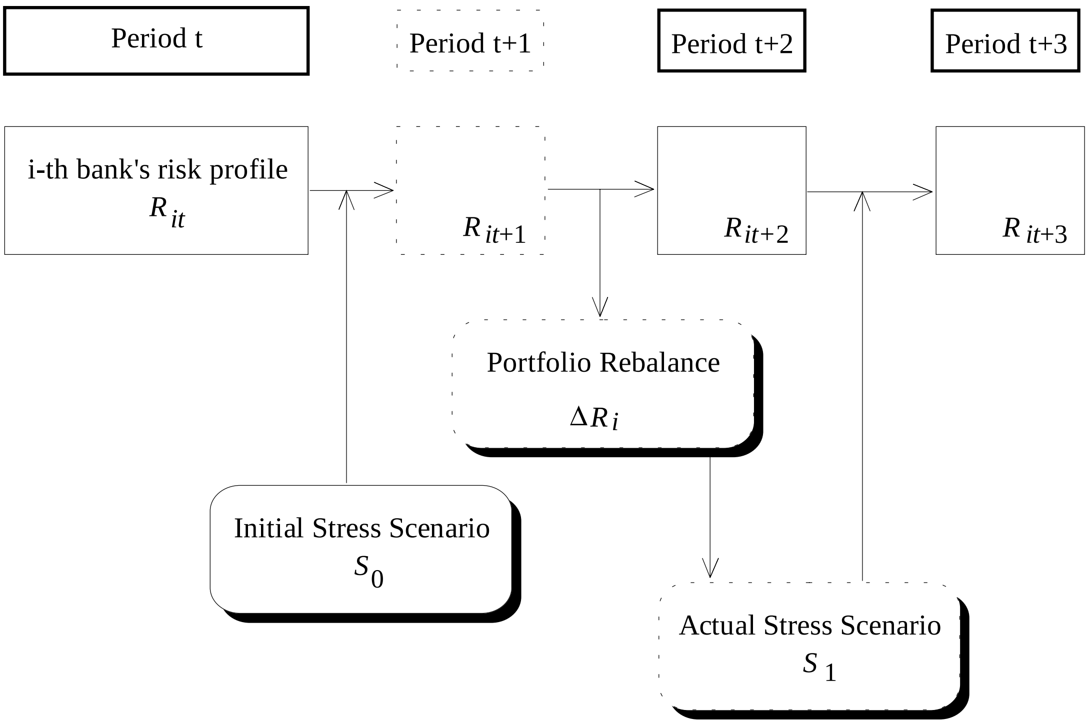


Figure 3

Figure 4-1

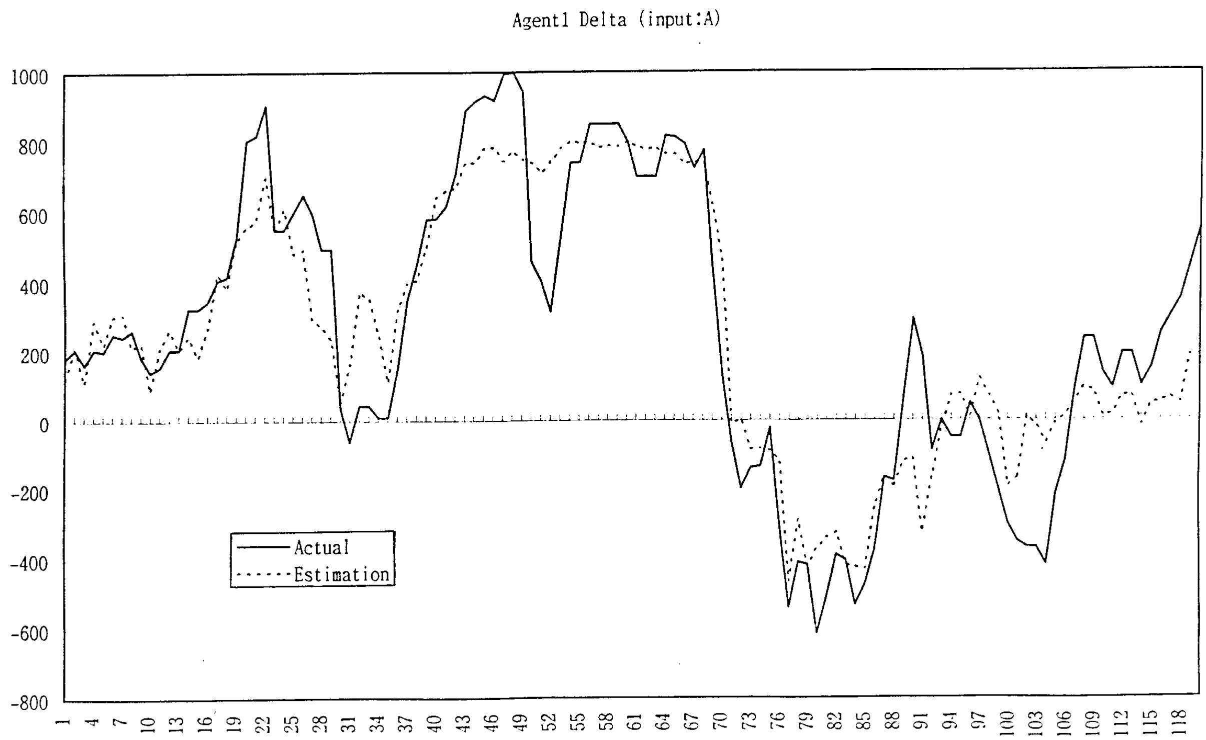


Figure 4-2

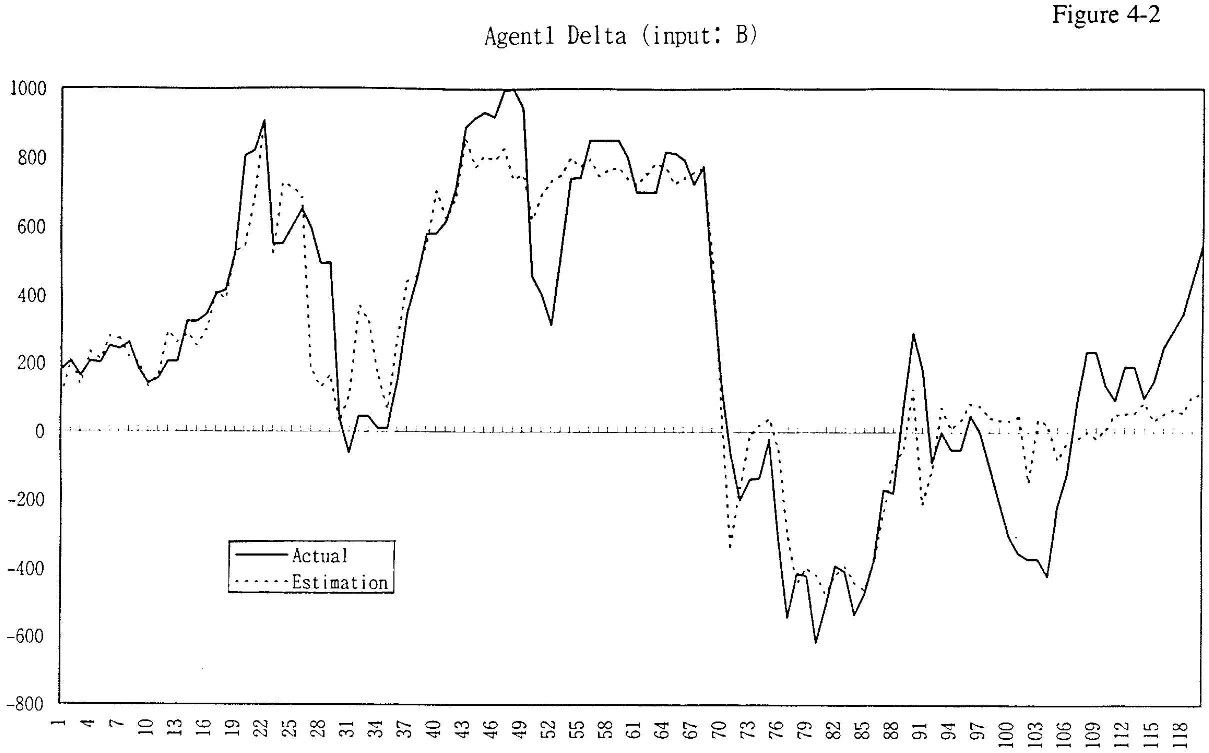


Figure 4-3

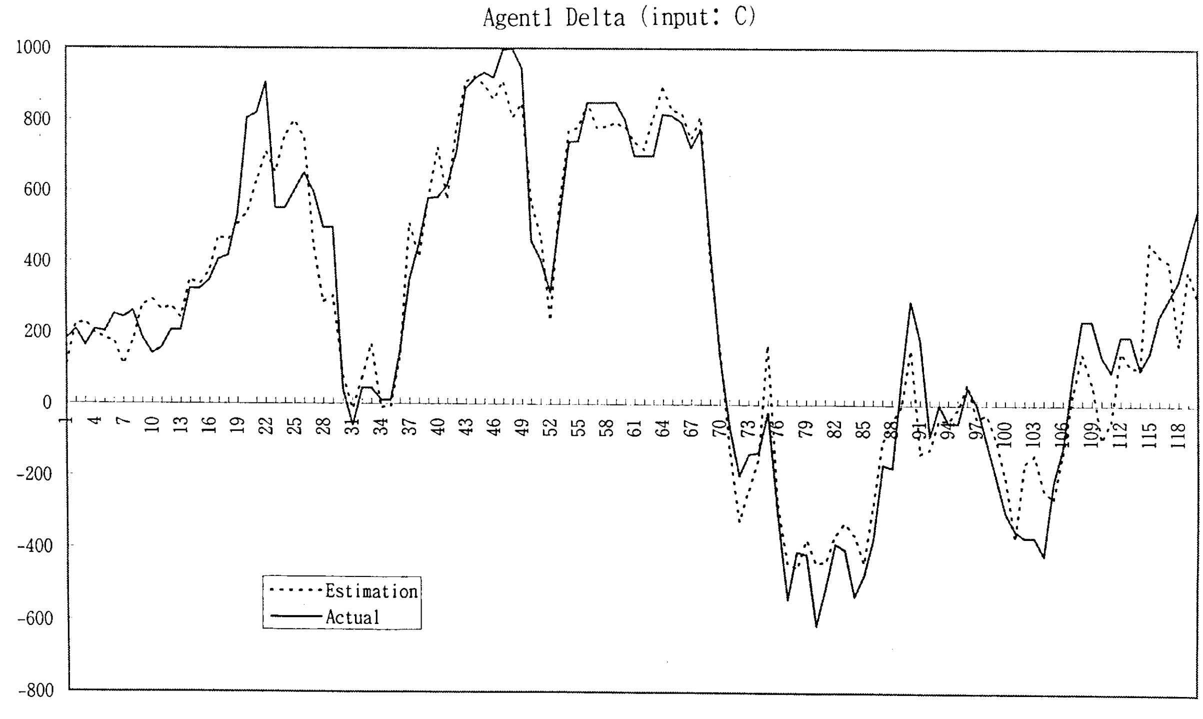


Figure 5

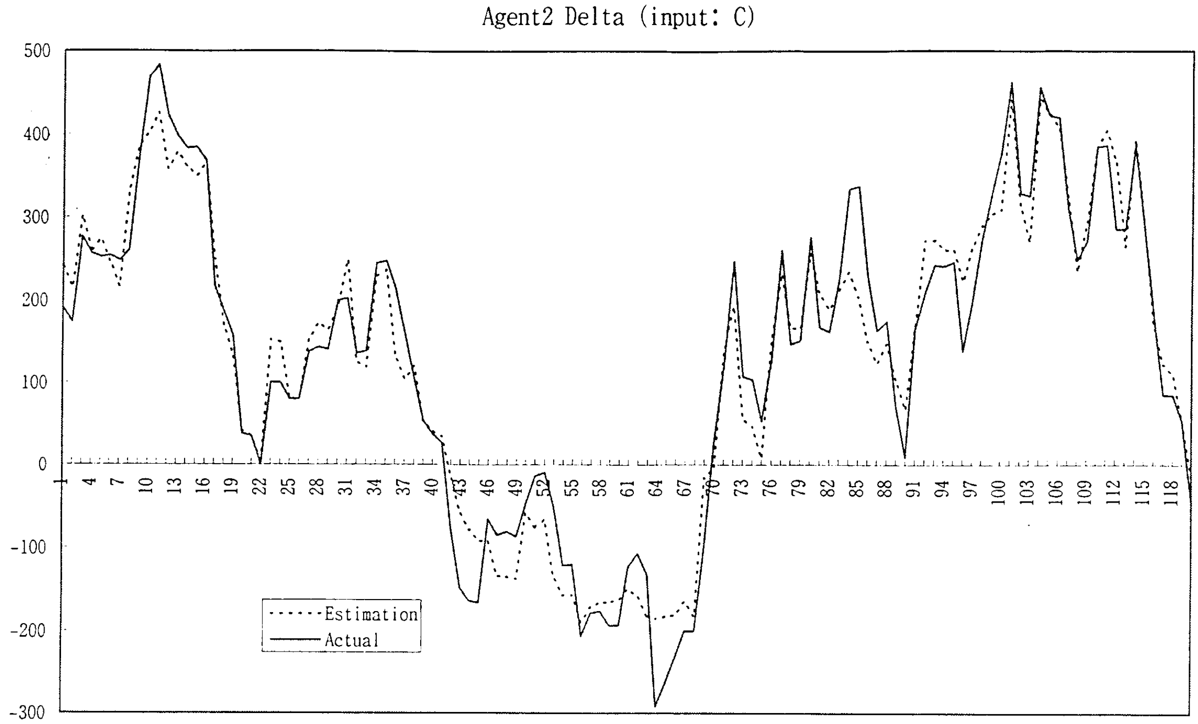


Figure 6

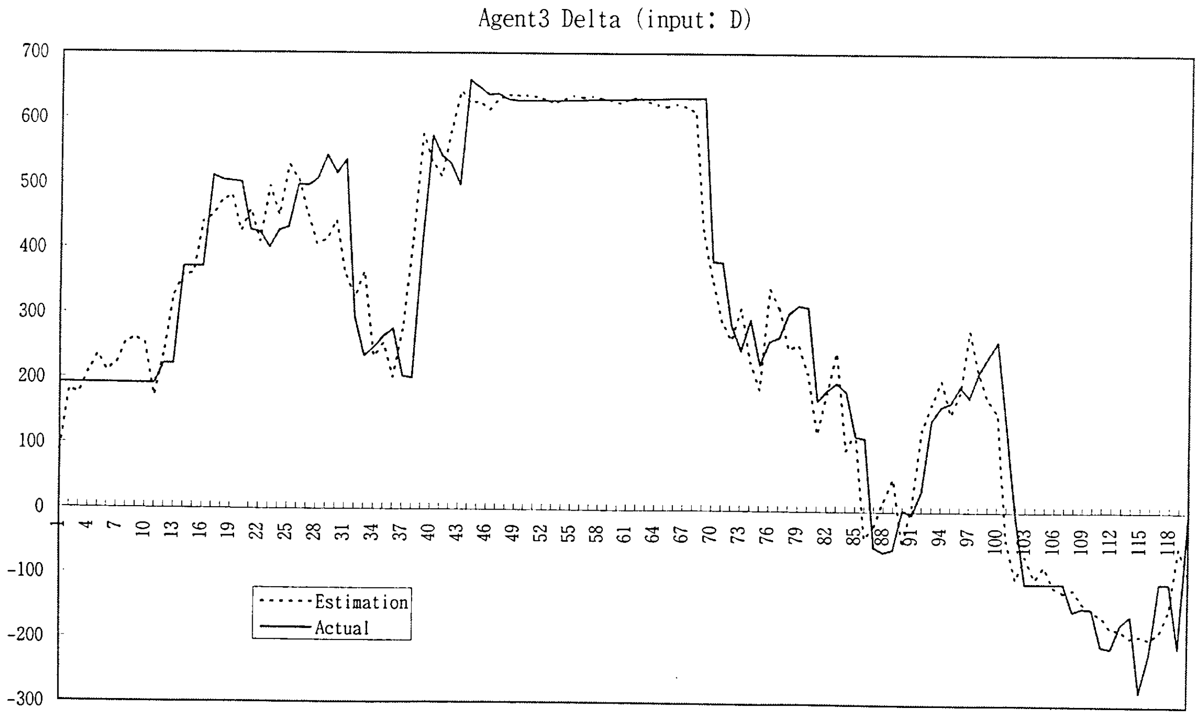


Figure 7-1

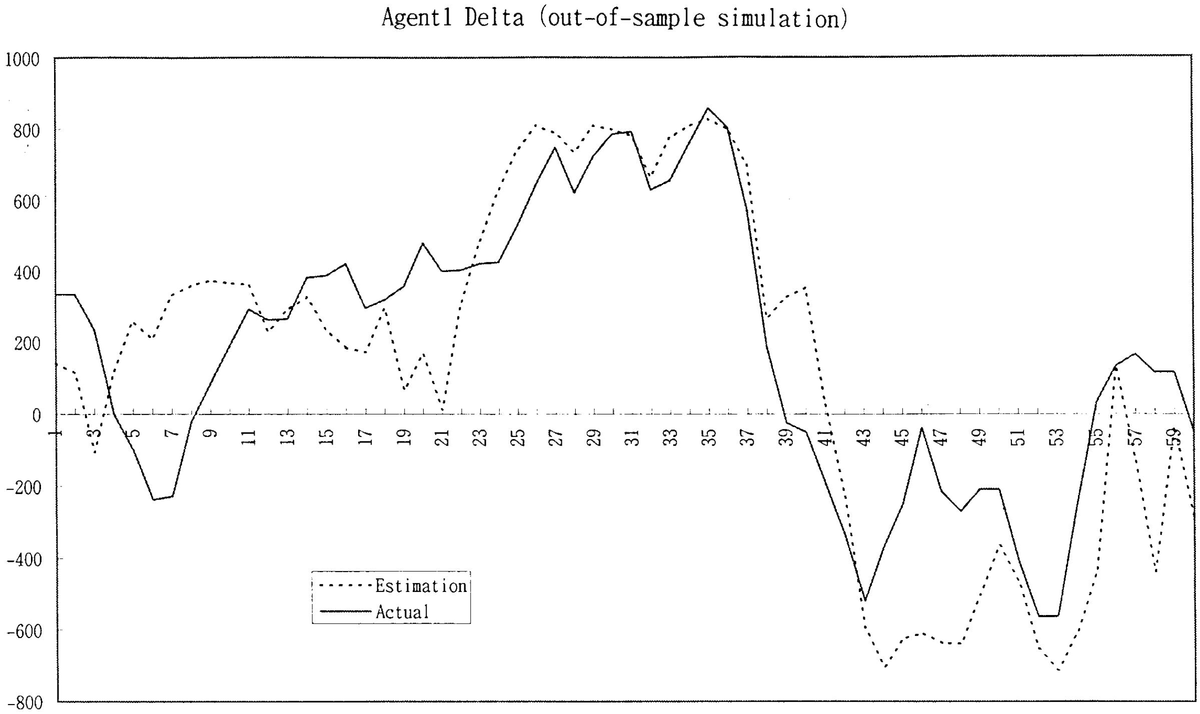


Figure 7-2

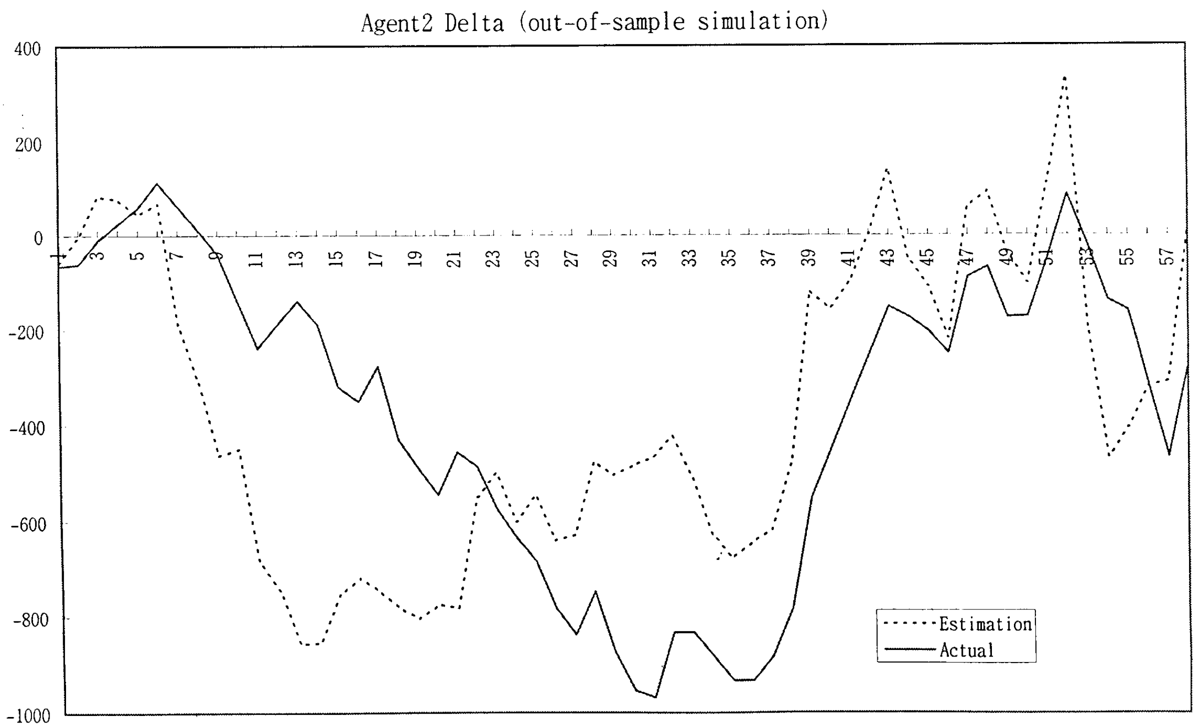
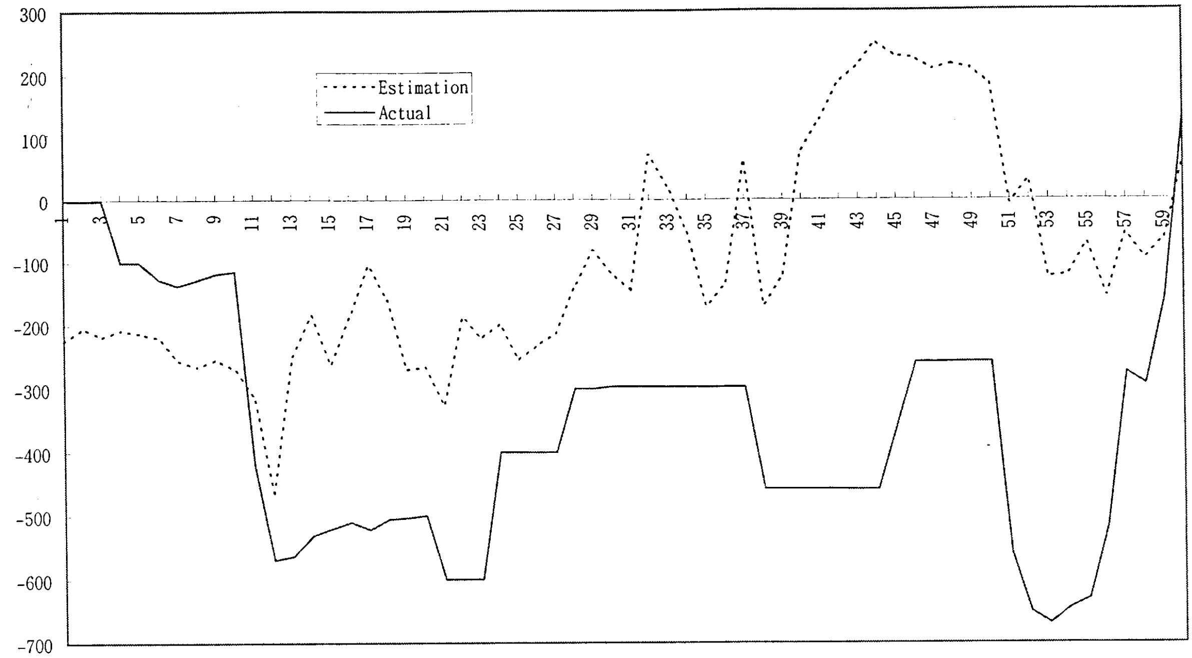


Figure 7-3

Agent3 Delta (out-of-sample simulation)



3y Swap Rate Daily Return

Figure 8-1

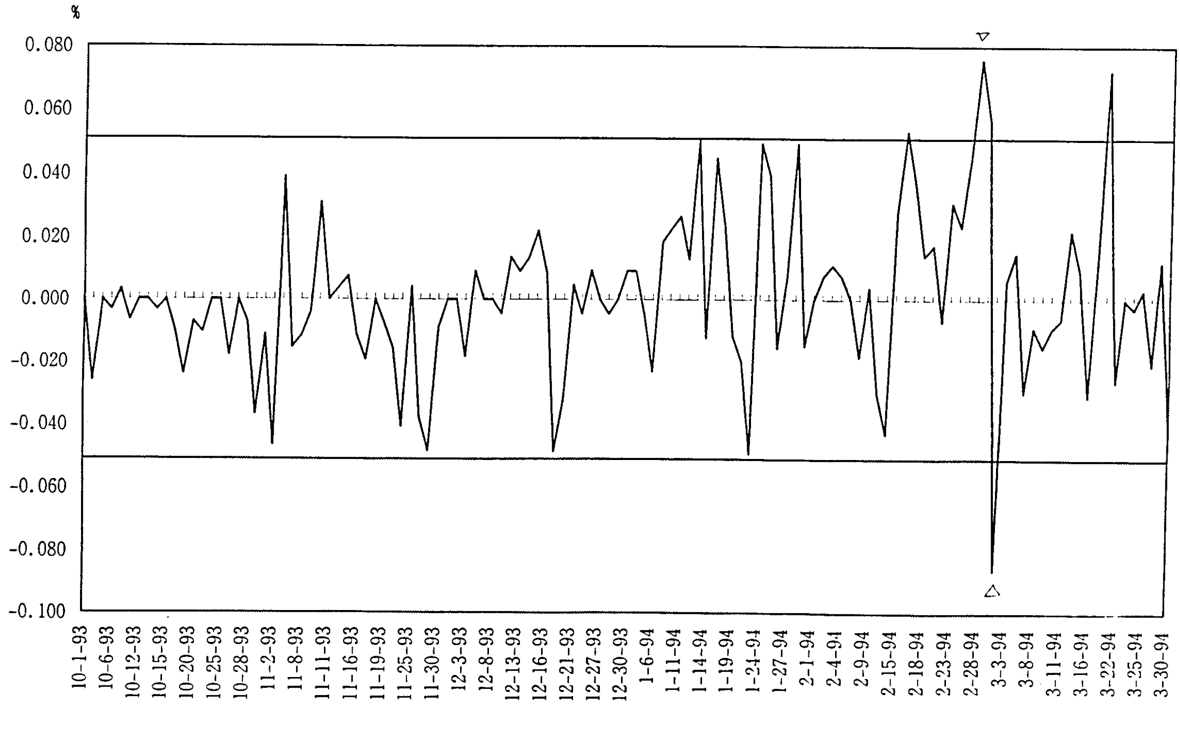


Figure 8-2

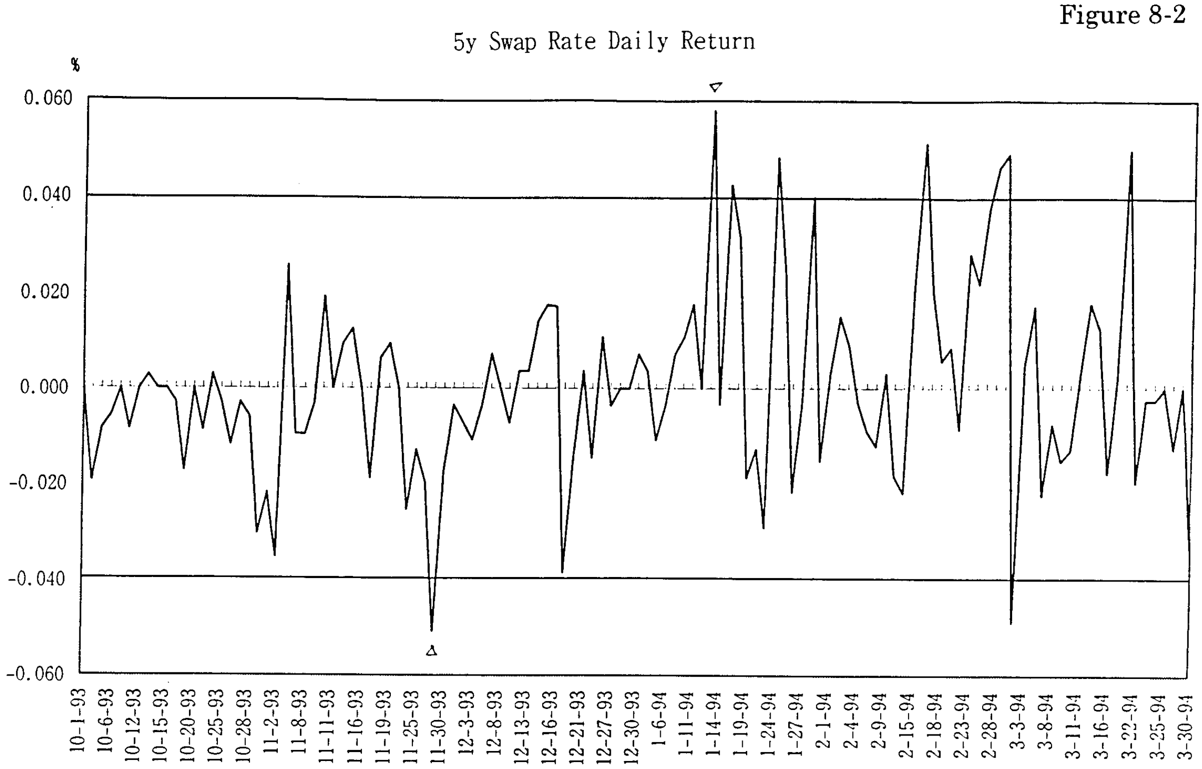


Figure 8-3

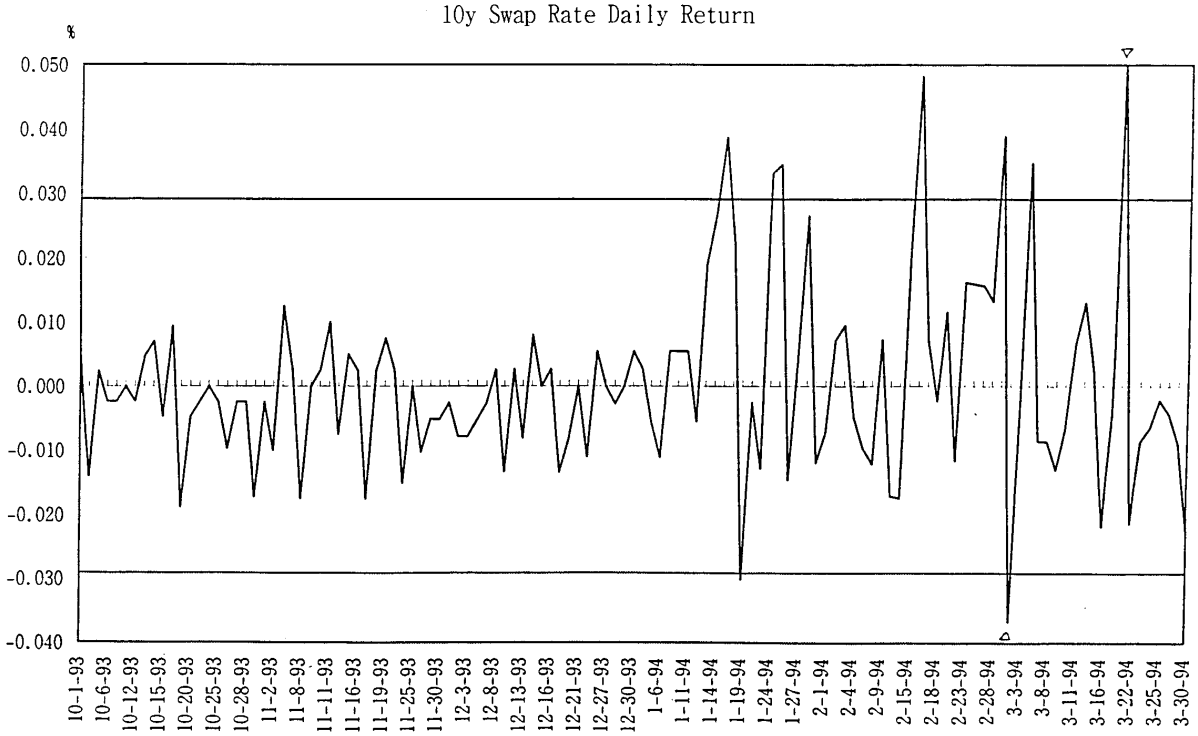


Figure 9

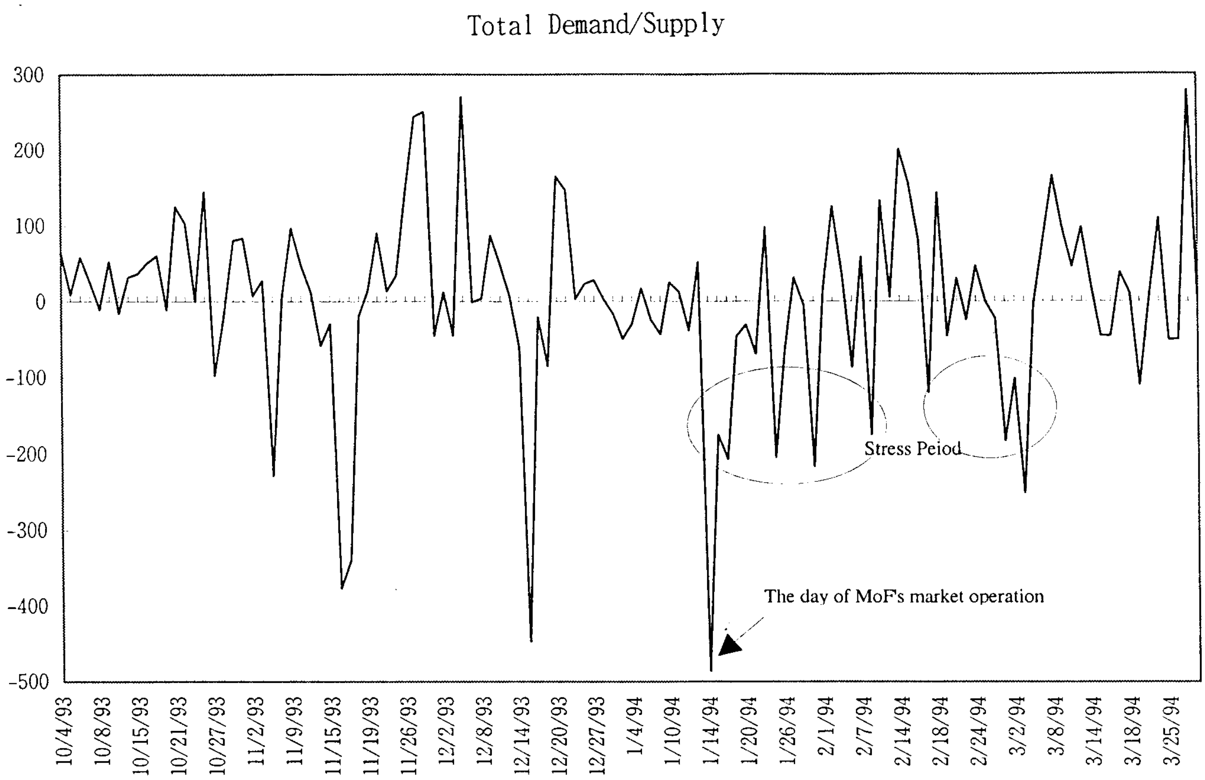

Total Demand/Supply
300
200
100
0
-100
-200
-300
-400
-500
Stress Peiod
The day of MoF's market operation
10/4/93
10/8/93
10/15/93
10/21/93
10/27/93
11/2/93
11/9/93
11/15/93
11/19/93
11/26/93
12/2/93
12/8/93
12/14/93
12/20/93
12/27/93
1/4/94
1/10/94
1/14/94
1/20/94
1/26/94
2/1/94
2/7/94
2/14/94
2/18/94
2/24/94
3/2/94
3/8/94
3/14/94
3/18/94
3/25/94

Figure 10-1

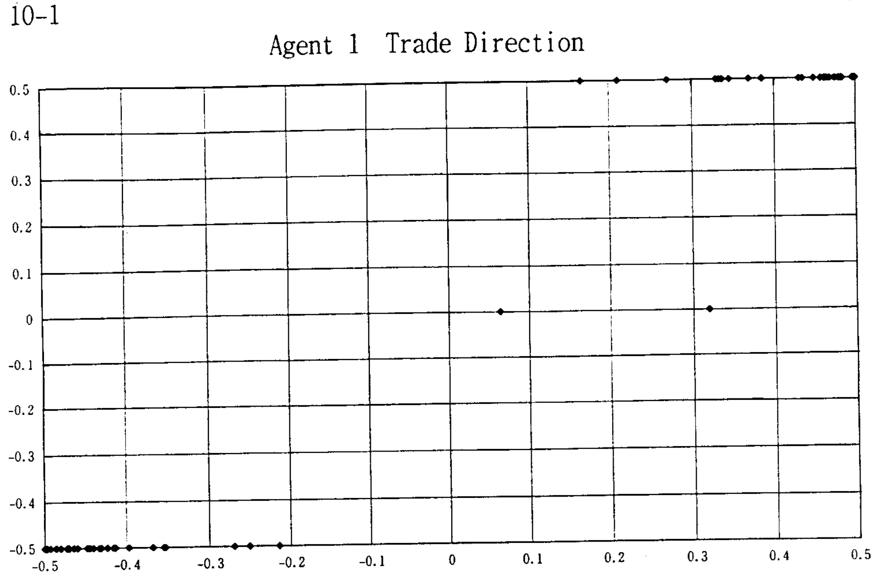


Figure 10-2

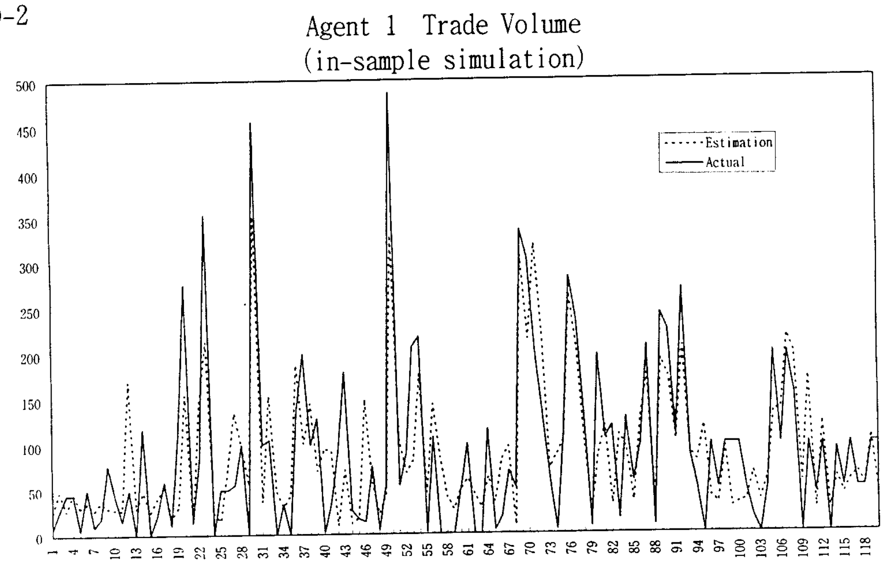


Figure 10-3

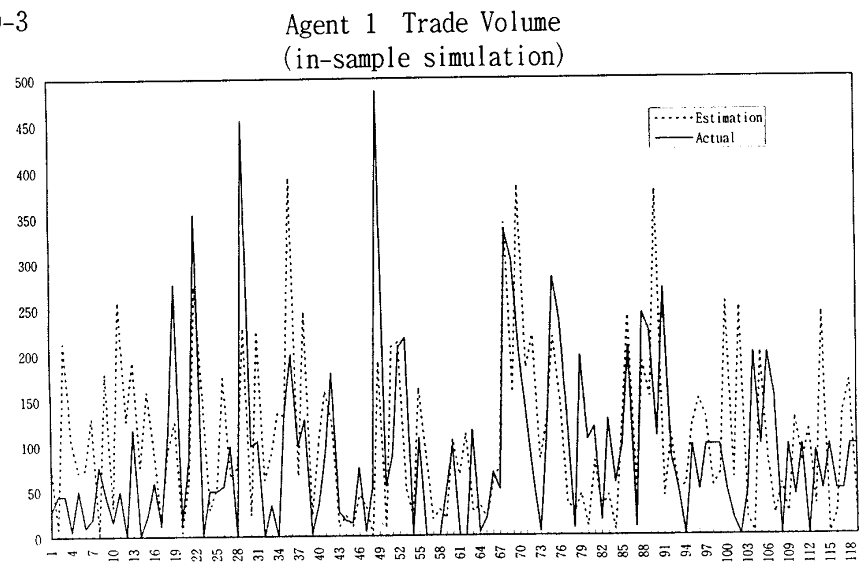


Table 1

Characteristics of the agents

	Key for trading	Fortrend/contrarian	Other characteristics	Targeted profit (loss limit), Position limit
Agent 1	Charts of market movements (chartist)	Fortrend	Positive correlation b/w P/L condition and trading volume	3 billion yen/half year, delta limit:± 100 billion
Agent 2	Charts of market movements (chartist)	Contrarian	Frequent writer of options	3 billion yen/half year, delta limit:± 100 billion
Agent 3	Fundamental events (fundamentalist)		Trading horizon is longer than the others (more an investor-type of trader than a dealer)	2 billion yen/half year, delta limit:± 70 billion

Table 2

Steps for inputs adjustments

Inputs	Agent 1	Agent 2	Agent 3
A: Market data (change on a trading day)	71.67%	82.08%	80.00%
B: A + accumulated P/L	76.67	82.08	82.92
C: B + recent market movements + news	85.42	87.92 (Figure 5)	87.92
D: C + market trend over a longer horizon (Agent 3)			90.83 (Figure 6)

Table 3

The magnitude of largest daily change

	Upward change	**Downward change**
3-year swap rate	3.01σ	-3.43σ
5-year swap rate	2.96σ	-2.59σ
10-year swap rate	3.46σ	-2.54σ

Table 4

Stress scenarios

	3y	**5y**	**10y**
Period t	3.33%	3.98%	4.49%
Scenario 1	3.59%	4.22%	4.72%
Scenario 2	3.05%	3.78%	4.33%
Scenario 3	3.59%	3.98%	4.49%
Scenario 4	3.05%	3.98%	4.49%
Scenario 5	3.33%	4.22%	4.49%
Scenario 6	3.33%	3.78%	4.49%
Scenario 7	3.33%	3.98%	4.72%
Scenario 8	3.33%	3.98%	4.33%

Table 5

Changes of each agent's delta under scenarios

(in billions of yen)

	Agent 1	Agent 2	Agent 3	Total D/S
Scenario 1	-8	26	-6	12
Scenario 2	-15	-9	-10	-34
Scenario 3	-3	33	-6	20
Scenario 4	-43	-3	9	-37
Scenario 5	-42	27	30	-15
Scenario 6	4	1	-4	1
Scenario 7	-4	14	-4	6
Scenario 8	21	20	31	72
Standard deviation				12

Table 6

Risk profiles of each agent

(in billions of yen)

	Agent 1	Agent 2	Agent 3	Total
Scenario 1	0.20	0.09	-0.03	0.26
Scenario 2	-0.51	0.18	0.13	-0.28
Scenario 3	0.45	0.25	-0.05	0.65
Scenario 4	-0.16	0.10	-0.12	-0.18
Scenario 5	0.08	0.13	0.18	0.39
Scenario 6	0.04	-0.00	-0.00	0.03
Scenario 7	0.11	0.02	-0.01	0.13
Scenario 8	1.98	0.40	0.80	3.18
VaR(t)	0.58	0.05	0.22*	

* Agent 3's VaR figure is measured at period t-1, since its position at period t is square.

References

Camargo, F.A. (1990): "Learning Algorithms in Neural Networks", *Working Paper*, The DCC Laboratory, Computer Science Department, Columbia University.

Glosten, L. and P. Milgrom (1985): "Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders", *Journal of Financial Economics*, 13, p.71-100.

Gennottee, G. and H. Leland (1990): "Market Liquidity, Hedges and Crashes", *American Economic Review 80*, p.999-1021.

Kawahara, J. (1996): "Credibility of Market Price and Market Impact", *Working Paper*, The Japanese Finance Association.

O'hara, M. (1995): "Market Microstructure Theory", Blackwell Publishers.

Shimizu, T. and T. Yamashita (1995): "Dynamic Micro and Macro Stress Simulation", *Discussion Paper*, 96-E-4, IMES, Bank of Japan.