Term structure and volatility shocks

by

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Abstract

This paper presents methodologies for generating scenarios for term structure and volatility shocks from historical data. These methodologies are selected to provide good approximations to data and to be easy to replicate. The approaches are based on principal components estimated from return data.

Government bond returns from seven countries are used to illustrate the methods. The country level data suggest that three to four principal components are sufficient to capture most variation in individual country term structures. The first principal components for bond returns are somewhat correlated across countries; there is less evidence of correlation for other components. Cross-country evidence suggests that correlations between components changed from 1990-93 to 1994-96. Generalised autoregressive conditional heteroskedasticity (GARCH) variance models for these principal components are estimated for each country. In several country models asymmetric responses of volatility to return surprises are detected.

The evidence suggests that variation in particular country term structures can be well described by relatively few common components. However, considerably more components are required to jointly describe movements in several country term structures, in part because returns of short maturity government securities were not highly correlated across the sample countries in 1990-1996.

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1. Introduction

Why measure exposure to asset prices? Management of financial firms must know their firm's exposure to asset price changes both to understand the riskiness of the firm's business and to appreciate which price changes would generate large losses at the firm. In order to understand the exposure of a financial firm, it is necessary to identify asset price movements that could cause large changes in market value of the firm or in the value of the firm's trading book.

Measures of aggregate exposure over a set of major market-making firms could identify events that would cause large losses at many firms. Aggregated exposure might also identify asset price movements that could generate unusual market dynamics by, for example, inducing additional hedging-related transactions that could exacerbate the initial price movement.

Whether at the firm level or aggregated over many firms, exposure could be measured in at least one of three ways: The first is historical, using large historical movements in asset prices to define scenarios. In this method, firms would price their books of actively traded instruments at each scenario. The change in value of their books would measure exposure to each scenario's shocks. The second approach to measuring exposure is based on firm sensitivities to asset price changes. Here firms estimate the sensitivities (defined as the derivatives) of the value of their books to changes in a set of prices. The third approach begins with the development of hypothetical future events that might create large losses. Plausible asset price responses to each event are constructed with the aid of market experts and used to define the scenario associated with the event. The implied change in value of firms' books defines the possible exposure generated by the event.

Each of these three approaches can generate scenarios that help evaluate how the market value of firms might change in response to asset price shocks. The first approach may provide a better measure of responses to large price changes than the second approach which probably better describes the impact of small asset price changes. The value of the third approach depends critically on the selection of the particular stressful events and the definition of their asset market consequences.

The two sections below outline methodologies for generating scenarios for term structure and volatility shocks from historical data. These methodologies are selected both to provide good approximations to data and to be easy to replicate. Both approaches are based on principal components estimated from return data.

The methodology are illustrated using returns across the government bond term structure from seven countries. The country level data suggest that three to four principal components are sufficient to capture most variation in individual country term structures. The first principal components are somewhat correlated across countries while there is less evidence of correlation for other components. Cross-country evidence suggests that correlations between components changed from 1990-93 to 1994-96. GARCH (generalised autoregressive conditional heteroskedasticity) models for the variance of these principal components are estimated for each country. In several country models asymmetric volatility responses to positive and negative return surprises are important for the components.

2. Term structure shocks

This section describes a methodology for summarising historical movements in interest rates across government term structures. The methodology could be used to construct scenarios of large historical changes in term structures. These scenarios could be used both for computing the exposure of a financial firm's trading book to term structure movements as well as for stress testing purposes. The outputs from the scenarios could also be aggregated over market-making firms to analyse the impact of large term structure shifts on aggregate profit & loss statements.

Introduction

Term structure research has established that much of the variability in government bond returns can be summarised by movements in a few underlying factors.[1] Further, if these factors have high predictive power across maturities, the factors may also capture extreme historical movements in the term structure.[2] I suggest using principal components to extract these factors from government zero coupon bond returns.[3] I also discuss methods for using extrema in the observed principal components to construct extreme historical scenarios for changes in rates across government term structures.

One disadvantage of principal components is that they are identified by a statistical procedure, and thus may be harder to interpret than factors defined by specific economic variables. However, most evidence suggests that principal components can provide a useful summary of variation across a term structure. Moreover, the principal components could also be used in modelling volatility across the term structure.

Zero coupon rates are natural building blocks to describe interest rate movements in a government securities market. Scenarios defined by changes in government zero coupon rates can easily be converted into scenarios for prices of other non-callable government securities. Finally, zero coupon rates can provide a dimension reduction if interpolation is used to give zero rates for maturities between those in the scenario.

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1 See Kahn (1989) and Litterman and Scheinkman (1991) for an introduction to US evidence; Steeley (1990) for UK evidence; and Murphy and Won (1995) and Chaumeton et al. (1996) for discussions of cross-country evidence.

2 Kambhu and Rodrigues (1997) present an example where excluding factors that account for variation across the term structure can lead to scenarios that miss substantial variation in the value of trading books.

3 This methodology and factor analysis are used extensively in the term structure literature and in practice. See Campbell et al. (1997) for a general introduction.

Other methodologies that could be used to summarise term structure movements include a variety of factor modelling approaches. While the principal components and factor approaches can give very similar results when variances are constant,[4] some form of time-varying variance model is more plausible for most asset returns. Consequently, factor models incorporating time-varying variances could give a more precise approximation to financial data than principal components approaches. Moreover, an explicit factor model might provide powerful tests of whether factor sensitivities are constant over time and whether an adequate number of factors has been specified to account for cross-maturity correlations. However, a potential drawback to factor approaches is that they must be estimated with computationally-intensive nonlinear optimisation and, thus, may be somewhat hard to implement and to replicate. Largely for computational simplicity, the current analysis uses a principal components approach.

Data

The analysis below uses data from 7 countries including Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), and the United States (US). Individual country analyses use returns on zero coupon securities derived from J.P. Morgan daily zero yields measured from January 1994 through September 1996 (see Murphy (1991) for a description of the methodology). The zero coupon maturities are 1, 2, 3, 4, 5, 7, 10, 15 and 20 years for all countries. In the recent period, the data include 25 and 30 year maturities for all countries but Japan and the United Kingdom. I also use data from January 1990 through September 1996 to conduct an aggregate analysis across countries. Maturities over 10 years are not available for Germany, Italy, and Japan in 1990-93.

There are two sources of measurement error in these returns: First, the zero coupon yields are estimated from bond prices and, thus, are not measured exactly. This error arises both because the estimated zero curve generally does not exactly fit all coupon bond prices and because some government bonds may not be traded frequently so their quotes may not indicate current market valuation. Second, the returns are computed from successive daily yields at the same maturity and, thus, do not reflect the return on a specific zero coupon security.

Country results

The principal component results are presented in Table 1. Three general conclusions can be drawn from the country data: First, all country term structures can be summarised by a small number of principal components. Second, two or three components are not always sufficient if we wish to describe close to 100% of term structure movements. Third, the natural ordering of principal

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[4] Campbell et al. (1997) discuss similarities between principal components and factor analysis. Loretan (1996) outlines conditions when principal component analysis might be expected to provide a valid summary of a multivariate distribution.

components does not always correspond to the most interesting economic order. Thus, some judgement is required in selecting principal components.

The same statistics are presented for each country. The first matrix reports the percentage of total variance of each maturity zero coupon return that is explained by a principal component. Each column in the matrix corresponds to a principal component - the columns are ordered from left to right; each row corresponds to the labelled maturity. The last row reports a measure of the overall explanatory power of the component for returns.[5] The second matrix for a country presents estimates of the responses of the zero coupon yields to a one-standard deviation increase in a particular principal component. As in the first matrix, columns correspond to principal components and rows correspond to maturities.

Starting with Canada results, the first (leftmost) principal component explains a large fraction of variance for each maturity, from a low of 46% up to 99% for the 25 year maturity. The second principal component adds to the explanatory power particularly for shorter maturity zeros. (Since the principal components are uncorrelated, these fractions can be added to determine the total fraction of variance explained by a set of components.) Note that the third component also improves explanatory power for short maturities but by less than the fourth component. The first, second, and fourth component together could explain at least 82% of variance at every maturity. More components would be required to cover 90% of variation in every maturity. The sensitivities at the bottom of the page, along with similar sensitivities for returns, give a method to convert extreme values of the principal components into extreme yield changes or returns. The sensitivities suggest that the first component represents shifts in the term structure, that the second component represents term structure steepening, and that the third component (and the fourth component except at 30 years) reflects greater curvature in the term structure.

The other country results are somewhat similar. In every case the first component accounts for considerable variance across the term structure and reflects a term structure shift. However, interpretation of the other components is not always identical. Further, for several countries (Germany, Italy, Japan, and the United Kingdom), four components are required to describe over 80% of variation at all maturities. In order to account for most of the variation across the term structure, this country-level analysis suggests that at least three or four principal components per country are required.

Results of an analysis of standardised returns (defined as the return for each maturity divided by its sample standard deviation) are shown in Table 2. The standardised returns lead to slightly neater results: Over 90% of variation in most countries standardised returns at all maturities are described by the first three components. As we observed in the analysis of unstandardised returns, the components for each country are associated with similar movements in yields across the term

[5] The measure of overall or total explanatory power is the ratio of variance of the component to the trace of the covariance matrix of returns. These values sum to one.

structure. Specifically, the first component for each country's data seems to represent a shift in the term structure while the second represents a twist and the third introduces greater curvature in the term structure. Note that the sensitivity of yields at various maturities to the components derived from standardised returns is often similar in size to sensitivities derived from unstandardised returns, implying that scenarios derived from both approaches will be similar.

Aggregate results

Combining several country term structures offers the possibility of further dimension reduction because there may be consistent, common influences across the markets. I consider two approaches to measuring possible gains by combining countries: First, I investigate correlations between principal components estimated for different countries. Second, I analyse combined data for the seven countries described above. These approaches both suggest that some dimension reduction is possible from combining data across countries - this reduction reflects correlations in returns across countries.

Evidence from correlations between country-level principal components suggests that there is some correlation between the first components extracted from different country bond return data. There is less evidence for correlation between second and third components. Table 3 compares correlations between first, second, or third principal components across countries over two time periods (1990-93 and 1994-96). Generally speaking the first principal components exhibit the highest correlations across countries with the second components less correlated and the third components typically weakly correlated. There is some evidence that correlations changed after 1993, with stronger correlations often observed between first principal components in 1994-96. Table 4 presents the same comparison estimated from standardised returns. While correlations between components are often more positive with the standardised returns, the standardised returns seem to give qualitatively similar results to the unstandardised returns.

Combining country data gives a different perspective on correlations across country bond returns. These calculations suggest that there are minor additional reductions in dimension from combining data across countries. The major positive result is that relatively few (5) principal components describe a large fraction of variance in the long ends of most country term structures. Japan is an exception; movements in Japanese rates are described by components that are not highly correlated with other country term structures. Moreover, term structure returns in the 1-5 year maturity range are not well described by the first 5 components that account for variation at long maturities. In fact, many principal components would be required to explain 1-5 year maturity variance across these countries. Thus, if our goals are both to reduce the number of variables that define scenarios as well as to choose variables that describe most of the movement in the term structures, joint modelling of term structures in these countries may require almost as many principal components as would be required for separate country models.

Two views of the explanatory power of the principal components illustrate these results: Table 5 presents the variance of each maturity's return explained by the first 5, 10, or 20 principal components estimated over 1994-96. These principal components were computed jointly from the covariances of all 73 returns. The first 5 principal components describe most of the variation in returns of maturities over 10 years with the exceptions of the U.K. returns (where around 60% is captured) and of the Japanese returns (where very little variation is captured). The first 10 principal components explain longer maturities well but only account for 30-50% of the variation in short maturities. Moving to the first 20 principal components helps but in five countries (Canada, Germany, Italy, Japan, the United Kingdom) is not sufficient to describe 80% of variation in short rates.

However, analysis of the 1990-93 period suggests that the relatively high recent correlation in returns may be exceptional. Table 6 reports the explanatory power of the first 5, 10, or 20 principal components for each maturity's return in 90-93. Note that the 15-30 year maturities are not available for Germany, Italy, and Japan in this earlier sample so the number of reported maturities is smaller for those countries. While the first 5 principal components continue to describe long maturity returns well in several countries, they capture less variation in German and Italian returns. Although the first 10 components describe variation in most country returns, the first 20 are required to also capture variation in Japanese returns. This comparison of 1990-93 to 1994-96 suggests that the role of the components changed somewhat from the first to the second period. These results are reinforced by a graphical analysis.

A second, graphical illustration of association across countries provides additional insight into return correlations across countries. Chart 1 is a contour plot for 1994-96 data that circles the combinations of principal components and of securities where a component explains at least 10% of a particular security's return variation. Thus, tight clusters of curves highlight components that account for substantial return variation in particular securities. In contrast, areas in the grid without any marking indicate components that do not describe those returns well. The chart illustrates that the first five components describe return variation for all countries except Japan. Component 6 describes several U.K. maturities while components 7 through 9 help explain the Japanese term structure. The small marks for higher numbered components indicate components that account for particular country shorter maturity returns. There is at least one unique "short-term" component for each country. An analysis of the return variation matrix suggests that at least 10 principal components are required to account for at least 90% of 1 year return variation in each country.

Chart 2 repeats this analysis for the 1990-93 data and illustrates that the correlations of returns were somewhat different than in 1994-96. The first five components no longer seem as important across countries with Japanese and Italian returns somewhat unrelated to other country returns. As in the later period, shorter and longer maturities are occasionally related to different sets of principal components.

Constructing term structure scenarios

Having derived a set of principal components that account for most variation in term structure returns, there are several approaches to deriving scenarios:[6] The first approach involves identifying large values of individual principal components and mapping those into a set of historical scenarios. In this approach, the large values of the components could be defined either by tail percentiles in the components' empirical distribution or, if the components are distributed symmetrically, by multiples of the components' standard deviations. Each actual scenario would be derived by multiplying the corresponding component value by the yield sensitivities reported earlier.

A second approach would combine movements in principal components to produce scenarios that might correspond more closely to actual term structure movements. Specifically, the approach involves creating a separate scenario for each possible combination of large increase, large decrease, or no change in the principal components.[7] Thus, with N principal components there would be 3^N-1 possible scenarios, illustrated below for a two component example:

		Component 1		
		–	0	+
Component 2	–	Scenario 1	Scenario 2	Scenario 3
	0	Scenario 4		Scenario 5
	+	Scenario 6	Scenario 7	Scenario 8

The extreme outcomes for each principal component could be selected using either observed values in the tails of the component's empirical distribution or multiples of the component's standard deviation.

A third approach would identify specific historical episodes and use the observed component shocks during those episodes to derive a scenario. An alternative to this approach would simply use the observed term structure movements during these episodes as a scenario.

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6 The analysis in this section assumes an objective of measuring the change in trading book value induced by large movements in term structures. A more detailed grid of scenarios would be required when the objective is measuring how trading book value varies with changes in interest rates over the whole range of historically observed rate movements.

7 This is a simplified version of the approach proposed by Jamshidian and Zhu (1996). A simpler alternative would construct scenarios using just large increases and large decreases in the components. This would create 2^N-1 possible scenarios. Jamshidian and Zhu (1996) present evidence that division of the historical range of component outcomes into several categories can lead to a set of scenarios that cover many likely term structure outcomes. The examples in Kambhu and Rodrigues (1997) imply that these risk measures should be constructed using all components whose risk is priced in the market. A separate analysis of portfolio sensitivity to shocks in sources of residual risk would also be appropriate if exposure to some sources of residual risk might be large.

Other topics

Several other issues need to be considered when constructing interest rate scenarios. These include how to measure credit quality shocks, whether some scenarios should include shocks that data suggest occurred over more than one day, and if the statistical models for returns are stable over the sample.

The discussion above has focused on modelling government security term structures. However, term structures for private sector debt with different credit quality are more relevant for pricing many instruments including swaps and most forwards and futures. Furthermore, academic and practitioner literature suggests that risky debt spreads over comparable government security yields vary somewhat predictably with the business cycle and with the level of short rates.[8] The limited data on private sector rates for different credit quality borrowers suggests that some compromises are necessary to model and generate scenarios for shocks to private sector rates. One approach would generate shocks for private sector rates by applying typical credit spreads to the government term structure shocks or by assuming that private sector rates retain their typical correlations with government rates. A major drawback of this approach is that large asset price movements might also lead to significant changes in credit spreads. A more realistic approach would use data on private sector rates by country in an analysis like that carried out above for government term structures.

My analysis has focused on modelling daily returns. An extension of this analysis would construct some scenarios from shocks over several days. While the results of applying this type of scenario could be misleading (because firms adjust hedges during an event), they may also represent a different type of stressful event from the single-day-shock. Another reason to consider multiple-day returns is that measured correlations would be less affected by non-synchronous daily measurements of yields.

Finally, a more detailed analysis of the stability of the principal components representations used to construct the scenarios is necessary.[9] If the models are not stable or if the models do not describe most of return variance, the scenarios may fail to map out large historical events.

3. Volatility shocks

Financial firms that trade options or financial instruments with option components will have typically have exposures both to asset prices and to volatility. Thus, analysis of their exposures

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8 See Duffee (1996), Knez, Litterman and Scheinkman (1994) or Litterman and Iben (1991) for examples.

9 The simplest test for stability, measuring whether the covariance matrix of returns is approximately constant over extended samples, is problematic with asset prices because conditional volatility of returns typically varies over time. However, it is possible that the responses of yields or returns to principal components or factors are stable. This could arise if variation in the conditional volatility of the principal components is the source of variation in the conditional volatility of returns.

requires measuring changes in the value of their trading books as both volatility and underlying prices and interest rates move.

There are several interpretations of changes in volatility: For example, a trader's view might be a change in the quote sheet of implied volatilities for options on a particular underlying by strike price and time-to-expiration. This is essentially equivalent to a change in the price of a group of options; without further structure such a change could be associated with a change in the variance of the underlying distributions of future prices or even with a change in the shape of underlying future distributions.[10]

Although volatility changes could be derived from options price data, I focus on models of the underlying variance of asset prices and specifically GARCH models. Such models provide estimates of the volatility of the underlying asset price and have the advantage of being easy to estimate.[11] Other models, such as stochastic volatility models, also provide good descriptions of movements in asset price volatilities.[12]

Introduction

This section outlines a fairly simple approach to measuring volatility movements, based on GARCH models. Specifically, if the set of underlying asset prices are well-described by a factor model, volatility models for the factors could describe most of the variation in volatility of the asset returns.[13] Volatility models for factors could provide a more parsimonious description of volatility and would require estimating far fewer models than direct modelling of volatility for each asset price.

The empirical section below illustrates an approximation to a full factor model. Rather than estimating a full factor model with GARCH volatilities, principal components are extracted from a set of asset returns and GARCH models are estimated for the principal components. This approach is easy to reproduce but would only provide consistent parameter estimates of the underlying factor model under quite restrictive assumptions. Future work is required to compare these estimates with factor model estimates to illustrate the size of estimation error arising from this approximation.

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10 There is a large literature proposing methods for extracting the implied distribution of the underlying price from a set of options prices. For examples, see Bates (1991), Derman and Kani (1994), Malz (1996), Melick and Thomas (1994), Neuhaus (1995), or Rubinstein (1994).

11 Andersen and Bollerslev (1997) suggest that evidence of poor volatility forecasts at high frequencies by ARCH class models may reflect the noise in daily volatility estimators and that ARCH models provide good forecasts of underlying volatility.

12 See Melino and Turnball (1991) or Campbell et al. (1997) for an introduction to stochastic volatility models. Nelson (1992) and Nelson and Foster (1994) demonstrate that ARCH models provide an approximation to stochastic volatility models. Jacquier et al. (1994) present a computationally attractive method of estimating stochastic volatility models.

13 This might correspond to the Factor ARCH model of Engle et al. (1990) or to a factor model with different volatility models for each factor as in Harvey et al. (1992) or King et al. (1994). If the sources of idiosyncratic risk also display time-varying volatility, then their volatilities must also be modelled.

When a set of volatility models have been derived, there are several approaches for developing scenarios for large historical changes in volatility: First, the models could be used directly to generate predicted changes in volatility in-sample; the extreme values of these changes could be used as scenarios. Second, scenarios for large factor movements could be combined with the GARCH models to generate predicted changes in volatility that are consistent with the changes in underlying prices. Finally, particular the predicted volatility changes during specific historical episodes could be computed. I present examples below of the first type of scenario.

Models

To illustrate this methodology, I estimated GARCH models for the first three principal components for government term structures of Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. The principal component procedure is described in the previous section. These models were estimated on daily J.P. Morgan zero coupon government bond return data from January 1994 through September 1996. (See Table 7.) The estimated models were GARCH(1,1) models with allowance for possible asymmetry in conditional variance (sometimes called the leverage effect in the literature). That is, negative returns are allowed to imply different future predicted volatility than positive returns. Results varied across term structure models but in many cases the models implied slowly decaying volatility in the first (shift) component; some country models also showed slow decay for the second (twist) and third (curvature) component. Leverage effects were most often important for the third component but in some countries are also observed in volatility of the first and second principal components.

Chart 3 displays several views of the predicted volatility across the term structures of each of the seven countries. The mean and range of predicted volatility across the maturities is reported in the top chart. As expected, volatility of returns typically rises with maturity. The volatility models appear to capture much of the underlying term structure variance because the mean variance predicted by the model is similar to an estimate of the mean unconditional variance. The middle and bottom charts show the largest one and five day changes in volatilities. Note that in some countries the largest five-day changes are slightly smaller than the one-day changes.

Scenarios

The predicted volatility changes provide one set of volatility scenarios. A full scenario would include changes in volatility for forecast horizons up to several months in addition to the daily changes exhibited in Chart 3. Another set of scenarios could be constructed from scenarios for the principal components by conditioning on the principal component shocks when constructing volatility scenarios. This second set of scenarios would be consistent with the scenarios for the term structure and so may be preferable to the first method. A third set of scenarios could be constructed from estimated conditional variance movements during specific historical episodes.

Future work

One check on the realism of this approach would be to compare volatility shocks constructed from GARCH models to movements in implied volatilities derived from options on the relevant asset. Practitioners often report that implied volatility tends to be less variable than historical volatility. Since GARCH estimates could be considered a type of historical volatility estimator, it is possible that the GARCH models generate volatility changes that are too large.[14]

Future work should compare volatility model estimates derived from principal components to factor model estimates. Such a comparison would provide an estimate of the likely size of estimation error from using the simpler principal component procedure.

4. Conclusions

This paper presented methodologies for summarising large movements in term structures and in volatilities. Empirical results suggest that several country government bond term structures can be described by three or four principal components. The principal components appear to have simple interpretations similar to those found by other researchers who have modelled the U.K. and U.S. term structures. The first principal component, which accounts for a large fraction of return variance at most maturities and which reflects roughly parallel shifts in the term structure, is more highly correlated across countries than other principal components. However, the cross country correlations seem to have shifted after 1993, suggesting that multiple country models are less stable than single country models. A methodology is also presented for constructing term structure shocks for risk measurement purposes.

The paper also proposes a standard GARCH methodology to derive conditional variances for the principal components. These models capture some aspects of the data well. Large changes in predicted variances from the GARCH models could be used to generate volatility scenarios.

[14] However, Andersen and Bollerslev (1997) present evidence that GARCH models may measure exchange rate volatility well even though they are not highly correlated with high frequency historical estimators of instantaneous volatility.

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Table 1a

Principal component analysis:
term structure returns in Canada (1994-96)

Component	1	2	3	4	5	6	7	8	9	10	11
Fraction of variance explained by component											
1	46%	23%	5%	13%	3%	7%	0%	1%	0%	0%	0%
2	58%	24%	6%	8%	1%	3%	0%	0%	0%	0%	0%
3	65%	23%	5%	6%	1%	0%	0%	0%	0%	0%	0%
4	71%	22%	4%	2%	0%	1%	0%	0%	0%	0%	0%
5	75%	21%	3%	1%	0%	1%	0%	0%	0%	0%	0%
7	82%	15%	2%	1%	0%	0%	0%	0%	0%	0%	0%
10	89%	7%	2%	2%	0%	0%	0%	0%	0%	0%	0%
15	91%	2%	6%	0%	0%	0%	0%	0%	0%	0%	0%
20	97%	0%	2%	0%	0%	0%	0%	0%	0%	0%	0%
25	99%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	97%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
Total	95%	3%	2%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	7.7	- 5.4	- 2.7	- 4.1	- 1.8	2.9	0.5	- 1.3	0.1	- 0.7	0.7
2	7.5	- 4.9	- 2.4	- 2.8	- 1.2	1.7	0.2	- 0.3	- 0.2	- 0.1	- 0.4
3	7.5	- 4.4	- 2.1	- 2.2	- 0.7	0.2	- 0.2	0.6	- 0.1	0.4	0.1
4	7.3	- 4.1	- 1.8	- 1.2	- 0.1	- 0.7	- 0.2	0.2	0.2	- 0.3	0.0
5	7.3	- 3.8	- 1.4	- 0.6	0.2	- 0.8	0.0	- 0.4	- 0.2	0.1	0.0
7	6.8	- 2.9	- 1.1	0.7	0.3	0.2	0.1	- 0.1	0.2	0.1	0.0
10	6.5	- 1.8	- 0.9	0.9	0.2	0.2	0.0	0.1	- 0.1	- 0.1	0.0
15	5.8	- 0.9	1.5	0.3	- 0.4	- 0.1	0.0	0.0	0.0	0.0	0.0
20	5.8	- 0.1	0.9	- 0.2	0.2	0.1	- 0.1	0.0	0.0	0.0	0.0
25	5.7	0.4	0.1	- 0.2	0.1	0.0	0.1	0.0	0.0	0.0	0.0
30	5.6	0.8	- 0.5	0.1	- 0.1	0.0	0.0	0.0	0.0	0.0	0.0

Table 1b

Principal component analysis:
term structure returns in France (1994-96)

Component	1	2	3	4	5	6	7	8	9	10	11
Fraction of variance explained by component											
1	26%	25%	30%	2%	3%	0%	11%	3%	0%	0%	0%
2	55%	22%	14%	1%	2%	0%	5%	0%	0%	0%	0%
3	66%	20%	12%	1%	1%	0%	0%	0%	0%	0%	0%
4	73%	17%	9%	0%	0%	0%	0%	0%	0%	0%	0%
5	79%	14%	5%	0%	0%	0%	1%	0%	0%	0%	0%
7	92%	5%	0%	1%	1%	0%	0%	0%	0%	0%	0%
10	95%	4%	0%	1%	0%	0%	0%	0%	0%	0%	0%
15	97%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%
20	99%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	99%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	98%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	4.5	- 4.4	4.8	- 1.1	- 1.6	0.6	- 2.9	1.5	0.4	- 0.2	0.2
2	4.8	- 3.1	2.4	- 0.7	- 1.0	0.1	- 1.5	- 0.1	0.3	- 0.1	- 0.1
3	5.3	- 2.9	2.3	- 0.6	- 0.5	- 0.1	- 0.2	- 0.3	- 0.4	0.3	0.0
4	5.6	- 2.7	1.9	- 0.4	- 0.2	- 0.1	0.4	- 0.1	- 0.2	- 0.3	0.0
5	5.7	- 2.4	1.5	- 0.2	0.1	0.0	0.6	0.2	0.2	0.1	0.0
7	5.9	- 1.4	0.0	0.6	0.6	0.2	- 0.1	0.3	- 0.1	0.0	0.0
10	5.9	- 1.2	0.0	0.5	0.3	0.0	- 0.1	- 0.3	0.0	0.0	0.0
15	6.0	- 0.7	- 0.5	0.3	- 0.3	- 0.2	0.0	0.0	0.0	0.0	0.0
20	6.1	- 0.4	- 0.4	- 0.2	- 0.1	0.2	0.0	0.0	0.0	0.0	0.0
25	6.3	0.0	- 0.1	- 0.2	0.1	- 0.2	0.0	0.0	0.0	0.0	0.0
30	6.5	0.6	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1c

Principal component analysis:
term structure returns in Germany (1994-96)

Component	1	2	3	4	5	6	7	8	9	10	11
Fraction of variance explained by component											
1	24%	0%	23%	22%	1%	16%	9%	1%	4%	0%	1%
2	44%	2%	35%	10%	0%	7%	1%	0%	1%	0%	0%
3	54%	2%	29%	12%	0%	2%	0%	0%	1%	0%	0%
4	65%	1%	26%	6%	0%	0%	1%	0%	0%	0%	0%
5	69%	2%	20%	7%	0%	1%	0%	0%	0%	0%	0%
7	79%	5%	15%	0%	2%	0%	0%	0%	0%	0%	0%
10	88%	0%	9%	2%	0%	0%	0%	0%	0%	0%	0%
15	92%	7%	1%	0%	0%	0%	0%	0%	0%	0%	0%
20	90%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	98%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	14%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	90%	8%	2%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	2.5	0.2	2.4	- 2.3	- 0.4	2.0	1.5	0.4	- 1.0	0.2	- 0.5
2	3.3	0.6	3.0	- 1.6	- 0.3	1.4	0.4	- 0.3	- 0.4	- 0.1	0.2
3	4.1	0.8	3.0	- 1.9	- 0.1	0.8	0.0	0.2	0.5	0.0	0.0
4	5.1	0.6	3.2	- 1.6	- 0.3	- 0.1	- 0.7	0.2	- 0.2	0.0	0.0
5	4.8	0.7	2.6	- 1.5	- 0.4	- 0.6	0.4	- 0.1	0.0	0.0	0.0
7	5.2	1.3	2.2	0.0	0.8	- 0.1	0.1	0.0	0.0	0.0	0.0
10	5.5	0.3	1.8	0.9	- 0.1	0.1	- 0.1	- 0.2	0.0	0.0	0.0
15	6.1	- 1.7	0.5	0.5	- 0.1	0.0	0.1	0.1	0.0	0.0	0.0
20	6.1	- 2.0	- 0.2	- 0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	5.8	- 0.8	- 0.4	- 0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	5.1	2.0	- 0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1d

Principal component analysis:
term structure returns in Italy (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	41%	11%	3%	22%	5%	13%	2%	2%	1%	0%	0%
2	53%	16%	9%	14%	2%	4%	2%	0%	0%	0%	0%
3	55%	16%	12%	12%	1%	1%	1%	2%	0%	0%	0%
4	61%	15%	12%	9%	1%	1%	0%	1%	0%	0%	0%
5	64%	15%	15%	4%	1%	1%	1%	0%	0%	0%	0%
7	50%	31%	16%	0%	2%	0%	0%	0%	0%	0%	0%
10	69%	12%	18%	1%	0%	0%	0%	0%	0%	0%	0%
15	93%	1%	7%	0%	0%	0%	0%	0%	0%	0%	0%
20	94%	5%	1%	0%	0%	0%	0%	0%	0%	0%	0%
25	98%	2%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	95%	3%	3%	0%	0%	0%	0%	0%	0%	0%	0%
Total	92%	4%	3%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	7.7	4.0	2.3	-5.6	-2.6	-4.4	-1.8	1.7	-1.3	0.4	0.0
2	7.8	4.3	3.2	-3.9	-1.6	-2.2	-1.3	0.5	0.6	-0.2	0.0
3	7.4	4.0	3.5	-3.5	-1.1	-0.8	1.1	-1.3	0.0	0.0	0.0
4	8.3	4.0	3.7	-3.1	-1.0	1.1	0.6	0.8	0.0	0.0	0.0
5	8.2	3.9	3.9	-2.0	0.8	0.8	-0.8	-0.4	0.0	0.0	0.0
7	7.6	6.0	4.3	0.0	1.4	-0.4	0.3	0.2	0.0	0.0	0.0
10	8.3	3.4	4.2	1.0	-0.5	0.0	0.0	0.0	0.0	-0.1	0.0
15	9.7	-0.7	2.6	0.4	-0.2	0.0	0.0	0.0	0.0	0.1	0.0
20	10.4	-2.4	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	10.5	-1.6	-0.4	-0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
30	9.8	1.7	-1.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1e

Principal component analysis:
term structure returns in Japan (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Fraction of variance explained by component									
1	40%	7%	13%	15%	19%	4%	0%	0%	0%
2	64%	12%	7%	10%	7%	1%	0%	0%	0%
3	72%	11%	7%	7%	1%	0%	0%	0%	0%
4	78%	11%	6%	5%	0%	0%	0%	0%	0%
5	82%	9%	5%	3%	1%	0%	0%	0%	0%
7	89%	5%	2%	4%	0%	0%	0%	0%	0%
10	93%	5%	1%	1%	0%	0%	0%	0%	0%
15	89%	0%	10%	1%	0%	0%	0%	0%	0%
20	91%	8%	1%	0%	0%	0%	0%	0%	0%
Total	89%	6%	4%	1%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	3.5	- 1.4	2.0	2.1	2.4	1.2	- 0.4	0.3	0.0
2	3.9	- 1.7	1.3	1.5	1.3	0.5	0.0	- 0.1	0.0
3	4.5	- 1.8	1.4	1.4	0.6	- 0.3	0.3	0.0	0.0
4	4.8	- 1.8	1.3	1.2	- 0.1	- 0.4	- 0.2	0.0	0.0
5	4.9	- 1.6	1.2	0.9	- 0.6	0.2	0.0	0.0	0.0
7	5.3	- 1.3	0.8	- 1.2	0.1	0.0	0.0	0.0	0.0
10	4.0	- 0.9	- 0.5	- 0.4	0.0	0.0	0.0	0.0	0.0
15	3.0	- 0.1	- 1.0	0.2	0.0	0.0	0.0	0.0	0.0
20	3.2	1.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0

Table 1f

Principal component analysis:
term structure returns in the United Kingdom (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Fraction of variance explained by component									
1	32%	23%	17%	11%	11%	4%	2%	0%	0%
2	62%	23%	7%	5%	3%	1%	0%	0%	0%
3	72%	18%	6%	3%	1%	0%	0%	0%	0%
4	80%	15%	4%	1%	0%	0%	0%	0%	0%
5	85%	12%	3%	0%	1%	0%	0%	0%	0%
7	92%	7%	0%	0%	0%	0%	0%	0%	0%
10	96%	2%	0%	1%	0%	0%	0%	0%	0%
15	98%	0%	1%	0%	0%	0%	0%	0%	0%
20	97%	2%	0%	0%	0%	0%	0%	0%	0%
Total	96%	3%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	3.9	- 3.3	2.8	- 2.2	- 2.3	1.3	- 0.9	- 0.4	0.3
2	5.2	- 3.2	1.8	- 1.4	- 1.2	0.5	- 0.3	0.0	- 0.2
3	5.8	- 2.9	1.7	- 1.2	- 0.6	- 0.2	0.4	0.3	0.0
4	6.4	- 2.7	1.4	- 0.6	0.2	- 0.3	0.2	- 0.3	0.0
5	6.6	- 2.5	1.1	- 0.2	0.6	- 0.2	- 0.3	0.1	0.0
7	7.2	- 1.9	0.0	0.5	0.2	0.4	0.1	0.0	0.0
10	7.4	- 1.1	- 0.5	0.6	- 0.2	- 0.2	0.0	0.0	0.0
15	6.5	0.1	- 0.8	- 0.4	0.0	0.0	0.0	0.0	0.0
20	6.1	1.0	0.4	0.1	0.0	0.0	0.0	0.0	0.0

Table 1g

Principal component analysis:
term structure returns in the United States (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	52%	34%	0%	10%	1%	3%	0%	0%	0%	0%	0%
2	64%	30%	0%	4%	0%	1%	0%	0%	0%	0%	0%
3	71%	26%	0%	3%	0%	0%	0%	0%	0%	0%	0%
4	78%	21%	0%	1%	0%	0%	0%	0%	0%	0%	0%
5	81%	18%	0%	0%	0%	0%	0%	0%	0%	0%	0%
7	88%	12%	0%	0%	0%	0%	0%	0%	0%	0%	0%
10	93%	6%	0%	0%	0%	0%	0%	0%	0%	0%	0%
15	98%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	99%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
25	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	99%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	98%	2%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	4.7	3.8	- 0.1	2.0	0.5	1.2	0.3	- 0.4	- 0.1	- 0.1	0.1
2	5.3	3.6	- 0.1	1.4	0.3	0.6	0.2	- 0.1	- 0.1	0.0	- 0.1
3	5.7	3.4	- 0.2	1.1	0.2	0.1	0.0	0.2	0.1	0.1	0.0
4	5.9	3.1	- 0.2	0.7	0.0	- 0.2	- 0.1	0.0	0.0	- 0.1	0.0
5	6.0	2.8	- 0.2	0.4	0.0	- 0.3	- 0.1	- 0.1	0.0	0.0	0.0
7	6.1	2.2	- 0.3	- 0.3	- 0.2	0.1	0.0	0.0	0.1	0.0	0.0
10	5.9	1.5	- 0.3	- 0.3	- 0.1	0.1	0.0	0.0	0.0	0.0	0.0
15	5.7	0.6	0.3	- 0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0
20	5.5	- 0.1	0.4	0.0	0.0	0.0	- 0.1	0.0	0.0	0.0	0.0
25	5.2	- 0.3	0.1	0.1	- 0.1	0.0	0.1	0.0	0.0	0.0	0.0
30	5.1	- 0.4	- 0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 2a

Principal component analysis:
standardised term structure returns in Canada (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	73%	20%	6%	0%	1%	0%	0%	0%	0%	0%	0%
2	85%	14%	1%	0%	0%	0%	0%	0%	0%	0%	0%
3	90%	8%	0%	0%	1%	1%	0%	0%	0%	0%	0%
4	93%	4%	1%	0%	1%	0%	0%	0%	0%	0%	0%
5	95%	2%	3%	0%	0%	0%	0%	0%	0%	0%	0%
7	95%	0%	3%	0%	1%	0%	0%	0%	0%	0%	0%
10	95%	1%	2%	0%	1%	0%	0%	0%	0%	0%	0%
15	84%	10%	0%	6%	0%	0%	0%	0%	0%	0%	0%
20	85%	12%	1%	1%	0%	0%	0%	0%	0%	0%	0%
25	85%	13%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	81%	14%	1%	4%	0%	0%	0%	0%	0%	0%	0%
Total	87%	9%	2%	1%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	9.7	- 5.1	- 2.8	- 0.3	- 0.9	0.6	0.2	0.2	- 0.2	0.0	0.0
2	9.1	- 3.7	- 1.1	0.0	- 0.2	- 0.4	- 0.2	- 0.3	0.3	0.0	0.0
3	8.8	- 2.7	0.1	0.1	0.9	- 0.7	- 0.1	0.0	- 0.3	0.0	0.0
4	8.4	- 1.7	1.0	0.2	0.9	0.2	0.2	0.3	0.2	0.0	0.0
5	8.2	- 1.1	1.4	0.0	0.5	0.6	0.0	- 0.4	- 0.1	0.0	0.0
7	7.3	0.0	1.4	0.1	- 0.8	0.0	- 0.1	0.1	- 0.1	- 0.2	0.0
10	6.7	0.7	1.0	0.4	- 0.8	- 0.2	0.0	0.0	0.0	0.2	0.0
15	5.5	1.9	- 0.1	- 1.5	- 0.1	- 0.1	0.3	- 0.1	0.0	0.0	0.0
20	5.4	2.0	- 0.6	- 0.5	0.2	0.1	- 0.3	0.1	0.0	0.1	- 0.1
25	5.2	2.0	- 0.6	0.4	0.2	0.1	- 0.1	0.0	0.0	0.0	0.1
30	5.1	2.1	- 0.7	1.1	0.0	0.0	0.2	- 0.1	0.0	0.0	- 0.1

Table 2b

Principal component analysis:
standardised term structure returns in France (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	51%	44%	3%	0%	0%	0%	0%	0%	0%	0%	0%
2	81%	18%	1%	0%	1%	0%	0%	0%	0%	0%	0%
3	89%	8%	1%	0%	0%	0%	0%	0%	0%	0%	0%
4	93%	4%	2%	0%	0%	0%	0%	0%	0%	0%	0%
5	96%	1%	3%	0%	0%	0%	0%	0%	0%	0%	0%
7	94%	3%	0%	2%	0%	0%	0%	0%	0%	0%	0%
10	95%	3%	0%	1%	0%	0%	0%	0%	0%	0%	0%
15	92%	7%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	91%	8%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	90%	9%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	10%	2%	1%	0%	0%	0%	0%	0%	0%	0%
Total	87%	10%	1%	1%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	6.3	- 5.9	- 1.6	0.3	0.6	- 0.2	- 0.1	0.1	0.0	0.0	0.0
2	5.8	- 2.7	- 0.5	0.1	- 0.6	0.1	0.2	- 0.2	0.0	0.0	0.0
3	6.2	- 1.9	0.6	- 0.4	- 0.3	0.3	- 0.4	0.1	0.1	0.0	0.0
4	6.3	- 1.2	1.0	- 0.5	0.1	- 0.1	0.1	0.1	- 0.3	0.0	0.0
5	6.3	- 0.6	1.0	- 0.2	0.4	- 0.3	0.2	- 0.1	0.2	0.0	0.0
7	6.0	1.0	0.4	0.9	0.4	0.1	- 0.1	0.0	0.0	0.0	0.0
10	5.9	1.1	0.2	0.5	- 0.1	0.3	0.1	- 0.1	0.0	0.0	0.0
15	5.8	1.6	- 0.1	0.3	- 0.4	- 0.3	0.1	0.4	0.0	0.0	0.0
20	5.9	1.7	- 0.4	- 0.1	- 0.2	- 0.3	- 0.2	- 0.2	0.0	0.2	0.0
25	6.0	1.9	- 0.6	- 0.4	0.0	0.0	- 0.1	- 0.2	0.0	- 0.2	0.0
30	6.1	2.1	- 0.9	- 0.7	0.4	0.3	0.1	0.2	0.0	0.1	0.0

Table 2c

Principal component analysis:
standardised term structure returns in Germany (1994-96)

Component	1	2	3	4	5	6	7	8	9	10	11
Fraction of variance explained by component											
1	51%	33%	10%	0%	0%	0%	0%	0%	0%	0%	0%
2	77%	21%	0%	1%	0%	0%	0%	0%	0%	0%	0%
3	85%	12%	0%	1%	1%	0%	0%	0%	0%	0%	0%
4	91%	4%	1%	1%	0%	0%	1%	0%	0%	0%	0%
5	92%	3%	1%	1%	2%	0%	0%	0%	0%	0%	0%
7	92%	0%	5%	1%	0%	1%	0%	0%	0%	0%	0%
10	91%	3%	1%	3%	0%	0%	0%	0%	0%	0%	0%
15	80%	14%	4%	0%	0%	0%	0%	0%	0%	0%	0%
20	74%	18%	8%	0%	0%	0%	0%	0%	0%	0%	0%
25	80%	17%	2%	1%	0%	0%	0%	0%	0%	0%	0%
30	76%	7%	9%	0%	0%	0%	0%	0%	0%	0%	0%
Total	81%	12%	4%	1%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	3.6	2.9	- 1.6	0.3	0.6	- 0.2	- 0.1	0.1	0.0	0.0	0.0
2	4.4	2.3	- 0.5	0.1	- 0.6	0.1	0.2	- 0.2	0.0	0.0	0.0
3	5.1	1.9	0.6	- 0.4	- 0.3	0.3	- 0.4	0.1	0.1	0.0	0.0
4	6.0	1.2	1.0	- 0.5	0.1	- 0.1	0.1	0.1	- 0.3	0.0	0.0
5	5.5	0.9	1.0	- 0.2	0.4	- 0.3	0.2	- 0.1	0.2	0.0	0.0
7	5.6	0.0	0.4	0.9	0.4	0.1	- 0.1	0.0	0.0	0.0	0.0
10	5.6	- 1.1	0.2	0.5	- 0.1	0.3	0.1	- 0.1	0.0	0.0	0.0
15	5.7	- 2.4	- 0.1	0.3	- 0.4	- 0.3	0.1	0.4	0.0	0.0	0.0
20	5.5	1.7	- 0.4	- 0.1	- 0.2	- 0.3	- 0.2	- 0.2	0.0	0.2	0.0
25	5.2	1.9	- 0.6	- 0.4	0.0	0.0	- 0.1	- 0.2	0.0	- 0.2	0.0
30	4.8	2.1	- 0.9	- 0.7	0.4	0.3	0.1	0.2	0.0	0.1	0.0

Table 2d

Principal component analysis:
standardised term structure returns in Italy (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	67%	11%	19%	0%	2%	0%	0%	0%	0%	0%	0%
2	85%	10%	4%	0%	0%	0%	0%	1%	0%	0%	0%
3	87%	8%	0%	0%	2%	2%	0%	0%	0%	0%	0%
4	91%	4%	0%	0%	2%	0%	1%	0%	0%	0%	0%
5	92%	3%	2%	0%	0%	1%	1%	0%	0%	0%	0%
7	80%	7%	10%	1%	1%	0%	0%	0%	0%	0%	0%
10	89%	0%	7%	0%	2%	0%	1%	0%	0%	0%	0%
15	85%	12%	0%	2%	0%	0%	0%	0%	0%	0%	0%
20	73%	24%	0%	1%	0%	0%	0%	0%	0%	0%	0%
25	74%	25%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	79%	11%	0%	9%	0%	0%	0%	0%	0%	0%	0%
Total	82%	10%	4%	1%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	9.9	- 4.1	- 5.3	0.2	1.5	0.2	- 0.1	- 0.6	- 0.3	0.0	0.0
2	9.9	- 3.4	- 2.1	- 0.1	0.5	0.2	0.1	0.9	0.5	0.0	0.0
3	9.3	- 2.8	- 0.4	- 0.6	- 1.5	- 1.5	0.6	- 0.1	- 0.1	0.0	0.0
4	10.0	- 2.2	0.5	- 0.6	- 1.7	0.6	- 1.3	- 0.3	0.2	0.0	0.0
5	9.8	- 1.7	1.6	- 0.3	- 0.6	1.2	0.8	0.3	- 0.4	0.0	0.0
7	9.5	- 2.8	3.4	1.3	0.9	0.1	0.5	- 0.6	0.4	0.0	0.0
10	9.4	- 0.4	2.7	- 0.2	1.4	- 0.7	- 0.7	0.3	- 0.3	0.0	0.0
15	9.4	3.4	0.6	- 1.5	0.7	- 0.1	- 0.1	0.0	0.0	- 0.1	0.0
20	9.2	5.3	- 0.8	- 1.3	0.1	0.1	0.3	- 0.2	0.1	0.0	0.0
25	9.1	5.3	- 1.2	0.2	- 0.3	0.1	0.2	- 0.1	0.1	0.0	0.0
30	9.0	3.3	- 0.6	3.0	- 0.6	- 0.2	- 0.2	0.2	- 0.1	0.0	0.0

Table 2e

Principal component analysis:
standardised term structure returns in Japan (1994-96)

Component	1	2	3	4	5	6	7	8	9
Fraction of variance explained by component									
1	65%	28%	5%	0%	1%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%
3	93%	5%	1%	0%	1%	1%	0%	0%	0%
4	95%	1%	2%	0%	1%	0%	0%	0%	0%
5	95%	0%	2%	1%	1%	1%	0%	0%	0%
7	88%	4%	2%	1%	4%	0%	0%	0%	0%
10	88%	8%	1%	1%	1%	0%	0%	0%	0%
15	76%	16%	1%	7%	1%	0%	0%	0%	0%
20	73%	12%	10%	5%	0%	0%	0%	0%	0%
Total	85%	10%	3%	2%	1%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	4.4	- 2.9	- 1.2	- 0.3	0.5	- 0.2	0.1	- 0.1	0.0
2	4.6	- 1.6	- 0.2	- 0.3	0.1	0.0	- 0.1	0.2	0.0
3	5.1	- 1.1	0.4	0.1	- 0.4	0.5	- 0.1	- 0.1	0.0
4	5.3	- 0.6	0.8	0.3	- 0.6	0.1	0.3	0.1	0.0
5	5.3	- 0.2	0.8	0.4	- 0.6	- 0.6	- 0.1	- 0.1	0.0
7	5.3	1.1	0.8	0.7	1.2	0.1	0.0	0.0	0.0
10	3.9	1.2	0.4	- 0.5	0.4	0.0	0.0	0.0	0.0
15	2.7	1.2	- 0.3	- 0.8	- 0.2	0.0	0.0	0.0	0.0
20	2.8	1.2	- 1.0	0.7	- 0.2	0.0	0.0	0.0	0.0

Table 2f

Principal component analysis:
standardised term structure returns in the United Kingdom (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Fraction of variance explained by component									
1	60%	36%	4%	0%	0%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%
3	94%	4%	1%	1%	0%	0%	0%	0%	0%
4	97%	1%	2%	1%	0%	0%	0%	0%	0%
5	97%	0%	2%	0%	0%	0%	0%	0%	0%
7	95%	3%	1%	1%	0%	0%	0%	0%	0%
10	91%	7%	0%	1%	0%	0%	0%	0%	0%
15	85%	12%	1%	0%	1%	0%	0%	0%	0%
20	80%	14%	4%	1%	0%	0%	0%	0%	0%
Total	88%	10%	2%	1%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	5.3	- 4.1	- 1.3	- 0.3	- 0.2	- 0.1	0.0	- 0.1	0.1
2	6.3	- 2.3	- 0.1	- 0.2	0.2	0.0	0.0	0.1	- 0.2
3	6.6	- 1.4	0.5	0.5	0.5	0.3	- 0.1	0.1	0.1
4	7.0	- 0.5	0.9	0.5	0.0	- 0.1	0.1	- 0.3	- 0.1
5	7.1	0.0	1.0	0.3	- 0.4	- 0.4	0.1	0.2	0.1
7	7.3	1.3	0.7	- 0.7	- 0.4	0.1	- 0.3	- 0.1	0.0
10	7.2	2.0	0.2	- 0.8	- 0.1	0.3	0.3	0.0	0.0
15	6.1	2.3	- 0.7	- 0.3	0.6	- 0.4	0.0	0.0	0.0
20	5.5	2.3	- 1.3	0.7	- 0.3	0.1	0.0	0.0	0.0

Table 2g

Principal component analysis:
standardised term structure returns in the United States (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	78%	19%	3%	0%	0%	0%	0%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3	93%	7%	0%	0%	0%	0%	0%	0%	0%	0%	0%
4	96%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%
5	97%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
7	98%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
10	98%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%
15	95%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	89%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	87%	12%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	13%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	91%	7%	1%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	5.8	- 2.9	- 1.1	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0
2	6.2	- 2.2	- 0.4	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
3	6.5	- 1.7	0.1	- 0.2	- 0.2	- 0.2	0.1	0.0	0.0	0.0	0.0
4	6.6	- 1.2	0.5	- 0.3	- 0.2	0.1	0.0	0.0	- 0.1	0.0	0.0
5	6.6	- 0.8	0.6	- 0.2	- 0.2	0.2	- 0.1	0.0	0.0	0.0	0.0
7	6.5	- 0.1	0.8	0.3	0.2	0.0	0.1	0.0	0.0	0.0	0.0
10	6.0	0.5	0.6	0.3	0.1	0.0	0.0	0.0	0.0	- 0.1	0.0
15	5.6	1.2	0.0	- 0.2	0.3	- 0.1	- 0.2	0.0	0.0	0.0	0.0
20	5.2	1.7	- 0.4	- 0.3	0.2	0.1	0.1	- 0.1	0.0	0.0	0.0
25	4.8	1.8	- 0.4	0.0	- 0.1	0.0	0.1	0.1	0.0	0.0	0.0
30	4.7	1.9	- 0.3	0.2	- 0.3	0.0	- 0.1	0.0	0.0	0.0	0.0

Table 3

Correlation between principal components

(derived from returns)

Correlations between first principal components (1990-93)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	0.63	0.20	-0.14	-0.09	-0.16	-0.21
CA	0.63	1.00	0.19	-0.16	-0.10	-0.20	-0.16
FR	0.20	0.19	1.00	-0.44	-0.24	-0.18	-0.28
GE	-0.14	-0.16	-0.44	1.00	0.16	0.26	0.13
IT	-0.09	-0.10	-0.24	0.16	1.00	0.08	0.23
JP	-0.16	-0.20	-0.18	0.26	0.08	1.00	0.13
UK	-0.21	-0.16	-0.28	0.13	0.23	0.13	1.00

Correlations between first principal components (1994-96)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	0.72	0.30	0.30	0.19	-0.02	0.39
CA	0.72	1.00	0.30	0.29	0.24	0.02	0.38
FR	0.30	0.30	1.00	0.60	0.40	0.09	0.65
GE	0.30	0.29	0.60	1.00	0.39	0.11	0.54
IT	0.19	0.24	0.40	0.39	1.00	-0.03	0.38
JP	-0.02	0.02	0.09	0.11	-0.03	1.00	0.02
UK	0.39	0.38	0.65	0.54	0.38	0.02	1.00

Correlations between second principal components (1990-93)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	0.27	0.01	-0.04	-0.01	0.07	0.06
CA	0.27	1.00	-0.05	0.02	0.03	0.00	0.10
FR	0.01	-0.05	1.00	-0.07	-0.08	0.01	-0.13
GE	-0.04	0.02	-0.07	1.00	0.05	0.09	0.14
IT	-0.01	0.03	-0.08	0.05	1.00	-0.01	0.25
JP	0.07	0.00	0.01	0.09	-0.01	1.00	-0.01
UK	0.06	0.10	-0.13	0.14	0.25	-0.01	1.00

Correlations between second principal components (1994-96)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	-0.23	-0.06	0.06	0.03	-0.05	-0.16
CA	-0.23	1.00	0.15	0.00	-0.04	0.01	0.11
FR	-0.06	0.15	1.00	0.07	-0.05	0.09	0.19
GE	0.06	0.00	0.07	1.00	-0.14	-0.08	0.07
IT	0.03	-0.04	-0.05	-0.14	1.00	-0.03	-0.05
JP	-0.05	0.01	0.09	-0.08	-0.03	1.00	0.10
UK	-0.16	0.11	0.19	0.07	-0.05	0.10	1.00

Correlations between third principal components (1990-93)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	-0.03	-0.02	-0.11	-0.16	-0.11	-0.01
CA	0.08	1.00	-0.05	-0.01	-0.01	-0.02	0.01
FR	0.02	-0.05	1.00	-0.02	-0.04	0.06	0.02
GE	-0.09	-0.01	-0.02	1.00	-0.09	-0.01	-0.02
IT	0.01	-0.01	-0.04	-0.09	1.00	0.07	0.11
JP	-0.01	-0.02	0.06	-0.01	0.07	1.00	-0.05
UK	-0.02	0.01	0.02	-0.02	0.11	-0.05	1.00

Correlations between third principal components (1994-96)

	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	0.14	0.09	-0.03	0.00	0.00	0.09
CA	0.14	1.00	0.14	-0.08	0.03	0.00	0.00
FR	0.09	0.14	1.00	0.03	-0.04	0.03	0.05
GE	-0.03	-0.08	0.03	1.00	-0.10	-0.10	0.03
IT	0.00	0.03	-0.04	-0.10	1.00	0.00	0.07
JP	0.00	0.00	0.03	-0.10	0.00	1.00	0.07
UK	0.09	0.01	0.05	0.03	0.07	0.07	1.00

Note: The table reports correlations between the first, second and third principal components derived from each country's government security zero coupon returns.

Table 4

Correlation between principal components

(derived from standardised returns)

Correlations between first principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.62	0.21	0.07	0.06	0.14	0.18
CA	0.62	1.00	0.20	0.11	0.07	0.14	0.16
FR	0.21	0.20	1.00	0.44	0.31	0.16	0.33
GE	0.07	0.11	0.44	1.00	0.22	0.16	0.20
IT	0.06	0.07	0.31	0.22	1.00	0.07	0.30
JP	0.14	0.14	0.16	0.16	0.07	1.00	0.14
UK	0.18	0.16	0.33	0.20	0.30	0.14	1.00

Correlations between first principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.76	0.30	0.35	0.20	0.01	0.42
CA	0.76	1.00	0.33	0.37	0.30	0.03	0.41
FR	0.30	0.33	1.00	0.64	0.54	0.13	0.66
GE	0.35	0.37	0.64	1.00	0.49	0.15	0.60
IT	0.20	0.30	0.54	0.49	1.00	-0.01	0.48
JP	0.01	0.03	0.13	0.15	-0.01	1.00	0.05
UK	0.42	0.41	0.66	0.60	0.48	0.05	1.00

Correlations between second principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.30	0.04	-0.06	-0.03	-0.08	0.10
CA	0.30	1.00	0.08	-0.09	-0.10	-0.07	0.12
FR	0.04	0.08	1.00	-0.22	-0.01	-0.04	0.12
GE	-0.06	-0.09	-0.22	1.00	0.01	0.17	-0.17
IT	-0.03	-0.10	-0.01	0.01	1.00	0.01	-0.35
JP	-0.08	-0.07	-0.04	0.17	0.01	1.00	-0.01
UK	0.10	0.12	0.12	-0.17	-0.35	-0.01	1.00

Correlations between second principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.35	0.13	-0.14	0.05	0.02	0.22
CA	0.35	1.00	0.13	-0.09	0.06	0.01	0.12
FR	0.13	0.13	1.00	-0.24	0.05	0.01	0.27
GE	-0.14	-0.09	-0.24	1.00	0.06	0.04	-0.05
IT	0.05	0.06	0.05	0.06	1.00	0.08	0.03
JP	0.02	0.01	0.01	0.04	0.08	1.00	0.00
UK	0.22	0.12	0.27	-0.05	0.03	0.00	1.00

Correlations between third principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.08	0.02	-0.09	0.01	-0.01	-0.02
CA	0.08	1.00	-0.05	-0.01	-0.01	-0.02	0.01
FR	0.02	-0.05	1.00	-0.02	-0.04	0.06	0.02
GE	-0.09	-0.01	-0.02	1.00	-0.09	-0.01	-0.02
IT	0.01	-0.01	-0.04	-0.09	1.00	0.07	0.11
JP	-0.01	-0.02	0.06	-0.01	0.07	1.00	-0.05
UK	-0.02	0.01	0.02	-0.02	0.11	-0.05	1.00

Correlations between third principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.14	0.09	-0.03	0.00	0.00	0.09
CA	0.14	1.00	0.14	-0.08	0.03	0.00	0.01
FR	0.09	0.14	1.00	0.03	-0.04	0.03	0.05
GE	-0.03	-0.08	0.03	1.00	-0.10	-0.10	0.03
IT	0.00	0.03	-0.04	-0.10	1.00	0.00	0.07
JP	0.00	0.00	0.03	-0.10	0.00	1.00	0.07
UK	0.09	0.01	0.05	0.03	0.07	0.07	1.00

Note: The table reports correlations between the first, second and third principal components derived from each country's standardised government security zero coupon returns.

Table 5

Explanatory power of principal components (1994-96)

(percent of variance explained)

Country	First 5	First 10	First 20	Country	First 5	First 10	First 20	Country	First 5	First 10	First 20
Canada 1	47%	48%	79%	**Italy 1**	43%	56%	78%	**UK 1**	27%	39%	65%
2	59%	59%	89%	**2**	57%	78%	92%	**2**	45%	65%	87%
3	67%	67%	94%	**3**	61%	83%	95%	**3**	51%	74%	91%
4	74%	74%	98%	**4**	68%	89%	97%	**4**	57%	82%	95%
5	77%	78%	98%	**5**	69%	95%	98%	**5**	60%	86%	97%
7	84%	84%	99%	**7**	59%	98%	98%	**7**	64%	93%	99%
10	91%	91%	99%	**10**	75%	99%	100%	**10**	64%	97%	99%
15	90%	91%	99%	**15**	94%	100%	100%	**15**	60%	99%	99%
20	97%	97%	100%	**20**	95%	100%	100%	**20**	58%	97%	100%
25	99%	100%	100%	**25**	98%	100%	100%				
30	97%	98%	100%	**30**	95%	100%	100%	**US 1**	52%	53%	87%
France 1	29%	31%	81%	**Japan 1**	4%	44%	63%	**3**	72%	73%	97%
2	56%	57%	91%	**2**	5%	68%	85%	**4**	78%	79%	99%
3	66%	67%	97%	**3**	5%	76%	92%	**5**	82%	82%	99%
4	74%	75%	98%	**4**	5%	82%	96%	**7**	88%	88%	100%
5	80%	81%	98%	**5**	5%	85%	97%	**10**	93%	94%	99%
7	92%	93%	98%	**7**	4%	90%	96%	**15**	97%	98%	100%
10	94%	95%	99%	**10**	4%	95%	99%	**20**	98%	99%	99%
15	96%	97%	99%	**15**	3%	89%	98%	**25**	98%	100%	100%
20	97%	99%	100%	**20**	4%	91%	100%	**30**	98%	99%	100%
25	98%	100%	100%								
30	97%	99%	100%								
Germany 1	26%	27%	54%								
2	45%	48%	83%								
3	56%	59%	87%								
4	66%	67%	93%								
5	71%	73%	92%								
7	79%	84%	98%								
10	88%	88%	98%								
15	91%	99%	100%								
20	90%	100%	100%								
25	98%	99%	100%								
30	85%	100%	100%								

Note: The table reports the percent of total variance of returns for government zero coupon bonds of the specified country and maturity explained by either the first 5, first 10, or the first 20 principal components derived from returns for all the securities.

Table 6

Explanatory power of principal components (1994-96)

(percent of variance explained)

Country	First 5	First 10	First 20	Country	First 5	First 10	First 20	Country	First 5	First 10	First 20
Canada 1	14%	57%	76%	**Italy 1**	15%	80%	82%	**UK 1**	11%	66%	90%
2	25%	80%	90%	**2**	27%	88%	90%	**2**	19%	80%	95%
3	33%	88%	95%	**3**	42%	89%	93%	**3**	26%	84%	99%
4	42%	93%	98%	**4**	62%	91%	93%	**4**	33%	88%	99%
5	46%	94%	98%	**5**	77%	90%	92%	**5**	41%	90%	99%
7	56%	95%	99%	**7**	87%	89%	100%	**7**	61%	93%	98%
10	71%	95%	100%	**10**	80%	99%	100%	**10**	78%	89%	100%
15	89%	93%	100%					**15**	81%	86%	100%
20	98%	98%	100%	**Japan 1**	2%	16%	43%	**20**	93%	99%	100%
25	99%	100%	100%	**2**	3%	35%	63%				
30	97%	100%	100%	**3**	5%	47%	72%	**US 1**	28%	39%	72%
				4	5%	64%	83%	**2**	39%	50%	86%
France 1	21%	37%	68%	**5**	6%	75%	87%	**3**	48%	57%	92%
2	34%	60%	88%	**7**	7%	94%	97%	**4**	56%	64%	96%
3	40%	66%	91%	**10**	7%	94%	98%	**5**	62%	69%	97%
4	47%	72%	94%					**7**	73%	77%	99%
5	50%	75%	96%					**10**	86%	88%	99%
7	61%	82%	95%					**15**	94%	95%	98%
10	74%	88%	97%					**20**	99%	99%	99%
15	85%	90%	99%					**25**	99%	100%	100%
20	94%	95%	99%					**30**	99%	99%	100%
25	98%	98%	100%								
30	97%	98%	100%								
Germany 1	5%	18%	58%								
2	8%	26%	70%								
3	10%	32%	78%								
4	13%	40%	85%								
5	15%	43%	87%								
7	20%	53%	94%								
10	22%	55%	98%								

Note: The table reports the percent of total variance of returns for government zero coupon bonds of the specified country and maturity explained by either the first 5, first 10, or the first 20 principal components derived from returns for all the securities.

Table 7: Garch Models

(Maximum Likelihood Estimates, January 1994 to September 1996)*

Country	**Principal Component**	**Constant**	**Lagged Conditioning Variables: Squared Error**	**Variance**	**Squared Error (Error < 0)**
Canada	1	0.326	0.206	0.556	-0.091*
	2	0.488	0.296	0.285	-0.026*
	3	0.875	0.129	0.194*	-0.135
France	1	0.105	0.214	0.750	-0.067*
	2	0.713	-0.014*	0.178*	0.219
	3	0.620	0.071	0.125	0.405
Germany **	1	0.190	0.400	0.476	0.261
	2	0.003	0.264	0.710	0.573
	3	0.302	0.212	0.417	0.342
Italy	1	0.023	0.135	0.880	-0.012*
	2	0.003	0.411	0.790	-0.205
	3	0.094	0.235	0.757	-0.068*
Japan	1	0.085	0.228	0.705	0.139
	2	0.285	0.375	0.359	0.059*
	3	0.260	0.162	0.555	0.111
United Kingdom	1	0.238	0.249	0.621	-0.160
	2	0.369	0.368	0.299	-0.017*
	3	0.111	0.214	0.693	0.074*
United States	1	1.560	-0.001*	-0.495	-0.104
	2	0.814	0.284	-0.051*	-0.034*
	3	0.131	-0.012*	0.774	0.296

* significant at the 5% level
** Germany's coefficients are estimated from March 1994 to September 1996.

CHART 1: EXPLANTORY POWER OF PRINCIPAL COMPONENTS (94-96)
Contour Lines Represent Explanatory Power in Steps of 10%

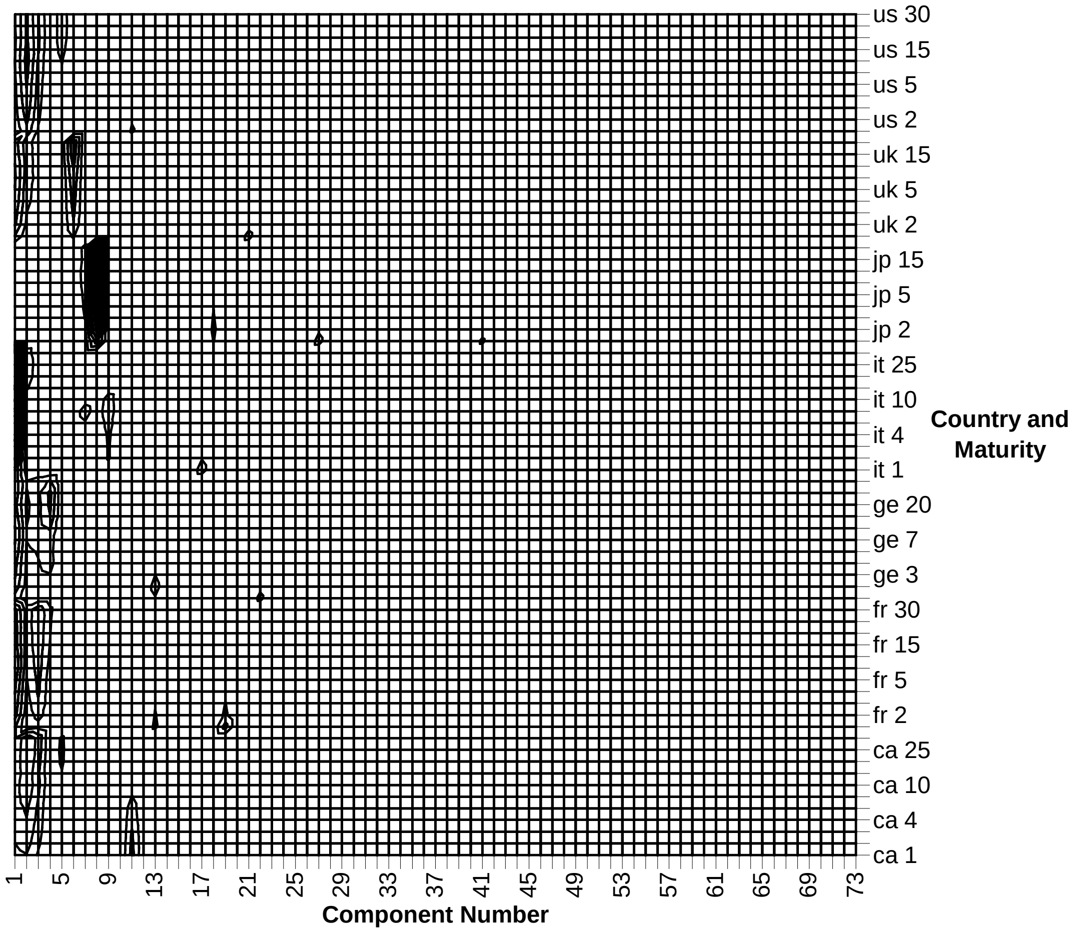


CHART 2: EXPLANATORY POWER OF PRINCIPAL COMPONENTS (90-93)
Contour Lines Represent Explanatory Power in Steps of 10%

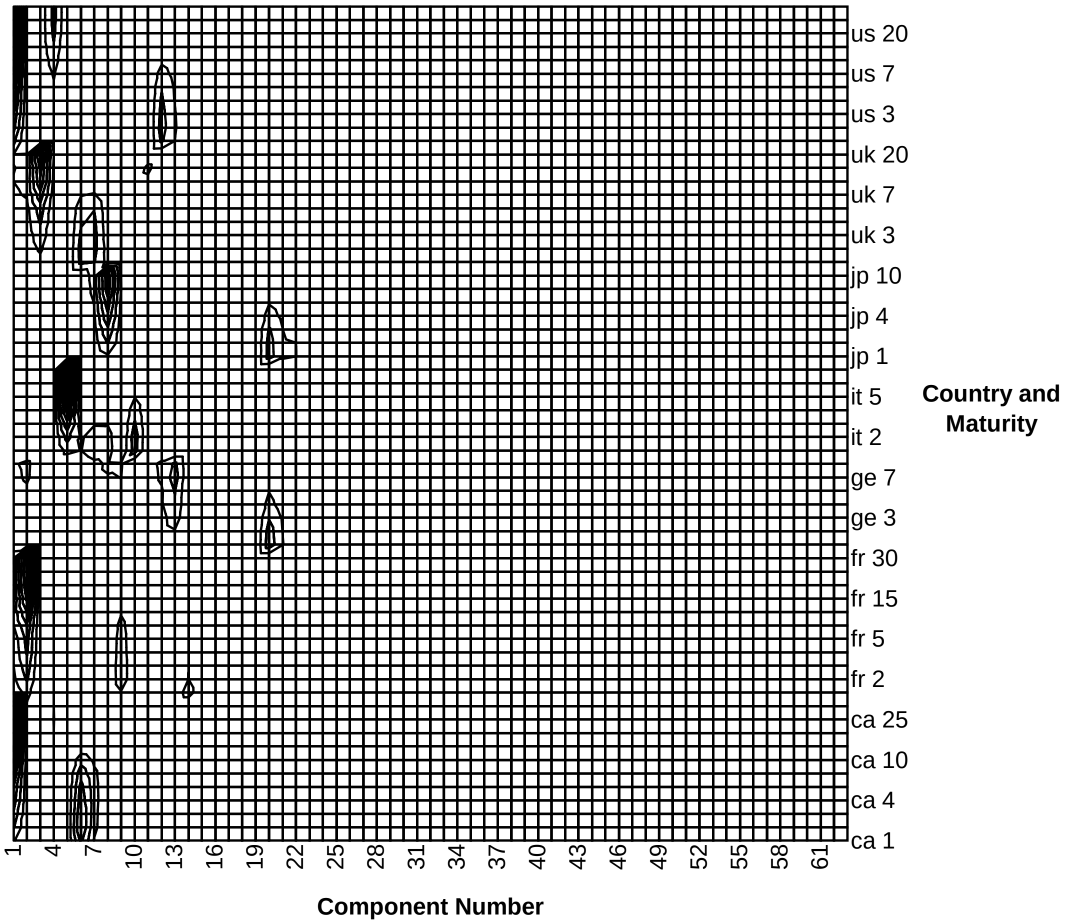
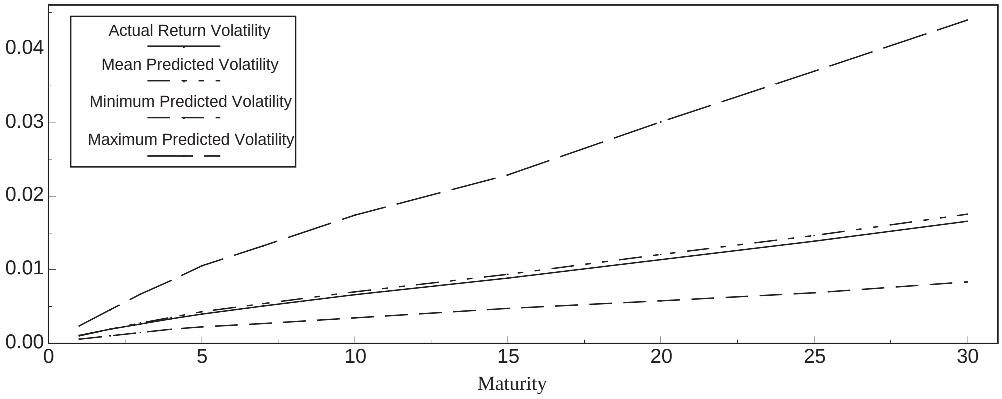
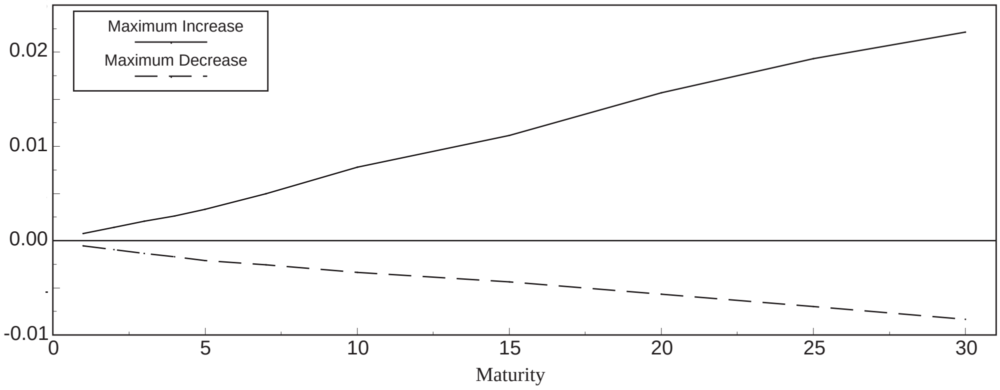


Chart 3a: Volatility Predictions, Canada

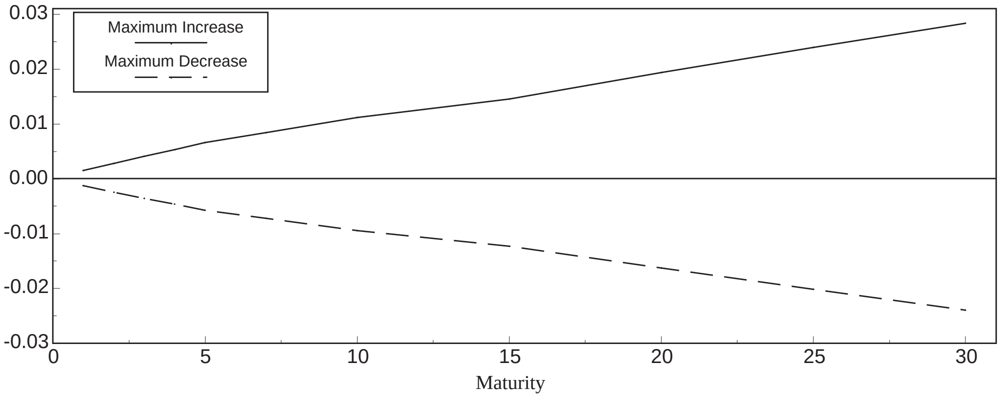
Canada Zero Return Volatilities (1994 to 1996)



One-Day Changes in Canada Zero Return Volatilities (1994 to 1996)

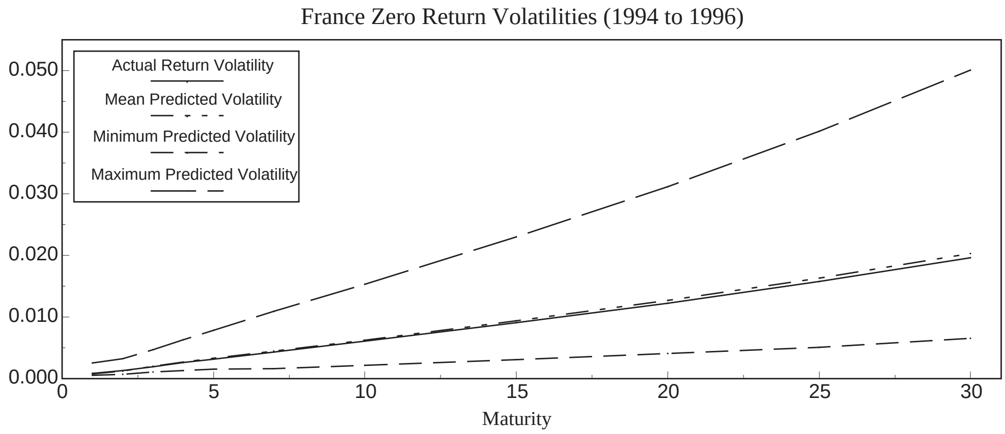


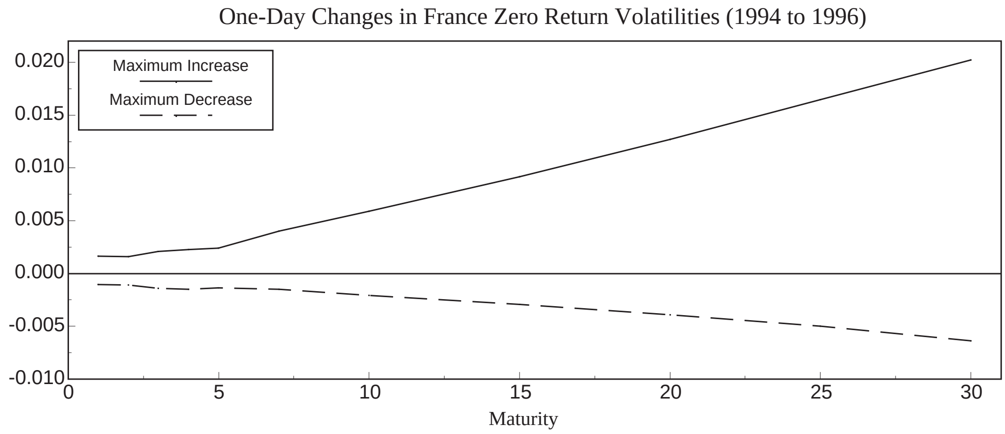
Five-Day Changes in Canada Zero Return Volatilities (1994 to 1996)

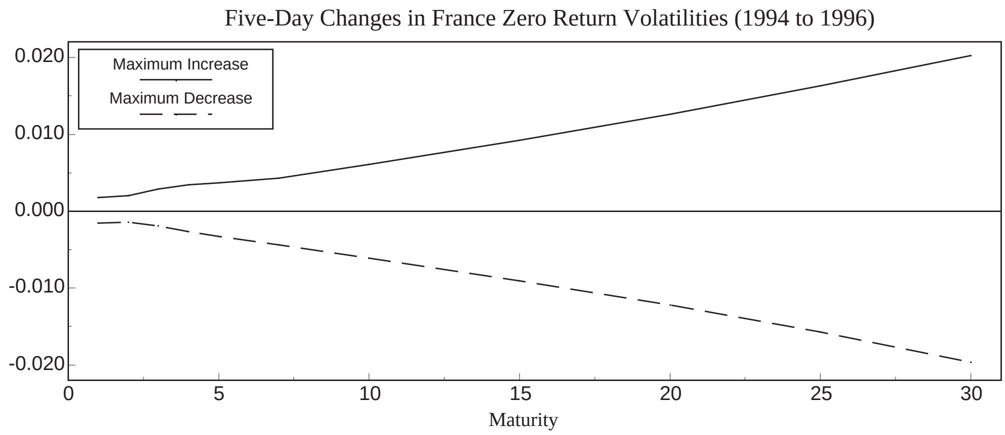


Volatility

Chart 3b: Volatility Predictions, France

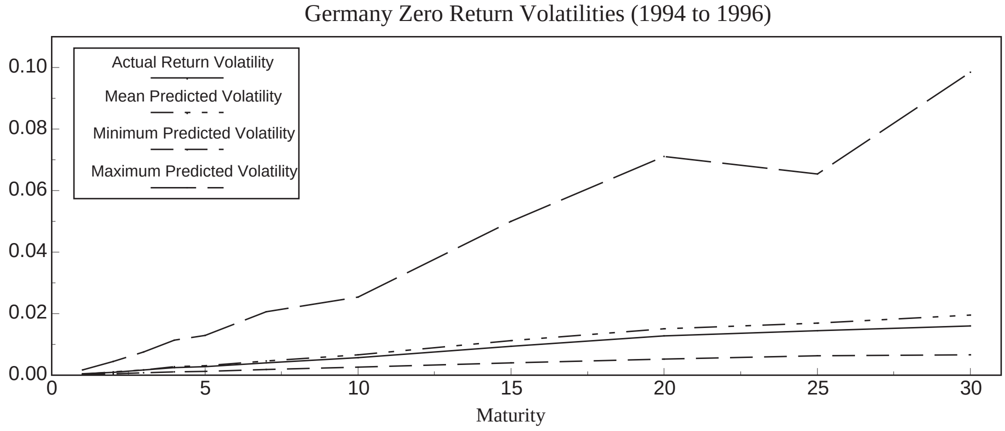


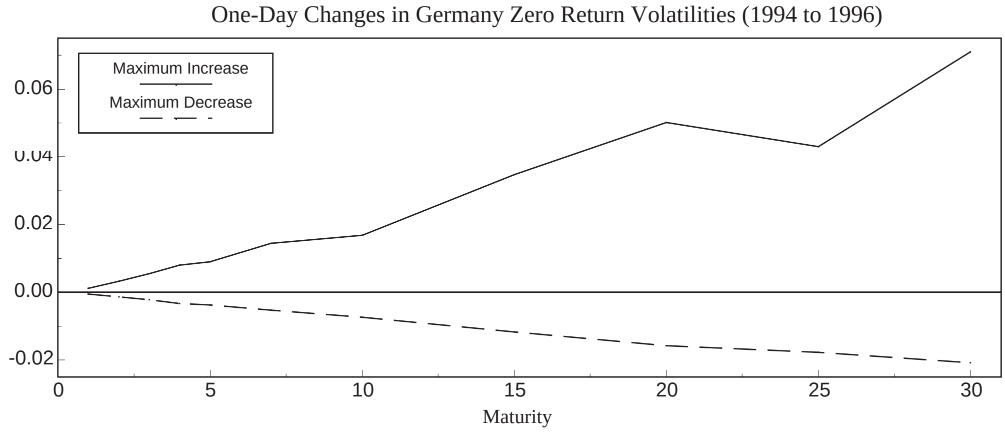


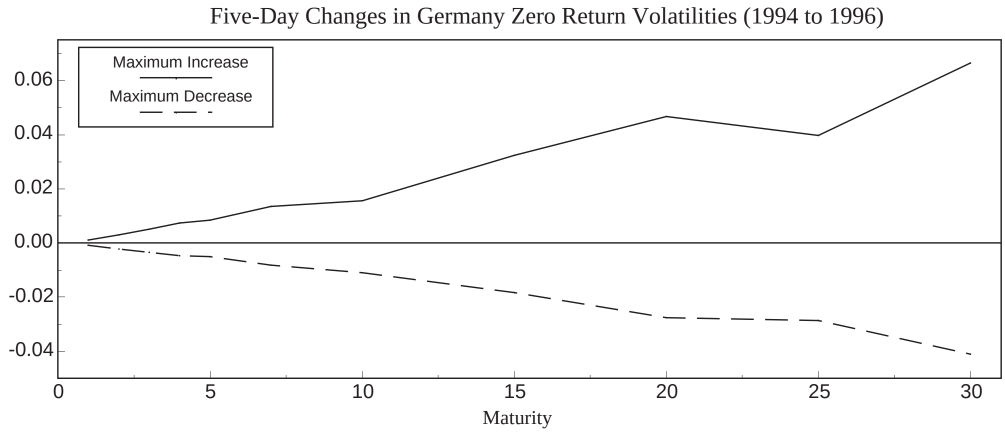


Volatility

Chart 3c: Volatility Predictions, Germany



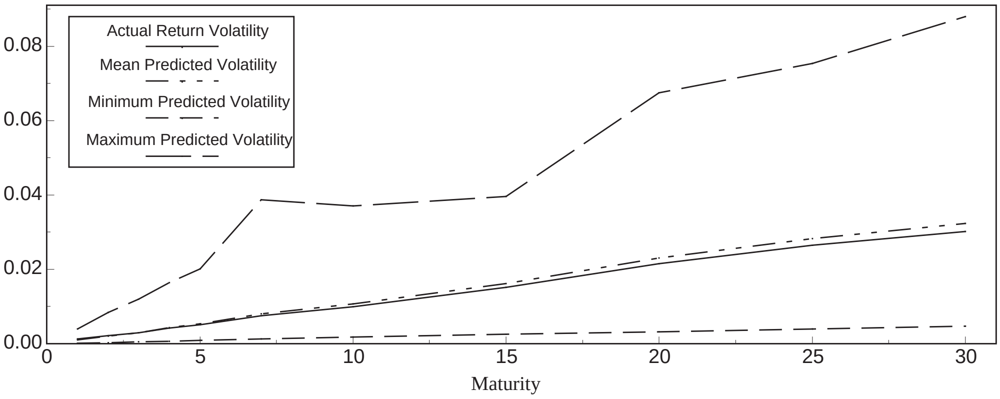




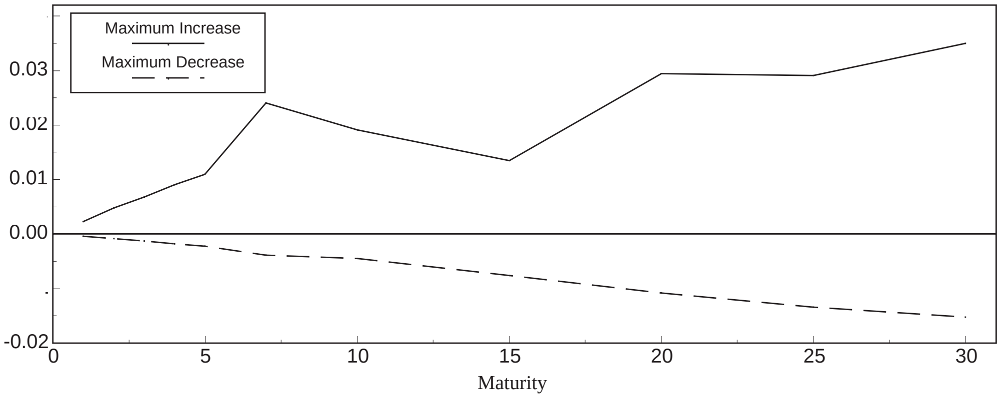
Volatility

Chart 3d: Volatility Predictions, Italy

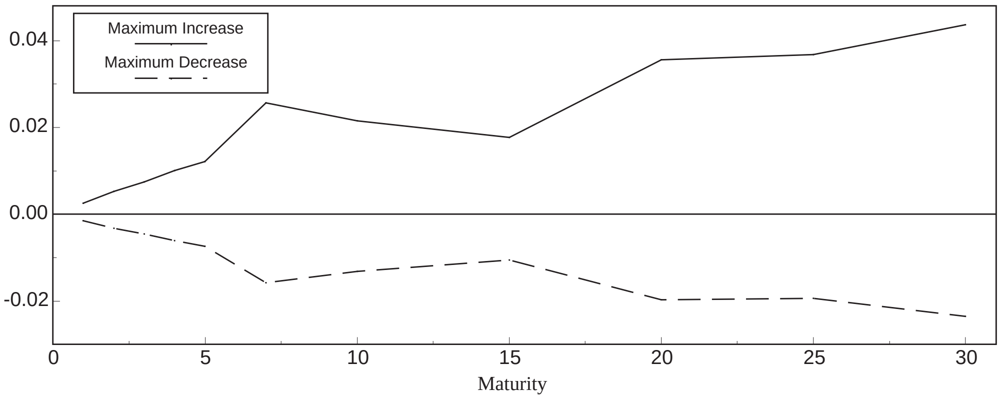
Italy Zero Return Volatilities (1994 to 1996)



One-Day Changes in Italy Zero Return Volatilities (1994 to 1996)



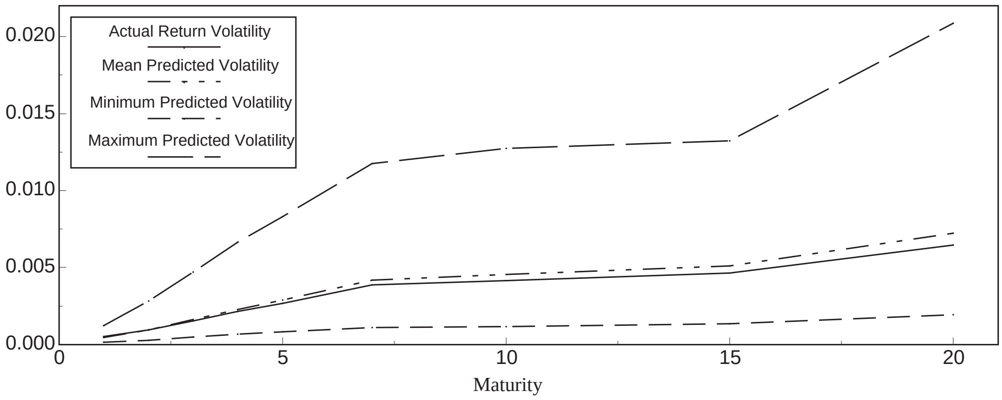
Five-Day Changes in Italy Zero Return Volatilities (1994 to 1996)



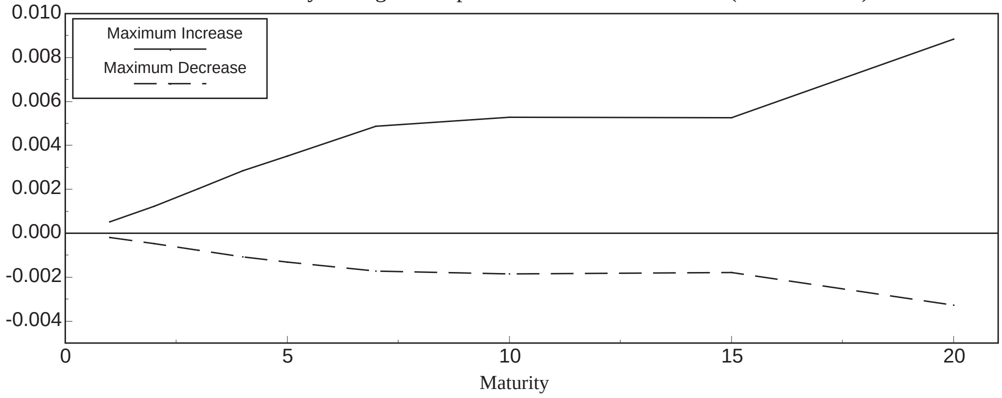
Volatility

Chart 3e: Volatility Predictions, Japan

Japan Zero Return Volatilities (1994 to 1996)



One-Day Changes in Japan Zero Return Volatilities (1994 to 1996)



Five-Day Changes in Japan Zero Return Volatilities (1994 to 1996)

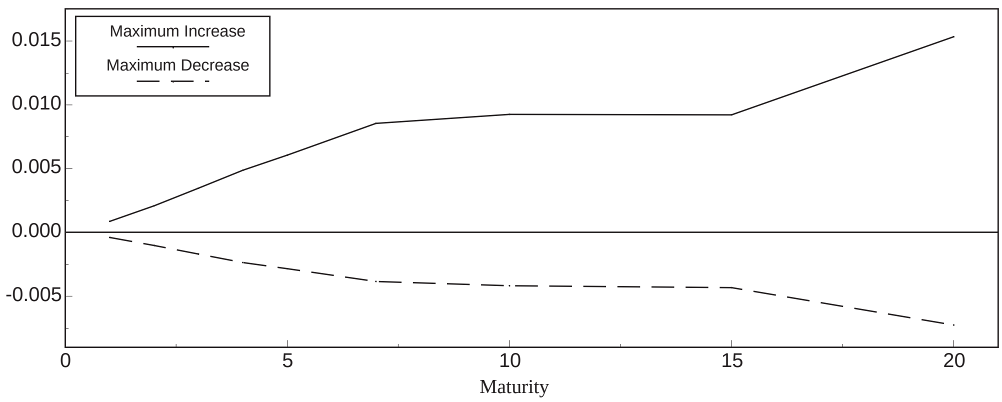
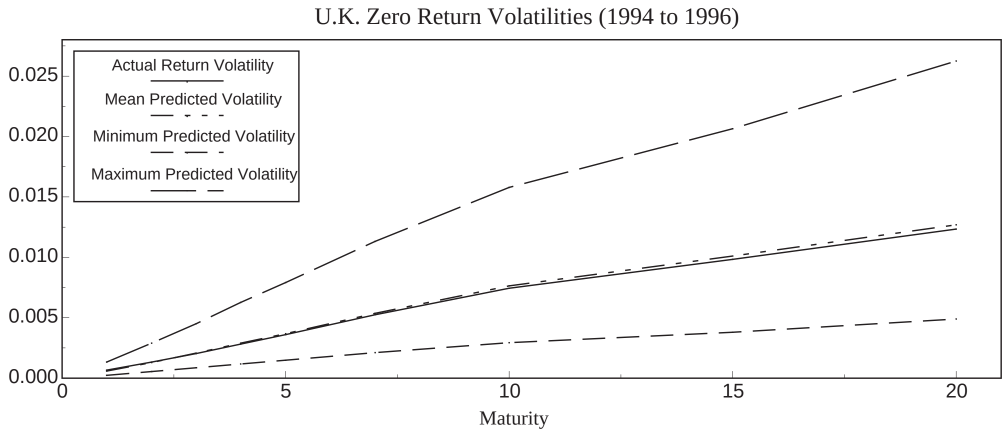
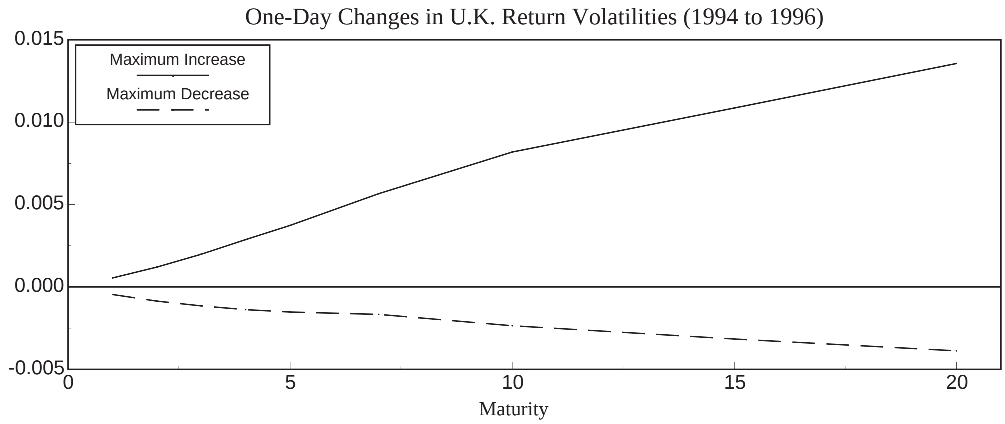


Chart 3f: Volatility Predictions, United Kingdom





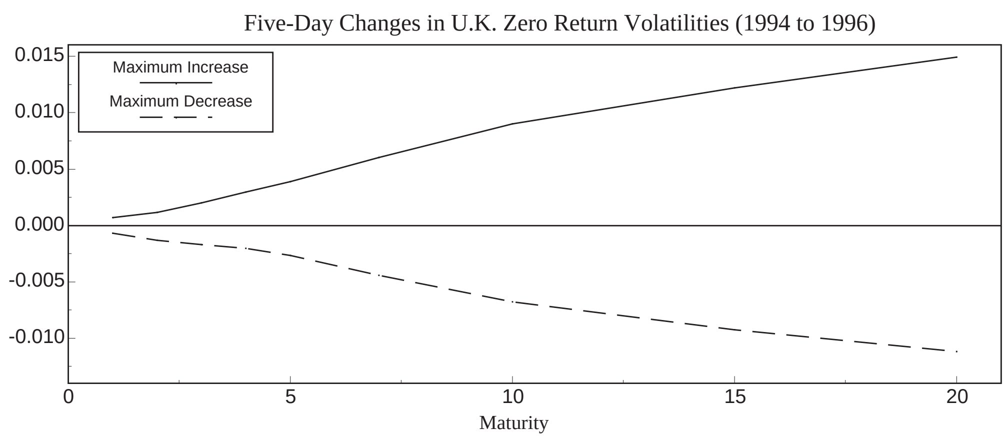
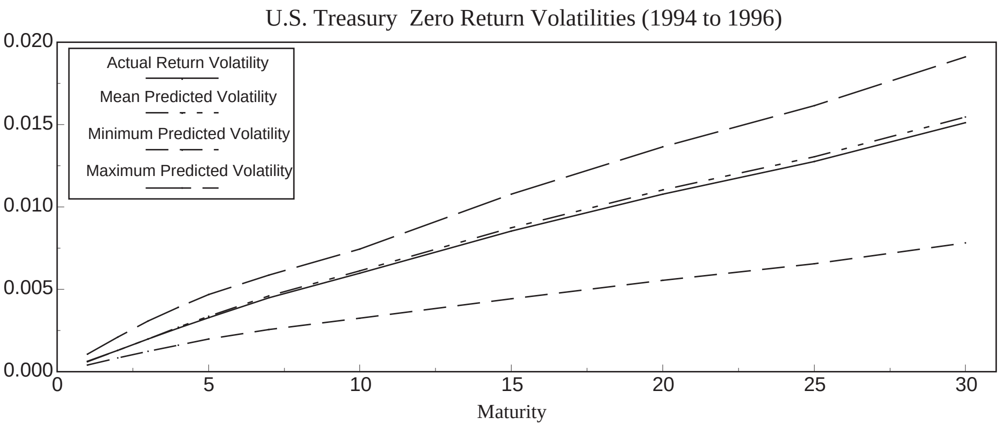
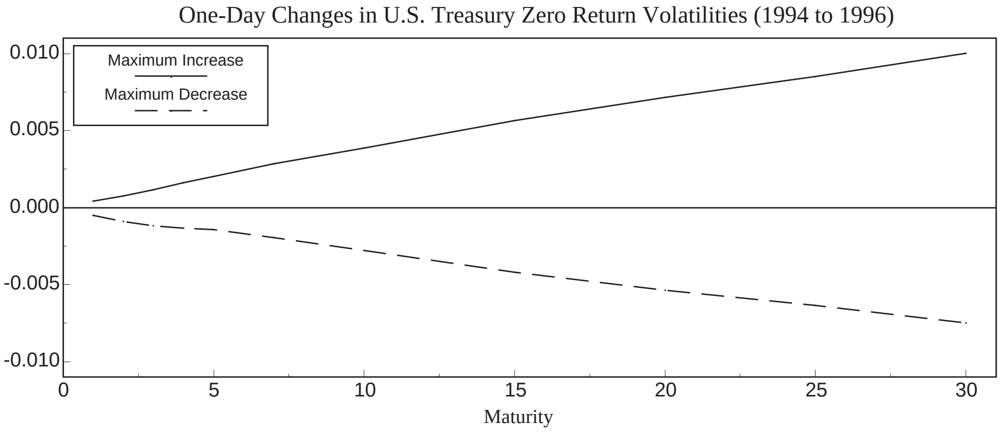
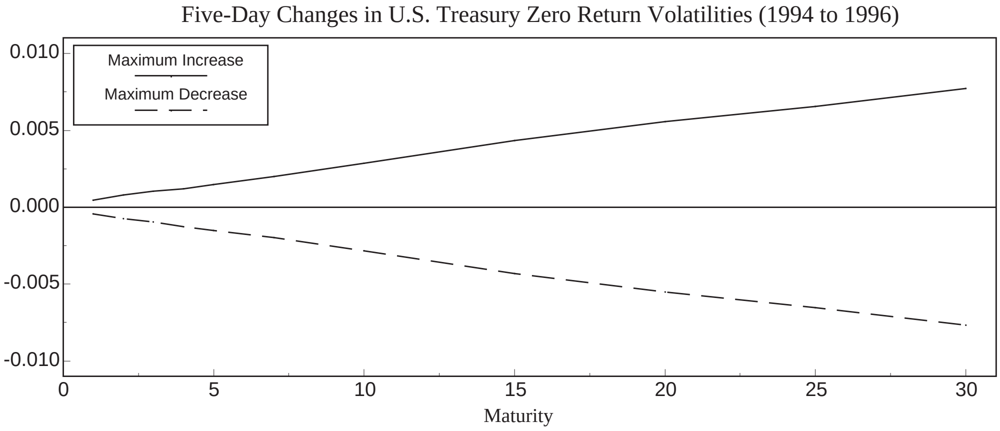


Chart 3g: Volatility Predictions, United States







Volatility