Generating market risk scenarios using principal components analysis: methodological and practical considerations

by

Mico Loretan*

Federal Reserve Board

March 1997

Abstract

In this paper, I study a number of statistical issues that arise in the formulation of stress scenarios for market risk in financial instruments. The possibility of reducing the number of scenarios through the use of data-based, statistical dimension reduction methods is explored. Using data on returns to spot exchange, stock market and interest rate products for a number of countries, I show that principal components analysis may be used to reduce the effective dimensionality of the scenario specification problem in several cases. Given the data dimensionality uncovered by PCA for the datasets considered, various methods for specifying stress scenarios are discussed.

---

* Views expressed in this paper need not reflect the views of the Board of Governors of the Federal Reserve System or of other members of its staff or of the Eurocurrency Standing Committee. Any errors are my own.

1. Introduction and general issues in market risk scenario specification

Market risk is commonly defined as the susceptibility of portfolio values to changes in asset prices, volatilities of prices, and related functions of asset prices. Measuring market risk may seem to require specifying a very large number of perturbations of prices and volatilities. However, in empirical practice, many asset price and volatility movements are highly correlated contemporaneously. The "effective dimensionality" of market risk is therefore often considerably less than the number of assets held in a typical portfolio. "Risk factors" are often defined and used to summarise observed changes in market prices and volatilities. This paper discusses some of the statistical issues that arise in the search for market risk factors and scenarios that describe stressful market risk events.

The remainder of this section discusses some general methodological considerations. The need for applying statistical methods for scenario specification is reviewed. Principal Components Analysis (PCA) is proposed as a tractable and easy-to-implement method for extracting market risk factors from observed data. Section 2 presents the returns series analysed in this paper, and tests whether the data are in fact amenable to PCA methods. Section 3 performs PCA on several groupings of these series. I find that the stock market and the exchange rate returns series are more highly correlated than, say, short term interest rates. This suggests that dimensionality reduction may apply for certain groups of series, but not for others. On the basis of the PCA results, I provide suggestions for stress scenarios for stock market and spot exchange rate shocks. Section 4 concludes and mentions several shortcomings of PCA not dealt with elsewhere in the paper. An appendix discusses some of the mathematical aspects of nonparametric density estimation and of PCA.

It is important to note that the dimensionality of the market risk scenario problem is, to a considerable extent, a choice variable for the researcher. Increasing the number of market risk factors tends to enhance descriptive accuracy or the amount of data variability captured by the scenarios, but also risks increasing the methodological complexity and unwieldiness of the study. An optimal cut-off for specifying additional risk factors will depend, in general, on the purposes for which the risk factors are being constructed.

When the number of series is small, say one or two, it is usually possible to simply "eyeball" scatter plots of the data and to decide heuristically what a relevant stress scenario might be. Unfortunately, "eyeball methods" become infeasible when the data are high-dimensional. To specify stress scenarios in such cases, it is necessary to resort to formal statistical methods. The statistical methods should provide answers to issues such as the effective dimensionality of the data and nature of data-coherent stress scenarios. Formulating market risk factors and extracting their distributions from the data is an intermediate step between assembling the data and specifying scenarios.

One may distinguish between model-driven and data-driven statistical methods for generating risk factors.[1] Model-driven methods rely heavily on hypothesised relationships between asset prices, returns, and volatilities (which are then estimated from the data). Examples of model-driven methods are the capital asset pricing model (CAPM) for returns and "GARCH" models for volatilities. Data-driven methods, on the other hand, impose less structure on the data. When a researcher is unwilling to impose a lot of structure on the data and would rather extract risk factors directly, data-driven methods are preferable. One method which is in widespread use among statistical practitioners is "Principal Components Analysis" (PCA). This method, whose technical details are described in the appendix to this paper, is frequently employed when one needs to reduce the data dimensionality to a tractable threshold without being willing to commit to strong hypotheses about the nature of the data generating process.

2. Data and preliminary data analysis

The data series I study in this paper are daily-frequency observations on spot exchange rates, stock market indexes, and long-term and short-term interest rates, and were obtained from the Federal Reserve Board's internal economic database. I consider data for nine countries: Belgium, Canada, France, Germany, Japan, the Netherlands, Switzerland, the United Kingdom, and the United States. The exchange rate series consist of the bilateral spot exchange rates of the first eight countries vis-a-vis the United States.[2] For each of the nine countries, a leading stock market index was chosen to represent movements in equity prices. Both short-term (3-month) and long-term (10-year) interest rates were collected for each of the nine countries. In addition, a nine-point term structure series for the US Treasury returns and four separate stock market indexes for the United States (S&P 500, Dow Jones Industrials-30 Average, Nasdaq Composite, and Wilshire 5000) were studied. The observations run from 2 January 1990 to 8 October 1996, or slightly less than 1,700 observations. Cross-sectional missing values, caused chiefly by differing national market holiday conventions, were deleted prior to further analysis. I first took natural logarithms of the exchange rate and stock market index series, and then first-differenced all series to induce stationarity.

Prior to applying PCA to these returns series, it is important to determine whether PCA is in fact a meaningful procedure given the distributional properties of the data. The main distributional requirement is "axis symmetry," i.e., that the joint distribution of the data be symmetric about its

---

1 In practice, of course, one finds that successful model-driven methods are congruent with the data, and successful data-driven methods can be interpreted to conform to certain statistical models.

2 The spot exchange rates are measured in units of foreign currency per US$ except for Sterling, where the inverse convention was applied.

axes.[3] Unfortunately, formal statistical techniques for testing axis symmetry are not well developed. To test the proposition of axis symmetry, I chose the following informal "eyeball method:" I computed the joint density of various pairs of series and graphed their contour plots. Significant deviations from axis symmetry are then readily apparent to the eye. Appendix B contains a brief discussion of the nonparametric density estimation methods that I employed to obtain the contour plots.

Contours of six (randomly chosen) bivariate densities are plotted in Figures 1 through 6. The returns data are standardised for this exercise. Density estimates are provided for a -5 to +5 standard deviations range from the joint mean of the data. The heights of the displayed contours are 0.40, 0.30, 0.20, 0.10, 0.04, 0.01, and 0.001.[4] The height of the outermost lines is only 1/400th of the height of the innermost "ring." For a practical assessment of axis symmetry, though, it is more practical to consider the shapes of the lines with heights between 0.01 and 0.40. The first three figures depict bivariate data sets that are not highly correlated; the second group of three figures depicts series that are highly correlated. In no case does failure of axis symmetry appear to be a prominent problem. Since axis symmetry cannot be rejected, at least not on the basis of the informal tests conducted, I conclude that we may indeed apply PCA methods to the data at hand.

3. Principal components analysis and effective dimensionality of the data

3.1 Fraction of variance explained by principal components

For a collection of returns series, the number of principal components (PCs) to be retained for further analysis is determined by the correlation structure of the data. If the data are all highly mutually correlated, one or two PCs will suffice to explain a large fraction of total data variation. On the other hand, if the data are either uncorrelated or only correlated across subgroups, more PCs need to be retained. By studying the fraction of the variance that is explained by successive PCs, one may obtain an estimate of the effective dimensionality of the data.

Since PCA is sensitive to the units of measurement of the data, we report our results both for the "raw" and for "standardised" (zero mean and unit variance) series. Standardisation is found to have little qualitative effect except when groups of series with differing group variances, such as exchange rates and interest rates, are analysed.

In Table 1, I list the fractions of the total variance explained by successive principal components. Numbers that exceed $1/N$ (where N is the number of series under consideration) are

---

[3] Intuitively, axis symmetry can be thought of as an absence of non-linear dependence among the series. Multivariate normality is sufficient but not necessary for axis symmetry. Other well-known distributions, such as the multivariate Student-*t*, are also elliptic and hence axis-symmetric.

[4] The height of 0.001 was chosen deliberately so that even a single data point would "show up" in the contour plots.

italicised, and numbers that exceed 2/*N* are underlined. I start with several more narrowly defined groups of series, and then go on to study larger data groups. For the eight groupings considered, I find:

(A) *Short Term Interest Rates, 9 Countries.* In the sample period, correlations among the nine short term interest rates were quite low. This is reflected in Panel (A) of Table 1: Whether standardised or raw interest rate changes are considered, the first two PCs barely explain 50% of total variance.

(B) *Long Term Interest Rates, 9 Countries.* In this case, the first PC alone explains ca. 50% of total data variability, and first three jointly explain about 75% of the variance.

(C) *9-Point US Term Structure Series.* Here, the first PC explains more than 80% of total variation, and the second explains about 10%. None of the other seven PCs explains more than about 3% of total variation.

(D) *Spot Exchange Rates, 8 Countries.* All of the series are very highly correlated, and the first PC explains more than 70% of the total variance. No other PC explains more than 15% of the variance.

(E) *Stock Market Indexes, 9 Countries.* The first PC explains about 40% of the variance, and the next two each contribute more than 10%.

(F) *4 US Stock Market Indexes.* All series are well known to be very highly correlated at daily frequencies; this is borne out in the PCA, where the first (of four) PCs explains close to 90% of total variance.

(G) *Combination of Stock Market Indexes and Exchange Rates, 17 Series.* For the raw data, only the first four PCs each explain more than a 1/*N* fraction of total variance, but none of these four is particularly dominant. A similar results applies for the standardised returns series.

(H) *Combination of Stock Market Indexes, Exchange Rates, and Long Term Interest Rates, 26 Series.* For the unstandardised series, the first PC explains 50% of the variance, and two more PCs explain more then a 2/*N* fraction of the variance. However, upon standardisation the influence of the first PC is diminished to 26%, and the second PC has roughly equal weight (21%).

From these numbers, it would appear that there is considerable scope for dimension reduction among the equity returns series and exchange rate series, as well as within the US term structure of interest rates. However, the two broad asset classes (G) and (H) are less mutually correlated, leading to a lower contribution to the total variance provided by the first few leading principal components.

3.2 Correlations of estimated principal components with observed time series

In the preceding subsection we found that, in several cases, one or two PCs suffice to explain most of the variability present in the data. This suggests that the effective dimensionality of the data groups is smaller than the number of series in the groups. However, this finding alone does not let us attribute an economic interpretation to the PCs, since it does not tell us whether the PCs are correlated with *all* of the series in the respective group, or only with a subset of the series

Since PCs are linear functions of the data, it is useful to study their correlations with the observed returns series to uncover their economic interpretation (if one exists). In Table 2, we list correlations for the first four PCs (computed from the raw as well as the standardised returns series) with the corresponding observed series. The discussion below focuses, to the most part, on the correlations between the observed series and the PCs obtained after first standardising the data. We find:

(A) *Short Term Interest Rates, 9 countries.* In keeping with the finding reported above that none of the PCs explains a large fraction of the total variance in the data, we find that each of the first four PCs is highly correlated with only one or at most two of the individual 3-month interest rate series. This finding precludes the use of PCA to reduce the dimensionality of the multivariate short-rate process.

(B) *Long Term Government Bond Interest Rates, 9 countries.* In contrast to the short rate case, the long rates (especially the six European series) are highly correlated with each other and with the first PC. The Canadian and US series are highly correlated with P2, and the Japanese long rate is highly correlated with P3. This suggests that for purposes of scenario specification, the nine series can be reduced to three "meta series:" one "European" dimension, one "North American" dimension, and one "East Asia" dimension.

(C) *9-Point US Term Structure Series.* For this group of time series, the first PC is highly correlated with all nine series, and the correlations are of the same sign. The second PC is negatively correlated with the short-maturity series and positively correlated with the long-maturity series. The third PC is positively correlated with the short- and long-maturity series, and negatively correlated with the intermediate-maturity series. This finding lets us interpret the first principal component as a factor that shifts the whole term structure, the second PC as a factor that tilts or rotates the yield curve, and the third as a factor that affects curvature. In many cases, it will be quite satisfactory to concentrate

(D) *Spot Exchange Rates, 8 Countries.* Here, all series except the Can$/US$ are highly correlated with the first PC. The Can$/US$ series is highly correlated with P2, and Yen/US$ series is highly correlated with P3 (as well as with P1). This means that these data show one dominant risk factor at work, *viz.* the joint comovements of all exchange rates (except the Canadian series) against the US$; the fluctuations of the Canadian currency vis-a-vis the US$ are governed by a separate risk factor, given by P2.

(E) Stock Market Indexes, 9 Countries. Concentrating on the standardised-PC correlations with the observed series, it is obvious that all but one of the series (the French stock market index returns) are highly correlated with the first PC. In addition, the Canadian and US series are also highly correlated with P2. Given these findings, one can easily conclude that there is one dominant global risk factor as well as a separate "North American" risk factor.

(F) 4 US Stock Market Indexes. All four series are highly correlated with P1; in addition, the Nasdaq Composite returns series is also somewhat correlated with P2. It seems, though, that it would suffice for many purposes to specify a single risk factor that governs the daily-frequency returns of all four indexes.

(G) Combination of Stock Market Indexes and Exchange Rates, 17 Series. (Here, it is definitely preferable to concentrate on the second part of panel (G) of Table 2, since the two types of series have differing levels of variance.) From the correlation numbers, P1 may be interpreted as an "exchange rate shock" and P2 as a "stock market shock." However, these first two principal components explain only 56% of the total data variability (*cf.* Table 1). Hence, a simple two-factor model may not be satisfactory for capturing a sufficiently large fraction of the variance in the data.

(H) Combination of Stock Market Indexes, Exchange Rates, and Long Term Interest Rates, 26 Series. Attributing economic significance to the PCs computed from the joint behaviour of all 26 series is even more difficult than in the previous case. P1 is negatively correlated with most stock market returns series; the exchange rate returns are negatively correlated with P1 but positively with P2; finally, the long term interest rates are positively correlated with both P1 and P2. These findings strongly suggest that it is not fruitful to study all 26 series jointly if the objective is reducing the dimensionality of the data.

To sum up, PCA applied to the various groupings of the data reveals that it is feasible to reduce the dimensionality of the scenario specification problem for certain groups of assets, especially for exchange rates and stock market index fluctuations. On the other hand, we also found groups of series—most notably the set of short-term interest rates—where there appears to be little scope for dimension reduction. Both the "positive" and the "negative" results are useful since they point out the types of groupings of the data for which dimension reduction is appropriate, as well as the ones for which it is not.

3.3 Stress scenarios based on principal components analysis

The preceding analysis suggests that several groupings of the data are well characterised as possessing only one or at most two "meta-dimensions." How does one specify scenarios that make use of this information? Consider first the case where a single principal component suffices to capture most of the variance of the data. Since the first PC is a one-to-one transformation of the observed data, it is possible to "reverse" the calculations and to compute the values of each of the series that correspond to given values of the first PC. Next, since the PC is a random variable we may pick tail-event quantiles of the empirical distribution of the PC to generate corresponding tail events of the observable series.

When more than one PC is required to describe a sufficient amount of the total variance in the data, one may proceed by specifying separate "shocks" in each of the directions given by the retained PCs, in analogy to the case of a single relevant PC. Alternatively, one may choose to form arbitrary linear combinations of the estimated PCs to generate "combined" shocks. Or, if the PCs are highly correlated with one of the observable series, one could simply sort the data by that series, and associate stress scenarios with particularly large realisations of that series.

Frye (1996) and Jamshidian and Zhu (1996) explain in detail how trading firms may use PCA as a basis for their risk management process. Once the "relevant dimensions" of market risk are established via PCA, scenarios are generated by taking various linear combinations of the first two or three PCs of the data.

In the remainder of this section, we report the results of specifying shock scenarios for the following four groupings of the data: spot exchange rates (8 series), the US T-Bond term structure (9 series), long-term government bond returns (9 series), and stock market indexes (9 series). For each of these datasets, four separate types of scenarios were generated. The first three are based on fluctuations in the direction specified by each of the first three PCs of the data; the fourth scenario is created by taking the direct sum of the first three scenarios. To indicate how the potential computational burden might be reduced for firms that would calculate their exposure to each of these shocks, "fluctuations" that do not exceed at least 0.5% per day or 1 basis point per day are set to zero.

For each of these four types of scenarios, the following quantiles of the resulting distributions are reported: 0.5%, 1%, 5%, 10%, 90% 95%, 99%, and 99.5%. By measuring the exposure to shocks of increasing severity-from 10% to 0.5%, and from 90% to 99.5%-it may be possible to determine if there is "curvature" in the exposure, i.e., if there is gamma risk that could lead to systemic breakdowns if these exposures are hedged by dynamic trading strategies. Note that the quantiles of the shock distributions should not be interpreted as meaning that any of these particular scenarios will occur with the specified probabilities; "real world" shocks are combinations of the shocks in the directions of the various PC-shocks. The results are listed in Table 3.

Turning first to the scenarios for the eight exchange rates (Panel A), we note that the shocks generated by fluctuations along the first PC affect mainly the European series; the second shock affects mostly the Can$/US$ exchange rate, and the third induces fluctuations in the Yen/US$ rate. The fourth shock, which is a weighted sum of the first three shocks, leads to fluctuations in all series except the Can$/US$ series

The scenarios for shocks to the US term structure, tabulated in Panel B of Table 3, show that shocks in the direction of the first PC-which was identified above as a "shift" factor-indeed lead to a shift in all rates, with the changes being largest for the longer-term bonds. The second scenario is a "tilt" of the yield curve, and the third serves to increase or decrease curvature. The numerical magnitude of the shocks, measured in basis points, may seem somewhat small. However, it should be remembered that they are "pure factor shocks," and that "actual" shocks are combinations of the "pure" shocks. To wit, the fourth scenario, which is a simple combination of the first three, does lead to fluctuations that exceed 20 basis points at either end of the distribution.

In Panel C of Table 3, various scenarios for fluctuation in long term bond rates across nine countries are presented. The first PC-shock leads to sizeable changes in all long rates except for Japan and the US; these two series are affected by PCs 2 and 3, respectively. Interestingly, a simultaneous shock to all three PCs leads to a scenario in which the Canadian and US long rates fluctuate strongly while the other series do not show much action.

Stock market shock scenarios are given in Panel D of Table 3. Here, the first PC induces shocks for all European series except France. The second shock affects US stock returns strongly, but has a smaller impact on the stock returns of Belgium, Canada, France and Germany as well. The third PC leads to large fluctuations in the series for France, and affects Japanese stock market returns as well. A combination of these three scenarios affects all stock markets except the ones for Canada and the United States.

We close this section by observing that the numerical values given in all of the scenarios confirm the qualitative interpretation of the nature of the PCs derived earlier in this paper. The numerical values presented here serve mainly to give a "flavour" of the severity of market risk scenarios that can be generated by PCA.

4. Conclusions

In this note, I have set out to discuss some of the technical issues that need to be addressed in the process of specifying scenarios that are based on data driven methods such as principal components analysis. The methodological points were illustrated empirically with a dataset that consists of daily-frequency observations on long- and short-term interest rates, stock market indexes, and exchange rates for nine industrialised countries. I find that the effective dimensionality of several subgroups of these time series is considerably smaller than the number of series included. These results would allow us to reduce the number of market risk scenarios to groups. Several methods for generating scenarios in terms of observables on the basis of the PCA-based results were discussed, and numerical values of several simple scenarios were presented.

We close by discussing some shortcomings of PCA that have not been mentioned up to this point. First, and most importantly, PCA is strongly affected by the choice of units of the series. An important consequence of this fact is that PCA will not detect risk factors that do not contribute significantly to the total variability of the data. This shortcoming could be remedied, at least in principle, by multiplying the series with appropriate portfolio weights. However, this requires knowledge of the actual asset holdings of participants in the reporting exercise.

A second shortcoming, less serious than the first, is that PCA is suitable for detecting risk factors that are linear functions of the data. Volatility factors, which are of interest for the valuation of options and of products with embedded-option characteristics, are more difficult to derive by PCA. To obtain volatility factors, it appears to be preferable to use a more model-driven approach to data analysis, say by specifying and estimating a multi-factor GARCH process. Third, by construction, the factors derived from PCA are mutually orthogonal. If the true market risk factors (assuming that there is such a thing as a "true" risk factor!) are not orthogonal, then the PCA-based factors will be linear combinations of the true factors, and it will be harder to give economic interpretations to the PCA-derived factors.

Appendix A

Technical exposition of principal components analysis

Consider a collection of T observations of N asset returns. Let X denote the resulting $T \times N$ data matrix, and assume without loss of generality that X has full column rank. (Otherwise, one or more of the returns series are redundant and may be omitted.) Our goal is to find a linear combination of the observed asset returns that "explains" as much as possible of the observed variability of the data. We will demonstrate that principal components analysis, PCA for short, achieves this objective.

The following discussion is based on Theil (1971, pp. 46–56). Let P denote the $T \times N$ matrix of the eigenvectors of XX' that correspond to the N non-zero eigenvalues (sorted in descending order) of XX'. (Since XX' is positive semi-definite, exactly N of its eigenvalues are positive and the remaining T-N are zero.) One can show that the first column of P, i.e., the first "principal component" (PC) of X, maximises the explained variance ("R^2") of the multivariate regression of X on any linear combination of the columns of X. Thus, the first PC solves the objective set out above. Similarly, the second column of P, i.e., the second PC, maximizes the explained variability in the data, given the explanation already provided by the first PC. Since the eigenvectors are mutually orthogonal, all of the principal components are uncorrelated with each other. Note that principal components are not unique up to sign, i.e., multiplying a PC by -1 has no effect on the explanatory power of the PC.

One may write $X = P\, A$, where A is the $N \times N$ matrix of "loadings" of the data on each of the principal components. This representation shows that PCA is a special form of the general statistical method of "factor analysis." In PCA, the "factors" are not directly observed, but are constructed by taking linear combinations of the data. Since each of the PCs is (in principle) a function of all N data vectors, PCA is a function of the *joint* distribution of all data points. This distinguishes it from regression analysis, which is concerned with the *conditional* distribution of the "dependent" variable(s) given observations on the "independent" variables. In PCA, one does not distinguish between dependent and independent variables.

The fraction of the data variance explained by each of the successive PCs is given by $\lambda_i / \left(\sum \lambda_i\right)$, where λ_i is the i'th (sorted) eigenvalue of XX', $i = 1,\ldots,N$. The cumulative fraction of the data variance explained by the first j PCs is given by $\left(\lambda_1 + \ldots + \lambda_j\right) / \left(\sum \lambda_i\right)$.

In empirical practice, when the data are correlated, the first few PCs tend to capture most of the variability. The leading PCs, then, can be used to represent the "meta-dimensions" in which the data fall. One could also say that the number of leading PCs, say, those that capture between 50% and 90% of the total variance, represents the effective dimensionality of the data, which will be well less than in general.

Appendix B

Nonparametric density estimation

Technical references to the field of nonparametric density estimation are Silverman (1986), Green and Silverman (1994) and Wand and Jones (1995) and the references contained in these works. The pieces cited explain both the intuition that underlies nonparametric density estimation methods as well as many of the mathematical subtleties and computational considerations that arise in this field in practice.

The key idea in nonparametric density estimation—as in other areas of nonparametric statistics—is to apply "local smoothing" techniques to obtain estimates of the probability density of the data. Local smoothing means that the estimate of the density at a point is influenced mostly by the number of observations close to that point, whereas it is little affected by the properties of the data far away from the point of interest. Generally, the local smoothing estimators are so-called "kernel methods." In all kernel methods, the crucial parameter is the "bandwidth." The bandwidth parameter determines the size of the region (around the point of interest) which is used to perform the smoothing operation.

The bivariate density estimates reported in the paper were computed using a two-dimensional Gaussian kernel and a (scalar) bandwidth chosen as $\sigma N^{-0.2}$, where σ is the average standard deviation of both series. The estimation routines were coded in the "Gauss" programming language by the author.

Table 1

Fractions of variance explained by successive principal components

Note: There are two lines for each group of series. Line 1 applies to the raw returns series, the second for the standardised returns series. Numbers greater than 1/N are italicised, numbers greater than 2/N are underlined, where N is the number of series included in the group.

(A) Short Term Interest Rates (9 countries)

0.355	0.232	*0.154*	0.090	0.069	0.046	0.032	0.014	0.008
0.202	*0.179*	*0.112*	0.102	0.097	0.093	0.080	0.074	0.060

(B) Long Term Government Bond Interest Rates (9 countries)

0.494	*0.159*	*0.094*	*0.084*	0.069	0.033	0.029	0.026	0.011
0.480	*0.122*	*0.100*	*0.087*	0.063	0.061	0.037	0.034	0.016

(C) 9-Point US Term Structure

0.843	0.093	0.028	0.011	0.007	0.007	0.005	0.004	0.003
0.810	*0.121*	0.031	0.013	0.009	0.006	0.004	0.003	0.003

(D) Spot Exchange Rates (8 countries)

0.812	0.084	0.043	0.022	0.022	0.010	0.007	0.001
0.716	*0.130*	0.079	0.042	0.017	0.008	0.006	0.001

(E) Stock Market Indexes (9 countries)

0.395	*0.192*	*0.164*	0.076	0.052	0.043	0.037	0.023	0.019
0.409	*0.130*	*0.113*	0.090	0.071	0.060	0.050	0.043	0.033

(F) US Stock Market Indexes (4 series)

0.868	0.107	0.018	0.007
0.883	0.090	0.019	0.008

(G) 9 Stock Market Indexes & 8 Exchange Rates

0.298	0.233	0.129	*0.113*	0.053	0.035	0.030	0.026	0.024
0.358	0.202	*0.069*	*0.064*	0.057	0.047	0.038	0.035	0.031

(H) 9 Stock Market Indexes, 8 Exchange Rates, & 9 Long Term Rates

0.500	0.147	0.087	*0.076*	*0.064*	0.032	0.027	0.025	0.010	0.009
0.257	0.213	*0.075*	*0.056*	*0.044*	*0.039*	0.034	0.032	0.028	0.027

Table 2

Correlations of the data series with the first four principal components, for various data groupings

Note: Two sets of correlations are reported for each group of returns, (i) between the data and the "raw-data PCs" and (ii) between the data and "standardised-data PCs." Correlations greater than 0.45 in absolute value are underlined.

(A) Short Term Interest Rates (9 countries)

Correlation between data and raw-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	-0.580	0.813	-0.008	0.031
CA	-0.184	-0.081	0.978	-0.001
FR	-0.899	-0.426	-0.099	0.039
GE	-0.218	0.070	0.043	-0.193
JA	0.021	0.021	0.042	-0.047
NE	-0.174	0.109	0.102	-0.182
SZ	-0.079	0.039	0.033	-0.278
UK	-0.130	-0.007	-0.044	-0.958
US	0.005	0.047	0.036	-0.080

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.433	0.403	-0.163	-0.077
CA	0.224	0.220	-0.274	0.742
FR	0.390	0.624	-0.143	0.024
GE	0.728	-0.077	0.227	-0.058
JA	0.114	-0.488	-0.332	0.319
NE	0.709	-0.193	0.164	0.077
SZ	0.537	-0.431	0.114	-0.065
UK	0.269	0.059	-0.394	-0.576
US	0.014	-0.228	-0.766	-0.107

Table 2 (cont.)

(B) Long Term Government Bond Interest Rates (9 Countries)

Correlation between data and raw-data PCs

Index	**P1**	**P2**	**P3**	**P4**
BE	0.777	-0.247	0.290	-0.015
CA	0.695	0.655	0.006	0.293
FR	0.814	-0.218	0.279	-0.053
GE	0.829	-0.215	0.285	-0.060
JA	0.315	-0.031	0.185	-0.309
NE	0.811	-0.227	0.289	-0.038
SZ	0.407	-0.100	0.184	0.015
UK	0.777	-0.325	-0.532	0.065
US	0.416	0.503	-0.190	-0.708

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.830	-0.180	-0.035	0.097
CA	0.579	0.525	-0.158	-0.094
FR	0.849	-0.110	-0.009	0.125
GE	0.894	-0.135	-0.055	0.094
JA	0.349	0.156	0.923	-0.011
NE	0.879	-0.163	-0.055	0.080
SZ	0.503	-0.239	-0.017	-0.819
UK	0.704	-0.036	-0.067	0.235
US	0.365	0.804	-0.113	-0.099

Table 2 (cont.)

(C) 9-Point US Term Structure

Correlation between data and raw-data PCs

Maturity	**P1**	**P2**	**P3**	**P4**
m03	0.624	-0.676	0.334	0.164
m06	0.807	-0.518	0.117	-0.119
y01	0.911	-0.294	-0.146	-0.198
y02	0.956	-0.109	-0.208	0.046
y03	0.975	0.005	-0.143	0.074
y05	0.979	0.120	-0.042	0.067
y07	0.960	0.223	0.070	0.041
y10	0.942	0.276	0.128	-0.004
y30	0.875	0.359	0.243	-0.134

Correlation between data and standardised-data PCs

Maturity	**P1**	**P2**	**P3**	**P4**
m03	0.664	-0.677	0.269	0.159
m06	0.836	-0.482	0.025	-0.208
y01	0.922	-0.224	-0.212	-0.109
y02	0.954	-0.039	-0.225	0.075
y03	0.968	0.067	-0.154	0.086
y05	0.969	0.170	-0.048	0.075
y07	0.949	0.261	0.067	0.042
y10	0.931	0.308	0.129	0.003
y30	0.865	0.385	0.258	-0.117

Table 2 (cont.)

(D) Spot Exchange Rates (8 countries)

Correlation between data and raw-data PCs

Country	P1	P2	P3	P4
BE	-0.956	0.074	-0.106	0.174
CA	0.009	0.188	0.191	-0.028
FR	-0.969	0.085	-0.054	0.100
GE	-0.985	0.062	-0.080	0.037
JA	-0.608	-0.790	0.070	0.017
NE	-0.981	0.065	-0.086	0.051
SZ	-0.947	0.038	-0.087	-0.301
UK	0.822	-0.148	-0.547	0.003

Correlation between data and standardised-data PCs

Country	P1	P2	P3	P4
BE	-0.955	0.023	0.080	0.126
CA	0.011	0.971	-0.232	0.050
FR	-0.969	0.030	0.091	0.075
GE	-0.983	0.006	0.076	0.094
JA	-0.617	-0.278	-0.735	-0.047
NE	-0.979	0.009	0.077	0.101
SZ	-0.941	0.008	0.046	0.091
UK	0.829	-0.130	-0.111	0.532

Table 2 (cont.)

(E) Stock Market Indexes (9 countries)

Correlation between data and raw-data PCs

Country	P1	P2	P3	P4
BE	0.587	0.229	-0.087	0.151
CA	0.436	0.129	-0.110	-0.569
FR	-0.012	-0.654	-0.758	0.012
GE	0.718	0.387	-0.316	0.338
JA	0.806	-0.479	0.346	0.017
NE	0.688	0.368	-0.228	-0.015
SZ	0.667	0.354	-0.241	0.053
UK	0.613	0.269	-0.232	-0.313
US	0.400	0.164	-0.155	-0.734

Correlation between data and standardised-data PCs

Country	P1	P2	P3	P4
BE	0.661	-0.303	0.017	0.189
CA	0.577	0.636	-0.139	0.070
FR	-0.060	0.241	0.930	-0.242
GE	0.766	-0.285	0.068	-0.138
JA	0.496	-0.020	0.313	0.763
NE	0.824	-0.201	-0.016	-0.188
SZ	0.759	-0.179	0.002	-0.162
UK	0.729	0.017	0.021	-0.224
US	0.538	0.680	-0.159	-0.023

Table 2 (cont.)

(F) US Stock Market Indexes (4 series)

Correlation between data and raw-data PCs

Index	**P1**	**P2**	**P3**	**P4**
djia30	<u>0.925</u>	0.329	0.191	0.017
nasdaqc	<u>0.890</u>	<u>-0.454</u>	0.033	0.025
sp500	<u>0.954</u>	0.225	-0.179	0.086
wilt5000	<u>0.982</u>	0.065	-0.080	-0.161

Correlation between data and standardised-data PCs

Index	**P1**	**P2**	**P3**	**P4**
djia30	<u>0.939</u>	0.280	0.198	0.018
nasdaqc	<u>0.862</u>	<u>-0.502</u>	0.053	0.041
sp500	<u>0.966</u>	0.170	-0.167	0.098
wilt5000	<u>0.986</u>	0.006	-0.072	-0.149

Table 2 (cont.)

(G) 9 Stock Market Indexes & 8 Exchange Rates

Correlations between data and raw-data PCs

Stock Market Index	P1	P2	P3	P4
BE	-0.520	-0.260	-0.236	0.112
CA	-0.372	-0.219	-0.137	0.123
FR	0.068	-0.118	0.693	0.709
GE	-0.628	-0.327	-0.386	0.352
JA	-0.609	-0.574	0.404	-0.368
NE	-0.713	-0.133	-0.308	0.239
SZ	-0.632	-0.224	-0.330	0.262
UK	-0.598	-0.180	-0.233	0.239
US	-0.325	-0.232	-0.183	0.174
Exchange Rate				
BE	-0.594	0.741	0.144	-0.024
CA	0.054	0.013	-0.011	-0.071
FR	-0.598	0.744	0.157	-0.056
GE	-0.617	0.753	0.146	-0.048
JA	-0.353	0.503	0.130	-0.006
NE	-0.614	0.750	0.148	-0.051
SZ	-0.650	0.685	0.113	-0.021
UK	0.485	-0.660	-0.103	0.036

Table 2 (cont.)

Correlations between data and standardised-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.241	-0.616	0.326	0.047
CA	-0.168	-0.563	-0.604	-0.213
FR	0.080	0.026	-0.289	0.553
GE	-0.262	-0.718	0.285	0.139
JA	-0.123	-0.491	0.046	0.130
NE	-0.466	-0.690	0.176	0.052
SZ	-0.336	-0.679	0.176	0.030
UK	-0.348	-0.640	-0.018	-0.026
US	-0.117	-0.545	-0.626	-0.266
Exchange Rate				
BE	-0.931	0.230	-0.015	-0.013
CA	0.043	0.074	0.254	-0.737
FR	-0.938	0.242	-0.023	-0.025
GE	-0.956	0.233	-0.014	-0.004
JA	-0.601	0.205	-0.098	0.240
NE	-0.952	0.234	-0.017	-0.008
SZ	-0.935	0.146	-0.014	-0.006
UK	0.800	-0.233	-0.009	0.126

Table 2 (cont.)

(H) 9 Stock Market Indexes, 8 Exchange Rates, & 9 Long Term Rates

Correlations between data and raw-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.380	0.071	-0.130	-0.023
CA	-0.306	-0.149	0.016	0.129
FR	0.044	-0.012	-0.025	0.026
GE	-0.477	0.140	-0.159	-0.047
JA	-0.185	0.006	-0.022	0.016
NE	-0.436	0.066	-0.030	-0.023
SZ	0.345	0.011	-0.052	0.002
UK	-0.411	0.076	0.086	0.047
US	-0.264	-0.166	0.055	0.309
Exchange Rate				
BE	-0.031	0.105	0.035	-0.100
CA	0.132	0.140	-0.008	0.106
FR	0.002	0.102	0.047	-0.100
GE	-0.029	0.107	0.043	-0.100
JA	0.026	0.023	0.055	-0.122
NE	-0.027	0.106	0.048	-0.101
SZ	-0.048	0.090	0.049	-0.094
UK	-0.016	-0.101	0.036	0.044
Long Term Rate				
BE	0.783	-0.248	0.292	0.027
CA	0.710	-0.642	-0.000	0.276
FR	0.828	-0.242	0.244	-0.018
GE	0.833	-0.244	0.272	-0.000
JA	0.349	-0.016	0.182	-0.414
NE	0.821	-0.250	0.277	0.017
SZ	0.450	-0.116	0.161	0.047
UK	0.787	-0.283	-0.546	0.015
US	0.464	0.492	-0.090	-0.669

Table 2 (cont.)

Correlations between data and standardised-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.512	-0.275	0.230	0.336
CA	-0.385	-0.259	0.475	-0.386
FR	0.082	-0.027	0.027	-0.066
GE	-0.602	-0.351	0.223	0.363
JA	-0.287	-0.197	0.387	0.217
NE	-0.707	-0.164	0.344	0.240
SZ	-0.560	-0.206	0.420	0.252
UK	-0.591	-0.208	0.356	0.098
US	-0.326	-0.266	0.522	-0.464
Exchange Rate				
BE	-0.615	0.730	-0.070	-0.048
CA	0.110	0.059	0.036	0.245
FR	-0.602	0.756	-0.050	-0.050
GE	-0.630	0.750	-0.066	-0.051
JA	-0.357	0.524	-0.043	-0.101
NE	-0.627	0.749	-0.064	-0.053
SZ	-0.643	0.693	0.004	-0.030
UK	0.508	-0.654	0.075	0.029
Long Term Rate				
BE	0.606	0.473	0.341	-0.053
CA	0.407	0.412	0.091	0.378
FR	0.637	0.478	0.333	-0.043
GE	0.622	0.514	0.408	-0.020
JA	0.178	0.345	0.079	0.070
NE	0.611	0.508	0.418	-0.027
SZ	0.358	0.291	0.315	0.040
UK	0.539	0.391	0.254	0.079
US	0.265	0.349	-0.134	0.643

Table 3

Market risk scenarios generated by PC shocks

A. Exchange Rate "Shock Scenarios"
(measured in percent per day; values less than 0.5% are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-2.25	-	-2.12	-2.28	-1.34	-2.28	-2.43	1.76
1%	-1.92	-	-1.80	-1.94	-1.15	-1.94	-2.07	1.50
5%	-1.09	-	-1.03	-1.11	-0.66	-1.10	-1.18	0.85
10%	-0.79	-	-0.74	-0.80	-	-0.80	-0.85	0.61
90%	0.79	-	0.74	0.80	-	0.80	0.84	-0.63
95%	1.10	-	1.03	1.11	0.64	1.11	1.18	-0.87
99%	1.87	-	1.76	1.90	1.10	1.89	2.01	-1.48
99.5%	2.17	-	2.04	2.20	1.27	2.19	2.32	-1.71

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-	-0.76	-	-	-	-	-	-
1%	-	-0.70	-	-	-	-	-	-
5%	-	-	-	-	-	-	-	-
10%	-	-	-	-	-	-	-	-
90%	-	-	-	-	-	-	-	-
95%	-	-	-	-	-	-	-	-
99%	-	0.82	-	-	-0.55	-	-	-
99.5%	-	0.93	-	-	-0.63	-	-	-

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-	-	-	-	1.30	-	-	-
1%	-	-	-	-	1.17	-	-	-
5%	-	-	-	-	0.76	-	-	-
10%	-	-	-	-	0.54	-	-	-
90%	-	-	-	-	-0.58	-	-	-
95%	-	-	-	-	-0.79	-	-	-
99%	-	-	-	-	-1.44	-	-	-
99.5%	-	-	-	-	-1.67	-	-	-

Simultaneous positive shock to first three PCs:
(sorted by value of GE column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-2.18	-	-2.05	-2.22	-1.65	-2.21	-2.37	1.68
1%	-1.90	0.57	-1.80	-1.93	-	-1.93	-1.99	1.48
5%	-1.12	-	-1.06	-1.13	-0.69	-1.13	-1.21	0.92
10%	-0.80	-	-0.76	-0.80	-	-0.80	-0.84	0.65
90%	0.79	-	0.75	0.82	-	0.81	0.84	-0.57
95%	1.09	-	1.03	1.11	-	1.10	1.13	-0.87
99%	1.88	-	1.76	1.90	1.58	1.89	2.04	-1.55
99.5%	2.18	-	2.05	2.22	1.67	2.21	2.36	-1.71

Table 3 (cont.)

B. US Term Structure "Shock Scenarios"
(measured in basis points; values less than 1 bp are suppressed)

Shock in direction of first PC:

Quantile	**m03**	**m06**	**y01**	**y02**	**y03**	**y05**	**y07**	**y10**	**y30**
0.5%	-11	-14	-18	-21	-22	-22	-21	-20	-16
1%	-8	-11	-14	-16	-17	-17	-16	-15	-12
5%	-5	-6	-8	-9	-9	-9	-9	-8	-7
10%	-4	-5	-6	-7	-7	-7	-7	-6	-5
90%	3	4	6	7	7	7	7	6	5
95%	5	6	8	9	10	10	9	9	7
99%	8	11	14	16	17	17	16	15	12
99.5%	10	13	17	20	21	21	20	18	15

Shock in direction of second PC:

Quantile	**m03**	**m06**	**y01**	**y02**	**y03**	**y05**	**y07**	**y10**	**y30**
0.5%	-11	-9	-5	-	1	4	6	7	7
1%	-9	-7	-4	-	1	3	4	5	6
5%	-5	-4	-2	-	-	2	2	3	3
10%	-4	-3	-1	-	-	1	2	2	2
90%	3	2	1	-	-	-1	-2	-2	-2
95%	5	3	2	-	-	-2	-3	-3	-3
99%	8	6	3	-	-1	-3	-4	-5	-5
99.5%	10	7	4	-	-1	-4	-5	-6	-6

Table 3 (cont.)

Shock in direction of third PC:

Quantile	m03	m06	y01	y02	y03	y05	y07	y10	y30
0.5%	-4	-	3	4	3	-	-1	-2	-4
1%	-3	-	3	3	2	-	-1	-2	-3
5%	-2	-	2	2	2	-	-	-1	-2
10%	-2	-	1	2	1	-	-	-1	-2
90%	1	-	-2	-2	-1	-	-	-	2
95%	2	-	-2	-2	-2	-	-	1	2
99%	3	-	-3	-4	-3	-	-	2	3
99.5%	4	-	-4	-4	-3	-1	1	2	4

Simultaneous positive shock to first three PCs:
(sorted by value of 30yr column)

Quantile	m03	m06	y01	y02	y03	y05	y07	y10	y30
0.5%	-15	-17	-19	-21	-21	-21	-20	-19	-15
1%	-8	-11	-14	-17	-18	-18	-18	-17	-14
5%	-4	-5	-6	-8	-9	-9	-10	-9	-8
10%	-3	-4	-5	-6	-6	-7	-7	-7	-6
90%	1	4	6	8	9	9	8	8	6
95%	2	3	3	5	6	8	8	9	8
99%	8	12	17	20	20	20	18	17	14
99.5%	11	17	25	29	29	27	25	22	17

Table 3 (cont.)

C. Long Term Interest Rate "Shock Scenarios"
(measured in basis points; values less than 1 bp are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-13	-14	-15	-13	-5	-13	-5	-17	-6
1%	-12	-13	-14	-12	-5	-12	-4	-15	-6
5%	-7	-8	-8	-7	-3	-7	-3	-9	-4
10%	-5	-5	-6	-5	-2	-5	-2	-6	-3
90%	5	5	5	5	2	5	2	6	2
95%	7	7	8	7	3	7	2	9	3
99%	13	14	15	13	5	13	5	16	6
99.5%	20	21	22	19	7	19	7	25	10

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-4	15	-3	-3	3	-3	-3	-1	17
1%	-3	12	-2	-2	2	-2	-2	-	13
5%	-2	7	-1	-1	1	-2	-1	-	8
10%	-1	5	-1	-	-	-1	-1	-	6
90%	-	-5	-	-	-1	-	-	-	-6
95%	1	-7	-	1	-1	1	1	-	-8
99%	2	-11	1	2	-2	2	2	-	-13
99.5%	3	-12	2	2	-2	2	2	-	-14

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-	-5	-	-1	16	-1	-	-2	-2
1%	-	-4	-	-	13	-	-	-2	-2
5%	-	-2	-	-	7	-	-	-1	-1
10%	-	-2	-	-	5	-	-	-	-
90%	-	1	-	-	-6	-	-	-	-
95%	-	2	-	-	-8	-	-	-	1
99%	-	3	-	-	-13	-	-	1	2
99.5%	-	4	-	-	-15	-	-	1	2

Simultaneous positive shock to first three PCs:
(sorted by value of US column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	3	-16	2	2	-3	3	3	-	-17
1%	-6	-17	-7	-6	-3	-6	-1	-9	-14
5%	-	-10	-1	-1	5	-	-	-3	-9
10%	-1	-8	-2	-2	3	-2	-	-3	-6
90%	4	8	5	4	7	4	1	6	6
95%	-	8	-	-	7	-	-	1	9
99%	3	16	5	4	13	4	-	6	15
99.5%	17	27	21	17	20	17	5	23	18

Table 3 (cont.)

D. Stock Market Shocks
(measured in percent per day; values less than 0.5% are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-1.82	-1.33	-	-3.37	-3.10	-2.32	-2.64	-2.43	-1.59
1%	-1.34	-0.98	-	-2.50	-2.31	-1.71	-1.94	-1.79	-1.17
5%	-0.75	-0.55	-	-1.41	-1.33	-0.95	-1.08	-1.01	-0.64
10%	-0.52	-	-	-0.97	-0.94	-0.65	-0.73	-0.69	-
90%	0.56	-	-	1.00	0.83	0.73	0.83	0.74	0.52
95%	0.75	0.55	-	1.35	1.15	0.97	1.11	1.00	0.69
99%	1.32	0.97	-	2.41	2.10	1.71	1.95	1.76	1.20
99.5%	1.53	1.13	-	2.81	2.45	1.98	2.26	2.05	1.39

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-0.66	1.23	0.99	-0.99	-	-	-	-	1.72
1%	-0.58	1.09	0.88	-0.87	-	-	-	-	1.52
5%	-	0.67	0.55	-0.52	-	-	-	-	0.94
10%	-	0.51	-	-	-	-	-	-	0.72
90%	-	-	-	-	-	-	-	-	-0.62
95%	-	-0.62	-	0.57	-	-	-	-	-0.85
99%	0.57	-0.94	-0.71	0.84	-	-	-	-	-1.29
99.5%	0.63	-1.05	-0.80	0.93	-	-	0.51	-	-1.43

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-	-	-4.11	-	-1.81	-	-	-	-
1%	-	-	-3.01	-	-1.34	-	-	-	-
5%	-	-	-1.78	-	-0.82	-	-	-	-
10%	-	-	-1.35	-	-0.64	-	-	-	-
90%	-	-	1.45	-	0.56	-	-	-	-
95%	-	-	1.95	-	0.77	-	-	-	-
99%	-	-	3.16	-	1.28	-	-	-	-
99.5%	-	-	3.61	-	1.47	-	-	-	-

Simultaneous positive shock to first three PCs:
(sorted by value of GE column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-1.83	-1.19	-2.00	-3.51	-4.08	-2.28	-2.63	-2.46	-1.37
1%	-1.62	-	1.27	-2.82	-1.87	-1.77	-2.00	-1.46	-
5%	-0.87	-	0.88	-1.52	-1.00	-0.96	-1.08	-0.81	-
10%	-0.53	-	-	-1.01	-1.12	-0.65	-0.75	-0.71	-
90%	0.59	-	0.66	1.05	1.01	0.65	0.75	0.56	-
95%	0.93	-0.51	-2.61	1.42	-	0.93	1.03	-	-0.72
99%	1.28	1.16	2.03	2.48	3.07	1.70	1.97	1.92	1.44
99.5%	1.85	-	-2.28	3.13	1.57	2.08	2.34	1.66	-

Figure 1

Nonparametrically Estimated Bivariate Density

Switzerland/US Exchange Rate vs. German Stockmarket

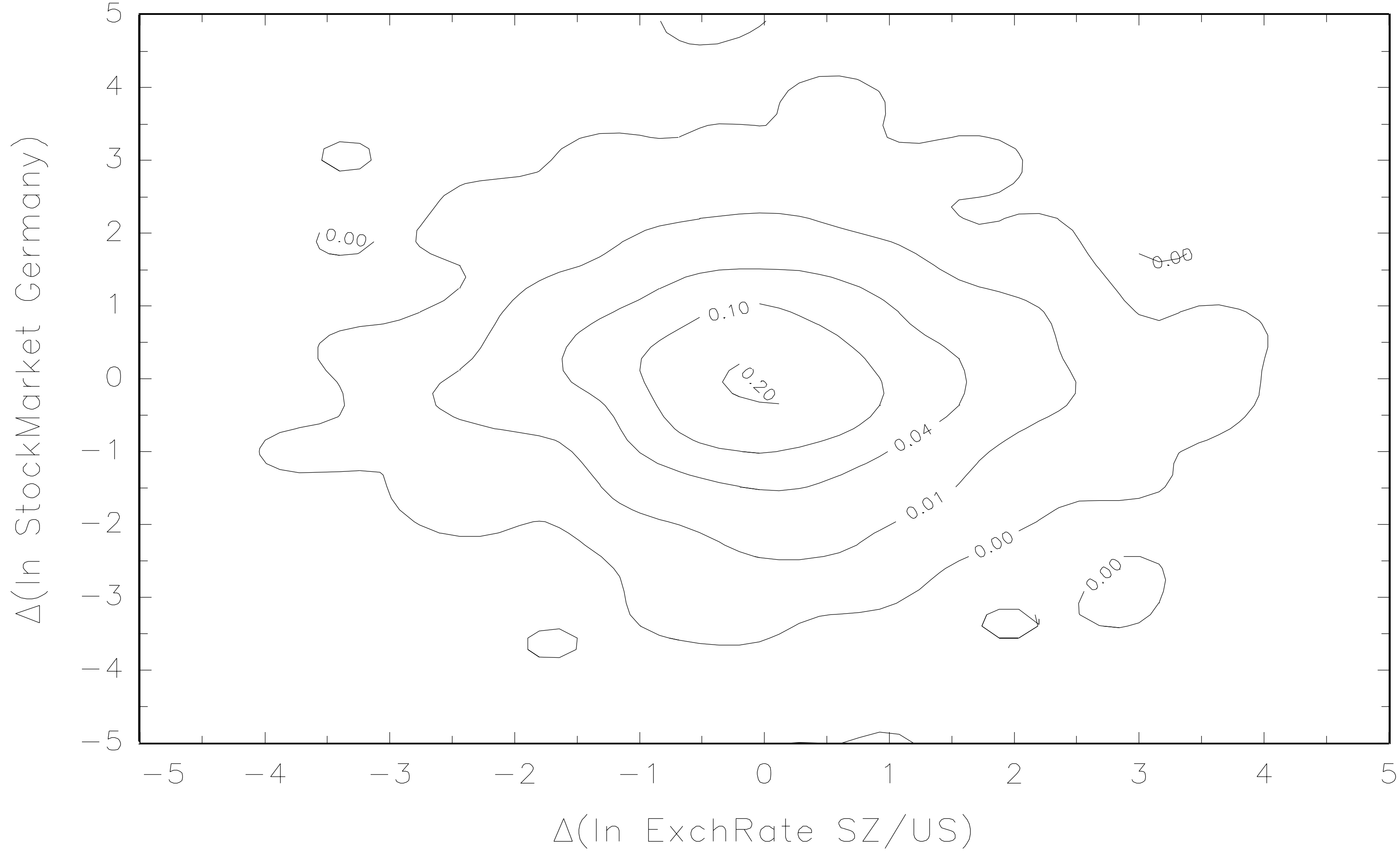


Figure 2

Nonparametrically Estimated Bivariate Density

UK/US Exchange Rate vs. British Stockmarket

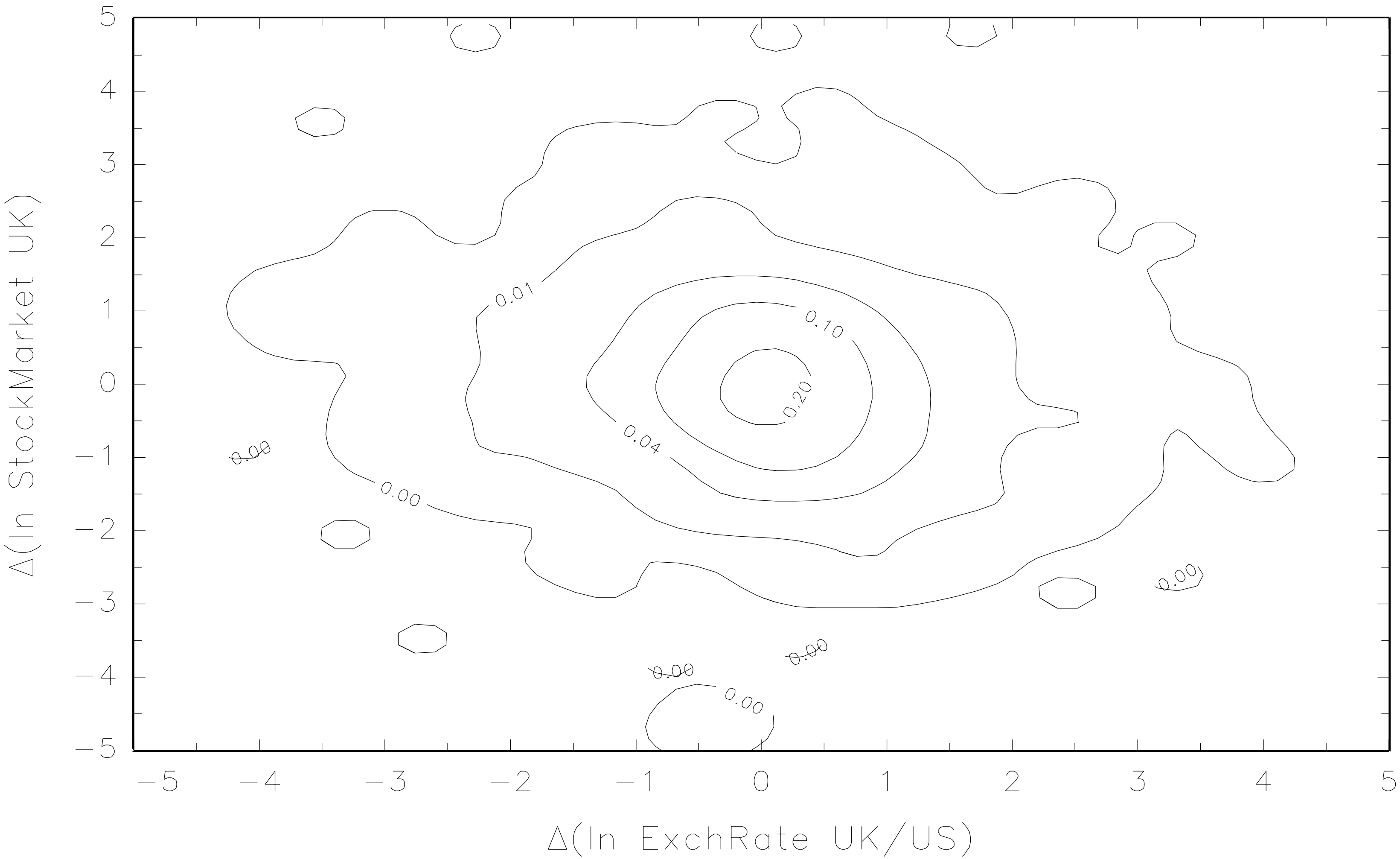


Figure 3
Nonparametrically Estimated Bivariate Density
Japanese/US Exchange Rates vs. Japanese Stockmarket

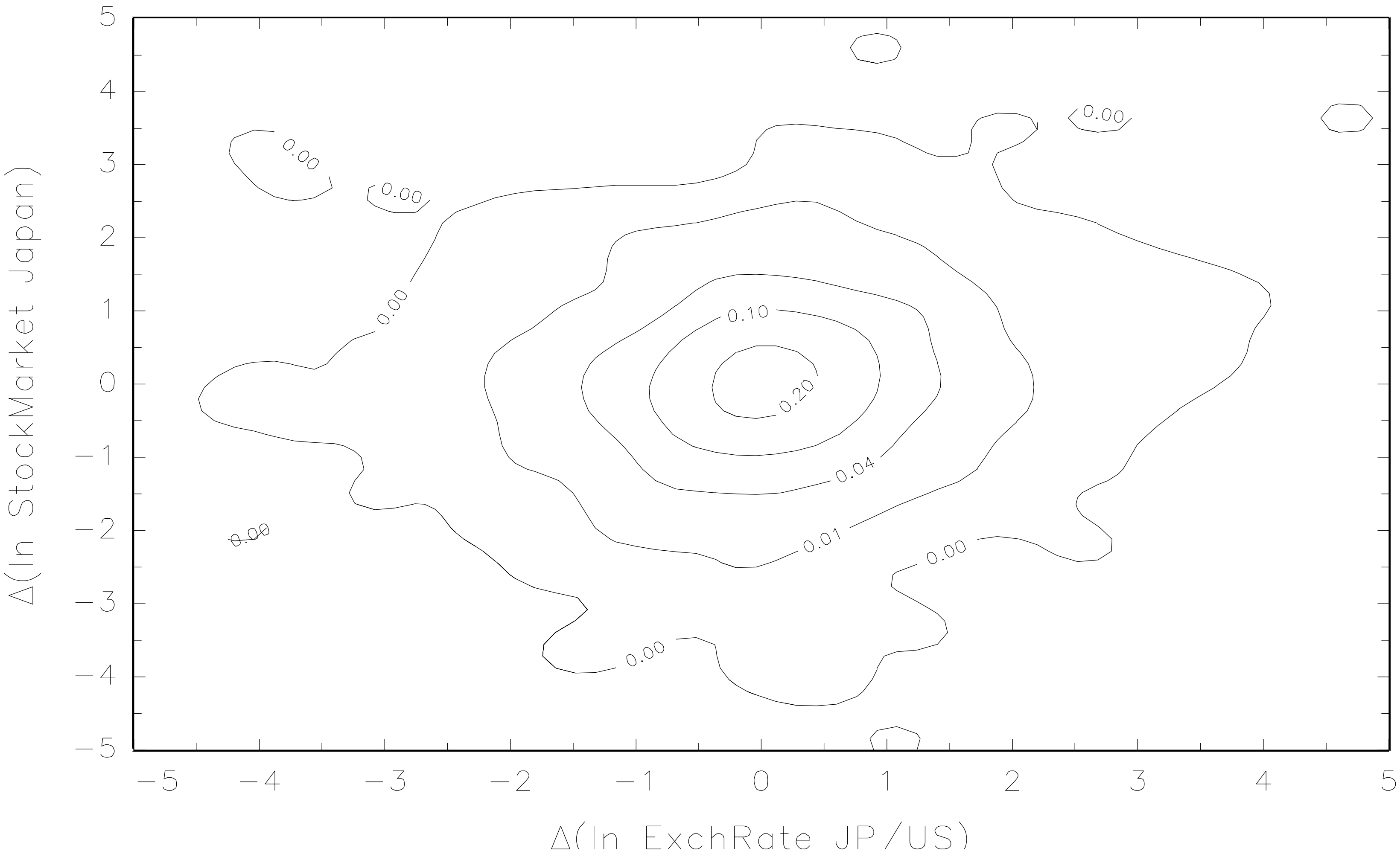


Figure 4
Nonparametrically Estimated Bivariate Density
Changes in 1-year and 30-year T-Bond Interest Rates

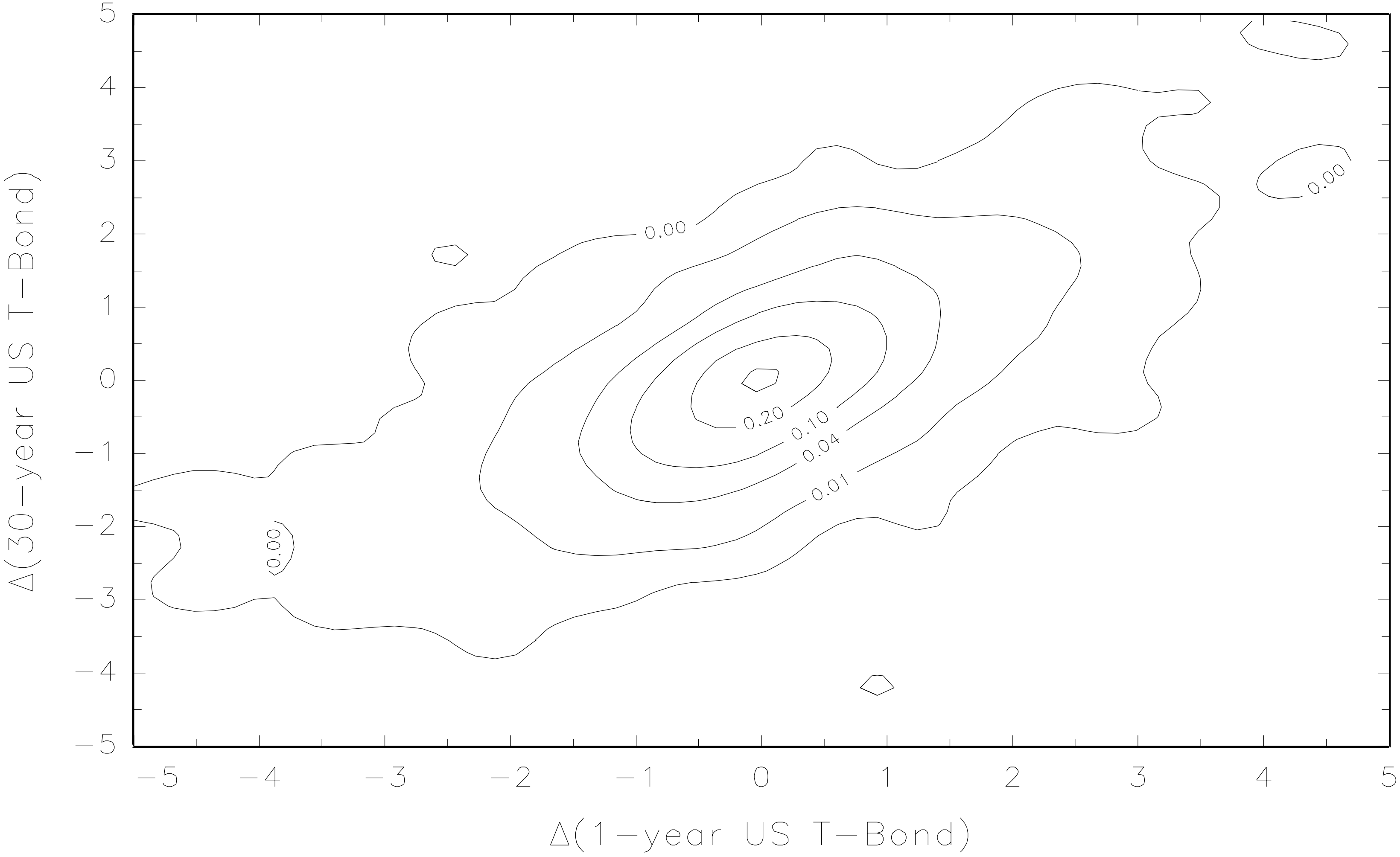


Figure 5

Nonparametrically Estimated Bivariate Density

Returns on Nasdaq–Composite and S&P–500

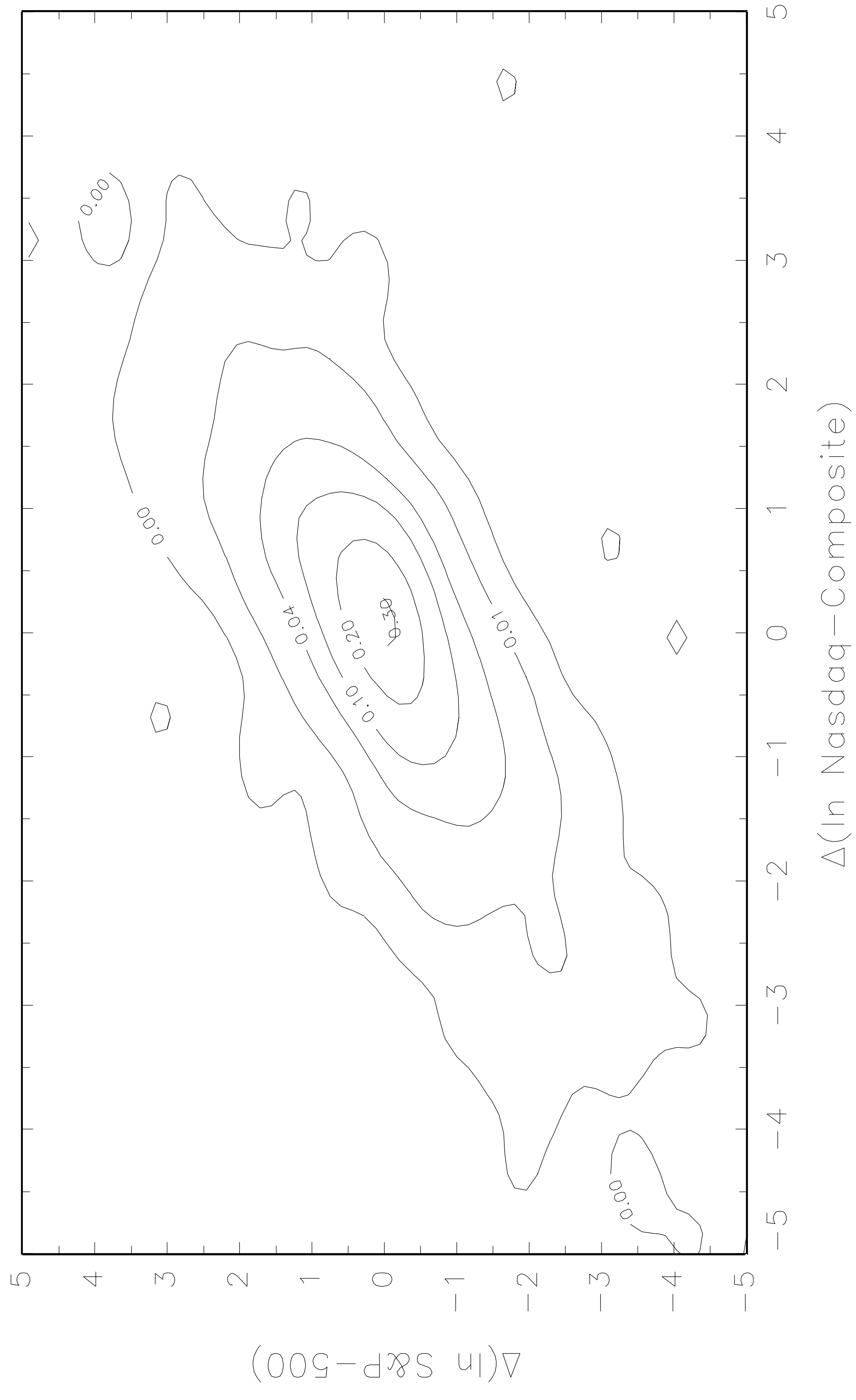
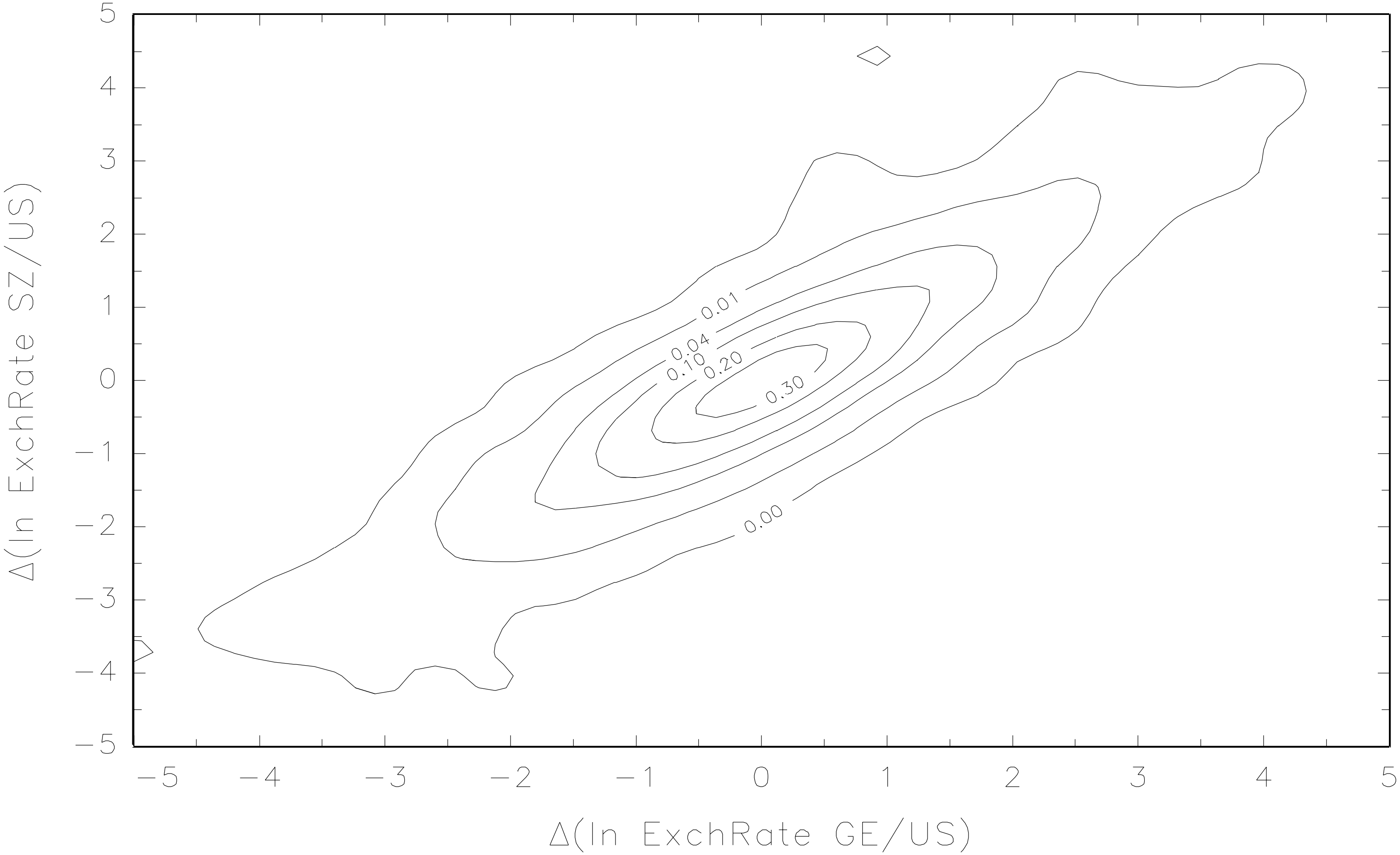


Figure 6
Nonparametrically Estimated Bivariate Density
German and Swiss Exchange Rates



References

Frye, Jon (1996): "Principals of Risk: Finding Value-at-Risk through Factor-based Interest Rate Scenarios, manuscript, Chicago: Nationsbanc-CRT (June).

Green, P.J. and B.W. Silverman (1994): *Nonparametric Regression and Generalized Linear Models.* Monographs on Statistics and Applied Probability No. 58. London: Chapman Hall.

Jamshidian, Farshid, and Yu Zhu (1996): "Scenario Simulation Model: Theory and Methodology." Manuscript, July. (Forthcoming, *Finance and Stochastics,* vol. 1, no. 1, 1997).

Silverman, B.W. (1986): *Density Estimation for Statistics and Data Analysis.* Monographs on Statistics and Applied Probability No. 26. London: Chapman Hall.

Theil, Henri (1971): *Principles of Econometrics*. New York: John Wiley.

Wand, M.P. and M.C. Jones (1995): *Kernel Smoothing*. Monographs on Statistics and Applied Probability No. 60. London: Chapman Hall.