BIS ECONOMIC PAPERS
No. 35 – January 1993

THE VALUATION OF
US DOLLAR INTEREST RATE SWAPS

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BASLE
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Introduction*

A swap is an agreement between two counterparties to exchange cash flows linked to two different indices at one or more dates in the future. Swaps have been used in conjunction with indices relating to interest and exchange rates as well as commodity and equity prices. With interest rate swaps, typically, the cash flows which are exchanged consist of interest payments having different characteristics but based on a common underlying or notional principal amount which in general is not exchanged. The most common ("plain vanilla") interest rate swap consists of one party undertaking payments linked to a short-term floating interest rate index such as LIBOR and receiving a stream of fixed interest payments; the other counterparty undertakes the opposite set of transactions. With currency swaps and commodity swaps the cash flows which are exchanged consist of payments indexed to interest rates (fixed or floating) in different currencies (and typically also include the exchange of the underlying principal amounts at maturity) and to prices of commodities respectively.

Since their inception in the early 1980s, various types of swap\(^1\) have come to dominate the markets for over-the-counter derivative

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\(^*\) I am indebted to P. Hainaut, P. van der Meulen, J.-B. Stuker, K. Brodhage and B. Allemann for helping to obtain the data for this study. M. Post and D. Simon, of the Federal Reserve Board, provided extremely useful comments on an earlier version of the paper. The comments by several colleagues, especially S. Arthur, C. Borio, W. Fritz, J.-M. Kertudo, C. Monticelli, M.-O. Strauss-Kahn and M. Takeda, are also gratefully acknowledged.

\(^1\) The generic term "swap" is applied to many other transactions in the financial markets. In the foreign exchange market a "swap" refers to a spot purchase and forward sale of one currency against another (see Grabbe (1986), Chapter 4). In the securities markets "swaps" consist of the sale of a security and the purchase of another security with somewhat different characteristics; typically, such swaps are undertaken between the most recent and actively traded long-term US Treasury securities, which are commonly referred to as "on the run", and other, "off the run" securities (see Homer and Leibowitz (1972)). A description of a specific episode of a pricing anomaly giving rise to potential arbitrage profits in the US government securities market is provided by Cornell and Shapiro (1989).
instruments and to rival in size and depth those for futures contracts traded on organised exchanges. The key parties to swap transactions are commercial and investment banks, though probably all major financial market participants have been counterparties to some form of swap.

Swaps have evolved from being initially linked to new issue activity in the capital markets, particularly the Euro-bond market, to being a more general instrument for financial risk management. This evolution has been accompanied by growing sophistication in the techniques utilised and by the development of new related financial instruments, such as caps, floors and swaptions (see Abken, 1991; Smith et al., 1990). At the same time, the basic swap transactions have become increasingly standardised, with two-way prices for swaps of various maturities being quoted by major commercial and investment banks and posted by brokers.

The purpose of this paper is to examine the valuation or pricing of interest rate swaps, specifically in the US dollar market. The first section provides a brief overview of the structure of the interest rate swap market and summarises some of the explanations which have been given for its growth.

The second section describes the main features of standard interest rate swaps, the principal market conventions and the pricing relationships with other closely substitutable financial instruments. Swaps are derivative instruments and their pricing is bounded by that of other financial transactions. In particular, Euro-dollar futures and new issues of fixed interest rate bonds can be used to obtain a structure of interest rate payments similar to that offered by swaps. However, like many other derivative instruments, arbitrage pricing may be difficult or imperfect, with the result that risk elements enter the pricing of swaps. The third section considers several potential explanatory factors for the prices of swaps when arbitrage is not perfect.

The remainder of this study examines the relationship between the pricing of US dollar interest rate swaps and other financial variables. The fourth section describes the statistical properties of a sample of interest rate swap spreads with different maturities in order to compare them with those of other commonly traded financial instruments. The fifth section outlines a model for the pricing of swaps in terms of other economic variables and provides evidence on the different factors which influence the pricing of swaps with various maturities.
1. The evolution of the interest rate swap market

1.1 The structure of the market

At end-1991 outstanding interest rate swaps in terms of their notional principal value — the underlying face value of the debt on which cash flows between swap counterparties are based — stood at over $3,000 billion, or nearly 350% higher than at end-1987. Interest rate swaps accounted for well over two-thirds of all swap and swap-related business reported by ISDA (International Swap Dealers Association).

Several changes have accompanied the expansion of the market for interest rate swaps. Firstly, the share of outstanding interest swaps accounted for by transactions between the main financial intermediaries, the members of ISDA, has risen from 30% at end-1987 to 44% at end-1991. This development reflects the changing use and character of swaps in recent years. The earliest swaps were undertaken on a one-off basis, which involved a search for matching counterparties in terms of amounts, maturities and currency. Interest rate swaps were also often directly linked to new issue activity in bond markets. With time, intermediaries began to accept swaps without searching for matching positions, taking the interest rate risk into their books. This process was accompanied by a growing standardisation of contracts as intermediaries began to place greater emphasis on being able to deal efficiently in swaps and to hedge positions temporarily until a suitable counterparty could be found. Swaps also became a more general instrument of risk management, especially for financial institutions, and to some extent replaced more traditional inter-bank activity. In the dollar sector, in particular, this development was reflected in the sharp decline in the weighted average original maturity of new swaps, from 4.1 to 2.5 years between the first half of 1987 and the second half of 1991.

Secondly, the share of the US dollar sector in total interest rate swaps outstanding contracted between end-1987 and end-1991 from 79 to 49%. The decline in the share of the US dollar reflects the gradual spreading of interest rate swaps to new markets. At end-1991 the most

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2 The term “notional” is used because the principal amount rarely, if ever, changes hands.
Table 1
Main features of the interest rate swap market, 1987-91
Notional principal value, in billions of US dollars

<table>
<thead>
<tr>
<th></th>
<th>News swaps arranged</th>
<th>Amounts outstanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-user</td>
<td>261.9</td>
<td>375.0</td>
</tr>
<tr>
<td>US dollar</td>
<td>192.6</td>
<td>237.9</td>
</tr>
<tr>
<td>Other currencies</td>
<td>69.3</td>
<td>137.1</td>
</tr>
<tr>
<td>Interbank (between</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISDA members</td>
<td>125.9</td>
<td>193.1</td>
</tr>
<tr>
<td>US dollar</td>
<td>94.1</td>
<td>128.4</td>
</tr>
<tr>
<td>Other currencies</td>
<td>31.8</td>
<td>64.7</td>
</tr>
<tr>
<td>Total</td>
<td>387.8</td>
<td>568.1</td>
</tr>
<tr>
<td>US dollar</td>
<td>286.7</td>
<td>366.3</td>
</tr>
<tr>
<td>Other currencies</td>
<td>101.1</td>
<td>201.8</td>
</tr>
</tbody>
</table>

Source: ISDA.

important non-dollar currency sectors were the yen and pound sterling segments.

Thirdly, the expansion of interest rate swaps in non-dollar currencies has coincided with a shift in the geographical and sectoral distribution of end-user business. Between end-1987 and end-1991 the share of European counterparties in total outstanding end-user swaps expanded from 30 to 45%, whereas that of US entities shrank from 45 to 35%.

1.2 Explanations for the growth of the market

The initial impulse for the growth of the swap markets can be found in the changes which the financial markets underwent in the 1980s. Both the demand for and supply of new financial instruments were stimulated by advances in technology, the process of deregulation and the volatility of asset prices. The emergence of novel arbitrage possibilities and, more importantly, the ability to exploit market inefficiencies were major factors in the rapid expansion of swaps. One would have expected, however, that by the beginning of the 1990s the growth of the market would have slowed down considerably because arbitrage opportunities due to market
imperfections would gradually have been eroded away in spite of the ever-increasing complexity of swaps and the widening of the range of swap-related transactions.\footnote{Under these circumstances swaps would merely prove to be redundant securities (see Turnbull, 1987).}

Several economic motivations have been put forward for the continuing growth of the market.

"Quality spreads" and comparative advantage. It is often argued that swaps owe their existence to the comparative advantage of different borrowers across segments of the credit market (Bicktsler and Chen, 1986). This argument stems in part from the observation that credit risk differentials exist between fixed and floating rate borrowing and that such "quality spreads" (e.g. the premia that lower-quality borrowers must pay over higher-rated takers of funds) tend to increase with the maturity of debt (see Section 2.4 below). Borrowers with a high credit standing appear to have an absolute advantage in all credit markets but a comparative advantage relative to other borrowers in raising funds in the fixed rate market at long maturities. Accordingly, swaps produce a gain to the two counterparties when the borrower with the lower credit standing takes up funds at a variable rate and the borrower with the higher credit standing taps the fixed-rate market.\footnote{See Das (1989) for a description of the benefits to both counterparties from this transaction.}

Comparative advantage is not a full explanation of swaps because it leaves unanswered the question of why "quality spreads" persist over time. Indeed, if interest rate differentials were simply due to market inefficiencies the use of swaps would tend to eliminate differences in "quality spreads" for securities of the same term (see Smith, Smithson and Wakeman, 1990). Even if "quality spreads" were due to different credit risk premia for short-term (variable rate) and long-term (fixed rate) debt such spreads could not necessarily be exploited by swaps if the cost of short-term debt rose over time to reflect the increasing probability of a borrower's default. The initial interest rate saving from a swap for the counterparty with the higher credit rating would merely be compensation for the risk of the other counterparty failing to perform its part of the swap agreement.

Information asymmetries and agency costs. Arak et al. (1988) suggest that swaps allow counterparties to separate interest rate risk from credit risk more effectively than do other instruments. For example, they argue
that borrowing short-term and swapping into a fixed rate liability would be preferred if a counterparty entered into a swap as a result of the following expectational asymmetries:6

a. it expected future risk-free interest rates to be higher than those of the market;

b. its risk aversion with respect to risk-free interest rates was greater than that of the market;

c. it expected future credit spreads to be lower than those forecast by the market;

d. it was less risk-averse with respect to changes in its own credit spread relative to the market.

Another explanation of the attractiveness of swaps relies on differences in agency costs between long and short-term debt.7 Wall (1989) argues that after a low-rated borrower issues long-term debt there is an incentive to make the firm riskier at the expense of the bondholder. Bondholders, in turn, perceive this incentive and attempt to protect themselves by requesting a larger premium than higher-rated companies which have already established a reputation. The agency problem, however, can be avoided if low-rated borrowers issue short-term debt and swap into fixed interest payments; the firm is monitored in each period that it enters the short-term debt market and is not required to pay the long-term premium for agency costs.

**Differential prepayment options.** Unlike most types of debt, interest rate swaps do not allow a counterparty to prepay its obligations at face value should interest rates move in its favour. For example, most bonds carry call provisions. Hence, if rates fall a borrower has an incentive to pay off the bond and refinance at a lower interest rate. As a consequence, the cost advantage on the fixed leg of an interest rate swap can be perceived as the cost saving from foregoing the interest rate option.

**Tax and regulatory arbitrage.** Even in very efficient markets tax and regulatory factors as well as investors’ own self-imposed constraints may limit the scope for a complete elimination of differentials in “quality spreads” across markets. For example, regulatory requirements such as

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6 Such expectational asymmetries imply the existence of information asymmetries. The possibility of credit quality signalling through the issuance of long-term debt and synthetic financing via short-term debt and swaps is discussed by Litzenberger (1992).

7 Agency costs arise in situations where contractual relationships give rise to divergences of interests between counterparties.
those imposed by the SEC (Securities and Exchange Commission) in the United States on bond issuance could be circumvented by using swaps and tapping the Euro-bond markets. Among the self-imposed constraints the most important are the requirements set out in charters that institutional investors hold only certain types of asset and the inability to take short positions in all types of security.  

*Interest rate exposure management.* As already mentioned, swap markets have evolved during a period characterised by major changes in other sectors of the financial markets, which have also grown dramatically in recent years. For example, in the 1980s large increases were recorded in turnover and open positions for financial futures and in the volume of transactions in the foreign exchange market.

Two developments accompanying these changes may help to explain the continuing growth of swaps. Firstly, very strong interlinkages have developed between the various segments of the financial markets. Shocks accordingly tend to be transmitted across various financial markets and generate transaction volume in different segments, whereas in the past they would affect only a narrow segment. Secondly, swaps have evolved into major tools for asset/liability management by allowing rapid modifications to exposures without entailing potentially costly changes in on-balance-sheet positions. This flexibility has meant that shocks flowing through the financial markets have inevitably tended to involve swap transactions.

### 2. Pricing of interest rate swaps

#### 2.1 The nature of interest rate swaps

As already mentioned, a standard interest rate swap is a contract that obligates one party to make payments linked to a fixed interest rate and obtain receipts based on a floating index, and the other party to assume the opposite set of obligations. Typically, one counterparty (A) agrees to pay interest based on a floating rate such as LIBOR, which is reset periodically throughout the life of the swap. The other counterparty (B) pays a

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8 An example of a tax-motivated set of swap transactions is provided by Smith, Smithson and Wakeman (1990), pp.220–224.

9 Bank for International Settlements (1992a), Chapter VIII.
known fixed interest rate set initially for the life of the swap. In a generic
swap the maturity and the notional principal on which the interest
payments are based are identical for both legs of the swap and no actual
exchange of principal takes place. The fixed rate payer (and floating rate
receiver) is said to have purchased a swap (or "gone long") whereas the
floating rate payer (and fixed rate receiver) is said to have sold a swap (or
"gone short").

The two underlying instruments to which interest rate swaps are most
closely related are a portfolio of loans and a portfolio of forward
contracts. In terms of cash flows, a swap can be viewed as nothing more
than a long and a short position in two loans of equal value with interest
being paid at a fixed rate and being received at a variable rate or vice
versa.\textsuperscript{10}

In contrast to parallel loans, which consist of two separate contracts,
interest rate swaps comprise only one contract. This means that the
periodic interest payments under swaps are on a net basis (with the net
amount being determined as the difference in interest rates) from the
party with the greater obligation to that with the lesser one. In the
example, the net amount for which A is responsible will increase in a rising
interest rate environment and fall with declining rates. It is also important
to note that the agreement to exchange the payment streams is independ-
ent of other assets or liabilities which either of the counterparties may
have contracted, but in practice swaps are for most end-users related
directly to other financial transactions, such as borrowing in the capital
markets.

2.2 Market conventions and "spreads"

The conventions for quoting swaps vary from one market to another. In
general, the market convention for quoting swap levels is to post the all-in
cost (the internal rate of return or yield to maturity) of the fixed side of
the swap versus the opposite flow of the floating index. In most markets
the fixed rate interest payments are quoted outright\textsuperscript{11} but in the US dollar

\textsuperscript{10} See Smith, Smithson and Wakeman (1990). Chapter II.

\textsuperscript{11} Differences with regard to quotation are analogous to those in fixed interest rate securities
markets, for example in respect of the length of the year (i.e. 360 or 365 days).
market the swap's all-in cost is expressed as a basis point spread to
the semi-annual bond equivalent interest rate taken from the US Treasury yield
curve. For example, a ten-year LIBOR swap might be quoted as "Treasury
yield curve plus 50–55 basis points versus six-month LIBOR flat". This
means that a dealer would enter into the swap to receive six-month
LIBOR flat and make fixed payments over the life of the swap at a rate 50
basis points above the semi-annual bond equivalent yield taken from the
Treasury yield curve at the ten-year maturity. Alternatively, the dealer
would offer to receive fixed (and pay floating) at 55 points above the yield.

The spread on a new swap can be non-zero if at least one of the parties
to the swap cannot borrow at the risk-free rate. This can be seen in the
following example. Suppose that the borrowing rates in the securities
market of one of the two counterparties to the swap are \( r_1 + h \) and \( r_2 + H \), where \( r_1 \) and \( r_2 \) are the benchmark risk-free short-term (Treasury bill)
and long-term (Treasury bond) interest rates respectively, and \( h \) and \( H \) are
the risk premia on the two securities. In this example, the par value of
issuing the two risky securities is identical but the risk premium over the
benchmark interest rate varies with maturity. If the floating payments
under the swap are based on \( r_1 \) and \( r_2 \), then the swap spread will be the
floating rate flat against the fixed rate plus \( H-h \).

The value of \( H-h \) can be interpreted as the default premium of longer-
term borrowing over the short-term benchmark rate. In other words, the
value of the spread is that which equates the cost of borrowing in the fixed
rate market with the cost of taking up floating rate funds and swapping
them with funds carrying a fixed rate. This equality can also be seen as the
outcome of borrowing at \( r_1 + h \), investing the proceeds at \( r_2 + H \) and
evaluating the return in terms of \( r_1 \) and against \( r_2 \) plus a spread. The
example also illustrates that swaps have a value only if \( H \) and \( h \) differ.\(^\text{13}\)

\(^{12}\) LIBOR is the most commonly quoted floating rate index in the US dollar market. Other
indices employed in US dollar interest rate swaps are the commercial paper index, the US Treasury
bill rate and the US prime rate.

\(^{13}\) It is also possible for a "spread" to arise in a swap between two riskless counterparties for
technical reasons. For example, if there are lags in determining the coupon rule used to set the
variable interest rate payments, i.e. if the interest payment rule does not match the maturity of the
investment in the short-term security to which the floating interest rate is tied, there will be an
interest rate mismatch or "basis risk". This will typically occur if the coupon rule is based on an
average of previously issued short-term securities. An analogous type of basis risk may arise if the
relative value of the floating and fixed rate instruments deviates from par owing to call provisions or
special features such as warrants or restrictive covenants attached to one of the securities.
2.3 Short-term interest rate swaps and Euro-dollar futures

The ability of intermediaries to accept a swap contract without an immediate matching transaction, or to manage a portfolio of swaps depends on the existence of other instruments which allow dealers to hedge their swap exposures. Hence, the valuation of swaps will be tied closely to the price of other instruments whose cash flows can be used to replicate those of a swap. In particular, the structure of forward rates implicit in the swap must in equilibrium conform with the market view in order to avoid arbitrage taking place with other markets.

The primary influence on shorter-dated swaps is the hedging costs in the Euro-dollar futures markets. Euro-dollar futures contracts which specify a forward rate on a three-month time deposit beginning on a specific future date can be used to hedge the exposure on short and medium-term (under three years) interest rate movements. A series or “strip” of futures contracts for different maturities creates the same exposure as an interest rate swap. For a given series of quarterly LIBOR-related payments, a strip consisting of the sale of Euro-dollar contracts with successive expiration dates would effectively lock in a fixed interest rate: an increase in LIBOR would be offset by the decline in the price of the Euro-dollar futures contract and vice versa.

Since LIBOR-based swaps perform much the same function as strips of interest rate futures, the yields obtained through the two instruments should be roughly aligned. However, there are quantitative and qualitative differences between the two instruments.\(^{14}\) These differences are analogous to those which exist between forward contracts and futures. Firstly, there is a “basis risk” in replicating a swap with a strip of futures contracts. This basis risk arises because futures contracts expire at specific dates (viz. the third Wednesday of March, June, September and December) whereas swaps, like forward contracts, are arranged for fixed maturities at varying expiration dates. Hedging a swap with a strip of futures may also entail some degree of basis risk if the interest rate on the Euro-dollar contract does not converge to LIBOR. Secondly, in the presence of random interest rates and with constant daily resettlement of futures contracts (“marking to market”), the prices of futures and forwards should not be

\(^{14}\) The practical problems involved in setting up a Euro-dollar strip hedge are discussed in Kawaller (1989).
Graph 1
Two-year interest rate swap and synthetic spreads
from strips of Euro-dollar futures*
In basis points; weekly averages

* The synthetic spread is the fixed rate created with a strip of Euro-dollar futures contracts (closing prices) expressed as a spread over US Treasuries. Transaction costs are not included. Both swap and synthetic spreads represent mid-market levels.


equal (see Cox, Ingersoll and Ross, 1981). Empirically, however, the prices of forward and futures contracts does not appear to have diverged markedly, at least as far as other markets, such as the foreign exchange market, are concerned (see Cornell and Reigenaum, 1981). Finally, a strip of futures carries little or no default risk in comparison with swaps, since clearing house risk in the futures markets is smaller than that of over-the-counter transactions. In addition, temporary differences in liquidity may affect arbitrage between the two markets.

In the light of these factors it is interesting to observe that the absolute swap rate and the synthetic fixed rate into which a LIBOR liability can be hedged using Euro-dollar futures contracts (expressed as a spread over
Treasuries) have converged markedly since the mid-1980s. Graph 1 illustrates that the differential between the two-year interest rate swap spread and the synthetic spread has narrowed from 40 basis points at the beginning of 1985 to on average less than 1 basis point at the end of September 1987.\textsuperscript{15} At present, two-year swap spreads are forced by dealers to converge within 5 basis points of the synthetic spread.\textsuperscript{16} Transaction costs and minor differences in the time of day at which swaps and synthetic spreads are priced remain the principal factors accounting for the differential between swaps and a strip of futures.

2.4 Longer-dated swap and bond prices

The prices of longer-dated swaps, the maturities for which futures contracts are not available, are determined on the basis of other instruments. As already mentioned, the relative credit ratings in the fixed and floating rate markets provide the incentive for interest rate swaps. At the same time, the prices of other instruments set the boundaries within which the swap rate can vary. Generic interest rate swaps have for the most part been arranged between highly-rated corporate counterparties which have access to funds in the longer-term markets and lower-rated counterparties which raise funds at floating rates, often in the international banking markets. Owing to risk premia associated with default experience in the bond markets, fixed rate funds tend to require a wider quality spread between higher and lower-rated counterparties than the floating rate markets.

The exact boundaries within which the longer term swaps fall are described in the following example. Suppose that a borrower with a lower credit standing were unable to access the longer-term fixed interest bond market without incurring a significant cost in terms of the spread (L) payable over a benchmark rate (T). Under these circumstances this borrower would end up seeking bank funds or another form of short-term indebtedness instead of longer-term funds. If swap prices are convenient the borrower with lower credit standing would seek to borrow at short

\textsuperscript{15} Although it is difficult to assess its direct impact, the growth of the market for interest rate swaps has been amongst the major factors stimulating the expansion in the volume of trading in Euro-dollar futures. The most immediate effect has been to bring about a lengthening of the maturity of Euro-dollar contracts, which since July 1987 have been traded at up to three years.

\textsuperscript{16} I thank Mark Wilkinson of Salomon Brothers for drawing my attention to this development.
term and roll over the loan, paying a spread of $L$ over the benchmark rate, and reverse the terms of borrowing through the swap.

In order for the swap opportunity to be attractive to the borrower with higher credit standing, the after-swap floating rate financing must be less than the cost of funding in the floating rate market, i.e.

\[
\text{LIBOR} + \text{[after-swap margin for high credit standing]} \leq \text{LIBOR} + h
\]

\[
\text{LIBOR} + [(T + H) - (T + S)] \leq \text{LIBOR} + h
\]

\[
H - h \leq S
\]  

(1)

where $T$ is the relevant Treasury bond rate, $H$ and $h$ are the spread over $T$ and LIBOR respectively for the high rated borrower, and $S$ is the swap spread.\(^{17}\) In other words, the lower boundary on the swap spread is given by the difference in the spread over the benchmark indices for borrowers of high credit standing in the fixed and floating rate markets. Typically, the value of $h$ will be very low and possibly negative. Similarly, the borrower of lower credit standing will take up “synthetic” fixed rate funds if:

\[
T + \text{[after-swap spread for low credit standing]} \leq T + L
\]

\[
T + [(\text{LIBOR} + L + S) - (\text{LIBOR})] \leq T + L
\]

\[
L - l \leq S
\]  

(2)

where $L$ and $l$ are the borrowing costs of the lower-rated credit in the fixed and floating rate markets. Combining expressions (1) and (2), the boundary conditions for the swap spread are given by the following inequalities:

\[
L - l \geq S \geq H - h
\]

The lower boundary on the swap spread is given by the difference in the potential cost of funds for the borrower with the higher credit standing in the fixed and floating rate markets, whereas the upper bound is set by

\(^{17}\) LIBOR carries a risk premium relative to the Treasury bill rate.
the difference in the potential cost of funding for the borrower with the lower credit standing.\(^\text{18}\)

Although the incentive to undertake swaps becomes greater the wider the relative differential for the high and low-rated borrower in the fixed and floating rate markets, swap activity has for the most part been restricted to investment grade borrowers.\(^\text{19}\) As shown in Graph 2, and for earlier periods by Evans and Parente (1987), swap spreads have almost consistently remained within the 20–30 basis point range defined by the spread of AA and A-rated industrial bonds over US Treasury securities.\(^\text{20}\) Indeed, these boundaries roughly correspond to those within which the commercial banks can borrow.

3. Determinants of swap spreads

3.1 Default premia and equilibrium swap “spreads”

The valuation of swap rates depends on the pricing of H and h. Unfortunately, there are very few models of the credit risk on securities. One possible pricing formula for default premia that does not rely directly on the risk aversion of investors is the elaboration by Merton (1974, 1990) of the correspondence between options and corporate securities. The formula obtained for the risk premium obtained by Merton is derived directly from a modification of the Black-Scholes formula for call options.

Suppose a company has assets V which have been financed by two types of security. The first security is debt (a discount bond) which matures at time t and has a face value of D. The second security is a share with a claim to the residual value of the underlying assets after the bondholders have been paid off. The contractual obligation implicit in the share can be viewed as a call option on the assets of the company with an exercise price

\(^{18}\) By borrowing in the short-term market and swapping into a fixed interest rate liability, the lower-rated borrower is accepting the risk that its credit premium may rise in the future in the floating rate market. If the default risk premia were to rise the lower-rated borrower might be worse off than if it had not undertaken the swap.

\(^{19}\) Lower-grade issuers are also able to raise synthetic funds through the swap market but they must often provide some form of credit enhancement.

\(^{20}\) In order to take account of the different borrowing costs in the floating markets, the upper and lower bounds of the arbitrage tunnel should be adjusted to include the spreads over LIBOR for AA and A-rated borrowers. However, if anything, the bounds would tend to be wider since AA-rated borrowers tend to be able to obtain funds at LIBOR, if not at lower rates, in the commercial paper market whilst A-rated borrowers must take up funds at above LIBOR rates.
D and a time to expiration $t$. Bondholders lay claim to their contractual promised payment (the "exercise price") or, in the event of default, to the full value of the firm. The value of the risky discount debt ($D^R$) with a promised payment $D$ and period to maturity $t$ is given by

$$D^R (V, t, D) = VN(d_1) + De^{-rt}N(d_2)$$  \hspace{1cm} (1)

where

$$d_1 = \frac{\log (De^{-rt}/V) - .5\sigma^2 t}{\sigma t^{1/2}}$$

$$d_2 = -d_1 - \sigma t^{-1/2}$$

and where $N(.)$ is the cumulative normal distribution function, $\sigma$ is the volatility of the underlying assets and $r$ is the riskless interest rate. $N(d_2)$ can be interpreted as the probability (assuming risk neutrality) that the firm will be solvent at maturity. Accordingly, the second term in this expression is the discounted expected value of receiving the promised payment $D$. The first term is the present value of receiving all the assets of the company conditional on $V$ being worth less than $D$. The yield to maturity on this bond ($Y$) is the solution to the problem $D^R = De^{-Yt}$. The risk
premium or spread $H$ is the difference between the yield ($Y$) and the riskless rate ($r$). From (1) the spread can be derived as

$$H = H(t,r,\sigma,De^{-rt}/V) = -\frac{1}{t} \log \left[ N(d_2) + \frac{V}{D} e^{-rt} N(d_1) \right]$$  \hspace{1cm} (2)

The comparative statics for this expression of the credit risk structure (see Merton, 1974) reveal that the risk premium $H(.)$ increases with the volatility of the company’s share price ($\sigma$) and the measure of “leverage” ($De^{-rt}/V$) but declines with the level of the riskless interest rate ($r$). A longer term to maturity, however, may imply an increasing or decreasing spread depending on the value of other parameters. With a high level of leverage the spread decreases with maturity; this reflects the “maturity crisis” of a debt issuer that is extremely likely to default on its maturing debt. In the more common situation where leverage ratios are much lower the yield spread initially rises with maturity but then levels off.

There are three problems with extending this analysis to the pricing of interest rate swaps. Firstly, the value of variable rate payments of the swap cannot be easily modelled in this framework. If the reset and payment dates occur in the future, the value of the riskless interest rate is stochastic, whereas in this framework the riskless interest rate is given. This is important because the measure of leverage in terms of the face value of the security is uncertain. Moreover, the volatility as well as the level of the riskless interest rate will affect the size of the default premium. Secondly, swaps comprise a strip of payments like a coupon bond. Whilst riskless coupon bonds can be valued as a portfolio of zero coupon payments, the same cannot be done with risky bonds because the payment of one coupon lowers the value of assets available for the payment of future coupons. As a result, there is an interdependency between coupon payments at successive dates. Finally, the option analogy cannot be fully carried over to the bond market because the underlying risk of the assets ($\sigma$), which is taken as exogenous to the writer or purchaser of call options, in the case of companies is not independent of the behaviour of shareholders and managers.

The latter is particularly important for determining the relationship between the maturity of the bond and the level of the expected default

\footnote{The modification of the Merton model suggested by Longstaff and Schwartz (1992) permits an easy extension to coupon-paying securities.}
premium. If there are agency problems in monitoring managers and shareholders the short-term rollover of credits should provide a mechanism for a constant reassessment of their risk-taking behaviour by the market. With longer-term bonds monitoring by bondholders becomes more difficult. This assumption tends to coincide with a priori reasoning: as the length of maturity grows shorter a degree of uncertainty is resolved. It also seems to coincide with casual evidence of ex-post default such as that shown in Graph 3 overleaf.\textsuperscript{22}

3.2 The yield curve and the default premium

Another aspect of the structure of default premia on swaps which is not captured by the Merton model is the relationship with the level and term structure of the riskless interest rate. The absolute level and the shape of the term structure may affect the value taken by the default risk premium by changing the perceived ability of borrowers to service their debt. As far as the level is concerned, high interest rates raise the probability of default and hence the default risk of both counterparties to the swap.\textsuperscript{23} However, it is not clear what the effect would be on swap spreads. As shown in our example, with parallel shifts in the yield curve (i.e. if both \( H \) and \( h \) rise by the same amount) the swap spread would remain unchanged.

The slope of the yield curve might also affect the level of the swap spread by altering the market perception of the relative ability of fixed and floating rate payers to service their debt. If the term structure of riskless rates is upward-sloping, market expectations are that the party paying fixed and receiving floating will pay out early in the term of the swap and receive net payments in later periods. This means that ceteris paribus the default risk of the floating rate payer is higher than that of the fixed rate payer if the term structure is upward-sloping. As a result, one would expect the default premium, and hence the swap spread, to vary inversely with the slope of the yield curve.

\textsuperscript{22} Fama (1986) presents evidence suggesting that the expected default premium tends to decline with maturity for money market securities. As already mentioned, one possible explanation is that short maturities may be riskier if the borrower faces difficulties in refinancing and meeting final redemption payments during redemptions — a “crisis at maturity”. This view is consistent with Fama’s finding that the expected default premia on longer-maturity securities are lower (and often negative) during recessions than during good times. It is more difficult to reconcile the “crisis at maturity” explanation with declining default premia during upswings in the business cycle.

\textsuperscript{23} This discussion assumes that nominal and real interest rates are equal.
Graph 3
Levels of default for various categories of borrower*
In percentages

* The percentage of borrowers defaulting on coupon or principal payments in the US bond market.
Source: Moody’s Financial Services.

3.3 Other determinants of swap “spreads”

Four other factors may interact with the expected default premium to affect the level of the swap spread over time. Firstly, if interest rates are subject to some form of reversion to a mean value, different levels of the mean to which securities tend to revert give rise to different values of long-term interest rates. In other words, since short-term risky rates (LIBOR + short-term spread) exceed the riskless rate (Treasury bill), for a constant default premium the term structure of private bonds need not lie above that for riskless securities by a constant amount. This result can easily be shown in an equilibrium stochastic model of the term structure such as that of Cox, Ingersoll and Ross (1985).
Secondly, the structure of the swap market is different from that for US Treasury securities. Pricing is less tight and quotes may be less indicative of the level at which transactions actually take place. The absence of depth and liquidity, particularly at specific periods, also means that pressures arising from changes in expectations in respect of the term structure may have a different impact on the Treasury securities markets than on that for long-dated swaps. As described in the next section, this means that the swap spreads will vary over time according to macroeconomic developments.

Thirdly, regulatory and tax factors may generate arbitrage opportunities which can be exploited through swaps and be capitalised into the spread. Finally, the spread on interest rate swaps may be driven by differential information or expectations, or by hedging needs. These asymmetries give rise to temporary supply and demand imbalances which would tend to be less systematic than other factors.

4. Characteristics of swap prices

4.1 The data

The sample of swap spreads used in this study consists of daily bid and offer prices of fixed/floating US dollar interest rate swaps for maturities of two, five, seven and ten years over the period July 1987 to July 1992. The data were posted by the money broker Fulton-Prebon at 11 a.m. standard eastern time and reported by Data Resources. The spreads are quoted on the basis of three-month LIBOR flat versus the Treasury security with a maturity equal to that of the swap.

The data exclude holidays and no interpolations were made for weekends. The two-year maturity was chosen as representative of short-term swap activity. At the longer end, the five, seven and ten-year maturities were chosen because ISDA data and other market information suggested that these were the representative contracts with most active trading.

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24 See, for example, Das (1989), pp. 255–256.
25 The empirical evidence for the existence of segmentation between the markets for government bonds and other securities is mixed. While there is some empirical evidence of segmentation for municipal bonds there is less evidence for corporate securities (see Jaffee (1975) and Cook and Hendershott (1978)).
### Table 2
**Characteristics of mid-rates of daily swap spreads**
*July 1987—July 1992*

<table>
<thead>
<tr>
<th>Item</th>
<th>Maturity of swap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 years</td>
</tr>
<tr>
<td>Average spread</td>
<td>54.8</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>18.2</td>
</tr>
<tr>
<td>Minimum</td>
<td>15.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>120.0</td>
</tr>
<tr>
<td>Significantly &gt; mean</td>
<td>34</td>
</tr>
<tr>
<td>Significantly &lt; mean</td>
<td>24</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.00</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.23</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td></td>
</tr>
<tr>
<td>at lag 1</td>
<td>0.988</td>
</tr>
<tr>
<td>at lag 2</td>
<td>0.978</td>
</tr>
<tr>
<td>Cross-correlations</td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td></td>
</tr>
<tr>
<td>5-year</td>
<td>0.803</td>
</tr>
<tr>
<td>7-year</td>
<td>0.794</td>
</tr>
<tr>
<td>10-year</td>
<td>0.742</td>
</tr>
</tbody>
</table>

**Memorandum:**
- Number of observations: 1,259

* Statistics based on daily swap rates expressed in basis points. Significantly > mean or < mean = number of observations different from mean by two standard errors. Skewness is measured as the third moment around the mean divided by the variance to the power of 3/2: positive (negative) values indicate skewness to the right (left). Kurtosis is defined as the fourth around the mean divided by the variance squared minus three.

**Sources:** Fulton Prebon and own calculations.

---

ISDA data for the second half of 1991 indicate that 33, 13, 4 and 4% of new US dollar interest rate swaps with a maturity exceeding one year were at these maturities.

Like all over-the-counter markets, the interest rate swap spreads quoted by a single dealer may not be representative of market prices. In order to examine this potential bias two and ten-year swap spreads quoted by Fulton-Prebon for end-week were compared with end-week spreads published by Salomon Inc. According to a two-tailed F-test the samples were not significantly different, with the maximum difference between the two series being about 10 basis points.
Graph 4

**Interest rate swap spreads over US Treasuries**
Spreads over Treasury bonds of equivalent maturity, in basis points; monthly averages

Source: Fulton Prebon.

4.2 The evolution of swap spreads since mid-1987

Table 2 provides summary statistics of the average of bid and offer rates for swap spreads over Treasury securities of different maturities for the period between July 1987 and July 1992, or a total of 1,259 trading days. During this period the average spreads varied from 55 basis points for two-year swaps to 72 basis points for ten-year swaps. Under a two-tailed t-test at a 1% confidence interval the two-year swap spread was lower than the spreads for all the longer-term swaps. However, the short-term spreads have at times been above longer-term spreads and the levels of the spreads also appear to depend on the sample period considered.\(^{26}\) As shown in Graph 4, the periods for which the two-year swap spread was higher than longer-term spreads occurred at the time of the stock market

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\(^{26}\) Evidence for earlier years is presented in Evans and Parente (1987). See also Graph 1 above for periods since 1987 in which short-term swap spreads exceeded longer-term spreads and Section 5 below for possible explanatory variables.
crisis (October 1987), a period of high volatility in financial markets, and in the period of inverted yield curves during the first half of 1989 and briefly in mid-1991.

There appears to be a positive correlation between the size of the swap spread and maturity, but there is no statistically significant difference amongst spreads for maturities of five or more years. The higher correlation between spreads on longer-term swaps may be due to the fact that in terms of present value the additional year of risk for assets with longer-term maturity is discounted more heavily at a longer maturity than at the shorter end. This means that the change in spread between a two and a three-year swap should exceed that between a three and a four-year swap. Moreover, differences in credit risk may be more easily perceptible between securities with two and five-year maturities than between those with maturities of five and ten years.

There are several other significant differences between the two-year and longer-term swap spreads. Firstly, as shown in Table 2 and Graph 4, the variability of the swap spread for two years is greater than that for longer maturities. In particular, the range of values over which the two-year swap spread has varied encompasses that of longer-term swaps. The volatility of the spread in the two-year swap market reflects the volatility of interest rates in the money markets. As already mentioned in the previous section, speculative and arbitrage activity characterise this segment of the market with a considerable amount of three-way arbitrage between the relevant Treasury note market, Euro-dollar futures and swaps.

Secondly, the distribution of swap spreads around the mean for two-year and longer-term swaps differs markedly. The distribution of longer-term swaps appears skewed to the left, with a high percentage of observations falling outside the range of two standard deviations. In addition, the measure of excess kurtosis shows that two-year swaps appear to be characterised by "fat tails", whereas longer-term swaps have a flat distribution.

Finally, the difference between the two-year and longer-term spreads is also clearly reflected in the value of the cross-correlations between the various spreads. The cross-correlation between the two-year and other swap spreads never exceeds 0.8, whereas all other cross-correlations are well over 0.95.

As shown in Graph 5, the swap spread has varied significantly over time and the level appears to be correlated with the slope of the yield curve.
Graph 5

Comparison of interest rate swap spreads and the slope of the yield curve

Spreads over Treasury bonds of equivalent maturity and yield differentials, in basis points; monthly averages

Source: Fulton Prebon.
Table 3
Swap spreads and the Treasury bond yield curve
July 1987–July 1992; in basis points

<table>
<thead>
<tr>
<th>Maturity of swap spread</th>
<th>Humped yield curve(^1) ((N=191))</th>
<th>Monotone yield curve ((N=1068))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 years(^2)</td>
<td>65.1</td>
<td>53.0</td>
</tr>
<tr>
<td>5 years(^2)</td>
<td>70.9</td>
<td>67.3</td>
</tr>
<tr>
<td>7 years(^2)</td>
<td>71.3</td>
<td>68.8</td>
</tr>
<tr>
<td>10 years(^2)</td>
<td>75.2</td>
<td>71.4</td>
</tr>
</tbody>
</table>

\(^1\) Dates on which the semi-annual yield of ten-year Treasury bonds was below that of the equivalent two-year Treasury bond. \(^2\) Swap spreads for humped and monotone yield curve significantly different at 95% level under two-tailed t-test.

Sources: Fulton Prebon and own calculations.

These impressions are confirmed in Table 3, which shows the average swap spread for periods in which the slope of the yield curve was negative ("humped yield curve") and upward-sloping ("monotone yield curve"). In periods during which the slope of the yield curve was positive, spreads were on average considerably higher than during periods in which the yield curve sloped downwards.\(^{27}\) This finding suggests a negative correlation between the term and default premia.

Table 4 presents a further analysis of the time series properties of the interest rate swaps in terms of daily-compounded returns which have been calculated as the first difference of the logarithm of daily observations. The returns change very markedly, occasionally varying by more than 30% on a single day; however, on balance they are not significantly different from zero. The daily percentage change in two-year swaps exhibits the greatest variability in terms both of the standard deviation and of the range of values observed, but only 6% of observations were beyond two standard errors. All the spreads exhibited a significant degree of autocorrelation,\(^{28}\) although of different order and, in the case of two-year swaps, of

\(^{27}\) Under a two-tailed t-test the difference between spreads for periods in which the yield curves were "monotone" and "humped" was statistically significant.

\(^{28}\) The test employed checked significance against twice the standard error. The standard error of the autocorrelation coefficient of order \(r\), \(r_n\), was measured according to Bartlett’s test:

\[
SE = \left(1 + 2 \sum \frac{r_i^2}{T} \right)^{1/2} \quad \text{where } T \text{ is the number of observations.}
\]
Table 4  
Characteristics of daily percentage changes of mid-rates of swap spreads*  
July 1987–July 1992

<table>
<thead>
<tr>
<th>Item</th>
<th>Maturity of swap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 years</td>
</tr>
<tr>
<td>Average daily change</td>
<td>0.067</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>5.742</td>
</tr>
<tr>
<td>Maximum</td>
<td>32.692</td>
</tr>
<tr>
<td>Significantly &gt; 0</td>
<td>44</td>
</tr>
<tr>
<td>Significantly &lt; 0</td>
<td>32</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.336</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td></td>
</tr>
<tr>
<td>at lag 1</td>
<td>-0.585</td>
</tr>
<tr>
<td>at lag 2</td>
<td>-0.035</td>
</tr>
<tr>
<td>at lag 3</td>
<td>-0.048</td>
</tr>
<tr>
<td>Cross-correlations</td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td>-</td>
</tr>
<tr>
<td>5-year</td>
<td>0.277</td>
</tr>
<tr>
<td>7-year</td>
<td>0.202</td>
</tr>
<tr>
<td>10-year</td>
<td>0.221</td>
</tr>
</tbody>
</table>

Memorandum:
Number of observations 1,259

* Statistics based on change in the logarithm of daily swap rates. Average returns and standard deviations are expressed in percentage terms per day. Significantly > 0 or < 0 = number of observations different from zero by two standard errors. Skewness is measured as the third moment around the mean divided by the variance to the power of 3/2; positive (negative) values indicate skewness to the right (left). Kurtosis is defined as the fourth moment around the mean divided by the variance squared minus three.

Sources: Fulton Prebon and own calculations.

Differing sign. As in the case of the absolute swap spreads, the cross-correlation between two-year and longer-term swap spreads was significantly lower than that across longer-term swaps.

The kurtosis measure shows a positive degree of leptokurtosis, meaning that too many observations occur around the mean and the tails compared with what would be expected for a normal distribution. The process generating the average swap spreads is characterised by relatively large deviations from the normal distribution in ways reminiscent of other
Table 5  
Summary statistics of bid-ask margins on US dollar interest rate swaps  
July 1987—July 1992; in basis points

<table>
<thead>
<tr>
<th>Maturity of swap</th>
<th>Average margin</th>
<th>Standard deviation</th>
<th>Range*</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 years</td>
<td>4.9</td>
<td>1.8</td>
<td>23.0</td>
</tr>
<tr>
<td>5 years</td>
<td>4.6</td>
<td>1.4</td>
<td>16.0</td>
</tr>
<tr>
<td>7 years</td>
<td>4.8</td>
<td>1.5</td>
<td>19.0</td>
</tr>
<tr>
<td>10 years</td>
<td>4.9</td>
<td>1.5</td>
<td>13.0</td>
</tr>
</tbody>
</table>

* Maximum—minimum bid-ask rate of swap spread against US Treasury securities of similar maturity.

Sources: Fulton Prebon and own calculations.

financial markets (see Fama, 1975). Finally, tests which are not reported in detail were carried out for potential weekend or day-of-week effects, but no evidence was found of systematic excess returns for changes in swap rates from one day to the next.

The difference between bid and offer rates on swap spreads exhibited a considerable degree of stability (see Table 5). The average spread between bid and offer rates varied from 4.6 basis points for five-year swaps to 4.9 basis points for two and ten-year swaps. The higher bid-offer spread on two-year swaps can be attributed to the greater volatility of two-year swaps which has already been discussed. Nevertheless, even in the case of the two-year swap, with the exception of the stock market break of 1987, the difference between the bid and offer rate remained within the bounds of one standard deviation from the mean of the bid and offer spread.

5. Estimating the determinants of swap spreads

5.1 Independent variables

The discussions in previous sections suggested several variables which might affect the determination of swap spreads: leverage, volatility of the asset values, the level of interest rates and the shape of the yield curve. It was also noted that long-term swaps could be interpreted as synthetic securities approximating the credit rating between AA and A bonds. Finally, casual observation of the data in the preceding section suggested
that swap rates have indeed often evolved in tandem with other financial variables.

This section examines the short and longer-term sensitivity of swap spreads against several basic explanatory variables without imposing any restrictions on the structure of estimated equations, except those suggested by the data series themselves. The econometric analysis was carried out on the weekly series of swap rates compiled from daily observations in order to eliminate the non-normality of the series.

The basic explanatory interest rate variables utilised in the regressions were the Treasury bond rate corresponding to the maturity of the swap, LIBOR, the "TED spread" and a measure of the slope of the yield curve (SLOPE). The TED spread — measuring the difference between the interest rate on Treasury bills and LIBOR — was used as a measure of the riskiness of LIBOR-based financing relative to the riskless government equivalent rate. Variations in the TED spread indicate changes in the perceived short-term credit rating of banks and this may be reflected indirectly in the level of the swap spread. As mentioned earlier, the relationship between the term structure of interest rates on government securities and swap spreads is complex since both the expected term and default premia are related to the business cycle. On the one hand, expected default premia should be higher during recessions, at times of sharp increases in interest rates or financial instability. On the other hand, term premia tend to increase with maturity when business activity is strong, whereas humps in the term structure of interest rates are common during recessions. The regression results presented below, where the slope of the yield curve is measured as the differential between interest rates on ten-year and two-year US government bonds, attempt to identify which of these hypotheses plays a stronger role in the determination of swap spreads.

The Standard and Poor 500 index (SP500) was included in the regression equations as a proxy for the creditworthiness of the corporate sector. Volatility (VOLA) of the stock market index was likewise included to proxy for greater counterparty risk. Higher volatility may also be considered to be associated with a greater dispersion of information amongst market participants.29 VOLA was measured as the rolling standard deviation of the daily change in the logarithm of the SP500 index.

29 See Arak et al. (1988).
5.2 Co-integrating restrictions and multivariate estimations

Testing for stationary series. The time series properties of the variables were examined in order to establish whether it was possible to identify a unit root in the individual variables. As shown in Table 6, for the levels of the series according to the Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests\(^{30}\) the null hypothesis of non-stationarity could not be

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF levels</th>
<th>ADF levels</th>
<th>DF first differences</th>
<th>ADF first differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2M</td>
<td>-2.236</td>
<td>-1.685</td>
<td>-16.557**</td>
<td>-6.017**</td>
</tr>
<tr>
<td>SSM</td>
<td>-1.018</td>
<td>-0.798</td>
<td>-15.059**</td>
<td>-5.698**</td>
</tr>
<tr>
<td>S7M</td>
<td>-1.040</td>
<td>-0.988</td>
<td>-14.115**</td>
<td>-5.880**</td>
</tr>
<tr>
<td>S10M</td>
<td>-0.915</td>
<td>-0.794</td>
<td>-15.902**</td>
<td>-5.287**</td>
</tr>
<tr>
<td>TR2</td>
<td>0.216</td>
<td>0.195</td>
<td>-14.935**</td>
<td>-4.719**</td>
</tr>
<tr>
<td>TR5</td>
<td>-0.716</td>
<td>-0.552</td>
<td>-15.546**</td>
<td>-4.641**</td>
</tr>
<tr>
<td>TR7</td>
<td>-1.101</td>
<td>-0.734</td>
<td>-16.480**</td>
<td>-4.548**</td>
</tr>
<tr>
<td>TR10</td>
<td>-1.644</td>
<td>-1.007</td>
<td>-16.490**</td>
<td>-4.616**</td>
</tr>
<tr>
<td>LIBOR</td>
<td>0.352</td>
<td>0.627</td>
<td>-17.379**</td>
<td>-4.646**</td>
</tr>
<tr>
<td>TBILL</td>
<td>0.524</td>
<td>0.346</td>
<td>-16.859**</td>
<td>-3.680*</td>
</tr>
<tr>
<td>SP500</td>
<td>-0.456**</td>
<td>-1.126</td>
<td>-17.015**</td>
<td>-6.021**</td>
</tr>
<tr>
<td>VOLA(^{5})</td>
<td>-5.491**</td>
<td>-5.805**</td>
<td>-11.208**</td>
<td>-7.929**</td>
</tr>
<tr>
<td>SLOPE</td>
<td>0.100</td>
<td>0.539</td>
<td>-18.463**</td>
<td>-5.218**</td>
</tr>
</tbody>
</table>

**Note:** The Dickey-Fuller (DF) test is based on the following regression:

\[
\Delta x_t = \beta_0 + \beta_1 x_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \epsilon_t
\]

The augmented \(\Delta_0 + \text{DF (ADF)}\) test is based on the following regression:

\[
\Delta x_t = \beta_0 + \beta_1 x_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \epsilon_t
\]

where \(\Delta\) is the first-difference operator and \(\epsilon\) is a stationary random error. The null hypothesis is that \(\beta_1\) is non-stationary series and it is rejected when \(\beta_1\) is significantly negative.

S2M, SSM, S7M and S10M are the average of the bid and offer swap spread over the US Treasury securities of equivalent maturity, where the number refers to the maturity of the swap in years. Similarly, TR2, TR5, TR7 and TR10 refer to the interest rates on US Treasury bonds. LIBOR and TBILL are the London Interbank Offered Rate and the Treasury bill rate respectively. SP500 is the Standard and Poor 500 index of share prices. VOLA refers to the 20-day standard deviation of daily changes in the SP500 index. SLOPE equals TR10 minus TR2.

\(^{**}\) Significant at the 1% level. \(^{**}\) Significant at the 5% level.

\(^{5}\) Some indication that stationarity could only be achieved by imposing a linear trend.

\(^{30}\) See Engle and Granger (1987).

34
rejected for most of the variables listed. The only variable appearing to display a linear trend (\(l(c) + \) linear trend) is VOLA, the standard deviation of the S&P 500 index. After first differencing, the null hypothesis of non-stationarity was rejected for all series.

Multivariate equations: long-run relationships. As shown in the top two sets of equations of Table 7 overleaf, a parsimonious representation of the long-run relationship between swap spreads and a small number of variables could only be achieved for two-year swaps. In the case of longer-term swaps, as can be seen from the bottom set of equations, a co-integrating long-term linear relationship was found with five explanatory variables -- the SP500 index, its volatility, the Treasury bill rate, the Treasury bond rate corresponding to the maturity of the swap, and the slope of the yield curve.\(^{31}\) Only in these cases was it not possible to reject the hypothesis of non-co-integration at the 5\% confidence level according to the DF and ADF tests (as extended by Engle and Yoo (1987) and Phillips and Ouliaris (1990)). In particular, other variables such as LIBOR and the “TED spread” did not appear to have any explanatory power in the long-term level of swap rates.

While the coefficients of the various independent variables differed across individual equations, their sign was consistently the same. As expected, the SP500 index, which is used as a proxy for the “leverage ratio” of market participants, has a negative value, indicating that the riskiness of swaps declines as the perceived creditworthiness of market participants increases. The volatility of this stock market index, which is taken as a proxy for the riskiness of financial markets, is positively correlated with the level of the spread. The negative coefficient on the slope of the yield curve has various interpretations. It may indicate that swap spreads tend to decline during boom periods, which themselves coincide with upward-sloping yield curves. It may also suggest that the riskiness of the fixed rate payer relative to the floating rate payer becomes lower because the upward-sloping yield curve reflects expectations of future increases in interest rates.

The coefficients on the Treasury bill rate as well as the reference Treasury bond yield corresponding to the maturity of the swap are more difficult to interpret. For longer-term swaps, higher interest rates appear

\(^{31}\) Individual variables were added one by one and in different sequential order.
## Table 7

### Cointegration regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant</th>
<th>Coefficients of:</th>
<th>R²</th>
<th>DW</th>
<th>DF</th>
<th>ADF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SP500</td>
<td>VOLA</td>
<td>TBILL</td>
<td>TR¹</td>
<td>SLOPE</td>
</tr>
<tr>
<td>S2M</td>
<td>10.600</td>
<td>-0.022</td>
<td>0.093</td>
<td>0.281</td>
<td></td>
<td>0.716</td>
</tr>
<tr>
<td>S5M</td>
<td>10.170</td>
<td>-0.018</td>
<td>0.104</td>
<td>0.309</td>
<td></td>
<td>0.628</td>
</tr>
<tr>
<td>S7M</td>
<td>10.420</td>
<td>-0.018</td>
<td>0.096</td>
<td>0.267</td>
<td></td>
<td>0.615</td>
</tr>
<tr>
<td>S10M</td>
<td>8.710</td>
<td>-0.015</td>
<td>0.010</td>
<td>0.418</td>
<td></td>
<td>0.613</td>
</tr>
<tr>
<td>S2M</td>
<td>3.835</td>
<td>-0.014</td>
<td>0.090</td>
<td>-0.530</td>
<td>1.221</td>
<td>0.754</td>
</tr>
<tr>
<td>S5M</td>
<td>18.176</td>
<td>-0.023</td>
<td>0.105</td>
<td>-0.582</td>
<td>-0.763</td>
<td>0.650</td>
</tr>
<tr>
<td>S7M</td>
<td>18.516</td>
<td>-0.024</td>
<td>0.100</td>
<td>0.506</td>
<td>-0.911</td>
<td>0.647</td>
</tr>
<tr>
<td>S10M</td>
<td>19.063</td>
<td>-0.024</td>
<td>0.104</td>
<td>0.615</td>
<td>-1.154</td>
<td>0.658</td>
</tr>
<tr>
<td>S2M</td>
<td>8.321</td>
<td>-0.015</td>
<td>0.091</td>
<td>-0.970</td>
<td>1.165</td>
<td>-0.994</td>
</tr>
<tr>
<td>S5M</td>
<td>28.679</td>
<td>-0.024</td>
<td>0.105</td>
<td>-1.115</td>
<td>-0.540</td>
<td>-3.393</td>
</tr>
<tr>
<td>S7M</td>
<td>28.365</td>
<td>-0.023</td>
<td>0.099</td>
<td>-1.116</td>
<td>-0.508</td>
<td>-3.196</td>
</tr>
<tr>
<td>S10M</td>
<td>27.431</td>
<td>-0.021</td>
<td>0.103</td>
<td>-1.002</td>
<td>-0.577</td>
<td>3.116</td>
</tr>
</tbody>
</table>

** ** Significant at the 1% level.  * Significant at the 5% level.

¹ Treasury bond rate corresponding to the maturity of the interest rate swap. ² Result for four lags of first differences of the error from cointegration equation. Box-Pierce and Lagrange multiplier tests on the residuals of the DF-test equation did not reveal autocorrelation of higher order.
### Table 8
**Error correction equations**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficients</th>
<th>T-statistics</th>
<th>Durbin-Watson</th>
<th>LM-test</th>
<th>Chow-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S2M = -0.0359 + 0.0088 \Delta SP500(t-2) + 0.0069 \Delta SP500(t-3) - 0.0291 \Delta VOLA(t-1) - 0.0275 \Delta VOLA(t-4)$</td>
<td>(-0.0286) (0.0045)</td>
<td>(0.0042) (0.0169)</td>
<td>(0.1883) (0.1898)</td>
<td>(0.1677) (0.0320)</td>
<td>0.0878 4.0470 2.0522 0.2882 0.0061</td>
</tr>
<tr>
<td>$\Delta S5M = -0.0419 + 0.0052 \Delta SP500(t-2) + 0.0074 \Delta SP500(t-3) - 0.0194 \Delta VOLA(t-2) - 0.1855 \Delta TR5(t-1)$</td>
<td>(-0.0172) (0.0025)</td>
<td>(0.0025) (0.0078)</td>
<td>(0.1912) (0.0936)</td>
<td>(0.2143)</td>
<td>0.1758 8.5705 1.9846 0.9248 0.0321</td>
</tr>
<tr>
<td>$\Delta S7M = -0.0316 + 0.0058 \Delta SP500(t-3) - 0.3527 \Delta TR7(t-1) - 0.2308 \Delta TR7(t-4) + 0.4705 \Delta SLOPE(t-1)$</td>
<td>(-0.0162) (0.0024)</td>
<td>(0.1199) (0.1041)</td>
<td>(0.0952) (0.0843)</td>
<td>(0.2049)</td>
<td>0.14716 7.1133 1.8580 0.92649 0.0331</td>
</tr>
<tr>
<td>$\Delta S10M = -0.0367 + 0.0049 \Delta SP500(t-3) - 0.3053 \Delta TBILL(t-1) - 1.1944 S10M(t-2) - 0.6151 u(t-1)$</td>
<td>(-0.0160) (0.0023)</td>
<td>(0.1202) (0.5474)</td>
<td>(0.1894)</td>
<td>0.09736 7.7147 2.0526 0.93472 0.0342</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Figures in parentheses refer to standard errors.
to reduce swap rates. In the case of short-term swaps only the effects of the Treasury bill rate enter the co-integrating equation.

Multivariate equations: error correction. The residuals from the estimated co-integrating equations were used to obtain a short-run dynamic relationship for changes in the swap spread (Table 8). The error correction term significantly affects changes in the swap spreads for all maturities. The dynamics of most swap spreads also appear to be affected by short-term changes in LIBOR — a variable not appearing in the cointegrating relationships — but the sign and magnitude of the impact varied across the maturity of the swaps.

6. Conclusions

This study has examined the determinants of the valuation of interest rate swaps in the US dollar market. It has shown how the pricing of swaps is closely connected by arbitrage relationships at short maturities to the market for Euro-dollar futures and at longer maturities to US domestic industrial bonds with AA ratings. There is evidence that short-term swap spreads are more variable than those of longer-term swaps. These findings confirm that the pricing for standard swaps is connected to conditions in markets for closely substitutable instruments. Furthermore, because of these arbitrage relationships with private sector markets swap spreads can be interpreted as default premia over Treasury securities of equivalent maturity. The econometric modelling of swap rates suggested that spreads tend to increase with maturity but the term structure of spreads varies with that of yields on government bonds. When the term structure of interest rates is humped the differential between swap spreads of differing maturity tends to narrow.

In the long run swap spreads are related to the yield curve, the level of long and short-term interest rates, the business cycle (leverage) as measured by the SP500 index and the perceived riskiness of the private

32 A Chow test for parameter stability revealed that there was no major structural change in the structure of the short-term adjustment mechanism.
33 This paper has not examined a number of factors which might affect the pricing of individual swaps between a dealer and a final user on the premise that such factors do not explain inter-dealer spreads. Such factors include the comparative advantage of raising funds in different markets or differentials in quality assessments of individual borrowers in separate markets. For example, "established" and "lesser known" firms raise funds at different rates.
sector. However, the effect of changes in these variables on swap spreads varies markedly according to the original maturity of swaps.

The relationships uncovered by these estimations are a further illustration of the interconnectedness of various financial instruments and of the sensitivity of the pricing of swaps to the underlying credit standing of market participants. In many respects these findings are not surprising but they help to explain why swaps have come to play such a central role in present day risk management.
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