Inflation versus monetary targeting in a P-Star model with rational expectations

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Introduction

In its report on the elements of the future monetary policy strategy of the European System of Central Banks in Stage III of monetary union, the European Monetary Institute rejected the strategies of exchange rate targeting, interest rate targeting and nominal income targeting as inappropriate (EMI (1997)). Inflation targeting and monetary targeting are the remaining possible strategies for future monetary policy in Europe. The choice between these alternative strategies crucially depends on the properties of the monetary transmission mechanism in the common currency area to be created.

In order to investigate how inflation targeting and monetary targeting work and how they relate to the properties of the monetary transmission mechanism, an appropriate theoretical framework is needed. From a purely logical point of view, it seems to be essential to let the money stock play an active role, especially with respect to its influence on the future price level. Numerous models on which the analysis of inflation targeting rests do not meet this requirement.[1] In these models, the money stock is determined only passively on the basis of a money demand function which is given as a recursive element of the respective model. Thus, a monetary policy strategy aimed at controlling the money stock is inefficient *a priori.*

Against this background, the paper presents a stylised model of a small open economy drawing on the P-Star approach which is considered to be a more adequate reference framework. According to the P-Star approach, which is generally used in isolation in the relevant literature, inflation is considered to be a monetary phenomenon in the long run which results from an excessive money supply by the monetary authority. The approach is empirically motivated by the fact that there is a long-term relationship between the money stock and the price level.[2] In view of the importance of forward-looking behaviour on the part of economic agents for the transmission of monetary impulses, forward-looking rational exchange rate and inflation expectations are taken into account.[3]

The alternative monetary policy strategies are implemented within the theoretical model by specifying appropriate feedback rules for monetary policy. According to the realisation of the respective monetary policy target, i.e. the inflation target in the case of inflation targeting and the monetary growth target in the case of monetary targeting, the policy rules determine the nominal short-term interest rate which is regarded as the monetary policy instrument. By endogenising monetary policy a nominal anchor is obtained for the forward-looking expectations of economic

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1 See, for example, Blake and Westaway (1994, 1996a), Britton and Whitley (1997), Fagan and Vega (1997), Fillion and Tetlow (1994), Fuhrer and Moore (1995a, 1995b) and Fuhrer (1997).

2 See the empirical findings in Issing and Tödter (1995) and Tödter and Reimers (1994).

3 Blake and Westaway (1994, 1996a) deal with the role of inflation expectations for the operation of inflation targeting in particular.

agents that are directed towards future monetary policy.[4]

The model of a small open economy with rational expectations based on the P-Star approach is developed first in Section 1. Then, in Section 2, the operation of monetary targeting and inflation targeting is analysed on the basis of two simulation scenarios – a transitory aggregate demand shock and a transitory money demand or velocity shock. Section 3 concludes with a short summary of the theoretical findings. These are discussed subsequently with respect to problems of economic policy which arise when inflation or monetary targeting is to be put into practice. An appendix describes the method employed in solving and simulating the model.

1. The model

In the short run, inflation is determined by monetary as well as by real factors. It is generally agreed, however, that inflation is a purely monetary phenomenon in the long run. Relying on a stylised IS/LM model of a small open economy based on the P-Star approach with rational inflation and exchange rate expectations, these features of monetary transmission will be illustrated below.

1.1 The IS/LM framework

Except for home and foreign interest rates, the model is specified in log-linear form:

$$y_t - y_t^* = \alpha_1(y_{t-1} - y_{t-1}^*) - \alpha_2(r_t - r_t^*) + \alpha_3\left[(e_t + p_t^f - p_t) - (e + p^f - p)_t^*\right] + \varepsilon_t^y \quad (1)$$

$$m_t - p_t = -\beta_1 i_t + \beta_2 y_t + \varepsilon_t^m \quad (2)$$

$$r_t = i_t - E_t[\Delta p_{t+1}] \quad (3)$$

$$\Delta p_t = \gamma_1 E_t[\Delta p_{t+1}] + (1-\gamma_1)\Delta p_{t-1} + \gamma_2(p_t^* - p_t) \quad (4)$$

$$e_t = E_t[e_{t+1}] + i_t^f - i_t \quad (5)$$

Here, y denotes real output, r the real interest rate, e ($e + p^f - p$) the nominal (real) exchange rate, p (p^f) the (foreign) price level, m nominal money, i (i^f) the (foreign) nominal interest rate and ε^y (ε^m) aggregate demand (money demand) shocks. Equilibrium values are marked with a star. All parameters are restricted to be non-negative. Furthermore, $\alpha_1, \gamma_1 < 1$ is assumed to hold.

Aggregate demand as well as money demand shocks follow a first order autoregressive process

$$\varepsilon_{t+1}^j = \rho^j \varepsilon_t^j + \eta_{t+1}^j, \qquad \left|\rho^j\right| < 1 \text{ for } j = y, m \quad (6)$$

where η^j are serially uncorrelated innovations with expectation zero realised in transition from period t to period t+1.

Finally, Δ denotes the difference operator and $E_t\,[\cdot] \equiv E\,[\cdot\,|\Omega_t]$ the expectation operator conditional on the information set Ω_t available in period t with $\Omega_t \supseteq \Omega_{t-1}$. The information set Ω_t contains the realisations of the exogenous variables and past endogenous variables which are

4 The necessity of endogenising monetary policy to guarantee the determinacy of the model solution in the presence of forward-looking expectations was recently re-emphasised by Fisher (1992), Chapter 6, and by Blake and Westaway (1994, 1996a).

predetermined.[5]

Equation (1) describes the deviation of real output y_t from equilibrium output y_t^*, i.e. the output gap $y_t - y_t^*$, which is determined by aggregate demand in the short run. The output gap depends on the output gap of the last period, on the deviation of the real interest rate r_t from the equilibrium real interest rate r_t^*, on the deviation of the real exchange rate $e_t + p_t^f - p_t$ from the equilibrium real exchange rate $(e + p^f - p)_t^*$, which is determined on the basis of purchasing power parity, and on the aggregate demand shock ε_t^y. In equation (2), real money demand $m_t - p_t$ depends on the nominal interest rate i_t, on real output y_t and on the money demand shock ε_t^m. As equation (3) shows, the real interest rate r_t is determined by means of the Fisher equation as the difference between the nominal interest rate i_t and the expected inflation rate $E_t\,[\Delta p_{t+1}]$.

In equation (4), the current inflation rate Δp_t is specified as a function of the *price gap* $p_t^* - p_t$ which measures the deviation of the price level p_t from the *equilibrium* price level p_t^*. Furthermore, it includes both a *backward-looking* element Δp_{t-1} which reflects the persistence of the inflation process to be observed in reality and a *forward-looking* expectation element $E_t\,[\Delta p_{t+1}]$ which is implicitly directed towards *future* price disequilibria as the cause of future inflation and which has an immediate effect on current inflation.[6]

The evolution of the nominal exchange rate e_t is governed by uncovered interest parity (equation (5)), i.e. arbitrage transactions of international investors lead to the equalisation of expected returns on home and foreign financial assets. The arbitrage transactions induced by the interest rate differential and the expectation of an exchange rate depreciation guarantee a continuous equilibrium in the international financial markets.

The specification of the inflation equation (4) guarantees the compatibility of the model with *any* equilibrium inflation rate.[7] In addition, as the nominal variables are homogeneous in the price level, the neutrality of monetary policy with respect to real variables holds in the long run.[8] The real interest rate and the real exchange rate assume their equilibrium values in the long run and, therefore, output cannot deviate permanently from its equilibrium value, i.e. the natural rate hypothesis holds.

As the factors underlying the real equilibrium values are not specified within the model, these are set equal to zero for the sake of simplicity, i.e. $y_t^* = r_t^* = (e + p^f - p)_t^* = 0$. For the same reason this simplification is carried out for the foreign variables, i.e. $p_t^f = i_t^f = 0$. The equilibrium price level p_t^* and the nominal interest rate i_t remain to be specified.

1.2 The P-Star approach

The determination of the equilibrium price level p_t^* and thus of the price gap $p_t^* - p_t$ is based on the P-Star approach.[9] The starting point is the equation of exchange (in logarithms) solved for p_t:

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5 For the definition of terms see the Appendix.

6 Fuhrer and Moore (1995a), for example, show in a model with a *staggered contracts* (real-) wage equation that a combination of backward-looking and forward-looking elements may generate a high degree of inflation persistence.

7 This is warranted by the restriction that the parameters of the expected future and of the past inflation rate add up to unity.

8 In the Appendix it is shown that the model has an equivalent representation in stationary real levels on account of its homogeneity in the price level.

9 For details see Hallman, Porter and Small (1991), Deutsche Bundesbank (1992), Tödter and Reimers (1994) and Issing and Tödter (1995).

$$p_t = m_t + v_t - y_t \qquad (7)$$

where v_t denotes *velocity*, i.e. the inverse coefficient of liquidity holdings.

The equilibrium price level p_t^* is defined as the price level which, for any given amount of money in circulation, is obtained when velocity and output assume their equilibrium values v_t^* and y_t^*:

$$p_t^* \equiv m_t + v_t^* - y_t \qquad (8)$$

It immediately follows from the preceding equations (7) and (8) that the price gap $p_t^* - p_t$ is composed of the output gap $y_t - y_t^*$ and the *liquidity gap* $v_t^* - v_t$:

$$p_t^* - p_t = (y_t - y_t^*) + (v_t^* - v_t) \qquad (9)$$

This decomposition illustrates that inflationary pressures exist not only when production capacity is excessively utilised but also when velocity is lower or liquidity holdings are higher than in equilibrium, i.e. a monetary overhang exists.

The liquidity gap is unobservable. However, according to Tödter and Reimers (1994) v_t can be obtained in terms of measurable quantities by replacing m_t in the equation of exchange by means of the money demand equation (2):

$$v_t = \beta_1 i_t + (1 - \beta_2) y_t - \varepsilon_t^m \qquad (10)$$

In view of this relationship, money demand shocks and *velocity* shocks are equivalent.

Analogously, equilibrium velocity v_t^* can be defined as a function of the equilibrium nominal interest rate i_t^* and the equilibrium output level y_t^*:

$$v_t^* \equiv \beta_1 i_t^* + (1 - \beta_2) y_t^* \qquad (11)$$

The equilibrium nominal interest rate is given by a Fisher-type identity $i_t^* \equiv r_t^* + (\Delta p)^*_{t+1}$ where $(\Delta p)^*_{t+1}$ denotes the future equilibrium *steady state* inflation rate.

Replacing the liquidity gap in equation (9) by means of equation (10) and (11), the price gap is given in reduced form by

$$p_t^* - p_t = -\beta_1 (i_t - i_t^*) + \beta_2 (y_t - y_t^*) + \varepsilon_t^m \qquad (12)$$

If the price gap in equation (4) is replaced in turn by equation (12), it is evident that the traditional Phillips curve is nested in the inflation equation. While the traditional Phillips curve traces price changes only back to existing output gaps, the liquidity gap within the P-Star model takes into account disequilibria in money holdings in addition to the output gap. These monetary disequilibria take effect on the *current* as well as on the *future* price level. The development of money holdings will become important for the transmission of monetary impulses only if these disequilibria are taken into account.

1.3 The monetary policy rule

The monetary policy rule determines the nominal interest rate and thus endogenises monetary policy. In general, a monetary policy rule may be specified as a feedback rule, according to which monetary policy reacts to deviations of a selected nominal target variable T from a given target value $T^{\#}$ by appropriately setting the nominal interest rate i given the equilibrium interest rate i^*.

Based on the work of Phillips (1954, 1957), this paper considers a general class of *simple* feedback rules:[10]

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[10] Instead of simple feedback rules, *optimal* feedback rules could be derived given the intertemporal loss function of a monetary authority. However, in the presence of forward-looking expectations the problem of time inconsistency of

$$i_t - i_t^* = \theta_P (T_t - T_t^{\#}) + \theta_I \sum_{\tau=1}^{t} (T_\tau - T_\tau^{\#}) + \theta_D \Delta (T_t - T_t^{\#}) \tag{13}$$

or after taking first differences and simple transformations:

$$\Delta i_t = \Delta i_t^* + (\theta_P + \theta_I + \theta_D)(T_t - T_t^{\#}) - (\theta_P + 2\theta_D)(T_{t-1} - T_{t-1}^{\#}) + \theta_D (T_{t-2} - T_{t-2}^{\#}) \tag{14}$$

(see Blake and Westaway (1994, 1996a)).

While the monetary policy rule is easily implemented using representation (14), the equivalent representation (13) offers an intuitive interpretation of the operation of this general class of policy rules. Representation (13) shows that the deviation of the current interest rate from the equilibrium interest rate depends on three components: the *proportional* (*P*–) component θ_P $(T_t - T_t^{\#})$ measures the feedback of the nominal interest rate on the *current* disequilibrium of the target variable, the *integral* (*I*–) component θ_I Σ_τ $(T_\tau - T_\tau^{\#})$ the feedback on the *cumulated* disequilibria, and the *differential* (*D*–) component θ_D Δ $(T_t - T_t^{\#})$ the feedback on the *change* in the disequilibrium.[11]

In view of the report of the European Monetary Institute on the alternative monetary policy strategies (inflation targeting and monetary targeting), policy rules are specified to control the inflation rate $(\Delta p)_t^{\#}$ or, alternatively, to control the growth rate of the money stock $(\Delta m)_t^{\#}$. As an extension of pure inflation targeting and pure monetary targeting, the deviation of current output from a target value $y_t^{\#}$ is additionally included in the specification of the policy rule. The target value $T^{\#}$ is then defined

(a) in the case of *pure* or *extended* inflation targeting as:

$T^{\#} \equiv (\Delta p)^{\#}$ or $T^{\#} \equiv ((\Delta p)^{\#}, y^{\#})'$

(b) in the case of *pure* or *extended* monetary targeting as:

$T^{\#} \equiv (\Delta m)^{\#}$ or $T^{\#} \equiv ((\Delta m)^{\#}, y^{\#})'$

The target variable T is defined accordingly. The parameters θ_P, θ_I and θ_D are scalars or vectors of dimension (1×2).

Due to the model's homogeneity in prices, the monetary policy rules guarantee that, given appropriate values of the parameters θ_P, θ_I and θ_D, any inflation rate $(\Delta p)^{\#}$ or money growth rate $(\Delta m)^{\#}$ is controllable. For their part, the target inflation rate and the money growth rate determine the equilibrium inflation rate $(\Delta p)^*$.[12] The policy rules impose restrictions on the time path of the inflation rate, but not on the time path of the price level. Nevertheless, the latter can be obtained recursively from the sequence of computed inflation rates given a starting value for the price level.

While the strategy of inflation targeting is aimed directly at the ultimate objective of monetary policy, the strategy of monetary targeting is directed towards controlling the money growth rate, which is an intermediate objective of monetary policy.[13] The target value of the money growth rate is given according to the equation of exchange (8) by:

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optimal monetary policy would be raised. This issue is not under investigation in the present paper. With regard to problems of time inconsistency, see, for example, Blake and Westaway (1994, 1996b).

[11] Specifically, the differential component increases the smoothness of the adjustment path of the target variable to its target level. See Phillips (1957) and Salmon (1982).

[12] Blake and Westaway (1994) show that, given the existence of steady state inflation, the integral component, but not the proportional and differential components, is necessary to control inflation. Taking into account the integral component, however, excludes any *base-drift* in monetary policy.

[13] See Haldane (1995) and Leiderman and Svensson (1995) for a discussion of the concept of inflation targeting and Deutsche Bundesbank (1995) for an exposition of monetary targeting.

$$(\Delta m)_t^{\#} = (\Delta p)_t^{\#} + \Delta y_t^{*} - \Delta v_t^{*}$$

or, if equation (11) is taken into account, by:

$$(\Delta m)_t^{\#} = (\Delta p)_t^{\#} - \beta_1 \Delta i_t^{*} + \beta_2 \Delta y_t^{*} \tag{15}$$

It can be shown within the present model that monetary targeting is a direct generalisation of inflation targeting. From the money demand equation (2) in first differences and the derived target for the monetary growth rate (15), the following relations are immediately obtained using equations (10), (11) and (9):

$$\begin{aligned} \Delta m_t - (\Delta m)_t^{\#} &= (\Delta p_t - (\Delta p)_t^{\#}) - \beta_1(\Delta i_t - \Delta i_t^{*}) + \beta_2(\Delta y_t - \Delta y_t^{*}) + \Delta\varepsilon_t^{m} \\ &= (\Delta p_t - (\Delta p)_t^{\#}) + \Delta(y_t - y_t^{*}) + \Delta(v_t^{*} - v_t) \\ &= (\Delta p_t - (\Delta p)_t^{\#}) + \Delta(p_t^{*} - p_t) \end{aligned}$$

Thus, monetary targeting not only reacts implicitly to a failure to achieve the current target inflation rate. It also responds to changes in the output gap and the liquidity gap, i.e. to changes in the price gap, which is the determinant of current as well as *future* inflation.[14] Hence, monetary targeting is already directed towards future inflationary pressures in a *forward-looking* manner.[15]

2. Model simulation

To illustrate the operation of inflation and monetary targeting within the P-Star model, two scenarios are investigated by means of impulse response analysis:

Scenario 1: A transitory aggregate demand shock.
Scenario 2: A transitory money demand or velocity shock.

The *impulses* of the dynamic system defined by the model are realised in transition from period $t = 0$ to period $t = 1$. The size of these impulses is equal to a unit of output and real money demand, respectively. The adjustment paths of the endogenous variables towards equilibrium, the *responses*, are reported for the periods $t = 1, 2, \ldots, 15$ in deviation from equilibrium[16, 17]

In the first stage, however, the model has to be parameterised. On the basis of empirical findings as well as considerations of plausibility and stability, the parameter values are calibrated at $\alpha_1 = 0.90$, $\alpha_2 = 0.25$, $\alpha_3 = 0.20$, $\beta_1 = 1.00$, $\beta_2 = 0.40$, $\gamma_1 = 0.90$ and $\gamma_2 = 0.20$. The parameter values of the autoregressive equations describing the transition of aggregate demand and velocity shocks are set equal to $\rho^y = \rho^m = 0.50$. As a result of these parameter values, output as well as money demand (in response to serially correlated velocity shocks) exhibit a relatively high degree of persistence. The values of the money demand elasticities are comparable with findings in empirical analyses of money

[14] It is obvious that monetary targeting is equivalent to controlling the equilibrium price level, i.e. P-Star: $\Delta m_t - (\Delta m)_t^{\#} = \Delta p_t^{*} - (\Delta p)_t^{\#}$.

[15] The *current* inflation rate, as a *non-predetermined* variable, reacts immediately to realised shocks, as expectations are formed in a forward-looking way. Therefore, the strategy of inflation targeting, as operationalised here, implicitly takes account of some elements of the *inflation forecast targeting strategy* proposed by Svensson (1997). Should the occasion arise, this strategy might be analysed within the P-Star model by including in the policy rule the deviation of the expected future inflation rate from the target inflation rate $E_t\,[\Delta p_{t+j}] - (\Delta p)^{\#}_{t+j}$, where j denotes the expectation horizon.

[16] The methods applied for solving and simulating the model are described in the Appendix.

[17] See Fillion and Tetlow (1994) and Blake and Westaway (1996a) for a description of running stochastic simulations with models under rational expectations.

demand based on broad monetary aggregates. The parameter values of the price equation guarantee that price disequilibria are removed fairly quickly without strongly oscillating adjustment paths.

The parameter values of the policy rule are uniformly chosen with $\theta_P = 0.50$, $\theta_I = 0.50$ and $\theta_D = 0.00$ for the inflation and the monetary growth target deviation as well as for the output disequilibrium. This assignment is motivated by the well-known *Taylor rule* which feeds back the deviation of the current inflation rate from the inflation target and the deviation of current output from equilibrium output to the deviation of the nominal interest rate from its equilibrium value with a parameter value of 0.50 each (see Taylor (1993)). Thus, the policy rule which underlies extended inflation targeting includes the Taylor rule as a special case. Note, however, that the Taylor rule takes into account only the proportional component of the general policy rule (14). The parameter θ_D of the differential component of the policy rule is uniformly set equal to zero as this component dampens the cyclical component of the adjustment paths which, as will be shown below, are already rather smooth without any further dampening.

Bearing in mind the controllability of any inflation rate and any money growth rate, a target inflation rate of $(\Delta p)^{\#} = 1.00\%$ and a target monetary growth rate of $(\Delta m)^{\#} = 1.00\%$ are assumed by way of example. Furthermore, if pure inflation and monetary targeting is extended by an output target, the target value chosen for output $y^{\#}$ is set equal to equilibrium output $y^* = 0$.[18]

2.1 Impulse responses to a transitory aggregate demand shock

Figure 1 below shows the impulse responses of selected endogenous variables to a positive transitory aggregate demand shock for pure inflation and pure monetary targeting. Because of the assumed steady state inflation of 1.00%, nominal levels are transformed into stationary real quantities by subtracting the price level. Both the equilibrium inflation rate and the equilibrium nominal interest rate are equal to one since the real interest rate is set equal to zero. The price gap, the real exchange rate and output are equal to zero in equilibrium, whereas real money demand is equal to minus one.

In response to the serially correlated demand shock a persistent price gap builds up. This induces an increase in the inflation rate, whose value is determined by the current and expected future price gaps in a forward-looking manner. According to the P-Star approach, the price gaps are composed of an output gap and a liquidity gap. Besides the output gap, the liquidity gap, in turn, depends on the deviation of the nominal interest rate from the equilibrium interest rate. In the case of inflation targeting, this deviation is determined by the deviation of current inflation from target inflation, and in the case of monetary targeting by the deviation of the monetary growth rate from its target value.

The real exchange rate, which is determined by the uncovered real interest parity, immediately appreciates. This appreciation as well as the change in the real interest rate feedback to output.[19] The responses of real money demand are determined by the nominal interest rate and by real output. The adjustments of the endogenous variables towards their (initial) equilibrium values take place with time-lags. These lags reflect the transmission mechanism of the model as well as the fact that the serially correlated demand shock diminishes only gradually.

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[18] Problems which result if the target value of output is higher than equilibrium output – in this case the realised equilibrium inflation rate is biased upwards – are discussed by Blake and Westaway (1994).

[19] The inflation rate and the real exchange rate are non-predetermined variables which immediately jump and put the dynamic system defined by the model on the saddlepoint stable adjustment path. See the exposition in the Appendix.

Figure 1

Impulse responses to a transitory demand shock

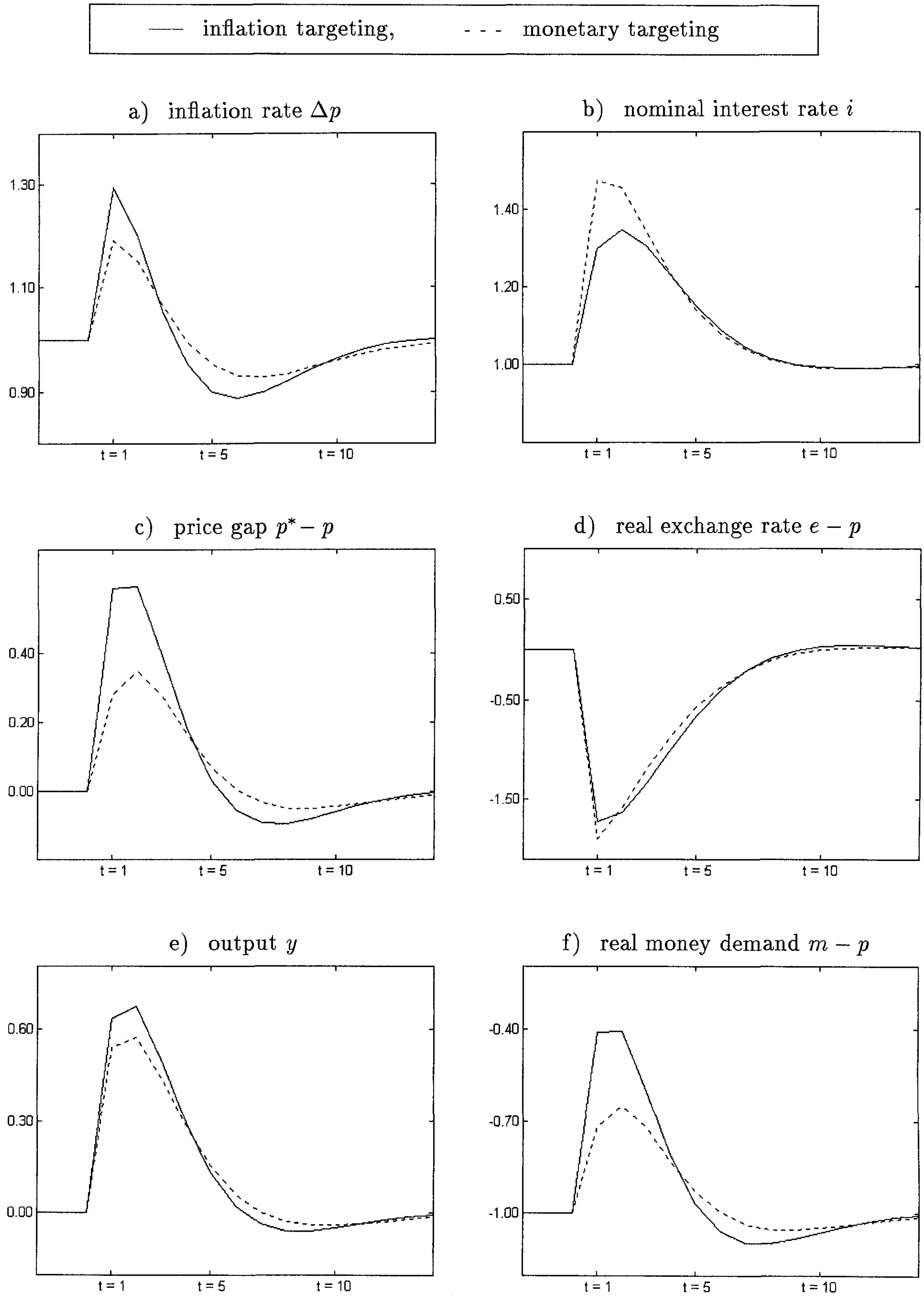


Figure 2

Impulse responses to a transitory demand shock

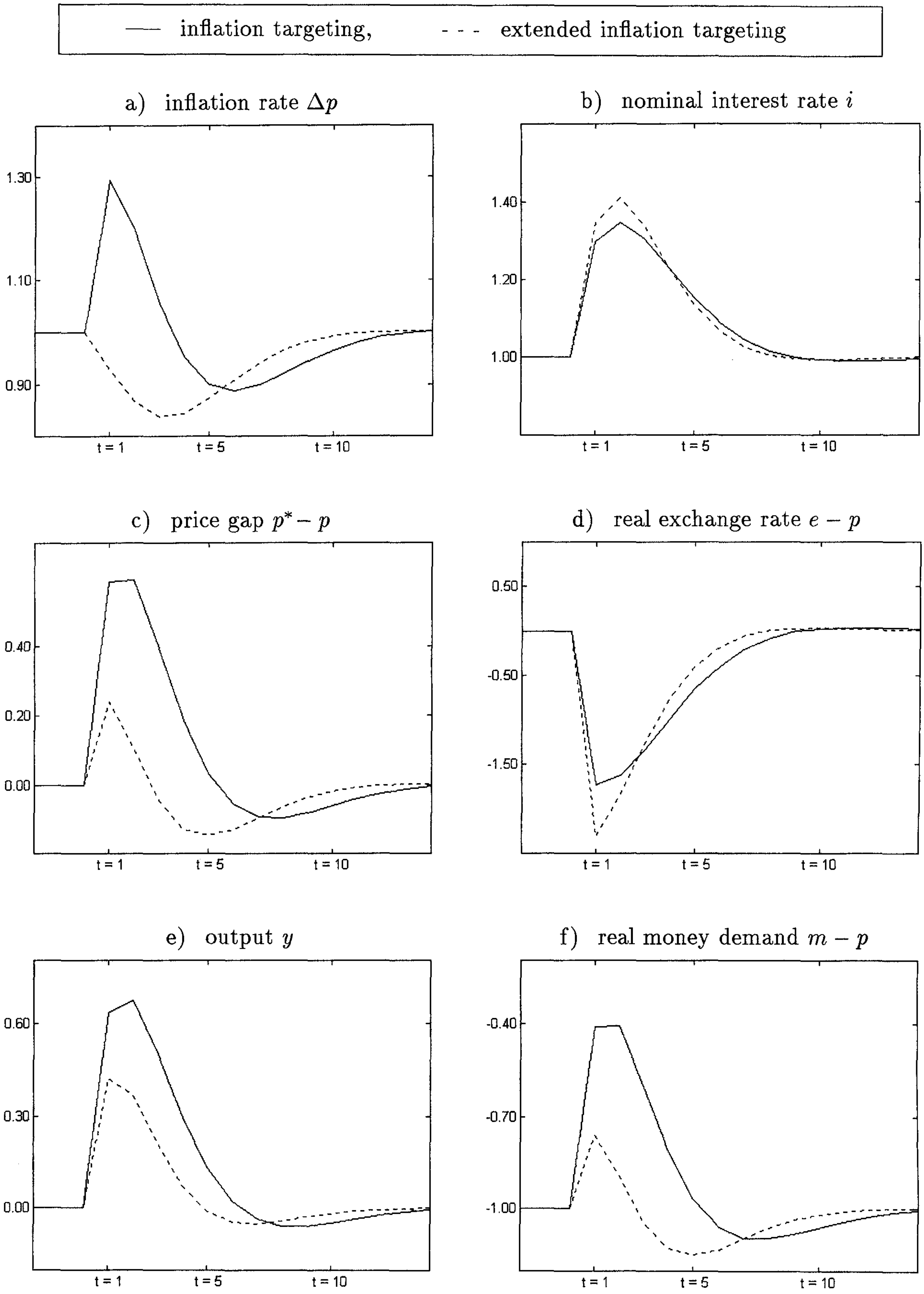


Figure 3

Impulse responses to a transitory demand shock

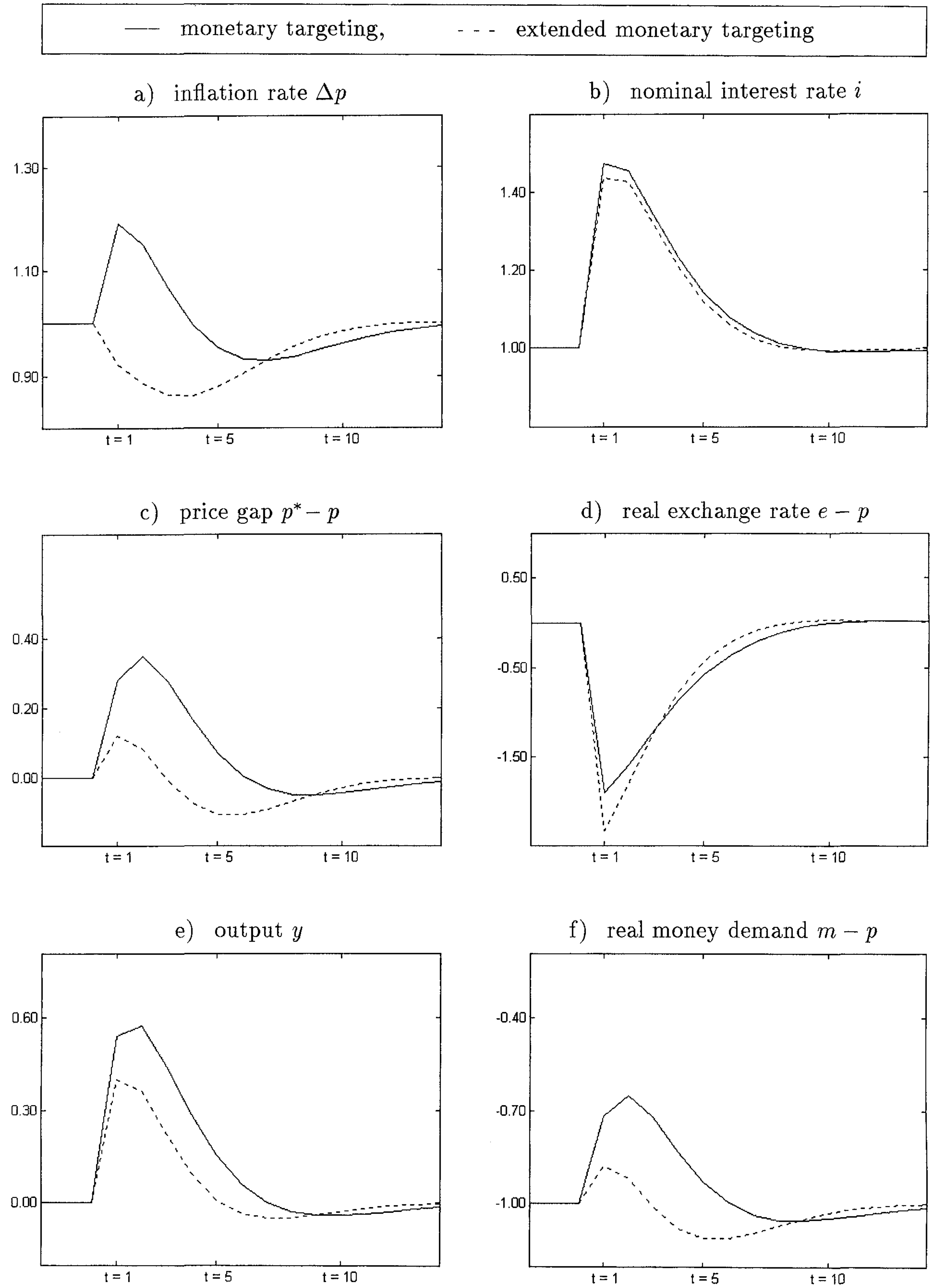


Figure 4

Impulse responses to a transitory velocity shock

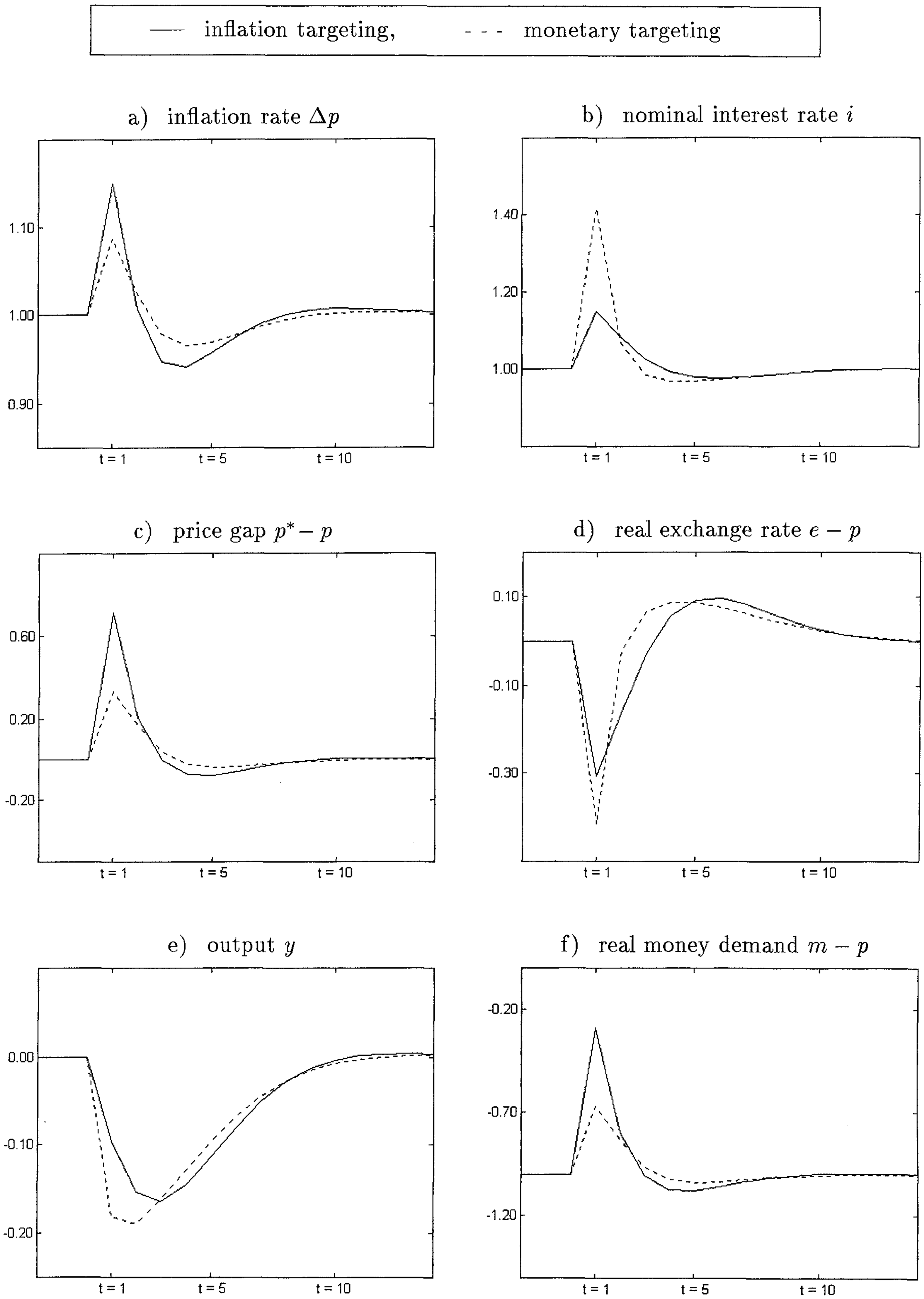


Figure 5

Impulse responses to a transitory velocity shock

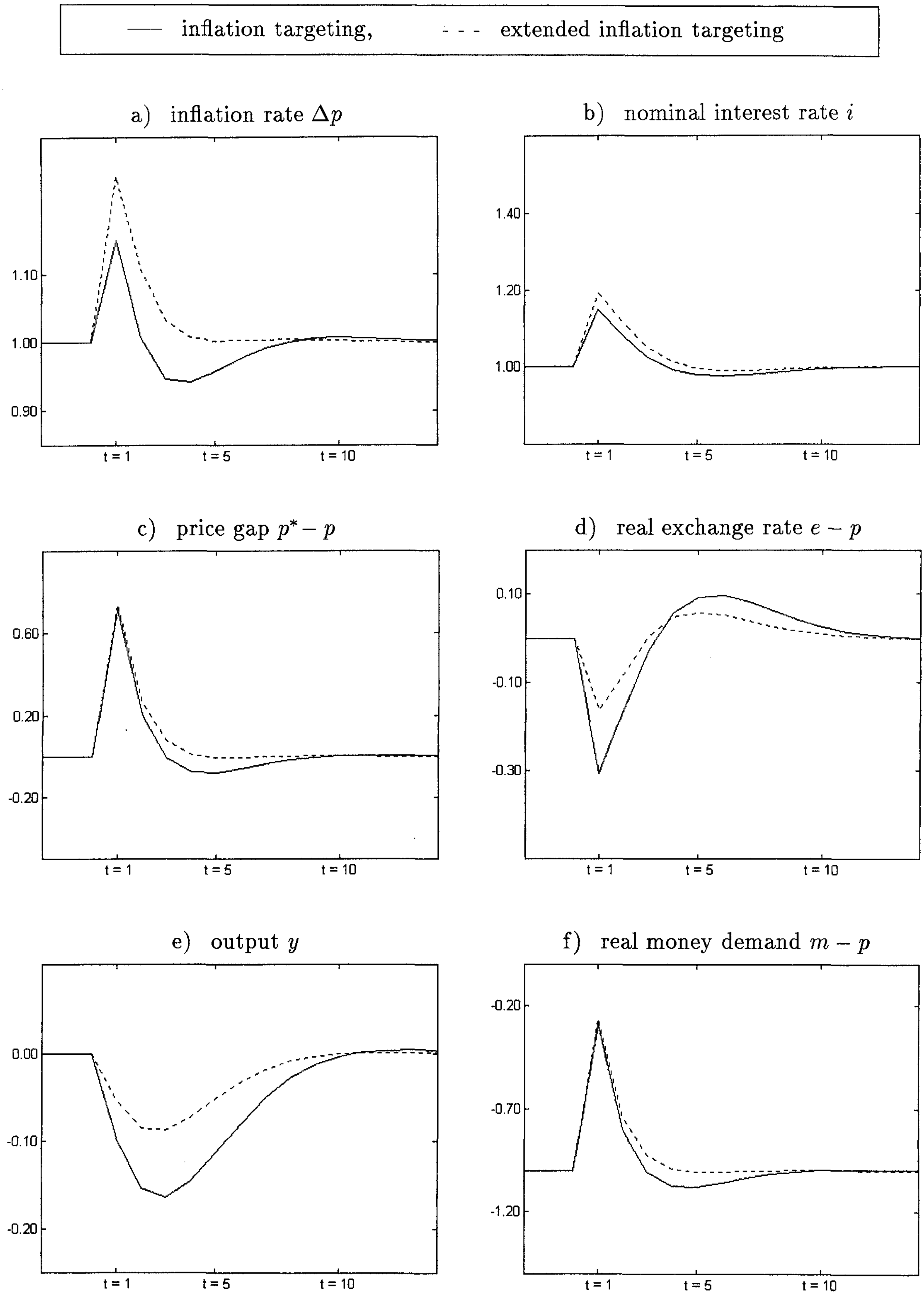
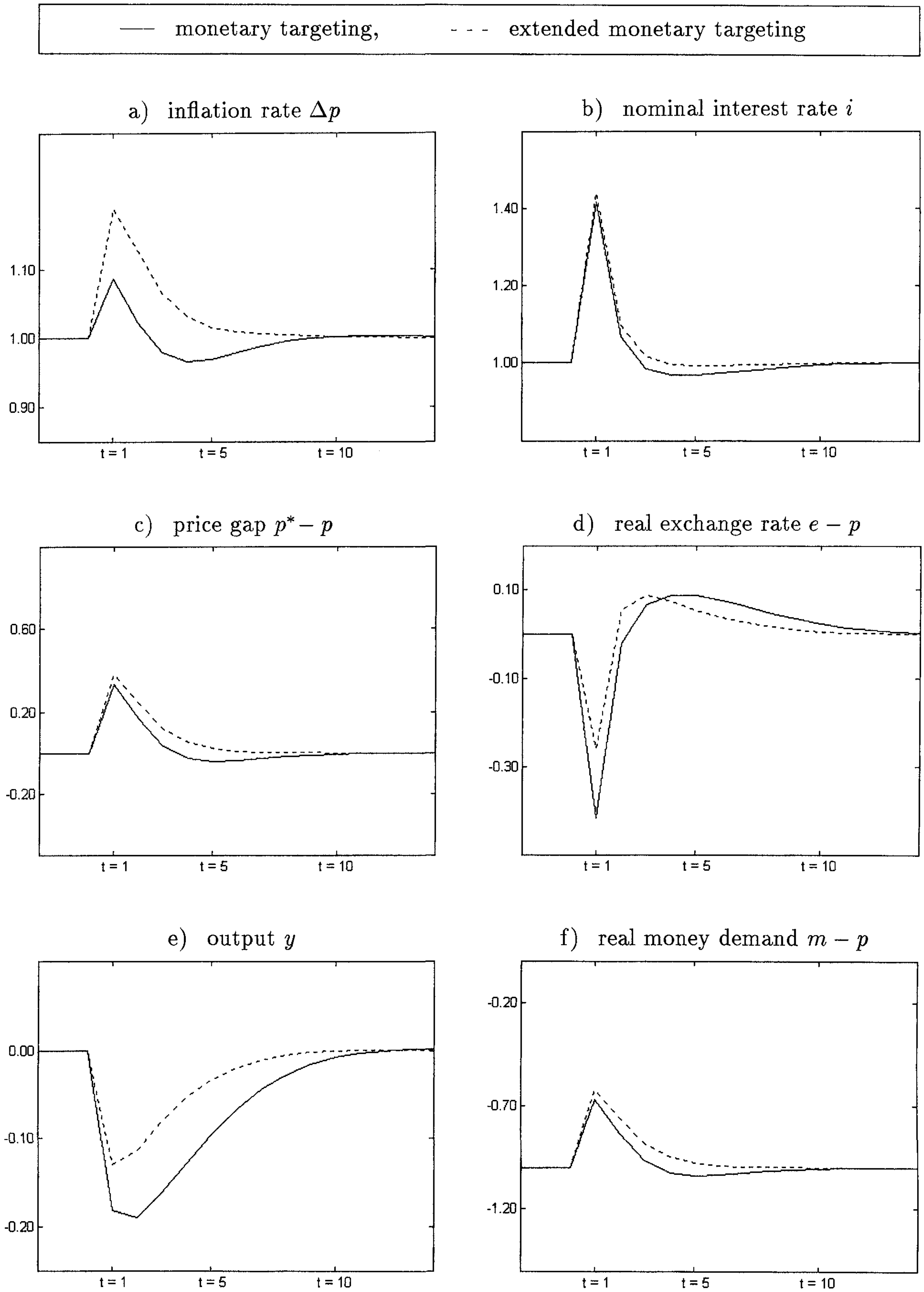


Figure 6

Impulse responses to a transitory velocity shock



If the adjustment paths of the endogenous variables are compared, it is obvious that the inflation rate displays lower volatility in the case of monetary targeting than it does in the case of inflation targeting. This result, however, is accompanied by a stronger response of the nominal interest rate, i.e. the monetary policy instrument. Inflationary pressures are weaker in the case of monetary targeting due to the stronger interest rate response; this reflects the fact that changes in the liquidity and output gap are taken into account in addition to the current inflation disequilibrium as shown in sub-section 1.2. Thus, the immediate increase in inflation is smaller. On the other hand, the stronger interest rate response induces a larger real appreciation.

Figures 2 and 3 show the impulse responses of the endogenous variables to the transitory demand shock for extended inflation and monetary targeting together with the impulse responses in the case of pure inflation and pure monetary targeting.

It is evident that letting the nominal interest rate depend on output disequilibria results in a faster return to equilibrium. This result holds for extended inflation targeting as well as for extended monetary targeting. However, it is accompanied by a transitory *decrease* in the inflation rate. This adverse reaction reflects the extreme assumptions underlying the forward-looking inflation expectations and, in particular, the selected parameter values in the inflation equation which heavily weights the future negative price gaps.

Accordingly, a *negative* demand shock or a business cycle trough would be countered by an interest rate decrease, i.e. an expansionary monetary reaction. This reaction would induce a transitory *increase* in the inflation rate. Hence, in the light of this finding, the extension of the monetary policy rule by output disequilibria should be judged critically as monetary policy is obliged to give priority to price stability.[20]

2.2 Impulse responses to a transitory money demand shock

The operation of inflation and monetary targeting in response to a transitory money demand or velocity shock is shown in Figure 4. The equilibrium values are identical to those of Scenario 1.

Owing to the serially correlated velocity shock, a persistent price gap emerges that induces an increase in the inflation rate. The inflationary impulse is counteracted by monetary policy by increasing the nominal interest rate according to the respective policy rule. Monetary targeting again responds by increasing the interest rate more sharply than would have been the case under inflation targeting and can thereby check inflationary impulses to a greater degree through a larger reduction in the liquidity gap. Analogous to the operation of inflation and monetary targeting in response to an aggregate demand shock, monetary targeting is again characterised by a lower volatility of the inflation rate. Furthermore, the volatility of the nominal interest rate is again higher.

Figures 5 and 6 show the impulse responses in the case of inflation and monetary targeting extended by output disequilibria.

Of course, in comparison with pure inflation and pure monetary targeting the feedback of interest rate changes on the output gap accelerates the reduction of output disequilibria. At the same time, the inflation rate increases more strongly. In other respects, however, the impulse responses do not differ fundamentally.

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20 If the Taylor rule including output disequilibria is compatible with empirical findings this result might be explained by the fact that monetary policy has historically been repeatedly used to stabilise business cycles.

Concluding remarks

In the present paper, the operation of inflation and monetary targeting has been analysed using a model of a small open economy based on the P-Star approach. The monetary policy regimes under investigation are the two alternative monetary policy strategies considered for the European System of Central Banks in Stage III of the European Monetary Union. Model-based simulations show that under a regime of monetary targeting the inflation rate has a lower volatility in response to demand shocks as well as in response to velocity shocks than under a regime of inflation targeting. The lower volatility of the inflation rate, however, is accompanied by a higher volatility of the nominal interest rate and, hence, of the exchange rate determined on the basis of uncovered interest parity. Thus, within the P-Star model, where inflation is a monetary phenomenon in the long run, there is much to be said for monetary targeting aimed at controlling the long-term determinant of inflation, i.e. the money stock.

If the simplifying assumptions which underlie the theoretical analysis are set aside, monetary policy makers are faced with the practical problem of operationalising monetary targeting. On the one hand, given a stable money demand, i.e. velocity is forecastable (a key assumption which underlies the specification of the P-Star model), monetary targeting ensures the controllability of money holdings with a fairly high degree of reliability. At the same time, monetary targeting offers a high degree of transparency to the general public. This transparency results not least from the timely availability of data on the current development of monetary aggregates. On the other hand, the recent instability of the financial sector in many countries renders the realisation of the ultimate goal of price stability by monetary targeting more difficult.

Inflation targeting which directly aims at the ultimate goal of price stability is often motivated by the failure of monetary targeting due to the instability of the financial sector. However, problems operationalising inflation targeting result from measuring inflation which is feasible only with a time-lag and which suffers from non-uniqueness. Furthermore, long and variable time-lags have to be taken into account when using monetary policy instruments to control inflation directly. Bearing that in mind, it would be advantageous to base inflation targeting on the *expected* future inflation rate instead of the *current* inflation rate, i.e. to follow a strategy of *inflation forecast targeting* which is proposed by Svensson (1997) in particular. However, inflation forecast targeting entails the problem of forecasting inflation with sufficient accuracy. As yet, this problem has not been tackled successfully.

Regarding the extension of pure inflation and pure monetary targeting by output disequilibria, it has to be pointed out that data on current output are only available with a time-lag and that the development of equilibrium output is uncertain. Furthermore, taking an output target into account could threaten the independence of monetary policy, whose main priority should, after all, be price stability.

Given the fact that the analysis in the present paper is confined to a stylised calibrated model, it has to be stressed that the analysis should be placed on a stronger empirical footing if it is to contribute to the discussion on the design of monetary policy beyond the theoretical findings documented here. Only a model which is firmly based on empirical grounds will provide a reliable framework for contrasting the operation of monetary and inflation targeting.

In particular, the parameterisation of the inflation equation and the monetary policy rule, which essentially determine the dynamic properties of the monetary transmission mechanism, needs further investigation. Against this background, the model under investigation should be estimated or at least calibrated taking a statistical criterion as a basis. Subsequently, the model could be evaluated using stochastic simulations to ascertain how far it matches empirical regularities measured in the data.

Appendix: solving and simulating the model

The solution of the model described in Section 1 is obtained using the method suggested by Blanchard and Kahn (1980).[21] Initially, the structural equations (1) – (5), (6), (12) and the policy rule (14) are written in *state space form*:

$$\begin{pmatrix} x_{1,t+1} \\ E_t\left[x_{2,t+1}\right] \end{pmatrix} = A \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + B\eta_{t+1} \tag{16}$$

with the *state vector* $x_t = (x'_{1,t}, x'_{2,t})'$ and the *transition matrix*

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

which is partitioned according to the dimension of the state vectors $x_{1,t}$ and $x_{2,t}$, where

(a) in the case of extended inflation targeting:

$$x_{1,t} = \left((\Delta p)_t^{\#}, (\Delta p)_{t-1}^{\#}, (\Delta p)_{t-2}^{\#}, i_{t-1}, y_{t-1}, y_{t-2}, \varepsilon_t^{y}, m_{t-1} - p_{t-1}, \varepsilon_t^{m}, r_{t-1}, p_{t-1}^{*} - p_{t-1}, \Delta p_{t-1}, \Delta p_{t-2}\right)'$$

$$x_{2,t} = \left(e_t - p_t, \Delta p_t\right)'$$

(b) in the case of extended monetary targeting:

$$x_{1,t} = \left((\Delta m)_t^{\#}, (\Delta m)_{t-1}^{\#}, (\Delta m)_{t-2}^{\#}, i_{t-1}, y_{t-1}, y_{t-2}, \varepsilon_t^{y}, m_{t-1} - p_{t-1}, m_{t-2} - p_{t-2}, \varepsilon_t^{m}, r_{t-1}, \Delta m_{t-1}, \Delta m_{t-2}, \right.$$
$$\left. p_{t-1}^{*} - p_{t-1}, \Delta p_{t-1}, \Delta p_{t-2}\right)'$$

$$x_{2,t} = \left(e_t - p_t, \Delta p_t\right)'$$

The *input matrix* $B = (B'_1, B'_2)'$ is partitioned according to the dimension of the state vectors $x_{1,t}$ and $x_{2,t}$ taking into account the dimension of the innovation $\eta_{t+1} = (\eta^{y}_{t+1}, \eta^{m}_{t+1})'$.

The vector $x_{1,t}$ contains the *predetermined* state variables of period t, the vector $x_{2,t}$ the *non-predetermined* state variables of period t. Non-predetermined are those variables whose realisations in the future period $t+1$ are subject to forward-looking expectations based on the information set Ω_t available in period t.[22] Thus, within the model under consideration, the (real) exchange rate and the inflation rate are non-predetermined irrespective of the monetary regime.

When writing the model in state space form, it has to be borne in mind that the nominal levels are trending by reason of the assumed steady state inflation. Therefore, as the method of Blanchard and Kahn presupposes the existence of a *stationary* equilibrium, these variables have to be transformed into stationary quantities by subtracting the price level, as is already shown by the definition of the state vectors under (a) and (b).[23] A necessary condition for this transformation is the homogeneity of the model in the price level.

The solution of the dynamic equation system (16) is *saddlepoint stable*, i.e. uniquely stable, if the number of the eigenvalues of the transition matrix A which lie outside the (complex) unit circle equals the number of the non-predetermined state variables and the number of the eigenvalues of the transition matrix A which lie inside the unit circle equals the number of the predetermined state variables (see Blanchard and Kahn (1980), Proposition 1).

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21 See also the exposition in Buiter (1984, 1986).

22 See Buiter (1982) for his amendment to the definition given by Blanchard and Kahn (1980).

23 See Buiter and Miller (1982).

The condition of saddlepoint stability is satisfied for the model irrespective of the monetary regime given the parameter values of Section 2 In view of two non-predetermined variables – the (real) exchange rate and the inflation rate – two (complex conjugate) eigenvalues lie outside the unit circle.

The saddlepoint stable solution of the model has to be determined on the basis of the information set Ω_t available in period t. If the conditional expectation operator $E_t[\cdot]$ is applied to the equation system (16) and account is taken of the fact that $E_t[x_t] = x_t$ and $E_t[\eta_{t+1}] = 0$, the following (deterministic) equation system is obtained:[24]

$$E_t[x_{t+1}] = Ax_t \tag{17}$$

The transition matrix A is transformed into the Jordan canonical form:

$$A = V \Lambda V^{-1} \tag{18}$$

with:

$$\Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}, \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}, \quad V^{-1} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}$$

where the matrices Λ, V and V^{-1} are partitioned to conform with the partition of the vector x_t. Assuming non-repeated eigenvalues, Λ is a diagonal matrix with the eigenvalues of the transition matrix A on its main diagonal; the matrix V is a matrix whose column vectors are the right eigenvectors corresponding to the eigenvalues and the matrix V^{-1} is a matrix whose row vectors are the left eigenvectors corresponding to the eigenvalues (see Golub and van Loan (1989), p. 339). The eigenvectors are ordered in such a way that the eigenvalues of the diagonal matrix Λ_1 lie inside the unit circle and the eigenvalues of the diagonal matrix Λ_2 lie outside the unit circle.

If the matrix of the left eigenvectors V^{-1} is multiplied from the left, the equation system (17) can be transformed into a system in the canonical variables $\tilde{x}_t = (\tilde{x}'_{1,t}, \tilde{x}'_{2,t})'$:

$$E_t\left[\tilde{x}_{t+1}\right] = \Lambda \tilde{x}_t \tag{19}$$

with $\tilde{x}_t \equiv V^{-1} x_t$.

Owing to the diagonal structure of matrix Λ, the transformed system (19) is decoupled. Hence, the subsystems:

$$E_t\left[\tilde{x}_{i,t+1}\right] = \Lambda_i \tilde{x}_{i,t} \quad i = 1,2$$

can be solved independently from each other.

As the eigenvalues on the main diagonal of Λ_2 lie outside the unit circle, the stable solution of the corresponding subsystem is to be determined by forward substitution of:

$$\tilde{x}_{2,t} = \Lambda_2^{-1} E_t\left[\tilde{x}_{2,t+1}\right] \tag{20}$$

After repeated substitution and application of the law of iterated expectations the solution is given by:

$$\tilde{x}_{2,t} = \lim_{\tau \to \infty} \Lambda_2^{-\tau-1} E_t\left[\tilde{x}_{2,t+\tau+1}\right]$$

It immediately follows that $\lim_{\tau \to \infty} \Lambda_2^{-\tau-1} E_t\left[\tilde{x}_{2,t+\tau+1}\right] = 0$ and thus:

[24] The solution for the vector of the non-predetermined variables $x_{2,t}$ is restricted to the class of linear functions of the vector of the predetermined variables $x_{1,t} \in \Omega_t$. Thus, the vector $x_{2,t}$ is implicitly an element of the information set Ω_t too.

$$\tilde{x}_{2,t} = 0 \tag{21}$$

Starting from (21) and taking into account the relationship $\tilde{x}_{2,t} = W_{21}x_{1,t} + W_{22}x_{2,t}$ the following result is obtained:

$$x_{2,t} = -W_{22}^{-1}W_{21}x_{1,t} \tag{22}$$

Thus, the vector of non-predetermined variables $x_{2,t}$ is given by a time-invariant linear function of the vector of predetermined variables $x_{1,t}$ depending on the left eigenvectors which correspond to the eigenvectors of A lying outside the unit circle.

If $x_{2,t}$ in system (16) is replaced by means of equation (22), the transition equation of the state vector $x_{1,t}$ is given by:

$$x_{1,t+1} = (A_{11} - A_{12}W_{22}^{-1}W_{21})x_{1,t} + B_1\eta_{t+1} \tag{23}$$

Reconsidering the decomposition of the partitioned transition matrix A according to (18), it follows from the formulae for inverting partitioned matrices (see Graybill (1983), p. 184) that:

$$\begin{aligned} A_{11} - A_{12}W_{22}^{-1}W_{21} &= V_{11}\Lambda_1(W_{11} - W_{12}W_{22}^{-1}W_{21}) \\ &= V_{11}\Lambda_1V_{11}^{-1} \end{aligned}$$

Hence, the transition equation (23) can be written equivalently as:

$$x_{1,t+1} = V_{11}\Lambda_1V_{11}^{-1}x_{1,t} + B_1\eta_{t+1}$$

Obviously, this transition equation is stable as the eigenvalues on the main diagonal of Λ_1 lie inside the unit circle.

By renewed application of the formulae for inverting partitioned matrices it can be shown that the identity:

$$-W_{22}^{-1}W_{21} = V_{21}V_{11}^{-1}$$

holds. Thus, the linear function (22) can be alternatively obtained using the right eigenvectors which correspond to the eigenvalues of A lying inside the unit circle.

If the preceding results are combined, the solution of the state space model (16) is:

$$x_{1,t} = Mx_{1,t-1} + B_1\eta_t \tag{24}$$

$$x_{2,t} = Nx_{1,t} \tag{25}$$

with $M \equiv V_{11}\Lambda_1V_{11}^{-1}$ and $N \equiv V_{21}V_{11}^{-1}$, where the transition equation of the predetermined variables given by (24) is shifted back in time one period.

If the solution formulae (24), and (25) are employed and the predetermined variables x_1 are given appropriate starting values $x_{1,0}$, model (16) can be easily simulated for $t = 1, 2, \ldots$ given a sequence of innovations η_t, $t = 1, 2, \ldots$. Here, the non-predetermined variables x_2 *jump* in each period $t = 1, 2, \ldots$ to reach a level $x_{2,t}$ that puts the vector of predetermined and non-predetermined variables $x_t = (x'_{1,t}, x'_{2,t})'$ on the saddlepoint stable adjustment path.

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