

# Forward interest rates and inflation expectations: the role of regime shift premia and monetary policy

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## Introduction

In recent years attention has been paid to the use of forward interest rates as monetary policy indicators of inflation expectations, see, for example, Dillén (1996), Svensson (1994), and Söderlind (1995). However, the Swedish experience of using forward interest rates as monetary policy indicators suggest that these rates are troublesome to interpret. The problem is essentially that fluctuations in forward rates often have been highly volatile and that it is difficult to relate these fluctuations to economic factors that usually are believed to affect interest rates, e.g. inflation expectations.

In this paper we argue that there are two major explanations for the relatively high and volatile development of forward interest rates: (i) investors' fears that the economy will switch to a high inflation regime give rise to a fluctuating regime shift premium; and (ii) expectations of monetary policy actions amplify the effect on forward interest rates originating from fluctuations in inflation expectations. In addition to these explanations we also include a time-varying term premium in the analysis. We show in an empirical analysis based on Swedish data the significance of adjusting for regime shift premia and taking the interaction between inflation expectations and expectations of monetary policy actions into account. Term premia are normally small, but occasionally they are of importance. In other words, it is essential to take regime shift premia (and sometimes term premia) and expectations of monetary policy actions into account when forward rates are used as monetary policy indicators.

In the literature it has been recognised that the usefulness of forward interest rates as indicators of future inflation expectations depends on the relative volatility and the correlation of inflation expectations and expected real interest rate. Several studies have tried to extract information about inflation expectations from the term structure. Fama (1990) finds that bond prices contain information about future values of a range of economic variables, such as future spot rates, inflation, real returns and expected term premiums. Mishkin (1990) analyses the information content of the term structure for future inflation and finds that nominal interest rates with maturities of nine to twelve months contain information about future inflation. In Söderlind (1995) "the forward rate rule", which states that all movements in forward interest rates reflect fluctuations in expected inflation, is evaluated on US and UK data. He finds that this rule performs reasonably well. Regime shift effects in the term structure were originally analysed by Hamilton (1988) but in a somewhat different context. This study is instead related to the kind of inflation regimes analysed by Dillén (1997), Evans and Wachtel (1993) and Evans and Lewis (1995).

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The outline of the paper is as follows: Section 1 is an overview of issues and concepts we aim to address in this study. In Section 2 the regime shift model is presented as well as a characterisation of the regime shift premium. Section 3 presents the estimates of regime shift premia in Sweden, which is an extended update of the estimates provided by Dillén (1996). The relation between inflation expectations obtained from surveys and forward interest rates is discussed and estimated in Section 4, which also includes an analysis of the role of monetary policy expectations in this context. The final section concludes.

## 1. Issues and concepts

The purpose of this study is to develop methods useful to extract and interpret information contained in forward interest rates, especially inflation expectations and expectations of future monetary policy. In this respect it is interesting to analyse *forward excess returns*,  $\eta$ , defined as the difference between forward interest rates and the future short-term interest rate:

$$\eta(t, \tau) = f(t, \tau) - i(t + \tau) \quad (1)$$

where  $f(t, \tau)$  is the forward interest rate<sup>1</sup> at time  $t$  with settlement at time  $t + \tau$ , and  $i(t + \tau)$  is the short-term interest rate at time  $t + \tau$ . Under the pure expectations hypothesis the excess forward return can be seen as a forecast error with  $E_t[\eta(t, \tau)] = 0$ , i.e., the forward interest rate is an unbiased indicator of the future short-term interest rate. Moreover, under the Fisher hypothesis the future short-term interest rate is the sum of the expected future short-term real interest rate and the expected future inflation rate.

A problem with using forward interest rates as monetary policy indicators is that they are probably affected by factors which make it difficult to extract and interpret information of expectations of future economic conditions. Forward interest rates may contain a time-varying term premia,  $\rho(t, \tau)$ , (such that  $E_t[\eta(t, \tau)] = \rho(t, \tau)$ ) which makes it more troublesome to extract information about expectations. Moreover, it is often desirable to separate different types of expectations. There might be Peso type problems, i.e. expectations of drastic, but unlikely, events (for example, devaluations or default) that might have impact on interest rates. Drastic events of this kind are sometimes modelled as regime shifts and in this study we consider the possibility that expectations of a shift to an inflationary regime will cause peso type problems. In other words we will assume that there is a regime shift component in forward interest rates. The above discussion leads us to consider the following decomposition of the forward interest rate:

$$f(t, \tau) = r^e(\tau) + \pi_0^e(\tau) + f_{rs}(t, \tau) + \rho(t, \tau) \quad (2)$$

where  $r^e(\tau)$  and  $\pi_0^e$  are the expected short-term real interest rate and inflation rate *within the regime* respectively,  $f_{rs}(t, \tau)$  is the regime shift component of the forward interest rate (or the regime shift premium) and  $\rho(t, \tau)$  is a time-varying term premium. By construction we have that  $E_t[i(t + \tau) \mid \text{no regime shifts}] = r^e(\tau) + \pi_0^e(\tau)$ , and regime shift expectations (or peso type problems in general) are represented by the regime shift component,  $f_{rs}(t, \tau)$ . Thus, the regime shift component will have a systematic effect on excess forward returns as long as no regime shift occurs, i.e.

$$\eta(t, \tau) = f_{rs}(t, \tau) + \rho(t, \tau) + e(t, \tau), \quad \text{with } E_t[e(t, \tau) \mid \text{no regime shifts}] = 0. \quad (3)$$

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<sup>1</sup> The interest rate agreed at time  $t$  for a short-term loan at time  $t + \tau$ .

## 2. Regime shift and term premia

### Regime shift premia

In order to characterise the regime shift premium we present a regime shift model that takes expectations of future shifts to an inflationary regime into account. Such expectations will give rise to a regime shift premium in forward interest rates, which can be seen as a compensation investors demand because they do not view the current price stability objective as fully credible and a switch to a higher inflation level might occur. The size of the regime shift premium depends on the probability of a shift to a high inflation regime assigned by the financial investors. It is likely that such probability assessments in turn depend on the political support for the target, the size and development of the national debt, the degree of central bank independence and the track record of inflation.

Let us first derive an expression for the regime shift component in inflation expectations by decomposing expectations of the future inflation rate,  $\tau$  years ahead, as

$$\pi^e(\tau) = \pi_0^e(\tau) + \pi_{rs}^e(\tau) \quad (4)$$

where

$\pi_0^e(\tau) = E[\pi(\tau) \mid \text{no regime shift}]$  = “normal” expectations about the future inflation rate within the current regime,

$\pi_{rs}^e(\tau)$  = the regime shift component of the expected future inflation rate representing the expected change of the inflation rate that regime shifts give rise to.

To obtain a more specific characterisation of the regime shift component, the inflation is assumed to fluctuate around certain levels,  $\pi_i$ , which follow a continuous time Markov chain  $\{S(t)\}$  (see Figure 1). There are three possible states:

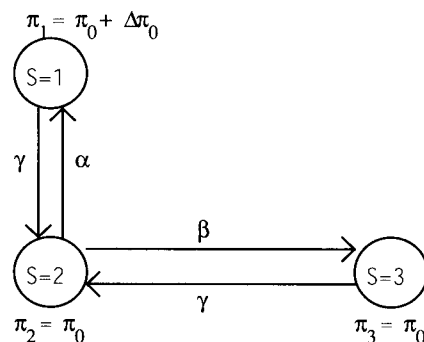
S=1 is characterised by high inflation;

S=2 is characterised by low inflation, but low credibility for the low inflation target;

S=3 is characterised by low inflation and high credibility for the low inflation target

Figure 1

### A high/low inflation regime shift model



In the following we will assume that the economy at present is in State 2. This is a critical state, because here the economy can either switch to the high inflation regime (State 1), in which case the inflation rate increases by  $\Delta\pi_0$ , or to the low inflation regime with high credibility

(State 3). The switching intensities for the states are  $\alpha$  and  $\beta$ , respectively, and the switching intensity back to State two from both State one and three is  $\gamma$ . The switching intensity  $\gamma$  is small, if we think that there exists a high degree of persistence of the regimes 1 and 3.<sup>2</sup> The low inflation level,  $\pi_0$ , is supposed to be identical to the declared inflation target.

Let  $P(\tau)$  be the probability of being in the high inflation regime  $\tau$  years ahead conditional on being in State 2 at present. It can be shown that the regime shift component,  $\pi_{rs}^e(\tau)$ , i.e. the expected increase in the future inflation level, takes the form:

$$\pi_{rs}^e(\tau) = P(\tau)\Delta\pi_0 \quad (5)$$

where<sup>3</sup>

$$P(\tau) = \frac{\alpha}{\lambda} [1 - e^{-\lambda\tau}] \quad \text{and} \quad \lambda = \alpha + \beta + \gamma$$

The regime shift premium in the forward interest rate (with settlement  $\tau$  years ahead) is defined, according to the Fisher Hypothesis, as the regime shift component of the inflation expectations:

$$f_{rs}(\tau) = \pi_{rs}^e(\tau) = \varphi(\tau) f_{rs}^\infty \quad (6)$$

$$\text{where } \varphi(\tau) = [1 - e^{-\lambda\tau}], \quad \text{and } f_{rs}^\infty = \frac{\alpha\Delta\pi_0}{\lambda}$$

Thus, the regime shift premium is the product of a pure *credibility factor*,  $f_{rs}^\infty$ , and a *credibility sensitivity factor*,  $\varphi(\tau)$ , showing the credibility effect on the forward rate curve for different horizons,  $\tau$ . Notice that the credibility sensitivity factor (and hence the regime shift premium) is increasing and concave in  $\tau$ . These properties as well as the specific functional form will be tested in the next section.<sup>4</sup>

In the model above the credibility (and thus also the regime shift premium) is constant over time. It is, however, possible to extend the set-up above and obtain models in which the credibility is varying stochastically over time. One possibility is to introduce more low inflation states and fluctuations in credibility can then be seen as switches between different low inflation regimes.<sup>5</sup> Another possibility is to let the switching intensities be positive stochastic processes. Since there are strong reasons to suspect that the regime shift premium does vary over time we will in the following extend the set-up above by allowing for a time dependent credibility factor, i.e.  $f_{rs}^\infty = f_{rs}^\infty(t)$ . The introduction of a fluctuating credibility factor then provides an explanation of the non-trivial

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<sup>2</sup> The low inflation regime with high credibility (regime 3) is included mainly for preserving the structure from the analysis in Dillén (1996) and Dillén and Lindberg (1997). One may think of regime 3 as an EMU-regime, and the credibility gains for Sweden of joining EMU can be analysed by considering a switch to regime 3. Credibility shocks of this type play an important role in Dillén and Lindberg (1997). In this paper very little is lost by disregarding regime 3 (i.e.  $\beta = 0$ ).

<sup>3</sup> See the Appendix for a derivation of the expression.

<sup>4</sup> Concavity of the regime shift premia is not a general implication of regime shift models of this kind and in a more general setting that takes sticky prices and exchange rate effects into account the regime shift premium can be more complex, see Dillén and Lindberg (1998).

<sup>5</sup> A switch to State 3 can be seen as a positive credibility shock.

phenomenon why long-term interest rates often appear to be more volatile than short-term rates since long-term rates are more sensitive to shocks in the credibility factor, i.e.  $\varphi(\tau)$  is increasing in  $\tau$ .<sup>6</sup>

To find a proxy for the credibility factor,  $f_{rs}^\infty$ , we use the fact that under the assumption that the long-term real interest rate essentially is given by a global real interest rate<sup>7</sup> the difference in long-term forward interest rates between two countries mainly reflects the expected difference in the long-run inflation rate. We assume that Germany has a credible inflation target of 2% (same as the Swedish inflation target) which means that the spread in long-term (10-year) forward interest rate between Sweden and Germany should be an approximation of the credibility factor,<sup>8</sup>  $f_{rs}^\infty(t)$ , i.e.

$$f_{rs}^\infty(t) \approx \delta_L(t) \tag{7}$$

Thus, the long-term forward interest rate differential can be seen as a quantitative measure of the degree of credibility of a low inflation policy. Moreover, the long-term forward interest rate differential relative to Germany is judged to be a more precise measure of (imperfect) credibility than the long-term forward interest rate itself, since the former should be less sensitive to international trends in long-term forward interest rates.

The long forward interest rate differential is likely to capture other risk factors than expectations of a shift to an inflationary regime, i.e. default risk or a time varying term premium.<sup>9</sup> In empirical investigations systematic fluctuations in excess returns have often been interpreted as a time varying term premium.<sup>10</sup> However, equation (3) suggests that fluctuations that *seem* to be systematic can also be attributed to fluctuating expectations of future regime shifts. Notice that if regime shifts are rare events it is very difficult (or even impossible if a regime shift does not occur) to statistically distinguish between these two competing explanations due to peso type problems. In other words, the reader is free to consider what the paper calls a regime shift premium as an additional component of the term premium.

### Term premia

It is natural to also incorporate a traditional term premium in the analysis. A standard view is that increased variability of interest rates means a higher risk of holding bonds with a maturity exceeding the investment horizon. A measure of risk in this sense is the variability of excess holding returns (the rate of return generated by capital gains during the holding period over the short-term risk free interest rate). To find a proxy for the term premium we first estimate an equation for the excess

<sup>6</sup> Two clarifications should be made about this statement: first, in more elaborated versions of the regime shift model, in which the switching intensities fluctuate, the variability will eventually decline with the term even though the decline will show up for very large  $\tau$  if  $\gamma$  is close to zero. However, the regime shift model still provides an explanation to why volatility increases (initially) with the term. Second, the variability discussed above is conditioned on that no regime shift occurs. The unconditional variability, which includes the variability that regime shifts cause, is decreasing with the term, as the expectation hypothesis implies.

<sup>7</sup> If there are no trends in real exchange rates then this assumption is justified by a real version of the uncovered interest parity.

<sup>8</sup> The assumption that the German (implicit) inflation target is two percent is not crucial for the analysis, since the long forward rate interest differential will appear in difference form in the regression.

<sup>9</sup> In a fixed exchange rate regime the long-term forward differential should reflect devaluation expectations..

<sup>10</sup> See e.g. Fama and Bliss (1987) or Engle, Lilien and Robins (1987) for empirical investigations on US data and Hördahl (1995), Dahlquist and Jonsson (1995) or Sellin (1995) for investigations on Swedish data.

holding return.<sup>11</sup> The generated variance,  $\omega(t,\tau)$ , gives an estimate of the risk factor measuring the variability of holding returns. We assume that the variance is of GARCH-type<sup>12</sup> and that the forward holding term premia can be written as a constant term,  $\rho_0(\tau)$  plus a time-varying term,  $\theta(\tau)\omega(t,\tau)$ :

$$\rho(t,\tau) = \rho_0(\tau) + \theta(\tau)\omega(t,\tau) \quad (8)$$

### 3. Estimation of regime shift and term premia

We now turn to the estimation of regime and term premia. Equations (3), (6), (7), and (8) suggest the following regression:

$$\Delta\eta(t,\tau) = \mu + \theta(\tau)\Delta\omega(t,\tau) + \varphi(\tau)\Delta\delta_L(t) + \varepsilon(t,\tau) \quad (9)$$

where  $E_t[\varepsilon(t,\tau) \mid \text{no regime shifts}] = 0$ . To reduce problems of serial correlation (due to the overlapping feature of the time series of forward excess returns) and non-stationarity we take first differences of the time series involved.<sup>13</sup> Weekly data from 9th December 1992 to 24th February 1998 is used. The short-term interest rate is represented by the marginal/repo rate. We assume that there has been no shift to a high inflation regime in this sample.<sup>14</sup> The credibility factor,  $\delta_L(t)$ , is measured as the ten-year forward interest differential between Germany and Sweden.<sup>15</sup> The variances,  $\omega(t,\tau)$ , are obtained from a GARCH-M estimation (see the Appendix). The model is estimated for  $\tau$  equal to 0.25, 0.5 and 1, i.e. for three, six and twelve months. All forward interest rates are continuously compounded and given as effective rates.

From Table 1 it can be seen that the estimated credibility sensitivity factor,  $\varphi$ , is indeed concave and increasing in  $\tau$ . However, the implied  $\lambda$  value in the case  $\tau = 1$  differs from those obtained when the horizon is three or six months even if the difference is not statistically significant. This indicates that the specific functional form of  $\varphi(\tau)$  is at best only a reasonable approximation for small  $\tau$ . The estimate of  $\varphi$  for  $\tau = 3$  months is essentially the same as in Dillén (1996), whereas it has decreased somewhat for  $\tau = 6$  months.<sup>16</sup> It is notable that the time varying part of the 6-month term

<sup>11</sup> See the Appendix for estimation of the excess holding return equation. The excess holding return equation is defined as the difference between the holding return during the period  $(t,t+s)$  of a zero coupon bond that matures at  $t+\tau$  ( $\tau > s$ ),  $h(t,t+s)$ , and the short-term interest rate,  $i(t)$ , at time  $t$ . Dillén (1996) discusses different estimation techniques based on excess forward return and excess holding return equations. The chosen estimation technique proved to be the most efficient one.

<sup>12</sup>  $\omega(t,\tau)$  is assumed to be of GARCH(1,1) type, i.e.  $\omega(t,\tau) = \text{Var}_{t-1}[(u(t,\tau))] = \alpha_0(\tau) + \alpha_1(\tau)u^2(t-1,\tau) + \beta_0(\tau)\omega(t-1,\tau)$ .  $u(t,\tau)$  is the error term from the excess holding return equation.

<sup>13</sup> To test for the appropriate dynamic specification we run regressions of the form:  $\eta(t,\tau) = \alpha + \rho(\tau)\eta(t-1,\tau) + \theta(\tau)\omega(t,\tau) + \theta_{-1}(\tau)\omega(t-1,\tau) + \varphi(\tau)\delta_L(t) + \varphi_{-1}(\tau)\delta_L(t-1) + \varepsilon(t,\tau)$ . In all regressions  $\rho$  is close to unity ( $\rho(3) = 0.911$ ,  $\rho(6) = 0.968$ , and  $\rho(12) = 0.995$ ) and augmented Dickey-Fuller tests of the time-series for excess forward return indicates that it is hard to reject the hypothesis of a unit root, especially for six and twelve months. Moreover, we also find that  $\theta_{-1}(\tau)$  is very close to  $-\theta(\tau)$  and that  $\varphi_{-1}(\tau)$  was very close to  $-\varphi(\tau)$  (the difference is statistically insignificant). The  $\theta$  and  $\varphi$ -estimates were also very close to the corresponding parameters reported in Table 1. The results of these regression suggest that (9) is an appropriate specification (see Hendry (1995) or Hendry et al. (1984)).

<sup>14</sup> A regime shift in Sweden would probably be an observable event where the current inflation target is given up.

<sup>15</sup> The forward interest rates are estimated by the extended Nelson-Siegel model, see Svensson (1994).

<sup>16</sup> Dillén (1996) did not estimate  $\varphi$  for  $\tau = 12$  months.

premium seems to be of less importance in these updated estimations.<sup>17</sup> The high values of  $R^2$  are remarkable in view of the fact that the excess forward return is an unpredictable forecast error under the expectation hypothesis.<sup>18</sup>

Table 1

**Estimation of the excess forward return equation:**

$$\Delta\eta(t,\tau) = \mu + \theta(\tau)\Delta\omega(t,\tau) + \varphi(\tau)\Delta\delta_L(t) + \varepsilon(t,\tau)$$

	$\hat{\mu}$	$\hat{\theta}$	$\hat{\varphi}$	$\hat{\rho}_0$	$\hat{\lambda}$	$R^2$	DW	SE
$\tau=3$ months	0.0014 (0.131)	0.0439 (7.616)	0.2117 (4.742)	-0.4922	0.09516 [0.530, 1.422]	0.273	2.395	0.176
$\tau=6$ months	0.0023 (0.197)	0.0119 (6.803)	0.3842 (8.124)	-0.6046	0.9696 [0.689, 1.296]	0.382	2.450	1.181
$\tau=12$ months	-0.0014 (-0.106)	0.0015 (6.048)	0.4786 (8.860)	-0.1134	0.6512 [0.0466, 1.296]	0.434	2.261	0.199

Note:  $\hat{\rho}_0$  is the implied value of  $\rho_0$  appearing in equation (7) and calculated as the mean of  $\eta(t,\tau) - \hat{\theta}\omega(t,\tau) - \hat{\varphi}\delta_L(t)$ ;  $\hat{\lambda}$  is the value of  $\lambda$  in the expression  $\varphi(\tau) = 1 - e^{-\lambda\tau}$  that the estimate of  $\varphi$  implies and the 95% confidence interval is given within brackets; t-values within parentheses. DW is the Durbin-Watson statistics for autocorrelation and SE denotes standard error of regression. The number of observations is 259 for 3 months, 246 for 6 months and 220 for 12 months.

As seen from Figures 2a-c the quantitative effects of regime shift premia are, with a few exceptions, much larger than those of the (time varying) term premia. The sizes of the six and twelve months regime shift premia have been above 1%, but they are below 0.5% today. At some occasions, e.g. the summer and fall of 1994, the term premia are substantial.

**Interpretation of the results**

As emphasised earlier, it is not possible with this estimation technique to formally test for the presence of regime shift premia due to peso type problems. What we have shown empirically is that the long forward interest rate differential relative to Germany appears to be an important factor for explaining systematic movements in excess forward returns. In addition, the impact from the long forward interest rate differential on excess forward return increases concavely with the time horizon. Regime shift models of the type presented in Section 2 are an attempt to provide an explanation for these new and non-trivial findings, but the results can not be seen as proof or strong evidence of the theory. The results can also be interpreted as if we have detected an additional component of a time varying term premium although it is difficult to justify such an explanation from economic theory.

The regime shift premium is also a way of formalising the wide spread use of long (forward) interest rate differential relative to Germany as an indicator of credibility problems. Indeed,

<sup>17</sup> For  $\tau = 3$  and 6 months the constant part of the term premium is negative and rather large in absolute terms. However, these estimates can be shown to be statistically insignificant.

<sup>18</sup> The high values of  $R^2$  do not imply that excess forward returns are unconditionally very predictable, but rather that they are predictable conditional on no regime shift.

Figure 2a

**3-month forward rates, regime shift premia and term premia (time varying part)  
(continuously compounded) (percent)**

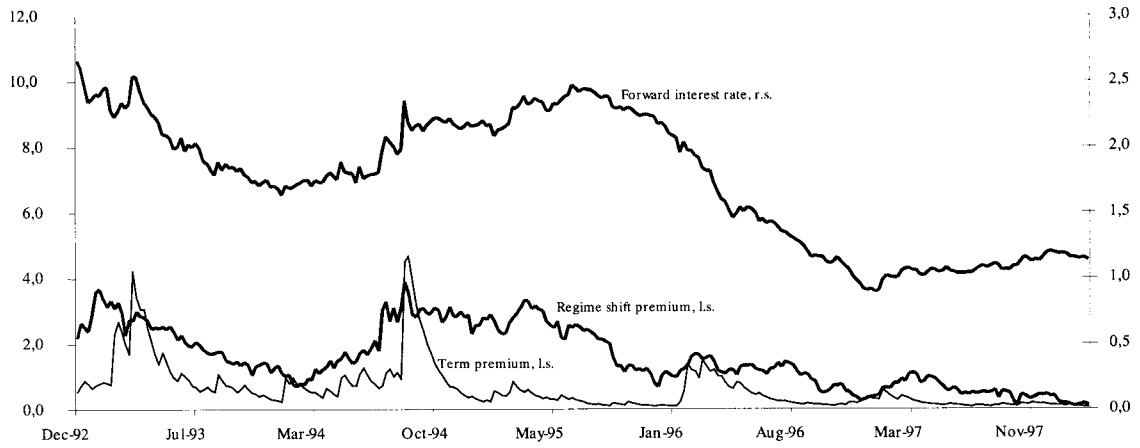


Figure 2b

**6-month forward rates, regime shift premia and term premia (time varying part)  
(continuously compounded) (percent)**

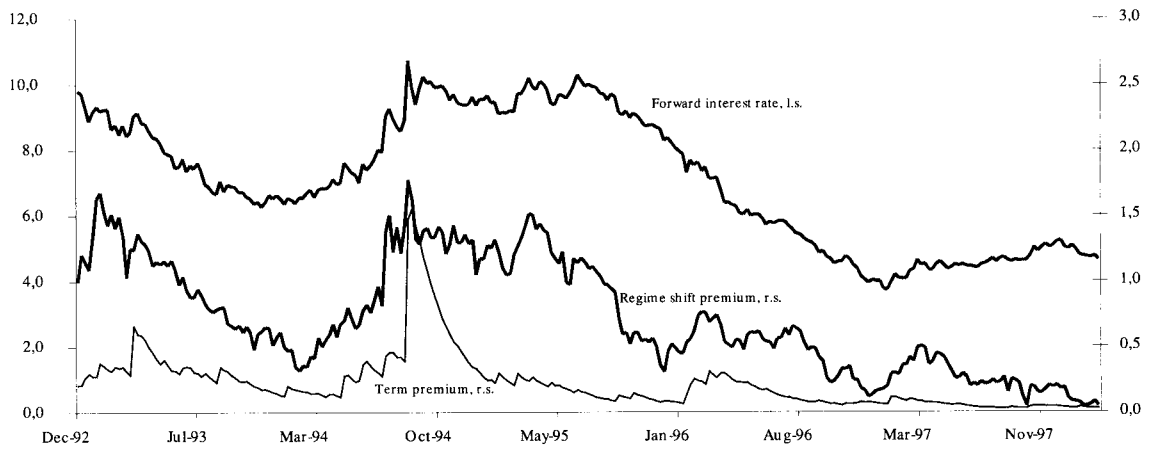
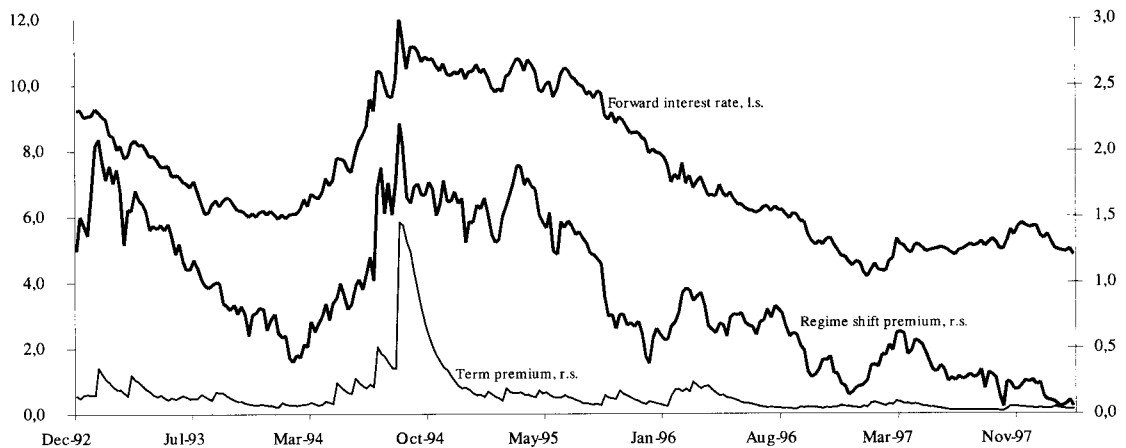


Figure 2c

**12-month forward rates, regime shift premia and term premia (time varying part)  
(continuously compounded) (percent)**





The regime shift premium is also a way of formalising the wide spread use of long (forward) interest rate differential relative to Germany as an indicator of credibility problems. Indeed, in periods when credibility problems were obvious in the public debate the regime shift premium has been large. Against this background we will use the term regime shift premium as a measure of the quantitative effects on the forward interest rate curve that credibility problems cause.

Having said that, it is natural to ask what kind of event a regime shift represents. One may argue that a switch to a more inflationary regime initially should imply lower interest rates (in order to generate expansionary conditions in the short run), i.e. negative estimates of  $\varphi(\tau)$  for small  $\tau$ . Indeed, this is a possibility in the regime shift model analysed by Dillén and Lindberg (1998), but a positive regime shift premium is the most natural case also in this model. The Dillén-Lindberg model is an extension of the one presented in Section 2 in that exchange rate effects are also incorporated, and these effects are important to understand why the regime shift premium is likely to be positive for small  $\tau$  after all. A regime shift would probably be some kind of institutional and political change that will lead to an economic policy that allows for a high and variable inflation rate without necessarily leading to an immediate change of the interest rate policy in an expansionary direction. If a regime shift of this kind occurs, the domestic currency will depreciate drastically when forward looking investors evaluate the implications of the new regime (see Dillén and Lindberg (1998)). Expectations of such a drastic jump in the exchange rate will put upward pressure on short-term interest rates.

#### 4. Investors' expectations of future inflation and monetary policy actions

In this section we will investigate how much information forward interest rates contain about expected inflation obtained from surveys and if investors take possible regime shifts into account when giving their assessment of expected inflation. It is not a priori obvious to what extent survey expectations incorporate regime shifts expectations. If participants in surveys report their inflation expectations, in a mathematical sense inflation expectations should fully reflect regime shift expectations. However, if participants report the most probable outcome of the future inflation then it is likely that regime shift expectations are incorporated to a very limited extent provided that a regime shift is considered to be an unlikely event. In the latter case one should be able to extract more information from forward interest rates by controlling for the regime shift premia. Also, we would expect the interaction between inflation expectations and expectations of monetary policy actions to influence forward interest rates. In order to analyse the effects of regime shift premia and of monetary policy expectations we will consider the following models:

$$\pi^e = \alpha + \beta f(\tau) \quad (10)$$

$$\pi^e = \alpha + \beta_{adj} f_{adj}(\tau) \quad (11)$$

$$\pi^e = \alpha + \beta_{rs} f_{rs}(\tau) \quad (12)$$

$$\pi^e = \alpha + \beta_{adj} f_{adj}(\tau) + \beta_{rs} f_{rs}(\tau) \quad (13)$$

where  $f_{adj}(\tau) = f(\tau) - \rho(\tau) - f_{rs}(\tau)$ , and  $\rho(\tau)$ , is the (total) term premium, i.e.  $\rho(\tau) = \rho_0(\tau) + \theta(\tau) \cdot \omega(t, \tau)$ . The regime shift premium,  $f_{rs}(\tau)$ , is calculated by taking the estimate for the credibility sensitivity factor,  $\varphi(\tau)$  from Table 1 and multiplying this estimate with the long-term forward interest rate differential between Germany and Sweden,  $\delta_L(t)$ .<sup>19</sup> Since we will only focus on one year forward interest rates ( $\tau = 1$ ) in what follows, the notation for the time horizon,  $\tau$ , will be dropped.

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<sup>19</sup> Since the survey of inflation expectations only comes (approximately) quarterly we have chosen the observations closest in time to Aragon's survey for our estimations. The sample consists of 21 observations, ranging from January 1993 to February 1998.

Equation (10) is the linear forward rate rule analysed by Söderlind (1995). The potential benefit of adjusting forward interest rates for regime shift (and term) premia can be assessed by comparing models (10) and (11). Since we want the real forward interest rate to reflect investors' expectations of the future monetary stance we adjust for the estimated term premium as well. Moreover, the parameter  $\beta_{rs}$  in equations (12) and (13) indicates to what degree regime shift expectations are reflected in investors' inflation expectations. As pointed out by Söderlind (1995), the estimate of  $\beta$  depends on the stochastic properties of the real interest rate.<sup>20</sup> If one of the following two pre-conditions hold: (i) the real interest rate (adjusted for the term premium) is stable or (ii) the real interest rate is negatively correlated with inflation expectations to such degree that this counteracts the estimation effects that a variable real interest rate gives rise to, then we expect:  $\beta \approx 1$ . Söderlind (1995) found that the forward rate rule (i.e.  $\beta \approx 1$ ) performs fairly well since pre-condition (ii) appears to be fulfilled to a large extent. This finding does not, however, necessarily imply that forward interest rates contain much information about investors' inflation expectations.

However, if investors think that changing prospects of future inflation will lead to a monetary policy response, it is not unreasonable that increasing (decreasing) inflation expectations are associated with increasing (decreasing) expectations of the future short real interest rates, i.e. inflation expectations are positively correlated with the real forward interest rate. In this case only some fraction of a change in the nominal forward interest rate reflects a change in inflation expectations. Thus, we expect:  $\beta < 1$ .<sup>21</sup> Notice, however, that a  $\beta$ -estimate less than unity might also reflect a high variability of the real interest rate, in which case fluctuations in forward interest rates contain relatively little information about inflation expectations.

### Expectations of monetary policy actions

To formalise the idea that investors' expectations of monetary policy are related to their inflation expectations we assume that investors' expectations of the future real interest rate can be written as

$$r^e = r_0 + \phi (\pi^e - \pi_0) + u, \quad E[u]=0, \quad Cov[u, \pi^e] = 0 \quad (14)$$

where  $r_0$  is the average expected future real interest rate when investors inflation expectations are on the inflation target ( $\pi_0$ ) and  $u$  is an error term (uncorrelated with  $\pi^e$ ) representing other factors affecting investors' expectations of the future real interest rate. The parameter  $\phi$  can be viewed as how sensitive investors' expectations of the future short-term real interest rate are to changes in investors' inflation expectations. With  $u$  set to zero, equation (14) is similar to the instrument rule under inflation targeting (see Svensson (1997)) whereby the real interest rate should increase (decrease) when inflation prospects are above (below) the inflation target. However, inflation prospects in this rule refer to the inflation forecast (conditional on an unchanged interest rate policy) of the central bank which is different from investors' (unconditional) inflation expectations.<sup>22</sup>

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<sup>20</sup> See Söderlind (1995) for a further analysis of how the  $\beta$ -estimate depends on the correlation between inflation expectations and the real interest rate.

<sup>21</sup> If regime shifts expectations are fully incorporated in investors' inflation expectations then it is reasonable that  $\beta_{rs} \approx 1$  since it should be inflation expectations within the regime that mainly affect investors' expectations of the future monetary policy actions.

<sup>22</sup> One may ask why investors' expectations deviate from the inflation target. In general investors' inflation expectations will only coincide with the inflation target under the assumptions of (i) strict inflation targeting with no imperfections e.g. model uncertainty, (ii) rational expectations, and (iii) the time horizon of investors' inflation expectations coincide with that of the central bank. In practice these assumptions are not fulfilled. Moreover, investors' inflation expectations often

By using results from single-equation regression analysis it is straightforward to show that the parameter  $\phi$  in equation (14) depends on  $\beta$  and the  $R^2$  from the regressions (10)-(12) in the following way:<sup>23</sup>

$$\phi = \frac{R^2}{\beta} - 1 \quad (15)$$

Thus, forward interest rates may contain much information of investors' inflation expectations even when the  $\beta$ -estimate is small if investors' inflation expectations are strongly linked to expectations about future monetary policy (i.e. future short-term real interest rates) according to (14). One can view formula (15) as a measure of the perceived link between monetary policy and inflation prospects. One implication of (15) is that *expectations of higher future inflation are associated with expectations of higher (lower) future real interest rates if  $R^2$  is larger (lower) than the  $\beta$ -estimate*. It is important to stress that formula (15) is valid only when models are estimated with only one regressor and therefore we apply formula (15) to equations (10), (11) and (12).

The interpretation of the  $\phi$ -estimate in these equations depends on which role regime shift expectations play. It is more natural to think that expectations of future monetary policy are related to normal inflation expectations (within the regime) rather than to expectations about a switch to a high inflation regime, and in the former case the  $\phi$ -estimate in model (11) is of interest. However, it is not unreasonable to assume that investors think that monetary policy will react to increasing regime shift expectations in order to restore credibility. On the other hand, credibility problems may arise due to expectations of a too expansionary monetary policy. Finally, the interpretation of the  $\phi$  is affected by the extent to which expectations of a regime shift are incorporated in inflation expectations obtained from surveys.

### Data and graphical inspection

To obtain inflation expectations ( $\pi^e$ ) we use Aragon's quarterly survey of financial investors' expectations of average inflation two years ahead for the period 1993q1 to 1998q1.<sup>24</sup> These expectations should normally be a quite good proxy for the expected one-year inflation rate, i.e. the expected rate of consumer price changes one year in the future. We have small sample problems (the data set consists of only 21 observations) and therefore strong quantitative conclusions should be avoided.

The data used in the following analysis is depicted in Figure 3. We see that the unadjusted as well as the adjusted forward interest rate has been more volatile than the inflation expectations. The higher volatility in the forward interest rate arises from the fact that increases in inflation expectations cause the nominal interest rate to rise, the Fisher effect, but also from the fact

refer to the average rate of change in consumer prices up to a certain point of time (and not the future inflation rate) implying that short-term deviations from the inflation target affect measured inflation expectations.

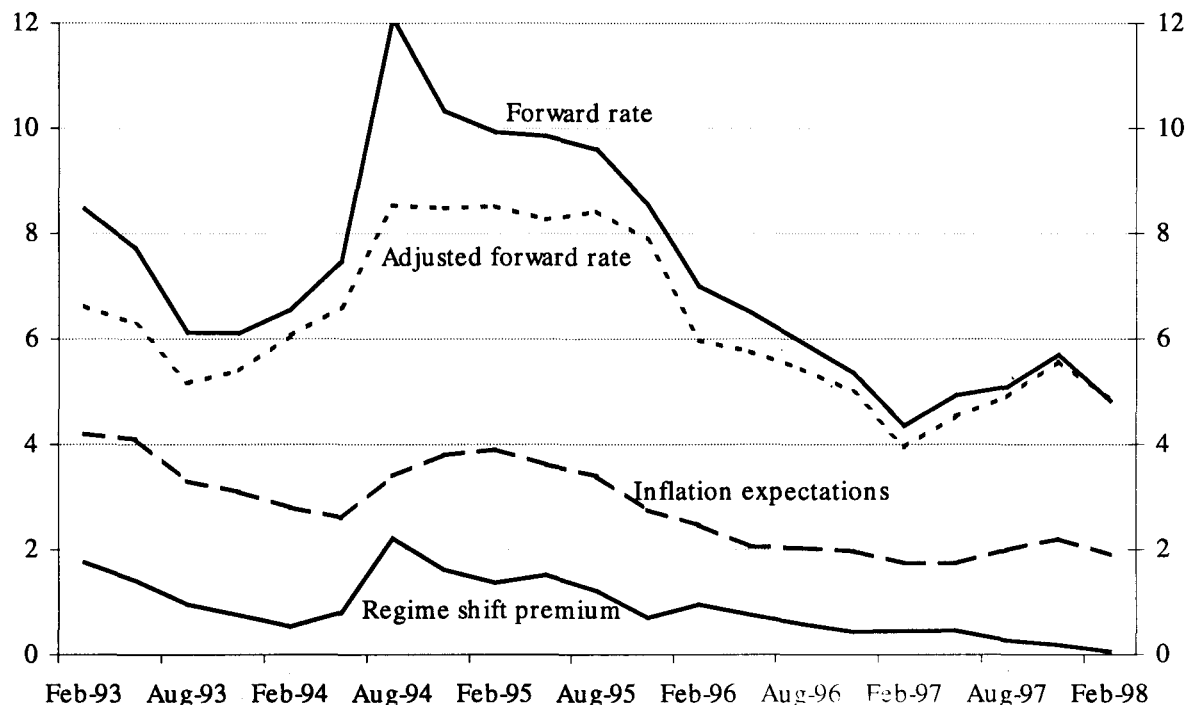
<sup>23</sup> In a regression of the type  $y = \alpha + \beta x + \varepsilon$  we have that  $R^2 = \hat{\beta}^2 \text{Var}(x)/\text{Var}(y)$  and  $\hat{\beta} = \text{Cov}(x,y)/\text{Var}(x)$  implying that  $R^2/\hat{\beta} = \text{Cov}(x,y)/\text{Var}(y)$  (i). In this case  $(y,x) = (\pi^e, f) = (\pi^e, r^e + \pi^e)$  and (14) implies  $\text{Cov}(\pi^e, f) = (1+\phi)\text{Var}(\pi^e)$  (ii). It then follows from (i) and (ii) that  $(R^2/\hat{\beta}) - 1 = \phi$ . One realises that this expression is valid also when  $(y,x) = (\Delta\pi^e, \Delta f)$  and for models (11) and (12).

<sup>24</sup> Aragon Securities Fondkommission AB measures since 1991 every quarter the average expected two-year Swedish inflation of the largest Swedish and foreign investors on the Swedish bond market. We choose not to extend the data to include data from 1991 and 1992 because during this period Sweden had a fixed exchange rate regime and the interest rates were occasionally very high due to devaluation expectations. Our set-up is closely related to credibility problems and monetary policy in an inflation target regime.

that with higher inflation expectations the market anticipates a future tightening of monetary policy, expressed in an increased short-term real interest rate in the future. The latter effect is also incorporated directly in the forward interest rates, which means that their fluctuations will be greater than the fluctuations in inflation expectations. Thus, in contrast to Söderlind (1995), the real interest rate appears to be positively correlated with inflation expectations. The finding that the regime shift premium is positively correlated with investors' inflation expectations may suggest that credibility aspects to some extent are reflected in surveys.

Figure 3

Swedish 12-month forward interest rate and inflation expectations,  $\tau = 1$



Sources: Aragon Securities Fondkommission AB and Riksbank.

Moreover, the real one-year forward interest rate (defined as the nominal forward interest rate minus inflation expectations) has fluctuated in the range of 3 to 9%, which is rather high. However, the adjusted real forward interest rate<sup>25</sup> has moved between 2 to 5%, which we think is a reasonable range for the short-term real interest rates in the absence of credibility problems.

### Results from regressions

In order to quantify how inflation expectations and forward interest rates interact we estimate equations (10)-(13). All equations are estimated in difference form to reduce problems with serial correlation and non-stationarity.

<sup>25</sup> The adjusted forward interest rate is also adjusted for the term premium, which normally is included in the real forward interest rate. However, if the real forward interest rate is used as a measure of the expected future real short-term interest rate then the term premium should be excluded. With the exception of a period in 1994 the size of the term premium is small, see Figure 2c.

As seen from Table 2 the adjusted  $\beta$ -estimates ( $\hat{\beta}_{adj}$ ) are approximately in the range of 0.20-0.30 depending on whether the regime shift premium is included or not. The significant estimate of  $\hat{\beta}_{rs}$  in estimation 3 suggests that regime shift expectations have an impact on investors' inflation expectations obtained from surveys. However, it is evident from Figure 3 that the regime shift premium is strongly correlated with the adjusted forward interest rate and therefore  $\hat{\beta}_{rs}$  comes out insignificant when also the adjusted forward interest rate is included as a regressor. We cannot, however, rule out that the presence of regime shift expectations can explain why inflation expectations of investors tended to be higher than for other groups in 1994-95.<sup>26</sup>

The  $R^2$  is larger when the adjusted forward interest rate is used in comparison with the unadjusted forward interest rate indicating that more information about investors' inflation expectations can be extracted from forward interest rates when they are adjusted for regime shift premia. The presence of a volatile regime shift premium in unadjusted forward interest rates is probably an explanation to why the unadjusted  $\beta$ -estimate ( $\hat{\beta}$ ) is smaller than the adjusted estimate ( $\hat{\beta}_{adj}$ ).

Table 2

**Estimates of forward interest rate rules (10) – (13)**

1.  $\Delta\pi_t^e = \alpha + \beta\Delta f + \varepsilon_t$

3.  $\Delta\pi_t^e = \alpha + \beta_{rs}\Delta f_{rs} + \varepsilon_t$

2.  $\Delta\pi_t^e = \alpha + \beta_{adj}\Delta f_{adj} + \varepsilon_t$

4.  $\Delta\pi_t^e = \alpha + \beta_{adj}\Delta f_{adj} + \beta_{rs}\Delta f_{rs} + \varepsilon_t$

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\beta}_{adj}$	$\hat{\beta}_{rs}$	$\hat{\phi}$	$R^2$ { $R^2$ adj}
1	-0.0855 (0.2321) [0.0691]	0.1617 (0.0003) [0.0364]			1.364	0.382 {0.348}
2	-0.0893 (0.2208) [0.0704]		0.2897 (0.0010) [0.0741]		0.525	0.442 {0.411}
3	-0.0779 (0.3100) [0.0746]			0.4369 (0.0030) [0.1273]	0.393	0.265 {0.224}
4	-0.0749 (0.2255) [0.0596]		0.2365 (0.0067) [0.0766]	0.2251 (0.2236) [0.1782]		0.497 {0.438}

Note: p-values within parentheses, standard errors within brackets. Both are corrected for autocorrelation according to the Newey-West method.

Turning to the interaction between inflation expectations and monetary policy we see that the  $\phi$  estimate of 1.4 when using the unadjusted forward interest rate as regressor suggests that increased inflation expectations lead to expectations of increases in the future short-term real interest rates.<sup>27</sup> The same conclusion holds when the adjusted forward rate is used as a regressor but the

<sup>26</sup> Investors' inflation expectations 2 years ahead were in the range of 3-4% in 1994 and 1995 whereas 1 year inflation expectations of households were between 2 and 3%.

<sup>27</sup> Experiments with Monte Carlo simulations indicate that this estimate as well as the other estimates of  $\phi$  are significant (the estimates deviate from zero by more than three standard deviations).

$\phi$  estimate is much smaller (0.5). One possible explanation for this result is that the adjustment for a volatile regime shift premium increases the  $\beta$  estimate much more than it increases the informative content ( $R^2$ ). Moreover, as discussed earlier, the interpretation of the  $\phi$ -estimates depends on the role played by regime shift expectations.<sup>28</sup> In spite of this and the problem of a small sample our results suggest that investors expect future monetary policy to counteract any increases in inflation expectations.

An interesting question is why other studies, e.g. Mishkin (1990) and Söderlind (1995), find evidence indicating a higher  $\beta$  value, closer to unity? One explanation can be that these studies use data that cover periods in which there were less transparent links between inflation prospects and monetary policy actions. The fact that Söderlind (1995) obtained  $\beta$ -estimates greater than the  $R^2$  (indicating a negative  $\phi$  in (14)) supports this view. This study uses data observed in an explicit inflation target regime and it is thus natural that inflation expectations of investors are closely related to expectations of future monetary policy (the real interest rate) and a  $\beta$ -estimate less than 0.5 is in accordance with this.

### A general dynamic specification

The models considered so far (i.e. the difference form of equations (10)-(13)) are more or less variants of traditional models in which the Fisher hypothesis can be addressed, and our results are comparable with results from other studies.<sup>29</sup> These models are probably misspecified from a statistical perspective, but the focus of the analysis has been on simple forward rate rules, i.e. to what extent do movements in (forward) interest rates reflect changes in inflation expectations.<sup>30</sup> In order to reduce potential problems of misspecification we extend the previous analysis to analyse the following more general models:

$$\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta f(t) + \gamma f(t-1) \quad (16)$$

$$\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{adj}f_{adj}(t) + \gamma_{adj}f_{adj}(t-1) \quad (17)$$

$$\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{rs}f_{rs}(t) + \gamma_{rs}f_{rs}(t-1) \quad (18)$$

$$\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{adj}f_{adj}(t) + \gamma_{adj}f_{adj}(t-1) + \beta_{rs}f_{rs}(t) + \gamma_{rs}f_{rs}(t-1) \quad (19)$$

Notice that the difference form models (10) – (13) are obtained if we impose the restrictions  $\delta = 1$ ,  $\gamma = -\beta$ . The specification in levels corresponds to the restrictions  $\delta = \gamma = 0$ .

In Table 3 the estimates of the general dynamic specifications are reported. The  $\beta$ -estimates associated with adjusted as well as unadjusted contemporaneous forward interest rates do not deviate much from corresponding estimates in the previous analysis even though  $\hat{\beta}_{adj}$  has dropped from 0.24 to 0.18 in the regression including both the adjusted forward interest rates and the regime shift premium. Moreover, we think that this regression generates the most reliable estimates

<sup>28</sup> The negative  $\phi$ -estimate in the regression that includes only the regime shift premium does not imply a negative correlation between the regime shift premia and inflation expectations since  $\text{Cov}(\pi^e_{f_{rs}}) = (1+\phi)\text{Var}(\pi^e) > 0$  (see footnote 23).

<sup>29</sup> Another reason to use simple models (with only one regressor) is that the interaction between inflation expectations and expectations of the future real interest rates easily can be analysed using equation (15).

<sup>30</sup> A correctly specified model may lead the conclusion that forward interest rates contain very little (if any) additional information about investors' inflation expectations that is not reflected in other regressors. This does not mean that forward interest rates do not contain information about investors' inflation expectations. Forward interest rates can therefore be simple useful indicators for detecting changes in investors' inflation expectations.

since all regressors, with one exception, appear to be significant, whereas the other equations therefore are subject to misspecification to a larger degree. The observation that  $\hat{\gamma}_{adj} \approx -\hat{\beta}_{adj}$  in the preferred regression indicates adjusted forward interest rates only reflect information of short-run changes of investors' inflation expectations.

Against this background, our interpretations of the results are that it is advisable to decompose forward interest rates into a regime shift component and an adjusted component. In addition,  $\hat{\beta}_{adj}$  should not be very far from 0.2, but a more cautious judgement is that this estimate should be in the range of 0.15 to 0.30.

Table 3  
Estimates of forward interest rates rules (16) – (19)

1.  $\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta f(t) + \gamma f(t-1) + \varepsilon(t)$
2.  $\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{adj} f_{adj}(t) + \gamma_{adj} f_{adj}(t-1) + \varepsilon(t)$
3.  $\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{rs} f_{rs}(t) + \gamma_{rs} f_{rs}(t-1) + \varepsilon(t)$
4.  $\pi^e(t) = \alpha + \delta\pi^e(t-1) + \beta_{adj} f_{adj}(t) + \gamma_{adj} f_{adj}(t-1) + \beta_{rs} f_{rs}(t) + \gamma_{rs} f_{rs}(t-1) + \varepsilon(t)$

	$\hat{\alpha}$	$\hat{\delta}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}_{adj}$	$\hat{\gamma}_{adj}$	$\hat{\beta}_{rs}$	$\hat{\gamma}_{rs}$	R <sup>2</sup> {R <sup>2</sup> adj}
1	-0.0798 (0.7118) [0.2122]	0.6970 (0.0000) [0.1149]	0.1702 (0.0000) [0.0193]	-0.0534 (0.4732) [0.0727]					0.920 {0.905}
2	-0.1123 (0.6741) [0.2622]	0.7636 (0.0000) [0.0924]			0.3004 (0.0000) [0.0545]	-0.1905 (0.0032) [0.0551]			0.916 {0.900}
3	0.5803 (0.0207) [0.2260]	0.5331 (0.0000) [0.0859]					0.5492 (0.0000) [0.0743]	0.1758 (0.1839) [0.1266]	0.927 {0.914}
4	0.5114 (0.0597) [0.2496]	0.5886 (0.0000) [0.0733]			0.1799 (0.0138) [0.0639]	-0.1792 (0.0004) [0.0386]	0.3686 (0.0371) [0.1600]	0.2574 (0.1454) [0.1669]	0.954 {0.937}

Note: p-values within parentheses, standard errors within brackets. Both are corrected for autocorrelation according to the Newey-West method.

The most important implication of extending the analysis to a more general dynamic specification is that the role of the regime shift premium has increased. The contemporaneous regime shift premium has a significant effect regardless of whether an unadjusted forward interest rate is included in the regression or not. Moreover, the long-run effect of a change in the regime shift component in the preferred regression is around 1.5.<sup>31</sup> If we ignore the insignificant impact from the lagged regime shift premium, the long-run effect is around 0.9 and not statistically significant from 1. We cannot exclude the possibility that  $\beta_{rs} \approx 1$ , i.e. that changes in the regime shift premium are associated with corresponding changes in investors' inflation expectations on a one-to-one basis. This observation suggests that the (long-run) decline of investors' inflation expectations from about 4% in 1993 to 2% in recent years mainly reflects a corresponding decline of the regime shift premium from 2 to 0%. An inspection of Figure 3 supports this view.

<sup>31</sup> The long-run effect is calculated as  $(\hat{\beta}_{rs} + \hat{\gamma}_{rs})/(1 - \hat{\delta})$ .

To summarise, we find that the simple forward rate rules in difference form (see Table 2) appear to reflect short-run changes in investors' inflation expectations *within the regime* fairly well, but that these rules are misspecified in the sense that they fail to accurately capture the long-run trend of improved credibility of the low inflation policy (quantified as a decline in the regime shift premium) in the sample period.

## Summary and conclusions

In this paper two arguments are put forward to explain the relatively high and volatile development of forward interest rates in Sweden. First, the paper presents a regime shift model, in which investors' fears that the economy will switch to a high inflation regime give rise to a regime shift premium for holding bonds. Estimates of regime shift premia in the forward interest rates are broadly consistent with the implications of the model. Fluctuating regime shift premia are one explanation why forward interest rates have been volatile in Sweden. The analysis also includes term premia, which occasionally are of quantitative importance, but normally appear to be small.

One may argue that the regime shift premium really is a component of a time-varying term premium although it is difficult to justify such an explanation from economic theory. However, there is evidence that investors' inflation expectations obtained from surveys reflect regime shift expectations indicating that the regime shift premium really has something to do with investors' inflation expectations. The analysis suggests that the observed long-run decline of investors' inflation expectations from 4 to 2% can mainly be attributed to gradually improved credibility, manifested by a decreasing (and disappearing) regime shift premium. Regardless of the interpretation of the regime shift premium its quantitative impact on Swedish forward interest rates is hard to ignore.

Another explanation for relatively volatile forward rates is that forward interest rates also reflect investors' expectations about future monetary policy actions (changes in the short-term real interest rate), which tend to amplify the effect on forward interest rates that fluctuating inflation expectations give rise to. An increase (decrease) in investors' inflation expectations is associated with an increase (decrease) in the future short-term real interest rate. This seems to be a new important mechanism that has not been considered in the literature, and an empirical analysis underscores its quantitative relevance. It is likely that this mechanism is more important today than in the eighties, since the increased focus on direct inflation targeting in recent years has created a more transparent link between inflation prospects and monetary policy actions.

From a monetary policy analysis perspective at least two conclusions can be drawn. First, during periods of fluctuating long-term forward interest rates (relative Germany) one should be careful of using short and medium-term forward interest rates as monetary policy indicators. In principle, forward interest rates should be adjusted for regime shifts premia, which reflect long-run trends in credibility for the inflation target rather than cyclical changes of inflation expectations (within the low inflation regime). Notice that this principle is valid even if the quantity that is called a regime shift premium really is an additional component of a time varying term premium. Second, one should only attribute about 20% of the movements in Swedish one-year term forward interest rates (adjusted for the regime shift premium) to changed inflation expectations of investors. Finally, even if the principles mentioned above are important when using forward interest rates as monetary policy indicators, another important aspect highlighted in this study, is the role of inflation expectations in the transmission mechanism. Expectations of higher inflation pressures in the medium term will lead to higher medium term real interest rates before the expected tightening of monetary policy takes place.



## Appendix

### A. Derivation of expression (7)

Let  $Q_i(t)$ ,  $i = 1, 2$  or  $3$ , denote  $\text{Prob}[S(t+t_0) = i | S(t_0) = i]$ . It follows from the theory of finite state continuous time Markov Chains (see e.g. Karlin and Taylor (1975) p.150-2) that  $Q = (Q_1, Q_2, Q_3)'$  satisfies

$$\frac{dQ}{dt} = \Lambda Q, \quad \Lambda = \begin{pmatrix} -\gamma & \gamma & 0 \\ \alpha & -(\alpha+\beta) & \beta \\ 0 & \gamma & -\gamma \end{pmatrix}, \quad Q(t_0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (\text{A1})$$

It is straightforward to verify that the eigenvalues of  $\Lambda$  are  $0$ ,  $-\gamma$  and  $-(\alpha+\beta+\gamma)$ , with corresponding eigenvectors (proportional to)  $v_1 = (1, 1, 1)'$ ,  $v_2 = (\beta, 0, -\alpha)'$  and

$v_3 = (-\gamma/(\alpha+\beta), 1, -\gamma/(\alpha+\beta))'$ , implying that the solution to the system (A1) is of the form

$$Q(t) = av_1 + bv_2 e^{-\gamma(t-t_0)} + cv_3 e^{-\lambda(t-t_0)}, \quad \lambda = \alpha+\beta+\gamma \quad (\text{A2})$$

The constants  $a$ ,  $b$ , and  $c$  can be determined from the initial condition  $Q(t_0) = (1, 0, 0)'$  to be

$$a = \alpha/\lambda, \quad b = 1/(\alpha+\beta), \quad c = -\alpha/\lambda \quad (\text{A3})$$

In particular we have  $P(\tau) = \text{Prob}[S(t_0+\tau) = 1 | S(t_0) = 2] = Q_2(t_0+\tau) = \frac{\alpha}{\lambda} [1 - e^{-\lambda\tau}]$ , which was to be shown.

## B. Estimation of excess holding return equation

In this appendix we report the estimates of the excess holding return equation (see Dillén (1996) for details) using the GARCH-M estimation technique (see Engle, Lilien and Robins (1987)). The purpose of this estimation is to obtain an estimate of the variance term ( $\omega(t,\tau)$ ), which is of GARCH(1,1) type and that is used in the excess forward return equation (eq. (5)). As discussed by Dillén (1996), the estimates of the regime shift and term premia (captured by the estimates  $\hat{\psi}$  and  $\hat{\theta}$ ) are highly inefficient since the excess holding return equation contains much more noise than the excess forward return equation.<sup>32</sup> The estimates in the variance equation ((eq. (ii)) appears to be more robust. The two-step procedure of using the estimated variance from a GARCH-M estimation in the excess forward return equation (which generates more efficient estimates) appears to be a novelty that generates more robust estimates of parameters.

### Estimation of excess holding return equation

(i)  $v(t,\tau) = \rho_{0h}(\tau) + \theta_h(\tau)\omega(t,\tau) + \psi(\tau)\delta_L(t) + u(t,\tau), E_{t-1}[u(t,\tau)] = 0$

(ii)  $\text{Var}_{t-1}[u(t,\tau)] = \omega(t,\tau); \omega(t,\tau) = \text{Var}_{t-1}[u(t,\tau)] = \alpha_0(\tau) + \alpha_1(\tau)u^2(t-1, \tau) + \beta_0(\tau)\omega(t-1, \tau)$

	$\hat{\rho}_h$	$\hat{\theta}_h$	$\hat{\psi}$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	$\ln L$
$\tau=3$ months	0.0477 (0.393)	0.0055 (0.193)	0.0411 (0.406)	0.0350 (4.506)	0.2060 (5.652)	0.8086 (14.693)	-246.3
$\tau=6$ months	0.3195 (1.051)	0.0250 (0.929)	-0.1701 (-0.437)	0.0230 (1.254)	0.1083 (5.170)	0.8981 (20.092)	-482.3
$\tau=12$ months	0.9429 (1.146)	0.0066 (0.538)	-0.1733 (-0.1500)	0.9539 (0.9137)	0.1464 (5.080)	0.8572 (19.034)	-717.3

Note: t-values in parentheses;  $\ln L$  is the maximised log likelihood value.

<sup>32</sup> The relation between the sensitivity factors  $\psi$  and  $\phi$  is explained in Dillén (1996).

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