The determination of interest rates and the exchange rate in the Bank of Canada's Quarterly Projection Model

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Introduction

In August 1993, the Quarterly Projection Model or QPM as it is known, replaced RDXF as the main model used by the staff of the Bank of Canada for economic projections and policy analysis. In this paper, our modest goal is to outline the determination of interest rates and the exchange rate in QPM, with particular emphasis on the interaction between these two key prices and the outcomes for real and nominal variables in the model. In doing so we also highlight some areas for future work.

The traditional approach to empirical macro modelling has been to estimate the individual equations for all the endogenous variables in the system, and then to combine these to form a macro model. Typically, empirical interest rate and exchange rate equations and the predictive power of these equations were a key ingredient in the model. Experience has shown, however, that the predictive power tends to deteriorate as new data are added outside the estimation period. This result is not very surprising and reflects a number of problems, including: simultaneity, the dangers of over-fitting, changes in policy regimes, and other structural shifts. Of particular relevance in the current context was the announcement of explicit inflation targets in Canada in February 1991. This marked a change in regime that should affect the way agents are forming their expectations, and therefore, the behaviour of interest rates and exchange rates.

In constructing QPM, the Bank staff broke from the past practice of building a model by using single-equation econometric techniques. QPM is not an estimated model; it is calibrated. Calibration has the important advantage that it allows the model builder to put more weight on features of the data that are thought to reflect "deep" structure, while putting relatively less weight on historical correlations that have more to do with shocks over history and the policy regime in place at the time. As monetary policy has increasingly emphasised medium to long-run objectives, the importance of a theoretically consistent tool for policy analysis has been reinforced.

The modelling of interest rates and the exchange rate in QPM reflects the model's overall objective - to meld the rigorous theoretical structure necessary for modern policy analysis with the practical requirements of a model designed to support economic projections. Interest rate and exchange rate determination in the model combines the essential elements of mainstream economic theory with a healthy respect for the short-run features of the data. The result is a model with a well-defined long-run equilibrium that produces dynamics that both converge on the long-run solution and replicate the stylised facts as captured by the short-run correlation structure of the data. Thus, although it is not estimated, we nevertheless consider QPM to be very much an empirical macro model.

In outlining how interest rates and the exchange rate are modelled in QPM, we begin in Section 2 with a brief overview of the model. This is followed in Section 3 by a description of the interest rate sector. Section 4 discusses the real and nominal exchange rates. With this as background, Section 5 illustrates the behaviour of interest rates and exchange rates under two policy shocks: a monetary disinflation shock and a fiscal shock that permanently raises the level of government indebtedness. We conclude by discussing some extensions to the base case model that would incorporate a richer description of interest rate and exchange rate behaviour.

1. Overview of QPM

In comparison to most other models used for similar purposes, QPM is relatively small. This reflects a conscious decision to abstract from the micro-sectoral details of the Canadian economy in order to focus on the core macro linkages in a theoretically consistent framework that takes full account of long-run budget constraints.

At the heart of the QPM is a steady-state model (see Black, Laxton, Rose and Tetlow 1994). The steady-state model describes the determinants of the long-term choices made by profit-maximising firms and overlapping generations of consumers, given the policy settings of the fiscal and monetary authorities, all in the context of an open economy with important relationships with the rest of the world. The economic behaviour of these agents, given their long-run budget constraints, and the market-clearing conditions of an open economy determine the long-run equilibrium or steady state to which the dynamic model converges.

The dynamic model has several important features[1]. First, agents in QPM are forward-looking. In particular, they act based on intertemporal optimisation, conditioned by expectations that are forward-looking, albeit not fully model-consistent. The evolution of expectations plays a key role in the overall dynamic response to shocks. In addition, adjustment of both quantities and prices is presumed to be costly, so there are also "intrinsic" elements to the model's dynamic properties. These include labour market contracts, the fixed costs associated with investment, and so on.

Second, the model provides a complete and consistent solution for all stocks and flows. When a shock affects the level of a stock, this often creates the necessity for cycles in flow variables, which can be an important contributor to overall dynamics.

Third, monetary policy is conducted using a forward-looking policy rule that calls for the monetary authority to adjust its policy instrument in such a way as to bring expectations into line with the targeted inflation rate. The instrument of monetary policy in QPM is the short-term interest rate, which has its influence on spending through the slope of the yield curve. Movements in the short-term nominal interest rate also affect the nominal exchange rate, and hence import prices and inflation, through an uncovered interest parity condition. Inflation is influenced directly by the state of excess demand and by expectations about future inflation.

Finally, fiscal policy in QPM, like monetary policy, is characterised by a set of objectives that are consistent with achieving a sustainable equilibrium. In particular, the fiscal authority picks a target level of government expenditures on goods and services as a proportion of output, and a target debt-to-GDP ratio. Taxes net of transfers and the deficit adjust to achieve these targets.

With this overview as background, we turn now to the main focus of this paper - modelling interest rates and the exchange rate.

2. Interest rates

2. 1 The yield curve and monetary stance

Canada is a relatively small economy with highly internationally integrated financial markets. As a result, over the longer term, real interest rates in Canada are largely determined in world markets. However, over the shorter term, domestic monetary policy exerts an important influence on real interest rates. Monetary actions affect short-term interest rates most directly, and these effects

1 For an overview of the QPM system, see Poloz, Rose and Tetlow (1994). See also Hunt, O'Reilly and Tetlow (1995) for a discussion of the model's simulation properties.

reverberate up the term structure and over to the exchange rate, all of which impact on aggregate spending and ultimately inflation.

The determination of interest rates in QPM reflects this characterisation of the transmission mechanism. Real interest rates in QPM are pinned to world real rates in the long run up to an exogenously specified risk premium. In the short run, monetary actions can affect real rates because prices are slow to adjust. The instrument of monetary policy in the model is the short-term nominal interest rate, and monetary actions are transmitted to real activity through the impact of changes in the short rate on the slope of the yield curve. Formally, the link between the yield curve and real activity in the model arises because consumer expenditures, housing and inventories (which are aggregated together) are a function of the yield spread - the short-term interest rate less the long rate.

The use of the yield spread as the key variable through which monetary policy is transmitted to real activity in QPM reflects two main considerations. First, it reflects the view that the yield spread provides a better indicator of the stance of monetary policy relative to the underlying momentum in the economy than do short-term real interest rates alone. Second, the use of the yield spread provides a parsimonious way of capturing the effects of the full term structure of interest rates on aggregate spending.

QPM is used for economic projections, and an important challenge in the projection exercise is to interpret the underlying shocks in the economy that are producing the incoming data. In this context, the yield spread has the attractive feature that it helps in isolating monetary influences on real interest rates. Movements in both long and short rates reflect fluctuations in the equilibrium or natural real interest rate (as determined by productivity and thrift in the world economy). Changes in the short rate also reflect changes in the stance of monetary policy, while long rates are relatively immune to changes in monetary conditions; thus to a large extent the yield spread serves to isolate the monetary component of changes in real interest rates.

For the monetary authority, there is useful information in long rates on the credibility of monetary policy which can serve as a useful guide to the changes in short-term interest rates that are required to control inflation. In the typical interest rate cycle, the long rate will initially rise with short rates when the central bank tightens monetary conditions to combat inflation, since, initially, credibility will tend to be low. As the central bank reveals its determination to reverse the rise in inflation, the long rate may begin to fall. This serves as a signal to the monetary authority that it can ease off a little on short rates. Measuring monetary stance in terms of the spread is thus a convenient way to summarise this relationship between policy actions and their credibility.

The yield spread also captures an intertemporal aspect of consumers' expenditure decisions. In particular, the spread provides information on the expected path of interest rates and this may influence the timing of expenditures and thus the dynamics of aggregate demand. For example, a consumer who is considering purchasing a car or a house may be enticed to do so sooner as opposed to later if the long rate is considerably above the short rate indicating that short rates are expected to rise in the future. Conversely, faced with an inverted yield curve, the consumer is likely to postpone major expenditures on the expectation that the cost of financing is going to fall.

2. 2 Modelling short and long rates

The interest rate sector in QPM comprises three main equations: a monetary policy rule, an equation for the representative long-term interest rate (10-year and over government of Canada bond rate) and an identity that describes the yield spread. One can think of the latter as solving for the short-term interest rate (the 90-day commercial paper rate), although in actual simulations, the yield curve, long and short-term interest rates are all determined simultaneously.

The approach that was used to calibrate the interest rate sector provides a good illustration of the general principles that were followed in the construction of QPM. In particular, it shows how one can combine traditional empirical methods with the prediction of theoretical models to seek to exploit the advantages of both approaches. This flexible approach has a strong appeal when building a model that is designed as both a projection and a research tool.

2.2.1 A forward-looking policy rule

In a forward-looking model, the role of the monetary authority is to provide a nominal anchor for expectations. Because inflation expectations depend, at least in part, on future monetary policy, a policy rule needs to be specified in terms of an attainable objective. Without an endogenous policy response to economic developments, agents do not have enough information to form their expectations, and nominal values become undefined (in other words, the model does not solve). An endogenous policy rule or reaction function is therefore an essential part of QPM. The rule specifies a path for the monetary policy variable in order to achieve an intermediate or final policy objective.

As discussed above, in QPM, the policy variable that is used is the yield spread. More specifically, the reaction function determines a path for the yield spread gap. The yield spread is defined as the difference between three-month and ten-year interest rates (50 basis points in steady state), while the yield spread gap is the difference between the actual yield spread and its equilibrium value. The reaction is specified in terms of the ultimate policy objective, that is to control inflation at some target level. In the simulations presented in the following sections, the target is assumed to be 2 per cent, the mid-point of the current official inflation target bands in Canada.

Because it takes time for monetary policy actions to have their effect on aggregate demand and inflation, monetary authorities are forced to look ahead when setting a path for their instrument. In the model, this is achieved with an explicit forward-looking policy rule. The policy instrument depends on the model's predictions of inflation in future periods. The base case reaction function used has the following form:

$$yieldgap_t = \alpha_1 \left\{ \sum_{k=6}^{7} 1/2 \left(\dot{P}_{t+k} - \dot{P}^{\text{targ}}_{t+k} \right) \right\} + \alpha_2 \, yieldgap_{t-1} \tag{1}$$

where $yieldgap_t = \left(i_t^s - i_t^l\right) - \left(i_t^s - i_t^l\right)_t^{ss}$ $\left(\dot{P} - \dot{P}^{\text{targ}}\right)$ is the deviation of inflation from its targeted rate, i_t^s and i_t^l are the short and long-term interest rates respectively, and ss denotes a steady-state value. As shown, the yield spread gap is a function of the deviation of inflation (based on the CPI excluding food and energy) from its target six to seven quarters in the future. The reaction function also includes a lagged dependent variable to smooth the movement of the policy instrument. If a shock tends to push inflation above (below) its targeted level in six to seven quarters, the authority increases (decreases) its instrument, the 90-day commercial paper rate, so as to achieve a level for the yield spread which will result in aggregate demand conditions that will bring inflation back towards its target.

Although this reaction function is an ad hoc rule, in the sense that it is not derived from an optimal control problem, the choice of parameters and the degree of forward-lookingness were not chosen arbitrarily. The six-to-seven quarters horizon is a good approximation of the sort of horizon over which monetary policy has a meaningful effect on trend inflation. Trying to hit an inflation target over a very short period of time would imply considerable volatility in interest rates (and the exchange rate), leading to an instrument instability problem. Even though the reaction function does not allow for secondary objectives other than smoothing of the policy instrument, the magnitude of the parameter, which is linked to the degree of inflation drift that the authorities are ready to accept, was also chosen by taking into consideration that the authorities may not be completely indifferent to the path of other macroeconomic variables.

The use of a forward-looking rule implies that the monetary authority has knowledge of the origin and nature of the shocks. In a model in which private agents are assumed to be (at least partly) forward-looking, it would be hard to argue that the authorities should not be characterised by the same behaviour. Because the world is plagued with uncertainty, it may nevertheless be very difficult for the authorities to extract information from volatile economic data. To take this into account, one can easily entertain shocks in QPM from which a more muted policy response is assumed. The important point to stress is that the model response to any shock depends importantly on the specification of the policy rule. In other words, in QPM, the policy regime matters.

2.2.2 Long-term interest rate

Given that the yield spread is the main monetary variable in QPM, the determination of the long-term interest rate plays a key role in the transmission mechanism. In previous models used at the Bank, the long-term interest rate was determined by a distributed lag of Canadian short-term rates and US long and short-term interest rates. This equation fit the historical data very well, as Canadian and US long-term interest rates have been strongly correlated over the post-war period. However, there is a presumption that this strong correlation reflects the fact that the two countries have generally faced similar shocks and have responded to them in similar ways. If one wants to consider scenarios with different inflation paths in the two countries, more structure needs to be added to the model.

The standard theory used in most policy simulation models is the expectations theory of the term structure, according to which the yield on the long-term bond should equal a weighted average of the current and expected future short-term interest rates, up to a term premium. However, when combined with pure model-consistent expectations, this theory is unable to replicate the historical behaviour of longer-term interest rates. The existence of time-varying term premia, the unpredictability of short-term rates, and the lack of credibility of macroeconomic policies are among the reasons that have been offered for explaining this apparent failure. Whatever the source of the failure, it would not seem appropriate to rely exclusively on this theory in a model designed in part for forecasting purposes.

For this reason, the QPM equation is constructed as a combination of both the expectations model and a reduced-form model, as follows:

$$i_t^l = \beta_1 \left\{ \theta \left[\prod_{k=0}^{39} i_{t+k}^s \right]^{\frac{1}{40}} + (1-\theta_1) i_t^s \right\} + termpremium$$
$$+ (1-\beta_1)\left(\gamma_1\left[i_t^{l,US} + \dot{P}^e - \dot{P}^{e,US} + risk_t^l\right] + (1-\gamma_1) i_t^s\right)$$

where $\dot{P}^e$ denotes the expected rate of inflation (based on the GDP deflator). The first part represents the expectations theory. As in other sectors of the model, expectations are expressed as a weighted combination of the model-consistent and extrapolative solutions. However, in this case, the extrapolative solution does not contain any lags. Moreover, it is represented by the contemporaneous short-term rate only, based on the view that financial markets respond quickly to new information. The second part is built from the traditional estimated equation. The Canadian rate is a function of the US long-term rate adjusted by the inflation expectations differential between the two countries and a risk premium, ($risk^l$). The contemporaneous short-term rate is included with a fairly large coefficient in order to mimic the historical sensitivity of long-term rates to movements in short-term rates.

The weight currently assigned to the reduced-form portion is, at 75 per cent, quite high. It could be reduced if, over time, more support develops for the expectations model, for instance, as a result of the adoption of clear policy targets by the authorities. The country risk premium and the term premium are currently exogenous. In steady-state, the long-term interest rate simply equals the short-term interest rate plus the term premium.

3. Real and nominal exchange rates

In addition to affecting interest rates, monetary policy in QPM influences the exchange rate with important effects on trade. The exchange rate also responds to external shocks, such as changes in world commodity prices, providing an important shock absorber through which the Canadian economy digests changes in external conditions. Over the longer term, the real exchange rate is the key relative price in the model that re-equilibrates the economy. Since real domestic interest rates are pinned to world rates in the long run, it is the real exchange rate that must ultimately adjust to bring aggregate demand in line with aggregate supply.

As a key relative price in the model, the real exchange rate is one of the "most endogenous" variables in the system in the sense that its determination reflects the simultaneous solution of all the essential elements of the model - monetary conditions, real allocations, prices and inflation, and international arbitrage. As a result, there is no single equation or even a small group of equations in the model that can be meaningfully described as determining the exchange rate. Having said this, there are some key building blocks in the determination of the exchange rate in QPM.

In the short run, the two key relationships influencing the nominal and real exchange rates are an interest parity condition and aggregate price adjustment. The interest parity condition requires investors in Canadian dollar assets to be compensated for expected changes in the value of the Canadian dollar:

$$i_t^s = i_t^{s*} + \left(s_t^e - s_t\right) + risk \tag{3}$$

where $\left(s_t^e - s_t\right)$ is the expected change in the nominal exchange rate (at annual rates), and *risk* is an exogenous country risk premium (set to 40 basis points in the steady state). The exchange rate is defined as the Canadian dollar price of foreign exchange, where foreign exchange is the trade-weighted basket of currencies of the rest of the G-7 countries (hereafter we will call this the G-6). Since 80 per cent of Canada's trade is with the United States, s_t is typically quite similar to the Canada-US exchange rate. The real exchange rate is defined as the nominal rate adjusted for relative prices:

$$e_t = \frac{s_t P_t^*}{P_t} \tag{4}$$

where e_t is the real exchange rate, and P_t and P_t^* are the domestic and foreign price levels respectively (measured by the GDP deflator). Parallel to the definition of the real exchange rate, the foreign price level is the trade-weighted price level in the G-6. Since the foreign price level is taken to be exogenous, the link between real and nominal exchange rates depends on the behaviour of domestic prices.

An important feature of exchange rate data is that nominal and real exchange rates tend to move together (see Mussa, 1986). This is captured in the model by sluggish adjustment of the aggregate price level. For example, a rise in domestic interest rates will result in an appreciation of the Canadian dollar vis-à-vis the G-6 currencies that is large enough to generate an expected depreciation in the future so as to satisfy the interest parity arbitrage condition (3). Since domestic price adjustments are gradual, due both to sluggish adjustment of expectations and to rigidities such as nominal contracts, the nominal exchange rate appreciation also results in a real appreciation from (4).

In the short run, the real exchange rate is, therefore, largely determined by the behaviour of the nominal rate together with the pace of price adjustments. Looking beyond the short run when price adjustments have caught up with nominal exchange rate changes, the real exchange rate adjusts so that the trade flows in the model will sustain the real equilibrium. More specifically, in the long run the real exchange rate adjusts to produce the trade balance surplus that is required to sustain the desired level of net foreign assets.

In the model, consumers hold three types of assets: the national capital stock, the consolidated federal, provincial and municipal government debt, and net foreign assets. The optimal level of physical capital is chosen by firms, and is essentially determined by the world real interest rate and the rate of labour-embodied technological progress. The level of government debt relative to GDP is chosen by the government, so this leaves net foreign assets as the residual component of non-human wealth through which consumers can adjust the level of wealth to be consistent with their desired flow of consumption expenditures. Overlapping generations of consumers accumulate non-human wealth so as to maximise the discounted present value of the utility of consumption over their expected lifetimes.

A permanent supply shock will, in general, alter the consumers optimal level of wealth and consumption, and this in turn will show up as a change in the level of net foreign assets. To sustain this level of net foreign assets, the trade balance of the economy must be consistent with the flows of interest payments on the outstanding stock of net foreign assets. This is achieved by the adjustment of the real exchange rate. For example, since Canada is assumed to face a downward sloping world demand curve for its exports, a real depreciation will raise exports, thereby improving the trade balance.

4. Two policy shocks

Perhaps the easiest way to understand how interest rates and the exchange rate are determined in QPM is to see the model in action. Below we consider two different policy shocks: a monetary disinflation shock, and a fiscal shock that permanently raises the level of government indebtedness. The disinflation shock highlights the transmission of monetary actions from interest rates to the exchange rate, and on to real behaviour and inflation. Since there is no long-run trade-off in the model between inflation and output, the disinflation shock does not influence the real economy in the long run.

4. 1 Disinflation shock

4.1.1 Base QPM

Figure 1 shows the results of a permanent one percentage point reduction in the rate of inflation targeted by the monetary authority. The solid lines show the response for the base calibration of QPM.

In order to reduce inflation, the authorities tighten monetary conditions. Short-term interest rates increase by almost 100 basis points on average during the first year. Long-term rates, however, increase only marginally in the first two quarters and start falling below control by year-end, as expectations of lower inflation develop rapidly. The resulting rise in the yield spread reduces consumption, and investment spending also falls due to the decline in expected output and the rising cost of capital.

The rise in short rates also results very quickly in an appreciation of the real value of the Canadian dollar which peaks at 0.9 per cent above its starting point. This rise in the value of the Canadian dollar leads to a marked decline in exports. In the very short run there is also a small rise in imports, so the trade balance deteriorates initially, thereby contributing to the emergence of excess supply. Beyond the very short run, imports also decline sharply despite the exchange rate appreciation, as consumption falls off in response to higher interest rates and declining employment and personal incomes. As a result, consistent with the stylised facts, the trade balance turns positive as the downturn gains momentum, and this contributes to the more rapid recovery of aggregate demand (as measured by the output gap) as compared to consumption.

Figure 1
Permanent 1 percentage point reduction in inflation target
Time in years

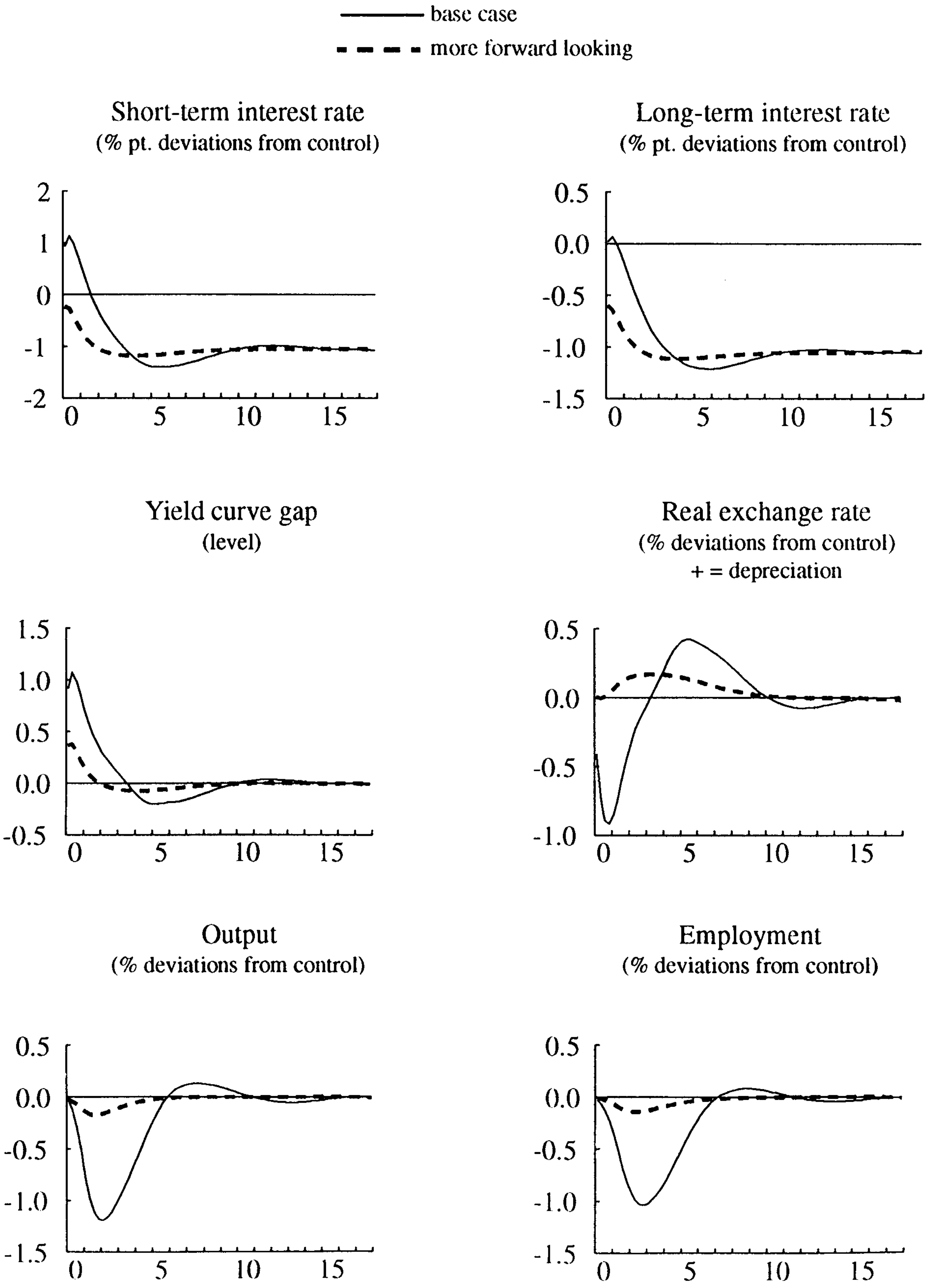
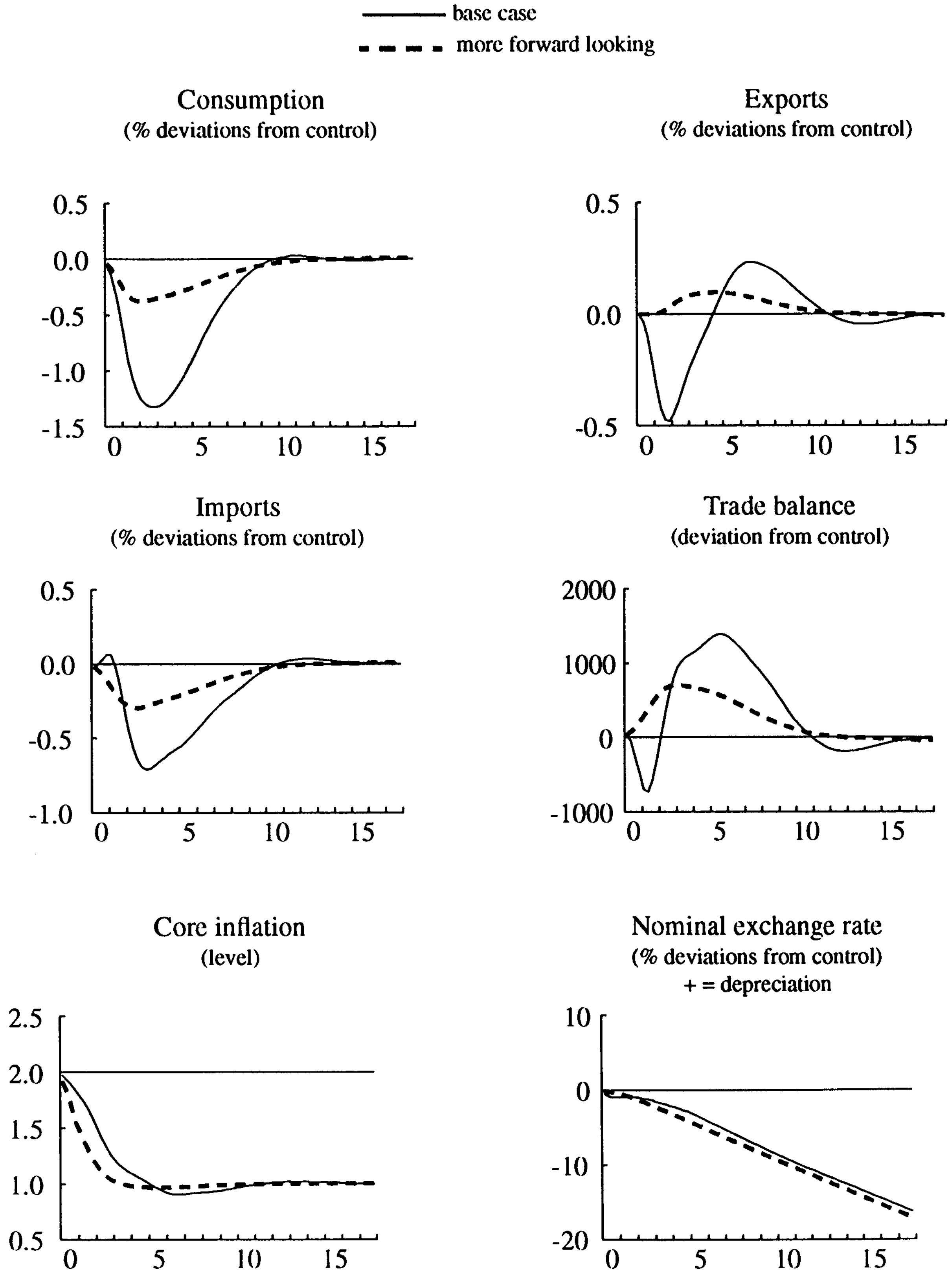


Figure 1 (cont.)

Permanent 1 percentage point reduction in inflation target

Time in years



The maximum effect on aggregate demand is felt in the third year, when the output gap averages 1.1 per cent. It takes almost five years for inflation to reach its new 1 per cent target level. Note that by then, interest rates have overshot their long-run equilibrium level. This is required to curtail the building of disinflationary momentum. The total output foregone in reducing inflation by 1 percentage point from steady state is about 3.0 per cent of one year's output.

As the economy settles into long-run equilibrium, all real variables in the model return to their initial steady-state levels, while inflation remains permanently lower. Since foreign inflation is unchanged this results in an on-going appreciation of the nominal exchange rate at a rate of 1.0 per cent per year, while the real exchange rate returns to its initial steady-state level.

4.1.2 More forward-looking behaviour

The base version of QPM puts a considerable weight on the backward-looking component in price and wage expectations. If one were to assume, instead, that expectations are to a large extent forward-looking, the costs of disinflating would appear extremely low, as can be seen by the dashed lines in Figure 1. In this alternative scenario, we increased the weight on the forward-looking component from 30 per cent to 70 per cent in the price equations and from 10 per cent to 50 per cent in the marginal cost and nominal wage equations.

Long-term rates fall significantly (60 basis points) on impact. This allows short-term rates to fall as well, but to a lesser extent, such that the yield spread tightens by about 30 basis points on average during the first year of the simulation. This small tightening is sufficient to bring inflation down to 1 per cent after three years. The cumulative output loss in this scenario amounts to only 0.6 per cent of one year's output. This small output loss reflects both the considerably smaller decline in consumption together with the rise in exports. The latter reflects the fact that the fall in short rates produces a small depreciation of the real and nominal exchange rates.

It is hard to believe that the monetary authority could, in fact, engineer a reduction in inflation without having to raise short-term interest rates or the value of the Canadian currency. The above results reflect the fact that in a deterministic model, forward-looking expectations become equivalent to perfect foresight. Assigning a significant weight to the extrapolative solution provides a source of propagation and allows one to produce a dynamic behaviour for the economy that seems to replicate the properties of the data fairly well. Although one might argue that agents are sophisticated enough and that their behaviour should not be represented by naive autoregressive assumptions, relying only on model-consistent expectations in a model without costs of uncertainty would produce results that are not judged to be reasonable.

4.1.3 Less credibility for lower inflation

With regards to long-term interest rates, one might in fact argue that the speed at which expectations are revised is too rapid in the core version of the model. The experience of the last few years would seem to suggest that it takes a long time for financial markets to revise their expectations for long-term inflation. Agents may have faith in the ability of the central bank to achieve its current inflation targets but, nevertheless, assign a certain probability to an outcome where the authorities will have to revert back to a high inflation regime, for instance to mitigate fiscal problems.

To illustrate how the expectations behaviour in the bond market could affect the outcome of a monetary shock, we modified the long-term interest rate equation such that the expected inflation differential does not play any role during the first five years of the simulation. This is equivalent to assuming that inflation expectations in Canada do not diverge from those in the United States over this period. Thereafter, the expected inflation differential term is phased-in rapidly, over a period of five quarters.

Figure 2

Permanent 1 percentage point reduction in inflation target

Time in years

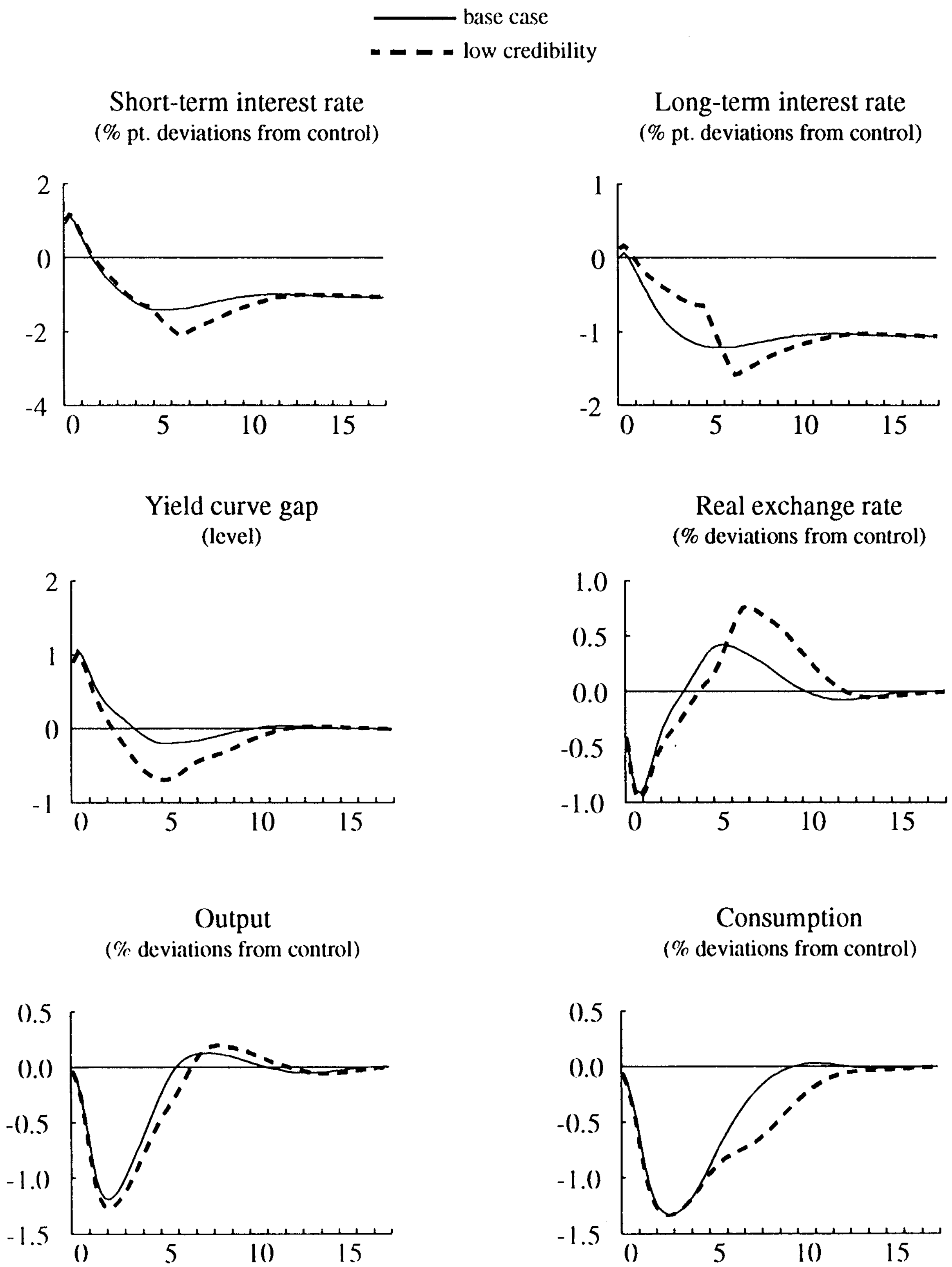
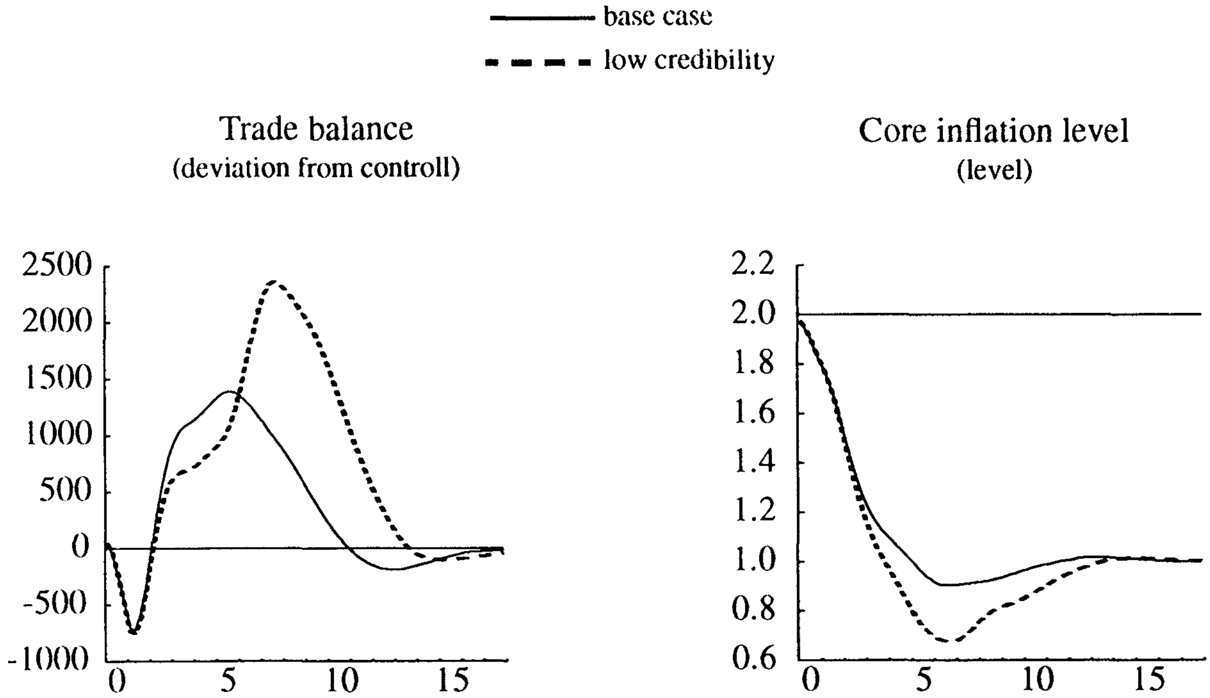


Figure 2 (cont.)
Permanent 1 percentage point reduction in inflation target
Time in years



The results of the disinflation shock under this assumption for the long-term interest rate behaviour are shown by the dashed lines in Figure 2 - the solid line continues to be the base QPM response. Initially, the results are very similar to what was obtained with the base version of the model, since the latter places a high weight on the autoregressive solution in the price expectations equation. By the third year of the simulation, long-term rates are about 50 basis points higher relative to the base-case scenario, and this difference persists for about three years. The cost of servicing the external debt is therefore higher and the negative effect on the current account balance and net foreign asset position is larger than in the base case. As a result, consumption remains below control for longer despite the bigger easing in monetary conditions. There is also more cycling in real output and inflation.

4. 2 A government debt shock

The disinflation shock has no long-run effects on real variables in the model. By contrast, since QPM is non-Ricardian, a change in the level of government debt (relative to GDP) does alter the economy's long-run equilibrium and result in permanent changes in consumption and the real exchange rate. The government debt shock also illustrates the short-run implications for interest rates when fiscal policy alters the level of aggregate demand and monetary policy must respond to achieve the inflation target.

Figure 3
Dynamic effects of a debt increase starting from the steady state
Time in years

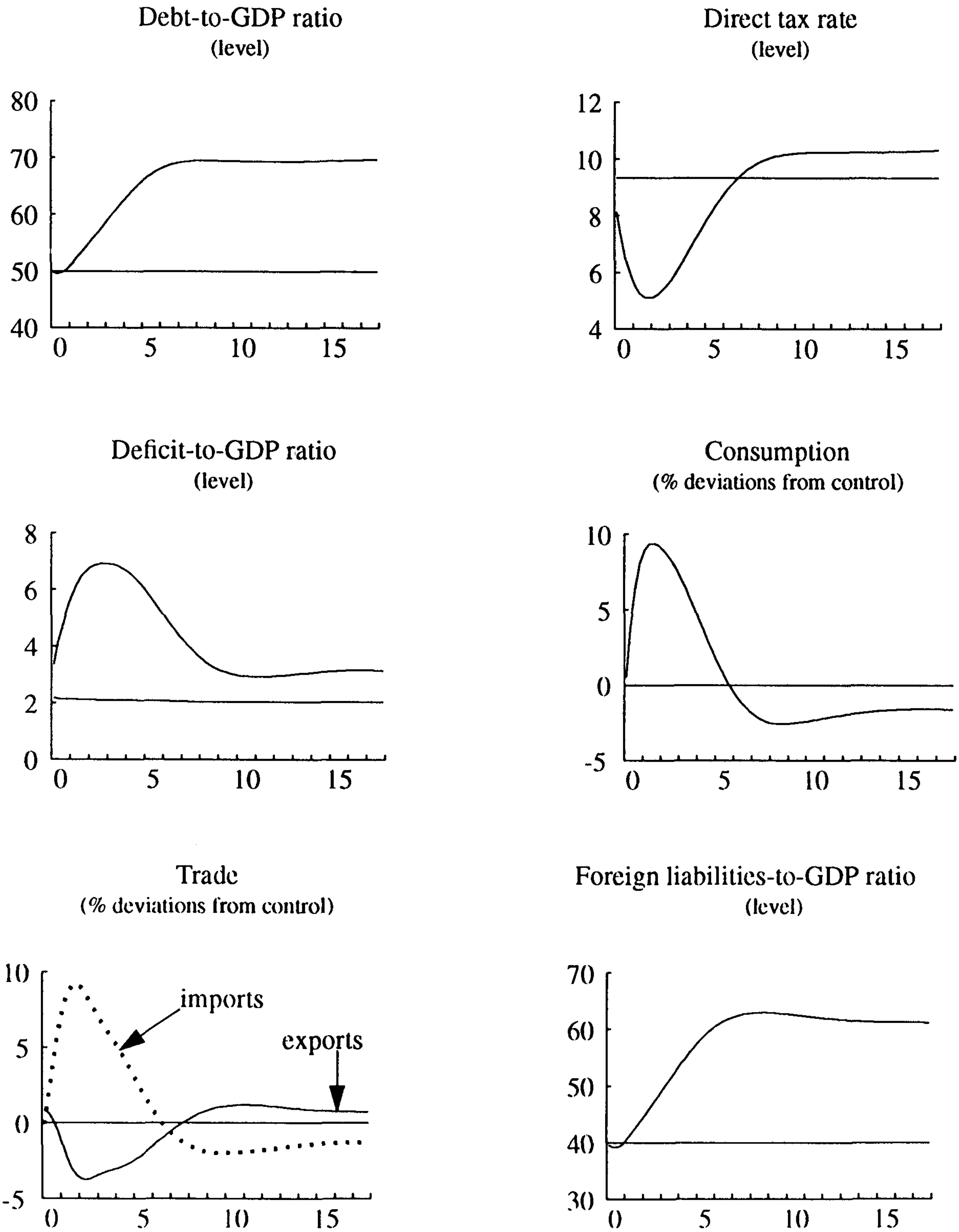
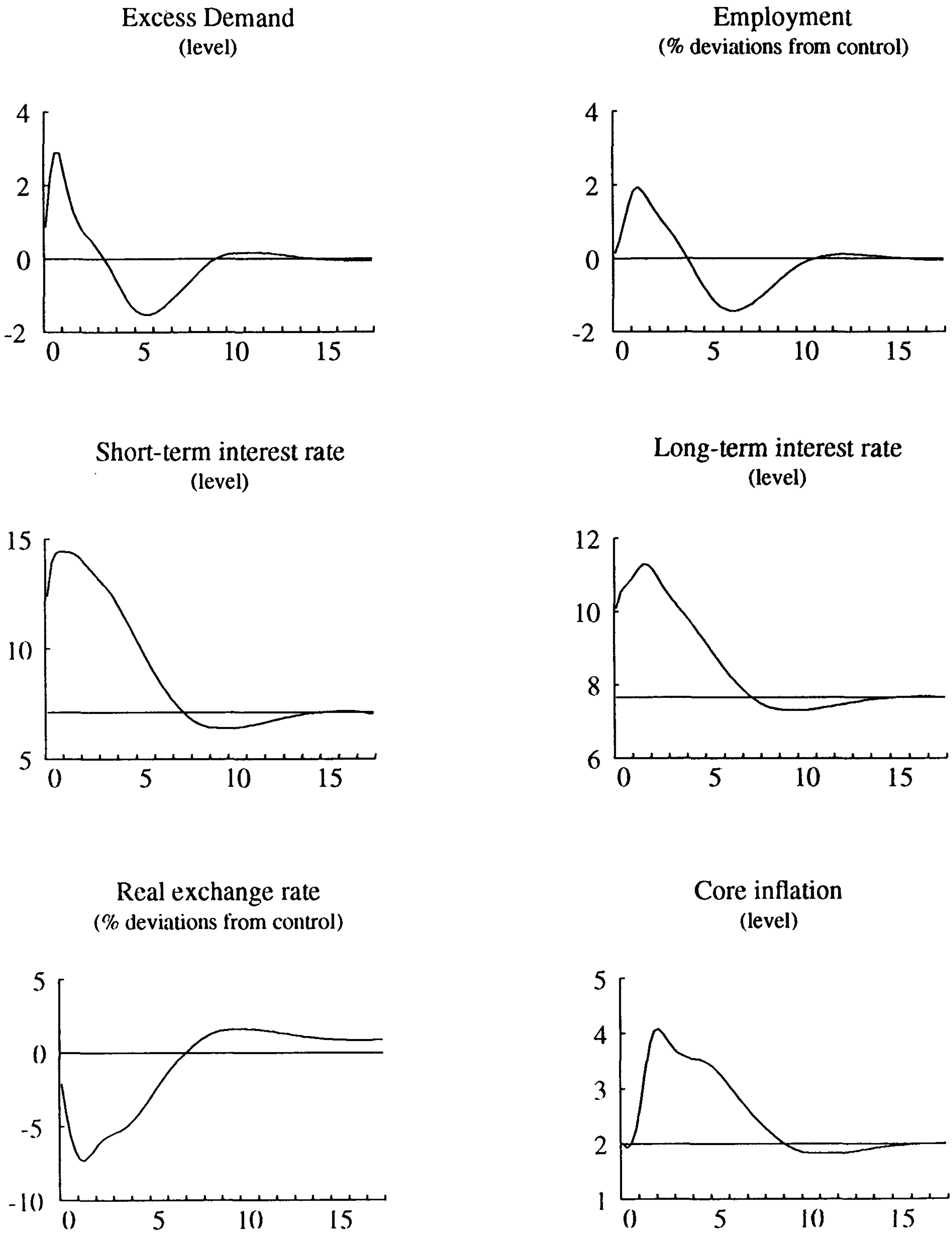


Figure 3 (cont.)

Dynamic effects of a debt increase starting from the steady state

Time in years



In QPM, government debt has real effects primarily for two related reasons. First, economic growth is fuelled by the birth (or immigration) of new consumers. Current consumers, therefore, act knowing that they will not be responsible for the full tax burden of servicing the debt, since some portion of this burden will automatically be assumed by future generations. Consumers also act knowing that they are mortal and may, therefore, not be around to pay even a reduced share of future taxes associated with current deficits. Thus, changes in government debt levels alter the real choices of households. Second, in the context of an open economy, there are consequences for net indebtedness to foreigners, which have an impact on the real exchange rate and this in turn feeds back to the level of output.

Figure 3 depicts the dynamic effects of a rise in the debt-to-GDP ratio from 50 to 70 per cent that is brought about by a cut in direct taxes. As shown, the tax cut is temporary, since in the new steady state with more government debt, taxes will be higher to support the higher debt service. However, current generations expect that some of the burden of higher future taxes will fall on future generations. As a result, for current generations the present value of the rise in disposable income during the temporary period with lower taxes is greater than the present value of the fall in disposable income thereafter. Households, therefore, increase their consumption in the short run rather than saving all the additional disposable income they receive. This rise in consumption increases imports and reduces exports as more output is absorbed domestically. The trade balance, therefore, deteriorates, and since Canada is a net international debtor, this results in a rise in net foreign liabilities. With more foreign liabilities, the steady-state trade surplus must be larger to cover the additional interest payments on the foreign debt. This requires a larger proportion of output to go to foreigners, leaving less for domestic consumption. Thus, while consumption rises in the short run, the impact of the higher level of foreign indebtedness is a permanently lower level of consumption thereafter.

Table 1

Steady-state effects of a rise in government debt

	Base QPM	**MRT extension**	**Country risk premium extension**
Shock to control (% change)			
Output	–0.29	–0.38	–0.48
Consumption	–1.54	–2.01	–0.99
Exports	0.78	1.02	0.08
Imports	–1.26	–1.65	–0.83
Capital stock	–0.84	–1.09	–1.39
Real exchange rate	0.90	1.17	0.28
Level as a percentage of GDP[1]			
Government debt [50.00]	70.00	70.00	70.00
Net foreign liabilites [40.00]	61.11	61.41	47.30
Government deficit [2.21]	3.10	3.10	3.10
Current account [–1.77]	–2.70	–2.72	–2.09
Interest payments to foreigners [2.67]	4.06	4.32	3.27
Taxes net of transfers [6.12]	6.84	7.13	7.05
Changes in rates in basis points			
Risk premium	0	34[2]	21[3]
User cost of capital	17	22	28

[1] The figures in square brackets are the intitial steady-state levels.

[2] This risk premium is applied only to the rate on government debt and net foreign liabilities.

[3] This risk premium is applied to the interest rates on government debt, net foreign assets and both long and short-term private domestic borrowing rates.

The short-run behaviour of interest rates and the exchange rate largely reflects the temporary rise in aggregate demand associated with the fiscal expansion together with the monetary reaction to this fiscal shock. Since the simulation starts from conditions of full economic capacity, the stimulus from fiscal policy pushes up inflation, which is inconsistent with the monetary objective of maintaining inflation at 2 per cent. The monetary authority, therefore, raises short rates, and via the interest parity condition, this leads to an appreciation of the Canadian dollar in the short run. This monetary reaction serves to dampen the consumption boom but, given the lags associated with the effects of monetary policy, inflation nevertheless rises to a peak of 4 per cent before turning the corner.

As the inflationary pressures abate in the medium term, short rates decline, eventually returning to their initial steady-state level. With the decline in interest rates, the real exchange rate depreciates but, unlike the real long and short-term interest rates, it does not simply return to its initial steady-state level. Rather in the long run, the real exchange rate must depreciate in order to stimulate exports and reduce imports enough to generate the larger trade balance surplus that is required to sustain the higher level of net foreign liabilities. The resulting real depreciation raises the cost of capital in Canada, since about 70 per cent of our machinery and equipment is imported. The impact of the long-run real depreciation is, therefore, to lower the steady-state capital stock, and thus output.

These long-run effects of the debt shock are summarised in the first column of results in Table 1. As shown, the real exchange rate depreciation is 0.9 per cent, which results in a relatively small decline in output of 0.3 per cent. With both lower output and higher foreign liabilities, consumption declines by 1.5 per cent.

5. Extending the base model: endogenous risk premiums

QPM has evolved since it went into regular use in August 1993, and it will continue to evolve as we learn how to improve it or add new features that might be particularly important for some issues. Looking ahead, one area that warrants more attention and is particularly relevant to interest rates and the exchange rate is the determination of the risk premiums in the model. In the base model, the risk premiums on different assets within Canada as well as the country risk premium for Canadian dollar-denominated assets are taken to be exogenous. However, the recent experience of Canada, as well as some other industrialised countries with high and rising levels of government debt, suggests that risk premiums can change substantially. By deferring taxes into the future, government deficits create uncertainty about how the government will ultimately deal with its obligations, the price of this uncertainty is a premium.

In a recent study, Macklem, Rose and Tetlow (1995) examine the implications of rising levels of government indebtedness using an extended version of QPM that considers one aspect of this risk premium issue. Based on the evidence reported by Alesina et al. (1993) for 12 OECD countries, they incorporate an endogenous risk premium in QPM which is applied to interest rates on government debt and net foreign assets. They find that while the risk premium itself is relatively modest, endogenising the risk premium has the effect of magnifying the long-run effects of higher government debts. The second column of results in Table 1 incorporates the risk-premium effect considered by Macklem, Rose and Tetlow (MRT) and illustrates their finding in the context of the debt shock considered above. Note, in particular, that the addition of the risk-premium effect increases the long-run decline in consumption from 1.5 per cent in the base model to 2.0 per cent. In addition, the real depreciation associated with the rise in debt is also slightly larger - 1.2 per cent in the extended model as compared to 0.9 in the base model.

In MRT's analysis the effects of government indebtedness are confined to interest rates on government debt and foreign borrowing - private domestic borrowing rates are not affected by the levels of government debt. Casual observation suggests, however, that larger debts, by increasing aggregate uncertainty, spill over to private borrowing rates and the exchange rate, although such effects have proven difficult to isolate with any precision.

In a preliminary effort to capture this type of effect in QPM, we extended the base model by specifying the country risk premium as a positive function of the government budget deficit and the current account deficit based on the pooled-time-series evidence for 17 OECD countries reported in Orr, Edey and Kennedy (1995). More specifically, the country risk premium in the model (which appears in the interest parity equation (3) among others) is assumed to increase by 17 basis points per percentage point increase in the government deficit-to-GDP ratio, and another 17 basis points per percentage point increase in the current account-to-GDP ratio. With this characterisation of the risk premium, all rates on Canadian-dollar assets are affected by changes in the level of government indebtedness, both directly due to the link between the deficit and the debt, and indirectly through the effects of government debt on net foreign liabilities and thus the current account. The final column of Table 1 reconsiders the effects of raising the level of government debt from 50 to 70 percent of GDP with this country-risk-premium effect.

Interestingly, this extension to the base model produces somewhat counterintuitive results. In broad terms, the effects of raising the level of government debt relative to GDP are similar to those discussed above, but surprisingly the long-run fall in consumption, as well as the real exchange rate depreciation, are now both smaller than in the base model with exogenous risk premiums. In particular, consumption now falls only 1 per cent as compared to a drop of 1.5 per cent in base QPM, and the real depreciation is 0.3 per cent as compared to 0.9 per cent in base QPM.

The reason is that endogenising the country risk premium in this way imposes a type of market discipline on behaviour, and the endogenous response of consumers to this discipline results in a better long-run equilibrium. The government debt shock raises the country risk premium and thus real interest rates (by 21 basis points in the steady state). Consumers, realising that higher debt levels mean higher interest rates, decide to consume less and save more, other things equal. As a result, whereas net foreign liabilities rise almost one-for-one with government debt in the base model, they now rise by considerably less than one-for-one - in the base model net foreign liabilities rise from 40 to 61 per cent of GDP, as compared to an increase from 40 to only 47 per cent of GDP in the extended model. With a smaller rise in net foreign liabilities, the long-run decline in consumption is smaller, as is the real depreciation that is required to produce the trade balance surplus that is consistent with the new level of interest payments to foreigners.

Note also that, as expected, adding the country risk premium channel to the model does increase the steady-state decline in output associated with the rise in government debt - from 0.3 per cent in the base model to 0.5 per cent with the country risk premium. This reflects the fact that the increase in the country risk premium feeds into the cost of capital, therefore reducing the optimal capital stock, and thus output. However, the additional increase in the cost of capital as a result of the country risk premium is quite small (only 11 basis points). This is because part of this effect is offset by the smaller rise in the price of imported capital that is associated with the smaller real exchange rate depreciation.

More generally, this experiment points out the discipline of a general equilibrium model. Since economic agents respond to incentives, raising interest rates increases saving. Moreover, for the current calibration of the model, this "savings effect" outweighs the "cost-of-capital effect". Output is, therefore, lower, but consumption is higher (relative to the exogenous risk premium case) because saving is higher. Whether, in practice, this saving effect is, in fact, bigger than the cost-of-capital effect is an empirical question. Casual empiricism suggests, however, that changes in risk premiums magnify the effects of government indebtedness, at least for some time, before inducing a response in behaviour that will mitigate the costs of larger government debts. In future work we plan to explore further how best to model risk premiums.

Conclusion

To conclude, the determination of interest rates and exchange rates in QPM combines several relatively simple concepts - international arbitrage, expectations theory, and sluggish price adjustments - together with some respect for the empirical evidence when implementing these features in the model. Our experience with QPM suggests that these relatively simple ingredients, when combined with forward-looking behaviour, complete stock-flow consistency, and endogenous reaction functions for both monetary and fiscal policy, can produce very rich dynamics for interest rates and the exchange rate. The main message that comes out of the model is that interest rate and exchange rate changes reflect the interplay between aggregate demand and supply, and the monetary and fiscal reactions to the implied real and nominal outcomes.

Like any model, QPM makes some important simplifications. Looking ahead, on-going work with the model will continue to explore areas in which the model may be usefully enriched. At the same time, the desire to improve different parts of the model must be balanced against the need to retain the usefulness of the model as a tool for economic projections and policy analysis. This objective argues for simplicity and transparency.

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Comments on paper by A. Côté and T. Macklem by R. Sean Craig (BIS)

Introduction

This excellent paper describes the financial sector of the Bank of Canada's model and explains the role of the model in the formulation of monetary policy. Essentially, the model is used to compute paths for the yield curve and short-term interest rates, which the Bank can influence, that would allow the Bank to hit its inflation target.

These comments will focus on the specification of the monetary policy reaction function in the model and some of the possible limitations inherent in using such a model for policy formulation. Also, some proposals for future work that address these limitations will be provided. First, however, it is useful to review the specification of the financial sector.

1. The financial sector and the specification of the monetary policy reaction function

This sector contains three basic equations: (1) An open interest parity equation; (2) an equation based on the expectations theory of the yield curve augmented to include the U.S. long term interest rate and the Canadian-U.S. inflation differential--variables that have been found to be empirically important determinants of Canadian long-term interest rates; (3) a monetary policy reaction function specifying an inflation rate of 2 percent as the inflation target for monetary policy. In this reaction function, the yield curve, the spread between the 3-month rate and the long-term rate, serves as the operational target for monetary policy. This operational target is influenced by the central bank through adjustments in the short-term interest rate.

This last equation is an essential feature of any model with forward looking expectations since financial markets react to anticipated future monetary policy. Without a monetary policy reaction function, nominal magnitudes would not be defined and the model could not be solved.

An appealing feature of this specification is that the yield curve plays a central role, performing several functions simultaneously: (1) it is an important link in the transmission of monetary policy changes to the real economy; (2) it serves an indicator of monetary stance; (3) it is the operational target for monetary policy in the monetary policy reaction function.

2. Relevance of the reaction function in the model to monetary policy in Canada

From the modelling point of view, use of a reaction function in which the yield curve is the operational target is attractive. However, this reaction function differs from that actually used by the Bank of Canada. As is well known, the Bank uses the monetary conditions index - which combines the short-term rate (the call money rate) and the exchange rate into an index - as its operational target and principle indicator of monetary stance.

The principle limitation of the yield curve-based reaction function is that in practice the yield curve is unlikely to be a completely reliable indicator of monetary stance, a consideration that limits its usefulness as an operational target. The reason is that it is not possible to know whether shifts in the yield curve due to changes in long-term interest rates are the result of changes in expected inflation, the real interest rate, or the risk premium. Consequently, it is not possible to unambiguously infer the stance of policy from the yield curve. This implies that if the yield curve serves as the operational target in a monetary policy reaction function, as in the Canadian model, it could give misleading signals to policy makers. For example, suppose the risk premium dropped, lowering long rates. If this were incorrectly interpreted as a drop in expected inflation this could lead the central bank to ease when it would not be appropriate to do so.

This limitation of the yield curve-based reaction function raises the question of whether use of this reaction function rather than one based on the MCI significantly reduces the usefulness of the model as a guide to policy? The answer depends on the policy issue that is being analysed in the model simulations. For exogenous policy shifts, such as the deterministic simulations of a reduction in the inflation target and an expansionary fiscal policy reported in the paper, the choice of reaction function is unlikely to have a significant effect. In these simulations long-term interest rates are completely endogenous and adjust to the policy change according to the dynamics of the model. The problem of interpretation noted above does not arise because movements in long-term interest rates are almost entirely due to the behaviour of expected inflation.

3. Factors limiting the use of the Bank of Canada model for policy analysis

There are, however, monetary policy issues likely to be of relevance to policy makers which would be hard to analyse using the model containing the yield curve reaction function. These arise from the fact that monetary policy operates in an environment of uncertainty where it has to respond to different stochastic shocks. These shocks can be shocks to the exchange rate or to long-term interest rates. In practice, a large proportion of the Bank of Canada's monetary policy actions probably are in response to such shocks so the capacity to simulate the effects of these shocks and the policy response would be quite useful. In particular, two problems are likely to arise if the Bank of Canada's model were used to simulate these shocks.

First, reliance on a yield curve-based reaction function could lead to an inappropriate monetary policy response in the case of a temporary shock to the term premium in long-term interest rates. Specifically, monetary policy would treat the resulting increase in long term interest rates as if it were a rise in expected inflation and tighten sharply, while the optimal response would be, at most, a slight tightening. One solution to this problem would be to make a corresponding temporary adjustment in the steady state value of the yield gap in the reaction function. However, such an adjustment would imply that the central bank knows the source of the rise in long-term interest rates which, as noted above, is unlikely to be the case in practice. It is worth noting that this problem would not arise with an MCI based reaction function.

Second, there can be significant differences in macroeconomic performance depending upon whether an MCI or yield curve reaction function is used in the case of shocks to the exchange rate (which can be interpreted as shocks to the risk premium in the open interest parity equation). Under an MCI reaction function, the central bank responds immediately to offset shocks to the exchange rate before they have much impact on the economy. In contrast, macroeconomic performance is likely to be quite different under a yield curve based reaction function. In this case, shocks are allowed to have their full impact on the economy, and the monetary policy response occurs only once the shock has its impact on the yield curve.

4. Deterministic versus stochastic simulation methods

The above argument suggested that the choice of reaction function can have a significant impact on the macroeconomic performance predicted by the model. It would be extremely useful to have some measure of the importance of this choice. A related question, likely to be of importance to monetary policy makers at the Bank of Canada, is which reaction function yields the best performance in terms of output and inflation? This question reflects the stochastic environment in which policy makers actually operate where frequent shocks to the economy make it impossible for the Bank to hit its inflation targets exactly. In this environment, it is important to know which policy reaction function will enable the Bank to hold the inflation rate close to its target (or within the target band) at the lowest cost in terms of output variance.

These issues cannot be addressed using deterministic simulations, such as those reported in the Bank of Canada paper, which can only calculate the effects of one-time shifts in exogenous variables. However, stochastic simulation of the model can be used to compare policy rules since it is possible to calculate the variance of output and inflation under the two alternative policy rules for a given distribution for the exogenous shocks to the model. This information should make it possible to rank policy rules and calculate how much of a difference the choice of policy rule makes to macroeconomic performance.

Stochastic simulations are difficult to implement in forward looking models, largely due to the computational cost. However, they represent an obvious avenue for future research. The Bank of Canada model is already quite useful for policy analysis and formulation. However, the set of policy issues that can be analysed is limited by the deterministic simulation method used. Stochastic simulation of the model would make it possible to analyse additional issues and would represent a significant step towards a more realistic representation of the stochastic environment in which policy makers actually operate.