# The behaviour of long-term interest rates in the FRB/US model

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## Introduction

For several years now, staff at the Federal Reserve Board have been engaged in a project to redesign its primary model of the US economy. Our goal in this project has been to produce an empirical model that clearly distinguishes the formation of expectations from other adjustment processes, under the paradigm that households and firms are rational optimising agents.[2] This project is now at an advanced stage, and this paper is in part a progress report on one facet of the modelling effort, the behaviour of bond rates.

The theoretical basis for the bond rate model is a version of the standard Expectations Hypothesis: The yield to maturity on a bond equals a weighted sum of future rationally-expected short-term interest rates, plus a risk premium that may be time-varying. To make the model operational for estimation work and forecasting, we employ a small-scale VAR system to generate expectations. The structure of the VAR is unconventional, in that it incorporates moving endpoints derived from market expectations of the long-term level of inflation and the real rate of interest. We believe that this specification has two advantages. First, it provides a more satisfactory characterisation of interest rates than conventional I(0) or I(1) formulations. Second, it allows us to distinguish between two primary forces influencing the level of long-term interest rates – a stationary element associated with the business cycle and monetary policy stabilisation, and a nonstationary component linked to long-term policy objectives.

The structure of the paper is as follows. We begin with a brief summary of the theoretical basis of the model. Next, we discuss our strategy to implement the model by using VAR-derived expectations. Here is where we discuss the drawbacks of standard VAR specifications, and introduce the concept of moving endpoints. From there, we turn to a closer look at endpoints, and consider the measurement and behaviour of long-term inflation expectations. The fourth section of the paper addresses the empirical model, and documents its behaviour and statistical properties. Finally, we conclude with a review of recent bond market developments in the US from the prospective of the model. An important theme in this discussion is the potential link between federal deficit reduction and recent declines in long-term interest rates.

## 1. Theory: RE models of the term structure

The theoretical basis of our bond rate model is the standard Expectations Hypothesis: The yield to maturity on a bond is equal to a weighted average value of the short-term rate (rationally) expected to prevail over the life of the bond, plus a risk premium. Depending on the capital asset pricing model or the arbitrage pricing theory used in the theoretical derivation, the risk premium may be constant or time varying (perhaps predictably so).

1 This paper draws heavily on work by a large number of people associated with the model reestimation project of the Federal Reserve Board. The opinions expressed here do not necessarily represent those of the Board of Governors of the Federal Reserve System or the staff of the Federal Reserve System.

2 For a description of a preliminary version of the new model (called FRB/US), see: Brayton and Tinsley (1995); French, Kozicki, Mauskopf and von zur Muehlen (1995); Kennedy, Reifschneider and Schuh (1995); and Bomfim, Brayton, Tinsley and Williams (1995).

Modern asset valuation theory[3] suggests that the price of a claim to a real payout of $\chi_{t+n}$ in period $t+n$ is determined by the Euler equation

$$P_{n,t} = E_t\left[x_{t+n}M_{t+n}/M_t\right] \tag{1}$$

where the ratio $M_{t+n}/M_t$ is an equilibrium discount factor. In the literature, $M$ is often functionally related to the marginal utility of consumption. Generally, it is assumed that the $m$-period log difference of $M$, $\left(1-L^m\right)\log M_t$, is a stationary stochastic process, where $L$ denotes the lag operator.

In the case of an $n$-period discount bond with a terminal price of one dollar, the no-arbitrage counterpart to equation 1 is

$$P_{n,t} = E_t\left[\frac{M_{t+n}P_t^c}{M_t P_{t+n}^c}\right] = E_t\left[\prod_{i=1}^{n}(\mu'_{i+1}/\pi'_{i+1})\right] \tag{2}$$

where $P_t^c$ is the consumption price level, and $\mu'_t$ and $\pi'_t$ are the period-to-period ratios, $M_t/M_{t-1}$ and $P_t^c/P_{t-1}^c$, or one plus the usual growth rates $(\mu'_t = 1+\mu_t)$. In models such as those developed by Rubinstein (1976) and Lucas (1978), $\mu$ is determined by the (stochastic) growth rate of consumption or household endowments.[4]

Under the assumption that $\mu$ and $\pi$ are lognormally distributed, the conventional rational expectations term structure can be expressed as:

$$r_{n,t} = \Phi'_n - \frac{1}{n}\sum_{i=1}^{n}\left[\mu_{t+i}^e - \pi_{t+i}^e\right] = \Phi'_n + \frac{1}{n}\sum_{i=0}^{n-1} r_{t+1}^e \tag{3}$$

where $r_{n,t} \equiv -\log p_{n,t}/n$ is the nominal yield to maturity of the $n$-period discount bond, $r_{t+i}^e$ denotes the one-period nominal rate expected in period $t$+1, and the superscript $e$ indicates agent expectations conditioned on information available at time $t$.

Although ignored for now, it will be important in later discussion to have an explicit definition of the term premium, $\Phi'_t$. A compact definition is obtained by stacking the component discount factors and inflation rates into $n\times 1$ vectors, $\mu$ and $\pi$, whose distributions are normal with the $n\times n$ variance-covariance matrices, $V_{\mu\mu'}$ $V_{\pi\pi'}$, and $V_{\mu\pi'}$. Denoting the $n$-element unit vector by $\iota_n$, the term premium for the $n$-period discount bond is:

$$\Phi'_n = -\frac{1}{2n}\left[\iota'_n V_{\mu\mu'}\iota_n + \iota_n V_{\pi\pi'}\iota_n + 2\iota'_n V_{\mu\pi'}\iota_n\right] \tag{4}$$

If variance-covariance matrices are stable over time, then $\Phi'_n$ is a constant. Otherwise the risk premium fluctuates in ways that may be correlated with changes in macroeconomic conditions.

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3 Historical contributions include Rubinstein (1976), Breedon (1979), and Cox, Ingersoll, and Ross (1985a).

4 In asset pricing models, it is frequently assumed that the utility function of the representative agent is characterised by constant relative risk aversion, i.e., $U(\circ) \equiv C^{\alpha}/\alpha$. This functional form implies that log differences of the pricing kernal $M$ are proportional to the expected growth rate of log endowments or consumption.

In the case of coupon bonds, the analogue to equation 5 developed by Shiller (1979) is

$$R_t^{(n)} = \Phi_n + \frac{1-B}{1-B^n} \sum_{i=0}^{n-1} B^i r_{t+i}^e \tag{5}$$

where the constant discount rate, $B = 1/(1+\overline{R})$, is determined by the sample average $\overline{R}$ of the yield to maturity on a coupon bond, $R_t^{(n)}$. Equation 5 is the standard version of the (rational) Expectations Hypothesis, under the assumption that the risk premium $\Phi_n$ is a constant.[5] Note that the risk premium $\Phi'_n$ in equation 3 is not equal to $\Phi_n$ in equation 5, owing to the different risk characteristics of discount and coupon bonds. However, the risk premium for an $n$-period coupon bond is a function of the same underlying variances and covariances that determine the premiums on 2-through-$n$ period discount bonds.

## 2. Implementation strategy: modelling expectations

The theoretical model developed in the previous section is incomplete. The expected path of future short-term interest rates is unobserved, and the size of the risk premium is not known a priori. If equation 5 is to be used in estimation, forecasting or policy simulation exercises, it must be augmented with equations that explain how $r_{t+i}^e$ evolves over time and responds to changes in the macroeconomic environment.

Because the equations used to determine bond rates are part of a general RE model of the economy, it is theoretically possible to use the full FRB/US model to derive expectations for use in estimation and forecasting. Such an approach is in principle preferable, because it would ensure that expectations conform with the behaviour of the overall system. A full-information maximum likelihood estimation procedure has been used by Leeper and Sims (1994) in their estimation of a small-scale macro model of the US economy. Unfortunately, the FRB/US model is too large to make this approach computationally feasible, at least at present. In addition to estimation and forecasting, the FRB/US model is also used in policy analysis, for which it is straight-forward to use Fair-Taylor or other algorithms to compute model-consistent RE solutions. This approach – an example of which is discussed in Section 4 – is now often used at the Federal Reserve Board to answer such questions as the likely macroeconomic impact of changes in fiscal and monetary policy.

As an alternative for estimation and forecasting, we have made the assumption that agents' expectations can be characterised by a small-scale VAR forecasting system. This system is used to generate all expectations used in the standard version of the FRB/US model, not just those associated with the prediction of bond yields.[6] The core portion of this system, which includes the short-term interest rate, inflation, and the output gap, can be expressed as:

$$Z_{t+i}^e = H^{i+1} Z_{t-1} \tag{6}$$

where $H$ denotes the companion matrix of the first-order representation of the VAR model, and the vector $z_{t-1}$ is a column stack of the relevant lagged values of the VAR model. $z_{t-1}$ thus summarises

5 Although, as Shiller notes, there is no inherent reason why $\Phi_n$ necessarily is constant over time.

6 Other important expectational variables used in the model include a variety of present value calculations (for household income and corporate profits), the future average rate of inflation, and the average level of resource utilisation expected to prevail in the near term.

the information set of agents. Substituting the predictions of equation 6 into the definition of the RE term structure in equation 5 provides a tractable linear formulation of the term structure:

$$R_t^{(n)} = \frac{1-B}{1-B^n}\left[\sum_{i=0}^{n-1}(BH)^i Hz_{t-1}\right] + \Phi_n \tag{5b}$$

which in turn simplifies to

$$R_t^{(n)} = \frac{1-B}{1-B^n}\iota_r'\left[I - BH\right]^{-1}\left[I-(BH)^n\right]Hz_{t-1} + \Phi_n \tag{7}$$

where $\iota_r$ is a column selector vector that contains a one to identify the position of the one-period rate $r_t$ in the information vector $z_t$, and zeroes elsewhere.

Because the annualised discount factor $B$ used in equation 7 equals 0.92, "distant" forecasts of $r_{t+i}^e = \iota_r' H^i z_{t-1}$ receive a relatively large weight in the calculation of $R_t^{(n)}$. For example, 50 percent of the value of a ten-year coupon bond is associated with the expected level of short-term interest rates after the first two years of the bond, and 20 percent after the first five years. The weight given to out-year forecasts means that the low-frequency characteristics of the VAR model are critical to the predicted behaviour of bond yields. However, this aspect of VAR specification is not usually given a great deal of attention by modellers, probably because VARs are typically regarded as short-run forecasting models.

To illustrate the importance of endpoint assumptions for bond rate forecasts, consider the three panels of Figure 1. All three panels display RE constructions of the 10-year bond rate using the formula in equation 7. To simplify the exposition for the moment, the forecast model is restricted to a simple m-order autoregression in the federal funds rate, which is selected as the effective one-period (monthly) rate. (In the empirical section below, we will return to a more complicated VAR system.) The models differ only in their characterisation of the long-run endpoint of the funds rate which, hereafter, we denote as $r_t^\infty \equiv \lim_{t\to\infty} r_t^e$.

### 2.1 A stationary I(0) format

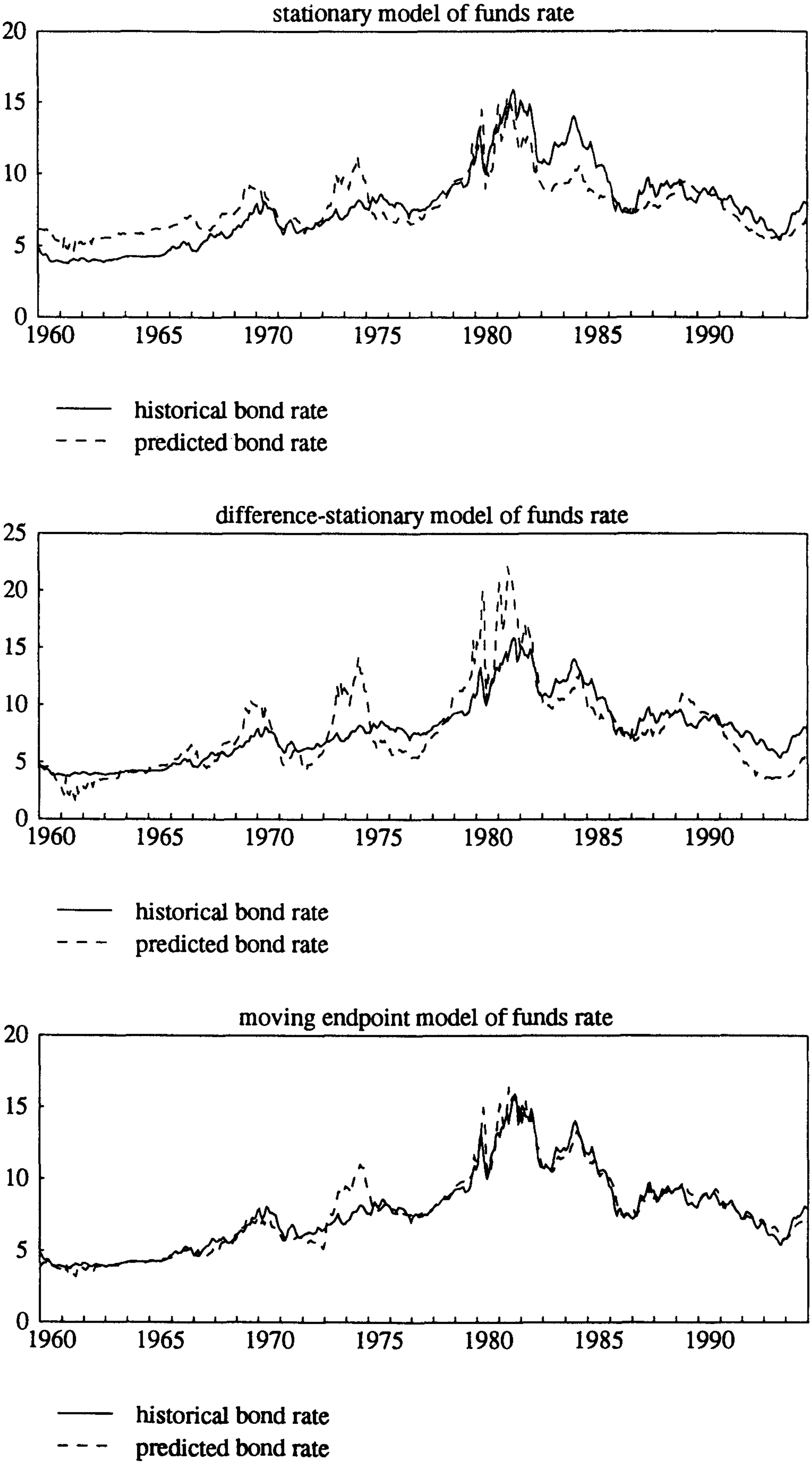
The top panel of Figure 1 illustrates bond rate predictions from a model in which the short rate is a stationary stochastic process – a common assumption in the finance literature.[7] In discrete time, the format is

$$\Delta r_t = \alpha_0 + \gamma r_{t-1} + A(L)\Delta r_{t-1} + \varepsilon_t \tag{8}$$

where *A(L)* denotes a finite polynomial in the lag operator, *L*. Estimates of the parameters of equation 8 are displayed in the first column of Table 1, where *A(L)* is a fourth-order polynomial and the sample is the 34-year span starting in 1960. One-month-ahead predictions of the 10-year Treasury bond rate are constructed by recasting equation 8 into companion form and substituting this into equation 7. Predictions of this autoregressive funds rate model display a tendency to lie above (below) the historical bond rate when the latter is below (above) its sample mean. This is because predictions of long-horizon instruments are eventually dominated by the limit or endpoint of the forward funds rate

7 For example, see Cox, Ingersoll, and Ross (1985b).

Figure 1
**RE predictions of the 10-year bond rate**



forecasts and, in the case of the stationary funds rate model, the funds rate endpoint is a constant, $r^{\infty} = -\alpha_0/\gamma$, which in large samples is the sample mean.[8]

A more intuitive view of the role of the constant endpoint is obtained by rewriting equation 8 as

$$\tilde{r}^e_{t+i} = (1+\gamma)\tilde{r}_{t-1} + A(L)\Delta\tilde{r}_{t-1} \tag{9}$$

where $\tilde{r}_t$ denotes the current displacement of the funds rate from the endpoint, $\tilde{r}_t = r_t - r^{\infty}$. The fading impact of the initial displacement on forecasts of forward funds rates can be gauged by the mean lag of an initial shock. According to the parameter estimates in the first column of Table 1, the mean lag of a displacement shock is 40.4 months.[9] In other words, the predicted forward rates have reached the neighbourhood of the funds rate endpoint by the fourth or fifth year of the forecast horizon, implying that the constant $r^{\infty}$ is a good approximation of the average expected funds rate in the second five years of the 10-year bond rate.

Table 1
**Autoregressive models of the funds rate**
$$\Delta r_t = \alpha_0 + \gamma r_{t-1} + A(L)\Delta r_{t-1} + \gamma_1 r^{\infty} + \varepsilon_t$$
Estimation period: January 1960 to December 1994

| Parameters | I(0) format | I(1) format | Endpoint format |
|---|---|---|---|
| $\alpha_0$ ........ | .149<br>(2.0) | .003<br>(0.0) | -.002<br>(-0.0) |
| $\gamma$ ........ | -.020<br>(-2.2) | | -.038<br>(-2.6) |
| $A(1)$ ........ | .173<br>(2.2) | .133<br>(1.8) | .173<br>(2.2) |
| $\gamma_1$ ........ | | | .038<br>(2.6) |
| $R^2$ ........ | .19 | .18 | .20 |
| SEE ........ | .687 | .691 | .686 |

### 2.2 A nonstationary I(1) format

In contrast to the stationary model of a representative short-term interest rate that is generally assumed in finance, a number of recent studies of the term structure in macrofinance, such as Campbell and Shiller (1987) and Mougoue (1992), are predicated on the assumption that all nominal interest rates are I(1). Indeed, because the format of equation 8 is the same as that required for an augmented Dickey-Fuller (ADF) test of stationarity, the first column of Table 1 indicates that the t-statistic associated with $\gamma$ is below the critical value (2.57 for a p-value of 10%) that would be required to reject the hypothesis that the funds rate contains a unit root. The second column of Table 1 contains the estimated parameters of the differenced funds rate model

8 Bond rate predictions in Figure 1 are adjusted for the difference between the sample mean of the funds rate and that of the bond rate.

9 In this case the mean lag equals $-(1+\gamma-A(1))/\gamma$.

$$\Delta r_t = \alpha_0 + A(L)\Delta r_{t-1} + \varepsilon_t \tag{10}$$

where the coefficient of the lagged level of the funds rate is restricted to zero. As indicated by the standard error of estimate (SEE) shown in the second column of Table 1, the deterioration in fit due to this restriction is negligible.

Bond rate predictions from the first-difference model of the funds rate are displayed in the second panel of Figure 1. These predictions differ markedly from those in the first panel and mirror rather closely the movements of the historical funds rate level, exceeding the 10-year bond rate in the early 1980s and remaining largely below the bond rate since the mid-1980s. In contrast to the bond rate forecasts in the first panel – which appear to be excessively damped relative to the movements of the historical bond rate – the bond rate forecasts in the second panel appear to be, if anything, too responsive to the last measured position of the funds rate.

The reason for the higher sensitivity of the bond rate predictions in the second panel to recent levels of the funds rate is that the unit root in equation 10 induces nonstationarity in the endpoint, as well as in the funds rate. Because the characteristic roots of $A(L)$ are stable, the forecasts of forward rate changes will approach a limit which is an ($m$+1)-order moving average of the funds rate. Then, summing over the forward rate changes and taking the limit indicates that the endpoint of the forward rate forecasts is also an ($m$+1)-order moving average of the funds rate, $r_t^\infty = r_{t-1} + \sum_{i=1}^{m} w_{i,t}\Delta r_{t-i}$. Thus, the endpoint is fixed in any given forecast period but will closely track the funds rate over time.

### 2.3. A moving endpoint format

So far, the discussion of endpoints indicates that both I(0) and I(1) formats are associated with undesirable low-frequency properties. In the I(0) case, the assumption of a fixed endpoint yields bond rate predictions that are too stable. By contrast, tying the system's endpoint to the current level of short-term interest rates produces forecasts that are too volatile. These results suggest that better predictive performance might be had from models that incorporate time-varying endpoints which are not too closely tied to current economic conditions.

To this end, we now consider the approach pursued by Kozicki and Tinsley (1995a), in which the forecast system is extended to include the effects of an explicit moving endpoint, $r_t^\infty$.

$$\begin{aligned} E_t\Delta r_t &= \alpha_0 + \gamma\left(r_{t-1} - r_{t-1}^\infty\right) + A(L)\Delta r_{t-1} \\ E_t r_t^\infty &= r_{t-1}^\infty \end{aligned} \tag{11}$$

Note that there are now two equations, the first describing the evolution of funds rate forecasts over the forecast horizon that begins in period $t$, and the second indicating that the endpoint forecast is fixed over the forecast horizon. It is important to observe also that (11) is not a closed system because the second equation is silent about the actual evolution of the interest rate endpoint over the historical sample. Indeed, as discussed later, the estimated endpoint appears to be a nonstationary process, indicating that its conditional moments have shifted over the historical sample. The second equation in (11) indicates only that the conditional expectation of the endpoint is fixed over the horizon of forecasts originating at time $t$.

Unlike most areas of macroeconomics, where agents' perceptions of the relevant transversality conditions or endpoints associated with Euler equation descriptions of optimal intertemporal behaviour are not observable, agents' current forecasts of the nominal rate endpoint are readily available from the observed term structure of nominal rates. One such measure, similar to that employed by Kozicki (1995), is the average of the forward rates from $t+m$ to $t+m'$, for $m' > m$:

$$\tilde{r}_t^{\infty} = \frac{D_{m'} r_{t+m'} - D_m r_{t+m}}{D_{m'} - D_m} \tag{12}$$

where the duration associated with an $m$-period coupon bond is estimated by $D_m \equiv \left(1 - B^m\right) / \left(1 - B\right)$.[10]

In the estimate of the funds rate equation in (11), displayed in the third column of Table 1 the endpoint regressor series, $r_t^{\infty}$, is a concatenation of monthly endpoint constructions based on the forward rates between the 10-year and the 30-year Treasury bonds.[11] Note that estimated characteristics of the moving-endpoint funds rate equation in the third column are very similar to those of the stationary rate equation shown in the first column of Table 1. This is because standard reporting statistics, such as $R^2$, are based on the one-step-ahead forecast properties of fitted equations and are relatively insensitive to assumptions about long-horizon endpoints. By contrast, bond rate predictions are long-horizon forecasts and the bond rates generated by the moving endpoint forecast system, shown in the third panel of Figure 1, track the historical 10-year bond rate much more closely than do the predictions in the first panel.

## 3. A closer look at endpoints

As one might suspect from the preceding analysis, the moving endpoint for the nominal short-term interest rate provides the lion's share of motion in the predicted bond rate. In fact, the squared correlation between $r_t^{\infty}$ and the historical 10-year bond rate indicates that the moving endpoint alone explains about 85% of the sample variation in the level of the 10-year bond rate. Of course, a consequence of the open design of (11) is that we must provide a plausible model of the economic determinants of $r_t^{\infty}$. Furthermore, because the forecasting system used to generate expectations of short-term interest rates in FRB/US is not autoregressive, but instead is a VAR model that includes inflation in the information set, we must also consider the related question of how a moving endpoint for this variable can be measured and explained.

An obvious place to start the analysis is the standard Fisherian decomposition of a nominal interest rate between the expected real rate and expected inflation. In the current context, the nominal rate endpoint can be partitioned into an expected real rate endpoint and an expected inflation endpoint:

$$r_t^{\infty} = \rho_t^{\infty} + \pi_t^{\infty} \tag{13}$$

Unfortunately, given the absence of indexed bonds in the United States, let alone an indexed term structure, both components – the expected real rate endpoint, $\rho_t^{\infty}$ , and the inflation rate endpoint, $\pi_t^{\infty}$ – are unobserved.[12]

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10 See the discussion in Shiller, Campbell, and Schoenholtz (1983) and Shiller (1990).

11 Prior to February 1977, estimates of the constant-maturity 30-year Treasury bond rate are not published and these observations were replaced by estimates of the constant-maturity 20-year rate. Recent work by Mark Fisher and Christian Gilles of the FRB staff on estimates of the daily term structure after the mid-1980s suggests that published estimates of the 20-year constant-maturity Treasury rate may contain significant measurement errors.

12 Throughout this discussion, we define the expected real rate component by $\rho_t^e = r_t - \pi_t^e$. As will be apparent, this definition of the real rate includes an assortment of term premium components, including those associated with the uncertainty of expected inflation

Survey evidence on expected inflation promises a way out of this conundrum, but for the United States such data are almost exclusively concerned with short-term expectations. However, there are two notable exceptions:

1) A survey of market participants conducted in the 1980s by Richard Hoey, an economist at Drexel Burnham Lambert, which asked for forecasts of inflation over a ten-year forecast horizon. The survey also distinguished between inflation expectations for the first and second five-year subperiods of the forecast period. Although this survey has been discontinued, a contiguous quarterly series of long-term expected inflation can be assembled for the span 1981 Q1 through 1991 Q1.[13]

2) A quarterly survey of professional forecasters conducted since late 1980 by the Federal Reserve Bank of Philadelphia, which queries participants for the expected average rate of inflation over the next ten years.

In principle, either survey could be used to decompose $r_t^\infty$ into its real and inflation components, at least over some portion of history. On the face of it, the Hoey survey is preferable to the Philadelphia survey, both because it contains information on expected inflation 5-to-10 years ahead, and because its participants are drawn from the investment community.[14]

Unfortunately, neither survey by itself is adequate to solve the $r_t^\infty$ decomposition problem, because we need a long time series for $\pi_t^\infty$ to estimate the VAR model. Therefore, we consider two indirect ways of estimating the inflation endpoint:[15]

- a regression decomposition of the nominal rate endpoint; and
- a learning model that extracts shifts in expected inflation from actual inflation.

In the first approach, the Hoey survey results are used directly in the analysis. Survey evidence is also useful for the other method, as it provides a check on the plausibility of our results.

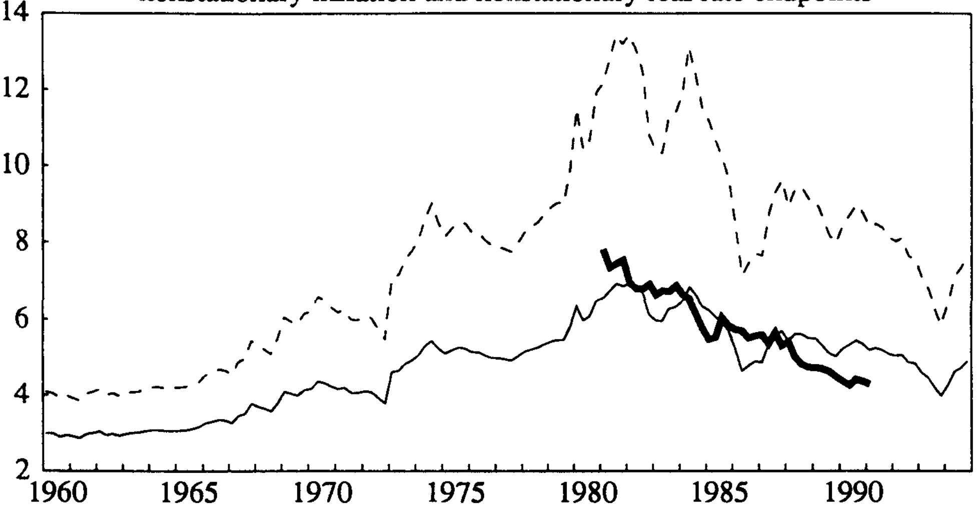
13 A missing observation in 1990 Q1 is estimated by linear interpolation.

14 In practice, results from the two surveys are quite similar for the 10-year expectation – perhaps because a large portion of Philadelphia survey respondents are economists who work for financial institutions, and a substantial portion of Hoey respondents were professional forecasters.

15 In addition to these two methods, we also investigated an unobserved components decomposition $r_t^\infty$. Under this approach, it is assumed that the unobserved real rate endpoint is stationary. Following the procedure developed by Harvey (1985) and Clark (1987), the inflation endpoint is then identified by assigning to it all the nonstationary movements in the nominal interest rate endpoint. Use is made of survey data in parameter identification by fitting the model to the Hoey survey over the 1981-1991 subsample period. Unfortunately, the measure of endpoint inflation produced by this method - shown in the bottom portion of Figure 2 - had the drawback of being highly sensitive to high-frequency movements in $r_t^\infty$. Furthermore, the measure had a tendency to be negative during the initial periods of the sample.

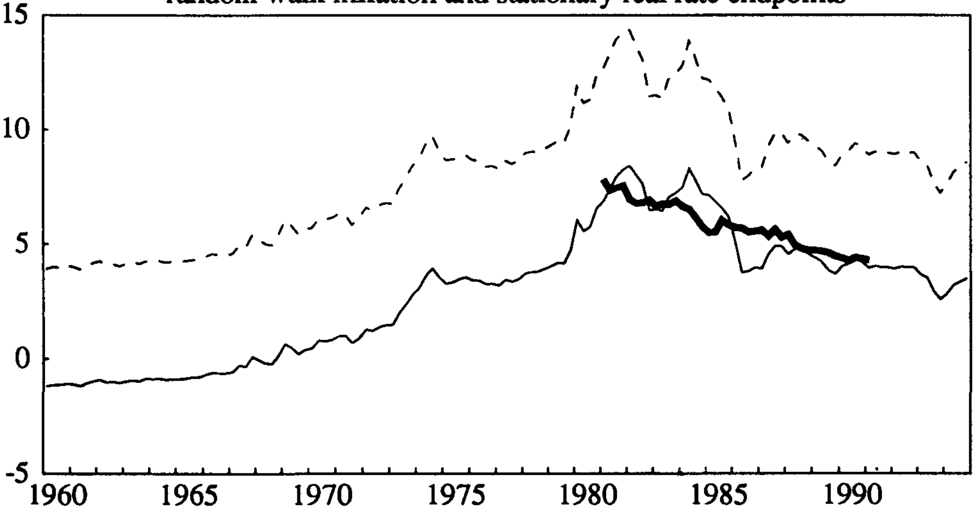
Figure 2
**Decompositions of nominal rate endpoints**

nonstationary inflation and nonstationary real rate endpoints




- - - nominal rate endpoints
—— inflation endpoints
▬▬ expected inflation 5-10 years ahead (Hoey)

random walk inflation and stationary real rate endpoints




- - - nominal rate endpoints
—— inflation endpoints
▬▬ expected inflation 5-10 years ahead (Hoey)

### 3.1 Regression decompositions of the nominal rate endpoint

To begin, consider the first equation in Table 2, which shows the results of the simple regression of $r_t^{\infty}$ on the Hoey estimate of long-term expected inflation, $\pi_{h,t}^{\infty}$. One interesting feature of this regression is that the coefficient of expected inflation is 1.44 and significantly greater than one. One possible explanation is that many holdings of US Treasuries are subject to taxation of earnings. Under this interpretation, the coefficient is $1/(1-t_x)$, where $t_x$ is the marginal tax rate. The value of the coefficient in Table 2 suggests a marginal tax rate around 0.31. Using flow of funds historical estimates of sectoral holdings of Treasury securities, Kozicki and Tinsley (1995b) estimate that the effective tax rate on Treasury securities faced by domestic households and businesses has fallen from around 0.39 in 1960 to around 0.21 in 1993. Of course, it is difficult to be definitive about the effect of tax rates (marginal or otherwise) on pre-tax bond yields, given that a large number of market participants – e.g., pension funds, foreign investors – pay no taxes.

Table 2
**Regression partitions of the nominal rate endpoint**

| | | |
|---|---|---|
| (1) | $r_t^{\infty} = \alpha_0 + \alpha_1 \pi_t^{\infty} + e_{1,t}$<br>1.81 1.44 .280[1]<br>$(1.5)^2$ $(6.8)^2$ (2.2) | $R^2 = .62$<br>SEE = 1.17<br>1981Q1 – 1991Q1 |
| (2) | $\Delta r_t^{\infty} = \alpha_0 + \gamma r_{t-1}^{\infty} + A(L)\Delta r_{t-1}^{\infty} + e_{2,t}$<br>.160 −.020 .280[1]<br>(1.8) (−1.7) (2.2) | $R^2 = .09$<br>SEE = .422<br>1954Q3 – 1994Q2 |
| (3) | $\log(\lvert e_{2,t}\rvert) = \alpha_0 + \alpha_1 \log(r_t^{\infty}) + e_{3,t}$<br>−4.80 1.53<br>(−10.9) (6.6) | $R^2 = .22$<br>SEE = 1.23<br>1954Q3 – 1994Q2 |
| (4) | $\log(r_t^{\infty} - \pi_t^{\infty}) = \alpha_0 + \alpha_1 \log(r_t^{\infty}) + e_{4,t}$<br>−2.04 1.51<br>$(-2.2)^2$ $(4.0)^2$ | $R^2 = .69$<br>SEE = .189<br>1981Q1 – 1991Q1 |

1 Coefficient sum, $A(1)$.
2 Standard errors adjusted for serial correlation using a Newey-West covariance construction with a band-width of ±3 quarters.

Another plausible interpretation is that some element of the real rate endpoint may be related to the level of the nominal interest rate endpoint. To develop this approach, rewrite the generic Euler equation 2 for a nominal return as

$$1 = E_r(\mu' r') = E(\mu')E_t(r') + v_{u',r'} \tag{14}$$

where, as earlier, the primes now indicate the discount factor and gross return $r' = 1 + r$, and $V_{\mu',r'}$ denotes the covariance. Condition 14 can be used to define a risk free rate, $r_f$, whose correlation with the discount factor is zero, and a market portfolio yield, $r_m$, whose correlation with the discount factor is one. Using these alternative yields, equation 14 (an arbitrage condition) can be restated as the standard expression for portfolio valuation of the return to an arbitrary asset, $r$:

$$r - r_f = \lambda_m V_{r,r_m} \tag{15}$$

where $\lambda_m$ is the market price of risk, $\lambda_m = \left(r_m - r_f\right)/V_{r,r_m}$. Equation 15 indicates that the expected real return includes the risk premium defined by the covariance between the asset yield and the return on the market portfolio.

Although a time series of the estimated return to the aggregate market portfolio is not easily constructed, it may be noted that the required covariance in equation 15 is equal to the product of the return standard deviations with the correlation between the asset and portfolio returns, $V_{r,r_m} = \rho_{r,r_m}\sigma_r\sigma_{r_m}$. Under the assumption of a constant correlation, some earlier studies of the term structure, such as Shiller, Campbell, and Schoenholtz (1983), use a moving standard deviation of the asset return, $r$, to capture time variation in the term premia of bonds.

More recent work in finance has suggested that the standard deviation of interest rates is a function of the level of interest rates.[16] For example, Chan, Karolyi, Longstaff and Sanders (1992) estimate a class of autoregressive interest rate models similar to that in equation 8 above, with the additional specification that the standard deviation of the residual innovation is proportional to $r_t^v$. In theoretical models of the term structure, $v$ has ranged from 0 to 1.5. The CKLS paper reports $v = 1.5$ in regressions using the monthly Treasury bill rate. Our own experience is that point estimates of $v$ can vary all over the map with different sample spans. However, the CKLS paper points out that the high-end 1.5 estimate has the considerable advantage that it can fit samples that contain the marked shift in 1979 of monetary policy without requiring additional dummy variables to capture any remaining rate volatility effects of the policy shift.

The second and third equations in Table 2 provide a regression approximation of a similar analysis for the nominal rate endpoint, $r_t^\infty$. The second equation describes a standard autoregressive model of the nominal rate endpoint, and the third equation summarises the results of a regression of the log absolute value of the autoregressive residual on the log level of the endpoint. Similar to the CKLS finding, the elasticity of endpoint volatility with respect to the endpoint level is 1.53 and significantly greater than one.

Although there is no necessary reason to expect an interest rate level indicator of rate volatility to adequately characterise the risk premium of the endpoint, the fourth equation of Table 2 presents an estimate of the regression

$$\log\left(r_t^\infty - \pi_{h,t}^\infty\right) = \alpha_0 + \alpha_1 \log\left(r_t^\infty\right) \tag{16}$$

Surprisingly, the estimated elasticity in the fourth equation, $\alpha_1 = 1.51$, is remarkably similar to the elasticity estimate in the third equation, although in this instance the estimate is not significantly different from one. In any event, equation 16 is the basis for our first decomposition of the nominal rate endpoint, $r_t^\infty$, into real and inflation endpoints. The resulting nominal rate and

16 A motivation in theoretical term structure models for a variance that is heteroskedastic in the level of the interest rate is to prevent Jensen inequality terms, the negative variance terms in equation 4's definition of the term premium, from predicting negative nominal rates.

inflation rate endpoints are plotted in the first panel of Figure 2. Note that the decompositions before 1981 and after 1991 are outside of the 10-year sample that is available for the Hoey estimate of expected long-term inflation. The low-frequency motion of the inflation endpoint identified by the log regression is not unreasonable, rising over the sample until the early 1980s and largely falling thereafter. However, the inflation endpoint is also relatively responsive to high-frequency movement in the nominal rate endpoint, such as the recent increase in 1994.

### 3.2 An agent learning model for shifts in expected inflation

Our second decomposition procedure is motivated, in part, by the observation that an I(1) description of inflation is problematic, if the real rate is assumed to be stationary: Under these conditions the inflation risk premium embedded in $\rho_t^\infty$ is unbounded in the limit.[17] Beyond this logical difficulty, I(1) characterisations of inflation and nominal interest rates – prevalent in the macrofinance literature – also would seem to be of questionable empirical relevance: After all, such a characterisation is only one subset of the general class of nonstationary time series,[18] and it is well-known that standard tests for unit roots have low power against other descriptions of nonstationarity, such as the episodic shifts analysed in Perron (1989).[19]

In particular, we now consider a model in which the nonstationarity of nominal interest rates is due (at least in part) to episodic shifts in the expected level of inflation. Following the analysis in Kozicki and Tinsley (1995a), consider two distributions of information between private and public agents. In the first case, information is symmetric and the inflation endpoint perceived by agents, $\pi_t^\infty$, is *identical* to the long-run inflation target of monetary policy, $\bar{\pi}$. Typically, central bankers of developed economies are cautious and slow to change either operational policies or strategic objectives. This suggests that policy changes are episodic with frequencies that are more appropriately measured in half-decades rather than months. This inference seems to be consistent with the small number of policy regimes typically identified in postwar analyses of US policy, such as Huizinga and Mishkin (1986).

In the second case, information is asymmetric. The long-run policy objective for inflation is not known (or believed), and private agents must infer that a shift in $\bar{\pi}$ has occurred by examining observable consequences of policy. A simple example of the latter is the following *changepoint* analysis of an autoregressive model of inflation:

$$\Delta\pi_t = \alpha_0 + \sum_k \alpha_k \delta_k + \gamma\pi_{t-1} + A(L)\Delta\pi_{t-1} + \alpha_t \tag{17}$$

17 As noted in equation 4, the term premium of nominal bond rates is decreasing in the variance of expected inflation rates. This variance term does not vanish even for risk-neutral representative agents (when the covariance between the marginal utility of consumption and inflation is zero). Because the variance of an I(1) process grows without bound over the forecast horizon, the variance of the inflation endpoint is also unbounded. But because the real rate contains the term premium for the variance of uncertain inflation, the real rate endpoint must fall without bound.

18 The shifting-regime model in Hamilton (1989) is an example of nonstationary behaviour that need not exhibit unit roots.

19 A nonstationary series is one whose moments are not constant over time. Changes in moments may be continuous and persistent, with expected absolute values that are predictable and small relative to current levels, as in most real aggregates identified as I(1) in macroeconomics, Nelson and Plosser (1982). Alternatively, changes may be infrequent, unpredictable in both sign and size, often large relative to the last observed level, and persistent only in the conditional sense that the next change cannot be predicted – characteristics that seem to describe well the long-term behaviour of inflation and nominal interest rates.

where each $\delta_k$ is a dummy variable that switches on in period $t+k$, $(k = k_1, k_2, \ldots)$. Both the size, $\alpha_k$, and timing, $t+k$, of changepoints are unknown to private agents. Analogous to the endpoint construction for the stationary autoregressive model discussed earlier, the perceived endpoint in period $\tau$ is

$$\pi_t^\infty = -\left[\alpha_0 + \sum_{t+k<\tau} \alpha_k\right] \Big/ \gamma \tag{18}$$

which includes all changepoints recognised by agents as of period $\tau$. If changepoints are assigned to each period, the endpoint is I(1).

The reliability of a detected changepoint varies inversely to the number of observations since the last changepoint. In the autoregressive changepoint problem discussed in Kozicki and Tinsley (1995a, 1995b), agents may choose among eight *minimum recognition lags*, ranging from one year (12 months) to eight years (96 months). The top panel of Figure 3 displays the inflation rate changepoints selected by agents using a minimum recognition lag of eight years. The thin solid line is the annualised monthly inflation rate of the personal consumption deflator. The dashed line is the concatenation of actual changepoints in the inflation endpoint detected by these agents, using 1% critical values. The thick solid line is the associated concatenation of virtual endpoints, allowing for the recognition lag between the month of the actual shift in the estimated inflation endpoint and the month when the shift was detected. The *virtual recognition lag*, the distance in months between the actual shift date and the virtual shift date, can sometimes greatly exceed the minimum recognition lag. However, this discrepancy is small, generally, for agents with lengthy minimum recognition lags, as in the case shown.

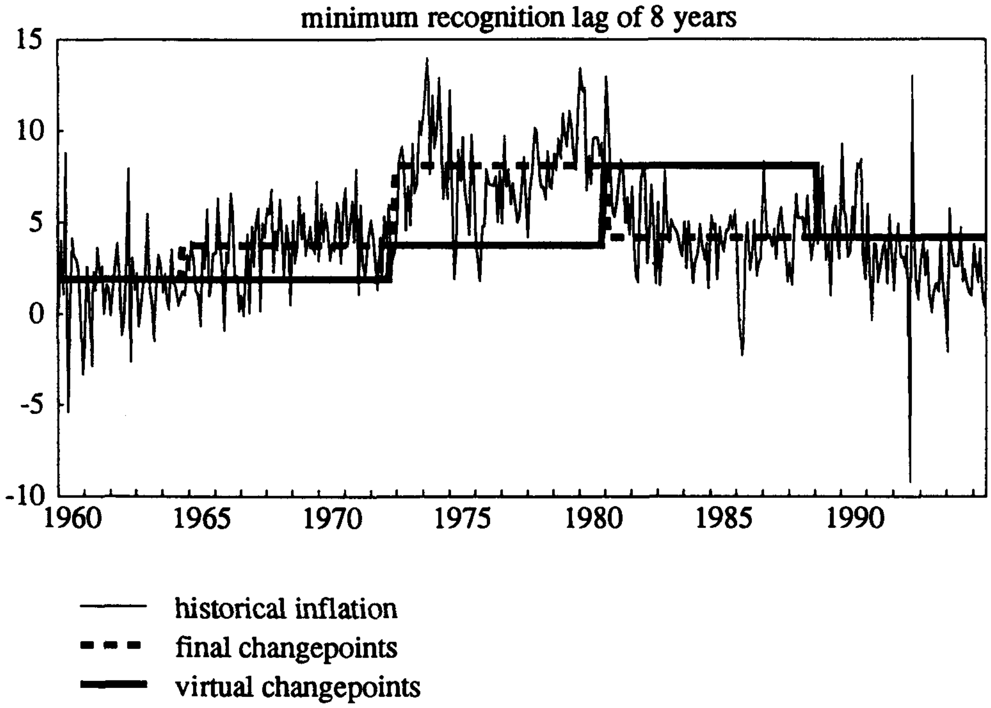
After estimating a series of concatenated virtual changepoints for each class of agents, the frequency distribution of agents is estimated by projecting the nominal rate endpoint, $r_t^\infty$, onto the full set of virtual changepoint series. The projection also yields the mean inflation endpoint series, $\pi_t^\infty$. The endpoints of both the nominal interest rates and inflation rates are plotted in the second panel of Figure 3, along with the Hoey estimate of expected long-term inflation. The remarkable feature of this figure is that the estimated inflation endpoint is aligned very closely with available Hoey estimates, even though no information in the latter survey was available to any of the learning agents (in contrast to the regression-based decomposition, which incorporates survey information in the estimation procedure). Another important feature of this model is that the estimated mean recognition lag exceeds five years, indicating that it can take years for agents to recognise shifts in the long-term inflation objective of policy.

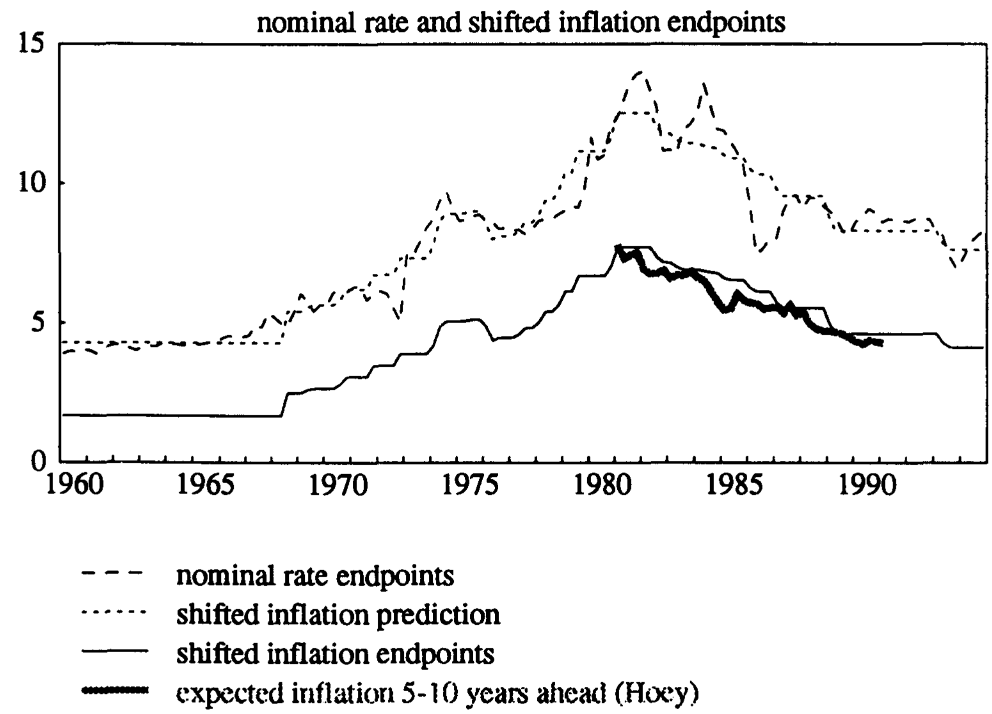
### 3.3 Multiple indicators

Among the two alternative methods for constructing historical measures of the inflation endpoint, we find the agent learning model the most promising. The learning model is also attractive because it provides a procedure for updating the inflation endpoint in forecasting and policy analysis. However, survey information provides alternative measures for the post-1980 period that are arguably superior to any constructed proxy, on the grounds that the surveys are direct measures of expected inflation for at least a subset of economic agents.

For this reason, we have chosen a splicing methodology that uses survey measures of $\pi_t^\infty$ where available, and predictions from the learning model elsewhere, in constructing a historical endpoint series for use in estimation and forecasting. A challenge under this approach is how best to model the inflation endpoint in forecasting and in policy simulations: As a static variable invariant to transitory shocks to the system, or as an expectation that responds dynamically to changes in the

Figure 3
**Shifted inflation endpoints**



macroenvironment? At this stage we are evaluating two solutions to this problem. The first is simply to assume that innovations in surveyed expectations evolve according to the predictions of the agent learning model. The second solution is to model the innovations in survey expectations empirically, by regressing changes in surveyed expectations on VAR equation innovations. Preliminary results suggest that VAR innovations – a measure of incoming news available to agents – can explain a significant portion of the historical path of expected long-run inflation.

## 4. The empirical model

As noted earlier, the actual procedure used to approximate interest rate expectations in the FRB/US model generalises the autoregressive funds rate model of equation 11 to a VAR system that incorporates moving endpoints for its nonstationary components. Forecasts from this system are then used, via the formulas presented in equations 6 and 7, to construct estimates of the bond rate, under the assumption that the risk premium, $\Phi_n$, is constant. The latter is approximated by the sample mean spread between the bond rate and the weighted sum of the projected short-term interest rates.

### 4.1 Specification and estimation of the VAR model

Since Sims (1980), a voluminous literature on VARs has arisen. It is filled with debates, both theoretical and empirical, about the proper specification of a macroeconomic VAR in terms of number and type variables to include. While there are a number of potentially important variables eligible for inclusion in our VAR model, to date we have restricted our work to simple specifications that include some version of the three basic variables used by Sims: (1) a measure of real economic activity; (2) a measure of price; and (3) a measure of monetary activity. Although our experience with parsimonious three-variable systems has been satisfactory, we leave open the possibility of adding other variables in the future.[20]

Based on a review of the VAR literature, the earlier discussion of moving endpoints, and a fairly extensive empirical investigation, we settled on the following specification of the VAR:

- Inflation $(\pi_t)$ – defined as the rate of change in the chain-weight price index for personal consumption in the National Income and Product Accounts. We selected this price inflation measure primarily because it was the measure least susceptible to the "price puzzle" – the positive impulse response in inflation following a positive interest rate shock – in a three-variable system.

- Output gap $(y_t)$ – defined as the log of real business (excluding farm and housing) output minus the log of a measure of potential output. The potential output series is consistent with the aggregate production function of the full FRB-US model. In particular, the potential output calculation assumes that the labour market is in equilibrium (unemployment equals the NAIRU), and that aggregate labour productivity is a function of the capital-labour ratio and exogenous technical progress. The latter is proxied by a split time trend to capture the post-1973 slowdown in productivity growth.

20 Some variables under consideration for an expanded VAR include: commodity price inflation; oil prices; the exchange rate; and fiscal policy variables (e.g. the deficit-to-GDP ratio).

- Interest rate $(r_t)$ – defined as the federal funds rate (effective annual rate basis). We use the funds rate for two primary reasons. First, it tends to outperform most other measures of monetary activity in VARs such as monetary aggregates and interest rate spreads according to conventional statistical criteria – see, for example, Bernanke and Blinder (1992) and Sims (1988). Second, it is often cited as the most reliable indicator of the stance of monetary policy.

For the two non-stationary components of the VAR system – inflation and the nominal funds rate – we control for moving endpoints using $r_t^\infty$ and $\pi_t^\infty$. The nominal interest rate endpoint is derived as described earlier from the observed term structure. As our measure of the inflation endpoint, we use a spliced estimate based on predictions from the agent learning model before 1981, and survey-based measures of expected long-run inflation thereafter.[21] By construction the third variable of the VAR, the output gap, is stationary with an endpoint fixed at zero.

The system of equations to be estimated can be written in compact form as

$$\tilde{z}_t - z_t^\infty = A\left[\tilde{z}_{t-1} - z_{t-1}^\infty\right] \tag{19}$$

where $\tilde{z}_t$ is a vector denoting the primary variables of the system, $z_t^\infty$ is a vector of the corresponding moving endpoints (zero in the case of $y_t$), and $A$ denotes a lag operator matrix. The order of $A$ – four lags of each variable – was selected based on standard information criteria tests. Variations in the lag length from four to eight periods do not substantially change the properties of the VAR. Tests for structural breaks in the VAR, which is estimated over the period 1960 Q1 to 1994 Q4, tend to suggest a relatively stable system.[22] Estimates of equation 19 are readily incorporated into the reduced-form expression for the bond rate (equation 7) by noting that $z_t = \left[\tilde{z}_t, z_t^\infty\right]$ and

$$H = \begin{vmatrix} A & -A \\ 0 & I \end{vmatrix}$$

The structure of $H$ ensures that forecasts of future $\tilde{z}_{t+i}$ made at time $t$ are based on the most recent available estimate of the endpoints, $z_t^\infty$. But if the VAR system is treated as a closed system and is used to generate future values of the endpoints (as we do next), it also implies that the endpoints follow a random walk. When endpoints are modelled in this naive fashion, VAR-based forecasts of nominal interest rates are very insensitive to how endpoint inflation is measured.[23] However, if the behaviour of the endpoints is modelled in a a more sophisticated manner – for example, by linking its behaviour to the overall FRB/US model – then the decomposition of the nominal rate endpoint can matter to the dynamic behaviour of the bond rate. We return to this issue in Section 5, where we consider some rudimentary models of the real interest rate endpoint.

21 To be specific, the survey portion of the spliced estimate equals the Hoey survey for 10-year ahead inflation expectations for the period 1981 Q1 to 1991 Q1, and the Philadelphia survey thereafter.

22 However, there are signs of instability in the funds rate equation, which fails a Chow test for a break around 1980.

23 For example, replacing the spliced measure with the learning model or regression-based estimates of endpoint inflation has little effect on the properties of the estimated system. As regards the bond rate, the key variable in the expectations generating system is $r_t^\infty$, not its decomposition.

## 4.2 Dynamic properties of the VAR and the bond rate models

Figures 4 and 5 show the impulse response functions for the estimated VAR model – dashed lines are one standard error bands – under the assumption that the two moving endpoints follow a random walk. We ordered the VAR variables as follows: $\pi_t$, $\gamma_t, r_t$, $\pi_t^{\infty}$ and $r_t^{\infty}$. The funds rate is ordered after $\pi_t$ and $\gamma_t$ on the argument that the monetary authority can respond to contemporaneous shocks to output and inflation, but that current activity does not react instantaneously to changes in short-term interest rates. With the funds rate third, the system is not particularly sensitive to alternative orderings of inflation and output. Endpoints are ordered last because they summarise agents expectations of the future, conditioned on all available information.

Figure 4
**Impulse response functions of the VAR model**

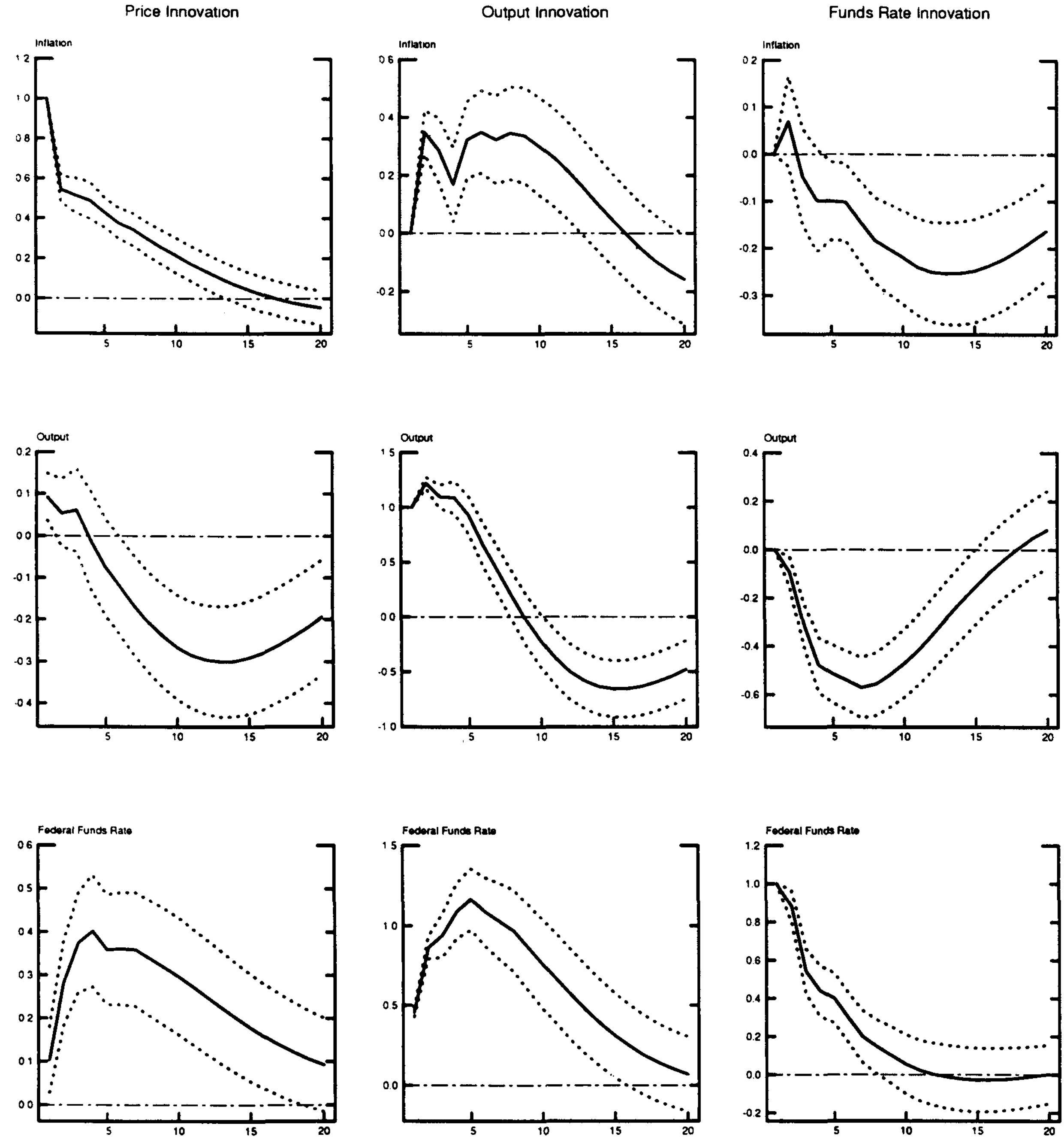


Figure 4 shows responses to innovations in $\pi_t$, $\gamma_t$ and $r_t$. As these variables are ordered prior to $\pi_t^\infty$ and $r_t^\infty$, no shocks to the endpoint are introduced. Thus, these impulse responses, while persistent, are stationary. Hence, shocks to the system gradually fade away and variables return to their original equilibrium (the zero line) in about 5 years.

Figure 5 shows the impulse response functions for output, inflation and the funds rate to innovations in the endpoints. Because of the random-walk nature of the endpoint equations, innovations to endpoints result in permanent shifts. The upper three panels show the effects of a unit change in endpoint inflation, under the assumption that $\rho_t^\infty$ is unaffected – implying that the nominal interest rate endpoint moves one-for-one with $\pi_t^\infty$. Both inflation and the funds rate exhibit overshooting, but eventually move one-for-one with the shift in their expected steady-state levels. By contrast, output initially expands following the shock to $\pi_t^\infty$, but after five years returns to equilibrium. Response patterns are roughly the same for a shock to $r_t^\infty$ alone (i.e., a permanent shock to the real rate), except that in this case the initial inflation surge fades away after a few years.

Figure 5
**Responses to innovations in endpoints**
Upper panel: innovation in the inflation endpoint; lower panel: innovation in real rate endpoint

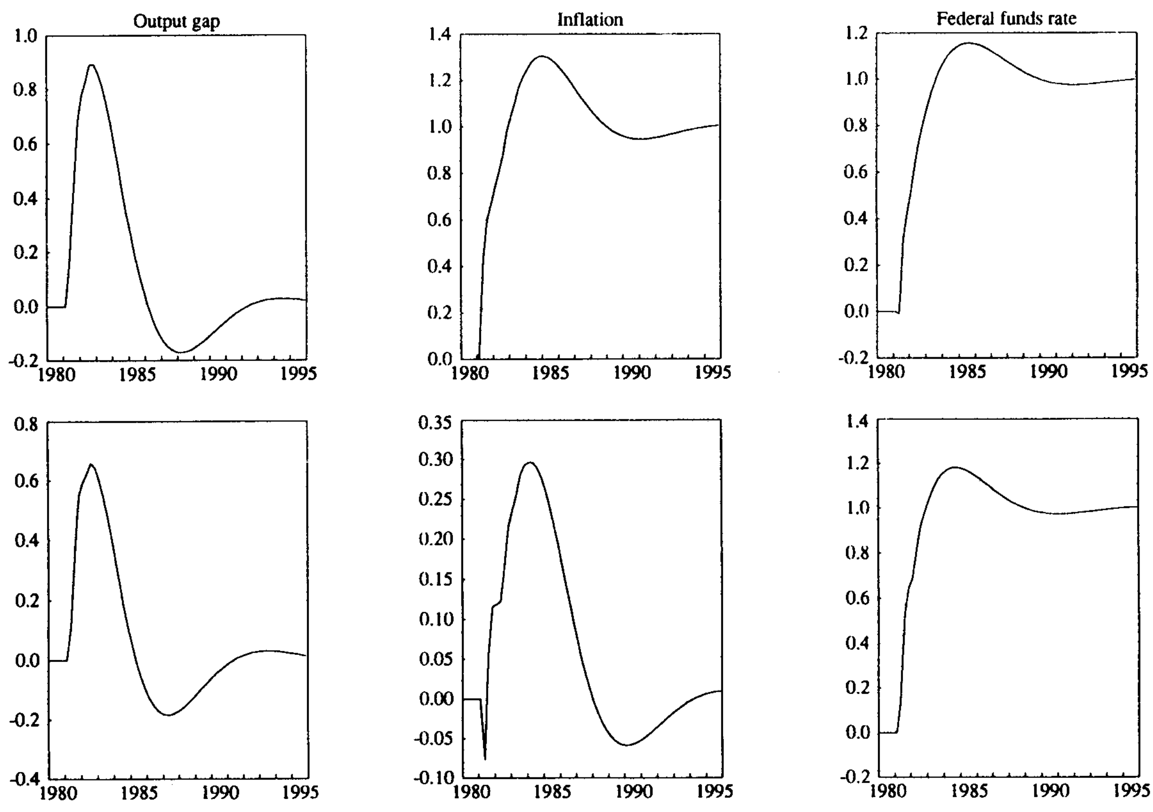



Figure 6 illustrates the implications of the behaviour of the VAR for the 10-year yield on Treasury bonds. As indicated by the solid lines in the upper panel, a unit innovation in the output gap yields an immediate jump in long-term interest rates of about 25 basis points. However, the rise is transitory and quickly fades away, in contrast to the response of the funds rate (the dashed line), which continues to build for a year or so before peaking at about a percentage point. Bond yields and the funds rate respond in a similar, but more muted, pattern to an innovation in inflation. But short and long-term rates behave quite differently following a shock to the funds rate. Because the VAR projects funds rate innovations to die out quickly, bond yields hardly respond to changes in short-term interest rates that are not driven by shocks to output or inflation.

Figure 6

**VAR-generated response of interest rates to innovations in output, inflation and the funds rate**

Solid: 10-year Treasury yield; dashed: funds rate

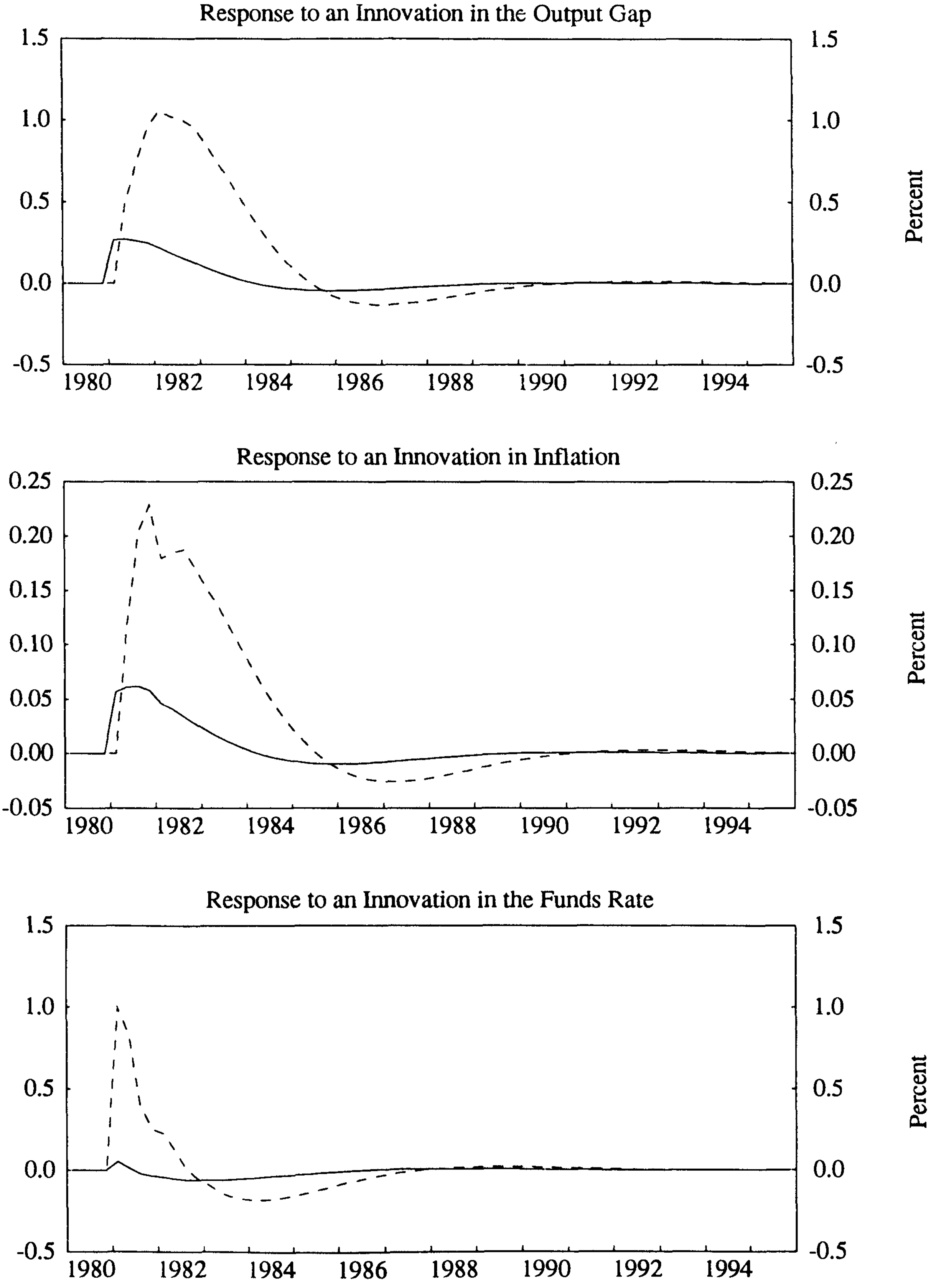


Figure 7

**VAR-generated response of interest rates to innovations in endpoint inflation and the real rate**

Solid: 10-year Treasury yield; dashed: funds rate

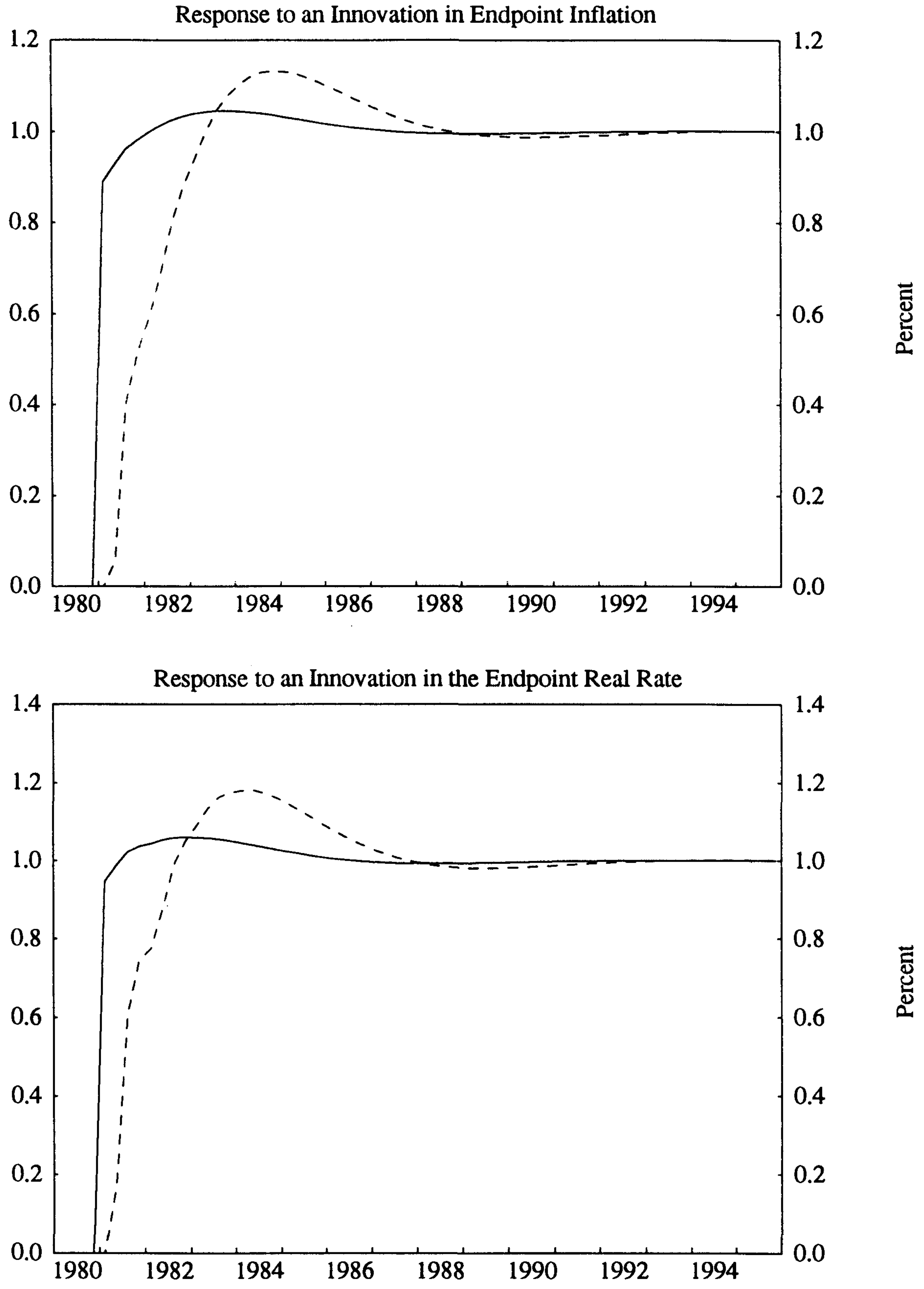
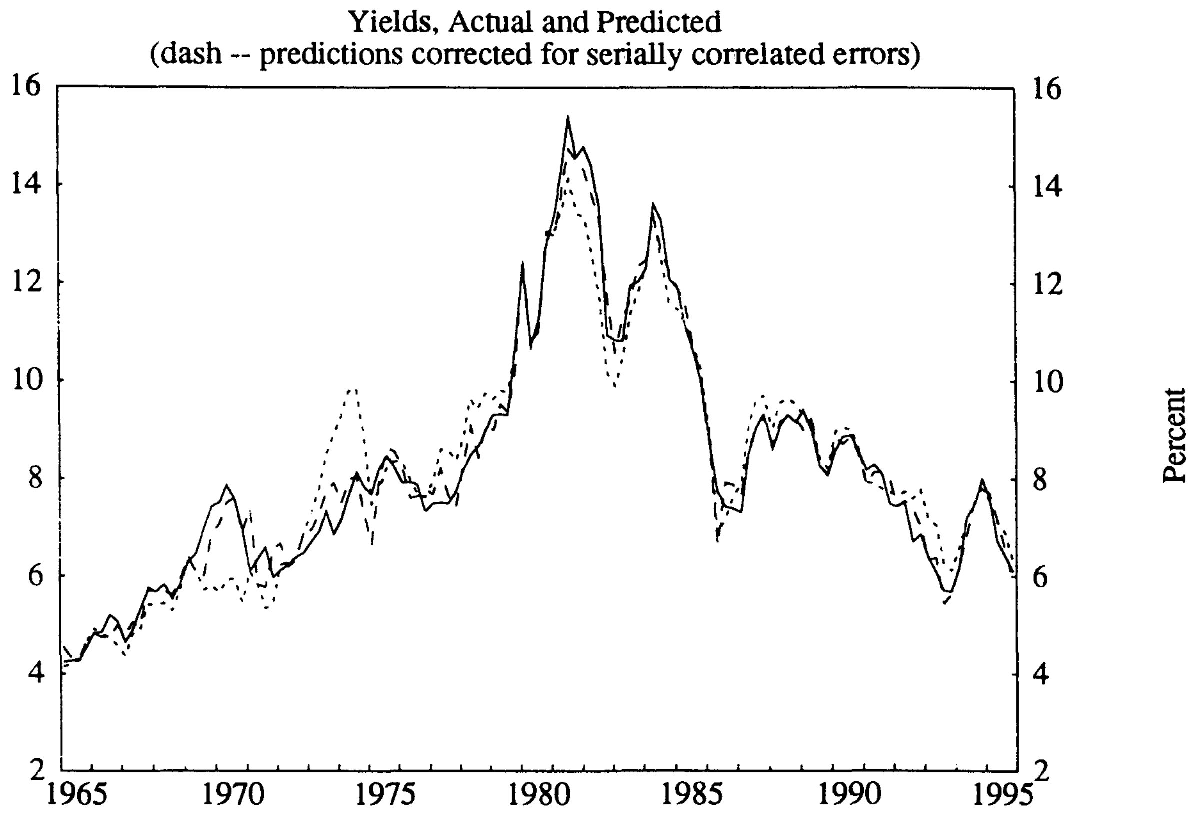
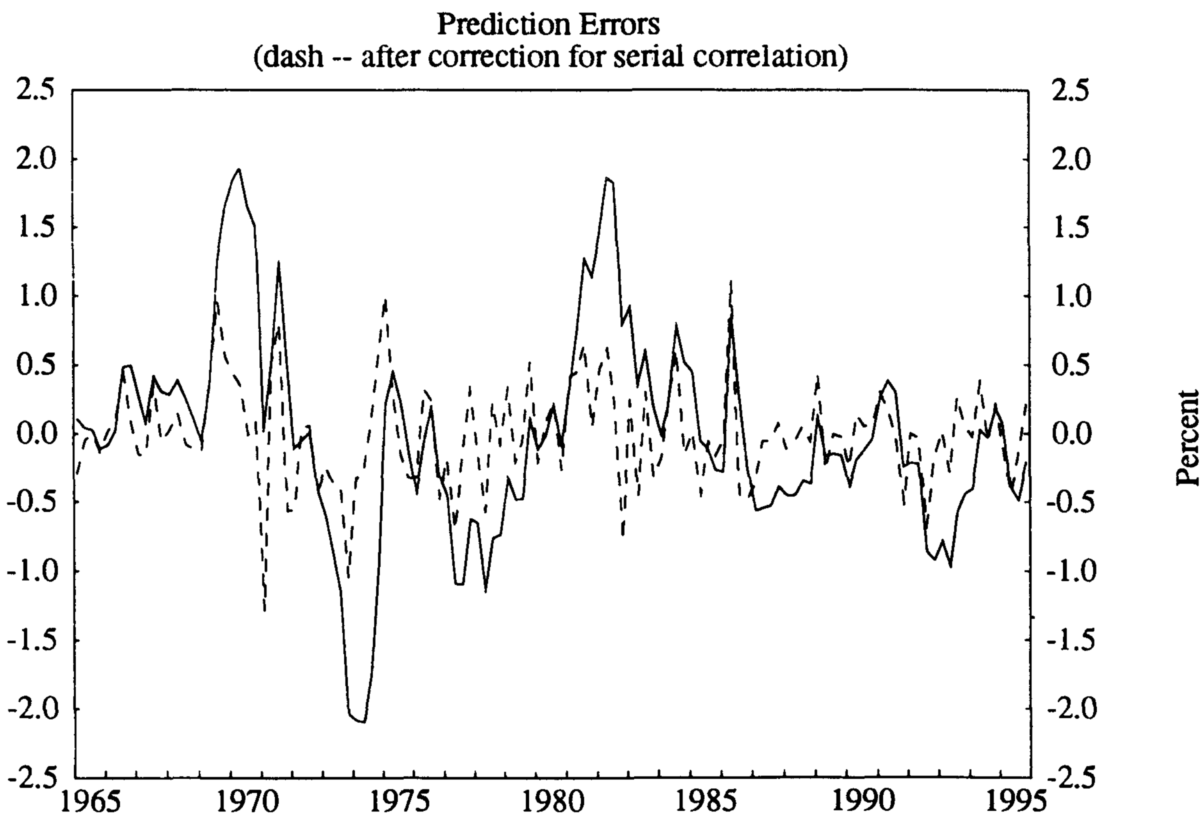


Figure 8

**VAR-based predictions of the yield on 10-year Treasury bonds to innovations in endpoint inflation and the real rate**



Although the model predicts that long-term rates are less responsive to transitory shocks than short-term rates, the situation is reversed for permanent shocks that alter the expected endpoint levels of inflation and the real interest rate. As shown in Figure 7, bond yields respond almost instantaneously (and fully) to a permanent shift in either endpoint. The funds rate, however, takes a full two years to respond as much as the bond rate to a shift in long-term expectations. Accordingly, positive endpoint shocks are associated with increases in the slope of the term structure, while positive shocks to output and inflation yield a less steep slope. Innovations in the funds rate are also associated with a decline in the slope of the yield curve, unless they are associated with changed expectations concerning the long-run target level of inflation.

### 4.3 Statistical evaluation of the bond rate model

The upper panel of Figure 8 compares the historical yield on 10-year Treasury bonds to that predicted by the VAR model, under the assumption that the term premium $\Phi_n$ is constant. The bottom panel displays the corresponding prediction errors. For both panels, results are plotted for two different predicted paths of the bond rate. The first path (the dotted line in the upper panel) represents the weighted sum of future short-term interest rates projected at each point in time, adjusted for the average 1965-1995 difference between the 10-year rate and the weighted sum. The second series (the dashed line in the upper panel) is the same as the first, except that account is taken of serial correlation in the prediction errors. Thus, the second series can be regarded as the one-step ahead prediction of the bond rate.[24]

As can be seen, both series do a reasonable job of capturing the overall historical path of the bond rate. Nonetheless, the lower panel demonstrates that the model makes significant prediction errors. However, after controlling for residual serial correlation, the model's in-sample tracking performance is competitive with that obtained from other models of the term structure. The standard error of our bond rate model is 38 basis points after correcting for residual serial correlation, as compared to 46 basis points for an equation styled after that used in the Federal Reserve's old MPS model, estimated over the same sample period.[25]

Aside from its magnitude, residual correlation in the bond model's errors is problematic because it indicates a violation of the underlying assumptions of the model: If the risk premium $\Phi_n$ is constant and expectations are rational, the prediction errors of the model should be white noise. However, a simple regression of the prediction error on its own lag yields an estimated first-order autoregressive parameter of 0.83. More formally, the hypothesis that the prediction errors do not display n-th order serial correlation can easily be rejected at the 5 percent level for n equal to 1, 4 or 12, using the Lagrange multiplier test developed by Breusch (1978) and Godfrey(1978).

Furthermore, the model fails a test of the rational expectations overidentifying restrictions imposed on the bond rate model. Following the procedure suggested by Hansen (1982), the unadjusted bond rate errors are regressed on the elements of the information set used to construct $\sum_{i=0}^{n-1} B^i r_{t+i}^e$ – that is to say, current and lagged observations on the funds rate, inflation, and the output gap, plus current observations on the two endpoints.[26] The value of $R^2$ from this regression, multiplied by the number of observations, is distributed $\chi^2$ with *k*-1 degrees of freedom, where *k*

24 Strictly speaking this statement is incorrect, because the information set includes contemporaneous observations on the funds rate, inflation, the output gap and the endpoints, and lagged information on the bond rate equation's errors.

25 The MPS-style equation is an error-correction model in which the change in the effective 10-year bond rate is regressed on lagged bond rate changes, the lagged level of the bond-funds rate spread, and current and lagged changes in the effective funds rate.

26 The number of lags allowed in the regression equals the number used in the construction of the bond rate.

equals the number of regressors. The p-value from this test is less than 0.01. Examination of the estimated coefficients from this regression reveals that the bond model underestimates the sensitivity of long-term interest rates to changes in the funds rate, and overestimates its sensitivity to movements in output or inflation.[27]

One must be cautious in interpreting the failure of the model to pass the tests for serial correlation and overidentifying RE restrictions. For example, one could interpret the evidence as a rejection of the rational expectations hypothesis. However, the test results may reflect an information set for the VAR model that is too restrictive – and in fact, preliminary work with expanded VARs does suggest that other variables, such as oil prices, are important. Furthermore, all the tests are conducted under a joint hypothesis of rational expectations and a constant term premium. As suggested by the finance literature, the correlation of bond model errors with macroeconomic factors and lagged errors may simply be the result of risk premiums that vary over time in a predictable fashion. We believe it is more fruitful from a modelling prospective to allow for this possibility, than to abandon the assumption of rationality.

As noted at the beginning of Section 3, a drawback of basing expectations on a small-scale VAR system is that, when embedded in a larger model such as FRB/US, the simulated behaviour of the VAR system is likely to be inconsistent with that of the broader system – a problem that does not affect model-consistent expectations. But this drawback is not necessarily a serious problem, if the moving endpoints in the VAR are consistent with those of the full model,[28] and the impulse response patterns of the two system are broadly similar. To minimise this problem, simulations of FRB/US are always designed to ensure endpoint consistency. However, gauging the similarity of impulse responses is more difficult. One approach to this problem is to compare the behaviour of the VAR system estimated using historical data, with a VAR estimated on a synthetic dataset derived from stochastic simulations of the FRB/US model itself. As discussed by Bomfim, Brayton, Tinsley and William (1995), the impulse responses generated by the two approaches are fairly similar. This result suggests that the expectations generated by the small-scale VAR are effectively rational within the context of the FRB/US model, since they are consistent with the behaviour of the overall system.

## 5. Explaining recent bond market behaviour

As a final test of our model, we consider what light it can shed on recent developments in the US bond market. As shown in the upper panel of Figure 9, long-term interest rates have fallen roughly 2¾ percentage points over the past five years. The decline has not been steady, but was marked by a major back-up in rates during 1994 that has since been reversed. Short-term rates experienced a similar decline on balance over the entire 1990-1995 period, but the co-movement between long and short rates has not been stable. In particular, the funds rate did not display anything like the 1992-1995 bond rate cycle. Furthermore, the typical lagging behaviour of long-term interest rates, which causes the yield curve to steepen (flatten) during periods of falling (rising) short-term interest rates, disappeared at times – most notably in the first months following the February 1994 change in monetary policy, when bond rates rose twice as much as the funds rate.

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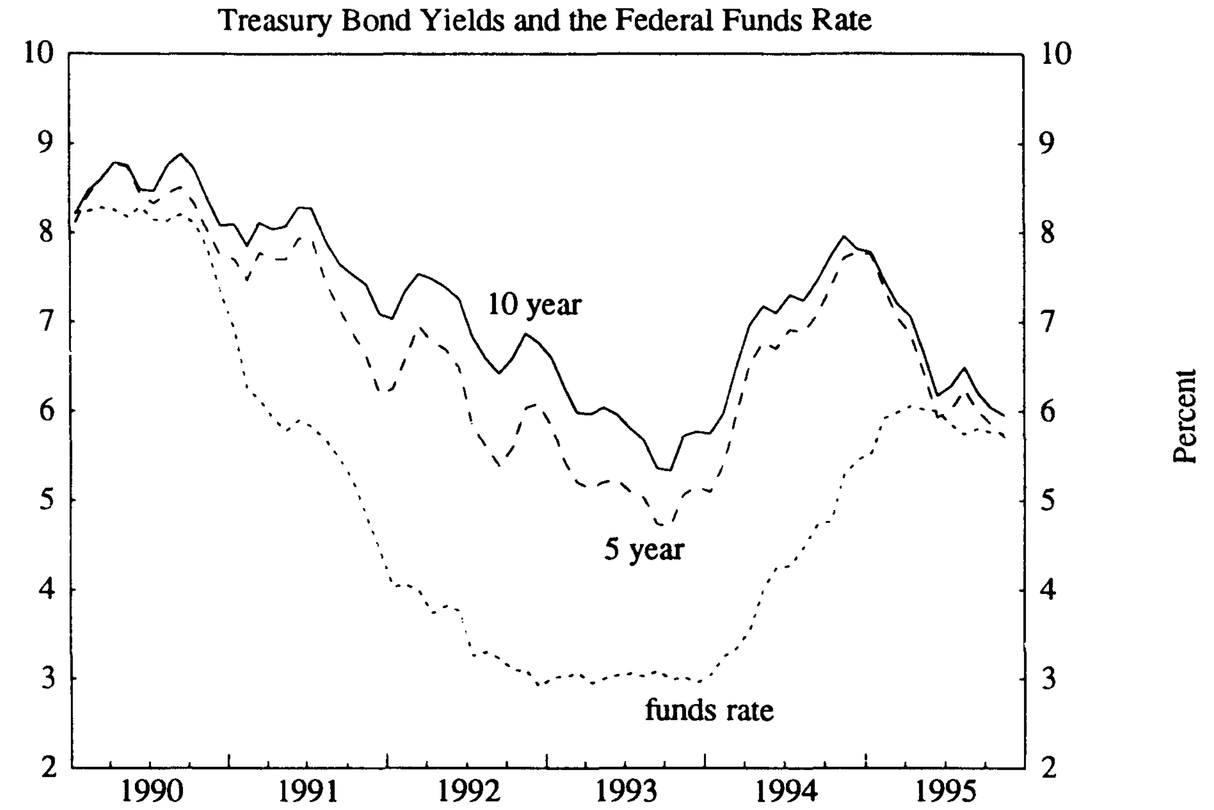
27 Because the bond rate prediction errors are serially correlated, the overidentifying restrictions test we carry out may not be appropriate. However, it is worth noting that even if the test is run with the errors corrected for serial correlation, the explanatory power of the VAR information set in the LM regression is still very high – $R^2 = 0.43$.

28 It should be noted that using model-consistent expectations does not allow one to avoid issues concerning the specification of endpoint conditions. As in the case of the VAR-based expectations, it is necessary to be specific about the determinants of terminal conditions – e.g., the inflation goals of the central bank and the target level of government indebtedness.

Figure 9

**Recent movements in US interest rates to innovations in endpoint inflation and the real rate**

Solid: 10-year Treasury yield; dashed: funds rate



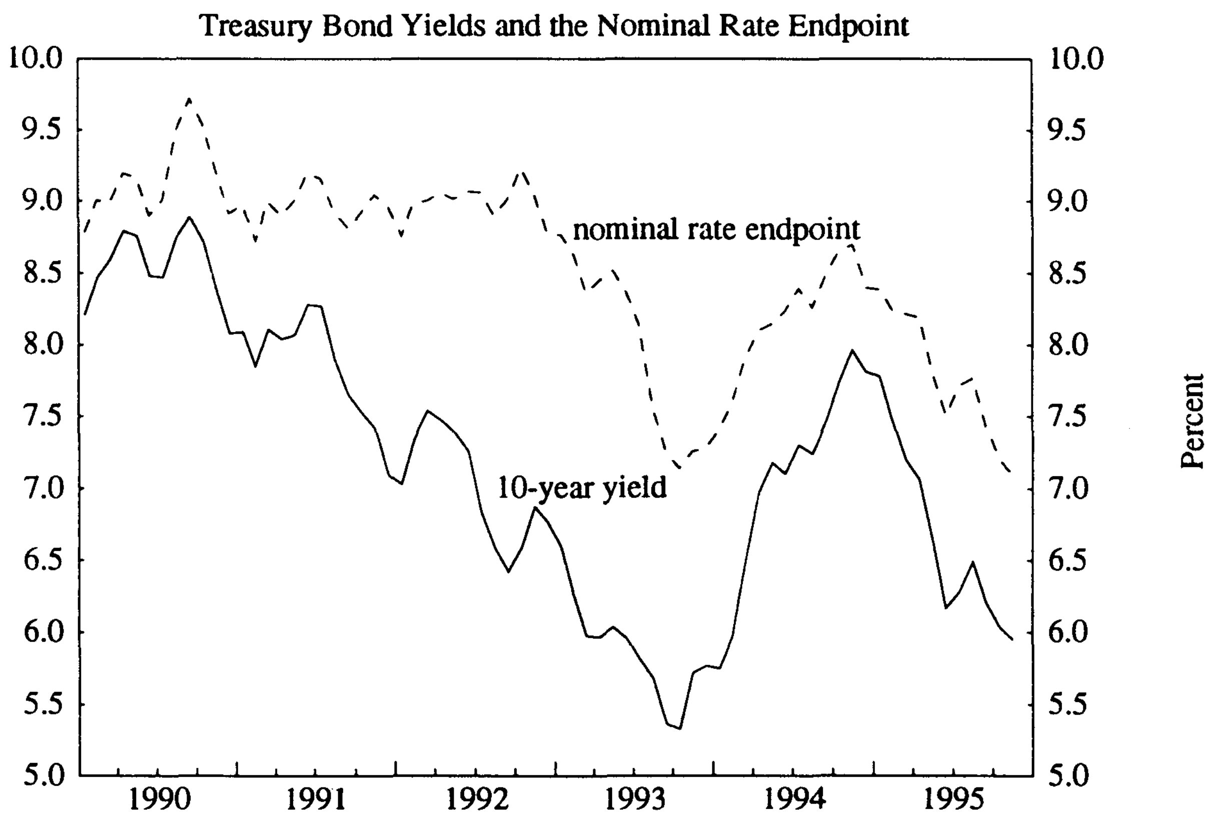
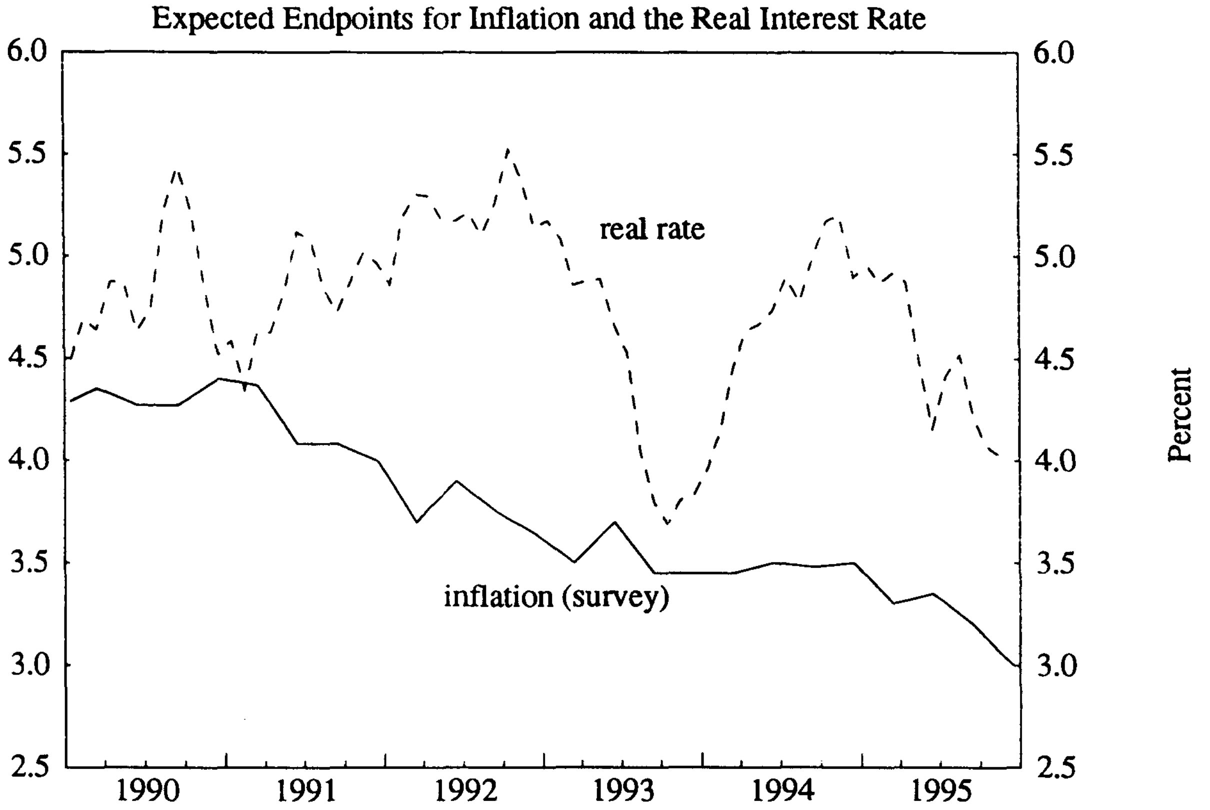
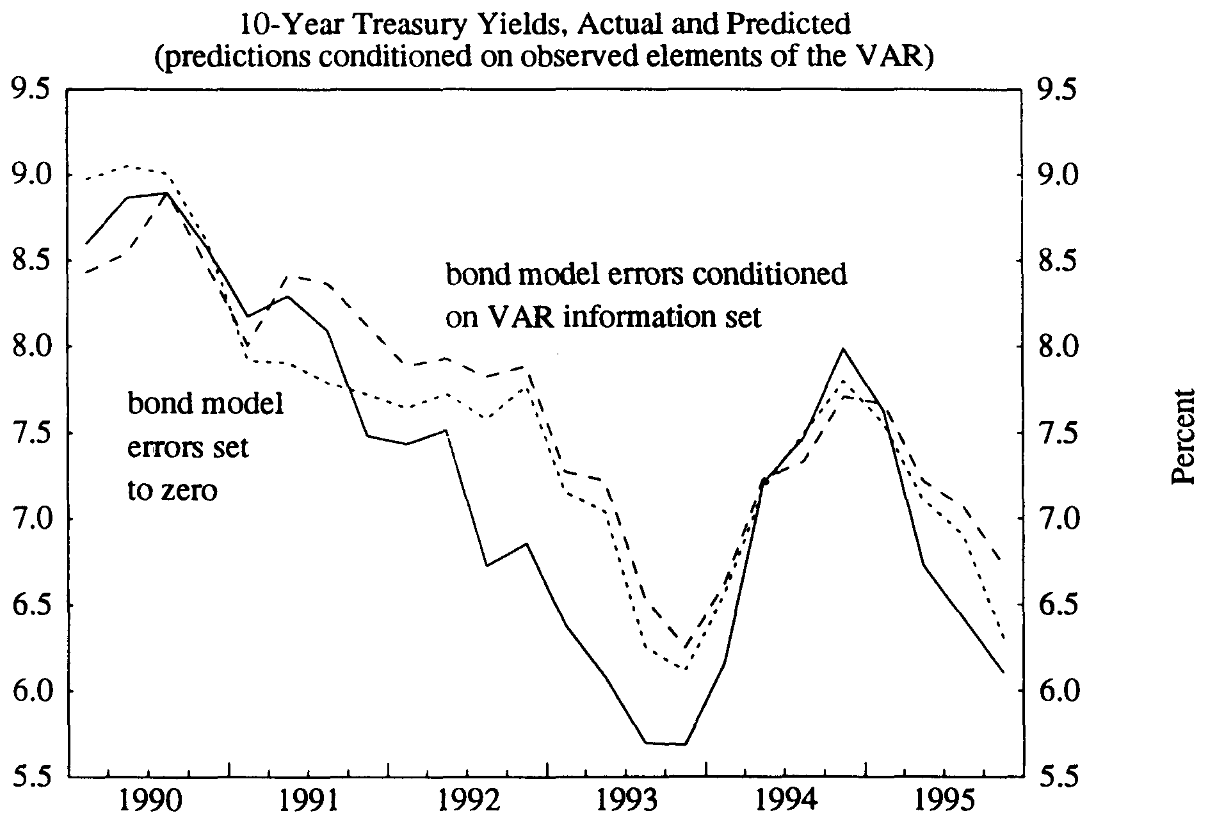



Figure 10

**Decomposition of recent bond rate movements to innovations in endpoint inflation and the real rate**

Solid: 10-year Treasury yield; dashed: funds rate



How well can the model account for this complicated pattern of interest rate movements? We begin by considering the accompanying behaviour of the nominal interest rate endpoint, illustrated by the dashed line in the bottom panel of Figure 9. As can be seen, market expectations were relative stable through 1992. During the first 9 months of 1993, however, there was a steep decline in $r_t^{\infty}$ of about 2 percentage points. Much of this decline was reversed over the next 12 months, but the nominal rate endpoint has since returned to its mid-1993 level.

As shown by the solid line in the top panel of Figure 10, essentially none of the 1993-1995 gyrations in $r_t^{\infty}$ can be attributed to a change in expected long-term inflation prospects – although a gradual fall in $\pi_t^{\infty}$ can account for a substantial portion of the overall decline in the nominal rate endpoint since 1990.[29] Rather, recent fluctuations in the bond rate reflect changed perceptions concerning the long-term level of real interest rates. This interpretation is the opposite of that reached by Campbell (1995), who attributes the bulk of the 1994 back-up in bond yields to a rise in inflation expectations. His conclusion appears to be largely based on an a priori assumption that the real rate is stationary and relatively constant. However, Campbell's interpretation is not necessarily at odds with ours, because his definition of the real rate may exclude risk premia. If so, the rise in our measure of the real rate endpoint, which includes a risk premium, could theoretically be attributed to market-perceived changes in the variance of inflation or in its covariance with other factors.

The lower panel of Figure 10 compares the actual bond rate path (solid line) to that predicted by the model (dotted line), conditioned on the actual path of output, inflation, the funds rate, and the two endpoints. Here, bond rate predictions are not corrected for residual serial correlation (as they were in the dashed line in the upper panel of Figure 8), but are instead defined as $\tilde{\Phi}_n + \frac{1-B}{1-B^n}\sum_{i=0}^{n-1} B^i r_{t+i}^e$. As it is, even uncorrected predictions do a good job of capturing the overall movement in bond rates since 1990. The panel also displays predictions (dashed line) that control for the economic structure of the bond equation errors, i.e., that incorporate an estimate of the dependence of model errors on the VAR information set.[30] The similarity between the two predicted bond rate series indicates that the correlation between model errors and economic conditions is not that quantitatively important, at least over the last few years.

As one might suspect from the upper panel of Figure 10, the key to explaining the recent cycle in bond rates hinges on the behaviour of the expected real interest rate endpoint. Unfortunately, we have just begun to develop an empirical model of $\rho_t^{\infty}$ . However, it is instructive to consider some preliminary results.

As noted in the prior section, the VAR system incorporates naive random-walk forecasting equations for the endpoints. The upper panel of Figure 11 compares the actual path of the real rate endpoint (here plotted at a quarterly frequency) to that which would have been predicted in 1989 Q4 using the VAR rule (dotted line). Conditioning on a constant value for $\rho_t^{\infty}$, but on the observed values of the other elements of the VAR, yields the projection of the bond rate illustrated in the bottom panel of Figure 10 by the dotted line. As suspected, under these conditioning assumptions the model does not predict a pronounced bond rate cycle.

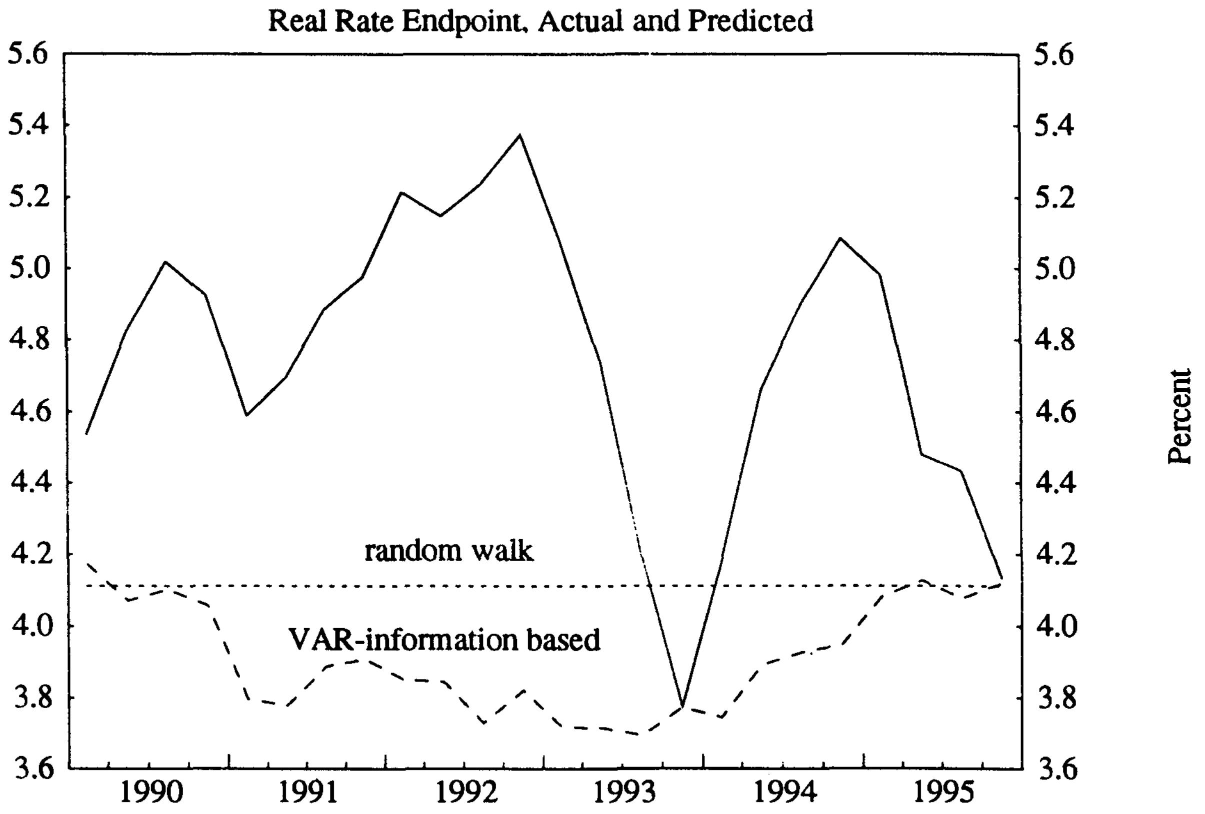
29 The upper panel of Figure 10 displays monthly data. A monthly series for expected inflation is constructed from quarterly survey data via cubic-spline interpolation.

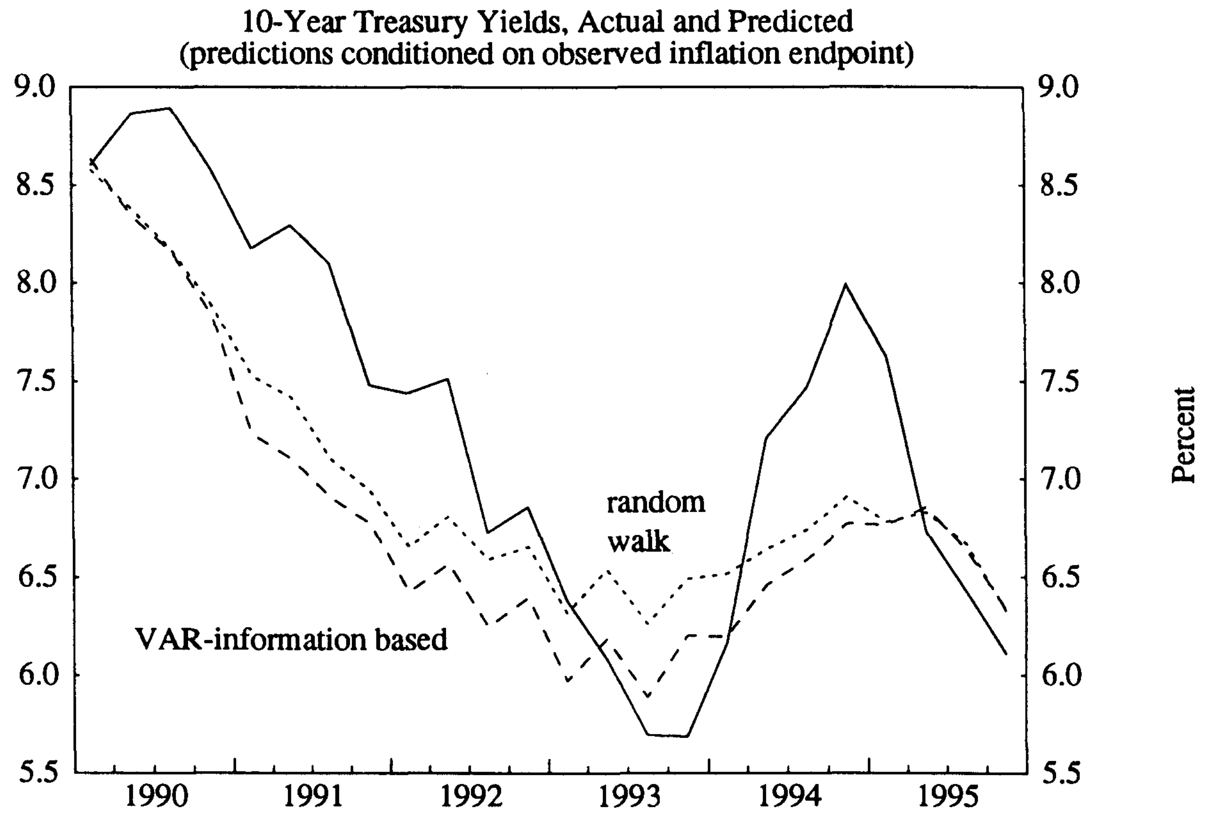
30 Specifically, bond model errors are regressed on the elements of the VAR, and then predictions of the errors are made conditional on the observed path of the VAR variables.

Figure 11

**Effect on bond rate predictions of alternative forecasts of the real rate endpoint**

Solid: 10-year Treasury yield; dashed: funds rate



Nor does the model do so if an attempt is made to control for the historical correlation of the real endpoint with aggregate macroeconomic conditions. Regressions of $\rho_t^{\infty}$ on current and lagged values of the other variables of the VAR (plus lags of itself) suggest a statistically significant link between the endpoint and changes in current economic conditions – particularly the funds rate. However, conditioning on this link does not appear to be that important from an economic standpoint. As indicated by the dashed line in the upper panel of Figure 11, a projection of the real rate endpoint based on this crude model differs only modestly from a random-walk projection.[31] This difference changes the projected bond rate (dashed line, lower panel) only marginally.

An alternative approach to this problem is to link forecasts of $\rho_t^{\infty}$ to the behaviour of the larger-scale FRB/US model. In simulation work with the full model under VAR-based expectations, our current practice is to employ an ad hoc adjustment equation that forces the expected real rate endpoint to converge slowly to the real funds rate generated by the overall model. Because the real interest rate produced in simulations of this sort is generated by a monetary policy reaction function that targets a specific rate of long-run inflation (using nominal short-term interest rates as an instrument), this practice is equivalent to forcing $\rho_t^{\infty}$ to converge to the overall model's steady-state real interest rate.[32] In the context of the FRB/US model, this implies that $\rho_t^{\infty}$ is a function of a large number of variables, including such fiscal variables as the level of government indebtedness, the mix of taxes and transfers, and marginal tax rates.[33] Unfortunately, although the overall macro model provides a framework for tying down $\rho_t^{\infty}$ in policy simulations, it cannot be used directly in estimation and forecasting. However, it can provide useful guidance in the specification of an empirical structural model of the real rate endpoint.

An example of such guidance is provided by work-in-progress on the link between real interest rates and government budget deficits. In non-Ricardian large-country open-economy models (such as FRB/US), a key theoretical determinant of the equilibrium real interest rate is the steady-state ratio of the government budget deficit to GDP: The deficit ratio determines the debt-to-GDP ratio, which in turn influences the private saving rate. The neoclassical growth model which is at the core of FRB/US suggests that a sustained 1 percentage point rise in the deficit ratio should boost the equilibrium real interest rate by ½ to ¾ percentage point.[34]

This model-generated estimate is in line with the historical correlation between real interest rates and the deficit-to-GDP ratio. As shown in the upper panel of Figure 12, annual averages of the cyclically-adjusted budget deficit and $\rho_t^{\infty}$ tend to move together on a contemporaneous basis.

31 The 1990-1995 predictions of the VAR-information model are conditioned on observations of $\rho_t^{\infty}$ through 1989 Q4 only, but on actual post-1989 observations for the other elements of the VAR.

32 Strictly speaking, real rate convergence also depends on $\pi_t^{\infty}$ converging to the policy target rate of inflation. In simulation this condition is met via updating rules similar to those implicit in the learning agent model, or through ad hoc adjustment equations.

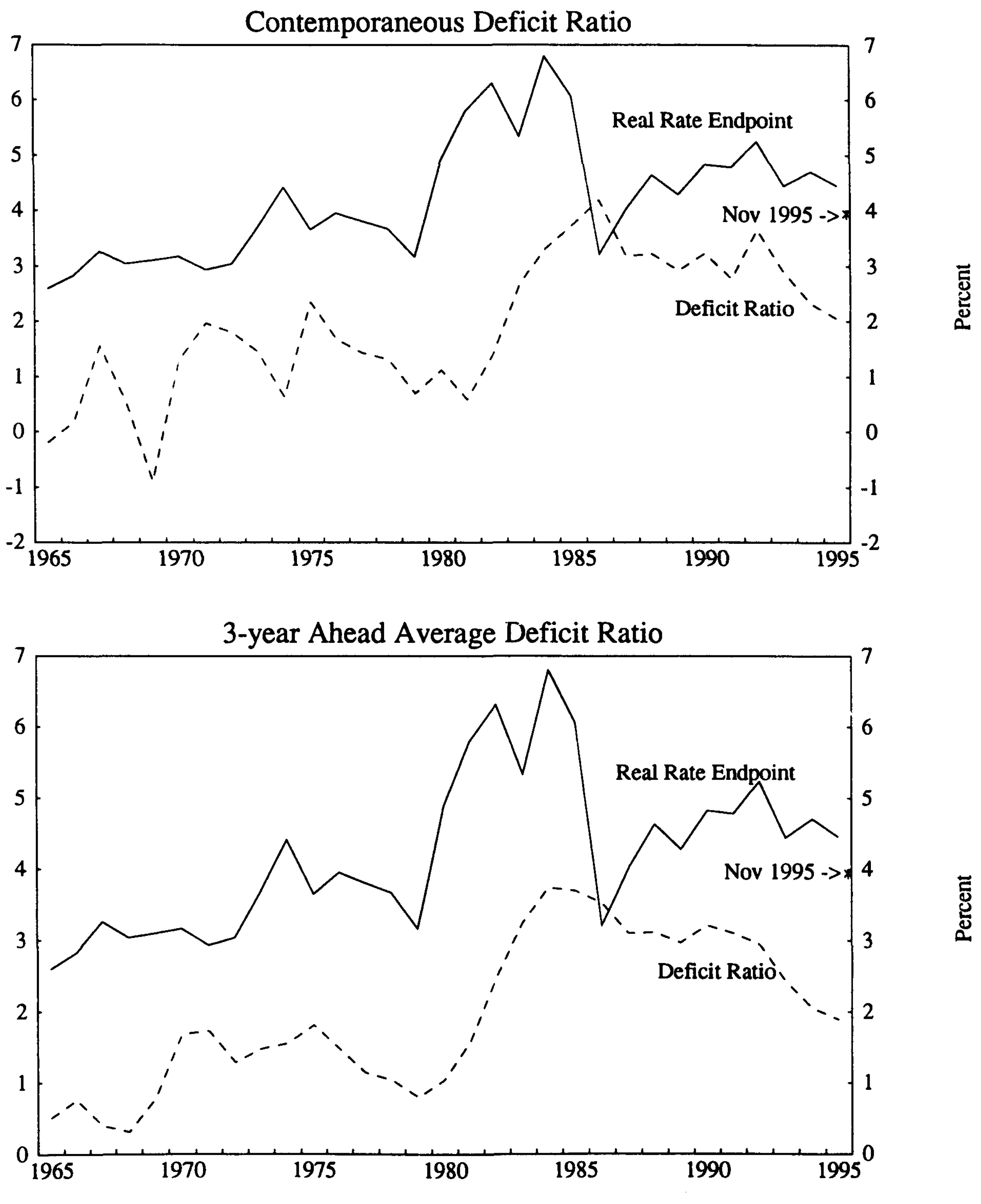
33 In the FRB/US model, the primary channel through which fiscal policy influences the equilibrium real interest rate is household wealth, owing to the fact that consumers are non-Ricardian. However, the mix of taxes and transfer payments also matters, since the propensity to spend out of the present value of transfer income is higher than that for after-tax labour or property wealth, implying that changes in the tax/transfer mix influence the aggregate saving rate, and thus the real interest rate. Finally, changes in marginal income tax rates influence the desired stock of capital per worker, and thus the aggregate productivity level. Changes in the latter influence the steady-state real interest rate, because the real rate is defined as that which equilibrates aggregate demand and supply in the long run.

34 The range of estimates is produced by simulating FRB/US under different assumptions about the responsiveness of foreign interest rates to a rise in domestic rates; the less responsive are foreign rates, the greater is the sensitivity of the domestic real rate to a change in government saving.

Figure 12

**Relationship between the real interest rate endpoint and the cyclically adjusted federal budget deficit**

Deficit expressed as a ratio to GDP



However, the two series are more closely correlated if the real rate endpoint is plotted against a forward moving average of the deficit ratio over the near future, as evidenced by the lower panel of Figure 12, which illustrates the relationship for a 3-year-ahead moving average.[35] That the fit is tighter for the average future deficit ratio is not surprising, given that $\rho_t^\infty$ is an estimate of the level of real rates expected to prevail five or ten years into the future.

Table 3
**Stationarity and regression tests**
**for the real interest rate endpoint and the deficit-to-GDP ratio**
Sample period 1965 Q1 to 1992 Q4

| **Stationary tests** | |
|---|---|
| **Variable** | **ADF statistic** |
| Real interest rate endpoint based on: | |
| Learning model inflation measure | -3.11 |
| Spliced inflation measure | -2.51 |
| Federal budget deficit-to-GDP ratio: | |
| Actual | -1.90 |
| Cyclically adjusted | -3.11 |

**Regression tests**

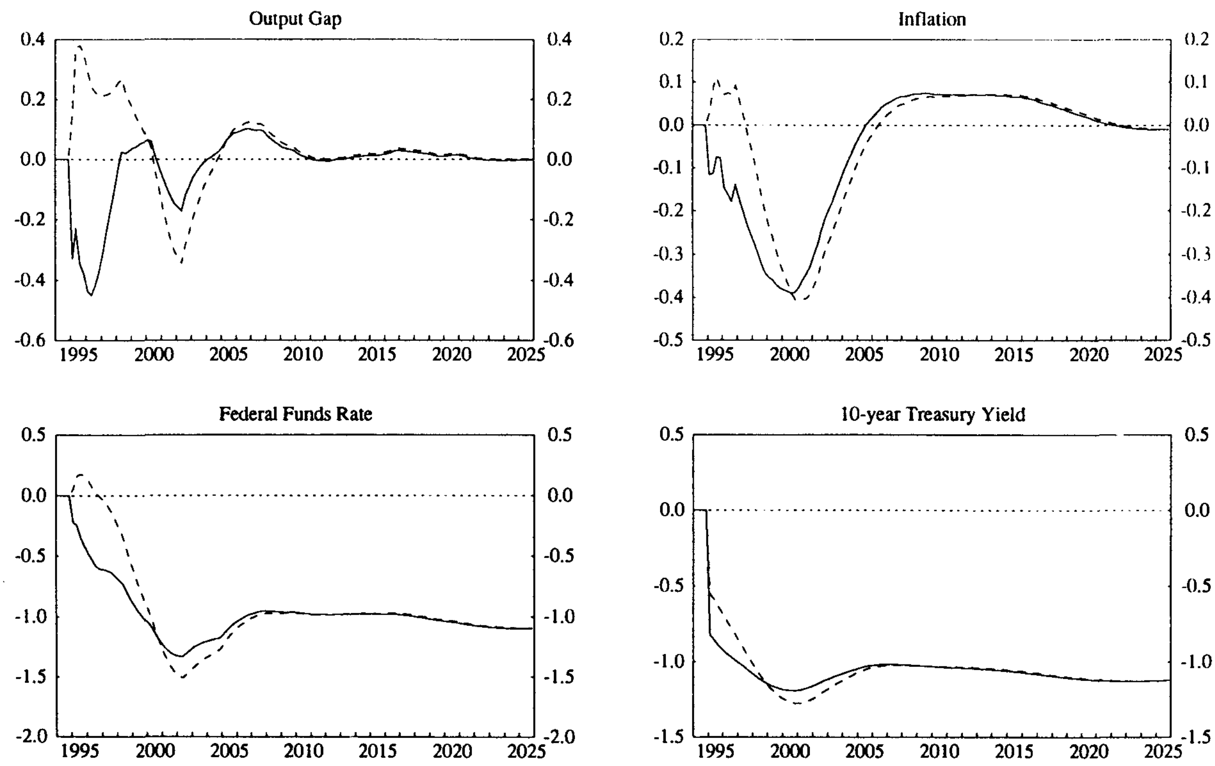
$$\rho_t^\infty = \alpha_0 + \alpha_1 def_t + \sum_{j=-8}^{8}\left[\beta_j \Delta\rho_{t+j}^\infty + \omega_j \Delta def_{t+j}\right]$$

| **Variable** | **Estimated $\alpha_1$** | **ADF statistic** |
|---|---|---|
| Actual deficit | | |
| Real interest rate endpoint based on: | | |
| Learning model inflation measure | 0.47 | -3.74 |
| Spliced inflation measure | 0.64 | -4.00 |
| Cyclically-adjusted deficit | | |
| Real interest rate endpoint based on: | | |
| Learning model inflation measure | 0.42 | -2.06 |
| Spliced inflation measure | 0.67 | -2.82 |

35 Forward-moving average estimates for the most recent period incorporate CBO projections of the federal deficit for 1996 to 1998.

Of course, pictorial evidence such as Figure 12 is subject to many criticisms.[36] But regression analysis suggests the degree of co-movement is remarkably similar to that suggested by the steady-state model. As shown in the lower portion of Table 3, regressions of $\rho_t^\infty$ on the deficit-to-GDP ratio ($def_t$), plus leads and lags of changes in $\rho_t^\infty$ and $def_t$, suggest that a 1 percentage point permanent increase in the ratio raises the real rate endpoint by 42 to 67 basis points. This result, which is estimated with a high degree of precision, holds whether or not the deficit is measured on an actual or cyclically-adjusted basis.[37]

Figure 13
**Medium-term effect of the congressional budget proposals on output, inflation and interest rates, based on RE simulations of the new FRB model of the US economy**
Solid: standard response; dashed: myopic consumers

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36 For example, the correlation would be more persuasive if it were based on real-time publicly-available projections of the deficit, not actual deficit outcomes. This proposition can be tested using forecasts of the deficit prepared and published by the Congressional Budget Office since the 1970s; we hope to conduct these tests in the near future. In addition, there is the issue of the direction of causality, given that the correlation may partly reflect the link between current interest rates and future government interest expense. However, this channel could at most account for only a small part of the correlation given the size of the net interest share of the budget.

37 It might be objected that the high t-statistics shown in Table 3 are an artefact of a possible trend in $\rho_t^\infty$ and $def_t$, given that both series appear to be only borderline stationary. If it was thought that both series are I(1), the regression results would be interpreted as a cointegration tests. Under this assumption, ADF statistics for the residuals from the regressions indicate that the actual deficit ratio is cointegrated with the real rate endpoint, but not the cyclically-adjusted deficit. However, because we don't view either series as I(1), we regard the ADF test results as moot.

These results suggest that fiscal policy may have been a possible cause of the recent cycle in the real rate endpoint. Indeed, 1993 saw enactment of a major fiscal package that significantly reduced projected budget deficits – the same year that the real endpoint fell 1½ percentage points. Similarly, 1995 saw both a large fall in $\rho_t^\infty$ and legislative action that has greatly increased the odds that the budget will be in approximate balance around the turn of the century. Experiments with FRB/US concerning the effects of the Congressional budget proposals currently under debate suggest that the magnitude of the proposed changes, if fully credible, are sufficient to explain almost all of the decline in the real endpoint experienced since late last year. As shown in Figure 13, simulations of the full model under model-consistent expectations indicate that the proposed budget savings would lower real interest rates in the long run by about 1¼ percentage points. The model would generate a similar decline in real rates from passage of the 1993 budget agreement, given that its savings are comparable to those currently being proposed.

Although changes in the stance of fiscal policy can help explain why bond rates fell sharply in 1993 and 1995, fiscal policy cannot account for the 1994 back-up in rates. For this episode, we must look for some other cause. One possibility is that the market perceived a shock to aggregate demand that would be highly persistent, and so raise the expected long-term level of real interest rates. In fact, during this time there was a very large upward revision to the expected level of future real output. As shown in Table 4, column 1, the 1994 Blue Chip consensus forecast of current year real GDP growth (4th quarter to 4th quarter) rose almost a percentage point between January and December. When coupled with the accompanying 1994 revision to 1995 growth (column 2), these revisions imply an upward revision in the projected level of output in 1995 Q4 of 1.1 percentage points. Simulations of FRB/US indicate that a shock of this magnitude to ex ante aggregate demand, if sustained, would raise steady-state real interest rates by 70 basis points – about half as much as the observed change in the real rate endpoint. Of course, it is difficult to say what sort of aggregate demand shock would display such persistence. Furthermore, inspection of Table 4 reveals that Blue Chip forecast revisions are poorly correlated with movements in $r_t^\infty$.

Table 4
**Revision to Blue Chip consensus forecasts of real GDP and movements in the real interest rate endpoint**

| | January to December GDP revisions in: | | | Q4 to Q4 change in real rate endpoint |
|---|---|---|---|---|
| | Current year growth | Next year's growth | Next year's Q4 level | |
| 1995 | 0.1 | 0.4 | 0.5 | –1.0 |
| 1994 | 0.9 | 0.2 | 1.1 | 1.3 |
| 1993 | –0.1 | –0.3 | –0.4 | –1.6 |
| 1992 | 0.4 | –0.3 | 0.1 | 0.4 |
| 1991 | –0.4 | –0.3 | –0.7 | 0.1 |
| 1990 | –0.7 | –1.9 | –2.6 | 0.8 |

## Conclusion

In this paper, we have presented a model of the bond rate that in some ways is quite traditional: As suggested by conventional rational expectation theories of the term structure, bond yields are modelled as a weighted sum of future expected short-term interest rates (plus a constant risk premium), with expectations derived from a VAR forecasting system. Where the model differs from standard practice is in its use of moving endpoints in the design of the VAR to help account for the observed nonstationarity of nominal interest rates and inflation. These moving endpoints, which denote investors' expectations on the long-term level of nominal rates and inflation, provide a means to decompose bond rate movements into two components – a stationary element associated with the business cycle and monetary policy stabilisation, and a nonstationary portion linked to longer-term monetary and fiscal policy goals.

VAR models that incorporate moving endpoints (derived from the term structure and surveys of inflation) provide more sensible predictions of the historical path of long-term interest rates than do models that assume interest rates to be stationary or I(0). In terms of goodness of fit, our approach also compares favourable with atheoretic error-correction models of the term structure, if allowance is made for serially-correlated movements in the term premium. However, our work in this area is still at a preliminary stage. In particular, more remains to be done to explore the empirical relationship between policy and other determinants of expected long-run inflation and the real rate of interest.

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**Comments on paper by S. Kozicki, D. Reifschneider and P. Tinsley by G. Sutton (BIS)**

The goal of this very interesting paper is to develop a model of long-term interest rate behaviour which is firmly grounded in economic theory and usable for policy analysis. This is a difficult task and the authors deserve credit for their clever efforts in this direction, breaking new ground close to the frontier.

The theoretical basis of the model is the expectations hypothesis of the term structure. Therefore, a key component of the model is a mechanism for generating expectations of future short-term interest rates. It is assumed that expectations are consistent with forecasts from a small scale VAR. The VAR is somewhat unconventional because it includes exogenous variables which influence the time path of the endogenous variables of the VAR.

The authors refer to the exogenous variables in the VAR as "moving endpoints". It is hoped that these moving endpoints contain information about the evolution of the endogenous variables in the VAR beyond that incorporated in their past behaviour. For example, one of the moving endpoints, the "nominal interest rate endpoint", is an average of forward interest rates. The conjecture is that this variable contains information about the future course of short-term interest rates above what is contained in the past history of short-term rates and the other endogenous VAR variables.

There are good reasons to believe that this is indeed the case. For many countries, the slope of the term structure contains information relevant for predicting the future course of short-term interest rates. The nominal interest rate endpoint – an average of forward interest rates – appears to be a useful variable for exploiting this information. Perhaps not surprisingly, empirical evidence reported in the paper supports the view that the nominal interest rate endpoint contains useful information about the future course of short-term interest rates.

The model is used to explain the recent behaviour of the US bond market. The conclusion of the exercise is that the bond market cycle of 1993-95 cannot be attributed to shifts in inflation expectations. Instead, these recent fluctuations in long-term interest rates reflect changes in the long-run level of real interest rates or in risk premia.

I will bring my comments to an end by raising several issues. First, the conclusion that the nominal interest rate endpoint contains information useful for forecasting the future course of short-term interest rates above what is contained in the past history of short-term rates is based on an examination of in-sample forecast performance. It would be interesting to test this hypothesis on the basis of out-of sample forecast performance. In particular, it would be useful to compare the authors preferred model of short-rate dynamics, which includes the use of a "nominal interest rate endpoint", with a very parsimonious alternative on the basis of out-of-sample forecasts.

Second, I am less optimistic than the authors that the serial correlation of the model's errors can be explained by time variation in risk premia. A paper that looks exactly at this issue is the recent study by Hardouvelis. He concludes that time variation in term premia is not an adequate explanation of deviations of ten-year government bond yields from the predictions of the expectations theory of the term structure, at least for the US market.

Serial correlation of the model's errors is most likely the result of the failure of the VAR to adequately capture market participants' expectations of future short-term interest rates. As the authors point out, a potential solution to this problem is to include more variables in the VAR. But there is a tradeoff here. As more variables are included in the VAR, more parameters are estimated and forecast performance may well deteriorate. Therefore, it might be useful to place additional restrictions on the VAR, perhaps through the use of Bayesian estimation, in order to reduce the number of estimated parameters with the goal of improving out-of-sample forecast performance.