THE DETERMINATION OF LONG-TERM INTEREST RATES AND EXCHANGE RATES AND THE ROLE OF EXPECTATIONS

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Introduction

During the conference on "Financial structure and the monetary policy transmission mechanism", held at the BIS in November 1994 (see CB 394), a general interest was expressed in having a regular annual meeting for central bank econometricians and model builders. The periods of turbulence during the past years, first in the world bond markets and later in foreign exchange markets, have generated further interest in the question of what drives movements in long-term interest rates and exchange rates. In particular, attention has focused on whether such changes are caused by revised expectations of the so-called fundamentals or are the result of overshooting due to "fads" or other herd-like behaviour. Since both the exchange rate and the long-term interest rate are important links in the transmission process of monetary policy, analyses and discussions of how to model the influence of unobservable expectations seem especially important in this context. In some countries, increasing government and foreign debt ratios may also have affected the risk premia on long-term interest rates and exchange rates as well as the interaction between interest rates and exchange rates.

Against this background the BIS invited central bank econometricians and model builders to a conference held at the BIS on 14th and 15th December 1995 on the following topic:

The determination of long-term interest rates and exchange rates and the role of expectations

The presentation and discussion of the fifteen contributions (including comments by discussants) took place in three separate sessions; a final and relatively brief session was mainly devoted to the need for and interest in regular meetings of this kind and to potential topics for future meetings. The contributions are reproduced on the following pages in the order in which they were presented, while the remainder of this introduction provides a summary of each paper. It concludes with a brief "cross-paper" discussion of the main themes of the meeting, including (i) the modelling of expectations and the results obtained; and (ii) the extent to which estimates of the two key equations (long-term interest rates and exchange rates) made use of and validate three principal theories in this area: purchasing power parity, uncovered interest parity and the expectation hypothesis of the yield curve.

1st Session: Determination of exchange rates

The paper by J. Ayuso and J.L. Vega (Bank of Spain) attempts to estimate the dynamics of the effective exchange rate of the peseta against ERM as well as non-ERM European countries. The model used builds on and extends two earlier models estimated within the Bank of Spain: one based on the period when the peseta was in the ERM which had estimated the size as well as the probability of "jumps" (realignments) against the DM; and a second model which had found PPP to hold in the long run, measured in terms of industrial goods prices.

The present paper combines these earlier models into one error correction equation (ECM) and extends the sample period to 1974-95. It augments the ECM equation with jumps, defined empirically from the size of residual deviations from the adjustment path towards long-run PPP. Hence, jumps involve depreciations as well as appreciations and the probabilities of the occurrence of a jump are estimated by two binomial probit models, which include a number of macroeconomic "fundamentals" among the determinants.

The estimates of the "pure" ECM equation are consistent with long-run PPP with a relatively slow speed of adjustment. The probit model for depreciations show that the probability of a jump increases with the cumulative current account deficit, the rate of economic growth and
deviations of the real effective rate from PPP, though the size of the parameters depends on whether the peseta is inside or outside the ERM. The probability also depends on the policy condition (or dilemma) that when the peseta is in the ERM the levels of domestic interest rates have to be consistent with the cyclical position as well as the requirement imposed by a fixed nominal exchange rate. Another key finding of the paper is that the jumps act as accelerators as unusually large depreciations significantly increase the speed of adjustment. Overall, the paper shows that, while the probabilities of large depreciations are relatively well explained by the model, an important degree of uncertainty remains with respect to the probabilities and size of unusually large appreciations.

The aim of the paper by F. Ettlin (Swiss National Bank) is to estimate a model for the Swiss franc/German mark exchange rate. Notwithstanding the rather disappointing results of previous empirical work on exchange rates, the model attempts to explain short and medium-term exchange rate movements by macroeconomic fundamentals and policies. The estimation results look promising. The model explains more than 85% of the variance of exchange rate changes in the sample and in post-sample prediction tests it clearly outperforms a random walk model.

The coefficient estimates are derived from an error correction model with the following determinants: relative consumer prices (which enter with a coefficient insignificantly different from unity), the difference between the Swiss and German discount rates (interpreted as an indicator of the comparative stance of monetary policies in the two countries), the difference between Swiss and German three-month Eurodeposit rates (as a measure of relative market rates), a term structure differential (as an indicator of expected future changes in monetary policy), the ratio of capacity utilisation rates (a measure of relative cyclical positions), the difference between current account positions (measured relative to GDP and intended to capture variations in exchange rate premia) and the exchange rate between the US dollar and the German mark (to capture the effects of shifts in capital flows and portfolio compositions). All coefficients are found to be significant and correctly signed and in most cases the parameters of the long-run cointegration equation and the short-run adjustment equations are remarkably close.

The paper by M.S. Astley and A. Garratt (Bank of England) attempts to identify the sources of UK exchange rate and relative price fluctuations between 1973 and 1994. It follows Clarida and Gali (1994) in using the Blanchard and Quah (1989) structural VAR method to identify the effects of three structural shocks -real aggregate supply (AS) shocks, real goods market (IS) shocks and nominal money market (LM) shocks- within a Dornbusch (1976)/Obstfeld (1985) model. Identification is achieved by imposing three theory-derived restrictions The first two are that both IS and LM shocks have zero long-run effects on the level of relative output (which is entirely supply-determined). The third one is that LM shocks have zero long-run effects on the level of the real exchange rate.

Astley and Garratt find that IS shocks constituted the main source of sterling real and nominal exchange rate movements. AS shocks were the secondary source of such fluctuations while nominal shocks played extremely limited roles. In contrast, the variance of UK relative prices was primarily attributed to LM shocks. These results indicate that sterling exchange rate fluctuations have not constituted an important channel through which exogenous shocks have affected UK relative price fluctuations. Moreover, when combined with the estimated impulse response functions, they indicate that the sterling exchange rate depreciations over the floating period have had largely benign relative price implications. The findings that: (i) the estimated impulse responses following each of the shocks are highly theory consistent and (ii) the periods for which the structural VARs indicate that particular shocks were most important correspond to observed macroeconomic developments, suggest that the structural VAR representations of the data have a high economic content.

The paper by C. Gartner and H. Glück (Austrian National Bank) starts by reviewing the key role of the exchange rate in the setting of Austria's economic policies; in particular the pegging of the schilling to the currencies of the most important trading partners in order to maintain low inflation and improve competitiveness. While the development of the real exchange rate is crucial in this
respect, the concept of PPP was never regarded as an essential element of policies, nor was it validated by earlier empirical studies. Against this background, the paper has two aims: applying more recent econometric techniques to testing whether PPP actually holds for Austria; and, to the extent that PPP is rejected, identifying factors that may explain movements in the real rate.

With regard to PPP, unit root as well as cointegration tests reject the hypothesis that the real rate is mean reverting; i.e. PPP is rejected. To explain movements of the real rate, Gartner and Glück first turn to real interest rates, but because central banks have little influence on real interest rates, they then move to nominal rates by, essentially, specifying and estimating a reaction function for the central bank. The results (both in level form and when derived from an error correction model) show that the call money rate is strongly influenced by foreign interest rates, the real exchange rates and relative prices, with a relatively fast adjustment of actual rates to their equilibrium path.

The final section of the paper tests the influence of productivity developments on the assumption that for broadly based price measures different rates of productivity growth in respectively tradable and nontradable sectors can affect the real exchange rate. Although promising, these tests are still preliminary, especially with regard to the data and the sample size.

W. Jahnke (Deutsche Bundesbank) first reviews the determination of interest rates and exchange rates in the Bundesbank's quarterly macroeconomic model of the German economy and then illustrates the dynamics of the model by simulating the response to various shocks. The equation determining the long-term bond rate is based on the Fisher equation and modelled as an error correction equation with only bond rates and expected inflation as cointegrating terms. Expectations of inflation are explained by an adaptive process which is very backward looking (past rates have a weight of 0.9). Because nominal bond rates in Germany have been nearly stationary while inflation has gradually declined since the early 1980s, the slow adjustment of expected inflation imparts a rising trend to the real rate.

The modelling of the money market rate is based on a two-stage procedure, which first links the repurchase rate to the spread between the discount and the lombard rate and the amount of unborrowed reserves. The second stage then links the money market rate to the repurchase rate by an error correction equation where the rate of inflation as well as the euro-dollar rate have transitory effects, while the repurchase rate and the money market rate are in the cointegration term.

The exchange rate equation also uses a cointegration relation. The cointegration term is based on two principal parity assumptions: uncovered interest parity and purchasing power parity. Both the interest rate differential and the relative price term are estimated without restrictions. Moreover, the exchange rate equation is estimated separately for three sub-components of the aggregate effective rate. As a result, the principal coefficients as well as the changes in interest rate differentials required to offset relative price shifts and shocks to the nominal exchange differ widely across variables and components:

- against the US$ the coefficient for relative prices is below unity while the interest rate elasticity is high. This means that only small changes in interest rates are required to offset external shocks to the nominal exchange rate whereas, in case of a relative rise in US prices, US interest rates have to be reduced compared with German rates to prevent an appreciation of the DM;
- against ERM currencies PPP is found to hold while the interest rate elasticity is relatively low. Hence, to maintain unchanged real exchange rates other ERM countries need fairly large changes in interest rate differentials to offset external shocks to their nominal exchange rate, whereas changes in relative prices require only small adjustments;
- against the remaining currencies in the basket, the elasticities on both relative prices and interest rates are well below unity, implying that rather large changes in interest rate differentials are required to maintain PPP.
Combined with actual developments in interest rates, relative prices and nominal exchange rates, these coefficients have, over time, led to a trend real appreciation of the DM, mainly due to developments in the US$ and the non-US and non-ERM currencies.

2nd Session: Determination of exchange rates and interest rates

M. Dombrecht and R. Wouters (National Bank of Belgium) derive and estimate equations for both long-term bond rates and the exchange rate, though only estimates for the former are shown. The specifications for both equations are based on an optimal inter-temporal model for the behaviour of consumption-saving-portfolio allocation in small open economies and applied to explain yield differentials against Germany as well as the DM/BF. The estimates in Table 2 are based on panel data for selected European countries. They show significant long-run impacts of short-term real interest rate differentials, inflation differentials and the ratios of both public and foreign debt to GDP. The cross-equation restriction of equality of long-run coefficients of the current account ratios is just accepted, whereas the same hypothesis with respect to the long-run coefficients of the public deficit ratios is only accepted when the coefficients are corrected for the standard errors of the country-specific equations. The estimated coefficients may, therefore, be considered as representing average EMS responses. Other equations and graphs in the paper suggest that risk premia tend to vary over time, depending on financial market volatility.

The final part of the paper discusses two alternative expectation schemes: an adaptive one (Table 4) and a forward-looking one (Table 5). While no discriminatory tests have yet been performed, they seem to lead to very similar conclusions.

The paper by P.J.A. van Els and P.J.G. Vlaar (Netherlands Bank) first discusses the specification of three equations (DM exchange rate, short-term interest rates and long-term interest rates) for the Netherlands and then proceeds to estimating the three equations and adding them to the Bank's macroeconomic policy model for the Netherlands (MORKMON) which is used regularly for the purpose of simulation and forecasting. The estimated exchange rate equation shows a combination of uncovered interest parity and less than complete PPP. At a first glance, the latter seems surprising but, unlike in the Swiss case discussed above, this seems a natural outcome when the nominal rate is an intermediate target and product markets determine price differentials with long lags. The equation for the short-term interest rate points to some, albeit moderate, degree of monetary independence, and in the equation for the long-term rate, the coefficient on the German long-term rate is significantly below unity. This result probably reflects that capital markets are not completely integrated, due, for instance, to transaction or information costs or restrictions on foreign portfolio investments by institutional investors. Another result is that a direct effect of the external balance on the long-term interest rate could not be established empirically. This may, in part, be ascribed to the persistent external surplus during the sample period; in addition, the external balance influences the long rate indirectly via the short-term interest rate.

From the simulation results it is worth noting that, as in the case of Australia, a rise in the public deficit generates an appreciation of the exchange rate; i.e. the interest rate effect seems to dominate the risk premia effect. Finally, as might be expected given the focus of monetary policy, interest rates in both the Netherlands and Germany tend to move together in response to shocks to either the German or the world economy.

A. Tarditi (Reserve Bank of Australia) starts her paper reviewing the financial sectors included in two (the "Murphy model" developed by Econtech and the TRYM model maintained by the Commonwealth Treasury) of the most widely used macroeconomic models for Australia. The models apply forward-looking expectations and impose strong and binding long-run conditions so that shocks tend to generate textbook-style, instantaneous "jump" responses from the exchange rate, long-
term bond rate and inflationary expectations. Simulations of shocks to these financial variables in such models are, therefore, of limited relevance to practical policy makers (or very accurate descriptions of the real world) and the paper next turns to the specification of single-equation behavioural models for the financial sector.

Starting with the real exchange rate equation, earlier empirical works had identified three key determinants of the real exchange rate for Australia: the terms of trade, net foreign liabilities and long-term interest rate differentials. Ms. Tarditi first tests the equation for missing variables and then replaces the last term by a yield curve differential on the grounds that this better captures the transmission of changes in policy interest rates via the exchange rate. A role for fiscal policy is also considered including a measure of changes in the government budget balance, though the a priori sign of the coefficient is ambiguous. The main results of the revised equation are: (i) for the post-float period, the yield curve differentials is very significant, whereas foreign liabilities are not; (ii) the budget deficit is also significant and the sign implies that a fiscal tightening leads to a depreciation of the exchange rate (i.e. as predicted by the Mundell-Fleming model); and (iii) terms-of-trade changes have a very strong effect; in fact it is "too" strong, confirming other empirical studies showing that the exchange market is not efficient.

The second behavioural equation discussed is the equation determining the long-run bond rate, for which the specification proposed in Orr et al. (1995) is used to select the fundamentals. The main contribution in this part of the paper is the use of a Markov switching model for deriving a forward-looking measure of the expected rate of inflation and then using this result in the bond rate equation. The final results, as summarised in Table 4, show that the forward-looking measure of expectations clearly improves the performance of the equation, including capturing the 1994 rise in bond rates. There is also a surprisingly strong and quick effect of changes in US bond rates, even though Australia has a floating currency.

The paper by E. Jondeau and R. Ricart (Bank of France) presents three tests of the term structure using French, German and US euro-rates. The first test is based on forward rates and the other two on the slope of the term structure. Moreover, because nominal interest rates tend be non-stationary, each test is carried out using both a standard specification based on first differences with only one right-hand side variable and an error correction model with two right-hand side variables. Among the many and important results found in the paper, the following are worth highlighting:

• the monetary turmoil that occurred during the sample period (1975-95) had a very large impact on the estimates for French rates. The impact was less noticeable for US rates and negligible for German rates;

• in contrast to tests based on the usual specification, the adoption of an error correction model implies that the expectation hypothesis is accepted when tested on variations in short-term rates. For forward rates or variations in long-term rates the advantages of the error correction model are far less evident;

• the contradictory results found in the literature on the sign of the slope of the two tests based on the spread of interest rates disappears when an error correction model is used;

• finally, regarding country-specific rates, the paper finds that the expectation hypothesis is generally accepted for French rates, regardless of the test and specification applied, whereas for US rates the adoption of an error correction model yields results that are more favourable to the expectation hypothesis.

The paper by S. Kozicki, D. Reifschneider and P. Tinsley (Federal Reserve Board) discusses the modelling of long-term interest rates in the new FRB model of the US economy. One feature of this model is that expectation processes are constructed under the paradigm that households and firms are rational optimising agents. Within this framework, the specific aim of the paper is to estimate long-term interest rates from expectations of future short-term rates, using a VAR model
with shifting end points to generate expectations. The introduction of shifting end points can be seen as a way of "getting around" the problem that short-term rates follow a random walk process. Moreover, it allows the modeller to distinguish between two elements that influence long-term rates: a stationary element associated with the business cycle and monetary policy stabilisation, and a non-stationary element associated with long-term policy objectives.

Sections 1 and 2 of the paper present the theoretical basis of the model and discuss the drawbacks of standard VAR models with fixed end points. Section 3 then introduces moving end points and extends this modelling concept to include a distinction between moving end points for real rates and the expected rate of inflation. While the former (including a risk premium) is assumed to be constant, the paper analyses various ways of modelling moving end points for expectations of inflation, in particular the use of survey data and experiments with an agent-learning-model for shifts in expectations of inflation.

Section 4 turns to the empirical model, documenting its behaviour and properties, while section 5 tests the ability of the model to explain recent developments in long-term bond rates. It appears that most of the 1993-95 changes in bond rates were related to shifts in the real rates and one particular issue addressed in this section is the potential link between changes in budget deficits and movements in the real component of long-term interest rates. Another problem is that the residual errors are highly autocorrelated which might suggest that the assumption of rational expectations is invalid. However, as argued in the paper, a more attractive alternative is to augment the set of variables in the VAR model for generating expectations (inflation, the output gap and the Federal funds rate) by other macroeconomic variables.

Because the paper introduces a novel approach to using the expectations theory in modelling long-term bond rates, many of the experiments discussed are at the "frontier" of econometric modelling. Consequently, some of the results are still preliminary and may be revised in the final FRB model. In particular, more remains to be done regarding the potential relationship between policy and other determinants of expected long-run inflation and real interest rates.

3rd Session: Estimation and application of models or financial market indicators

The paper by A. Côté and T. Macklem (Bank of Canada) begins by summarising the main features of the Quarterly Projection Model (QPM) and then turns to a more detailed description of the interest rate and exchange rate sectors of the model. The former has two main equations: a long-term interest rate equation and a forward-looking monetary policy reaction function. The former combines elements of real uncovered interest rate parity and the expectations hypothesis, with a large weight assigned to the short-term interest rate in order to replicate historical properties of the data. The monetary authority primarily influences short-term interest rates which in turn affect the yield spread between long and short rates. The monetary reaction function is written in terms of this yield spread and is specified so as to minimise deviations of inflation from its targeted rate six to seven quarters in the future. The real exchange rate, as one of the "most endogenous" variables in the system, reflects the simultaneous solution of the full model. In the long run, the real exchange rate plays the role of the key relative price that adjusts to equilibrate the economy. In the short run, its determination is largely influenced by the assumptions of uncovered interest rate parity and sluggish price adjustment.

To better understand the model's dynamics, the paper next presents simulations of two shocks: a disinflation shock and a fiscal shock. The first set of simulations serve in part to illustrate the standard result in a deterministic model that the costs of disinflating are very low if one ("unrealistically") assumes that all agents have perfect foresight. A combination of backward and forward-looking expectations, as is currently assumed in the model, leads to a more reasonable solution. The fiscal shock – a permanent rise in the public debt/GDP ratio – illustrates that the short-run appreciation of the Canadian dollar associated with a fiscal expansion eventually gives way to a
permanent depreciation which is necessary to generate a higher trade surplus to finance larger foreign liabilities. The final section of the paper examines the effects of making risk premia embodied in interest rates a function of the level of government indebtedness. The impact of a permanent rise in the debt/GDP ratio is found to be larger if the risk premium applies only to interest rates on government debt; when it also applies to private borrowing rates, economic agents start to adjust and the impact on consumption declines compared with the case of exogenous risks. However, the higher risk premia on private borrowing rates leads to a larger decline in investment.

Following the presentation and discussion of the paper from the Bank of Canada, M. Apel and Y. Lindh (Bank of Sweden) presented an oral review on their work on implementing a model similar to the QPM for Sweden, the main problems encountered and solved so far and the technical, theoretical and policy-related issues to be dealt with in the near future.

T. Watanabe and H. Matsuura (Bank of Japan) also discuss the determination of long-run interest rate and exchange rate equations and illustrate their findings by model-based simulations. The paper first explains the specification of three equations in the Bank of Japan model (long-term interest rates, the USS exchange rate and equity prices), with estimation results for the long-term interest rate including elements of the expectation hypothesis, influences of the US rate, exchange rate expectations as well as expectations of inflation and government debt. The proxy used for expectations is not significant and the positive coefficient on the lagged change of the exchange rate could suggest that a depreciation leads markets to expect a rise in long-term interest rates, presumably due to expectations of a policy-induced rise in short rates. The equation for equity prices is based on current profits and the long-term interest rate on the assumption that expectations of future profits are formed adaptively. In the real exchange rate equation, a rise in the interest rate differential against US rates as well as a higher cumulative current account surplus (net of foreign direct investment and measured relative to other G-10 countries) tend to produce a real appreciation of the yen. The model assumes that PPP holds for the real yen/US$ rate and the relative current account position is regarded as capturing a risk premium. Alternatively, however, the external surplus could be one of the fundamentals driving the real exchange rate and leading to a rejection of long-run PPP.

The second part of the paper presents simulations which, given the backward-looking nature of the expectation schemes found in the model, attempts to take account of the Lucas critique by using the innovation-simulation technique. Essentially, this technique corrects for possible effects of the simulation shocks on key parameters of the model but, overall, Watanabe and Matsuura find that this method does not produce any major changes or surprises. In part 3 of the paper, the simulations are repeated assuming that expectations are forward-looking. As implemented in the model, this implies that market makers know the dynamic structure of the exogenous variables used for the simulations and, from the graphs, it is easily seen that the two sets of simulations yield virtually identical results.

The final section of the paper attempts to use model simulations to analyse the sensitivity of the effects of monetary policy changes to three alternative expectation formation processes: adaptive-regressive, rational and the process estimated from the expectation survey. While a major difference between rational expectations and the estimated process is that only the former imposes terminal conditions, the simulated effects of monetary policy changes appear to be very similar under the two schemes.

The paper by E. Gaiotti and S. Nicoletti-Altimari (Bank of Italy) reports on ongoing work, which addresses the issue of determining the lira/DM exchange rate and the long-term interest rate when adding explicit expectation mechanisms (modelled from a quarterly survey conducted by Forum-Mondo Economico) to the Bank's quarterly model. The first section of the paper briefly reviews the different methods used to quantify expectations in the model while section 2 discusses the problem of endogenising exchange rate expectations and their role in determining the exchange rate. The main empirical findings of this section show that, in the short run, exchange rate expectations are strongly adaptive and this tends to amplify the effects of shocks to the spot rate and to increase
persistence. It is further found that the uncovered interest parity condition is a useful tool in modelling the exchange rate and that the risk premium on domestic short-term interest rates is positively correlated with exchange market volatility.

Section 3 of the paper presents estimates of the long-term interest rate using, as the principal determinants, domestic short rates, foreign yields, exchange market volatility and empirical estimates of expectations of inflation (further discussed and explained in section 4). Foreign yields are found to have a significant influence on domestic long-term yields while the effect of domestic short rates is only marginal. Expectations of inflation have a very significant influence on long-term interest rates and clearly outperform actual rates of inflation.

The final section of the paper attempts to use model simulations to analyse the sensitivity of the effects of monetary policy changes to three alternative expectation formation processes: adaptive-regressive, rational and the process estimated from the expectation survey. While a major difference between rational expectations and the estimated process is that only the former imposes terminal conditions, the simulated effects of monetary policy changes appear to be very similar under the two schemes.

The paper by A. Estrella and F.S. Mishkin (Federal Reserve Bank of New York) looks at the predictive power of the spread between 10-year and 3-month US Treasury papers, compared with other forward-looking indicators. Rather than attempting to predict future growth rates, the paper focuses on predicting the probability of a recession (as defined by the NBER) $k$ quarters ahead. The main findings and conclusions are: (i) the predictive performance of the spread exceeds that of all other indicators, except for the stock market index, which is better in predicting 1-2 quarters ahead. Hence, the best predictive performance is obtained when combining the stock market index with the spread; (ii) the performance of an indicator can change substantially when moving from in-sample to out-of-sample predictions; and (iii) the failure of other indicators, such as the Stock-Watson index and the Commerce Department's index of leading indicators, can be ascribed to overfitting; i.e. wrongly including indicators which have no predictive power and only obtain a significant weight because of estimation errors.

A. Levin (Federal Reserve Board) reviews recent modifications of the Federal Reserve's Multi-Country Model (FRB/MCM) that have facilitated the comparison of alternative monetary rules under model-consistent or "rational" expectations as well as under VAR-based or "adaptive" expectations. Using dynamic simulations of the model in response to US aggregate supply and demand shocks, the paper evaluates three specific monetary policy rules, each of which prescribes a short-term interest rate target based on current output deviation from potential and either the current price level deviation from a specified target path or the current inflation deviation from a specified target rate. The results generally confirm the favourable properties of a policy rule considered by Henderson and McKibbin. By targeting inflation rather than the price level, this rule generates greater output stability and similar inflation stability compared with a policy rule based on nominal GDP targets. Moreover, when prescribing larger interest rate adjustments in response to the current output gap and current inflation deviation from target, the rule by Henderson and McKibbin generates more stable economic activity and inflation compared with the monetary policy rule analysed by Taylor. However, similar experiments for Germany and Japan do not yield such clear-cut differences and highlight the crucial role of the expectation formation mechanism in comparing alternative monetary policy rules.
Concluding remarks

Most of the discussion during the relatively brief fourth session focused on possible future topics and conference dates and there was little time to review and evaluate the wide range of issues – policy related as well as econometric – that had been covered at the meeting. However, to provide the reader with a broader perspective, this Introduction concludes with a comparative overview of some of the principal themes and issues presented and discussed.

(i) The role and modelling of expectations of inflation

Strategies to model inflation expectations and to evaluate their effects in simulation exercises have broadly covered the main approaches suggested by the literature and offered new promising insights. Yet the evidence produced in the present collection of studies does not lead to the conclusion that one particular approach, or one particular class of models, possesses features that are generally superior to those of alternative approaches. In the paper by W. Jahnke inflation expectations are estimated according to a backward-looking scheme that is consistent with declining trends of German inflation. As the large weight of past inflation on expected inflation imparts a rising trend on the real interest rate, it is quite natural for the discussant to wonder how the long-term interest equation - and the whole model - would behave under alternative hypotheses.

However, the presumption that different price expectation models produce different simulation results is not always confirmed by evidence. For example, E. Gaiotti and S. Nicoletti-Altimari find that monetary policy shocks produce very similar effects when rational expectations or adaptive regressive inflation expectations are used in the model. Similarly, M. Dombrecht and R. Wouters conclude that adaptive and forward-looking models generate broadly equivalent results. The paper by A. Tarditi, on the contrary, is a witness to the fact that the Markov-switching model for inflation expectations dominates the textbook-style forward-looking scheme as a means of replicating real world situations. In fact, by using the Markov model to estimate the change in the inflation regime and using it in the bond rate equation, the author captures well the recent puzzling rise of the real bond rate. The fact, however, that the inflation expectations derived from the Markov model are similar to the Australian survey data of inflation expectations, makes it legitimate to wonder - as the discussant does - whether the Markov model dominates survey data. More generally, one can wonder whether, in cases where inflation expectations show less abrupt changes than in Australia, a Markov model supplemented by the assumption that transition probabilities (i.e. changes in inflation expectations) vary with fundamentals is a potentially new approach to model expectations. This was suggested by the discussant of the paper by J. Ayuso and J.L. Vega in the context of modelling the probability of jumps in the exchange rate for the Spanish peseta.

The use of survey data as a direct measure of inflation expectation of economic agents features prominently in the Banca d'Italia paper as a means to separate the analytical problem of detecting what expectations are from that of evaluating what are their effects on other economic variables. This paper offers promising results and the availability of similar data for a variety of countries suggests that a more general investigation and application is possible. One shortcoming of survey data, however, is that long-term inflation expectations are not generally available. In the case of the United States, where one such survey exists, long enough series are less satisfactory than one would like. To overcome this problem, S. Kozicki, O. Reifsneider and P. Tinsley have implemented a very innovative strategy to model long-run inflation expectations with a VAR model supplemented by moving endpoints. As noted below, this model seems to track well the historical path of long-term inflation expectations of the survey and helps to explain the recent behaviour of bond rates. The authors suggest that the VAR model with moving end points could be augmented with the use of other macro variables to improve its performance as well as our understanding of how inflation expectations are formed.
(ii) The role and testing of economic hypotheses

Considering the rather mixed empirical results obtained over the last 10-15 years concerning purchasing power parity (PPP) as a principal determinant of exchange rates, it was, perhaps, surprising that so many of the papers discussing and estimating exchange rate equations included PPP among the principal long-run determinants. Even more surprising, several papers came to the conclusion that PPP holds in the long run. For instance, long-run PPP is found to hold completely in the case of the bilateral Swiss franc/German mark rate (F. Ettlin) and also for the peseta/German mark rate (J. Ayuso and J.L. Vega), though in the latter case only when the price measures excludes non-tradables. Moreover, as implied by the discussant of that paper, the finding of absolute PPP could reflect that other variables are not included in the exchange rate equation. Complete PPP also holds for the German mark against an average of ERM currencies, whereas against the US dollar and non-ERM currencies the relative price term is significant but with a coefficient significantly below unity (W. Jahnke). The same is true for the Dutch guilder against the German mark (P.J.A. van Els and P.J.G Vlaar), suggesting that a fixed nominal exchange rate does not necessarily lead to fixed real rates, even for neighbouring countries with very similar structures and monetary policies. This conclusion is even more evident in the paper by Ch. Gartner and H. Glück which rejects long-run PPP even though a fixed nominal exchange has for many years been a main feature of Austrian policies. As suggested by the discussant, one way of identifying the sources of the absence of mean reversion might be to look at the behaviour of the Austrian schilling against different sub-groups of countries. PPP also appears to be rejected in the Banca d'Italia model as the coefficient on the relative price term is insignificant. However, as the discussant notes, it is not clear why the specification included a relative price term since PPP was not imposed on the equation.

Long-run PPP is also found to hold (or underlies the specification of the exchange rate equation) in several other papers, although in these cases the effects of long-run PPP tend to be dominated by other influences. For instance, T. Watanabe and H. Matsuura model the yen/US dollar rate on the basis of long-run PPP while, at the same time, interpreting the influence of the growing current account surplus as a gradually declining risk premium. As pointed out by the discussant, it is slightly odd to start with the notion that the market is "itching to get back to a fundamental purchasing power rate and is temporarily dragged away by an accumulating international asset position". A main feature of the real effective exchange rate for Australia (A. Tarditi) is the very significant and permanent influence of changes in the terms of trade. As the discussant observes, this excessively strong influence could reflect that international investors tend to buy Australian stocks, currency and even bonds as insurance against the adverse effects of higher commodity prices on bond returns and corporate profits in other industrial countries. Given the nature of the Canadian model (A. Côté and T. Macklem) and the role of the exchange rate as the principal equilibrating mechanism, PPP is neither imposed nor tested. The same applies to the paper by M. Astley and A. Garratt) which finds that nominal exchange rate variations are mainly caused by monetary shocks while relative price variations can mostly be ascribed to demand shocks. Even after taking account of the discussant's concern about the identification of shocks it remains an open the question whether long-run PPP holds.

One issue, which was raised by several discussants of the various exchange rate equations and influences the interpretation of long-run PPP, concerns the precise role played by net foreign assets or liabilities in determining exchange rate movements. Do they, notably in the case of net liabilities, reflect a solvency concern with implications for exchange rate expectations? Or is the role derived from a portfolio balance model on the assumption that net foreign assets or liabilities are denominated entirely in the domestic currency?

The specification and testing of the expectation hypothesis of the term structure of interest rates plays a particularly prominent role in the papers by E. Jondeau and R. Ricart and by Kozicki et al. In the first case an attempt is made to reconcile the different empirical tests (and results) found in the literature, using first difference ("standard") as well as cointegration specifications. As noted by the discussant of this paper, it is interesting that the former specification rejects the expectation hypothesis more frequently than the latter. The second paper, by contrast, focuses on the
modelling of expectations of future short-term interest rates on the assumption that exogenously determined moving endpoints contain information additional to the past history of short-term rates. While this approach appears to improve the model's ability to explain recent movements in long-term bond rates and thus the evidence favouring the expectation hypothesis, serially correlated errors could, as noted by the discussant, indicate that market participants' expectations of future rates have not been fully captured.

The expectation hypothesis also enters the specification of bond rate equations for a number of other countries, but in several instances the influence of expected short-term interest rates is "swamped" by the dominating role of foreign bond rates. Indeed, one common theme of the meeting was the evidence pointing to strong international linkages of long-term bond rates and supporting the notion of uncovered interest parity (UIP). One exception to this was the German bond rate which entirely depends on expectations of inflation and changes in German short-term rates. By contrast, while clearly supporting an important role for the expected future course of short-term rates, the bond rate equation for Japan finds a significant role for US bond rates. US bond rates have an even stronger impact, notably in the case of Canada but also for Australia. In fact, in the latter case actual or expected domestic short-term rates are not even included.

The dominating role of UIP relative to the expectation hypothesis of the term structure is even more evident for countries adhering to a fixed exchange rate regime, as seen in the bond rate equations for Italy, the Netherlands and the various ERM countries modelled in the Belgian paper. Nonetheless, as pointed out by the discussant of the Italian paper, some caution is called for regarding the strength of international linkages, especially when they are based on contemporaneous foreign rates. It is also worth noting that UIP receives less support when tested on short-term rates. In fact, as noted by the discussant of the Dutch paper, there are even reasons to doubt what little evidence there is of short-run UIP.

It is also relevant to note that, quite apart from the influence of UIP, there are a number of other variables affecting long-term rates in addition to expected short rates. This is, perhaps, most clearly seen in the modelling of long-term interest rate differentials in the Belgian paper. In all cases public sector deficits and current account balances are found to have a significant influence on differentials. As pointed out by the discussant of this paper (and by the discussant of the Dutch paper as well), it is, however, questionable whether these influences should be interpreted as risk premia within the context of a portfolio balance model. An alternative, and equally valid, interpretation would be that they mainly reflect exchange rate expectations and thus belong in the exchange rate equation rather than the interest rate equation.

These last observations also point to a final common theme of the meeting; viz. the risks and biases involved in testing specific hypotheses or modelling procedures using single equations. Obviously, such questions (for instance, the respective role of respectively UIP and the expectation hypothesis) can only be settled within a model that includes both exchange rate and interest rate equations and is estimated by a procedure that takes account of the simultaneities and respects the cross-equation parameter restrictions.
Participants in the meeting

Australia: Ms. Alison TARDITI
Austria: Ms. Christine GARTNER
          Mr. Heinz GLÜCK
Belgium: Mr. Michel DOMBRECHT
         Mr. Raf WOUTERS
Canada: Ms. Agathe CÔTÉ
       Mr. Tiff MACKLEM
France: Mr. Eric JONDEAU
       Mr. Roland RICART
Germany: Mr. Wilfried JAHNKE
       Mr. Sergio NICOLETTI-ALTIMARI
       Mr. Eugenio GAIOTTI
Italy: Japan: Mr. Tsutomu WATANABE
       Mr. Peter J.A. VAN ELS
       Mr. Peter J.G. VLAAR
Spain: Mr. Juan AYUSO
       Mr. Juan Luis VEGA
Sweden: Mr. Mikael APEL
       Mr. Yngve LINDH
Switzerland: Mr. Franz ETTLIN
            Mr. Erich SPÖRNDLI
United Kingdom: Mr. Mark ASTLEY
                Mr. John WHITLEY
United States: Mr. Arturo ESTRELLA (New York)
              Mr. Andrew T. LEVIN (Washington)
              Mr. David REIFSCHNEIDER (Washington)
BIS: Mr. William WHITE
     Mr. Renato FILOSA
     Mr. Zenta NAKAJIMA
     Mr. Joseph BISIGNANO
     Mr. Palle ANDERSEN
     Mr. Claudio BORIO
     Mr. Benjamin COHEN
     Mr. Sean CRAIG
     Mr. Gabriele GALATI
     Mr. Stefan GERLACH
     Mr. Robert McCauley
     Mr. Frank SMETS
     Mr. Greg SUTTON
     Mr. Kostas TSATSARONIS
An empirical analysis of the peseta's exchange rate dynamics

Juan Ayuso and Juan L. Vega

Introduction

In the early 1980's Meese and Rogoff (1983) puzzled most economists by showing that despite the existence of several competing theories to explain freely floating exchange rates\(^2\), none is able to reliably improve the forecasts from a simple random walk model. More than ten years later their results remain in place. In a recent survey, Frankel and Rose (1994) conclude that standard theoretical models still fail to predict future exchange rate changes in the short and medium term.

Empirical results are also disappointing regarding our ability to explain future exchange rate movements for currencies that belong to managed exchange rate regimes like the Exchange Rate Mechanism (ERM) of the European Monetary System (see Garber and Svensson, (1994), in spite of the convincing theoretical work pioneered by Krugman (1991).

The recent periods of turbulence in the foreign exchange markets have renewed interest in identifying the driving forces of exchange rate movements in the short and medium term. In this paper we estimate a model explaining the dynamics of the effective exchange rate of the peseta vis-à-vis the currencies of other OECD countries\(^3\). Our model takes into account that this exchange rate is neither under the direct control of the monetary authorities (as it includes bilateral exchange rates against currencies that are, or have been, outside the ERM) nor completely flexible (because it includes bilateral managed exchange rates). It also pays special attention to the role of the "jumps" in the exchange rate that we observe from time to time.

The empirical model relies, on the one hand, on the results in Pérez-Jurado and Vega (1994), who showed that purchasing power parity (PPP) holds in the long run when tradable-good prices are considered. On the other hand, the model builds on the work by Ayuso and Pérez-Jurado (1995) where unusual jumps in the exchange rates of ERM currencies are explained in terms of real exchange rate deviations from a reference value and different variables that determine the costs for the monetary authorities of maintaining a given exchange rate.

In particular, the starting point of the analysis is an error correction model (ECM) for the first difference of the peseta's (log) effective exchange rate. This model is enlarged with terms which take into account the possibility of a jump in the exchange rate. Following Ayuso and Pérez-Jurado (1995) the size of the jumps is assumed to be a function of PPP deviations. The probability of the jumps is also estimated using Probit models that allow us to investigate to what extent macroeconomic variables may help to predict such jumps.

According to the estimate of our modified ECM equation, exchange rate jumps act to accelerate the speed of adjustment to the long run equilibrium. On the other hand, although a number of macroeconomic variables can help to explain why exchange rates jump, their predictive power is rather low.

The structure of the paper is the following: after this introduction, Section 1 depicts the

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1 We are grateful to W. Melick for his excellent discussion and to the participants at the meeting of central bank econometricians and model builders held at the BIS. We also thank O. Bover and J.J. Dolado for helpful comments.

2 Surveys on this topic are legion. See, for example, MacDonald and Taylor (1989).

3 See Bajo and Sosvilla (1993) for a survey on the empirical evidence on different theoretical models to explain the peseta's exchange rate dynamics.
basic model. Section 2 deals with the estimate of the modified ECM equation and Section 3 is devoted to estimating the jump probabilities. The final section summarises the main results in the paper.

1. **Econometric framework**

Our starting point is the work by Ayuso and Pérez-Jurado (1995). This paper decomposes the expected devaluation rate into the likelihood of a devaluation and its expected size and puts forward, in the context of the ERM, the following univariate model for the bilateral peseta-Deutschmark exchange rate:

\[
s_t = k + \Gamma(L)s_{t-1} + d_t + \varepsilon_t
\]

\[
d_t = \begin{cases} d_t^* & \text{with prob } Pr_{t-1} \\ 0 & \text{with prob } 1 - Pr_{t-1} \end{cases}
\]

where \( s_t \) is (the log of) the exchange rate; \( \Gamma(L) \) is a general lag polynomial; \( d_t^* \) is the size of the exchange rate jump in the event of a devaluation; and \( Pr_{t-1} \) is the likelihood, at time \( t-1 \), of a devaluation occurring at time \( t \).

It is also assumed that \( d_t^* \) depends on the vector of variables \( x_{t-1}^d \) and that a devaluation takes place when a given indicator \( c_t^* \) becomes positive. This indicator can be interpreted as the cost perceived by the government of maintaining the current parity. This cost depends on a vector of fundamentals \( x_{t-1}^c \). Therefore:

\[
d_t^* = \beta^c x_{t-1}^c + u_t^c
\]

\[
c_t^* = \beta^x x_{t-1}^x + u_t^x
\]

\[
Pr_{t-1} = \text{prob. } \left( u_t^c > -\beta^c x_{t-1}^c \right)
\]

According to the results in Ayuso and Pérez-Jurado (1995), \( d_t^* \) depends exclusively on the deviations of the real exchange rate from a reference level, so that equation (2) can be rewritten as:

\[
d_t^* = \beta (t_c r_{t-1} - t_c r^*) + u_t^d = \lambda - \beta t_c r_{t-1} + u_t^d
\]

Neither \( c_t^* \) nor \( d_t \) are observable. The only information available to the econometrician is whether or not a devaluation has occurred and, conditional on its occurrence and on an estimate of \( k \) and \( \Gamma(L) \), its size \( (d_t^*) \). However, by defining a binomial variable:

\[
\omega_t = \begin{cases} 1, & \text{if } c_t^* > 0 \\ 0, & \text{if } c_t^* \leq 0 \end{cases}
\]

the parameters \( \beta^c \) can be estimated from a probit model for \( \omega_t \). Given the probit estimates, \( \beta_d \) can also be obtained by including in equation (2) the well-known Heckman lambda. Nevertheless, Ayuso and Pérez-Jurado (1995) confined their attention to the direct estimation of \( \beta^d \) from a non-linear transformation of equation (2) which exploits the uncovered interest rate parity assumption and the information contained in the interest rate differentials.

In this paper the aforementioned framework is extended in a number of directions. First,
a more general process for the exchange rate is allowed for by using the results in Pérez-Jurado and Vega (1994). In a multivariate-multicountry framework based on the Johansen procedure, Pérez-Jurado and Vega (1994) found evidence that in the long run prices in the tradable sector (as proxied by the industrial price index) in Spain, Italy, France, the United Kingdom, Germany and the United States, expressed in the same currency, tend to converge. This convergence implies that the bilateral and multilateral real exchange rates follow processes that tend towards a constant long-run equilibrium. Hence PPP holds in the long run when prices of non-tradable goods are excluded from the analysis.

This cointegration property allows us to extend equation (1) by estimating the following ECM:

\[ \Delta s_t = \mu - \delta (\Delta p_t - p^*_t) - \alpha tcr_{t-1} + \sum_{i=1}^{p} \xi_i \Delta s_{t-i} + \sum_{i=1}^{p} \beta_i \Delta^2 p_{t-i} + \sum_{i=1}^{p} \delta_i \Delta^2 p^*_{t-i} + \epsilon_t \]  

(6)

where \( s_t, p_t \) and \( p^*_t \), (all variables in logs) stand for respectively, the nominal exchange rate index vis-à-vis OECD countries (foreign currency/pesetas), the domestic industrial price index, and a weighted index of industrial prices in OECD countries and \( tcr_t = s_t + p_t - p^*_t \) is the real exchange rate. The following statistical properties of the data are implicit in the specification of equation (6)\(^4\):

\[ p_t \sim I(2) \quad , \quad p^*_t \sim I(2) \]
\[ s_t \sim I(1) \quad , \quad (p_t - p^*_t) \sim I(1) \]
\[ \Delta(p - p^*) \sim I(0) \quad , \quad tcr_t = s_t + p_t - p^*_t \sim I(0) \]

The second extension is related to the concept of exchange rate jumps. Ayuso and Pérez-Jurado (1995) confined their analysis to official devaluations of the peseta - i.e. realignments - during the ERM period (1989:6 onwards). In this paper the analysis is extended to also including these cases where, although no devaluations occur, there are abrupt changes (both positive and negative) in the exchange rate. Such episodes will be labelled as jumps.

Because extended concept increases the number of observations on jumps, it allows us to include both depreciation and appreciation episodes and it is readily extended to the free-floating period. But it also presents some shortcomings. First, variable \( c^*_t \) must be reinterpreted as the short-term economic costs that agents, both public and private, perceive from maintaining a given level of the nominal exchange rate. Secondly, a problem of econometric identification arises as variable \( \omega_t \) is no longer observable. In this latter respect the adoption of a fairly empirical approach is suggested by assuming that the exchange rate jumps whenever the absolute value of the residuals in equation (6) exceed some arbitrary critical value (\( \theta \% \)).

In accordance with the extended concept of a jump, two variables (\( Q_t \) and \( D_t \)) are defined:

\[ Q_t = \begin{cases} 0, & \text{if } \hat{u}_t < \theta \\ 1, & \text{if } \hat{u}_t \geq \theta \end{cases} \]

\[ D_t = \begin{cases} 0, & \text{if } \hat{u}_t > -\theta \\ 1, & \text{if } \hat{u}_t \leq -\theta \end{cases} \]

---

\(^4\) See Pérez-Jurado and Vega (1994) for a detailed description of unit root test results.
The first variable \((Q_t)\) captures positive jumps, i.e. unusual appreciations of the exchange rate, while the second \((D_t)\) captures negative jumps, i.e. unusual depreciations. These variables enable us to estimate two probit models in Section 3 relating the likelihood of jumps, both positive and negative, to economic fundamentals. Moreover, they make it possible to estimate the parameters in equation \((2')\) explaining the size of the jumps\(^5\).

Residuals from equation \((6)\) can be decomposed into two components: one capturing abrupt changes in the exchange rate \((d_t)\), and the other a homoscedastic innovation \((v_t)\): \(u_t = d_t + v_t\).

Noting further that \(d_t = (D_t + Q_t)d^*_t\) and substituting equation \((2')\) into equation \((6)\) yields:

\[
\Delta_t = \Phi'Z_{t-1} - \alpha tcr_{t-1} + \lambda (D_t + Q_t) - \beta (D_t + Q_t)tcr_{t-1} + \eta_t
\]

\[
\eta_t = (D_t + Q_t)u^d_t + v_t
\]

where the vector \(Z_{t-1}\) groups all variables in \((6)\) other than \(tcr_{t-1}\) and the residuals \(\eta_t\) are no longer homoscedastic. Instead:

\[
E(\eta_t^2) = \begin{cases} 
\sigma^2_\eta & \text{if } (D_t + Q_t) = 1 \\
\sigma^2_\eta & \text{if } (D_t + Q_t) = 0 
\end{cases}
\]

In the next section we estimate the exchange rate equation by GLS\(^6\) using monthly data over the sample 1974:7-1995:9. In order to test for asymmetries in the effects of positive and negative exchange rate jumps, we estimate a slightly different version of equation \((6')\):

\[
\Delta_t = \Phi'Z_{t-1} - \alpha tcr_{t-1} + \lambda^+ D_t + \lambda^- D_t - \beta^+ D_t tcr_{t-1} - \beta^- Q_t tcr_{t-1} + \xi_t
\]

where:

\[
E(\xi_t^2) = \begin{cases} 
\sigma^2_\xi & \text{if } Q_t = 1 \\
\sigma^2_\xi & \text{if } D_t = 1 \\
\sigma^2_\xi & \text{otherwise}
\end{cases}
\]

2. Exchange rate dynamics

As described above, the proposed econometric strategy begins by estimating the error correction model for the changes in the (log) exchange rate given by equation \((6)\). When this equation is estimated by OLS using monthly data spanning the period 1974:4-1995:9, the coefficient \(\hat{\alpha} = -0.046\) (t-ratio = \(-2.3\)), on the error correction term turns out to be consistent with the low speed of adjustment towards the PPP long-run equilibrium underlined in Pérez-Jurado and Vega (1994). More importantly, as expected, the estimated residuals, \(u_t\), show strong signs of heteroscedasticity and non-normality. Conversely, no signs of autocorrelation or ARCH are detected.

\(^5\) In Vlaar (1994), jump probabilities and jump effects on the exchange rate dynamics are jointly estimated inside the ERM. Nevertheless, he has to assume that jump sizes are constant.

\(^6\) Note that although \((Q_t + Q_t), (D_t + Q_t)tcr_{t-1}\) and \(\eta_t\) are different functions of \(u_t\), the chosen functional forms are such that neither regressor is correlated with the noise, thus making IV estimation unnecessary.
Chart 1 shows the scaled residuals from the estimation and Table 1 summarises some diagnostic tests on these residuals. The White (1980) HET test rejects unconditional homoscedasticity. The Doornik and Hansen (1994) $N_2$ statistic strongly rejects normality, indicating a distribution which is skewed to the left and has fatter tails than the normal distribution, i.e. extreme values are more common than in the normal distribution.

**Chart 1**

**Scaled residuals from equation (6)**

![Chart 1](image)

**Table 1**

**Some diagnostic tests on the residuals from equation (6)**

<table>
<thead>
<tr>
<th></th>
<th>OLS estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample: 1974/7-1995/9</td>
</tr>
<tr>
<td>$LM_{12.216} = .892$</td>
<td>$ARCH_{7,214} = .058$</td>
</tr>
<tr>
<td>$N_2 = 304.3^{**}$</td>
<td>$Sk = -3.738$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Positive</th>
<th>Negative</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 2.0%$</td>
<td>8 (3.1%)</td>
<td>12 (4.7%)</td>
<td>20 (7.8%)</td>
</tr>
<tr>
<td>$\theta = 1.75%$</td>
<td>11 (4.3%)</td>
<td>14 (5.5%)</td>
<td>25 (9.8%)</td>
</tr>
<tr>
<td>$\theta = 1.5%$</td>
<td>17 (6.7%)</td>
<td>17 (6.7%)</td>
<td>34 (13.4%)</td>
</tr>
</tbody>
</table>

Notes: See the Appendix for a description of test statistics. * and ** stand for, respectively, rejection at the 5% and 1% significance level.
The latter observation provides some support for the proposed decomposition of the residuals into two components: the first \((d_t)\) capturing abrupt changes in the exchange rate -jumps-, and the second \((v_t)\) a homoscedastic innovation. The bottom part of Table 1 shows the number of jumps in the sample depending on the empirical definition of jumps \((\theta)\): there are 20 jumps for \(\theta=2\%\), 25 for \(\theta=1.75\%\) and 34 for \(\theta=1.5\%\), representing, respectively, 7.8\%, 9.8\% and 13.4\% of the sample.

The variables \(D_t\) and \(Q_t\) were defined as dummies which take values equal to one whenever there is a jump and zero otherwise. Again, depending on \(\theta\), we have three pairs \((D_t, Q_t)\). Results for GLS estimates of the preferred specification of equation \((6'')\) are summarised in Table 2. The bottom part of the table reports some diagnostic tests on the transformed residuals that are shown in Chart 2.

### Table 2

**Estimation of \((6'')\) and some diagnostic tests**

<table>
<thead>
<tr>
<th>Exchange rate equation: GLS estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample: July 1974 - September 1995</td>
</tr>
</tbody>
</table>

\[
\Delta_s = \mu + \alpha_1 \Delta s_{t-1} + \alpha_2 \left( \Delta^2 s_{t-1} + \Delta^2 s_{t-3} \right) + \alpha_3 \left( \Delta^2 p_{t-1} + \Delta^2 p_{t-2} \right) + \delta(\Delta p - \Delta p^*_{t-1}) + \alpha tcr_{t-1} + \lambda D_t + \beta^\prime D_t * tcr_{t-1} + \lambda^\prime Q_t + \beta^\prime Q_t * tcr_{t-1} \]

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(\theta = 2%)</th>
<th>(\theta = 1.75%)</th>
<th>(\theta = 1.5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu)</td>
<td>(0.1023 (2.09))</td>
<td>(0.0874 (1.97))</td>
<td>(0.1047 (2.68))</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>(0.1915 (4.37))</td>
<td>(0.2070 (5.36))</td>
<td>(0.2552 (6.71))</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>(0.0792 (3.35))</td>
<td>(0.0760 (3.48))</td>
<td>(0.0958 (4.81))</td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>(0.1641 (2.22))</td>
<td>(0.1778 (2.62))</td>
<td>(0.2496 (4.19))</td>
</tr>
<tr>
<td>(\delta)</td>
<td>(-0.2417 (2.07))</td>
<td>(-0.1958 (1.98))</td>
<td>(-0.2928 (3.29))</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(-0.0225 (2.07))</td>
<td>(-0.0192 (1.96))</td>
<td>(-0.0232 (2.69))</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>(-0.0000 (1.42))</td>
<td>(-0.0002 (1.40))</td>
<td>(-0.0004 (1.53))</td>
</tr>
<tr>
<td>(\beta^\prime)</td>
<td>(-0.1879 (1.54))</td>
<td>(-0.1425 (1.48))</td>
<td>(-0.1331 (1.60))</td>
</tr>
<tr>
<td>(\lambda^\prime)</td>
<td>(-0.1410 (0.66))</td>
<td>(-0.0625 (0.56))</td>
<td>(-0.0579 (1.65))</td>
</tr>
</tbody>
</table>

| \(R^2\) | \(= .58\) | \(= .57\) | \(= .53\) |

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>(LM_{12,234})</th>
<th>(ARCH_{1232})</th>
<th>(HET_{16,228})</th>
<th>(RESET_{1,245})</th>
<th>(N_2)</th>
<th>(H^1)</th>
<th>(H^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(LM_{12,234})</td>
<td>(= .63)</td>
<td>(= .51)</td>
<td>(= .53)</td>
<td>(ARCH_{1232})</td>
<td>(= .98)</td>
<td>(= .26)</td>
<td>(= 1.19)</td>
</tr>
<tr>
<td>(ARCH_{1232})</td>
<td>(= .36)</td>
<td>(= .46)</td>
<td>(= 1.01)</td>
<td>(HET_{16,228})</td>
<td>(= .36)</td>
<td>(= 1.66)</td>
<td>(= 3.02)</td>
</tr>
<tr>
<td>(HET_{16,228})</td>
<td>(= 1.12)</td>
<td>(= 1.66)</td>
<td>(= 3.02)</td>
<td>(RESET_{1,245})</td>
<td>(= 5.20)</td>
<td>(= .90)</td>
<td>(= 2.39)</td>
</tr>
<tr>
<td>(RESET_{1,245})</td>
<td>(= .90)</td>
<td>(= 1.51)</td>
<td>(= .90)</td>
<td>(N_2)</td>
<td>(= 5.01)</td>
<td>(= 2.31)</td>
<td>(= 2.62)</td>
</tr>
</tbody>
</table>

Notes: See the Appendix for a description of test statistics. T-ratios in brackets.
Chart 2
Scaled residuals from equation (6")

\[ \Theta = 2\% \]

\[ \Theta = 1.75\% \]

\[ \Theta = 1.5\% \]
Some features are worth mentioning. Firstly, the point estimate of $\alpha$, the parameter that measures the speed of adjustment towards the long-run equilibrium in the absence of jumps, is somewhat above 2% (with t-ratios ranging from 2.0 to 2.7), and thus smaller than in equation (6). The remaining point estimates are quite similar to those of equation (6).

Secondly, exchange rate jumps act as an accelerator mechanism towards restoring the long-run equilibrium defined by PPP. For negative jumps - i.e. unusual depreciations - the parameter $\beta$ that measures how much of the accumulated gain or loss in competitiveness is reverted when there is a jump is estimated between 13% and 19%, depending on the definition of jump: this is close to that estimated in Ayuso and Pérez-Jurado (1995) when the most restrictive definition is used ($\theta=2\%$). For positive jumps - i.e. unusual appreciations - this accelerator mechanism is weaker. The $\beta^+$ parameter ranges from 0, for the most restrictive definition of jump ($\theta=2\%$), to 6%, when $\theta$ equals 1.5%. In the intermediate case ($\theta=1.75\%, \lambda, \lambda^+$ and $\beta^+$), t-ratios are well below 1, although the point estimates imply that the normal speed of the adjustment towards PPP equilibrium is doubled. In general terms, the precision of these estimates is low because of the lack of degrees of freedom. This leads to low t-ratios, but the effects are economically meaningful.

Finally, diagnostic tests performed on the transformed residuals reveal no signs of autocorrelation, ARCH, unconditional heteroscedasticity or misspecification as reported, respectively, by the LM [Harvey, 1990], ARCH [Engle, 1982], HET [White, 1980] and RESET [Ramsey, 1969] tests. Normality is not rejected at standard confidence levels, even in columns 1 and 2 where only negative jumps are added to equation (6). The normality test statistic decreases from more than 300 to values around 5. Also, $H^1$ and $H^2$ [Hansen, 1992] tests show no signs of within-sample parameter instability.

Overall, the results from estimating the exchange rate equation given by (6") seem quite satisfactory, especially when $\theta$ is equal to 1.5%. The estimates point to an exchange rate characterised by a slow adjustment towards the long-run equilibrium determined by relative prices in the tradable sector. Occasionally, unusual abrupt changes occur, acting as an accelerator mechanism of this adjustment process. This accelerator effect is stronger when the jump implies an unusual depreciation.

Exchange rate jumps, both positive and negative, take place when economic agents perceive that maintaining a given level of the nominal exchange rate is costly in the short run. Which macroeconomic fundamentals affect this perception is analysed below.

3. **Jump probabilities**

In this section we analyse to what extent fundamental macroeconomic variables can help anticipate future jumps in the peseta's effective nominal exchange rate.

The probability that agents assign to a future jump in the exchange rate plays an important role in explaining the credibility of exchange rate commitments like the ERM. Nevertheless, the literature has paid more attention to credibility indicators that take into account not only probabilities but also the expected size of the jump. Only a few papers have focused on estimating jump or realignment probabilities inside the ERM (see, for instance, Mizrach, 1993 and Gutiérrez, 1994) and they do not include the peseta. Recently, Ayuso and Pérez-Jurado (1995) estimated the probability of a realignment of the bilateral exchange rate of the peseta (and other ERM currencies) against the Deutschemark, using an empirical model that explains this probability in terms of the general performance of the ERM, a reputation effect, and a policy condition requiring an

---

7 It should be clear that our approach is a parsimonious modelling of jumps and does not involve the usual jump by jump intervention analysis.
interest rate level consistent with a country's position in the economic cycle. In any case, in all these papers jumps in exchange rates are associated with central parity realignments and always imply an unusual depreciation of the currency considered against the Deutschemark. Compared with that approach, jumps in the peseta's effective exchange rate are more difficult to define.

As explained in earlier sections of this paper, we define exchange rate jumps empirically and consider different critical sizes which allow for a reasonable number of jumps (between 8% and 14% of the sample size). In our case, jumps are both positive and negative and it is worth noting that jumps over the ERM period other than those associated with changes in central parities are included, as well as jumps over the non-ERM period that were not preceded by any official announcement.

We fit the probabilities of both an unusual depreciation, and an unusual appreciation in the exchange rate over the next month by estimating two probit models, one for positive jumps and the other for negative ones. This approach merits some comment. Strictly speaking, the exchange rate can show a positive jump, a zero jump or a negative jump at any time. Thus, we face a multinomial qualitative variable taking three possible values. However, as can be seen in McFadden (1984), multinomial qualitative response models are rather rigid and restrictive, like the multinomial Logit model, or have high computational requirements, like the multinomial Probit model. Instead, our approach relies on binomial Probit models that are both flexible and easier to implement. Nevertheless, it does not guarantee that the sum of negative and positive jump probabilities is below 1. Our results show, however, that this restriction has not been binding at any time in our sample.

Regarding the choice of the explanatory variables, we consider a relatively wide set of macroeconomic variables which, according to economic theory and to the results in the above-mentioned papers, could be arguments in the cost function described in Section 1 and, therefore, help to explain the probability of exchange rate jumps: real exchange rate, current-account deficit, inflation differential and variables capturing the relative position in the business cycle such as the unemployment rate, output growth, the real interest rate or the capacity utilisation index. Naturally, these variables are appropriately lagged in order to avoid simultaneity problems.

The maximum likelihood parameter estimates of the Probit models are shown in Tables 3 and 4. Charts 3, 4 and 5 show the fitted probabilities. The parameter estimates in Table 3 exhibit correct signs although, in several cases, they are only marginally significant. According to these estimates, the better the cyclical position (the higher the capacity utilisation is) the lower the probability of an unusual depreciation. On the other hand, the higher the accumulated real appreciation (over the last 12 months), the higher the negative jump probability, although this effect is less important after the entry of the peseta into the ERM. In the same vein, the higher the current-account deficit, the higher the probability of an unusual depreciation. This effect, however, also disappears after the peseta's entry into the ERM. Finally, the exchange rate regime change in June 1989 increased the probability of an unusual depreciation and opened the door to a new variable capturing the policy requirement (or dilemma) that the domestic interest rate needs to be considered with the new exchange rate commitment as well as the cyclical position. The greater this dilemma, the greater the probability of an abrupt depreciation.

If we focus on the probability corresponding to months in which jumps have effectively occurred, the mean for these months is clearly higher than the mean probability for the remaining months. Histograms (not-provided) show that probabilities are distributed quite differently for the months in which jumps are observed. This is also the case for positive jumps.

Point estimates in Table 4 show, however, some wrong signs. This is the case for the cyclical position and for the accumulated real appreciation, during the period when the peseta was outside the ERM, although the first one is not statistically significant and the second is only marginally significant. After June 1989, however, both variables are correctly signed and are

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8 To be more precise, the parameter changes its sign and is not statistically significant.

9 Other variables have t-ratios below 1 and, sometimes, the wrong sign.

- 9 -
significant: the probability of an unusual appreciation increases if the cyclical position improves or the real exchange rate has depreciated in the last 12 months. Contrary to Table 3, the entry of the peseta into the ERM reduced the probability of positive jumps. Again, the mean probabilities corresponding to months in which positive jumps have been observed are well above those for the remaining months.

Table 3
Probit model for the probability of an unusual exchange rate depreciation

<table>
<thead>
<tr>
<th></th>
<th>Probability of a jump higher than</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2%</td>
</tr>
<tr>
<td>Constant</td>
<td>6.74</td>
</tr>
<tr>
<td>(0.93)</td>
<td></td>
</tr>
<tr>
<td>Cyclical position</td>
<td>-0.13</td>
</tr>
<tr>
<td>(-1.38)</td>
<td></td>
</tr>
<tr>
<td>Accumulated real appreciation</td>
<td>17.03</td>
</tr>
<tr>
<td>(2.12)</td>
<td></td>
</tr>
<tr>
<td>CA deficit</td>
<td>0.05</td>
</tr>
<tr>
<td>(2.61)</td>
<td></td>
</tr>
<tr>
<td>ERM</td>
<td>1.67</td>
</tr>
<tr>
<td>(2.08)</td>
<td></td>
</tr>
<tr>
<td>Accumulated real appreciation times ERM</td>
<td>-16.39</td>
</tr>
<tr>
<td>(-1.87)</td>
<td></td>
</tr>
<tr>
<td>Policy dilemma time ERM</td>
<td>0.07</td>
</tr>
<tr>
<td>(1.63)</td>
<td></td>
</tr>
<tr>
<td>pseudo-R²</td>
<td>11%</td>
</tr>
<tr>
<td>(1.44)</td>
<td></td>
</tr>
<tr>
<td>RM</td>
<td>5.16</td>
</tr>
<tr>
<td>(5.3%)</td>
<td></td>
</tr>
<tr>
<td>RF</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

The model includes 246 observations corresponding to the period February 1975 to July 1995; t-ratios in brackets.
1 Capacity utilisation index.
2 Over the last 12 months.
3 As a percentage of GDP until May 1989, and 0 thereafter.
4 Dummy variable that takes unit value as from June 1989.
5 1-month interest rate differential divided by 12-month output growth differential (proxied by industrial output growth).
6 Ratio between mean probabilities in months with and without jumps.
7 Relative frequency of the corresponding jumps in the sample.

In Tables 3 and 4, results are very similar for jumps higher than 2%, 1.75% or 1.5%, although they are slightly better in the second case. Nevertheless, the pseudo-R² (see Estrella, 1995) range from 4% to 13% and are particularly poor for the positive jump models. The low predictive power of the Probit models is also confirmed by Charts 3, 4 and 5 which show that fitted probabilities are, in general, small, which relatively frequent peaks in periods in which the exchange rate has not jumped. Again, the picture is worse for positive than for negative jumps.

10 Over the ERM period, the estimated probability of an unusual depreciation is of the same order of magnitude as the realignment probability found in Ayuso and Pérez-Jurado (1995).
Table 4
Probit model for the probability of an unusual exchange rate appreciation

\[ Pr_{i-1}(Q_t = 1) = \Phi(X^O_{i-1} \beta^O) \]

<table>
<thead>
<tr>
<th>Probability of a jump higher than</th>
<th>2%</th>
<th>1.75%</th>
<th>1.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.77</td>
<td>7.88</td>
<td>2.91</td>
</tr>
<tr>
<td>(0.22)</td>
<td>(-80)</td>
<td>(0.40)</td>
<td></td>
</tr>
<tr>
<td>Cyclical position 1</td>
<td>-0.06</td>
<td>-1.12</td>
<td>-2.06</td>
</tr>
<tr>
<td>(-0.39)</td>
<td>(-0.99)</td>
<td>(-0.61)</td>
<td></td>
</tr>
<tr>
<td>Accumulated real appreciation 2</td>
<td>7.73</td>
<td>7.11</td>
<td>4.09</td>
</tr>
<tr>
<td>(1.83)</td>
<td>(2.04)</td>
<td>(1.63)</td>
<td></td>
</tr>
<tr>
<td>ERM 3</td>
<td>-28.9</td>
<td>-34.0</td>
<td>-31.5</td>
</tr>
<tr>
<td>(-1.56)</td>
<td>(-2.03)</td>
<td>(-2.08)</td>
<td></td>
</tr>
<tr>
<td>Accumulated real appreciation times ERM</td>
<td>-30.1</td>
<td>-29.5</td>
<td>-25.6</td>
</tr>
<tr>
<td>(-2.78)</td>
<td>(-2.79)</td>
<td>(-2.59)</td>
<td></td>
</tr>
<tr>
<td>Cyclical position times ERM</td>
<td>0.38</td>
<td>0.44</td>
<td>0.40</td>
</tr>
<tr>
<td>(1.59)</td>
<td>(2.04)</td>
<td>(2.09)</td>
<td></td>
</tr>
<tr>
<td>Pseudo-R²</td>
<td>6%</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>RM 4</td>
<td>4.95</td>
<td>2.65</td>
<td>1.94</td>
</tr>
<tr>
<td>(1.63)</td>
<td>(2.04)</td>
<td>(2.09)</td>
<td></td>
</tr>
<tr>
<td>RM 5</td>
<td>3.3%</td>
<td>4.5%</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

The model includes 246 observations corresponding to the period February 1975 to July 1995; t-ratios in brackets.
1 Capacity utilisation index.
2 Over the last 12 months.
3 Dummy variable that takes unit value as from June 1989.
4 Ratio between mean probabilities in months with and without jumps.
5 Relative frequency of the corresponding jumps in the sample.

Chart 3
Fitted jump probabilities: jumps higher than 2%

Note: Vertical lines correspond to observed jumps.
Chart 4
Fitted jump probabilities: jumps higher than 1.75%

Note: Vertical lines correspond to observed jumps.

Chart 5
Fitted jump probabilities: jumps higher than 1.5%

Note: Vertical lines correspond to observed jumps.
All in all, it can be said that according to our results, agents can hardly anticipate these unusual exchange rate jumps on the single basis of the macroeconomic fundamentals mentioned. This difficulty is especially clear when we look at the unusual appreciations. If agents were able to anticipate exchange rate jumps correctly, other factors such as expectations about political events or speculative bubbles should also play an important role. Unfortunately, these variables are difficult to measure and, therefore, difficult to include in a model like ours. Hence, not too much can be said about the timing of the exchange rate jumps, though some information is provided with respect to the macroeconomic fundamentals that may help to reduce this uncertainty.

Conclusion

In this paper we investigate the dynamics of the peseta's effective exchange rate vis-à-vis the currencies of other OECD countries over the period from January 1974 to September 1995. The proposed empirical model extends the results in Pérez-Jurado and Vega (1994) and Ayuso and Pérez-Jurado (1995). The former found that PPP holds in the long run when only prices in the tradable sector are considered. The latter estimated a model for the realignment probabilities inside the ERM and for the related jumps in the exchange rates. The results of both papers are embraced in our analysis by estimating an equation for exchange rate dynamics that combines the features of an ECM and the possibility of unusual jumps. The size and the probability of these jumps are also estimated.

Jumps are defined empirically and include not only "official" devaluations as in Ayuso and Pérez-Jurado (1995) but also other abrupt depreciations or even appreciations that are above a given threshold. Several thresholds are considered with a view to testing the robustness of the results.

The size of these unusual jumps depends on the deviation of the real exchange rate from its PPP value. Therefore, jumps enter the ECM as 'accelerators' in the path towards the long-run equilibrium. In particular, negative jumps, i.e. unusual depreciations, multiply the speed of the adjustment process by a factor ranging from 10 (for the most restrictive definition of a jump) to 7 (for the least restrictive one). This accelerator effect is less clear for unusual appreciations. Only for the less restrictive definition of a jump is that effect significant, multiplying by 4 the speed of the adjustment.

Regarding the perceived probability of exchange rate jumps, two Probit models were estimated, one for each sort of jump. The results underscore that jump probabilities react to changes in certain fundamental macroeconomic variables: the current-account deficit (over the period when the peseta was outside the ERM), the accumulated real appreciation over the last twelve months and the position of the economy in the business cycle. Nevertheless, estimated probabilities are small and show relative peaks in periods in which exchange rate jumps have not occurred. Therefore, an important degree of uncertainty remains in predicting the timing of jumps.
Appendix

All the calculations in the paper have been made using TSP 4.2B and PcGive 8.0. The following is a list of the test statistics reported in Tables 1 and 2:

$\text{LM}_{ij}$ = the Lagrange Multiplier F-test for residual autocorrelation up to $i^{th}$ order. See Harvey (1990) for a description.

$\text{ARCH}_{ij}$ = the Autoregressive Conditional Heteroscedasticity F-test reported in Engle (1982).

$\text{HET}_{ij}$ = the White (1980) F-test for heteroscedasticity. In this test, the null is unconditional homoscedasticity, and the alternative is that the variance of the residual depends on the levels and squared levels of the regressors.

$\text{RESET}_{ij}$ = the Regression Specification F-Test due to Ramsey (1969). This test may be interpreted as a test for functional form.

$\text{Sk}$ = skewness.

$\text{Ek}$ = excess kurtosis.

$N_2$ = the Doornik and Hansen (1994) $\chi^2$-test for normality.

$H_1$ = the Hansen (1992) within-sample parameter instability statistic for the residual variance $\sigma^2$.

$H^2$ = the Hansen (1992) joint statistic for within-sample stability of all the parameters in the model.
References


Gutiérrez, E., 1994, "Un modelo de devaluaciones para el SME", CEMFI, Documento de Trabajo, No. 9416.


Comments on paper by J. Ayuso and J.L. Vega by W. Melick (BIS)

The paper "An Empirical Analysis of the Peseta's Exchange Rate Dynamics" represents an innovative and interesting attempt to realistically model variables such as exchange rates that are subject to "jumpy" behaviour. I would like to highlight the paper's strengths and contributions and offer two constructive criticisms.

The paper's insights really spring from one source, namely the authors use of a general definition of an exchange rate jump. The easy approach of defining a jump as a realignment of an official zone or parity is avoided, allowing for three significant contributions. First, this general definition of a jump allows, in the case of Spain, a longer time series to be analysed, not just the period over which Spain has participated in the ERM. Second, the general definition of a jump gives the paper a wider applicability. The modelling strategy developed here can be applied to countries with a floating regime as well as to those with a fixed or target regime. Therefore, the technique and results are of interest under any set of circumstances. Finally, the general definition of a jump allows for interesting tests when countries transition from one exchange regime to another, as was the case for Spain in 1989. To my mind the most interesting parts of the paper are the results from the probit estimations when comparing periods before and after June 1989. The disappearance of a current account effect after entry into the ERM is a finding worthy of further study.

Unfortunately, the general definition of a jump is not without problems. The general definition gives rise to an unobserved or latent variable (the jump) that complicates any estimation. The authors handle this problem using a two-stage estimation procedure. In the version presented at the December meeting, the procedure was somewhat flawed, resulting in biased and inefficient estimates, as pointed out in my comments at the meeting. In this revised version of the paper, a clever and simple modification of the two-stage procedure removes the bias in estimated coefficients. However, the inefficiency remains. I offer an alternative strategy. The model could be estimated using the regime switching technique of Hamilton, augmented with the assumption that transition probabilities (the jumps) are determined by fundamentals. That is, Markov switching variables could be defined, with the probability of being in a jump state determined by the fundamentals currently used in the probit estimation. Two such variations on the Hamilton technique have been developed, one by Diebold, et al (1994) and the other by Filardo (1994). This alternative strategy would allow for a simultaneous estimation of the model, avoiding the inefficiency problem.

Moving from econometrics to economics, my second constructive criticism involves the choice of variables used to explain the exchange rate. It seems somewhat restrictive to include only home and foreign prices as determinants of the exchange rate. It seems reasonable that there might be other short-run determinants of the exchange rate that ought to be included. I am curious if variables such as interest rates were part of an initial specification and rejected, or if they were not considered from the outset. Given the findings of some of the other papers presented at the meeting, it seems a longer list of determinants should be included or at least examined.

By way of conclusion, the paper provides a specification for exchange rates, be they fixed or floating, that allows for the jumps commonly seen in the data. Such a specification should be valuable and widely applicable. In the context of model building, the paper raises questions on the implication of such jumpy behaviour for the modelling of rational agents expectation formation.

References


On the fundamental determinants
of the Swiss franc exchange rate for the D-mark

Franz Ettlin

Introduction

The reaction patterns of the foreign exchange markets are macroeconomic phenomena of
great concern to central banks and other economic policy makers as well as to important parts of the
financial and business community. Unfortunately, economists have, in the past, not done well in
reliably tracing and quantifying these reaction patterns of exchange rates. As a consequence they have
become rather pessimistic in regard to the development of successful econometric exchange rate
models based on fundamental determinants; i.e. standard macroeconomic variables. Out-of-sample
predictions of such fundamental models have usually been either not much better, no better or – in
most cases – still more inaccurate than the no-change predictions of the unpretentious simple random
walk model, even when the actually realised rather than forecasted values of the fundamental
explanatory variables were used. This predictive failure justifies a sceptical attitude in regard to the
theoretical and practical relevance of much of existing exchange rate theory. It appears that
economists do not yet understand the determinants of short to medium-run movements in exchange
rates. This paper attempts to bring optimism back to this issue by presenting – as an alternative to the
established but empirically unsuccessful monetary models – a behavioural type of fundamental
exchange rate model which clearly beats the no-change predictions of the random walk time series
model for a relatively wide range of time horizons.

1. Specification of an alternative fundamental model

In developing the fundamentals-based behavioural model of the Swiss franc exchange
rate for the D-mark the focus was on a set of variables from the financial and goods markets which
economic agents might actually use as relevant signals. The univariate stationarity properties of the
chosen data set provided essential information for model construction. Stationarity tests, the results of
which are summarised in footnote 4, suggest that the level form of the stochastic data series are all
individually integrated of order one. This led, in particular, to the specification of a relationship
between the levels – rather than the changes – of the exchange rate and the interest rate differential
between the countries concerned. Moreover, on the basis of preponderant observed reaction patterns in
the foreign exchange markets, the model envisages, on balance, a positive – rather than negative –
partial relationship between the domestic level of interest rates and the external value of the domestic
currency (given by the inverse of the exchange rate, defined as the domestic price of foreign
currency). Both of these features are contrary to the uncovered-interest-parity framework, which
economists continue to use as a guiding principle in their monetary exchange rate models in spite of
its poor empirical record. Apparently, the risk-neutral arbitrage behaviour under rational

1 Revised and updated version of Ettlin (1995a).
2 This conclusion is drawn, for example, by Meese (1990, p. 132), after reviewing the empirical performance of
monetary exchange rate models based on the asset market approach, which have been dominant in the literature for
most of the last two decades.
3 For negative empirical results regarding the application of that framework to the implied relationship between Swiss
and German interest rates, as well as the presentation of an alternative approach, see Ettlin and Bernegger (1994).
expectations, on which uncovered interest parity builds, is not decisive for the movements of the foreign exchange markets in general.

The specification of the single-equation behavioural model of the nominal Swiss franc exchange rate for the D-mark (represented in the model by the mnemonic symbol LSFDM) contains two interest rate differentials among its explanatory variables. These are the differential between the official discount rates in Switzerland and Germany (RDISD) as well as the differential between the three-month Swiss franc and D-mark Eurodeposit rates (R3MSD). The former variable serves as a robust current indicator of relative monetary policy while the latter represents a key measure of relative market interest rates. There is also a term structure differential (R3YSD - R3MSD) regarding the relative steepness of the yield curve between three-year and three-month Euromarket rates for the two currencies. This variable contains information on forward, i.e. expected future short-run interest rate differentials and thus also on the relative stance of future monetary policy. It should be observed that in contrast to most fundamental exchange rate models in the recent literature, no measure of relative money supply is included among the relevant fundamentals, since the discount rate differential provides a more autonomous and econometrically much more reliable measure of the relative stance of monetary policy.

Furthermore, the natural logarithm of the ratio of the consumer price indices in the two countries concerned (LPCSD) is generally recognised as a representative measure of the relative purchasing power of the two currencies. The logarithm of the (lagged) ratio of industrial capacity utilisation (LCUSD) is intended to serve as an indicator of the comparative cyclical state of real economic activity. A positive difference of the current account balance to GDP percentage ratio (CAGSD) is interpreted by the foreign exchange market as a sign of relative strength of the local currency. The implied larger relative capital outflow or smaller inflow apparently requires a differential foreign exchange risk premium on account of imperfect substitutability between (changes in) domestic and foreign assets. Finally, the logarithm of the lagged US-dollar exchange rate for the D-mark (LUSDM) is also included as a fundamental signal. The choice of this variable derives from the fact that US-dollar investments are a substitute for D-mark or Swiss franc investments, whereby it is empirically observed that the Swiss franc tends to be proportionately more affected than the D-mark by fund flow pressure out of or into US-dollars. This can be attributed to the relatively smaller liquidity in the Swiss financial market.

2. Estimation procedure and results

The econometric exchange rate model based on the above-mentioned set of fundamental determinants is estimated for the sample period 1979Q2 – 1991Q2. The starting point coincides with the beginning of the institutional framework of the Exchange Rate Mechanism of the European Monetary System, in which Germany participates. The early termination of the in-sample period in 1991Q2 allowed the inclusion of four quarters of observations following the structural break related to the German unification in mid-1990, while still leaving up to 19 calendar quarters for extended post-sample prediction tests. Regarding methodology, the single equation two-step ordinary least squares procedure developed by Engle and Granger (1987) is applied.4

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4 This requires, as a preliminary task, the testing of the univariate time-series properties of the data used in the study. It was done by means of augmented Dickey-Fuller tests, with the critical values derived from MacKinnon (1991). For none of the stochastic level variables in the model was the null-hypothesis of the presence of a unit root rejected at the 1% size of a one-tail test, whereas for all the corresponding first differences the unit root hypothesis was rejected by the same criteria. This implies that the individual data series should be treated as being I(1), i.e. integrated of order one; they need to be differenced once to become stationary.
The cointegration equation in Table 1 shows – as the first of the two main steps of the estimation procedure – the equilibrium relationship between the logarithm of the nominal exchange rate and the chosen set of fundamental variables. The estimated coefficient of the discount rate differential is -0.025. This implies that the Swiss franc will, ceteris paribus, ultimately strengthen by 2½ percent when the Swiss discount rate is raised by 1 percentage point, or when the German discount rate is lowered that much. The response parameter of the three-month Euromarket rate differential is -0.012, i.e. about half of the one related to the discount rates. Finally, the relative term structure between three-year and three-month Euromarket rates affects the logarithm of the exchange rate with a coefficient of -0.018.

The estimated coefficient for the logarithm of the ratio of consumer prices is 1.026; it practically coincides with unity. Thus, as the equilibrium nominal exchange rate adjusts fully to the relative price level, the real exchange rate is by implication not influenced, ceteris paribus, by such aggregate price movements.

The logarithm of the lagged ratio of industrial capacity utilisation captures the influence of the comparative cyclical state of real activity. The coefficient of -0.49 indicates that for each percentage point of relatively higher (lower) capacity utilisation in Switzerland than in Germany the Swiss franc will appreciate (depreciate) by 0.5 percent. At this time, it remains undetermined by which potential path this cyclical effect mainly arises. It could be, for example, via profitability, share prices, or anticipated monetary policy reaction.

The logarithm of the lagged US-dollar exchange rate for the D-mark has a long-run coefficient of -0.061. This implies, for example, that a 10 percent increase in that exchange rate, i.e. a corresponding depreciation of the US-dollar, will lead to an appreciation of the Swiss franc vis-à-vis the D-mark by 0.6 percent. As already explained, both the D-mark and the Swiss franc tend to appreciate (depreciate) when the effective dollar weakens (strengthens), but the relative impact on the Swiss franc is usually somewhat larger. On account of simultaneity problems, the presumably larger unlagged response within a calendar quarter would be traceable only with a system of at least two equations, in which the reaction patterns of the US-dollar vis-à-vis the D-mark are also modelled.

The difference between Switzerland and Germany regarding the current account to GDP ratio shows a response parameter of -0.009. This means, for example, that with a +7 percent ratio for Switzerland and a +2 percent ratio for Germany the Swiss franc would on that account alone be some 4½ percent [i.e. -0.009 (7-2)=-.045] stronger vis-à-vis the D-mark.

The economic and monetary unification of Germany in 1990 introduced a large structural break into the German current account data. Because of that break the relative current account variable in the model is considered reliable only until April 1990. After that date only the mean impact of the
relative current account balance is included in the form of the negative coefficient of the level dummy variable $D90UNIF$. If the entire value of that coefficient of -0.05 is attributed to the mean of the difference between the scaled current account balance of Switzerland and Germany, then this would correspond to a pre-unification effect of an average excess of the Swiss current account to GDP ratio of some 5½ percentage points, whereas the actual data for the period 1990Q3 - 1996Q1 show an average excess of more than 7 percentage points. The German current account data for the 1990s are subject to some further problems as the introduction of interest rate taxation led to large-scale tax evasion which induced considerable outflows of capital and inflows of interest income. It seems that the latter type of interest receipts has so far not been adequately registered in the German current account data. This implies a corresponding downward bias in the latter during the most recent years.

The transitory dummy variable $D89WALL$ is intended to capture the temporary appreciation of the D-mark in connection with the international euphoria created by events symbolised by the fall of the Berlin wall. This is estimated to have resulted in a depreciation of some 2 percent of the Swiss franc vis-à-vis the D-mark in the last quarter of 1989 and the first quarter of 1990. Finally, the model contains a seasonal shift which is related to the current account variable, the seasonal adjustment of which is not appropriate for the current purpose.

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The equation shows a cointegration regression Durbin-Watson statistic of 1.89. This result is close to the ideal value of the Durbin-Watson statistic for a stationary white-noise stochastic process, and it is obviously very significantly different from zero, which is the expected value of this statistic under the null hypothesis of non-cointegration. As the latter hypothesis is thus rejected, the estimated relationship can be considered as stationary and to have a valid error-correction representation according to Engle and Granger (1987). The null hypothesis of non-cointegration seems also rejected by an Engle-Granger unit root test on the cointegration residual, but in this case the critical values can only be derived by approximate extrapolation from MacKinnon (1991), as criteria for potential cointegration equations with more than six stochastic variables are not provided.

The estimated error-correction equation for the first differences of the logarithm of the Swiss franc exchange rate for the D-mark ($ALSF_{DM}$), which represents the second step of the estimation procedure, is summarised in Table 2. The last variable listed in this equation is the lagged error-correction term ($LSFDM - LSFDM*_{-1}$). Its coefficient of -1.147 implies that any difference between the actual level of the exchange rate and its equilibrium value according to the cointegration equation in Table 1 will practically be fully corrected after one calendar quarter. Both the large absolute magnitude and the high t-value of the error correction coefficient confirm the stationary character of the cointegration equation and the validity of the error-correction representation. Moreover, the summary statistics of this equation suggest a good approximation to the unknown data generation process. The standard error of the regression is 0.007, i.e. about 0.7 percent of the exchange rate level. The adjusted R-square of 0.86 indicates that only 1/7 of the total variance of the exchange rate changes in the sample period remains unexplained. The Durbin-Watson statistic of 1.84 lies close to the ideal value of 2.0 for a stationary white-noise residual.

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5 It seems that, in fact, after about two years of transition following reunification, the pre-unification type of sensitivity to the changing current account situation was re-established. Pursuing that line of approach in the present paper, however, would have left much fewer observations for the following post-sample prediction tests, which are a crucial part of the paper.

6 Actually, the point estimate, which is in excess of unity in absolute value, suggests some initial overcorrection. The difference to -1 is, however, not significant according to all standard test criteria. In any case, as the feedback is negative, only a coefficient value of -2 or smaller for the error correction term would imply an unstable adjustment process. It can also be observed that the quarterly unit period of the estimated model is relatively long for the fast reacting foreign exchange markets; this may tend to lead to a large absolute value of the error-correction coefficient, provided also that the cointegration equation is quite well specified.
Table 2
Error-correction equation*

$$\begin{align*}
LSFDM &= 1.002 \Delta LPCS - 0.019 \Delta RDI - 0.015 \Delta M2RMSD - 0.019 \Delta R3YSD - R3MSD - 0.457 \Delta M2LCUSD_3 \\
& \quad - 0.055 \Delta USDM_{-1} - 0.011 \Delta \left[ M2CCAGSD_{-1} \left[ 1 - D90UNIF \right] \right] - 0.029 \Delta D89WALL - 0.030 \Delta D90UNIF \\
& \quad + 0.094 \left[ SD24 \left[ 1 - D90UNIF \right] \right] - 1.147 \left[ LSFDM_{-1} - LSFDM_{-1} \right] \\
\end{align*}$$

<table>
<thead>
<tr>
<th>Sample period</th>
<th>1979Q2 – 1991Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin-Watson statistic</td>
<td>1.838</td>
</tr>
<tr>
<td>Standard error of regression</td>
<td>0.007</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.864</td>
</tr>
</tbody>
</table>

Note: Absolute values of the standard t-statistic are shown below the coefficients.

The short-run response coefficient of the nominal exchange rate with regard to price level changes is unity, i.e. the same as in the cointegration equation. This means that even in the short run the real exchange rate remains, ceteris paribus, unaffected by changes in relative consumer prices between Switzerland and Germany. It does not, however, imply a constant real exchange rate in agreement with purchasing power parity, since the other explanatory variables affect the nominal and real exchange rate in equal proportions, both in the short and longer run. The estimated immediate responses to changes in those other explanatory variables do not differ in sign but to some extent in magnitude from the corresponding equilibrium responses in the cointegration equation. Any remaining difference will be almost fully corrected after one calendar quarter via the lagged error correction term.

The in-sample period of the model was chosen to go only to the second quarter of 1991. The choice of this early endperiod left up to 19 calendar quarters for post-sample testing, which is crucial for determining the validity of an empirical exchange rate model.

3. Tests of the accuracy of post-sample predictions

Figure 1 visually illustrates the quarterly development of the Swiss franc/D-mark exchange rate for the period 1979Q2–1996Q1 as a thick solid line and the corresponding model-based in-sample predictions for the period 1979Q2–1991Q2 as a broken line. The tracking performance looks rather good. The post-sample predictions for the period 1991Q3–1996Q1 are indicated by a dash-dotted line. They correspond to the post-sample projection values from the cointegration equation. These predictions are somewhat less accurate than the in-sample results. But their overall tracking performance can be judged as quite satisfactory considering that in this case the post-sample horizon extends over 19 calendar quarters and that no information on the actual exchange rate between 1991Q3 and 1996Q1 was used for these out-of-sample predictions. The largest prediction errors appear in 1995Q4 and 1996Q1, after the Swiss franc appreciated on account of a much discussed speculative surge away from the D-mark. The surge which began in the second half of 1995Q3 related to fears concerning price stability and interest rate levels in the future European Monetary Union. Although the present version of the model is not sufficiently complete to endogenously explain this speculative movement, it does permit to derive an ex post estimate of around 5 percent for the magnitude of this appreciation.
In agreement with the well-known type of prediction tests for exchange rates originally associated with Meese and Rogoff (1983a, 1983b), the out-of-sample predictions of the estimated model are based on actual rather than on predicted values of the fundamental explanatory variables. As no data on the dependent variable from within the respective prediction horizon were to be used, the results shown in Table 3 neglect the error-correction equation and thus are done with the cointegration equation only. The post-sample predictions for 1991Q3–1996Q1 illustrated in the graph on the preceding page provide the first set of fundamental-model predictions. They are based on the parameters of the cointegration equation estimated to 1991Q2. Then the observations for 1991Q3 are added to the in-sample data and the cointegration equation is re-estimated to generate predictions for 1991Q4–1996Q1. After successively adding one quarter to the in-sample period until the latter extends to 1995Q4 a total of 19 rolling cointegration regressions are estimated from which a set of predictions for the respective post-sample time periods is calculated. These predictions are compared with the naive (no change) forecasts of the random walk model.

The neglect of not only the error-correction term but also the short-run dynamics incorporated in the error-correction equation can be expected to decrease the accuracy of the fundamental model predictions. But since the quarterly unit period of the present empirical application is relatively long in comparison with the fast reactions of the foreign exchange market to fundamental news, there are only minor differences in the response coefficients between the cointegration equation, which was used, and the error-correction equation, which was not used for the predictions.
Table 3
Out-of-sample prediction statistics for the level of the Swiss franc exchange rate of the D-mark

<table>
<thead>
<tr>
<th>Horizon (quarters)</th>
<th>Number of prediction samples (rolling regressions)</th>
<th>Root mean square error of prediction samples</th>
<th>Random walk (percent)</th>
<th>Fundamental model (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16 (19)</td>
<td>1.9 (1.8)</td>
<td>1.0 (1.8)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13 (16)</td>
<td>4.4 (4.2)</td>
<td>1.2 (2.2)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9 (12)</td>
<td>6.7 (6.6)</td>
<td>0.9 (2.2)</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>5 (8)</td>
<td>6.3 (8.3)</td>
<td>1.1 (2.8)</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1 (4)</td>
<td>2.9 (7.6)</td>
<td>1.5 (3.2)</td>
<td></td>
</tr>
</tbody>
</table>

The latter uses no information on the fundamental variables but sets the exchange rate forecast for any period equal to the actual exchange rate value preceding the beginning of the forecast period. Despite the very naive character of the random walk model used as a benchmark for comparison, predictions of fundamental exchange rate models (generally some variants of the monetary asset market approach) using the actual future values of the fundamentals have, in the past, mostly failed to dominate the simplistic random walk forecasts for different time horizons. The corresponding prediction comparisons regarding the exchange rate model of this paper show, however, a clear dominance of the fundamental model over the random walk scheme.

In Table 3, the prediction results for the quarterly level of the Swiss franc rate for the D-mark are summarised for horizons of 1, 4, 8, 12 and 16 quarters. The table gives averages for predictions ending in 1995Q2 as well as, in brackets, for predictions which also include the subsequent three quarters, when the Swiss franc was subject to the above-mentioned speculative appreciation surge. The out-of-sample prediction statistic used for comparison is the root mean square error (RMSE), i.e. the square root of the average of the squared forecast errors. Except for the one-quarter horizon predictions extending to 1996Q1, which indicate a draw, the fundamental model has much smaller RMSE values than the random walk model. The differences range from 0 for the one-calendar-quarter horizon predictions ending in 1996Q1 to 6.2 percentage points for the twelve-quarter horizon predictions ending in 1995Q2. The random walk model's RMSEs are preponderantly several times as large as those of the fundamental model.

Conclusion

Of course, actual forecasting with this fundamentals-based model will be less accurate than out-of-sample prediction, because the explanatory variables themselves have to be forecasted as well. It still remains to be shown that the random walk can be beaten also in an actual forecasting context for short as well as longer-run horizons. Nevertheless, even without proof of such superior forecasting ability the model presented should be quite helpful for developing much improved scenarios for past and future developments of the Swiss franc exchange rate of the D-mark.

Corresponding sets of macroeconomic fundamentals to the ones chosen to model the Swiss franc/D-mark exchange rate from the late 1970s to the mid-1990s will not necessarily be sufficient for other exchange rate contexts. But they may still prove to form an essential part of similarly successful fundamental-based models of other flexible exchange rates as well.
The results of the fundamental model of the Swiss franc rate for the D-mark also provide some suggestions about the potency of monetary policy for exchange rate developments. For example, if the Swiss National Bank lowers its average discount rate in comparison with the discount rate of the Bundesbank by one percentage point, the average Swiss franc rate will, as a direct effect, depreciate by almost 2 percent in the same quarter and by about another ½ percent in the following quarter. The cut in the discount rate (accompanied by a similar reduction in the day-to-day rate of interest) could exert some additional depreciation pressure indirectly via its effects on the market rates of interest. When previously estimated reaction patterns of three-month and three-year Swiss franc interest rates are also taken into account, this indirect effect turns out to be negligible on balance. The initial exchange rate depreciation will, at first, be re-enforced and later weakened via changes in the current account balance. When, after some delay, be reduced also on account of improvements in industrial capacity utilisation in Switzerland.

In conclusion, the present behavioural type of exchange rate model seems to be a promising alternative fundamental approach. Its success in the post-sample prediction tests of this paper stands in marked contrast to the experiences with applied monetary models based on the asset market approach, which by now have been dominant in the literature for almost two decades. But the single-equation specification in this paper should in the future be succeeded by a multi-equation system. Thereby, problems arising from simultaneity and dynamic interdependence among the variables could be taken into account more effectively. In the present model such issues were dealt with only partially by lagging some regression variables and by excluding, in particular, any measure of relative money supply in favour of the more autonomous discount rate differential. In future applications it should also be worthwhile to shorten the unit period of the empirical model to one month or less, since, as already mentioned, the quarterly unit period seems long in comparison with the speed of reaction to fundamental news in the foreign exchange market.

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8 Ettlin and Bernegger (1994).
9 See footnote 5.
10 For an attempt at a comprehensive quantitative assessment of the effects of a change in central-bank interest rates in Switzerland, see Ettlin (1995b).
List of variables

**LSFDM**  
Natural logarithm of the Swiss franc exchange rate for the D-mark; quarterly average of daily spot rates.

**LSFDM**  
Equilibrium level of LSFDM according to the cointegration equation.

**LPCSD**  
Logarithm of the ratio of the consumer price index of Switzerland to that of Germany; quarterly average of monthly data.

**RDISD**  
Difference between the official discount rate in Switzerland and Germany; quarterly average of beginning and end-of-month rates expressed as percentage points per annum.

**R3MSD**  
Difference between the three-month Euromarket deposit rate for the Swiss franc and the D-mark; quarterly average of daily data expressed as percentage points per annum.

**R3YSD**  
Difference between the three-year Euromarket deposit rate for the Swiss franc and the D-mark; quarterly average of daily data expressed as percentage points per annum.

**LCUSD**  
Logarithm of the ratio of industrial capacity utilisation in Switzerland to that in Germany; once-per-quarter observations.

**LUSDM**  
Logarithm of the US-dollar exchange rate for the D-mark; quarterly average of daily spot rates.

**CAGSD**  
Difference between the current account to nominal GDP ratio of Switzerland and that of Germany; percentage points based on seasonally adjusted quarterly data.

**D89WALL**  
Dummy variable related to the fall of the Berlin wall; one for 1989 Q4 and 1990 Q1 and zero otherwise.

**D90UNIF**  
Dummy variable related to the German Economic and Monetary Unification in 1990; zero until 1990 Q1, two-thirds for 1990 Q2 and one thereafter.

**SD24**  
Dummy variable with the value 1 in the second quarter and -1 in the fourth, implying a seasonal shift between the second and the fourth quarter.

**M2...**  
Two-period moving average of the subsequently indicated variable.

**M2C...**  
Centred two-period moving average of the subsequently indicated variable.

**Δ**  
First backward difference of the indicated variable.
References


Comments on paper by Franz Ettlin by G. Galati (BIS)

The objective of this paper is to build a model of the Swiss franc-DM exchange rate based on economic fundamentals that performs well as predictor of short- and medium run future exchange rate movements. It tries to improve on the poor forecasting performance of exchange rate models based on fundamentals compared to a simple random walk model, as documented in the literature (Meese and Rogoff, 1983).

The Swiss franc-DM exchange rate is represented by a single equation which has on the right hand side variables that economic agents might plausibly look at when they form their views on future exchange rate movements. These variables include two interest rate differentials (between discount rates, 3-month Eurorates) and a yield curve differential, the ratio of the CPI in the two countries, the ratio of capacity utilisation, the current account-GDP ratio, and the DM-US dollar exchange rate lagged one period. Furthermore, two dummies are included - one to capture the "international euphoria" following the collapse of the Berlin wall in 1989, and the other to capture the German monetary unification in 1990. The presence of two interest rate differentials is dubious on grounds of multicollinearity, while the inclusion of two dummies makes this approach more difficult to extend to other exchange rates. It would be interesting to see how well this approach can work for other exchange rates.

The model is estimated in error correction form using the Engle-Granger two step procedure with quarterly data from 1979.II (the start of the ERM) to 1991.II. All the coefficients turn out to be significant and the fit of the error-correction equation is judged to be good. However, the number of explanatory variables (eleven in the error-correction equation and ten in the cointegration equation) looks high compared with the number of observations. It would be useful (as the author admits) to estimate the model with data of monthly or higher frequency.

The model is then used to compute in-sample predictions as well as rolling out-of-sample forecasts over different horizons. The out-of-sample predictions are computed using actual values for the explanatory variables and only past values of the exchange rate (consistent with the approach followed by Meese and Rogoff). They are based on the cointegration equation only, whereas the whole error correction model should be used. Using the root mean square error as a criterion, the model is found to dominate the random walk model over all horizons beyond one quarter, and especially over longer horizons. The model, however, performs poorly during periods of tension in European markets and dollar weakness, for example in the fourth quarter of 1995.

An interesting finding of the paper is that even after controlling for macroeconomic and financial variables and indicators of monetary policy, changes in the Swiss franc-DM exchange rate are influenced by changes in the dollar-DM exchange rate: a 10% depreciation of the dollar vis-à-vis the DM leads to a 0.6% appreciation of the Swiss franc vis-à-vis the DM. This is consistent with BIS (1996) which estimates the elasticity of dollar exchange rates of different European currencies, the Australian and the Canadian dollar with respect to the dollar-DM exchange rate from bivariate regressions. Using daily data for rolling samples of 125 days over the period 1994 to 1996, it finds a differentiated response to dollar-DM exchange rate changes: at one side of the currency spectrum, the Swiss franc appreciates by 1.1% with respect to the dollar following a 1% appreciation of the DM with respect to the dollar. The coefficients of other European exchange rates lie between 0 and 1, with coefficients of currencies like the Dutch Guilder or the Belgian franc closer to 1 and those of the Italian lira and the British pound closer to 0. At the other side of the spectrum, the Australian and the Canadian dollar fall against the US dollar when it falls against the DM.

Moreover, the elasticities are not stable over time: periods of dollar weakness (strength) are associated with falling (rising) elasticities of the European currencies and a rising (falling) elasticity of the Canadian dollar. It would be interesting to see how the coefficient of the DM-dollar exchange rate in Ettlin's model changes in 1995.IV.

Although there are a number of studies that have looked at the links between exchange rates, these results are more recognised than understood in the literature. The author's view can be
identified with what is known as the *moka Tasse* effect, i.e. shifts in asset demand having larger effects on the exchange rate the larger the size of the shift relative to the underlying asset stock. Earlier work on this interpretation by Giavazzi and Giovannini (1989) looks at the offshore market size by currency of denomination and compares it with the economic importance of a country (proxied by its GNP share). They argue that in countries that have relatively small financial markets because of transaction costs arising from capital controls, the DM (dollar) exchange rate is more (less) exposed to movements in the value of the dollar. Ongoing research at the BIS finds that the order of sensitivities of each dollar exchange rate is significantly correlated with the order of international banking intensity as measured by the ratio of international and Eurodeposits to GDP.

However, there are other possible explanations of the observed exchange rate links. The same factors used by the author to explain the Swiss franc-DM exchange rate may also drive the correlation coefficients of the Swiss franc-DM or other DM exchange rates with the DM-dollar rate. These include the relative cyclical position of the home country, Germany and the US, and the relative stance of monetary policy. Another interpretation looks at the structure of trade as a determinant of exchange rate links (Brown, 1979).

**References**


Sources of sterling real exchange rate fluctuations, 1973-94

Mark S. Astley and Anthony Garratt

Introduction

"There is no simple relationship between exchange rate changes and subsequent inflation"

What are the price (inflation) implications of an exchange rate movement? Several factors have to be borne in mind in answering this question. First, exchange rates and prices are both endogenous variables. As such, exchange rate changes constitute one (potentially important) channel through which exogenous shocks affect prices. But they do not constitute an independent source of price fluctuations unless the authorities allow wage bargaining and price setting behaviour to be affected by such changes – the "second round" effects. Second, and directly following from the above, we need to identify the (unobservable) source of any exchange rate change to answer the question. This is especially important as both the sign and magnitude of the direct ("first round") price effects depend on the type of shock underlying the exchange rate change.

The sources of real exchange rate movements is a long-debated issue. The "disequilibrium" approach (Dornbusch (1976), Mussa (1982)) posits that sluggish price adjustment means that nominal shocks will play a large role. Another prominent theory is the "equilibrium" approach of Stockman (1987, 1988). This stresses that real shocks, with large permanent components, are likely to be the source of real exchange rate fluctuations.

But have exchange rate changes actually constituted an important channel through which exogenous shocks have affected prices? To determine this we also need to identify the sources of price movements. The answer is clearly "no" if price fluctuations are attributable to different types of shocks to exchange rate movements. A priori the types of shocks that might potentially underlie price movements are similar to the potential sources of exchange rate movements; an exchange rate is, after all, a relative price.

To investigate these issues we follow Clarida and Gali (1994) in estimating UK-centred two country open economy macro models in the spirit of Dornbusch (op cit). The analysis is conducted with the United States, Japan, Germany and France in turn as the foreign countries. We use the Blanchard and Quah (1989) structural VAR (SVAR) approach to identify three structural shocks: (i) real AS (aggregate supply) shocks, which include all labour market factors, such as

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1 Monetary Assessment and Strategy Division, Bank of England. The views expressed are those of the authors and not necessarily those of the Bank of England. The following has benefited from comments by Danny Quah, Clive Briault, Andrew Haldane and Frank Smets at the BIS. Remaining errors are, of course, entirely our responsibility. Our thanks go to Siobhan Phillips for excellent research assistance.

2 The well documented strong positive correlation between real and nominal exchange rate movements supports the disequilibrium view. But the Meese and Rogoff (1988) empirical rejection of the predicted strong correlation between real interest differentials and real exchange rate changes called the approach into question.

3 Other popular theories include the monetary approach (which is the long-run solution to Dornbusch (op cit), the portfolio balance approach and the currency substitution approach.

4 The Huizinga (1987) finding that a high proportion of real exchange rate variation is due to permanent shocks (real exchange rates contain unit roots) supports the equilibrium view.

5 Clarida and Gali (op cit) paid less attention to this issue than we do.
differential productivity developments, that shift the aggregate supply curve; (ii) real IS (goods market) shocks, encompassing exogenous changes to real relative domestic absorption due to shifts in consumption, investment, government expenditure and home/foreign goods tastes; and (iii) nominal LM (money market) shocks, reflecting shifts in both relative money supplies and relative money demands. As the models are relative ones, we only consider the effects of asymmetric shocks.

To identify the model we impose three theory-derived long-run restrictions. The first two restrictions are that both IS shocks and LM shocks have zero long-run effects on the level of relative output (which is entirely supply determined). The final restriction is that LM shocks have zero long-run effects on the level of the real exchange rate. The strength of these restrictions is their generality and uncontentious nature. The remaining responses – long-run and short-run – are entirely data determined, rather than being imposed.

The framework adopted is highly suited to answering the questions at hand because: (i) it takes account of both real and nominal shocks (AS and IS shocks represent real perturbations, while LM shocks are nominal ones); and (ii) it allows us to uncover the contribution of each of the unobservable structural shocks to the observed exchange rate and relative price (UK consumer prices minus their foreign equivalents) movements.

Our main findings are as follows. First, IS shocks constituted the main source of sterling real and nominal exchange rate movements. AS shocks were the secondary source of these fluctuations, while LM shocks played extremely limited – and usually statistically insignificant – roles, even at short horizons. The dominance of real (IS and AS) shocks as sources of sterling exchange rate movements is more consistent with the Stockman (op cit) equilibrium view than the Dornbusch (op cit) disequilibrium approach. And combined with the estimated impulse response functions these results imply that the sterling exchange rate depreciations over the floating rate period have had largely benign relative price implications. In particular, we find that a 10% nominal sterling depreciation is most likely to be associated with a small (around 1%) fall in UK relative prices.

Second, the variation of UK relative prices was due mainly to LM shocks. Of the real shocks, the influence of AS shocks was most apparent. This strong contrast with the exchange rate results indicates that sterling exchange rate fluctuations have not constituted an important channel through which exogenous shocks have been translated into price fluctuations.

Third, the estimated dynamic responses of the variables to each of the three shocks are highly theory consistent. Fourth, the periods in which the SVARs indicate that particular shocks were most important correspond to observable relative productivity, domestic demand and monetary aggregate developments. Both these findings indicate that the SVAR representations of the data have a high economic content.

The remainder of the paper is organised as follows. Section 1 describes the rational expectations open economy stochastic exchange rate model that underlies the empirics and outlines the structural VAR approach. Section 2 presents the results, the implications of which are discussed in Section 3. Section 4 examines how the SVARs explain sub-period exchange rate and price movements, while the final section concludes.

These restrictions exactly identify the model. They cannot, therefore, be directly tested. We determine the economic content of the SVARs by implementing several informal "overidentifying" tests commonly used in the literature.

This similarity of the real and nominal exchange rate results reflects the fact that these two series closely tracked each other.
1. Method

1.1 Structural exchange rate model

The Obstfeld (1985) stochastic two country version of the Dornbusch (op cit) model underlies our empirics. This serves the two usual purposes in the SVAR literature. First, it provides the economic underpinnings of the long-run identifying restrictions imposed. Second, it supplies the theoretical priors to compare the estimated dynamic responses against – the important "overidentifying" test of SVARs. But, importantly, the empirical strategy is not tied to this particular model. A number of mainstream models display the same long-run conditions and predicted short run responses.

The Obstfeld (op cit) model is a relative one, defined in terms of home country (UK) variables minus foreign country ones. This formulation means that only the effects of asymmetric shocks are considered. Four equations make up the model. First, an open economy goods market relationship, where IS shocks are introduced. Second, a relative money market equilibrium condition, where LM shocks are introduced. Third, a price setting rule. Finally, a nominal UIP condition. Appendix A outlines the model in more detail.

Table 1 summarises the long-run and short-run model solutions, which are derived in Appendix A. The long-run solution occurs when prices become perfectly flexible and rational expectations hold. Relative output is then determined entirely by supply shocks – a vertical long-run aggregate supply curve. The zero long-run effects of IS and LM shocks on the level of relative output constitute two of the three restrictions required to achieve identification. These restrictions allow us to distinguish AS shocks from the other two shocks. The final identifying restriction is that LM shocks have no long-run effect on the real exchange rate, which allows us to distinguish between IS and LM shocks.

<table>
<thead>
<tr>
<th>Shock/variable</th>
<th>Relative output</th>
<th>Real exchange rate</th>
<th>Relative prices</th>
<th>Nominal exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR SR</td>
<td>+ (&lt; LR)</td>
<td>+ (&lt; LR)</td>
<td>− (&lt; LR)</td>
<td>? (+)</td>
</tr>
<tr>
<td>IS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR SR</td>
<td>Zero TempComp</td>
<td>− PermComp</td>
<td>+ TempComp</td>
<td>? (−)</td>
</tr>
<tr>
<td>LM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR SR</td>
<td>Zero</td>
<td>Zero</td>
<td>+ equal</td>
<td>+ equal</td>
</tr>
</tbody>
</table>
| Key: +(−) = increase (decrease); ? = ambiguous response; >(<) = greater (less) than; equal = response equals size of shock; TempComp (PermComp) = only temporary (permanent) component of shock; exchange rate increases depreciations.

Both AS and IS shocks are capable of affecting the long-run real exchange rate. Positive AS shocks unambiguously produce long-run real depreciations; an improvement in competitiveness is required to stimulate demand for the extra output generated by such shocks. Conversely, positive IS shocks produce real appreciations. But the real exchange rate movements will only be permanent to the extent that the IS shock is permanent. This is because the foreign exchange market discounts the reversal of the temporary component of IS shocks (Appendix A shows this algebraically).
The long-run relative price/output responses summarised in Table 1 are mainly intuitive. The exception is that only the temporary component of IS shocks affect long-run relative prices. This is because the permanent component of IS shocks moves world real/nominal interest rates, leaving relative interest rates unchanged. And relative output, the other argument in the LM curve, is, by assumption, unchanged. For the LM relationship to continue holding requires relative prices, and hence real money balances, to be invariant to the shock. Positive permanent IS shocks thus induce home and foreign prices to increase by the same proportion in the long-run, leaving relative prices unchanged.

Only LM shocks have unambiguous long-run nominal exchange rate responses — positive shocks producing depreciations that equal the size of the shock. The indeterminate long-run IS and AS shock effects reflect the real exchange rate and price effects working in opposite directions. But intuitively we expect both nominal exchange rates and relative prices to move to facilitate a required real exchange rate movement. Positive (negative) AS (IS) shocks will then produce nominal depreciations.

In the short-run, when prices are sticky, all shocks potentially affect all endogenous variables. Appendix A proves the intuitive result that the short run price effect of each of the shocks is less than the long run equivalents. We also confirm the usual result that positive LM shocks depreciate the real exchange rate when prices are sticky. And price stickiness means that real exchange rates undershoot their long-run responses following AS and IS shocks. The nominal exchange rate may either undershoot or overshoot in the short-run, depending upon parameters such as the responsiveness of relative output to the real exchange rate and interest rate differentials. Relative output is demand determined in the short-run, with positive LM shocks and the temporary component of positive IS shocks raising relative output. Finally, price stickiness reduces the output effect of AS shocks.

### 1.2 Structural VAR overview

The Blanchard and Quah (op cit) SVAR approach enables us to transform a VAR into its structural moving average representation. The impact of three shocks on the joint long-run behaviour of the three endogenous variables — relative output, the real exchange rate and relative prices — are exploited to achieve identification. The method has several benefits. First, the short-run dynamics, about which there is considerably less agreement in the literature, are left completely unconstrained (data determined). Second, the method side-steps the well-known problems with VARs: the need to impose contemporaneous restrictions, ordering problems and the Cooley and LeRoy (1985) critiques. Finally, the forecast error variance decompositions, impulse responses, historical decompositions and shock series generated can be given structural interpretations. To take account of the fact that the structural impulse responses/variance decompositions are based upon estimated VAR coefficients we use Monte Carlo techniques to put error bands on the point estimates. Appendix B provides a full description of the mechanics of identification.

---

8 Obtained by combining the long-run relative price and real exchange rate expressions.

9 Appendix A details the factors that determine the extent of this nominal exchange rate response.

10 The nominal depreciation following negative IS shocks is intuitively due to the fall in domestic interest rates induced by an inward shift in the IS curve provoking a capital outflow.

11 Keating (1992) is a good introduction to the SVAR literature. SVARs, however, are not without their detractors. Faust and Leeper (1994) outline several potential problems with SVARs identified with purely long-run restrictions.
2. Results

2.1 Estimation

We implement the above procedures by estimating trivariate SVARs of relative output ($y_t$), the real exchange rate ($q_t$) and relative prices ($p_t$). Each of the variables is defined in terms of home (UK) variable minus the foreign equivalent. For example, $p_t = p^h_t - p^f_t$, where superscript $h$ ($f$) denotes a home (foreign) variable. Since we take natural logarithms of all the individual country variables, the relative measures constitute ratios. The real exchange rate ($q_t$) is constructed by subtracting relative prices from the nominal exchange rate ($s_t$). As $s_t$ is defined as the number of units of domestic currency required to purchase a unit of foreign currency, rises in $q_t$ ($s_t$) constitute real (nominal) depreciations. The model was estimated on quarterly data between 1973 Q1 and 1994 Q4. Real GDP is the output measure used, while consumer price indices constitute the relative price measure and are used in the construction of the real exchange rates. The nominal exchange rate component of the real exchange rates are quarterly average spot rates.

The ADF tests (Tables 2 and 3) indicate that the variables are all I(1). Theory does not suggest that we can expect the three variables to be cointegrated (the matrix determining the long-run effects of the shocks on the endogenous variables is lower triangular). And the Johansen tests (Table 4) support this prediction. The null of no cointegration is only rejected in the UK-Japanese case. But we concluded that no meaningful long-run relationship was present even here because: (i) the rejection of the null only occurred at the 90% confidence level; (ii) the resulting residuals appeared non-stationary; and (iii) the long-run coefficients had no economic content. The first stage VARs were, therefore, estimated in first differences.

<table>
<thead>
<tr>
<th>Country/variable</th>
<th>Relative output</th>
<th>Real exchange rate</th>
<th>Relative prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom-United States</td>
<td>-2.2</td>
<td>-2.6</td>
<td>-3.0</td>
</tr>
<tr>
<td>United Kingdom-Japan</td>
<td>-1.9</td>
<td>-2.5</td>
<td>-1.6</td>
</tr>
<tr>
<td>United Kingdom-Germany</td>
<td>-2.1</td>
<td>-2.1</td>
<td>-1.6</td>
</tr>
<tr>
<td>United Kingdom-France</td>
<td>-1.9</td>
<td>-1.8</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

* ADF(4) with trend test (95% critical values = -3.5).

The first stage VAR lag lengths were selected using a combination of sequential likelihood ratio tests and the Akaike information criteria. We attached higher weight to the former because of the DeSerres and Guay's (1995) finding that Akaike (or Schwartz) criteria tend to select an insufficient number of lags. Our approach eliminates the possibility of too short a lag length biasing the estimates of the structural parameters (DeSerres and Guay (op cit)). The tests indicated that 3 lags were appropriate in the UK-US system, 1 lag in the UK-Japanese and UK-German systems and 4 lags in the UK-French systems. Running the systems with higher number of lags (up to 8) produced only minor changes in results and meant that shorter periods of data could be examined.

12 Except interest rates.

13 The approach of King, Plosser, Stock and Watson (1989) would need to be applied if cointegration were found.
Table 3

ADF tests on first differences of variables*

<table>
<thead>
<tr>
<th>Country/variable</th>
<th>Relative output</th>
<th>Real exchange rate</th>
<th>Relative prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom-United States</td>
<td>-5.2</td>
<td>-4.8</td>
<td>-3.9</td>
</tr>
<tr>
<td>United Kingdom-Japan</td>
<td>-3.1</td>
<td>-4.6</td>
<td>-4.4</td>
</tr>
<tr>
<td>United Kingdom-Germany</td>
<td>-4.2</td>
<td>-3.2</td>
<td>-4.1</td>
</tr>
<tr>
<td>United Kingdom-France</td>
<td>-4.2</td>
<td>-4.8</td>
<td>-3.9</td>
</tr>
</tbody>
</table>

* ADF(4) without trend test (95% critical values = -2.9).

Table 4

Johansen cointegration tests

<table>
<thead>
<tr>
<th>Country/variable</th>
<th>Eigenvalue test</th>
<th>Trace test</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom-United States</td>
<td>7.51</td>
<td>14.78</td>
</tr>
<tr>
<td>United Kingdom-Japan</td>
<td>19.80</td>
<td>29.22</td>
</tr>
<tr>
<td>United Kingdom-Germany</td>
<td>13.20</td>
<td>24.54</td>
</tr>
<tr>
<td>United Kingdom-France</td>
<td>12.63</td>
<td>22.47</td>
</tr>
</tbody>
</table>

1 Four lags in VAR.
2 95% critical value = 21.07.
3 95% critical value = 31.52.

We investigated possible VAR instability by undertaking several variants of recursive Chow tests. Policy regime changes in both the UK and abroad constitute one potential source of instability. The one step ahead tests indicate that outliers are present, especially in the exchange rate equations; Figure 1 presents the United Kingdom-United States system plots. But the n-step tests indicate that these outliers did not translate into regime shifts — see Figure 2 for the UK-US system — which we are more concerned about. Moreover, these benign results — which are not unusual in the literature — were not a function of poorly specified VARs.

14 Interestingly, the major £/$ outlier occurs around 1985, tying in with the Evans (1986) finding that the £/$ was subject to a speculative bubble between 1981-84 and the general perception of dollar misalignment around this period.

15 For example, Evans and Lothian (op cit) and Sarantis (1993) uncovered no evidence of instability in their dollar and sterling based analyses over periods similar to our own.

16 Full VAR diagnostics are available on request from the authors. Importantly, serial correlation was never a problem.
Figure 1
1-step ahead recursive Chow test (outlier test): United Kingdom-United States system

1-step recursive Chow test (regime shift test): United Kingdom-United States system

Figure 2
N-step recursive Chow test (regime shift test): United Kingdom-United States system
2.2 Forecast error variance decompositions (FEVDs)

FEVDs tell us which shocks were the primary sources of movement in the endogenous variables over the sample period. In each case we calculate the FEVDs on the levels of the endogenous variables, as these correspond most closely to the questions we wish to address. The results presented in Tables 5-8 detail, in the top row for every horizon, the point estimate of the proportion of the variation in each variable attributable to each shock. The two standard errors associated with these point estimates appear in the lower row in smaller font. This allows us to determine whether the contribution of a particular shock is significantly different from zero at a 95% confidence level.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>£/$</th>
<th></th>
<th></th>
<th>£/Yen</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
</tr>
<tr>
<td>1</td>
<td>0.021</td>
<td>0.966</td>
<td>0.014</td>
<td>0.222</td>
<td>0.762</td>
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</tr>
<tr>
<td>2</td>
<td>0.012</td>
<td>0.143</td>
<td>0.019</td>
<td>0.139</td>
<td>0.145</td>
<td>0.058</td>
</tr>
<tr>
<td>4</td>
<td>0.035</td>
<td>0.959</td>
<td>0.005</td>
<td>0.181</td>
<td>0.816</td>
<td>0.003</td>
</tr>
<tr>
<td>8</td>
<td>0.075</td>
<td>0.923</td>
<td>0.002</td>
<td>0.172</td>
<td>0.827</td>
<td>0.001</td>
</tr>
<tr>
<td>12</td>
<td>0.122</td>
<td>0.129</td>
<td>0.039</td>
<td>0.160</td>
<td>0.161</td>
<td>0.006</td>
</tr>
<tr>
<td>16</td>
<td>0.088</td>
<td>0.911</td>
<td>0.001</td>
<td>0.168</td>
<td>0.832</td>
<td>0.001</td>
</tr>
<tr>
<td>20</td>
<td>0.091</td>
<td>0.908</td>
<td>0.001</td>
<td>0.167</td>
<td>0.833</td>
<td>0.000</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Horizon</th>
<th>£/DM</th>
<th></th>
<th></th>
<th>£/FFr</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
</tr>
<tr>
<td>1</td>
<td>0.093</td>
<td>0.720</td>
<td>0.187</td>
<td>0.217</td>
<td>0.782</td>
<td>0.001</td>
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<tr>
<td>2</td>
<td>0.112</td>
<td>0.749</td>
<td>0.139</td>
<td>0.200</td>
<td>0.799</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.136</td>
<td>0.782</td>
<td>0.082</td>
<td>0.244</td>
<td>0.743</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.157</td>
<td>0.803</td>
<td>0.040</td>
<td>0.249</td>
<td>0.743</td>
<td>0.007</td>
</tr>
<tr>
<td>12</td>
<td>0.167</td>
<td>0.809</td>
<td>0.025</td>
<td>0.252</td>
<td>0.742</td>
<td>0.005</td>
</tr>
<tr>
<td>16</td>
<td>0.171</td>
<td>0.811</td>
<td>0.018</td>
<td>0.252</td>
<td>0.744</td>
<td>0.004</td>
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<tr>
<td>20</td>
<td>0.174</td>
<td>0.812</td>
<td>0.014</td>
<td>0.252</td>
<td>0.745</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Key: Top rows detail fraction of variation in variable attributable to each shock. Bottom rows give empirical two standard errors, computed by Monte Carlo simulation.

---

17 FEVDs on first differences produced similar results.

18 Calculated using 100 draws of Monte Carlo simulations. For computational simplicity, these error bands are portrayed as symmetric. Runkle (1987) and Blanchard and Quah (op cit) illustrate that this is not necessarily the case when bootstrapping methods are used.
2.2.1 Real exchange rates

Table 5 presents the strong result that IS shocks were the main source of movements in each of the four sterling real exchange rates considered. IS shocks were most important in determining real £/$ movements, where they accounted for over 90% of movements at all horizons. But they also accounted for at least 75% of the fluctuations in the three other rates, with their importance often rising at longer horizons. AS shocks were usually the second most important source of real sterling movements. Their effect was most pronounced, and statistically significant, in the £/Yen and £/FFr cases, where they accounted for around 20% of movements at most horizons.

LM shocks were usually unimportant sources of real sterling fluctuations at all horizons. The only exception is the £/DM rate. But the effect is limited even here – a maximum of 19% – and is only apparent at short horizons. Though an identifying restriction underlies the unimportance of LM shocks at long-horizons, their extremely limited role at short horizons is entirely data generated.

Clarida and Gali (op cit) similarly concluded that LM shocks were unimportant determinants of real $/£ fluctuations. But they found that they played larger roles in real $/DM and $/Yen movements. This might initially suggest that different factors underlie sterling and dollar movements. But there are several reasons for not overplaying these differences. First, movements in both currencies primarily reflect IS shocks. Second, considering a broader range of bilateral rates might blur the above distinction. Indeed, it is noticeable that, on our dataset, LM shocks played virtually no role in real $/FFr fluctuations. And other sterling exchange rates might replicate the higher, though still small, importance of LM shocks in real £/DM fluctuations.

The Rogers (1995) application of the Lastrepes (1992) framework to the $/£ rate also produced results consistent with those presented in this paper. Real (permanent) shocks were found to be the main source of real and nominal $/£ movements. And these results ties in with those of Lastrepes (op cit) on five other dollar rates. But Rogers (op cit) observed that the simplistic model may have been driving these results: nominal (LM) shocks were attributed a higher role in real $/£ fluctuations in his more structured (trivariate) model. In particular, nominal shocks constituted the most important source of real $/£ movements at short horizons (accounting for around 50% of the fluctuations). Evans and Lothian (1993) also concluded that temporary (nominal) disturbances played a significant role in sub-sample £/$ movements. But this role was usually small (a maximum of around 15%). A useful cross-check of the robustness of our results, which we postpone for future research, is to apply these alternative frameworks to the sterling exchange rates analysed in this paper.

2.2.2 Relative prices

UK relative price movements were mainly due to LM shocks (Table 6). The role of LM shocks was most pronounced in UK-US prices, where they accounted for 80% of movements at the shortest horizon and 97% inside a year. But they also accounted for approximately 70% of the variation of UK-Japanese and UK-German prices, with comparatively little variation across horizons. Finally, LM shocks were the second most important determinants of UK-French price movements at every horizon, accounting for up to 44% of the fluctuations.

19 Clarida and Gali found that LM shocks accounted for up to 36% (53%) of real $/Yen ($/DM) movements. The point estimates we obtain on our (longer) dataset are lower, but not significantly different.

20 A maximum of 0.6%.

21 Lastrepes (op cit) excluded the $/£ rates from his dollar-centred work because of evidence of them being I(0). On our longer dataset this is not a problem.
Table 6
Variance decomposition of relative prices

<table>
<thead>
<tr>
<th>Horizon</th>
<th>United Kingdom-United States</th>
<th>United Kingdom-Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>IS</td>
</tr>
<tr>
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<td>0.149</td>
</tr>
<tr>
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<tr>
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<td>0.002</td>
</tr>
<tr>
<td></td>
<td>0.133</td>
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<tr>
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<td>0.002</td>
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<td>0.135</td>
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</table>

<table>
<thead>
<tr>
<th>Horizon</th>
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<th>United Kingdom-France</th>
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</thead>
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<tr>
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<td>8</td>
<td>0.196</td>
<td>0.156</td>
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<tr>
<td></td>
<td>0.174</td>
<td>0.164</td>
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<tr>
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<td>0.155</td>
</tr>
<tr>
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<td>0.173</td>
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<tr>
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<td>0.155</td>
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<td>0.154</td>
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<tr>
<td></td>
<td>0.172</td>
<td>0.164</td>
</tr>
</tbody>
</table>

Key: Top rows detail fraction of variation in variable attributable to each shock. Bottom rows give empirical two standard errors, computed by Monte Carlo simulation.

AS shocks also played large, and statistically significant, roles. They were the main source of UK-French price movements (up to 66% at short horizons) and the second most important source of fluctuations in the remaining series. IS shocks were uniformly the least important source of relative price fluctuations. Their role was most pronounced at long horizons, where they accounted for at least 10% of the observed movements (except in the UK-US case).

2.2.3 Nominal exchange rates

LM shocks played a larger role in sterling nominal exchange rate movements (Table 7) than in the real exchange rate equivalents. But this role was still small. The maximum effect was 35% (£/DM), but was more frequently under 15%. This larger role obviously reflects the dominant role that LM shocks played in relative price movements. But their effect remains extremely limited because the nominal exchange rate paths largely mirrored their real rate equivalents. This close tracking means that IS shocks again constituted the main source of nominal rates movements. This dominance was most pronounced in the £/$ and £/FFr rates. AS shocks also often underlay some of the nominal rate movements, especially of £/DM and £/Yen rates.
### Table 7

Variance decomposition of nominal exchange rate

<table>
<thead>
<tr>
<th>Horizon</th>
<th>£/$</th>
<th>£/Yen</th>
<th>£/DM</th>
<th>£/FFr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
<td>AS</td>
</tr>
<tr>
<td>1</td>
<td>0.006</td>
<td>0.911</td>
<td>0.083</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>0.088</td>
<td>0.176</td>
<td>0.169</td>
<td>0.108</td>
</tr>
<tr>
<td>2</td>
<td>0.009</td>
<td>0.903</td>
<td>0.089</td>
<td>0.121</td>
</tr>
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<td>0.169</td>
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<tr>
<td>Horizon</td>
<td>£/DM</td>
<td>£/FFr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
<td>AS</td>
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<tr>
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<td>0.124</td>
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<tr>
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<td>0.645</td>
<td>0.303</td>
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<tr>
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<td>0.155</td>
<td>0.155</td>
<td>0.128</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Key: Top rows detail fraction of variation in variable attributable to each shock. Bottom rows give empirical two standard errors, computed by Monte Carlo simulation.

#### 2.2.4 Relative output

UK relative output fluctuations were primarily attributable to AS shocks (Table 8). This ties in with the Holland and Scott (1995) results. The first and second identifying restrictions (see Section 1) obviously underlie this finding at long horizons. But it is again data generated at shorter horizons. AS shocks accounted for over 80% of movements in most of the output series after two quarters. The only exception was the large (60%) role that LM shocks played in short horizon UK-French movements.
Table 8
Variance decomposition of relative output

<table>
<thead>
<tr>
<th>Horizon</th>
<th>United Kingdom-United States</th>
<th>United Kingdom-Japan</th>
<th>United Kingdom-Germany</th>
<th>United Kingdom-France</th>
</tr>
</thead>
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<tr>
<td></td>
<td>AS</td>
<td>IS</td>
<td>LM</td>
<td>AS</td>
</tr>
<tr>
<td>1</td>
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<td>0.057</td>
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<tr>
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<td>0.969</td>
<td>0.014</td>
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<td>0.071</td>
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<td>0.005</td>
</tr>
</tbody>
</table>

Key: Top rows detail fraction of variation in variable attributable to each shock. Bottom rows give empirical two standard errors, computed by Monte Carlo simulation.

2.3 Impulse responses

Figures 3-6 present the estimated dynamic responses of the variables to each of the structural shocks. The dark line in the figures represent the point estimates of the response of the levels of each of the variables to a one standard deviation perturbation to each the three shocks. The lighter lines on either side of these point estimates represent the two standard deviation error bands. Like Clarida and Gali (op cit) we find that the signs of these responses are highly consistent with our theoretical priors. Moreover, the relative magnitudes of the responses are also sensible: exchange rates respond by more than relative prices which in turn respond by more then relative output. Our results, therefore, pass the important SVAR "overidentifying" test. This means that we can be confident of the economic content of the FEVDs.
Figure 3
United Kingdom-United States responses

Figure 4
United Kingdom-Japanese responses
Figure 5
United Kingdom-German responses

Figure 6
United Kingdom-French responses
2.3.1 Responses to AS shocks

AS shocks produce dynamic responses which are highly theory consistent. Positive (benign) AS shocks usually generate falls in relative prices, real exchange rate depreciations and rises in relative output. The only counterintuitive response is the real £/$ appreciation. Interestingly, Clarida and Gali (op cit) also uncovered exactly this "perverse" real $/£ response.

Relative prices respond sluggishly to AS shocks, uniformly taking at least 8 quarters to approach their new long-run equilibria. This price stickiness is most apparent in UK-French prices, which take 12 quarters to "level off". Relative prices usually fall by between 1.1%-1.6% in the long-run following positive AS shocks. The exception is the much smaller UK-US response. The long-run real exchange rate responses are, at between 1.9% and 3.5%, considerably larger and more dispersed. Though real exchange rates adjust quicker than relative prices, this adjustment is again comparatively slow (full adjustment taking up to 7 quarters). These responses mean that nominal exchange rates, as expected, deprecate (slowly) following positive AS shocks. The long-run relative output responses are, at between 1.0% and 1.5%, fairly uniform.

2.3.2 Responses to IS shocks

The responses to IS shocks require a little more interpretation. IS shocks usually produce, across countries and variables, counterintuitive responses – falls in relative prices and output and real exchange rate depreciations. But these results are actually benign. This is because, as Faust and Leeper (1994) note, the SVAR method does not tie down the sign of each of the elements on the principal diagonal of the structural impulse response matrices. This indeterminacy arises from having to solve what is essentially a quadratic expression – which can produce either a positive or a negative solution.\(^2\) This means that we can only conduct the "overidentifying" in terms of the consistency of the relative responses. The uniformly "incorrectly" signed responses indicate that negative IS shocks have been identified. The responses to these negative shocks are, therefore, "correctly" signed. Positive IS shocks results which are easier to interpret, can be obtained by simply multiplying the associated responses and IS shock series\(^2\) by -1.

This is an important point to appreciate because IS shocks have been found to underlie the majority of sterling exchange rate movements. But there is also a corollary. Because the "incorrectly" signed £/FFr response is not matched by counterintuitive output and price responses, we have less grounds for suspecting that negative UK-French IS shocks have been identified. Our finding that IS shocks underlay most £/FFr movements may, therefore, be on shaky ground.

Relative prices again rise sluggishly following positive IS shocks, taking up to nine quarters to approach their long-run responses. Interestingly European (UK-German and UK-French) prices appear stickiest. The UK-US responses again constitute the main outlier, their long-run movements lying considerably below the 0.8% to 1.3% range of the remaining relative prices.

The real exchange rate appreciations following positive IS shocks are again large and quite dispersed – the long-run responses lying between 3.3% (£/FFr) and 7.8% (£/Yen). The adjustment to the new long-run is usually smooth and comparatively protracted; it takes up to 6 quarters for steady state to be reattained. Interestingly, there is some evidence of real £/FFr overshooting. But this probably reflects the comparative volatility of this response. As expected, positive IS shocks also produce nominal appreciations. The increases in relative output following positive IS shocks are uniformly small, peaking at 0.4%.

\(^{22}\) In particular the signs of the each of the principal diagonals of \(C_0\) solved in equation (B4) are indeterminate. If \(C_0^a\) satisfies (B4) then so will \(C_0^b = C_0^a H\), where \(H\) is a diagonal matrix with either 1 or -1 on the diagonal.

\(^{23}\) The fact that that transformed IS structural shock series (not shown) have considerably more intuition adds further weight to the argument that negative IS shocks have been uncovered.
2.3.3 Responses to LM shocks

LM shocks generate responses that uniformly accord with our theoretical priors. A positive LM shock produces a temporary rise in relative output, a temporary real exchange rate depreciation\(^{24}\) and relative price increases. Finally, as expected, such shocks produce nominal depreciations.

Relative prices adjust slowly to LM shocks, typically taking around 10 quarters to adjust more or less fully.\(^{25}\) The long-run response are, at between 1.7% and 2.9%, reasonably consistent across the country pairs. The temporary real exchange rate depreciations are, except for the £/DM rate, relatively short-lived – reaching their zero long-run effects within 6 quarters.\(^{26}\) The short lived real exchange rate responses mean that the nominal exchange rate responses largely mirror the relative price responses at all but short horizons.

Our estimates of the speed of adjustment of nominal sterling exchange rates to LM shocks differ from the existing dollar based findings. Clarida and Gali (op cit) and Eichenbaum and Evans (1993) found that dollar rates take around two years to respond fully to LM shocks and monetary policy shocks respectively. While we uncover a similar lag in the £/$ responses, this is not a general feature of our results. In particular, the £/Yen rate adjusts quickly and the £/DM and £/FFr rates overshoot slightly in the short-run. This suggests that the dollar based results may not hold for other currencies. Clearly further work is required on this issue.

3. Implications

So what are the price implications of an exchange rate movement? Our results have confirmed the theoretical proposition that what matters is the type of shock underlying the exchange rate/price movements. In particular, they indicate that the common perception of exchange rate depreciations producing potential inflationary pressures – through their impact on import prices etc. – is usually misplaced. This is because this malign scenario only holds if LM shocks underlie the exchange rate/price movements. In contrast, the AS and IS shocks required to produce depreciations bring forth relative price falls. Moreover, these are precisely the shocks that our results indicate have been the major sources of sterling exchange rate movements.\(^{27}\) And the major source of UK relative price movements, LM shocks, have been unimportant sources of sterling exchange rate fluctuations. This means that, according to our results, sterling exchange rate movements have not been a major channel through which shocks have affected UK relative prices.

The impulse responses allow a quantification of these arguments. We consider the relative price implications of a 10% nominal\(^{28}\) sterling depreciation due entirely to each of the shocks in turn. To avoid any perverse short-run dynamic effects and side-step problems with long-run restrictions being imposed, we consider the effect of the depreciation occurring 3 quarters after the

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24 These temporary responses reflect the second and third identifying restrictions respectively (see Section 1).

25 UK-US prices appear stickiest, taking over three years to reach any kind of plateau.

26 Again these short-run responses are entirely data generated. They do not, in particular, reflect the long-run horizon or the VAR lag lengths employed. For example, the long-lived £/DM response arises from a VAR with only one lag.

27 The short-term role of nominal shocks in £/DM movements is the minor exception.

28 The fact that the nominal responses in our model are derived from the real exchange rate and relative price responses is potentially problematic. This is especially the case when LM shocks are considered – the fact that real exchange rate responses often quickly asymptote to zero means that the nominal exchange rate responses mirror the relative price responses at all but short horizons. A framework that directly models the nominal exchange rate would side-step this problem.
shock hits the economy. We also exclude incorrectly signed responses and only consider the average of the point estimate responses. In the unlikely case that (positive) $LM$ shocks underlie the depreciation, it is accompanied by an initial 7.5% rise in relative prices, increasing to 10% after a further 3 quarters. In the more likely case of (positive) $AS$ shocks producing the depreciation, relative prices fall by 9.2% on impact and by 12% two years after the shock. In the most likely case of $IS$ shocks causing the depreciation, the accompanying relative price fall is much smaller – around 1% on impact, rising to 1.2% in the long-run.

What are the implications for policymakers? Is a monetary policy response called for when exchange rates move? The most commonly advanced rationale for such responses is that the exchange rate changes alter inflationary pressures, which should be offset. This is particularly important in the United Kingdom, as it could imply breaching the Government's inflation target. The aim of any monetary policy response should then be to prevent the exchange rate movement being built into wage setting and pricing behaviour – eliminating the "second round" effects. It should not aim to offset the direct ("first round") effects, which shift the price level and so only affect recorded inflation for a limited period.

In deciding the appropriate direction and magnitude of any policy response the authorities should, of course, recognise that the direction and magnitude of the price changes associated with an exchange rate movement depend on its source. Unfortunately, identifying the type of shock that generated a particular exchange rate as it happens is very difficult. This could lead to incorrect policy responses. Our finding that past sterling depreciations have largely been associated with small falls in relative prices suggests that depreciations should, if anything, induce small official interest rate reductions. But we have also shown that past sterling fluctuations have not constituted a major channel through which inflationary pressures are transmitted. This suggests that the optimal policy response is to leave interest rates unchanged.

But these conclusions are necessarily provisional because: (a) the above are average results, and will not necessarily apply to every exchange rate movement; (b) they are based upon past relationships that will not necessarily hold in the future – a large potential Lucas critique; and (c) endogenous monetary policy responses may already be included in the results. Unfortunately, there are ambiguities about where monetary policy shows up in the model. Clarida and Gali (op cit) allocated monetary policy shocks to LM shocks. This may be motivated by the textbook descriptions of monetary policy in terms of monetary aggregate shifts and the traditional description of monetary policy as a nominal perturbation. But there are several arguments for including monetary policy in IS shocks. First, monetary policy actually operates through interest rates, which then affect domestic demand. Second, Eichenbaum and Evans (1993) demonstrated that monetary policy shocks have long-run real exchange rate effects. Both these are characteristics of IS shocks. Importantly, IS shocks remain unambiguously real phenomena even if monetary policy is included in them; in our framework monetary policy only affects domestic demand if it moves real interest rate differentials. These ambiguities reflect the fact that our framework is not intended to identify monetary policy shocks, which the literature as a whole has difficulty doing.

The dominance of real shocks as determinants of sterling exchange rate movements makes our results most consistent with the Stockman (op cit) equilibrium exchange rate theory. However, we have also uncovered evidence of substantial price stickiness. Yet this has not translated into LM shocks constituting major sources of sterling real exchange rate movements – the

29 We thus omit the UK-US from the AS shock analysis and UK-France from the IS shock analysis.  
30 Our empirical framework, however, is incapable of separating out the first round and second round effects.  
31 Twelve months if the price level shifts immediately.  
32 See equation 1 of Appendix A. Sluggish price expectations are obviously required for this to hold.  
disequilibrium view. This suggests that either LM shocks were less prevalent than real shocks over the floating exchange rate period or that they had a lower variance.

4. Sub-period analysis

How do the SVARs rationalise the sterling real exchange rate and UK relative price movements that occurred (Figure 7) over the sample period? We use historical decompositions (HDs) to plot separately the historic paths that the endogenous variables would have followed in response to each of the structural shocks. This allows us to determine the importance of each of the shocks in exchange rate and price developments over historic episodes. We simply examine how closely the endogenous variable movements due to each of the shocks (the light lines in Figures 8 to 15)

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34 This contrasts with the full sample FEVD results.
Figure 8
Historical decomposition £/$ exchange rate

Figure 9
Historical decomposition £/Yen real exchange rate
Figure 10
Historical decomposition £/DM exchange rate

Figure 11
Historical decomposition £/FFr real exchange rate
Figure 12
Historical decomposition of UK-US prices

Figure 13
Historical decomposition of UK-Japanese prices
Figure 14
Historical decomposition of UK-German prices

Figure 15
Historical decomposition of UK-French prices
correspond to the actual movements (the dark lines).\textsuperscript{35,36} But can these predictions be linked to observed economic developments? This is an important cross-check of the economic content of the results. We use relative domestic demand as the equivalent to IS shocks (Figure 16), relative productivity developments\textsuperscript{37} as the AS shock measure (Figure 17) and relative broad\textsuperscript{38} money growth rates for the LM shocks (Figure 18). We find a high correspondence between these observable developments and the SVARs' predictions in our examination of 1990s exchange rate and price movements. Moreover, this correspondence is also apparent throughout much of the earlier period (not reported to conserve space).

Figure 16 indicates that relative domestic demand shifted away from the UK in the 1990s. The HDs indicate that these negative IS shocks played a large role in the sharp post-1992 real sterling depreciations, especially of the £/$ rate. This ties in with the large role that IS shocks played in sterling movements over whole sample (FEVD results).

The improvement in UK relative productivity, apparent in the 1980s, accelerated in the 1990s (Figure 17). The origin of these positive AS shocks varies between country pairs. The negative short term effects of German reunification and the bursting of the Japanese asset price bubble are likely candidates in the German and Japanese systems. The HDs indicate that these positive AS shocks also played a large role in the post-1992 real sterling depreciations. And the neglible role of

\textsuperscript{35} The actual path (dark line) with which we compare the decompositions is that which was forecast by the structural VAR model on the basis of a few initial periods of shocks – the "base projection". Endogenous variable movements in excess of this base projection constitute the "news" occurring after the few initial periods. This news must obviously be due to realisations of the three structural shocks after the few initial periods of shocks.

\textsuperscript{36} Evans and Lothian (op cit) offer the alternative of testing the significance of the constructed path as a determinant of the actual movements over sub-periods. De Arcangelis (1995) implements a similar procedure.

\textsuperscript{37} As measured by manufacturing output/industrial production per head of employment in manufacturing.

\textsuperscript{38} Using narrow money aggregates produced similar results.
Figure 17
UK – overseas relative productivity developments*

* UK minus overseas manufacturing output or industrial production per head of employment in manufacturing. Increases (decreases) represent positive (negative) relative supply shocks.

Figure 18
UK – overseas relative broad money¹ growth² developments

¹ Measures used: UK M4, US M3, Japanese M2 + CDs, German M3 and French M3.
² UK M4 four-quarter growth rate minus overseas equivalent. Increases (decreases) represent positive (negative) nominal shocks.
AS shocks in the real £/$ depreciation tie in with UK-US productivity differentials being virtually unchanged (around zero) over this period. According to our results, these productivity improvements also played a large role in the flattening of UK relative prices in the 1990s. And their effect again has the intuitive appeal of being least apparent in UK-US price movements.

The September 1992 suspension of sterling's ERM membership might be expected, according to the Mussa (1986) analysis, to constitute a nominal (LM) shock. But the HDs provide no evidence of LM shocks playing a role in the post-1992 real sterling depreciations. This is not, however, surprising. The observed fall in UK relative monetary aggregate growth rates (Figure 18) constitutes a negative (less positive) LM shock. These, of course, produce real appreciations (slower depreciations), rather than the observed depreciation. LM shocks did, however, contribute to the observed flattening of UK relative prices. And, noticeably, relative money growth rates slowed most at the start of the 1990s – exactly when LM shocks appear to have had their largest price impact.

**Conclusion**

This paper has presented a number of strong results. The main ones are that IS shocks underlay most of the variance of sterling real and nominal exchange rates and that LM shocks were the main source of UK relative price fluctuations. The most important implications for policymakers are that: (i) sterling depreciations usually have, counter to common perceptions, had benign relative price implications; and (ii) sterling exchange rate movements per se have not constituted a major channel through which exogenous shocks have fed into UK relative prices. Both these points testify to the importance of uncovering the underlying source of exchange rate and price movements. We have also argued that the SVAR representations of the data appear to have a high economic content. We believe that our results are sufficiently interesting to merit further investigation in the Lastrapes (op cit), Rogers (op cit) and Evans and Lothian (op cit) frameworks.

The following four structural equations make up the Obstfeld (op cit) model:

\[ y^d = \eta q_t - \sigma \left( i_t - E_t(p_{t+1} - p_t) \right) \]  
(A1)

\[ m_t^i - p_t = y_t - \lambda i_t \]  
(A2)

\[ p_t = (1 - \theta) E_{t-1} p_t^* + \theta p_t^* \]  
(A3)

\[ i_t = E_t(s_{t+1} - s_t) \]  
(A4)

The open economy IS relationship (A1) states that relative output demand \((y^d)\) rises with: (i) real exchange rate \((q_t)\) increases (depreciations); (ii) narrowings of the real interest differential in favour of the home country; (iii) rises in all other exogenous changes to relative domestic absorption \((d_t)\) such as government expenditure and home/foreign goods taste shifts.

The money market equilibria condition (LM curve) (A2) specifies real relative money demand as a positive function of relative output and a negative function of nominal interest rate differentials. The price setting rule (A3) specifies prices in period \(t\) as being set as an average of the output market clearing price that was expected in \(t-1\) to prevail in period \(t\) \((E_{t-1} p_t^*)\) and the price that would actually clear the output market in period \(t\) \((p_t^*)\). The \(\theta\) parameter determines the degree of price flexibility, full flexibility holding when \(\theta = 1\). Finally (A4) represents a UIP condition linking nominal interest rate differentials to expected nominal exchange rate changes:

The shocks are introduced by specifying the following stochastic processes for the exogenous variables in equations (A1) to (A3). We assume that the AS \((z_t)\) and LM shocks \((v_t)\) follow simple random walks, being solely permanent in nature. But relative IS \((\delta_t)\) shocks have both permanent and transitory components, the latter of which is offset in the following period, that is:

\[ y_t^* = y_{t-1}^* + z_t \]  
\[ d_t = d_{t-1} + \delta_t - \gamma \delta_{t-1} \]  
\[ m_t = m_{t-1} + v_t \]  
(A5)

The long-run model solution, presented in equations (A6) to (A9), occurs when prices become perfectly flexible and rational expectations hold. Relative output is entirely determined by AS shocks - a vertical long-run supply curve. The absence of LM shocks from (A6) represents money neutrality.

\[ y_t^* = y_{t-1}^* + z_t \]  
(A6)

\[ q_t^* = \left( y_t^* - d_t \right) \left( \eta + (\eta + \sigma) \right)^{-1} \sigma \delta_t \]  
(A7)

\[ p_t^* = -y_t^* + \lambda \left( 1 + \lambda \right)^{-1} (\eta + \sigma)^{-1} \gamma \delta_t + m_t \]  
(A8)

39 Demand switches towards home goods as they become more competitive.

40 Reflecting the effect on interest sensitive aggregate demand components such as investment.

41 The \(C_{12}(1) = C_{1j}(1) = 0\) restrictions outlined in appendix B.
The long-run real exchange rate expression (A7) is obtained by substituting the stochastic processes for AS and IS shocks into the IS equation and solving for \( q_t^* \). Positive AS shocks produce long-run real depreciations (rises) – an improvement in competitiveness is required to stimulate demand for the extra output supply generated by the AS shock. Conversely, the real exchange rate appreciates (falls) following positive IS shocks. But the market’s discounting of the partial reversal of the IS shock in the following period, represented by the coefficient on the temporary component of the IS shock (\( \gamma \)), offsets this appreciation. This means that the real appreciation will only be permanent to the extent that the IS shock is permanent. The main text outlines the economics of this result. Finally, LM shocks have no long-run real exchange rate effect; we show below that the associated relative price and nominal exchange rate responses exactly offset each other.

Inverting the LM curve produces the long run price expression (A8). Positive AS shocks reduce relative prices, by shifting the (vertical) AS curve to the right. Positive LM shocks and the temporary component of positive relative IS shocks (\( \gamma \delta_t \)) both raise relative prices, the former equiproportionately, by shifting the AD curve up the vertical AS curve. The permanent component of IS shocks has no long-run effect on prices. The main text again outlines the intuition of this result.

Expression (A9) demonstrates that only LM shocks have unambiguous long-run nominal exchange rate effects. In particular, positive LM shocks produce equiproportionate nominal depreciations in the long-run. The indeterminacy of the long-run responses to IS and AS shocks is due to their real exchange rate and price effects working in opposite directions. But the main text outlines why in general we expect positive AS (IS) shocks to produce long-run nominal depreciations (appreciations).

In the short-run, when prices are sticky, all shocks potentially affect all endogenous variables. Equations (A10) to (A13) represent the short-run solution. The short-run relative price expression (A10), obtained by substituting (A8) into (A3), illustrates that higher price stickiness (decreases in \( \sigma \)) reduces the short run price effect of each of the shocks below their long-run effects.

\[
p_t = \theta p_t^* - (1 - \theta)(v_t - z_t + \alpha \gamma \delta_t) \tag{A10}
\]

\[
q_t = q_t^* + \nu(1 - \theta)(v_t - z_t + \alpha \gamma \delta_t) \tag{A11}
\]

\[
s_t = y_t^*(1 - \eta \theta)\eta^{-1} - d_t \eta^{-1} + \left[ \frac{(\eta(\eta + \sigma))^{-1} + \theta \lambda (1 + \lambda)^{-1}(\eta + \sigma)^{-1} - \nu(1 - \theta)}{(1 + \alpha \gamma \delta_t)} \right]^2 \tag{A12}
\]

\[
y_t = y_t^* + (\eta + \sigma)\nu(1 - \theta)(v_t - z_t + \alpha \gamma \delta_t) \tag{A13}
\]

The short-run real exchange rate expression (A11) is obtained by substituting (A1) and (A4) into (A2) and using (A10) to represent the difference between actual and market clearing price levels. The positive coefficient on \( v_t \) illustrates the usual result that positive LM shocks depreciate the real exchange rate when prices are sticky. The negative coefficient on \( z_t \) shows that price stickiness means that the real exchange rate will undershoot its long-run appreciation following positive AS shocks. Likewise the positive coefficient on \( \gamma \delta_t \) illustrates that the real exchange rate

---

42 The \( C_{25}(l) = 0 \) restriction of appendix B.

43 Where \( \nu = (1 + \lambda)(\lambda + \sigma + \eta)^{-1} \).
undershoots its long-run depreciation following positive IS shocks. Again the extent of this undershoot is related solely to the temporary component of the IS shock \((\gamma \delta_t)\).

Equation (A12) presents the short run nominal exchange rate expression. Clarida and Gali (op cit) show that LM shocks produce short run nominal exchange rate overshooting if \((1-\sigma-\eta) > 0\). And this condition implies short-run nominal exchange rate undershooting following AS and IS shocks.

Finally the short-run relative output expression (A13) is obtained by inserting the sticky price real exchange rate expression (A11) into (A1) and solving for \(y_t\). Relative output is demand determined in the short-run, with positive LM shocks \((v_t)\) and the temporary component of positive IS shocks \((\gamma \delta_t)\) raising relative output. The negative coefficient on \(z_t\) demonstrates that price stickiness reduces the output effect of AS shocks.

Appendix B: The Blanchard and Quah (1989) structural VAR identification method

The structural model formulates movements of endogenous variables \((y_t - \text{relative output}, \text{the real exchange rate and relative prices in our case})\) as a moving average of past structural shocks \((e_t)\):

\[
y_t = C(L)e_t \\
\text{Var}(e) = I
\]

Where \(C(L) = \begin{bmatrix} C(L)_{11} & C(L)_{12} & C(L)_{13} \\ C(L)_{21} & C(L)_{22} & C(L)_{23} \\ C(L)_{31} & C(L)_{32} & C(L)_{33} \end{bmatrix}\) and \(e_t = [z_t \delta_t v_t]\)

\(z_t\) represents the AS shocks, \(\delta_t\) the IS shocks and \(v_t\) the LM shocks. We first estimate the VAR (in first differences):

\[
A(L)y_t = \epsilon_t \\
\text{Var}(\epsilon_t) = \Omega
\]

where \(A(0) = I\) and \(\epsilon_t\) is the vector of reduced form residuals. Inverting (B2)\(^{44}\) produces the moving average representation:

\[
y_t = A(L)^{-1}\epsilon_t
\]

To move from (B3) to (B1), we first assume that a non-singular matrix \(S\) exists that links the structural shocks \((e_t)\) and the reduced-form disturbances \((\epsilon_t)\) i.e. \(\epsilon_t = Se_t\). Comparing (B3) and (B1) reveals that \(C_0 = S\). It is also clear that:

\[
C_0 C_0^\top = \Omega
\]

\(^{44}\) We assume that the MA representation is invertible. See Lippi and Reichlin (1993) and Blanchard and Quah (1993) for discussions of the consequences of non-invertibility.
To identify \( C_0 \), the key to the procedure, we need to impose \( n^2 \) restrictions are imposed (\( n \) is the number of variables in the system, three in our case). The usual assumptions of orthogonality and unit variance of the structural shocks \( (\varepsilon_t) \) provides \( n(n+1)/2 \) (six) of these restrictions. This means that (B4) is a system of \( n(n+1)/2 \) (six) equations in \( n^2 \) (nine) unknowns. Thus \( n(n-1)/2 \) (three) further restrictions are required to achieve (exact) identification. We follow Blanchard and Quah (op cit) in employing long-run theory-based restrictions zero to complete the identification.

We denote the sum of the structural MA matrices\(^{45} \) by \( C(1) \). The restriction that shock \( j \) has zero long-run effect on the level of endogenous variable \( i \) requires the restriction \( C_{ij}(1)=0 \) to be imposed. We follow the Clarida and Gali (op cit) formulation of the three required long-run restrictions. First, the shock which we label as "IS" \( (\delta_t) \) has zero long-run relative output effects: \( C_{12}(1)=0 \). Second, the shock we label as "LM" \( (\nu_t) \) also has zero long-run relative output effects: \( C_{13}(1)=0 \). Long-run relative output is thus entirely determined by the first shock, which we label as "AS" \( (\xi_t) \) – a vertical long-run aggregate supply curve. Finally, LM shocks are constrained to have zero long-run effect on the real exchange rate: \( C_{23}(1) = 0 \). These restrictions mean that, as in Blanchard and Quah's (1989) bivariate case, the \( C(1) \) matrix is lower triangular.

The procedure to obtain an estimate of \( C_0 \) parallels that outlined in Blanchard and Quah (op cit). First calculate:

\[
(I - A(1))^{-1} \Omega (I - A(1))^{-1}
\]

(B5)

It is easily shown that \( C(1) \) obeys the following equality:

\[
(I - A(1))^{-1} \Omega (I - A(1))^{-1} = C(1)C(1)'
\]

(B6)

But we can also compute the lower triangular Choleski decomposition of (B5), which we denote by \( H \). As \( C(1) \) is also lower triangular, it may clearly be equated to \( H \). Combined with the fact that \( (I-A(1))-1 = C(1)C_0 \), we obtain a \( C_0 \) as follows:

\[
C_0 = (I - A(1))H
\]

(B7)

From (B2) and (B4) it is clear that:

\[
C_j = (I - A_j)^{-1}C_0
\]

(B8)

showing that identifying \( C_0 \) allows the computation of the dynamic responses of the variables to the structural shocks. The time series of structural shocks are also easily obtained \( (\varepsilon_t = C_0^{-1}\varepsilon_t) \). And the orthogonality and unit variance of the structural shocks makes it simple to compute the structural forecast error variance decompositions. Finally, historical decompositions may also be straightforwardly obtained.

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\(^{45}\) That is \( C(1) = C_0 + C_1 + C_2 + \ldots + C_n + \ldots \).
References


Comments on paper by M.S. Astley and A. Garratt by Frank Smets (BIS)

This paper analyses the sources of sterling real and nominal exchange rate fluctuations using a structural VAR model proposed in Clarida and Gali (1994). The paper is well-motivated. Knowledge about what drives the exchange rate is important for monetary policy makers as it may determine the inflationary consequences of an exchange rate change and therefore the appropriate policy response. The main finding of the paper is that most of the nominal and real exchange rate movements are caused by real demand shocks, which have only limited effects on relative prices. The tentative policy conclusions the authors draw from this is that the optimal policy response may be to leave interest rates unchanged in the face of exchange rate changes. My comments will be in two parts. In the first part I deal with the identification problem in structural VARs and how this may have affected the results presented in the paper. The second part deals with the policy implications the authors draw from their analysis.

The goal of structural or identified VAR analysis is to interpret some of the correlations in the data in terms of a limited number of structural shocks. In the spirit of Sims (1980) only a minimum number of assumptions is used to identify these structural shocks. One problem with this approach is that in many cases there is no obvious one to one relationship between the kind of shocks one wants to examine and the identification scheme. This may be problematic because differences in identifying assumptions are known to have nontrivial effects on the impulse responses and the historical and variance decompositions. This uncertainty puts the burden of the proof with respect to the structural interpretation on the SVAR practitioners. The authors do a good job in making their case. They follow Clarida and Gali (1994) in motivating the long-run identifying assumptions on the bases of a standard open economy AS-AD model whereby only supply shocks have long-run effects on output and nominal (LM) shocks are neutral in the long run. They show that the estimated impulse responses to these shocks by and large satisfy the over-identifying implications of the model and they try to use historical decompositions to convince the reader that the estimated structural shocks also correspond to plausible actual events. Nevertheless, in what follows I will argue that the model may be misspecified in a way which could affect the main results and that therefore some caution is necessary in interpreting the results.

Let me first comment on the plausibility of the long-run identifying assumptions, i.e. a long-run vertical supply curve and long-run money neutrality, that are used to identify the three fundamental shocks (AS, IS and LM). The authors claim "the strength of the restrictions is their generality and uncontentious nature". While I would like to believe in these long-run assumptions, they are not that uncontentious. With respect to the first assumption, I know at least one SVAR study (Bayoumi and Thomas (1994)) which assumes exactly the opposite long-run identifying assumption to distinguish supply from demand shocks. As Bayoumi and Thomas wanted to compare factor market integration between the states in the United States and countries in Europe, they had a very good reason for not imposing a long run vertical supply curve. Factor market integration would tend to make the long-run supply curve upward-sloping. Fortunately for the authors, Bayoumi and Thomas find that the supply curve in European countries is almost vertical, in contrast to the supply curve in the US states, probably reflecting differences in factor market integration.

With respect to the second assumption of money neutrality, the authors themselves point to the fact that there is quite a lot of evidence that monetary policy shocks have real exchange rate effects even in the medium to long run (see Section 3). To the extent that this misspecification implies an underestimation of the exchange rate effects of nominal (LM) shocks, some of the sharp results about the near dichotomy between exchange rate changes and relative price changes may disappear. Another way of looking at these issues is from the perspective of recent research on long-run PPP. More and more papers (e.g. Oh (1996)) are able to reject the hypothesis of a unit root in the real exchange rate using more powerful econometric techniques or longer data. If indeed long-run PPP holds, then the assumptions in the model would again imply a serious misspecification and would
tend to underestimate the exchange rate effects of nominal shocks.\textsuperscript{46} Even if the structural shocks one wants to analyse satisfy the identifying assumptions, one should realise that other shocks may also do so. In that case the impulse responses one estimates are an amalgam of the effects of each of these shocks, leading to a misspecification of the impulse responses, variance decompositions, etc. This will in particular be a problem in small-scale VARs where the number of identified shocks is inevitably limited. In what follows I consider two examples which may affect the conclusions drawn from the paper.

First, consider the estimated LM shocks. These shocks are not only a mixture of money supply and money demand shocks, they may also incorporate temporary real demand shocks, which similarly have no long run effect on real output and the real exchange rate. The reason why this is important is that while a temporary real demand shock has a similar short run effect on output and prices as an increase in the money supply or a reduction in money demand, it has a different impact on the exchange rate. A positive temporary demand shock will most likely lead to an appreciation of the exchange rate, while an expansionary monetary policy shock will initially lead to a depreciation. If the so-called LM shocks are a mixture of both shocks, then this will tend to bias downward the effect on the exchange rate, which, again, may explain the main result in this paper.\textsuperscript{47}

Second, consider the IS shocks. One of the surprising results in the Astley-Garratt paper is that these shocks which are, for example, meant to capture permanent increases in government spending have very limited output and price effects. How can this be? One possible interpretation is that these shocks do not primarily reflect aggregate demand shocks but rather permanent shocks to the risk premium required on sterling investments. One could easily extend the theoretical model in the paper to incorporate such shocks and find that a permanent rise in the required risk premium would lead to a permanent real exchange rate depreciation. Moreover, if the central bank allows the real interest rate to immediately adjust to this shock, then one would also find that the effects on output and prices are very limited, suggesting that the so-called IS shocks could be interpreted as risk premium shocks. This brings me to the policy interpretation of the results.

In Section 3 the authors state "... we have shown that past sterling fluctuations have not constituted a major channel through which inflationary pressures are transmitted. This suggests that the optimal policy response is to leave interest rates unchanged". Above we have argued that this main result may be due to various identification problems. However, even if it holds, it would not necessarily justify the policy implication. The reason for this is that the estimated impulse response functions incorporate the endogenous reaction of the monetary authorities to the underlying shock. Take, for example, the case in which the estimated IS shocks would partly represent risk premium shocks. As mentioned above a likely reason why such shocks may not turn into relative output and price movements is that the monetary authorities lean against them by changing the policy-controlled interest rates. This would, for example, be the case if the central bank targets some form of monetary conditions index as currently in Canada. An extension of the Clarida-Gali model which includes the short-term interest rate differential does indeed show that the interest rate differential rises sharply in response to an expansionary IS shock, presumably reducing the inflationary effects. Clearly, following the authors' policy advice of leaving interest rates unchanged could very well result in relative price effects, which were to be avoided in the first place.

\textsuperscript{46} As an aside I should mention that using a limited number of lags with long-run restrictions, will tend to reduce the importance of temporary shocks in the variance decomposition. For example, using four lags instead of one in the sterling/DM model one increases the contribution of the LM shocks to the real exchange rate forecast error variance from 8% to 40%. Such a dramatic increase in the contribution of nominal shocks may change the conclusions of the paper.

\textsuperscript{47} Indeed, some attempts to distinguish between temporary demand shocks and monetary policy shocks by including the short-term interest rate differential while maintaining the long-run restrictions à la Clarida and Gali, raised the joint importance of these shocks in explaining real and nominal exchange rate changes from close to zero to over 50% in the short term.
Introduction

In the concept of Austria's exchange rate policy, the pegging to stable currencies of important trading partners is regarded as an intermediate target in order to maintain low inflation and to improve competitiveness. In the longer view, the credible implementation of a policy like this will stabilise expectations and reduce uncertainties. What is crucial in this context is the development of the real exchange rate. There exists, as is well known, a close link between the evolution and the time-series properties of a country's real exchange rate and the concept of purchasing power parity (PPP). However, in the 1970s and 1980s most empirical studies rejected the validity of this concept. Consequently, in the course of the evolution of Austria's exchange rate policy, PPP was never regarded as an essential element nor as a source of potential contradiction to actual policy.

Recent years, however, have seen a new and increasing interest in PPP. This revival may among other things, have two reasons: First, the relative simplicity and intuitive clarity of this concept, and, second, the development of new econometric methods, especially time-series analysis, which offered new tests to evaluate the validity of PPP. The results, however, are still quite tentative, but generally point to the fact that at least in the very long run PPP probably cannot be rejected (see, for instance, Kim 1990).

Thus, the purpose of this paper is twofold: First, as there are very few studies using Austrian data, we look at the time-series properties of the schilling's real effective exchange rate in the light of these new developments. As will be shown, the results are not supportive for PPP; therefore, in a second step, we try to identify other (or additional) factors which may influence the evolution of the real exchange rate.

We proceed as follows: In Section 1 the Austrian exchange rate policy and its relation to PPP are reviewed. Section 2 discusses some recent research on PPP, and in Section 3 we present the empirical results for Austria on the real effective exchange rate. The last section concludes the paper.

1. Austria's exchange rate policy and the Schilling's real effective exchange rate

There are various articles on Austria's exchange rate policy (Gartner 1995, Glück, Proske and Tatom 1992, Glück 1994, Gnan 1995, Hochreiter and Winckler 1995, Pech 1994, and others). In a nutshell, this policy and its evolution can be summarised as follows:

Since the end of World War II, Austria has consistently followed a policy of fixing the exchange rate of the Austrian schilling. First, during the Bretton Woods era the schilling was fixed to the US$ between 1953 (unification of the exchange rate) and August 1971. During this time there was only one parity change, namely a revaluation of the schilling against the US$ by 5.05% in May 1971. Second, when the United States closed the gold window in August 1971, Austria's exchange rate policy had to be adapted. A free float was not considered feasible by the Austrian authorities because of the exchange rate uncertainties connected with it and because of a perceived underlying speculative
threat due to the lack of market depth and width which might threaten the stability of the currency and the economy. Instead, Austria pioneered a new concept by pegging its exchange rate against a basket of currencies. In the following period, the composition of the basket in terms of currencies and base dates was frequently adjusted. As the importance of the DM as reference currency rose, a peg exclusively to this currency emerged in the second half of the 1970s. Finally, since the end of 1981 the schilling has remained fixed to the DM with practically no fluctuation margin.

What is particularly interesting with regard to Austria's economic policy in general and exchange rate policy in particular is that the authorities in the 1970s (specifically related to the evolution of the schilling with regard to the DM in May 1974) explicitly accepted a real appreciation of the schilling in order to get domestic inflation on a lower path. At the same time, it was recognised that this policy could entail considerable costs, in particular for the exposed sector of the economy. Hence, it was attempted to mitigate the initial costs of the real appreciation by an expansionary fiscal policy and, to some extent, also through temporary subsidies. However, the authorities were confident that over the longer term the economy would benefit because wage pressures would be reduced in the wake of lower inflation; consequently profit margins and employment could be restored.

It is important to note that Austria's specific institutional framework, i.e. the social partnership, has a significant role to play in order to ensure that wage developments are commensurate with productivity increases and that economic policy is designed in such a way as to be conducive to an improvement of the supply-side and to foster general economic flexibility. Sound fiscal policy, innovative supply side policies and an institutional framework enhancing the overall flexibility of the economy go far towards explaining the success of Austria's virtually fixed single currency peg against a stable anchor currency in the face of occasional shocks, even of severe real asymmetric shocks.

In short, the concept of the hard-currency option can be described as follows:

- It provides the possibility of importing stability via the pass-through from the prices of imported goods to consumer prices or to the prices of production inputs.
- The tough performance of the currency causes a profit squeeze in the exposed sector which leads to rationalisation, innovation, rising productivity, and improved structures. It also prevents excessive wage increases in this sector which keeps the wage level low in the sheltered sector, too.
- By these mechanisms - lower inflation rates as a precondition for a moderate incomes policy and a profit squeeze in the exposed sector leading to structural improvements - "virtuous circle" effects are brought into play.

Assuming PPP, this can simply be interpreted in the following way (Handler 1989): In relation to the anchor country, diverging price developments are not used as explanatory variable for the exchange rate but as an equilibrium condition by means of which the domestic price level \( p \) is determined:

\[
p = p^* + s^2
\]

with

\[
s = \log \text{ of nominal exchange rate}
\]

\[
p = \log \text{ of domestic price level}
\]

\[
p^* = \log \text{ of foreign price level},
\]

By fixing the exchange rate to the anchor country's currency, i.e. \( s = 0 \), stability from this country is imported, and the implicit inflation target is defined as \( p = p^* \).

This, of course, implies also a constant \textit{real} exchange rate vis-à-vis the anchor country. If, on the other hand, the anchor country's inflation rate is the goal which is aimed for, but which - as

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2 Relations of this kind have been tested empirically by Ardeni and Lubian (1991).
was frequently the case for Austria vis-à-vis Germany - cannot be attained, i.e. $p$ tends to be higher than $p^*$, this would imply a continuous real appreciation of the pegging country's currency.

The case is different when we look at the pegger's exchange rate vis-à-vis the weighted average of exchange rates of its trading partners, i.e. the nominal effective exchange rate. As the exchange rate of the average Austrian trading partner tended to be weaker than the DM and the schilling, a nominal revaluation of the latter was the effect. As, on the other hand, inflation rates in Germany (and in Austria) were lower than for the trading partners, the real effective revaluation generally was smaller than the nominal one, or, in the ideal case, even a real effective devaluation could be the outcome, a favourable effect for international competitiveness.

It is characteristic for Austria's exchange rate policy that over the course of the years it has been developing in a rather pragmatic way which sometimes was in contradiction to textbook wisdom. Similarly, there has been no concern about mean-reverting by which - after a shock - nominal and real exchange rates would be forced back to equilibrium levels as determined by PPP. On the contrary, there was (and is) much more the intuitive belief that by the very absence of mean-reversion exchange rates could be used in order to achieve specific economic goals even in the long run. In the following we try to investigate whether recently developed tests applied to Austrian data justify this view.

2. Tests of PPP and time series analysis

Most recent literature describes the idea of long-run PPP as the hypothesis that there exists a stationary equilibrium real exchange rate. Before the virtual explosion of PPP tests that followed the introduction of econometric techniques designed to handle non-stationary data, it had become more or less a stylised fact that PPP was rejected in empirical tests. Since the concepts of integration and cointegration became common knowledge, a large number of empirical tests have been presented. Alexius (1995) gives an overview of the empirical literature on PPP and finds that the results are mixed, so that no final verdict has been reached concerning the validity of the PPP doctrine. She argues that while there is widespread agreement that PPP does not hold in the short run, the disagreement basically concerns the question whether it holds in the long run and how long the long run is. She also finds that a rejection of PPP depends partly on the choice of countries, the length of the sample period and the econometric techniques used. Studies covering less than 15 years of data almost always reject PPP, while those covering a entire century usually do not. Furthermore, rejections of the PPP hypothesis are much more frequent for the United States and Canada than for European countries. Since most tests of PPP have focused on bilateral exchange rates between major industrial nations like USA, Japan and Germany, a case can be made that these countries have rather different economic structures and the real exchange rate between them is less likely to be stationary than the real exchange rates between more homogenous European countries.

There are two popular approaches testing the validity of PPP. One approach has been to investigate whether the real exchange rates contain a unit root, which is incompatible with PPP. The existence of a unit root in the real exchange rate would imply that shocks to the real exchange rate have not only temporary, but permanent effects: If the real exchange rate is pushed below (above) its equilibrium level, it cannot be expected to return.

The second approach has been to investigate whether nominal exchange rates and price levels are cointegrated. Studies using cointegration techniques have quite often found cointegration among nominal exchange rates and price levels. But the existence of a stationary linear combination of exchange rates and prices does not necessarily mean that PPP holds. According to PPP, it is the

---

real exchange rate that should be stationary. This implies certain restrictions on the cointegration vector(s).

Bilateral PPP has been tested much more often than multilateral PPP. Possible reasons could be that the choice of weights is rather arbitrary and that the hypothesis of stationary effective real exchange rates is not testable within multivariate systems of price levels and bilateral exchange rates.

3. Modelling the long-run real exchange rate for Austria: empirical results

This section presents our empirical findings on determinants of the real exchange rate of the Austrian schilling. Following the line of other recent empirical studies on long-run exchange rate modelling, we will make use of time-series analysis, especially cointegration and unit root testing. In a first step we are interested in knowing whether the PPP doctrine is valid for Austrian data. This seems particularly appealing since the Austrian case is hardly included in the various papers testing PPP. Furthermore, we will concentrate on the examination of the real effective exchange rate, whereas the great bulk of former PPP tests have focused on bilateral real exchange rates. The paper by Johansen and Juselius (1992) and the one by Alexius (1995) represent exceptions to this rule. In agreement with Alexius (1995) we are convinced that if the mechanism driving PPP has to do with international competitiveness, it may be more relevant to study multilateral than bilateral PPP.

In a second step we investigate if real interest rate differences contribute to the modelling of the long-run real exchange rate. However, with a fixed exchange rate regime as in the Austrian monetary policy concept, it seems more appropriate to test the reaction function of the Austrian national bank in which the short-term interest rate is the dependent variable. The test results show that the real interest rate difference is not important for the determination of the real effective exchange rate.

In a third step we look for other determinants of the real effective exchange rate than its own past development. The long-term interest rate difference between Austrian and German government bonds and the productivity differential between the two countries' industrial sectors are regarded as promising candidates.

3.1 Alternative tests of PPP

As mentioned in Section 2 there a two alternative approaches to investigating the validity of PPP. In our empirical examination we made use of both of them.

3.1.1 PPP and cointegration

First, we test the following equation:

\[ s_t = \beta + \alpha_0 p_t + \alpha_1 p_t^* + \varphi_t \]  

An informal way to get a first impression of the characteristics of a time series is to inspect the plots of a variable in levels and differences. We show the time plots for the three series in the Figures 1 to 3. As one would expect, all variables show a trending behaviour in the levels. That means that once there is a level change in these series, it remains for a longer time span. The differences of the series appear to be stationary around zero or a constant. This shape indicates the presence of a difference-stationary data generating process. However, it is difficult to distinguish between a trend-stationary and a difference-stationary process by means of time plots of finite sample length. Other and more formal tools exist for this purpose. Since the variables contained in this equation are likely to be nonstationary, our tests like most tests by other authors, have concentrated on exploiting the cointegration methods proposed by Engle and Granger (1987).
Figure 1
Nominal effective exchange rate

Figure 2
Domestic consumer price index
We used monthly, seasonally unadjusted data of the Austrian nominal effective exchange rate, the Austrian CPI and the so-called "foreign CPI" (which is a basket of trade-share weighted consumer price indices of trading partner countries). To specify the test correctly, we first had to check whether all variables entering the above equation were integrated of order one, I(1). We tested the stationarity properties of the variables by means of rather informal visual inspection and more formal unit root tests, i.e. the augmented Dickey-Fuller test.

Analysing the autocorrelation function of the levels, the differences and the residuals of a regression against a time trend should reveal more information about whether the time series belong to one of the two model classes. We found that the autocorrelation functions of the levels start at a value of around 0.9 and die out very slowly. In contrast to the levels, the autocorrelation functions of the differences die out quickly, with the exception of the domestic consumer price index, which shows some significant autoregressive components at 6 and 12 lags and multiples of these lags, indicating a seasonal pattern. Since we wanted to avoid the shortcomings induced by seasonal filtering of the time series and also wanted to treat all series alike (we did not find any seasonal pattern in the other variables), we refrained from any of the popular seasonal adjustment transformations.

A third step towards differentiating between trend-stationary and difference-stationary representations of macroeconomic time series could be the calculation of residuals from a regression of the variables against a constant and a linear time trend. Uri and Wehinger (1990) pointed out that the distinction between the individual models is difficult, so it was necessary to carry out some formal tests.

We used the econometric software package RATS (version 4.2) to compute the Dickey-Fuller test statistics, after having specified the number of autoregressive correction terms (arcorts) \( p \) by inspection of the series’ ACF. We chose the number of autoregressive correction terms as small as
possible, because it is general knowledge that the test's power is reduced with an increasing number of "arcorrs". This procedure also tests for possible constant or linear time trends. The results are given in Table 1.

Table 1

<table>
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<th>Variable</th>
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<th>Arcorrs p</th>
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</tr>
<tr>
<td>p*/p</td>
<td>T</td>
<td>1</td>
<td>-1.91</td>
<td>-3.96</td>
</tr>
<tr>
<td>q</td>
<td>T</td>
<td>1</td>
<td>-2.70</td>
<td>-3.96</td>
</tr>
<tr>
<td>Δ12P</td>
<td>N</td>
<td>1</td>
<td>-1.71</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δ12P*</td>
<td>N</td>
<td>1</td>
<td>-2.53</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δ(r-r*)</td>
<td>T</td>
<td>1</td>
<td>-3.74</td>
<td>-3.96</td>
</tr>
<tr>
<td>First differences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δs</td>
<td>N</td>
<td>0</td>
<td>-11.29</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δp</td>
<td>N</td>
<td>0</td>
<td>-6.34</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δp*</td>
<td>N</td>
<td>0</td>
<td>-4.72</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δ(p*/p)</td>
<td>N</td>
<td>0</td>
<td>-13.73</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δq</td>
<td>N</td>
<td>0</td>
<td>-12.48</td>
<td>-3.43</td>
</tr>
<tr>
<td>(Δ12P)</td>
<td>N</td>
<td>0</td>
<td>-8.63</td>
<td>-3.43</td>
</tr>
<tr>
<td>(Δ12P*)</td>
<td>N</td>
<td>0</td>
<td>-9.42</td>
<td>-3.43</td>
</tr>
<tr>
<td>Δ12(r-r*)</td>
<td>N</td>
<td>0</td>
<td>-10.96</td>
<td>-3.43</td>
</tr>
</tbody>
</table>

* Sample for (r-r*) is from January 1980 to December 1995.
N = constant; T = linear trend.

For some cases the tests give empirical evidence that the null hypothesis of a unit root in the log-transposed levels of the series cannot be rejected which, loosely speaking, means that most processes are not trend-stationary. However, the tests also suggest that the null hypothesis of a unit root can be rejected for the first differences of the series, which indicates that the variables are integrated of order 1. The results of those unit root tests are not uncontroversial for the domestic and the foreign price level. A special unit root test (Hylleberg et al.) for seasonally unadjusted data would have been more appropriate. Uri and Wehinger (1990) applied the Hylleberg-Engle-Granger-Yoo test to the unadjusted quarterly CPI data for Austria and found that the unit root hypothesis was confirmed for the log-levels, which means that they are not stationary. When testing for seasonal roots, they stated the presence of an annual root in the CPI series. Having in mind the empirical evidence of price indices being I(2)-processes4, we used the (logarithm of the) price ratio as a variable in equation (1). We were unable to reject the hypothesis that this variable is I(1). The result is not really surprising, because it could well be that the Austrian and the foreign price levels are cointegrated and that there exists a linear combination of the two I(2)-series that follows a I(1)-process. Also the theoretical underpinning of the Austrian exchange rate concept suggests that this long-run relationship exists. So equation (1) was transformed to equation (1a).

\[
s_t = \beta_0 + \alpha_2 \left( \frac{p_t}{p_t^*} \right) + \varphi_0 t \quad (1a)
\]

In a next step we tested for a possible long-run relationship between the nominal exchange rate and the price ratio. The Engle-Granger method simply entails estimating the

4 See for example MacDonald (1995), footnote 17, p. 453
coefficients of equation (1a) by OLS and subjecting the residuals to a variety of diagnostic tests of which the most popular has proven to be the augmented Dickey-Fuller test. If there is no long-run relationship between the variables, the residual series of the cointegrating equation would be nonstationary. If there is a long-run relationship as the traditional PPP doctrine would suggest, then, despite the variables entering the equation being individually nonstationary, there would exist some linear combination that transforms the residuals to an I(0) series.

The augmented Dickey-Fuller test of the residuals amounts to estimating an equation of the form:

$$\Delta \varphi_i = \nu_1 \varphi_{i-1} + \nu_2 \sum_{i=2}^{p} \Delta \varphi_{i-1} + \varepsilon_i$$

(2)

If the null hypothesis of no cointegration is valid - the residuals are I(1) - then \(\nu_1\) should be insignificantly different from 0, and this may be tested using a t-test, denoted \(\tau\). Under the alternative hypothesis of stationarity, \(\nu_1\) is expected to be significantly negative. As the distribution of \(\tau\) is not standard, Engle-Granger have tabulated the appropriate critical values. Our empirical results refer to the critical values by Engle and Yoo (1987). The paper tabulates critical values for \(\tau\) from a cointegrating regression of up to five variables and for smaller samples. The initial paper by Engle and Granger (1987) only computed critical values for \(\tau\) for an equation with two variables and samples of more than 100 observations.

### Table 2a
**Cointegrating regression of equation (1a):**

\[ s_t = \beta_0 + \alpha_2 \left( \frac{p_t}{p_{t-1}} \right) + \varphi_{0t} \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_0)</td>
<td>4.62</td>
<td>0.004</td>
<td>1,250.18</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>1.65</td>
<td>0.022</td>
<td>76.33</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; monthly data from 68:07 to 95:12; usable observations: 330. Degrees of freedom: 328; \(R^2 = 0.95\); Durbin-Watson statistic = 0.048.

### Table 2b
**Unit root tests of residuals (\(\varphi_{0t}\))**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Arcorrs p</th>
<th>DF test statistics</th>
<th>Critical value* 1% sign. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi_{0t})</td>
<td>4</td>
<td>-1.57</td>
<td>-3.78</td>
</tr>
</tbody>
</table>

Number of variables in regression (N) = 2.
In the context of the cointegration literature, the existence of long-run PPP amounts to satisfying three conditions. Besides the stationarity of the errors, \( \varphi_0 \) of the cointegrating regression, MacDonald (1995) also mentions the condition of symmetry and the condition of proportionality. The condition of symmetry means that the \( \alpha_0 \) and \( \alpha_1 \) coefficients in equation (1) should enter the cointegration equation with an (equal and) opposite sign. The condition of proportionality means that both coefficient should equal plus and minus unity. A reformulation of the last two conditions for equation (1a) amounts to the requirement that \( \alpha_2 \) equals 1.

As expected, we did not find empirical evidence of (absolute or relative) PPP holding for Austria. The first and most important condition, namely the stationarity of the residuals of the cointegrating equation (1a), could not be confirmed (Table 2a). The results of the unit root test are presented in Table 2b. The estimation results also reject the validity of the two other conditions.\(^6\)

### 3.1.2 A random-walk real exchange rate model

As mentioned in Section 2 there is an alternative to testing for cointegration between a nominal exchange rate and relative prices, and examining the PPP theorem. This approach tests the null hypothesis that the real exchange rate follows a random walk against the alternative that PPP holds in the long run. In contrast to the test applied above, these tests impose - rather than test - the hypothesis that \( \alpha_1 = 1 \) and test - rather than impose - that the (log of the) real exchange rate \( \log q \) is stationary.\(^7\)

In more technical terms, the test checks the null hypothesis of a random walk (equation 4) against the alternative of a trend-stationary process (equation 5):

\[
\Delta q_t = \alpha + \omega_t \tag{4}
\]

where \( \Delta \) is the first difference operator, \( \alpha \) is a drift term, which captures, perhaps, the failure of real interest rates to be equalised across countries and \( \omega_t \) is a stationary process.

The alternative hypothesis to the above equation would be that the real exchange rate exhibits temporary deviations around a trend, i.e. it is trend-stationary:

\[
q_T = \gamma_0 + \gamma_1 T + \epsilon_t \tag{5}
\]

where \( T \) denotes the time trend.

The modern literature uses three main techniques for testing whether the real exchange rate is a random walk. The first - and most commonly used - are the Dickey-Fuller and the augmented Dickey-Fuller tests. The second commonly used technique is that of variance ratios. And the third is that of fractional integration, which encompasses a broader class of stationary processes under the alternative hypotheses.\(^8\) We have used the augmented Dickey-Fuller test, because Taylor (1990), using

\(^5\) If the errors are not I(0), there will be a tendency for the exchange rate and the relative prices to drift apart without bound, even in the long run.

\(^6\) Some authors (e.g. Arden and Lubian 1991) suggest the estimation of equation (3) below for fixed-exchange rate-regimes. Testing this hypothesis also seemed meaningful with regard to the Austrian exchange rate concept:

\[
p_t = \beta_0 + \alpha_2 s_t + \alpha_3 r_t^* + \delta_t \tag{3}
\]

The estimation results for equation (3) were not that exciting, so we abstain from reporting them. One major reason could be that the ATS is primarily pegged to the DEM as an anchor currency and not to a basket of currencies as represented by the effective exchange rate.

\(^7\) See Froot and Rogoff (1994).

\(^8\) See Froot and Rogoff (1994) for a more detailed description of the various techniques.
a Monte Carlo analysis, found the test to be quite powerful against a range of stationary local alternatives.

Given the above results, it seemed rather unlikely that the Austrian real effective exchange rate would be trend-stationary. The Dickey-Fuller test with one autoregressive correction term (number of arcorrs was chosen by ACF; see Figure 4) gives no indication that the Austrian real effective exchange rate is trend-stationary and mean-reverting. The time trend in the estimated regression is not significant. The results of Table 1 also show that the null hypothesis of a unit root in the log-levels cannot be rejected, which means that the real exchange rate exhibits persistent deviations from a trend. We found, however, that the first differences of the time series can be considered stationary. The findings can therefore be interpreted as an ex-post empirical confirmation of the rather intuitive assumptions underlying the Austrian exchange rate concept. As mentioned in the introduction, Austrian monetary policy makers tried to exploit the possibility of a non-mean-reverting real exchange rate in order to import price stability.

**Figure 4**

**Real effective exchange rate**

---

### 3.2 The real interest rate/exchange rate link

As mentioned above, testing for a unit root in real exchange rates may be interpreted as a rather strict test of PPP. In particular, the condition that forces the real exchange rate to be stationary is that, ex ante, real exchange rates are equalised across countries. In the most recent literature (e.g. MacDonald, 1995) we found that persistent deviations of real exchange rates from their long-run equilibrium path are often explained by the development of real interest rate differential. The null
hypothesis of no cointegration is once again tested using the cointegration technique developed by Engle and Granger:9

\[ q_t = \alpha_4 + \beta_2 (r - r^*) + \eta_t \] (6)

with \( r \) the domestic short-term interest rate less the 12-month domestic CPI inflation rate and \( r^* \) the foreign short-term interest rate less the foreign CPI inflation rate over 12 months.

\[ q_t = \alpha_4 + \beta_2 (r - r^*) + \eta_t \]

Figure 5
Real interest rate differential

The estimation results were not very conclusive (Tables 3a and 3b). Moreover, the specification of equation (6) may be inappropriate in the context of the Austrian monetary policy concept. We therefore tested a somewhat modified and a more sensible hypothesis. Because interest rates are instrumental to the Austrian exchange rate target, we reformulated equation (6) to the new specification of equation (7), which seems plausible especially for fixed exchange rate regimes. In the new formulation, equation (7) looks very much like a reaction function of a central bank. We actually estimated equation (7a), where the domestic and foreign inflation rates are attached as additional explanatory variables12:

\[ r_t = \alpha_5 + \beta_5 q_t + \beta_6 r^*_t + \sigma_t \] (7)

\[ i_t = \alpha_6 + \beta_5 q_t + \beta_6 r^*_t + \beta_7 \Delta_i + \beta_8 \Delta_2 p^*_t + \sigma^*_t \] (7a)

9 We tested the stationarity properties of the real interest rate differential, too. Figure 5 gives an informal indication that the time series follows an I(1)-process. We could confirm the hypothesis by the Dickey-Fuller test.

10 The foreign interest rate is a trade-share weighted average of trading partners' short-term interest rates.

11 The foreign CPI is a trade-share weighted average of trading partners' CPIs, which is also used to compute the ATS' real effective exchange rate.

12 We refrained from estimating equation (7) in real terms, because it is very unlikely that a central bank can influence the real short-term interest rate.
with \( i \) the Austrian call money rate and \( i^* \) the foreign call money rate, while \( \Delta_{12}p \) and \( \Delta_{12}p^* \) represent the domestic and the foreign 12-month inflation rate, respectively. What we found was exactly the result we had expected (Tables 4a and 4b). The ADF test suggests a long-run relationship between the variables (the residuals are 1(0) at a 1%-significance level) and the \( R^2 \) (0.9) is rather high. Therefore, we also estimated a so-called short-run reaction function (equation 8) including an error correction term (ECT), which turned out to be significant and had the expected negative coefficient:

\[
\Delta_i = \beta_0 \Delta q_i + \beta_{10} \Delta i^* + \beta_1 \Delta (\Delta_{12}p)_t + \beta_{12} \Delta (\Delta_{12}p^*)_t + \beta_{13} ECT_{1-1} + \xi_t
\]

The estimation results are reported in Table 5.

### Table 3a
**Estimation of cointegrating equation (6):**

\[ q_t = \alpha_4 + \beta_2 (r - r^*)_t + \eta_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_4 )</td>
<td>-0.025</td>
<td>0.004</td>
<td>-6.85</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-0.007</td>
<td>0.003</td>
<td>-2.80</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; monthly data from 80:01 to 95:12; usable observations: 192. Degrees of freedom: 190; \( R^2 = 0.39 \); Durbin-Watson statistic = 0.030.

### Table 3b
**Unit root tests of residuals (\( \varphi_{0t} \))**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Arcorrs p</th>
<th>DF test statistics</th>
<th>Critical value* 1% sign. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_t )</td>
<td>4</td>
<td>-0.24</td>
<td>-3.73</td>
</tr>
</tbody>
</table>

Number of variables in regression (\( N \)) = 2.
Table 4a

Estimation of equation (7a):

\[ i_t = \alpha_6 + \beta_5 q_t + \beta_6 \Delta p_t + \beta_7 \Delta q_t p_t + \beta_8 \Delta_2 p_t + \sigma_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_6 )</td>
<td>-2.18</td>
<td>0.251</td>
<td>-8.67</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>10.24</td>
<td>1.373</td>
<td>7.46</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>1.18</td>
<td>0.049</td>
<td>24.14</td>
</tr>
<tr>
<td>( \beta_7 )</td>
<td>0.19</td>
<td>0.071</td>
<td>2.70</td>
</tr>
<tr>
<td>( \beta_8 )</td>
<td>-0.10</td>
<td>0.058</td>
<td>-1.74</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; monthly data from 80:01 to 95:12; usable observations: 192.
Degrees of freedom: 187; \( R^2 = 0.90 \); Durbin-Watson statistic = 0.67.

Table 4b

Unit root tests of residuals (\( \varphi_t \))

<table>
<thead>
<tr>
<th>Variable</th>
<th>Arcorrs p</th>
<th>DF test statistics</th>
<th>Critical value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_t )</td>
<td>1</td>
<td>-5.44</td>
<td>-5.18</td>
</tr>
</tbody>
</table>

Number of variables in regression (N) = 5.

Table 5

Estimation of equation (8):

\[ \Delta_i = \beta_9 \Delta q_t + \beta_{10} \Delta p_t + \beta_{11} \Delta(\Delta_2 p_t) + \beta_{12} \Delta(\Delta_2 p^*) + \beta_{13} ECT_{t-1} + \xi_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_9 )</td>
<td>10.21</td>
<td>5.364</td>
<td>1.90</td>
</tr>
<tr>
<td>( \beta_{10} )</td>
<td>0.67</td>
<td>0.122</td>
<td>5.54</td>
</tr>
<tr>
<td>( \beta_{11} )</td>
<td>0.18</td>
<td>0.114</td>
<td>1.56</td>
</tr>
<tr>
<td>( \beta_{12} )</td>
<td>1.09</td>
<td>0.197</td>
<td>5.55</td>
</tr>
<tr>
<td>( \beta_{13} )</td>
<td>-0.28</td>
<td>0.053</td>
<td>-5.22</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; monthly data from 80:02 to 95:12; usable observations: 191.
Degrees of freedom: 186; \( R^2 = 0.26 \); Durbin-Watson statistic = 1.99; Significance level of Ljung-Box Q-statistic = 0.314.
3.3 Other determinants of the real exchange rate

In a recent study by Deutsche Bundesbank (1995) the differences in the evolution of productivity between the trading partners were emphasised as potential influences on the real exchange rate as well as long-term interest rates. As pointed out by Balassa (1964), when using broadly defined price indices (including prices for tradables as well as non-tradables - as we used them here) a productivity-bias may arise, inducing a systematic tendency towards revaluation for countries with higher productivity increases in the sector producing tradables.

Following the arguments of the Bundesbank's study, we tested the hypothesis of a long-run relationship between the real effective exchange rate, the productivity differential and the long-term interest rate differential by OLS-estimation of the following equation (9):

\[ q_t = \alpha + \gamma_2 (pd - pd^*) + \gamma_3 (lr - lr^*) + \phi_t \]  

The results can only be regarded as tentative due to two major problems. First, there is a data problem. We found it very difficult to find long and/or high frequency time series for productivity and long-term interest rates. The problem was solved by using German data as a proxy for foreign productivity and long-term interest rates, respectively. With Germany being Austria's main trading partner, this can be regarded as an appropriate solution. The second problem concerns the sample size. We used annual data on industrial productivity \((pd\) and \(pd^*\)) and real government bond yields \((lr\) and \(lr^*\)) from 1971 up to 1995. A sample size of 25 observations is too small for time series analysis\(^{14}\).

Table 6 reports the regression results of equation (9). Measured in log-levels the productivity differential clearly seems to have some positive and significant influence on the log-levels of the real effective exchange rate. The influence of the long-term real interest rate turned out to be less significant, but positive. The \(R^2\) (0.76) appears rather high, but this could also be a sign of spurious conclusion. Although it does not make too much sense to test for cointegration within a small sample, we added an ECT to generate a short-term adjustment equation:

\[ \Delta q_t = \psi_1 \Delta q_{t-1} + \psi_2 \Delta (pd - pd^*) + \psi_3 ECT_{t-1} + \mu_t \]  

The results (Table 7) look encouraging to us and will, therefore, be a gravitation point of our further research.

### Table 6

**Estimation of equation (9):**

\[ q_t = \alpha + \gamma_2 (pd - pd^*) + \gamma_3 (lr - lr^*) + \phi_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.13</td>
<td>0.012</td>
<td>-10.79</td>
</tr>
<tr>
<td>(\gamma_2)</td>
<td>0.57</td>
<td>0.073</td>
<td>7.79</td>
</tr>
<tr>
<td>(\gamma_3)</td>
<td>0.01</td>
<td>0.007</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; annual data from 1971 to 1995; usable observations: 25. Degrees of freedom: 22; \(R^2 = 0.76;\) Durbin-Watson Statistic = 0.52.

---

\(^{13}\) They were deflated by CPI inflation rates.

\(^{14}\) Most critical values reported for unit root tests refer to samples with over 50 observations.
Table 7

Estimation of equation (10):

\[ \Delta q_t + \psi_1 \Delta q_{t-1} + \psi_2 \Delta (pd - pd^*)_t + \psi_3 ECT_{t-1} + \mu_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 )</td>
<td>0.47</td>
<td>0.185</td>
<td>2.56</td>
</tr>
<tr>
<td>( \psi_2 )</td>
<td>0.32</td>
<td>0.225</td>
<td>1.43</td>
</tr>
<tr>
<td>( \psi_3 )</td>
<td>-0.31</td>
<td>0.116</td>
<td>-2.69</td>
</tr>
</tbody>
</table>

Estimation by Ordinary Least Squares; annual data from 1972 to 1995; usable observations: 24.
Degrees of freedom: 21; R\(^2\) = 0.22; Durbin-Watson statistic = 2.16; Ljung-Box Q(6) = 4.002;
significance level of Q = 0.68.

Conclusion

Starting from the concept of the so-called "hard currency strategy" of the Austrian monetary authorities which includes the exploitation of deviations of exchange rates from an equilibrium path (which could be defined by PPP) in order to reduce inflation, we tried to find out whether in the long run, this policy might be eroded by an unexpectedly powerful working of PPP. Our findings suggest that the Austrian real exchange rate follows a random walk, implying the persistence of shocks to the exchange rate. This further implies that no mean-reversion is taking place, and that longer-term deviations from an equilibrium path, as defined by PPP might well be possible and sustainable. An exploitation of this fact for policy purposes, therefore seems justified. Tentative results for other determinants of the real exchange rate like interest rate and productivity differentials seem promising, but need further research.
References


Comments on paper by C. Gartner and H. Glück by P.S. Andersen (BIS)

It is probably well known to most sitting around this table that, for many years, a nominal exchange rate anchor has been a principal component of macroeconomic policies in Austria. Many (myself included) probably also thought that the nominal anchor would generate a mean-reverting real rate, in particular given widespread evidence that the exchange rate anchor has influenced and been taken into account in wage negotiations. However, as demonstrated in the paper, this not the case and the absence of a mean-reverting real rate is, apparently, not bothering policy makers. By showing that PPP does not hold for Austria, the paper fills out an important gap in the empirical literature on exchange rates. At the same time, the absence of long-run PPP raises the question as to why it does not hold. The authors attempt to include productivity developments, but I am not surprised that this does not give very promising results as the data on sectoral productivity developments are poor and unreliable. I would rather urge Gartner and Glück that to disaggregate the exchange rate with respect to country groups as done in the paper by Dr. Jahnke. Identifying the sources of failing PPP might give an important clue as to the direction of further research.

The authors also played with the idea of using real interest rate differentials as a determinant, but quickly came to the conclusion that it would be more fruitful to specify this equation in nominal terms and interpret it as a policy reaction function. I find their estimates of equation (8) in Table 5 convincing and, except for some "fine tuning", there is probably not much more to be done on this. However, it might be an idea to go one step further and estimate a yield curve and then return to the problem of explaining movements in the real exchange by including bond yield differentials among the determinants. In its current form the paper does not use the UIP condition, implying that there is an important source of information which could prove useful.
Long-term interest rates and exchange rates in the Bundesbank macroeconometric model of the German economy

Wilfried Jahnke

Introduction

Long-term interest rates and exchange rates constitute two main channels in the transmission process of monetary policy to financial markets and the real economy. The determination of these rates, therefore, plays an important role in analysing the effects of monetary policy measures. Recent turmoil in bond and foreign exchange markets have stressed again the influence which these asset prices exert on the stability or instability of economic developments. Moreover, the Maastricht treaty underlines the importance of stable exchange rates and relatively low long-term interest rates as convergence criteria on the way to the European Monetary Union.

Estimated equations explaining long-term interest rates and exchange rates are integrated into the Bundesbank's macroeconometric model of the German economy which has recently been reduced to a size of about 140 equations. This model is based on quarterly data from the first quarter of 1975 to the fourth quarter of 1995, with figures after the third quarter of 1990 extended to total Germany, i.e. including eastern Germany. Monetary policy is exogenous to the model, with no reaction function or monetary policy rule relating official interest rates to target variables. The following sections of the paper describe the determination of interest rates and exchange rates within the model as well as the dynamic properties of the equations. An annex reproduces the estimated equations and gives a list of the variables.

1. Determination of interest rates

In the model, the determination of interest rates in the long run is based on the so-called Fisher equation which relates the nominal long-term interest rate \( r \) to real returns from the stock of physical capital \( \rho \), the expected inflation rate \( \pi^e \) and a risk premium \( \varepsilon \):

\[
r = \rho + \pi^e + \varepsilon
\]

It is assumed that in the long run when all adjustments have occurred expected inflation is fully reflected in nominal interest rates. Apart from the nominal long-term interest rate, which is approximated in the model by the yield on government bonds with residual maturities of 9 to 10 years, none of the remaining variables in this equation can be observed. The long-run real return from physical capital depends on time preferences of economic agents and on various marginal rates of substitution and transformation. It moves only very slowly and can be approximated either by the growth rate of potential output or a constant. In the model, the real return has been estimated as a constant. Inflation expectations can be formed in a rational way by using all informations available in the model or in an adaptive way by correcting expectation errors, i.e. deviations between the actual and the expected inflation rate in the previous period. In fact the model uses the following adaptive expectation formation process:

\[
\pi^e = \pi^e_{-1} + \lambda (\pi^e_{-1} - \pi^e_{-1}) = \lambda \pi^e_{-1} + (1 - \lambda) \pi^e_{-1}
\]

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1 A previous version of the model has been described in Deutsche Bundesbank, *Macroeconometric model of the German economy*, Frankfurt am Main, April 1994.
The best fit could be obtained by setting the coefficient $\lambda$ to 0.1, which results in a fairly slow adjustment to previous inflation rates. The risk premium proved very difficult to estimate. Relating it to the government debt to GDP ratio which has increased over the estimation period led to implausible estimates of the real interest rate. Therefore, it was assumed in the model that the risk premium was constant over the estimation period which does not seem an implausible assumption for the past German development. The actual nominal long-term interest rate adjusts to the long-run rate which equals the sum of a constant (real returns and the risk premium) and the (expected) inflation rate. In this adjustment process, influences from monetary policy as well as from foreign capital markets seem to be of some importance. Monetary policy impulses are transmitted to long-term interest rates through changes in short-term interest rates ($i_l$). But these direct influences are of a temporary nature only. In the long run, monetary policy affects long-term interest rates through its impact on the growth of the money stock and, thereby, on the inflation rate. Apart from domestic factors, foreign long-term interest rates exert some influence on German rates. But, probably due to multicollinearity problems, it was not possible to estimate the size of these effects with plausible results. Fears of a heavy burden on capital markets from German unification increased long-term rates in the first half of 1990. This has been considered in the equation by including a dummy variable $DWU$. Thus the adjustment process in the determination of long-term interest rates is described by the following equation:

$$\Delta r = \alpha_1 + \alpha_2 DWU + \alpha_3 \Delta i + \alpha_4 \Delta r_{-1} + \alpha_5 (\pi_{-4}^e - r_{-4})$$

In the long-run when expected inflation equals actual inflation, the long-term interest rate is determined by a constant and the inflation rate:

$$r = \frac{\alpha_1}{\alpha_5} + \pi$$

The value of the constant which approximates real returns from capital and risk premia has been estimated at 3.28%. As Chart 1 shows interest rates have been nearly stationary in the past twenty years. Inflation rates, on the contrary, have followed a decreasing trend, so that "real interest rates" have increased. As there are no reasons for an increase in real returns from physical capital, this development can be interpreted either as a rise in risk premia or as a very slow adjustment of nominal long-term interest rates to lower inflation rates.

Short-term interest rates on the money market are mainly determined by monetary policy. The Bundesbank uses rediscount facilities which are charged at the discount rate, $DIS$, to provide central bank money on a longer-term basis. Marginal refinancing needs, on the other hand, are satisfied by Lombard loans which form the most expensive way of refinancing at the Lombard rate, $LOMS$. The repurchase rate, $z$, for regular open market transactions normally ranks between these two rates, depending nonlinearly on the liquidity situation which has been approximated in the model by the ratio of excess reserves of banks, $ZBGD$, to the total stock of central bank money supply, $ZEBA$:

$$z = DIS + GMST \times (LOMS - DIS)$$

$$\ln \left( \frac{GMST}{1.25 - GMST} \right) = \alpha_1 + \alpha_2 \ln \left( \frac{GMST_{-1}}{1.25 - GMST_{-1}} \right) + \alpha_3 \sum_{i=0}^{2} \frac{ZBGD_{-1}}{ZEBA_{-1}}$$

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Chart 1  
Interest rates and inflation in Germany from 1975 to 1995  
In % p.a. or in percentage changes

Lombard rate and discount rate

Average liquidity ratio

Repo rate and short-term interest rate

Long-term interest rate

Actual and expected inflation rate

"Real interest rate"

Deutsche Bundesbank

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Chart 2

Interest rate equation: yield on government bonds
With residual maturities of 9 to 10 years, % p.a.

Actual values
Estimates values

Residuals

Deutsche Bundesbank
Money market rates for three month funds mainly depend on the repurchase rate. Additionally their development is influenced by short-term rates in the Euro-dollar market \(i_2\) and by the inflation rate. Interest rates on the money market are thus described in the model by the following equation:

\[
\Delta i = \alpha_2 \Delta z + \alpha_3 \Delta i_2 + \alpha_4 \pi + \alpha_5 (z_{-1} - i_{-1})
\]

Changes in official rates as well as changes in liquidity policy are transmitted, in the first stage, to short-term money market rates and, in a second stage, to long-term interest rates. (Chart 2).

2. Determination of exchange rates

The effective exchange rate of the D-Mark against foreign currencies is described in the model by a weighted index, the so-called external value of the D-Mark against the currencies of 18 industrial countries. This index has been disaggregated into the external value against the US-dollar (Chart 3), the external value against the currencies of the countries participating in the exchange rate mechanism of the European Monetary System (ERM), and the external value against the currencies of the remaining countries, the respective weights being the trade shares\(^3\) (equation 7 in the annex). The external value of the D-Mark is the equivalent of the inverse of the domestic price of foreign currencies. An increase (decrease) of this value represents an appreciation (depreciation) of the D-Mark.

The determination of exchange rates in the model is based on interest rate parities as well as on purchasing power parities\(^4\). Comparing investments in assets denominated in domestic or in foreign currencies the following applies:

\[
i - (i + \beta) = e^\varepsilon - e
\]

After all arbitrage transactions have occurred the difference between domestic and foreign interest rates plus a risk premium \(\beta\), resulting e.g. from imperfect capital mobility or risk-averse investors equals the expected change in the exchange rate (where \(e\) is the natural logarithm of the exchange rate and the superscript "\(\varepsilon\)" denotes the expected value). In the long run exchange rate expectations in the model converge to the relation between foreign and domestic prices, i.e. to purchasing power parity (where \(p\) and \(p^*\) are the natural logarithms of foreign and domestic price deflators for final demand respectively):

\[
e^\varepsilon = \alpha_1 + \alpha_2 (p^* - p)
\]

By inserting and rearranging the following estimated exchange rate equation has been derived, where the coefficient \(\alpha_3\) takes into account that the interest rate differential has been approximated by short-term interest rates whereas the expectations apply to the long run:

\[
e = \alpha_1 + \alpha_2 (p^* - p) + \alpha_3 (i^* - i) + \beta + u
\]

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\(^4\) The long run validity of purchasing power with respect to single currencies as well as the interaction of purchasing power and interest rate parity has been analysed in Deutsche Bundesbank, "Trends and determining factors of the external value of the Deutsche Mark", Monthly Report, November 1993. See also MacDonald, R., "Long-Run Exchange Rate Modelling - A Survey of the Recent Evidence", IMF Staff Papers, 42, 1995.
Chart 3
Exchange rate equation: external value of the D-mark against the US-dollar
Logarithmic change against the previous year, in %

Actual values
Estimated values

Deutsche Bundesbank
The coefficient \( \alpha_1 \) deviates from zero mainly because the foreign price deflators and the external values of the D-Mark are based on end-1972 = 100 whereas domestic prices are based on the year 1991 = 100. Furthermore the existence of transportation costs and tariffs may have some importance. The coefficient \( \alpha_2 \) equals 1 for the ERM currencies, but is below 1 for the US-dollar and the other currencies which means that price differentials are not fully compensated in exchange rate changes, at least not over the medium term. Real exchange rates will change accordingly. This could, in part, be explained by the fact that the price deflators for domestic and foreign total demand contain different and non-neglectable amounts of nontraded goods. Moreover, when adjustment processes are slow, the sample available over the recent floating period seems to be relatively short.

The interest sensitivity of the US-dollar is found to be much higher than the reaction of the other currencies to changes in interest rate differentials. Attempts to estimate the risk premia \( \beta \) by introducing the net foreign assets to GDP ratio into the equation failed. Therefore it was assumed that \( \beta \) is constant. The short-run adjustment of exchange rates to the longer-term relations has been estimated by an error correction process, depending on changes in price and interest rate differentials:

\[
\Delta e = \alpha_1 \Delta (p^* - p) + \alpha_2 \Delta (p^*_{t-1} - p_{t-1}) + \alpha_3 \Delta (i^*-i) + \alpha_4 \Delta e_{t-1} + \alpha_5 \Delta e_{t-2} + \alpha_6 \sum_{i=1}^{4} u_i
\]

3. Effects of shocks in official rates and in inflation on interest rates and exchange rates

The various interest rate and exchange rate equations described in the previous sections build, together with expectation formation, a small bloc of the complete macroeconometric model of the Deutsche Bundesbank for the German economy. The main exogenous variables to this small bloc model, consisting of 15 equations, are the domestic official interest rates, i.e. the lombard rate and the discount rate, the Euro-dollar rate for three-month funds as well as domestic and foreign price deflators for final demand. To demonstrate the dynamic properties of the estimated equations two different shocks have been simulated with this bloc model. The first one consists of an increase in official interest rates by 100 basis points for two years (1988 and 1989 as an example) and a return to base line values, i.e. to actual values, thereafter. The second simulation describes the effects of a temporary two-year increase in the domestic inflation rate by 1 percentage point, all other exogenous variables being unchanged, as in the first simulation.

A dynamic base line simulation of the estimated equations building the small bloc model over the whole estimation period from 1975 to 1995 shows, no doubt, that there are periods of large deviations from the actual values, especially in the exchange rate of the Deutsche Mark (DM) against the US dollar (Chart 4). These deviations are not systematic, however, and in the long run, the variables tend to return to their observed values.

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5 Similar results with respect to the external assets ratio have been described in Deutsche Bundesbank, "Overall determinants of the trends in the real external value of the Deutsche Mark", Monthly Report, August 1995.

6 In the complete model the price deflator for final demand is an endogenous variable.
Chart 4
Dynamic simulation of the interest rate and exchange rate equations in the econometric model of the German economy

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Actual values
---

Simulated values
---

Short-term Interest rate

Long-term Interest rate

US dollar

ERM-currencies

Other currencies

Nominal external value of the D-Mark

Deutsche Bundesbank
The increase in official interest rates, followed by a decrease to base line levels, is transmitted almost completely and contemporaneously to short-term money market rates (Chart 5). The yield on government bonds, on the contrary, reacts only in a restricted manner. Its increase amounts merely to 30 basis points at the most. When short-term rates have returned to their base line levels long-term rates fall by 17 basis points below their base line. But in the long run the long-term interest rates, like the short-term rates, return to their base line levels. In reaction to the increase in interest rates the DM appreciates by 0.8 % at the peak (2.2 % against the US dollar, 0.7 % against the ERM currencies and 0.5 % against the other currencies). When the short-term interest rates have returned to their base lines in the third year after the shock the DM depreciates afterwards by the same amount. The level of the exchange rate, therefore, returns to its base line.

A temporary increase in the inflation rate by 1 percentage point for two years raises the price level by 1 % in the first year after the shock and permanently by 2 % from the second year on (Chart 6). Expected inflation follows the change in actual inflation with considerable delay. After two years inflation expectations are 0.6 % higher than in the base line. But as the actual inflation rate then returns to its base line, inflation expectations likewise return to the base line in the long run. The higher inflation expectations raise the long-term interest rate by 30 basis points at the most. In the long run government bond yields will return to their base line levels too. Only a permanent change in the inflation rate will be transmitted completely to the level of long-term interest rates. The assumed increase in domestic inflation and the induced changes in interest rates with unchanged foreign prices and foreign interest rates depreciate the DM by 2½% at the most. As the inflation rate and interest rates after two years return to their base line levels the depreciation rate returns to zero. After all adjustments have taken place the DM has depreciated by 1.6 %. Since the domestic price level has increased by 2 % and the foreign price level has been assumed unchanged, the real exchange rate (which depreciates temporarily because the domestic price level increases more slowly than the nominal exchange rate depreciates) will be changed slightly in the long run (incomplete purchasing power parity).

In the complete model of the German economy, domestic prices are endogenous. But as the model does not contain a monetary policy reaction function, official interest rates are still exogenous. In addition to the two simulation experiments with the small bloc model, a temporary increase in official interest rates by 100 basis points (in the years 1988 and 1989) has been simulated with the full model. The reaction of long-term interest rates and exchange rates corresponds completely to the reaction in the small bloc model. As Chart 7 shows, a temporary change in monetary policy only results in temporary changes in real variables. In the long run the real long-term interest rate, the real effective exchange rate, the real stock of money, real GDP and real wages return to their base line levels. This is true also, regarding prices, wages and other nominal variables, although wages lag more than other variables due to the prevailing rigidities in the labour market. After the economy has been exogenously shocked by the temporary change in interest rates the system returns to the base line with damped oscillations. In a model with adaptive expectation formation it seems necessary to change official interest rates permanently to obtain a permanent success in reducing the stock of money and the level of prices.
Chart 5
Effects of an increase in Bundesbank interest rates by
100 basis points for two years on market interest and exchange rates
Deviation from base line in % or in percentage points

Lombard and discount rate

Repurchase rate

Long-term interest rate

Money market interest rate

Effective exchange rate change

Effective exchange rate level

Deutsche Bundesbank
Chart 6

Effects of an increase in the inflation rate by 1 percentage point for two years on market interest and exchange rates

Deviations from base line in % or in percentage points

Actual and expected inflation rate

Price level

Long-term interest rate

Money market interest rate

Nominal and real exchange rate change

Nominal and real exchange rate level

Deutsche Bundesbank
Chart 7
Effects of an increase in Bundesbank interest rates by
100 basis points for two years in the complete model
Deviation from base line in % or in percentage points

Nominal and real long-term interest rate

Nominal and real effective exchange rate

Nominal and real money stock M3

Nominal and real GDP

GDP deflator

Nominal and real wage rate

Deutsche Bundesbank
Conclusions

Interest rates and exchange rates are determined in the Bundesbank macroeconometric model of the German economy according to traditional lines using the Fisher equation in explaining the development of long-term interest rates as well as purchasing power parity and uncovered interest rate parity in explaining exchange rates. Moreover, expectation formation is based on adaptive adjustment processes. The empirical relevance of rational expectations seems to be - at least - questionable. Neither in the case of interest rates nor in the case of exchange rates could a firm empirical basis be found in Germany for an integration of intertemporal, i.e. time-consistent stock-flow constraints and their effects on risk premia, into the determination of these asset prices.

7 Even in the new quarterly project model of the Bank of Canada which uses a mixture of adaptive and model-consistent expectations, "considerable weight is in fact put on the backward-looking portion in order to capture the slow adjustment of expectations apparent in economic data". (p. 29). See Poloz, S., D. Rose and R. Tetlow, (1994).
Annex

1. Interest and exchange rate equations in the Bundesbank macroeconometric model of the German economy

1.1 Repurchase rate

\( RPEN = DIS + GMST \times (LOMS - DIS) \)

\[
\ln \left( \frac{GMST}{1.25 - GMST} \right) = 0.24 + 0.56 \ln \left( \frac{GMST_{-1}}{1.25 - GMST_{-1}} \right) - 9.51 \sum_{0}^{3} \frac{ZBGD_{-1}}{ZEB_{-1}} \times 0.25
\]

\( R^2 = 0.45 \quad DW = 1.99 \quad SEE = 1.16 \)

1.2 Three-month money market interest rate

\( \Delta RGD = 0.92 (14.98) \Delta R PEN + 0.15 (4.64) \Delta RG DE + 0.03 (1.93) \times 100 \Delta \ln (PEV) \)

+ 0.31 (2.88) \((RPEN_{-1} - RGD_{-1})

\( R^2 = 0.80 \quad DW = 2.05 \quad SEE = 0.35 \)

1.3 Yield on ten-year government bonds

\( \Delta RFUO = 0.45 (2.81) + 0.87 (3.22) \Delta DWU_{-2} + 0.14 (4.23) \Delta RG D + 0.62 (8.96) \Delta RFUO_{-1} \)

+ 0.14 (3.51) \((100 * PEVD_{-4} - RFUO_{-4})

\( R^2 = 0.81 \quad DW = 0.91 \quad SEE = 0.49 \)

1.4 External value of the DM against ERM-currencies

a) \( \ln (AUWS) = 3.93 + 1.01 \ln \left( \frac{PEVE}{PEV} \right) - 0.66 (RGDE - RGD) \times 0.01 \)

- 0.02Q1 - 0.03Q2 - 0.03Q3 + ECAUWS

b) \( \Delta \ln (AUWS) = 0.67 (2.67) \Delta \ln \left( \frac{PEVE}{PEV} \right) - 0.58 (2.28) \Delta \ln \left( \frac{PEVE_{-1}}{PEV_{-1}} \right) \)
\[
+ 0.96 \Delta_4 \ln(AUWS_{-1}) - 0.14 \Delta_4 \ln(AUWS_{-2}) - 0.24 \sum_{i=1}^{4} ECAUWS_{-1} \cdot 0.25
\]

\[R^2 = 0.88 \quad DW = 1.75 \quad SEE = 1.29\]

1.5 External value of the DM against the US dollar

a) \( \ln(AUUS) = 4.27 + 0.86 \ln \left( \frac{PEVU}{PEV} \right) - 1.88 (RGDE - RGD) \cdot 0.01 + ECAUUS \)

b) \( \Delta_4 \ln(AUUS) = \frac{3.00}{(3.57)} \Delta_4 \ln \left( \frac{PEVU}{PEV} \right) - \frac{2.92}{(3.50)} \Delta_4 \ln \left( \frac{PEVU_{-1}}{PEV_{-1}} \right) - \frac{0.38}{(1.27)} \Delta_4 (RGDE - RGD) \cdot 0.01 + \frac{1.01}{(9.80)} \Delta_4 \ln(AUUS_{-1}) - \frac{0.19}{(1.77)} \Delta_4 \ln(AUUS_{-2}) - \frac{0.09}{(1.93)} \sum_{i=1}^{4} ECAUUS_{-1} \cdot 0.25 \)

\[R^2 = 0.81 \quad DW = 1.91 \quad SEE = 5.67\]

1.6 External value of the DM against other currencies

a) \( \ln(AUSO) = 4.30 + 0.66 \ln \left( \frac{PEVS}{PEV} \right) - 0.71 (RGDE - RGD) \cdot 0.01 + ECAUSO \)

b) \( \Delta_4 \ln(AUSO) = \frac{2.44}{(6.38)} \Delta_4 \ln \left( \frac{PEVS}{PEV} \right) - \frac{2.77}{(6.11)} \Delta_4 \ln \left( \frac{PEVS_{-1}}{PEV_{-1}} \right) - \frac{0.03}{(0.19)} \Delta_4 (RGDE - RGD) \cdot 0.01 + \frac{1.00}{(11.66)} \Delta_4 \ln(AUSO_{-1}) - \frac{0.26}{(2.89)} \Delta_4 \ln(AUSO_{-2}) - \frac{0.17}{(2.61)} \sum_{i=1}^{4} ECAUSO_{-1} \cdot 0.25 \)

\[R^2 = 0.84 \quad DW = 1.74 \quad SEE = 2.46\]

1.7 External value of the DM against 18 currencies

\( AUDM = AUWS^{0.39794} \cdot AUUS^{0.14151} \cdot AUSO^{0.46055} \)

1.8 Exchange rate of the DM against the US dollar

\( ER = 100.633 \cdot \frac{3.203}{AUUS} \)
1.9 Price expectations

\[ PEVD = 0.9PEV_{-1} + 0.1\Delta_t \ln(PEV_{-1}) \]

1.10 Price deflator of final demand in 18 industrial countries

\[ PEVF = PEVE^{0.39794} \cdot PEVU^{0.14151} \cdot PEVS^{0.46055} \]

1.11 Real external value of the DM against 18 currencies

\[ AUDR = AUDM \cdot \frac{PEV}{PEVF} \]

2. List of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUDM</td>
<td>External value of the Deutsche Mark against the currencies of 18 industrial countries, end-1972 = 100, Deutsche Bundesbank, Monthly Report, Table X.9, Series WU5879.</td>
</tr>
<tr>
<td>AUDR</td>
<td>Real external value of the Deutsche Mark, end-1972 = 100. Defined: [ AUDR = AUDM \cdot \frac{PEV}{PEVF} ]</td>
</tr>
<tr>
<td>AUSO</td>
<td>External value of the Deutsche Mark against the currencies of other countries, end-1972 = 100. Defined: [ AUSO = AUDM \cdot \frac{1}{PEV} \cdot AUUS^{0.14151} \cdot AUWS^{0.39794} ]</td>
</tr>
<tr>
<td>AUUS</td>
<td>External value of the Deutsche Mark against the US dollar, end-1972 = 100, Deutsche Bundesbank, Monthly Report, Table X.9, Series WU5409.</td>
</tr>
<tr>
<td>AUWS</td>
<td>External value of the Deutsche Mark against currencies of countries participating in the exchange rate mechanism of the European Monetary System, end-1972 = 100, Deutsche Bundesbank, Monthly Report, Table X.9, Series WU5690.</td>
</tr>
<tr>
<td>DIS</td>
<td>Discount rate of the Deutsche Bundesbank, per cent p.a., Deutsche Bundesbank, Monthly Report, Table VI.1, Series SU0110.</td>
</tr>
<tr>
<td>DWU</td>
<td>Dummy variable for German unification, from third quarter of 1990 = 1, before = 0.</td>
</tr>
<tr>
<td>ER</td>
<td>Exchange rate of the Deutsche Mark against the US dollar. Defined: [ ER = 100.633 \cdot \frac{3.203}{AUUS} ]</td>
</tr>
<tr>
<td>GMST</td>
<td>Variable fixing the repurchase rate within the discount/lombard rate band. Defined: [ GMST = (RPEN - DIS) / (LOMS - DIS). ]</td>
</tr>
</tbody>
</table>
LOMS Lombard rate resp. special lombard rate of the Deutsche Bundesbank, per cent p. a., Deutsche Bundesbank, Monthly Report, Table VI.1., Series SU0111.

PEV Price deflator of final demand, 1991 = 100.

PEVD Price expectations.

Defined: \[ PEVD = 0.9 \times PEVD_{-1} + 0.1 \times A_4 \ln(PEV_{-1}) \]

PEVE Price deflator of final demand in ERM countries, end-1972 = 100, Series YQD723.

PEVF Price deflator of final demand in 18 industrial countries, end-1972 = 100, Series YQD720.

PEVS Price deflator of final demand in other countries, end-1972 = 100.

Defined: \[ PEVS = \frac{PEV_{-1}^1}{PEF_{0.45055}^4} \]

PEVU Price deflator of final demand in the United States, end-1972 = 100, Series KA7115.

Q1, Q2, Q3 Seasonal dummy variables for the first, second and third quarter.

RFUO Yield on government bonds with residual maturities of 9 to 10 years, per cent p.a., Deutsche Bundesbank, Monthly Report, Table VII.5, Series WU8612.

RGD Money market interest rate for three-month funds in Frankfurt am Main, per cent p.a., Deutsche Bundesbank, Monthly Report, Table VI.4, Series SU0107.

RGDE Money market interest rate at the Euro-dollar market for three-month funds, per cent p.a., Deutsche Bundesbank, Monthly Report, Table VI.7, Series IV1212.

RPEN Interest rate for Bundesbank's open market transactions in securities under repurchase agreements (repurchase rate), per cent p.a., Deutsche Bundesbank, Monthly Report, Table VI.3, Series VQ7225.

ZBGD Excess reserves of banks, DM bn, Deutsche Bundesbank, Monthly Report, Table II.3 and Table V.2, Series AU0715 (unused refinancing facilities) less Series AU0800 (lombard loans) plus Series AU0710 (excess reserves).

ZEBA Supply of central bank money, DM bn, Deutsche Bundesbank, Monthly Report, Table II.3 and Table IV.1, Series AU0024 (central bank money) plus Series OU0313 (cash in hand of credit institutions) plus ZBGD.
References


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Comments on paper by Dr. Jahnke by P.S. Andersen (BIS)

This is a concise and well written paper which does not leave much for a discussant to add. The presentation of the econometric work is clear and all financial sector equations are supported and illustrated by simulations. Yet, I do have a few comments on each of the three equations presented in the paper.

*Long-term bond rate:* The structure of the long-term interest rate equation is remarkably simple and transparent, since only the Fisher effect is present. The short-run ECM version is also very simple but I wonder if it might not be too simple:

(i) in Chart 1 the level of the nominal bond rate looks very much like a stationary process, whereas actual and expected rates of inflation are I(1); thus the ECM equation might be misspecified;

(ii) the very low DW statistic might also point to a specification or missing variable problem. Although I have no reason to doubt that Dr. Jahnke carefully tested the influence of foreign bond rates, it is, indeed, surprising that the general trend towards internationally converging bond rates is not confirmed in the German equation;

(iii) when faced with a trend rise in the real bond rate, one always wonders whether this reflects slowly adjusting expectations of inflation or a gradually rising risk premium. Considering the overall economic development of the German economy I share Dr. Jahnke's view that the trend rise is due to highly adaptive expectations. Yet, it might be interesting to see how the equation performs if expectations were taken from surveys or modelled by a Markov switching process as explained in the paper by A. Tarditi.

*Short-term interest rate:* I found the three-stage explanation of the three-month interest rate very interesting and instructive. However, I "missed" a fourth stage where reactions by the Bundesbank to deviations between actual developments and the target of monetary policy are explained and "fed" into the model.

*Exchange rate:* I really liked this part of the paper as the three-part equation captures key theoretical arguments (PPP and UIP) while, at the same time, the country disaggregation provides a convincing identification of the sources of the real appreciation of the DM. The variation in parameter size across country groups is also very interesting, and it could well be that the high sensitivity of the DM/US dollar exchange rate to changes in the interest rate differential against US rates provides the "missing link" in the bond rate equation. By contrast, the rather low interest rate sensitivity of the effective value of the DM against other ERM currencies is not "good news" to Germany's main trading partners.
On the determination of long-term interest rates and exchange rates

Michel Dombrecht and Raf Wouters

Introduction

Empirical investigations of the behaviour of interest rates are mostly based on a loanable funds theory. Well known examples of this approach are Evans (1987), Hoelscher (1986), Barro and Sala-i-Martin (1991). In these models the real interest rate is determined by the equilibrium between investment demand and desired saving in the economy. Following this approach, expected economic growth or profitability, inflation surprises, public deficits and public consumption are considered to be the main variables explaining the behaviour of interest rates. Especially the impact of public deficits was, however, the subject of contradictory results. Public deficits, by reducing the available funds, were expected to raise real interest rates, except in those cases where private agents would increase their private wealth accumulation to offset future tax liabilities. This last argument illustrates that the loanable funds approach by lacking rigorous microeconomic and intertemporal underpinnings, is not the best possible theoretical model to analyse the behaviour of interest rates.

Another shortcoming of the loanable funds approach, which was partly responsible for the contradictory results, was the absence of a distinction between the determination of the short and long-term interest rates. Most authors who did not find a significant impact of government deficits on interest rates, were actually concentrating on short-term interest rates, while others who found strong influences of deficits were explaining long-term interest rates, including the short-term rate as an explanatory variable in the equation (Hoelscher, 1986, Correira-Nunes and Stemitiotis, 1995). The empirical investigation should therefore start from a model that incorporates an explanation of the term structure, and distinguishes the determinants of short and long-term rates. Following this reasoning it is important to introduce uncertainty in the model to avoid the simplistic expectations theory and to allow for time varying risk premia in the determination of returns on risk-bearing assets.

The loanable funds approach also led to overemphasising the role of public deficits, and to ignoring the role of the current account balance in the determination of interest rates. This asymmetric treatment of two macroeconomic imbalances is also reflected in the public discussion. Following the loanable funds approach one should expect a surplus on the current account, if determined exogenously by the competitiveness of the economy, to increase the interest rate as the domestic economy is lending to the rest of the world. But the empirical results, which incorporate current account balances, show a negative effect on the real interest rate (OECD 1995). This result was interpreted as reflecting expectations of exchange rate appreciation allowing lower interest rates. Such an argument, however, is more appropriate in a portfolio diversification approach, than in a loanable funds context.

In this paper, we adopt an alternative framework for analysing the determinants of long-term interest rates and exchange rates. The theoretical model is based on optimal intertemporal behaviour of the consumption-saving-portfolio allocation in small open economies and then applied to the explanation of bond yield differentials in a number of European countries the German bond vis-à-vis yield giving special emphasis to the treatment of expectations and uncertainty. The complementary analysis of exchange rate determination is applied to the DEM/BEF exchange rate.
1. Theoretical framework

The representative consumer maximises the expected value of a discounted expected logarithmic utility function which depends on consumption:

\[ \text{Max } E \sum_{k=0}^{\infty} \rho^k U(C_{t+k}) \]  

subject to a budget constraint (here written as in Lee, 1995):

\[ \sum_{i=k+1}^{\infty} b_{t+k}^{t+i} B_{t+k}^{t+i} + \sum_{i=k+1}^{\infty} f_{t+k}^{t+i} s_{t+k} F_{t+k}^{t+i} = \]

\[ B_{t+k-1}^{t+k} + s_{t+k} F_{t+k-1}^{t+k} + \sum_{i=k+1}^{\infty} b_{t+k}^{t+i} B_{t+k-1}^{t+i} + \sum_{i=k+1}^{\infty} f_{t+k}^{t+i} s_{t+k} F_{t+k-1}^{t+i} + Y_{t+k} - C_{t+k} \]

where \( E(\cdot) \) is the mathematical expectation conditioned on information available at time \( t \), \( \rho \) is the subjective discount factor, \( U \) is the utility function, \( C \) is consumption, \( b_{t+k}^{t+i} \) is time \( t \) price of a domestic discount bond which pays one unit of domestic currency at time \( t+i \), \( B_{t+k}^{t+i} \) is the number of \( i \) period domestic discount bonds held by the household at time \( t \), \( f_{t+k}^{t+i} \) is time \( t \) price of a foreign discount bond which pays one unit of foreign currency at time \( t+i \), \( s \) is the price of one unit of foreign currency in domestic currency, \( F_{t+k}^{t+i} \) is the number of \( i \) period foreign discount bonds held by the household at time \( t \), and \( Y \) is income.

The dynamic Lagrangean to be maximised is:

\[ L_t = \text{Max } E \sum_{k=0}^{\infty} \rho^k \left( U(C_{t+k}) + \lambda_t (Y_{t+k} - C_{t+k} + B_{t+k}^{t+k} - s_{t+k} F_{t+k}^{t+k} + \sum_{i=k+1}^{\infty} b_{t+k}^{t+i} B_{t+k}^{t+i} + \sum_{i=k+1}^{\infty} f_{t+k}^{t+i} s_{t+k} F_{t+k}^{t+i}) \right. \]

\[ \left. + \sum_{i=k+1}^{\infty} f_{t+k}^{t+i} s_{t+k} F_{t+k}^{t+i} - \sum_{i=k+1}^{\infty} b_{t+k}^{t+i} B_{t+k}^{t+i} - \sum_{i=k+1}^{\infty} f_{t+k}^{t+i} s_{t+k} F_{t+k}^{t+i} \right) \]

In period \( t \), the first order conditions w. r. t. the five decision variables are:

\[ \frac{\delta L_t}{\delta C_t} = E \left[ U'(C_t) - \lambda_t \right] = 0 \]  

\[ \frac{\delta L_t}{\delta b_{t+1}^{t+1}} = E \left[ -\lambda_t b_{t+1}^{t+1} + \rho \lambda_{t+1} \right] = 0 \]  

\[ \frac{\delta L_t}{\delta b_{t+2}^{t+2}} = E \left[ -\lambda_t b_{t+2}^{t+2} + \rho \lambda_{t+1} b_{t+1}^{t+2} \right] = 0 \]

1 The transversality conditions accompanying this maximisation problem are not discussed here. For an infinite horizon model and no uncertainty, these conditions would imply that the present value of future public and current account surpluses would equate current deficits. However under the assumption of imperfect substitution between different assets (and liabilities), supply and wealth effects will still influence the consumption and allocation decision.
\[
\frac{\delta L_t}{\delta r_{t+1}} = E_t \left[ -\lambda_t f_{t+1} s_t + \rho \lambda_t s_t \right] = 0
\] (7)

\[
\frac{\delta L_t}{\delta r_{t+2}} = E_t \left[ -\lambda_t f_{t+2} s_t + \rho \lambda_t s_t \right] = 0
\] (8)

1.1 The holding return on domestic bonds

Because period \( t \)-values are known with certainty in period \( t \), it follows from equation (6) and

\[
\rho E_t \left[ \frac{\lambda_{t+1} b_t^{t+2}}{\lambda_t b_t^{t+2}} \right] = 1
\] (9)

Assuming that \( \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \) and \( \left( \frac{b_t^{t+2}}{b_t^{t+2}} \right) \) are jointly lognormally distributed, then (9) can be solved as:

\[
\ln \rho + E_t \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \right] + \frac{1}{2} \text{var} \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \right] + E_t \left[ \ln \left( \frac{b_t^{t+2}}{b_t^{t+2}} \right) \right] + \frac{1}{2} \text{var} \left[ \ln \left( \frac{b_t^{t+2}}{b_t^{t+2}} \right) \right] + \text{cov} \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right), \ln \left( \frac{b_t^{t+2}}{b_t^{t+2}} \right) \right] = 0
\] (10)

Using similar assumptions, (5) can be written as:

\[
\ln \rho + E_t \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \right] + \frac{1}{2} \text{var} \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \right] + E_t \left[ \ln \left( \frac{1}{b_t^{t+1}} \right) \right] + \frac{1}{2} \text{var} \left[ \ln \left( \frac{1}{b_t^{t+1}} \right) \right] + \text{cov} \left[ \ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right), \ln \left( \frac{1}{b_t^{t+1}} \right) \right] = 0
\] (11)

Eq. (4) implies:

\( \lambda_t = U'(C_t) \)

and hence:

\( \lambda_{t+1} = U'(C_{t+1}) \)

In case of a logarithmic utility function, the marginal rate of substitution equals the negative of the growth rate of consumption, so that:

\[
\ln \left( \frac{\lambda_{t+1}}{\lambda_t} \right) = \ln \left[ \frac{U'(C_{t+1})}{U'(C_t)} \right] = -g_C
\] (12)
For a discount bond, the expected one period holding return \( (H) \) should correspond to its expected price change over the corresponding period:

\[
E(H_{t+1}^t) = E\left( \ln \frac{b_{t+1}^{t+2}}{b_{t+1}^{t+1}} \right)
\]

(13)

where the price of such a bond depends on the one period rate of interest \((i)\):

\[
\ln b_{t+1}^{t+1} = -i_{t+1}^{t+1}
\]

(14)

From (10) and (11) and making use of (12), (13) and (14):

\[
E(H_{t+1}^t) = i_{t+1}^{t+1} - \frac{1}{2} \text{var}(H_t^{t+1}) + \frac{1}{2} \text{var}(i_{t+1}^{t+1}) + \text{cov}(g_{C_i}, H_t^{t+1} - i_{t+1}^{t+1})
\]

(15)

For a logarithmic utility function, the growth rate of consumption equals the growth of wealth \((g_w)\). However, wealth itself grows with the one period total return on the wealth portfolio \((T)\) and the savings ratio:

\[
g_w = T_{t+1}^t + \left( \frac{Y - C}{W} \right)_t
\]

Total portfolio return can be expressed as the weighted sum of holding returns on riskbearing assets (having maturity longer than one period) and the one period interest rate (the remuneration on the one period asset, assumed to be the riskless asset in the absence of price risk):

\[
T_{t+1}^t = s' \left( \tilde{H}_{t+1}^t - 1_i^{t+1} \right) + i_{t+1}^t
\]

where: \( \tilde{s}' = \) vector of shares of riskbearing assets in the total portfolio and \( \tilde{H}_{t+1}^t - 1_i^{t+1} = \) vector of one period risk premia on riskbearing assets (\( I \) being the identity nature).

Therefore:

\[
g_w = s' \left( \tilde{H}_{t+1}^t - 1_i^{t+1} \right) + i_{t+1}^t + \left( \frac{Y - C}{W} \right)_t
\]

(16)

By substituting (16) into (15), neglecting the risk premia in terms of variances and considering the savings rate and the one period interest rate to be non-stochastic, the following expression for the expected holding period return on the two period domestic discount bond is obtained:

\[
E(H_{t+1}^t) = i_{t+1}^{t+1} + \text{cov} \left( \tilde{s}' \left( \tilde{H}_{t+1}^t - 1_i^{t+1} \right), (H_t^{t+1} - i_{t+1}^{t+1}) \right)
\]

which can be written as:

\[
E(H_{t+1}^t) = i_{t+1}^{t+1} + \tilde{V}' \tilde{s}_t
\]

(17)

where: \( \tilde{V}' = \) variance-covariance nature of expected risk premia.
Equation (17), shows that the expected holding period rate of return on domestic bonds equals the sum of the one period rate of interest with unitary coefficient and a risk premium that depends on the shares of domestic and foreign bonds in the total portfolio, premultiplied with the variance-covariance nature of expected returns on these assets.

This interpretation of the intertemporal CAPM stresses the importance of the supply and wealth effects in the risk premia. A more general model would include additional risk components, such as the variance of inflation and the covariance of the latter with expected returns. If then the rate of inflation is correlated with its volatility, this would suggest a positive relation between the past realised inflation rates and expected real returns on bonds.

1.2 The exchange rate

From eq. (7) follows:

\[ \rho E\left( \frac{\lambda_{t+1}}{\lambda_t} \cdot \frac{S_{t+1}}{S_t \cdot f_t^{t+1}} \right) = 1 \]

Assuming lognormal distributions, this equation can be rewritten as:

\[ \ln \rho + E\left( \ln \frac{\lambda_{t+1}}{\lambda_t} \right) + \frac{1}{2} \text{var} \left( \ln \frac{\lambda_{t+1}}{\lambda_t} \right) + E\left( \ln \frac{S_{t+1}}{S_t \cdot f_t^{t+1}} \right) + \frac{1}{2} \text{var} \left( \ln \frac{S_{t+1}}{S_t \cdot f_t^{t+1}} \right) + \text{cov} \left( \ln \frac{\lambda_{t+1}}{\lambda_t}, \ln \frac{S_{t+1}}{S_t \cdot f_t^{t+1}} \right) = 0 \]  \hspace{1cm} (18)

From (18) and (11) and neglecting risk premia in terms of variances, the following exchange rate equation is obtained:

\[ \ln s_t = E \ln (s_{t+1}) + i^* t^{t+1} - \tilde{W}_t^r s_t \]  \hspace{1cm} (19)

where \( i^* \) denotes the foreign short term interest rate, and
\( \tilde{W}_t^r \) is the variance-covariance nature of expected returns.

Equation (19) implies that the exchange rate depends on the expected future exchange rate, the short-term interest rate differential and a risk premium that depends on the shares of domestic and foreign bonds in the total portfolio, premultiplied with the covariance-variance vector of expected returns on these assets.

2. Empirical application

2.1 Bond yields

2.1.1 From holding period returns to bond yields

The theoretical derivation in Section 1.1 resulted in an expression for the expected holding period return on domestic bonds. In the empirical application, we want to explain the bond yield. Therefore the link between holding return and yield has to be clarified. Furthermore, for estimation purposes, we necessarily have to focus exclusively on a discrete time approach.
The return, \( R \), on a perpetuity paying a coupon of one unit of domestic currency, depends inversely on its price, \( P \):

\[
R_t = \frac{1}{P_t}
\]

In discrete time the expected holding return on such a bond can be approximated as:

\[
E(H_t^{t+1}) = R_t - \frac{E(R_{t+1}) - R_t}{R}
\]  

(20)

where \( \bar{R} \) is interpreted as an average return (see e.g. Mankiw, 1986, and Mankiw and Summers, 1984). Substituting (20) into (17), assuming rational expectations and applying recursive forward solution:

\[
R_t = (1 - \gamma) \sum_{k=0}^\infty \gamma^k E_i[i_{t+k} + (\tilde{Y}^{t+k})]
\]  

(21)

where \( \gamma = \frac{1}{1 + \bar{R}} \)

Equation (21) can be rewritten as:

\[
R_t = (1 - \gamma) \sum_{k=0}^\infty \gamma^k E(i_{t+k}) + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E(i - i^*)_{t+k} + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E[(\tilde{Y}^{t+k})]
\]

where \( i^* \) is the German short-term interest rate.

Taking the same expression as (21) for German interest rates, but assuming absence of a risk premium in the German long-term bond yield, results in the following domestic bond yield equation:

\[
R_t = R^*_t + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E(i - i^*)_{t+k} + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E[(\tilde{Y}^{t+k})]
\]  

(22)

where \( R^* \) is the German long term bond yield.

In the presence of transaction costs, the bond yield will not instantaneously react to its new equilibrium level \( \left( R^* \right) \). Therefore we assume a partial adjustment mechanism:

\[
R_t = (1 - \varepsilon)R^*_t + \varepsilon R_{t-1}
\]

such that eq. (22) can be rewritten as:

\[
R_t = (1 - \varepsilon) \left( R^*_t + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E(i - i^*)_{t+k} + (1 - \gamma) \sum_{k=0}^\infty \gamma^k E[(\tilde{Y}^{t+k})] \right) + \varepsilon R_{t-1}
\]  

(23)

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Eq. (23) can easily be reparameterised as a forward looking error correction model:

\[ \Delta R_t = -(1 - \varepsilon) \left( R - R^* - i + i^* - \tilde{v'} \tilde{z}' \right)_{t-1} + (1 - \varepsilon) \sum_{k=0}^{\infty} \gamma^k E \left[ \Delta i_{t+k} \right] + \sum_{k=0}^{\infty} \gamma^k E \left[ \alpha \left( \tilde{v'} \tilde{z}' \right)_{t+k} \right] + \Delta R^*_t \]  

(24)

Equations (4) and (5) clearly show that the short-term nominal interest rate differential must be related to differential growth rates in nominal consumption expenditures, or to inflation and real growth differentials taken separately. We will split expected future nominal interest rate differentials into expected future real interest rate differentials (which should be related to growth prospects) and expected future inflation differentials.

Furthermore, in the presence of transaction costs, portfolio reallocations will occur at the margin through the allocation of new savings. Therefore, the risk premia can be restated as a function of the public deficit (instead of the public debt) and in terms of the current balance of payments (instead of net foreign assets). Another reason for substituting the deficit and current account variables for the stock variables, is the forward looking character of the flow concepts. The future development of the debt ratio or the net foreign asset position is crucially dependent on the actual and expected future deficit and current account balances. It is precisely this information on the future evolution of the asset composition that is relevant for the financial markets (see also Blanchard and Fisher, 1989).

Under these conditions, eq. (24) can be rewritten as:

\[ \Delta R_t = -(1 - \varepsilon) \left\{ \left[ R - R^* - a(r - r^*) - b(I - I^*) - cB - dA \right]_{t-1} - \Delta R^*_t \right\} \]

(25)

where:
- \( r \) = real short term interest rate
- \( I \) = inflation rate
- \( B \) = government budget deficit (revenues - outlays) as a percentage of GDP
- \( A \) = current account balance (revenues - outlays) as a percentage of GDP

Equation (25) explains the domestic nominal government bond yield in terms of:
- the German long term interest rate;
- actual and expected future inflation differentials;
- actual and expected future real short-term interest rate differentials (or growth differentials);
- actual and expected future course of the domestic government budget deficit;
- actual and expected future course of the domestic current account balance;
- variances and covariances of expected returns, which are mainly related to uncertainty.
2.1.2 Estimation results

Table 1 contains the results of unit root tests for the variables that enter into the long-term part of equation (25). Dickey-Fuller and augmented Dickey-Fuller tests show that the null hypothesis that the series contain a unit root cannot be rejected, whereas the null that their first differences contain a unit root is rejected, except for the Dutch-German short-term interest and inflation differentials. Also all growth differentials seem to be stationary.

Table 1
Dickey–Fuller (DF) and Augmented Dickey Fuller (ADF) tests for unit roots
Sample 1980 I – 1995 II

<table>
<thead>
<tr>
<th>Variable</th>
<th>Levels</th>
<th>First differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
</tr>
<tr>
<td>Long-term interest difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITL-DEM</td>
<td>-1.36</td>
<td>-1.45</td>
</tr>
<tr>
<td>DKK-DEM</td>
<td>-1.18</td>
<td>-1.35</td>
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<tr>
<td>FRF-DEM</td>
<td>-1.02</td>
<td>-0.86</td>
</tr>
<tr>
<td>BEF-DEM</td>
<td>-1.02</td>
<td>-0.68</td>
</tr>
<tr>
<td>NLG-DEM</td>
<td>-2.17</td>
<td>-1.32</td>
</tr>
<tr>
<td>Short-term interest difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITL-DEM</td>
<td>-2.03</td>
<td>-1.58</td>
</tr>
<tr>
<td>DKK-DEM</td>
<td>-2.74</td>
<td>-2.46</td>
</tr>
<tr>
<td>FRF-DEM</td>
<td>-2.61</td>
<td>-1.54</td>
</tr>
<tr>
<td>BEF-DEM</td>
<td>-1.89</td>
<td>-1.22</td>
</tr>
<tr>
<td>NLG-DEM</td>
<td>-3.59 *</td>
<td>-2.66</td>
</tr>
<tr>
<td>Inflation difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITL-DEM</td>
<td>-0.99</td>
<td>-1.22</td>
</tr>
<tr>
<td>DKK-DEM</td>
<td>-1.33</td>
<td>-1.58</td>
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<tr>
<td>FRF-DEM</td>
<td>-1.21</td>
<td>-0.92</td>
</tr>
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<td>BEF-DEM</td>
<td>-1.31</td>
<td>-1.41</td>
</tr>
<tr>
<td>NLG-DEM</td>
<td>-3.81 *</td>
<td>-2.61</td>
</tr>
<tr>
<td>GDP growth difference</td>
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<td></td>
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<tr>
<td>ITL-DEM</td>
<td>-8.41 *</td>
<td>-3.61 *</td>
</tr>
<tr>
<td>DKK-DEM</td>
<td>-7.33 *</td>
<td>-3.72 *</td>
</tr>
<tr>
<td>FRF-DEM</td>
<td>-8.73 *</td>
<td>-4.03 *</td>
</tr>
<tr>
<td>BEF-DEM</td>
<td>-27.76 *</td>
<td>-10.57 *</td>
</tr>
<tr>
<td>NLG-DEM</td>
<td>-8.47 *</td>
<td>-6.97 *</td>
</tr>
<tr>
<td>Public deficit/GDP ratio</td>
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<tr>
<td>ITL</td>
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<td>DKK</td>
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<tr>
<td>FRF</td>
<td>-2.71</td>
<td>-2.59</td>
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<tr>
<td>BEF</td>
<td>-1.43</td>
<td>-2.93 *</td>
</tr>
<tr>
<td>NLG</td>
<td>-2.55</td>
<td>-3.38 *</td>
</tr>
<tr>
<td>DEM</td>
<td>-2.83</td>
<td>-3.16 *</td>
</tr>
<tr>
<td>Current account/GDP ratio</td>
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<td></td>
</tr>
<tr>
<td>ITL</td>
<td>-2.76</td>
<td>-3.91 *</td>
</tr>
<tr>
<td>DKK</td>
<td>-0.76</td>
<td>-4.04 *</td>
</tr>
<tr>
<td>FRF</td>
<td>-1.72</td>
<td>-4.59 *</td>
</tr>
<tr>
<td>BEF</td>
<td>-0.73</td>
<td>-6.55 *</td>
</tr>
<tr>
<td>NLG</td>
<td>-2.82</td>
<td>-4.98 *</td>
</tr>
<tr>
<td>DEM</td>
<td>-1.84</td>
<td>-3.16 *</td>
</tr>
</tbody>
</table>

* Indicates significant at 95 per cent. The 95 per cent critical value for the DF and the ADF–test is -2.91.
We first estimated the long-term part of eq. (25) for a number of EMS currencies: Belgium, Denmark, France, Italy and the Netherlands. The results of these time series regressions are summarised in Table 2. All coefficients have the correct sign. The null hypothesis of no cointegration is rejected.

Table 2

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Nominal long-term interest rate differential RL - RLDEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ITL-DEM</td>
</tr>
<tr>
<td>RRS differential</td>
<td>0.36</td>
</tr>
<tr>
<td>INF differential</td>
<td>0.66</td>
</tr>
<tr>
<td>CURACC / GDP</td>
<td>-0.47</td>
</tr>
<tr>
<td>PUBDEF / GDP</td>
<td>-0.32</td>
</tr>
<tr>
<td>DF-test</td>
<td>-4.81</td>
</tr>
<tr>
<td>ADF-test</td>
<td>-3.05</td>
</tr>
<tr>
<td>SER</td>
<td>1.14</td>
</tr>
</tbody>
</table>

RL long-term interest rates
RRS real short-term interest rates
INF consumer price inflation
CURACC current account of the balance of payments
PUBDEF public deficit
GDP gross domestic product.

The 95 per cent critical values are -4.7 for the DF-test and -4.15 for the ADF-test. (The hypothesis of cointegration is acceptable for NLG-DEM as not all explanatory variables are I(1).)

* Including a dummy from 1991 I to account for the discontinuity in the long-term interest rate series.

Test for equality of the risk premium coefficients over equations:

\[
\begin{align*}
\text{CURACC/GDP} & : -0.25 \quad \chi^2(4)=17.86 \quad p = 0.00 \\
\text{PUBDEF/GDP} & : -0.20 \quad \chi^2(4)=49.82 \quad p = 0.00
\end{align*}
\]

Test for equality of the risk premium coefficients over equations after multiplication with the standard error of the equation:

\[
\begin{align*}
\text{CURACC/GDP} & : -0.43 \times \text{SER} \quad \chi^2(4)=5.39 \quad p = 0.25 \\
\text{PUBDEF/GDP} & : -0.34 \times \text{SER} \quad \chi^2(4)=8.22 \quad p = 0.08
\end{align*}
\]

Theory suggests that the impact of the current account and the public deficit ratio on the risk premium in the long-term rate should depend on the degree of uncertainty about the expected returns. Therefore, it is interesting to compare the impact of these variables between countries and over different time periods.
The hypothesis of cross-country equality of coefficients of the current account ratio and the public deficit ratios is not accepted. This result possibly indicates differences in market participants' conditional degree of uncertainty across countries. If these ratios are multiplied with the standard error of the country-specific equation, as a measure of the differences in uncertainty across countries, the same hypothesis is (just) accepted.

Table 3 shows the joint-estimation results of the long-term equations after imposing equality of all coefficients across all countries and also imposing equal effects of both public deficits and current account balances (to prevent a possible multicollinearity problem). It provides information on an average EMS response of long term interest rate differentials to all explanatory variables. These results indicate that in the long run, nominal domestic bond yield differentials w.r.t. the German bond yield depend on:

- the real short term interest rate differential which, in principle, reflects differences in expected growth rates. A positive real short term interest rate differential of one percentage point increases the bond yield differentials by 22 basis points;
- the inflation differential. A positive inflation differential by one percentage point increases the bond yield differential by 61 basis points;
- the government budget deficit. Lower budget deficits, with constant current account balance, reduce the supply of bonds and, therefore, tend to lower interest rates. The impact is different across countries as it depends on the standard error of the equations. Each one percent deficit reduction in terms of GDP reduces the bond yield differential by 33 basis points multiplied by the standard error of the regression;
- the current account balance. Increasing current account balances, with constant budget deficits, augments liquidity in domestic financial markets and tends to lower domestic interest rates. Each one percent improvement of the current account balance in terms of GDP reduces the bond yield differential by 33 basis points multiplied by the standard error. This means that countries, like Belgium, where lower budget deficits are accompanied by higher current account surpluses, would tend to experience a fast narrowing of the bond yield differential.

Table 3
Restricted joint-estimation (SUR) of the static equations for different sub-periods

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Nominal long-term interest rate differential RI-RLDEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>79 IV - 95 II</td>
</tr>
<tr>
<td>RRS differential</td>
<td>0.22</td>
</tr>
<tr>
<td>INF differential</td>
<td>0.61</td>
</tr>
<tr>
<td>SER* (CURACC+PUBDEF)/GDP</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

Mean volatility for five currencies for the

| Nominal long-term interest rate differentials | 0.73 | 0.41 | 0.39 |
| Exchange rate w.r.t. DEM                     | 1.06 | 0.47 | 1.21 |

Although the static relations passed the cointegration test, it is interesting to consider the pooled regression results over different sub-periods. The long-run equation was estimated over three sub-sample periods. Table 3 compares the estimation results over the period 1986 I to 1992 II, which was characterised by relative exchange rate stability within the EMS, with those for the periods 1979 IV to 1985 IV and 1992 III to 1995 II. These results indicate that the coefficients of the risk
premia (current account and public deficit) have been markedly different in those periods. These coefficients are related to market participants’ uncertainty concerning the expected returns on domestic and foreign bonds. In a stable environment as to interest and exchange rates, these risk premia would tend to disappear. The ultimate case of stable exchange rates would occur in a monetary union. In such a world returns would, therefore, converge. During the middle period, credibility in the EMS was relatively high up to the point where some authors raised the question: "The European Monetary System: Credible at Last?" (Frankel, Phillips, 1992). Since mid-1992 uncertainty in the EMS re-emerged and the influence of risk premia led to divergences among bond yield differentials especially in those countries with relatively poor performance in terms of government budget and current account balances.

Charts 1 and 2 illustrate the relation between uncertainty and the influence of risk premia for a sample of European countries, including Belgium (B), Netherlands (N), France (F), Denmark (DK), United Kingdom (UK), Italy (IT), Spain (E), Portugal (P), Ireland (IR), Austria (A) and Sweden (S). During the period 1980 - 1994, the differentials of long-term interest rates in these countries w.r.t. Germany are strongly correlated with the aggregate risk premium. This correlation is much weaker in the relatively calm period 1986 - 1991.

Chart 1
European long term interest rate differentials and sum of public and current balances
1980 - 1994
Of course, eq. (25) illustrates that short term variations in bond yields are not only related to actual values of these long-term determinants, but equally so to market participants' expectations concerning their future evolution. The question then arises as to how these expectations are formed. In this context we should pay extra attention to stability over the different periods distinguished above. Different formulations of the expectations can probably solve the instability problem.

We investigated two major alternative assumptions in this respect: the use of an autoregressive forecasting rule, on the one hand and of a forward looking device, on the other hand.

If expectations on short-term real interest rates, inflation rates, government budget deficit ratio's and current account balance ratio's are each based on a second order autoregressive scheme containing a unit root, then eq. (25) reduces to a traditional error correction mechanism (eq. (26)). The latter can then be interpreted as a reduced form of a structural forward looking model with rational expectations and autoregressive processes generating the expectations. This formulation would be sensitive to the Lucas critique, but the real issue in this respect is the stability of the autoregressive processes, which can be tested for.

\[
\Delta R_t = -\mu [R - R^* - a(r - r^*) - b(I - I^*) - cB - dA]_{t-1} + \tau_1 \Delta (r - r^*)_t + + \tau_2 \Delta (I - I^*)_t + \tau_3 \Delta B_t + \tau_4 \Delta A_t + \tau_5 \Delta R^*_t
\]

The estimation results for this equation are shown in Table 4. The diagnostic statistics are acceptable, except for the stability test: three countries show a significant structural break after 1986. However, the most recent period does not form a special problem for the relations.
## Table 4
**Error correction model (two-step estimation)**
Sample 1980 I - 1995 II

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>ITL</th>
<th>DKK</th>
<th>FRF</th>
<th>BEF</th>
<th>NLG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.04</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(.09)</td>
<td>(.09)</td>
<td>(.05)</td>
<td>(.04)</td>
<td>(.03)</td>
</tr>
<tr>
<td>ECM-coefficient</td>
<td>-0.26</td>
<td>-0.58</td>
<td>-0.54</td>
<td>-0.46</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td>(.10)</td>
<td>(.10)</td>
<td>(.11)</td>
<td>(.11)</td>
<td>(.11)</td>
</tr>
<tr>
<td>Δ RLDEM</td>
<td>0.88</td>
<td>0.33</td>
<td>0.88</td>
<td>0.66</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(.17)</td>
<td>(.16)</td>
<td>(.10)</td>
<td>(.08)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Δ RRS-RRSDEM</td>
<td>0.13</td>
<td>0.15</td>
<td>0.12</td>
<td>0.18</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
<td>(.05)</td>
<td>(.02)</td>
<td>(.04)</td>
<td>(.05)</td>
</tr>
<tr>
<td>Δ INF-INFDEM</td>
<td>0.37</td>
<td>0.15</td>
<td>0.19</td>
<td>0.33</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(.10)</td>
<td>(.11)</td>
<td>(.08)</td>
<td>(.08)</td>
<td>(.08)</td>
</tr>
<tr>
<td>Δ CURACC/GDP</td>
<td>-0.23</td>
<td>-0.52</td>
<td>-0.74</td>
<td>-0.15</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(.24)</td>
<td>(.27)</td>
<td>(.21)</td>
<td>(.08)</td>
<td>(.07)</td>
</tr>
<tr>
<td>Δ DEFPUB/GDP</td>
<td>-0.20</td>
<td>-0.78</td>
<td>-0.11</td>
<td>-0.21</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(.12)</td>
<td>(.20)</td>
<td>(.23)</td>
<td>(.12)</td>
<td>(.12)</td>
</tr>
<tr>
<td>Δ RL{−1}</td>
<td>0.16</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.10)</td>
<td>(.10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ RLDEM{−1}</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.16)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy 1991 I</td>
<td>2.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.64)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Statistics**

<table>
<thead>
<tr>
<th></th>
<th>ITL</th>
<th>DKK</th>
<th>FRF</th>
<th>BEF</th>
<th>NLG</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.66</td>
<td>0.53</td>
<td>0.74</td>
<td>0.65</td>
<td>0.82</td>
</tr>
<tr>
<td>SER</td>
<td>0.62</td>
<td>0.67</td>
<td>0.39</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>DW</td>
<td>1.90</td>
<td>2.06</td>
<td>1.88</td>
<td>1.66</td>
<td>2.12</td>
</tr>
<tr>
<td>AR(1) : χ²(1)</td>
<td>2.97</td>
<td>1.77</td>
<td>0.51</td>
<td>3.81</td>
<td>5.42</td>
</tr>
<tr>
<td>probability value</td>
<td>(.08)</td>
<td>(.18)</td>
<td>(.48)</td>
<td>(.05)</td>
<td>(.02)</td>
</tr>
<tr>
<td>Ljung-Box : χ²(15)</td>
<td>7.73</td>
<td>13.20</td>
<td>10.25</td>
<td>18.56</td>
<td>18.50</td>
</tr>
<tr>
<td>probability value</td>
<td>(.93)</td>
<td>(.59)</td>
<td>(.80)</td>
<td>(.23)</td>
<td>(.24)</td>
</tr>
<tr>
<td>ARCH(2) : χ²(2)</td>
<td>0.63</td>
<td>1.60</td>
<td>1.22</td>
<td>4.58</td>
<td>2.11</td>
</tr>
<tr>
<td>probability value</td>
<td>(.73)</td>
<td>(.44)</td>
<td>(.54)</td>
<td>(.10)</td>
<td>(35)</td>
</tr>
<tr>
<td>Norm test : χ²(2)</td>
<td>0.15</td>
<td>3.43</td>
<td>0.18</td>
<td>4.86</td>
<td>7.71</td>
</tr>
<tr>
<td>probability value</td>
<td>(.93)</td>
<td>(.18)</td>
<td>(.91)</td>
<td>(.09)</td>
<td>(.02)</td>
</tr>
<tr>
<td>CHOW test 86:1*</td>
<td>3.59</td>
<td>2.54</td>
<td>1.46</td>
<td>3.74</td>
<td>0.74</td>
</tr>
<tr>
<td>probability value</td>
<td>(.00)</td>
<td>(.02)</td>
<td>(.20)</td>
<td>(.00)</td>
<td>(.63)</td>
</tr>
<tr>
<td>CHOW test 92:3*</td>
<td>1.13</td>
<td>0.68</td>
<td>0.46</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>probability value</td>
<td>(.36)</td>
<td>(.70)</td>
<td>(.86)</td>
<td>(.58)</td>
<td>(.58)</td>
</tr>
</tbody>
</table>

* Based on F-test, with critical values determined by F(c,n-2c), with c = number of coefficients and n = number of observations.
To give some indication of the origin of the stability problem, the dynamic equations were jointly estimated with equal coefficients over equations. The results are summarised in Table 5. Four remarks are obvious:

Table 5
Restricted joint-estimation (SUR) of the Error Correction Model (two-step estimation)

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Change in the long-term interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80 I - 95 II</td>
</tr>
<tr>
<td>ECM-coefficient</td>
<td>-0.38 (.04)</td>
</tr>
<tr>
<td>Δ RLDEM</td>
<td>0.80 (.04)</td>
</tr>
<tr>
<td>Δ RRS-RRSDEM</td>
<td>0.15 (.01)</td>
</tr>
<tr>
<td>Δ INF-INFDEM</td>
<td>0.21 (.03)</td>
</tr>
<tr>
<td>Δ SER*[CURACC/GDP+DEFPUB/GDP]</td>
<td>-0.24 (.06)</td>
</tr>
</tbody>
</table>

- the short-term impact of the DEM long rate on the other countries' long rates increased substantially and was not significantly different from one after 1986 I. This result reflects the increasing capital mobility between countries;
- the short-term interest differentials and the inflation differentials did have a stronger short-term impact on the long-term interest rates during the period of relative stability;
- the direct impacts of changes in the deficit and current account ratios were less important during the second period, but regained their impact during the most recent period;
- the adjustment speed toward the long run equilibrium was lower during the second period; this may reflect the smaller importance of the fundamental determinants of the risk premium during this period. After 1992 II, the adjustment speed increased again.

The alternative to the ECM is to estimate eq. (25) with forward looking expectations directly (using all restrictions on the coefficients) with non-linear instrumental variables. It seems that the explanatory power of the equations incorporating the forward looking expectations assumption drops dramatically, in comparison with the alternative hypothesis which retains all the dynamic restrictions included in equation (25). Therefore, the short-term coefficient for the German long rate was estimated freely (instead of estimating changes in interest differentials), and the lagged dependent variables were included to prevent a possible autocorrelation problem. Table 6 contains the results. There remains a stability problem for the long BEF-rate (Table 7). The results for DKK and ITL also indicate that at least the short-term coefficient for the DEM rate still poses a problem for stability.

We did not make any formal discriminatory test between the two assumptions concerning expectation formation. Nonetheless, it seems to us that the results in Tables 3, 4 and 6 largely support the same conclusions. The stability problem for the risk premium coefficients suggests the introduction of time-varying second moments in the equations (the absence of ARCH and the use of quarterly data prevent us from applying a GARCH-M specification).
Table 6
Estimation with forward-looking expectations (non-linear instrumental variables)*
Sample 1980 IV - 1994 III

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Change in the nominal long-term interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ITL</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.41</td>
</tr>
<tr>
<td></td>
<td>(.72)</td>
</tr>
<tr>
<td>γ discount factor (imposed)</td>
<td>0.96</td>
</tr>
<tr>
<td>ε RL-RLDEM</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
</tr>
<tr>
<td>a RRS-RRSDEM</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
</tr>
<tr>
<td>b INF-INFDEM</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
</tr>
<tr>
<td>c CURACC/GDP</td>
<td>-0.53</td>
</tr>
<tr>
<td></td>
<td>(.18)</td>
</tr>
<tr>
<td>d DEFPUB/GDP</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(.12)</td>
</tr>
<tr>
<td>ε1 Δ RLDEM</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(.23)</td>
</tr>
<tr>
<td>ε2 Δ RL{-1}</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(.10)</td>
</tr>
<tr>
<td>Dummy 1991 I</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>(.45)</td>
</tr>
<tr>
<td>Statistics</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>0.64</td>
</tr>
<tr>
<td>SER</td>
<td>1.00</td>
</tr>
<tr>
<td>DW</td>
<td>1.93</td>
</tr>
<tr>
<td>CHOW test 86 I**</td>
<td>1.07</td>
</tr>
</tbody>
</table>

* Instruments: 4 lags of all variables. Truncation after two leads.
** Based on F-test, with critical values determined by F(c,n-2c), with c = number of coefficients and n = number of observations.

Table 7
Restricted joint-estimation with forward-looking expectations

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Change in the long-term interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80 I - 95 II</td>
</tr>
<tr>
<td>γ discount factor (imposed)</td>
<td>0.96</td>
</tr>
<tr>
<td>ε RL-RLDEM</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
</tr>
<tr>
<td>a RRS-RRSDEM</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>b INF-INFDEM</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>c SER*[CURACC/GDP+DEFPUB/GDP]</td>
<td>-0.32</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>ε1 Δ RLDEM</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
</tr>
</tbody>
</table>
Chart 3
European long-term interest rate differentials and current account balances
1980-1984

Chart 4
European long-term interest rate differentials and public balances
1980-1994
Charts 3 and 4 confirm our estimation results in Table 6 by showing for some countries a weaker relationship between long-term interest rate differentials and public balances in comparison with the correlation with current account balances.

Chart 5 contains the most recent evolution of long-term interest rate differentials. In 1994 interest differentials were up again, especially in France, Denmark and Italy, but not in Belgium. Even more so, the long-term interest differential BEF/DEM declined dramatically since early 1995, in contrast to most of the other countries under review. An explanation may be advanced in terms of expectations. The Belgian strategy of linking its exchange rate to the DEM kept inflation expectations low, while its increasing current account surplus and lower public deficit led to a considerable reduction of the risk premium on Belgian bonds.

Chart 5
Long-term bond yield differentials w.r.t Germany
Benchmark data

2.2 The exchange rate

The DEM/BEF exchange rate equation in the Quarterly model of the NBB, is based on eq. (19). The explanatory variables are the expected exchange rate, the short-term interest rate differential and one risk premium: net foreign assets, approximated by the cumulated current account balance (CCA) and the cumulated official interventions in the exchange market (CINT), multiplied by a variable conditional variance (H). The latter was constructed in a rather ad hoc way. It depends on the probability, as perceived by market participants, that the monetary authority is of the hard currency type. The longer the time span since the last devaluation against the DEM, the higher this probability and therefore, the lower H. The same type of reasoning applies to the modelling of the expected exchange rate. Before 1990 the expected exchange rate is determined by the slowly
increasing probability of the Belgian monetary authorities evolving towards a strong currency policy, from time to time interrupted by a devaluation against the DEM. Thereafter, the expected exchange rate is affected by the official announcement of the DEM-link, such that the expected exchange rate gradually converged towards the DEM-EMS parity rate.

The long-run exchange rate equation is estimated as follows:

\[ \ln s_t = E(s_{t+1}) + 0.36(t^*$ $- i_t) + 2.59H_t CCA_t - 1.05H_t CINT_t + \mu_t \]

Conclusion

Starting from an intertemporal optimal consumption - saving - portfolio allocation model, it was shown that the holding period return on domestic and foreign bonds depends on the short term risk free rate of interest (which is risk free because it does not contain any price risk) and on risk premia. These risk premia depend on the degree of uncertainty with which market participants hold their expectations concerning future returns; or, more generally, on the volatility in the financial markets. These premia also depend on the shares of domestic and foreign bonds in the total portfolio.

This analysis was applied to the explanation of bond yield differentials w.r.t. German yields (in the perspective of an application to EMS currencies). When allowing for transaction costs, it was shown that these differentials depend on actual and expected future inflation differentials, actual and expected future real short-term interest rate differentials, which are theoretically related to growth differentials, the actual and expected future course of government budget ratio’s and of current account balance ratio’s and finally on the degree of uncertainty or financial market volatility.

We estimated the average long run EMS responses of long-term interest rate differentials to all of these explanatory variables. The estimation results seemed to accord with theory. One result indicated that the average EMS response of bond yield differentials to the public balance and current account balance ratio’s were about equal across countries after multiplication of these coefficients by the standard error of the equations. The influence of the risk premium, however, disappears in periods of low exchange rate and interest rate volatility, such as from 1986 to mid 1992. In such period the inflation differential was found to be the most important factor of bond yield differentials.

As far as expectation formation is concerned, we investigated two alternative assumptions. The first one assumes that market participants base their forecasts of the determining variables on autoregressive processes. A traditional ECM mechanism then describes the dynamic adjustment of bond yields. The alternative assumption relies on forward looking expectations and a dynamic equation in terms of forecasts was estimated with non-linear instrumental variables. We did not perform rigorous discriminatory tests between the two assumptions, but they both lead to the same conclusions.

The theoretical analysis concerning exchange rate determination revealed dependence of the exchange rate on the expected future exchange rate, short-term interest rate differentials, and the same type of risk premium as was found for holding period returns on domestic bonds. Estimation of the DEM/BEF exchange rate confirms the importance of the short-term interest rate differential as well as important effects of the current account balance and financial market volatility.
References


Comments on paper by M. Dombrecht & R. Wouters by Frank Smets (BIS)

In this paper the authors examine both theoretically and empirically the main determinants of bond yield differentials in Belgium, the Netherlands, Denmark, France and Italy vis-a-vis Germany. The most interesting result in the empirical work is that both the public deficit and the current account are important determinants of bond yield differentials, in particular in periods of higher uncertainty. While similar results have been found previously, the robustness across countries is striking. In my comments I will first discuss the adequacy of the theoretical framework the authors present to motivate their estimated equations. I will then propose a different framework to think about the parameter estimates and discuss within that framework the results with respect to the effects on bond yield differentials of the current account, inflation differentials and the government budget deficit. Finally, I will say a few words about the importance of credit or default risk.

A. The authors motivate the inclusion of the current account in their estimated equations in terms of a portfolio balance model in which the risk premium is a function of the variance-covariance matrix of the excess returns on the various risky assets and the shares in the total portfolio of each of the risky assets.

I have doubts on whether this is the appropriate theoretical framework to motivate the estimated equation for bond differentials for two reasons. First, we know from more direct tests of the international CAPM model that it is hard to make it work. A recent survey by Charles Engel on the foreign exchange risk premium, for example, lists six or seven studies which test the implications of this model and find very poor results. The fit is terrible and sign errors are everywhere. Second, while the asset pricing equations are rigorously derived from an intertemporal saving and portfolio allocation model, a partial adjustment argument is necessary to derive the estimated equation. Given the efficiency of international asset markets it seems to me that the assumption of a relatively slow adjustment of asset prices is rather implausible. In the alternative framework which I discuss below a dynamic adjustment model, as considered in the paper, may be justified when credibility is imperfect and there is learning about the true type of the government.

B. This brings me to the second major point. I find it a bit strange that in the theoretical framework that the authors present there is almost no mention of the role of the exchange rate regime. Given that all the countries analysed in the paper were members of the ERM and attempted to fix the exchange rate with respect to the DM, I would expect that most of the variations in the long-term interest rate differential are determined by changes in the credibility of the respective exchange rate parity. Thus, a more appropriate theoretical framework would try to model such devaluation expectations. My interpretation of the empirical results the authors present is that each of the variables that enter the bond yield differential equation have their primary effects because they affect devaluation expectations. This can also explain why the significance of the effects varies across periods when the overall credibility of the fixed parities in the ERM differs. If the fixed exchange rate is fully credible, then the interest rate differential should be close to zero (primarily reflecting a default premium) and all the parameters should be insignificant. In what follows we elaborate on how the current account, the inflation differential and fiscal variables may affect devaluation expectations.

1. The current account

The current account and the size of the net external debt are important factors in the determination of devaluation expectations. The link between bond yields and net external indebtedness is illustrated in the following graph. The main reason for such a link is clear. If a country has an external sustainability problem, one of the easiest ways of solving this is to devalue the exchange rate which, if the pass-through in domestic prices is imperfect would improve the trade balance and stop the accumulation of external debt. That this is not just a theoretical possibility was visible in 1994 when there was a clear positive correlation between the degree of exchange rate
overvaluation (as measured by deviations from purchasing power parity) and the current account balance.

**Debt, deficits and long-term interest rates**

17 industrial countries

![Graphs showing relationship between long-term interest rate and government deficit vs. net external debt](image)

2. The inflation differential

One would expect that in a floating exchange rate regime and with a long enough sample, the coefficient on both the real interest rate difference and the inflation difference would be insignificantly different from one. However, in a fixed exchange rate regime, this is not necessarily the case. There can be temporary factors that drive a wedge between inflation rates in the two countries (e.g., the German reunification boom). However, if the fixed exchange rate parity is credible, this should not lead to an interest rate differential. For example, Halikias (1993) finds that over the period 1982-1992 the inflation differential is significant in Belgium (with a coefficient of 0.45), but insignificant in the Netherlands and Austria where the credibility of the fixed exchange rate parity was higher. He also shows that Belgium has been moving towards this strong version of credibility during the period under consideration, as the inflation differential becomes insignificant towards the latter part of the period. Finally, Halikias (1993) also shows that it is really competitiveness that explains the significance of the inflation differential, which again indicates the appropriateness of the devaluation expectations hypothesis.

3. The importance of fiscal variables

Although the time series data do not give a lot of evidence in favour of a clear link between deficits and bond yield differentials, there is quite a lot of cross-country evidence that government deficits matter as e.g. illustrated by the above graph. For the evidence on Belgium, I would again like to refer to the study by Halikias (1993) who finds that both the relative debt and the relative primary deficit turn out to be a statistically significant determinant of the bond yield differential with Germany. Moreover, he shows that this effect remains, even if one controls for its impact on inflationary expectations and hence expected exchange rate movements.
C. This brings me to a last point which concerns the presence of default risk. Several pieces of evidence suggest that fiscal variables have an impact on the bond yield differential beyond their impact on inflation or exchange rate devaluation expectations. Next to the evidence presented above under B.3., it appears that it is total government debt, and not necessarily local currency denominated debt that matters for long-term interest rate differentials. Second, high-debt countries typically face higher interest rates on foreign currency bonds than e.g. comparable bonds issued by the World Bank. While the authors interpret this premium as a portfolio balance premium, I would prefer to call this a credit or default risk premium. Some evidence in favour of the latter interpretation is that one can find a positive correlation between measures of such a premium, debt variables and indicators of political stability.
The determination of long-term interest rates in the Netherlands

Peter J.A. van Els and Peter J.G. Vlaar

Introduction

The determination of long-term interest rates in the Netherlands presents a case which may be characteristic for small open economies maintaining a fixed exchange rate with an anchor country. In the typical standard textbook situation, under the assumption of perfectly integrated capital markets, the spread between the domestic and the anchor country's nominal long-term interest rates will reflect expected exchange rate changes and risk premia. In this paper on the Dutch long-term interest rate, the assumption of perfectly integrated capital markets is not imposed a priori, but viewed rather as a hypothesis which has to be confirmed by empirical evidence. As we will argue, in the Dutch case with Germany as the anchor country, it is difficult to find a satisfactory empirical specification for this model of long-term interest rate determination, at least for the entire period since the establishment of the EMS (1979-1994). The empirical evidence on the Dutch nominal long-term interest rate presented here does not point to perfectly integrated Dutch and German capital markets, although the German long-term interest rate is found to be by far the most dominant factor in explaining its Dutch counterpart. The failure to find fully integrated capital markets may be due, for instance, to transaction and information costs, the existence of restrictions on foreign portfolio investments by institutional investors, differences in the taxation of capital income and the higher liquidity of the German bond market. As a result, the Dutch nominal long-term interest rate is partly affected by domestic economic conditions as signalled by variables such as the short-term interest rate, the inflation rate, the government financial deficit, and the current account. Indeed, there exists a large empirical literature of models of the long-term interest rate in open economies, the Netherlands in particular, explaining a role for domestic economic conditions (e.g. Fase and Van Nieuwkerk, 1975; Knot, 1995; Correira-Nunes and Stemitsiotis, 1995; Fase and Van Geijlswijk, 1996).

In the approach pursued in this paper, the short-term interest rate is one of the domestic variables affecting the long-term rate. Hence, developments and sentiments in the exchange market affect the determination of the long-term interest rate through the response of the short-term interest rate, which is closely linked to the policy-controlled interest rate. In view of the interdependencies between exchange, money and capital markets a three equation system is presented featuring the guilder/D-mark exchange rate, the short-term interest rate and the long-term interest rate as endogenous variables. The equations are estimated using quarterly data. Section 1 provides a further analysis and background of the empirical results, including an investigation of simulation properties, a decomposition analysis of the direct causes of movements in the exchange rate and interest rates, and a comparison with other studies. Section 2 presents some impulse response exercises, showing the response of interest rates to changes in domestic and foreign fundamentals. In order to allow for various feedback mechanisms, the three equations are embedded in a larger model of the Dutch economy, i.e. the Bank's quarterly macroeconomic policy model MORKMON (Fase et al., 1992). For the analysis of a change in the German price level, the accompanying response of the German interest rates is computed using the Bank's new model EUROMON of the EU-countries (Boeschoten et al., 1995). The final section concludes the paper.
1. Empirical results

1.1 The guilder/D-mark exchange rate

The Netherlands has a long monetary policy tradition in fostering exchange rate stability. Since Germany is by far the most important trading partner of the Netherlands and the Deutsche Bundesbank has a solid low-inflation reputation, maintaining a stable guilder/D-mark rate, in accordance with relative competitiveness, has always been, and still is, considered of major importance (e.g. Wellink, 1994). With the collapse of the Bretton-Woods system in the early seventies, the guilder/D-mark peg was enhanced by the so called "Snake Agreement". Within the snake, in which seven other European countries also participated, bilateral exchange rate movements were limited to stay within relatively narrow bands of plus or minus 2.25%. In March 1979 the Snake was replaced by the European Monetary System (EMS).

Although exchange rate stability, and, therefore, a stable guilder/D-mark rate, has been the focus of Dutch monetary policy for several decades, the way real exchange rate stability is achieved has changed with the introduction of the EMS. Until 1979, inflation rates were higher in the Netherlands than in Germany (Figure 1). From time to time the central parities were realigned to (partly) offset price differentials. Within the EMS, the Netherlands pursued a strict guilder/D-mark peg and more emphasis was laid on economic convergence, to avoid parity realignments. As a consequence, Dutch inflation rates converged to German ones (e.g. Berk and Winder, 1994). Realignments became rare and from March 1983 on they were even absent. Since the end of the eighties, Dutch inflation rates have on average been lower than German ones, resulting in a slight real depreciation of the guilder/D-mark rate. By the end of 1994 the real guilder/D-mark rate was still lower than the one in the early seventies, however (Figure 1).

Figure 1
Price differential between the Netherlands and Germany and nominal and real D-mark rate

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal D-mark rate</th>
<th>Real D-mark rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>1980</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>1990</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>2000</td>
<td>0.8</td>
<td>0.7</td>
</tr>
</tbody>
</table>
The model for the guilder/D-mark rate $e_{DM}$, i.e. the value of the D-mark measured in guilders, is given by equation (1) below. It is based on both purchasing power parity (ppp) and uncovered interest parity (uip). According to the ppp-framework the expected long-run exchange rate depends on the price level ratio $p_c/p_c^{GE}$. The uip-condition implies that the difference between the Dutch and German interest rate equals the expected change of the exchange rate plus a risk premium. As could be expected from Figure 1, the hypothesis of relative purchasing power parity, here interpreted as a coefficient of 1 for the log of the price ratio, has to be rejected. In the long run, two thirds of a price differential is compensated for by a change in the exchange rate. A possible explanation for the less than complete compensation is that the consumer price indices used in this study are not representative of the price of tradables. Another explanation might be that authorities did not want to fully offset price differentials by means of parity realignments in order to enhance domestic policy discipline and to prevent a further divergence of inflation performances between the two countries due to imported inflation. In any case, changes in price differentials do not have to result in exchange rate changes as long as one is willing to maintain a higher (lower) short-term interest rate $r_k$ relative to the German short rate $r_i\text{E}$ in case of a positive (negative) price differential, thereby offsetting the exchange rate risk for international investors.

$$\Delta \ln e_{DM} = -0.0051 (r_k - r_i^{GE}) - 0.1725 \ln e_{DM,-1} - 0.6662 \ln \left( \frac{p_c}{p_c^{GE}} \right)_{-1} - 0.1645 \text{dum}_{7083} \Delta \ln e_{DOL} + 0.215$$

Sample: 1972Q1-1994Q4 SE = 0.0082 Q(12) = 17.10

Until 1983 the guilder/D-mark rate was also affected by the strength of the dollar. As international investors preferred the D-mark to the Dutch guilder, a depreciation of the D-mark/dollar rate, $e_{DOL}^\text{DM}$, resulted in a higher demand for D-mark investments rather than guilder investments, thereby weakening the guilder relative to the D-mark. After the last devaluation of the guilder, this link could no longer be detected.

Other potential explanatory variables not included in (1) are the central parity and the current account. A significant impact of the current account on the exchange rate could only be found for the unlagged one quarter current account balance. However, although insignificant, the coefficients of the one year balance had the wrong sign. Since current account data are published with a long time lag, just including the unlagged one quarter deficit would be undesirable on economic grounds. In addition, the fact that export and import data display clear seasonal differences makes the one quarter deficit hard to interpret. The influence of the lagged central parity was not significant, probably because the parity has been realigned several times over our sample. Moreover, the impact of the price differential could no longer be found if the parity was included. Neither could it be detected if the sample was restricted to the EMS-period. This is probably due to the small changes in the differential over this sample.

In Figure 2, a dynamic simulation of the guilder/D-mark rate, $Ee_{DM}$, is shown together with its actual realisation, $e_{DM}$, and the central parity, $c_{DM}$. The dynamic simulation gives a good prediction of the actual exchange rate movements, in particular since 1988.

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1 To avoid simultaneity problems, the equation was estimated by two-stage least squares. The one period lagged interest rate differential was used as an instrument for the interest rate differential.
1.2 The short-term interest rate

The short-term interest rate in the Netherlands (three-month euro-deposit rate) is to a large extent controlled by the central bank. As the direct or intermediate target of Dutch monetary policy since the start of the EMS has been a stable guilder/D-mark exchange rate, Dutch interest rate policy is primarily dictated by German monetary policy and the strength of the guilder relative to the D-mark. Hence, changes in the official German interest rates almost always lead to similar changes in the Netherlands. If the strength of the guilder, measured by the distance of the guilder/D-mark rate from its central parity, diminishes, the short-term interest rate differential with Germany has to rise. Also, if there are signs that the exchange rate is, or will become, overvalued, interest rates may have to rise since international investors will then demand a higher risk premium. Therefore, an increase in inflation relative to Germany and a weakening of the current account are likely to increase Dutch interest rates. Due to the fluctuation margin around the central parity, the Dutch central bank has some room for manoeuvre left. If the guilder is strong relative to the D-mark, the Dutch central bank may lower its policy-controlled interest rates independently from the Bundesbank. Further requirements here are that inflation is (expected to remain) low - the ultimate objective of monetary policy - and that the position of the current account is appropriate. Likewise, if inflation performance is (expected to be) poor, the Dutch central bank may raise interest rates independently from Germany.

\[
\Delta \eta_k = \Delta \eta_k^{GE} - 70.42 \Delta \left( \frac{\sum_{j=0}^{3} (B - M)_{t-j}}{\sum_{j=0}^{3} (B + M)_{t-j}} \right)
\]

\[+ 96.92 \Delta \left( (e_{DM} - c_{DM})/c_{DM} \right)_{t-1} - 0.8238 (r_{k-1} - 0.8019 \eta_{k-1}^{GE}) \]

\[+ 26.68 \left( \frac{\sum_{j=1}^{4} (B - M)_{t-j}}{\sum_{j=1}^{4} (B - M)_{t-j}} \right) - 121.24 \left( (e_{DM} - c_{DM})/c_{DM} \right)_{t-2} \]

\[-0.1786 \hat{p}_{c-1} - 2.3158 \]

Sample: 1979Q2-1994Q4  \quad SE = 0.3525  \quad Q(12) = 10.76
Given these considerations, the following reaction function for the short-term interest rate, given by (2), is postulated. In the short run, changes in German short-term interest rates are fully transmitted to Dutch short-term interest rates. In the long run, due to the limited room for manoeuvre provided by the fluctuation margin, the hypothesis of complete domination of the German interest rate has to be rejected. Only 80% of a change in the German interest rate level is ultimately transmitted directly to the Dutch rate. In addition, a higher current account surplus, defined as the one year exports, \( B \), minus imports, \( M \), of goods and services scaled by exports plus imports, results in a lower short-term interest rate. This effect may either reflect a risk premium, demanded by international investors, or the central bank policy not to lower interest rates independently from the Bundesbank in case of a possible current account deficit. The short-run effect is higher than the long-run effect, which could point towards a learning effect of market participants concerning the importance given by the authorities to the current account deficit. No direct effect of the deviation of the D-mark rate from its central parity \( c_{DM} \) could be found. When included, the sign of the coefficient was wrong. This may be due to a simultaneity problem. If the guilder is weak, the Dutch interest rate is expected to rise, but if the Dutch interest rate rises the guilder strengthens. With a time lag of one quarter the strength of the guilder is a very important determinant of the Dutch short-term interest rate, however.

Finally, the Dutch inflation rate \( \hat{p}_c \) affects the Dutch short-term interest rate only in the long run. No effect could be found for the German inflation rate. This probably indicates that inflation differentials were no cause for risk premia, which could be explained by the small magnitude of this differential over the sample period. On the other hand, cumulative inflation differentials, resulting in price level differentials, do affect the short-term interest rate through the response of \( e_{DM} - c_{DM} \). The separate domestic inflation effect reflects the high priority the authorities give to inflation as the ultimate objective of monetary policy.

In Figure 3, the short-term interest rate differential between the Netherlands and Germany is shown, together with the dynamically simulated differential, \( Er_k - r_k^{GE} \), and the German rate. Although the German interest rate is by far the most important determinant of the Dutch interest rate, the graph clearly illustrates the significance of the other variables as well. The dynamically simulated interest rate differential closely resembles the actual differential, which in turn clearly deviates from zero most of the time.

Figure 4 shows the contribution of the domestic explanatory variables to the dynamically simulated short-term interest rate (as deviation from the average contribution). The strength or weakness of the guilder, measured by \( e_{DM} - c_{DM} \), was very important until the mid eighties. The difference between its highest and its lowest contribution to the determination of the short-term interest rate is about 3.5 percentage points. For the current account, \( B: - M \), this difference amounts to almost 2.0 percentage points, whereas for the inflation rate it is 1.5 percentage points.

---

2 The equation for the short-term interest rate is estimated together with the one for long-term interest rates, by means of iterated three stage least squares. The same set of instruments was used for both equations. For the unlagged variables, other than the German interest rates, the one period lagged equivalents were used as instruments.
Figure 3
German short-term interest rate and actual and simulated differential with the Netherlands

Figure 4
Contribution of domestic influences to the dynamically simulated short-term interest rate
As a deviation from the average contribution
1.3 The long-term interest rate

The Dutch long-term interest rate $r_t$, represented by the yield on ten-year government bonds, is largely determined by its German counterpart $r_{t,GE}$. As there are no capital controls effective in the markets for either Dutch or German bonds, one should expect all deviations between Dutch and German long-term interest rates to be accounted for by expected depreciations and risk premia. These risk premia may represent both a devaluation risk or other factors such as for instance liquidity. As the liquidity of the Dutch bond market is not as high as that of Germany, Dutch interest rates will be slightly higher. In practice however, the markets for Dutch and German bonds are not integrated completely. Many institutional investors, for instance, are restricted in the relative amounts they are allowed to invest abroad. Also, the presence of transaction and information costs will contribute to some degree of segregation of bond markets. The fact that world capital markets are less than perfectly integrated in practice can also be deduced from the well documented fact that the share of domestic assets in the portfolios of investors is much too high according to diversification motives (e.g. Hatch and Resnick, 1993). Owing to the segregation, domestic economic conditions still play an important role in the formation of long-term interest rates, over and above the role they play in the determination of the risk premium. In our model the less than perfect integration of Dutch and German bonds markets results in coefficients for the German interest rates that are significantly smaller than 1.

In equation (3) below, which is based on a loanable funds framework, the relevant domestic factors determining the long-term interest rate are the short-term interest rate, inflation and the one year government deficit, $D_t$, scaled by gross domestic product, $Y_t$. The relevance of the short-term interest rate also follows from the term structure theory, according to which the long-term interest rate reflects the expected development of future short-term interest rates. The current inflation rate reflects the expected future inflation rate which is an important component of nominal interest rates. In the loanable funds approach, the government deficit is an important determinant of the demand for long-term funds. Unless the supply of funds schedule is infinitely elastic with respect to the long-term interest rate (i.e. through perfect substitutability between domestic and foreign assets) and unless full Ricardian equivalence holds, a higher demand for long-term funds by the government ceteris paribus increases the long-term interest rate.

$$
\Delta r_t = 0.1343 \Delta r_k + 0.8361 \Delta r_{k,GE} - 0.4238 (r_{t-1} - 0.1844 r_{k-1}) \\
- 0.6804 r_{t-1}^{GE} - 0.1869 p_{t-1} + 15.72 \left( \frac{\sum_{j=0}^{3} D_{t-j}}{\sum_{j=0}^{3} Y_{t-j}} \right)_{t-1} \\
- 0.2977
$$

(2.7) (13.1) (5.4) (3.5) (7.0) (4.5) (2.6) (0.8)

Sample: 1979Q2-1994Q4  SE = 0.1781 Q(12) = 12.53

Moreover, a high government deficit may induce future governments to inflate the debt burden. This both increases the risk premium demanded by foreign investors and the nominal interest rate demanded by domestic investors.

Apart from these variables, others were included as well, but were found to be insignificant. The influence of the German inflation rate turned out to be negligible. The irrelevance of this variable means that the effect of domestic inflation cannot be explained by a loss of competitiveness. Segregation of bond markets seems to be more important than exchange rate risk premia for the Netherlands. Another possible candidate for the exchange rate risk premium, the current account, was not significant either. A possible explanation for the lack of significance of this variable could be that there were no sustained periods of current account deficits over the sample period. Finally, the influence of interest rate volatility turned out to be insignificant as well. This
might be due to the resemblance of the volatility patterns of Dutch and German bonds. Therefore, the influence of volatility on Dutch interest rates is already captured by the German rate.

Figure 5 depicts the dynamically simulated long-term interest rate differential between the Netherlands and Germany, \(Er_t - r_t^{GE}\), together with the realised differential and the German long rate. Although the German interest rate is by far the most important determinant of the Dutch rate, domestic influences cannot be discarded. The actual differential was positive most of the time, and in line with the model predictions.

Figure 5

German long-term interest rate and actual and simulated differential with the Netherlands

In Figure 6 the contributions of the three domestic factors to the Dutch long-term interest rate are shown. The short-term interest rate and the Dutch inflation rate are the most important. Their contributions show fairly similar patterns, which is not very surprising since monetary authorities will change short-term interest rates in response to (anticipated) changes in inflation rates. It is interesting to see that the interest rate effect precedes the inflation effect by almost a year most of the time. The influence of the government deficit is also substantial, as the contribution of the deficit was over 0.5 percentage point higher in 1983 than it was in 1992.

Table 1 provides a summary of recent empirical research with respect to the Dutch long-term interest rate. The coefficients reported refer to the long-run or equilibrium impact of the explanatory variables on the level of the long rate. In five out of the eight studies considered (including the present), the German long-term interest rate is the dominating explanatory factor. For this variable, Knot (1995) reports the highest coefficient (0.96), but this perhaps reflects the fact that his model does not allow for domestic term structure effects. Boeschoten (1989) reports the lowest coefficient for the German long rate (0.60), but also allows for a separate effect of the US long rate.
Figure 6

Contribution of domestic influences to the dynamically simulated long-term interest rate
As a deviation from the average contribution

![Graph showing contributions of different factors to the long-term interest rate.]

Table 1

Comparison of recent estimated long-term interest rate equations for the Netherlands

<table>
<thead>
<tr>
<th>Author, model and sample</th>
<th>Long-term coefficient of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long rate GE</td>
</tr>
<tr>
<td>Boeschoten (1989)</td>
<td>0.60</td>
</tr>
<tr>
<td>1980Q1-1987Q3</td>
<td></td>
</tr>
<tr>
<td>MORKMON II</td>
<td>0.83</td>
</tr>
<tr>
<td>1979Q1-1987Q4</td>
<td></td>
</tr>
<tr>
<td>Douven (1995)</td>
<td>..</td>
</tr>
<tr>
<td>1960-1991</td>
<td></td>
</tr>
<tr>
<td>Knot (1995)</td>
<td>0.96</td>
</tr>
<tr>
<td>1960-1991</td>
<td></td>
</tr>
<tr>
<td>Fase/Van Geijlswijk (1995)</td>
<td>0.75</td>
</tr>
<tr>
<td>1979Q2-1991Q4</td>
<td></td>
</tr>
<tr>
<td>EUROMON</td>
<td>..</td>
</tr>
<tr>
<td>1971Q2-1992Q4</td>
<td></td>
</tr>
<tr>
<td>Correira-Nunes/Stemitsiotis (1995)</td>
<td>..</td>
</tr>
<tr>
<td>1979-1993</td>
<td></td>
</tr>
<tr>
<td>This study</td>
<td>0.68</td>
</tr>
<tr>
<td>1979Q2-1994Q4</td>
<td></td>
</tr>
</tbody>
</table>

¹ Coefficient of inflation differential with Germany.
² Effect of an increase in the current account by 1% of GNP.
³ Effect of an increase in the net excess demand for funds in the domestic capital market by 1% of GNP.
⁴ Effect of a 1% increase in the capacity utilisation rate.
Douven (1995) and EUROMON (Boeschoten et al., 1995) concentrate on term structure and inflation effects in long-run equilibrium, with foreign long-term interest rates having a direct impact in the short run only. Correira-Nunes and Stemitsiotis (1995), too, focus on the domestic short-term interest and inflation rate as explanatory factors. In the equations featuring the inflation differential with Germany, the direct impact of domestic inflation on the long-term interest rate is rather weak compared to the equations which include the domestic inflation rate only. In the equations which include both domestic term structure effects and the German long-term interest rate, the coefficients reported for the short-term interest rate are close to 0.2. Both the present study and those by Knot and by Correira-Nunes and Stemitsiotis report a significant positive influence of the government financial deficit on the long-term rate. The fact that in the latter two a much stronger impact has been found (0.56 and 0.50, respectively, versus 0.16 in the present study) may in addition to the use of a different specification and annual data, be attributed to the longer sample period, which also covers the sixties (Knot) and seventies. Indeed, these findings are in line with the simulation effects of a 1% higher budget deficit (relative to GNP) on the long-term interest rate according to a range of Dutch econometric policy models whose sample periods only include the sixties and seventies (Van Loo, 1984). In those decades, capital mobility and the international integration of capital markets were still fairly limited. Hence, domestic economic conditions had a relatively large impact on interest rates. The equation of the Nederlandsche Bank's model of the Dutch economy MORKMON II (Fase et al., 1992) also allows for a small effect of public financial policy on interest rates via the response of the net excess demand for funds in the domestic capital market.

Apart from the equations reported in Table 1, which are of the reduced-form type, various models of the Netherlands' economy exist in which the long-term interest rate clears the domestic capital market. Examples of these models are the Central Planning Bureau's model FREIA-KOMPAS (Van den Berg et al., 1988), CESAM (Kuipers et al., 1990), DUFIS (Sterken, 1990), and more recently the IBS-CCSO model (Jacobs and Sterken, 1995). According to these models, changes in foreign long-term interest rates have a strong impact on Dutch long rates, as is the case for most of the equations presented in Table 1. Moreover, for FREIA-KOMPAS and CESAM, an increase in the government financial deficit by 1% of national income leads to a rise in long-term interest rates of about 0.35 percentage points and 0.20 percentage points, respectively. According to DUFIS, the increase in the long rate amounts to over 1.7 percentage points, which may be considered a rather extreme result. A similar exercise with the IBS-CCSO model is not available. Other benchmark simulations based on that model, however, indicate rather weak interest rate responses to domestic policy actions.

1.4 Dynamic system simulations

Dynamic simulations with the three-equation system presented above provide further information on its stability when shocks to one equation are allowed to influence all three dependent variables over time, as is the case in reality. If the dynamic interdependencies between the exchange and interest rates in the model system are such that lasting or systematic differences between the simulated and observed values occur, this would question the quality of the model. Figures 7 to 9 show the observed and simulated paths of the exchange rate, the short-term and the long-term interest rates, respectively. Of particular interest are Figures 7 and 8, as they indicate substantial forecast errors in the 1979-1985 period for the guilder/D-mark exchange rate and the Dutch short-term interest rate. These errors mainly originate from relatively large residuals in the exchange rate equation. However, the deviations, though persistent in the short run, are by no means systematic and die out in the course of time. Since the mid eighties, the deviations for the exchange rate and short-term interest rate almost never exceed the level of 1 per cent and 1 percentage point, respectively. For the long-term interest rate, the simulated values are quite close to the observed ones. It must be noted, however, that the simulations are based on the strong assumption that all explanatory factors other than the exchange rate and domestic interest rates are exogenous and deterministic variables.
Figure 7
Actual and dynamically simulated guilder/D-mark exchange rate in the three-equation system

Figure 8
Actual and dynamically simulated short-term interest rate in the three-equation system
2. Changes in fundamentals: evidence from impulse responses

This section analyses the impulse responses of Dutch interest rates and the exchange rate of the guilder vis-à-vis the D-mark to changes in fundamentals, concentrating on domestic fiscal policy and a German price increase. The impulse responses are computed by including equations (1)-(3) presented above in the Bank's macroeconometric model MORKMON II. Hence, the endogeneity of the domestic factors affecting the exchange and interest rates is explicitly taken into account. In the case of the German price increase, accompanying responses of the German short- and long-term interest rates have been computed using the Bank's model of the EU-countries EUROMON. The simulation period is 1990,Q1-1994,Q4, being the most recent period for which actual data are available on a consistent basis. Owing to the nearly linear character of the model, the effects reported in the tables below would be very much the same for other simulation periods.

Table 2 presents the impulse responses to an increase in government expenditure by 1% of GDP. This increase is attended by a lower current account balance by 0.8% of GDP. For this reason, the short-term interest rate rises by about 10 basis points. Since a plausible and significant impact of the current account on the exchange rate of the guilder could not be established empirically, a depreciation of the exchange rate does not occur. Instead, the higher short-term interest rate leads to a small appreciation of the guilder vis-à-vis the Deutsche mark, which in turn mitigates the increase in the short rate. The fiscal impulse also leads to a increase by the government financial deficit by 0.75% of GDP and a gradual rise of the price level, which stabilises at about 0.20 per cent above base level. As a result, the long-term interest rate rises, also reinforced by the term structure effect of the increase in the short-rate. Eventually, the long-term interest rate is 15 basis points above base level. This result is broadly in line with the outcomes for the models FREIA-KOMPAS and CESAM mentioned earlier.

---
3 Boeschoten and Van Els (1995) analyse the model's monetary transmission channels.
Table 2

Effects of a permanent increase in government expenditures by 1% of GDP
Effects measured in percentages, unless stated otherwise

<table>
<thead>
<tr>
<th>Variable</th>
<th>Effects after</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year</td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.46</td>
</tr>
<tr>
<td>Private consumption deflator</td>
<td>0.02</td>
</tr>
<tr>
<td>Unit labour costs, enterprises</td>
<td>-0.22</td>
</tr>
<tr>
<td>Government financial deficit (% of GDP)</td>
<td>0.76</td>
</tr>
<tr>
<td>Current account balance (% of GDP)</td>
<td>-0.69</td>
</tr>
<tr>
<td>Guilder/D-mark exchange rate*</td>
<td>0.09</td>
</tr>
<tr>
<td>Short-term interest rate (% points)</td>
<td>0.09</td>
</tr>
<tr>
<td>Long-term interest rate (% points)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

* + = appreciation of guilder.

Table 3 summarises the results of a permanent increase in the German price level by 1 per cent. According to the model EUROMON, this impulse is attended by an increase in the German short and long rates by 62 and 21 basis points, respectively, in the first year. The increase in the German price level relative to Dutch prices leads to a small appreciation the guilder vis-à-vis the D-mark, despite the fact that the rise of the German short rate exceeds that of its Dutch counterpart. In the second year, the German short rate approaches its base level again, as inflation returns to base value. Due to the strong position of the guilder, the Dutch short-term interest rate remains 13 basis points below the German short rate. In the first year, the Dutch long-term interest rate rise is slightly higher than the rise of the German long rate. The aggregate impact of both the domestic short rate and the German long rate implies somewhat stronger term structure effects in the Netherlands. From the second year on, when inflation stabilises, the same mechanism results in lower Dutch long-term interest rates relative to their German counterparts. All in all, Dutch and German long-term rates move closely in line. An additional sensitivity analysis shows that the outcomes in Table 3 are robust to changes in the semi-elasticity of the German short-term interest rate with respect to inflation. Indeed, doubling the long run value of this elasticity in EUROMON from 0.65 to 1.3, which typically has been reported by others in the literature (Willms, 1983; Vlaar, 1994; Stokman and Schächter, 1995), only leads to minor changes.

Table 3

Effects of a permanent increase in the German price level by 1 percent
Effects measured in percentages, unless stated otherwise

<table>
<thead>
<tr>
<th>Variable</th>
<th>Effects after</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year</td>
</tr>
<tr>
<td>Assumptions</td>
<td></td>
</tr>
<tr>
<td>Private consumption deflator, Germany</td>
<td>1.00</td>
</tr>
<tr>
<td>Short-term interest rate, Germany</td>
<td>0.62</td>
</tr>
<tr>
<td>Long-term interest rate, Germany</td>
<td>0.21</td>
</tr>
<tr>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.03</td>
</tr>
<tr>
<td>Private consumption deflator</td>
<td>0.18</td>
</tr>
<tr>
<td>Unit labour costs, enterprises</td>
<td>0.11</td>
</tr>
<tr>
<td>Government financial deficit (% of GDP)</td>
<td>-0.03</td>
</tr>
<tr>
<td>Current account balance (% of GDP)</td>
<td>0.10</td>
</tr>
<tr>
<td>Guilder/D-mark exchange rate*</td>
<td>0.04</td>
</tr>
<tr>
<td>Short-term interest rate (% points)</td>
<td>0.51</td>
</tr>
<tr>
<td>Long-term interest rate (% points)</td>
<td>0.26</td>
</tr>
<tr>
<td>Interest differentials with Germany</td>
<td></td>
</tr>
<tr>
<td>- short rate (% points)</td>
<td>-0.11</td>
</tr>
<tr>
<td>- long rate (% points)</td>
<td>0.05</td>
</tr>
</tbody>
</table>

* + = appreciation of guilder.
Conclusion

The main conclusions from this paper are the following:

1. The guilder/D-mark exchange rate over the period 1972-1994 can be explained by a combination of (less than complete) purchasing power parity and short-term uncovered interest parity.

2. The short-term interest rate in the Netherlands is determined by the German interest rate, the strength of the guilder, the current account balance and the domestic inflation rate.

3. The long-term interest rate in the Netherlands is significantly influenced by the German long rate, the domestic short-term rate, the domestic inflation rate and the government financial deficit.

4. In the long-run interest rates in the Netherlands do not respond 100% to changes in German interest rates. For the money market, this points to some room for manoeuvre for monetary policy provided by the existence of fluctuation margins around the central parity. For the bond market, this probably means that the Dutch and German markets are not perfectly integrated in practice.

5. Econometric evidence of a direct influence of German inflation rates on interest rates in the Netherlands could not be found. This suggests that risk premia are not based on inflation differentials. On the other hand, differences in price level movements between the Netherlands and Germany have an impact on the short-term interest rate through the response of the strength of the guilder. The fact that inflation-based risk premia are hard to find underlines the credibility of the guilder/D-mark peg over most of the sample period.

6. Impulse response simulations show that shocks to domestic fundamentals of regular magnitude have only a modest impact on Dutch interest rates and the exchange rate.

7. Despite the fact that we did not find a one-to-one relationship between German and Dutch interest rates empirically, simulation exercises show that interest rates in both countries tend to move together in the presence of shocks to the German (or world) economy.

8. The magnitude of changes in the spread between German and Dutch interest rates caused by shocks to domestic and foreign fundamentals is consistent with the magnitude of fluctuations in the spread observed in reality.
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This paper offers a good illustration of the strengths and weaknesses of the style of econometric forecasting which tries a large number of variables and keeps the ones that are significant. The primary strength of this method is that one is more likely to pick up unexpected patterns and correlations, without being constrained by a theory that may or may not be plausible. It is thus interesting to see certain textbook relationships confirmed by the model, given that the model's parameters are based solely on past statistical relationships. The primary weakness is that parameters estimated by past experience may not be very informative about the results of a hypothesised policy experiment.

The first part of this paper presents parameter estimates of structural equations for quarterly changes in the Netherlands guilder/Deutsche Mark exchange rate, quarterly changes in the difference between Dutch and German short term interest rates, and quarterly changes in long-term Dutch interest rates. These equations seem to fit the data fairly well, though their forecasting ability improves markedly from the mid-1980's onward.

Significant effects on changes in the exchange rate are found for lagged differentials between Dutch and German short-term interest rates and prices, for lagged levels of the exchange rate, and, before 1983, for changes in the US dollar/DM exchange rate. The effect of price differentials, however, is not strong enough to indicate purchasing power parity; price differentials do not lead to equivalent compensating nominal exchange rate movements.

The authors attribute the negative effect of short interest rate differentials on the contemporaneous exchange rate movement -- an interest rate differential in favour of the guilder is accompanied by the guilder's appreciation -- as evidence for uncovered interest parity. To correct for simultaneity problems -- such as, perhaps, that a currently weak guilder might lead the central bank to raise rates -- the previous quarter's interest rate differential is used as an instrument. It is not clear to me that this is an adequate test of the uncovered interest parity hypothesis. I would be more convinced if relatively higher three-month interest rates in the Netherlands on the last day of the previous quarter were followed, on average, by an equivalent depreciation of the guilder in the current quarter, and lower rates were followed by an appreciation; this would suggest that investors' expectations regarding the guilder's movements were correct on average.

Significant effects on changes in the Dutch-German short rate differential are found for the change in and lagged level of the trade surplus, the lagged change in and lagged level of the exchange rate, the lagged levels of short rates in the two countries, and lagged inflation. The authors test the differential, rather than the level of the Dutch rate alone, because they find the two countries' short rates to have been so highly correlated as to drown out other effects.

It is somewhat curious that the model assigns current account conditions a role in interest rate determination but no role in exchange rate determination. It is also curious that the short- and long-term rate equations are estimated simultaneously, but not the exchange-rate equation, even though exchange rates enter into the short-term rate equation and, via short rates, the long-rate equation as well.

Even though the short-rate differential is the variable being modelled, the lagged values of the two country's short-rates enter the model separately on the right-hand side. The authors explain this as an attempt to separate long and short term effects of German rates on Dutch rates. I would think there are easier ways to do this, for example comparing the coefficient from a regression using quarterly changes to the coefficient using annual or multi-year changes. The results presented here suggest that the two rates are closely, but imperfectly correlated, through the "backdoor" method of demonstrating that they have different serial correlation coefficients, but these results do not seem especially informative as to the time horizon over which this correlation is effective.
Significant effects on changes in the long-term interest rate are found for changes in and lagged levels of the Dutch short rate and the German long rate, for the lagged Dutch long rate, for inflation, and for the government budget deficit.

Having decided previously that the differential between Dutch and German short rates, rather than the level of either, is the relevant short-rate variable, the authors look only at the Dutch short rate here. This, too, makes their results difficult to interpret, because it is unclear whether long rates respond only to the level of short rates, as they would in a naive expectations-based term-structure hypothesis, or also to the Dutch-German spread, which may indicate exchange rate or inflation trends.

The second part of the paper, after revealing that these three equations form part of the Netherlands Bank's macroeconomic forecasting model, presents the model's forecast results for two policy changes: a permanent, debt-financed increase in government spending, and a permanent increase in the German price level.

The exchange rate effects of the fiscal experiment follow orthodox macroeconomic theory (though not the current "journalistic" consensus) in that a spending increase leads to an appreciation of the guilder. The results for long-term interest rates also accord with textbook macroeconomics, in that more borrowing raises rates. An expansive fiscal policy also leads to higher short-term interest rates. The authors explain that the government spending increase leads to a current account deficit, which has historically led to higher short rates, either because sustainability issues lead to a higher risk premium or because it leads the central bank to tighten policy. Higher prices in Germany lead, as one might expect, to a stronger guilder and lower Dutch interest rates.
Introduction

Australia, like many countries, has undergone extensive market deregulation and internationalisation. More than two decades have elapsed since the initial relaxation of domestic interest rate controls and just over one decade since the float of the Australian dollar. Interest rates and exchange rates now constitute two of the most important channels through which macroeconomic policy can affect the broader economy. Over the longer run, their influence extends to the efficient allocation of capital and resources. The need for policymakers to better understand the forces that determine the behaviour of these two variables motivates this research. In particular, it is now widely recognised that expectations play a critical role in these mechanisms, affecting both the timing and speed by which interest and exchange rates transmit shocks through to real activity and prices. While theoretical discussions of the role of interest and exchange rates often incorporate forward-looking expectations, it has been difficult to model this type of behaviour within an empirical framework. This paper makes that attempt by developing behavioural models of the Australian real exchange rate and the long bond yield which explicitly incorporate some forward-looking behaviour.

Section 1 begins with a review of existing macroeconomic models of the Australian economy. These large-scale models offer the convenience of an internally consistent link between the financial sector variables and the real economy and typically embody forward-looking expectations. But their exchange rate and long bond yield equations reflect orthodox theoretical relationships; they are not estimated equations. This conclusion that, for the purposes of practical policymaking, a more complete analysis of the determinants of financial prices is required. The remaining sections of the paper proceed to develop single equation, behavioural models.

Section 2 builds on the wealth of earlier applied econometric studies of the Australian real exchange rate. This previous literature identifies roles for the terms of trade, net foreign liabilities and long-term interest differentials in determining exchange rate movements. Direct roles for macroeconomic policy and forward-looking expectations have, to date, been ignored. Herein, these omissions are redressed. The explanatory performance of the real exchange rate equation developed in this paper is found to be superior to earlier specifications.

In contrast, very little work has been undertaken in Australia on modelling the behaviour of long bond yields. Section 3 attempts to address this gap. Firstly, a model of the Australian \textit{ex ante} real long bond yield, deflated with the customary backward-looking measure of inflationary expectations is specified. This draws heavily on Orr, Edey and Kennedy (1995) who identify a comprehensive list of the fundamental determinants of real long-term yields across a 17 country panel data set, including Australia. This time-series model suffers several inadequacies and raises the question of how best to transform nominal bond yields into real magnitudes. Inflation expectations are largely unobservable and the paper spends some time exploring a suitable methodology for their measurement.

In practice, inflationary expectations can be heavily conditioned on a country's historical inflation performance. In Australia, successful inflation reduction policies in the early 1990s appear to have been accompanied by falls in existing measured inflationary expectations series. Section 3.2 discusses the inadequacies of these existing measures and estimates an alternative, forward-looking inflationary expectations series. For this purpose, a Markov switching technique is used. This methodology endogenises shifts in the series and produces estimates of the probabilities associated with remaining in particular (high or low) inflationary regimes. A model of the long-bond yield,
deflated with this unconventional forward-looking series, performs quite well. The final section concludes.

I. The macroeconomic model approach

The two most widely quoted macroeconomic models of the Australian economy are the models developed by the private consulting firm, Econtech (the "Murphy" model)\(^1\) and the model developed by the Australian Commonwealth Treasury (the "TRYM" model)\(^2\). These macro models embody similar philosophies, sharing many common features of design and specification. They have similar theoretical underpinnings, with Keynesian properties in the short run (prices are sticky and output is demand determined) and neoclassical properties in the long run. Equations describing the exchange rate and the long-term bond yield are elements of the financial sectors of these models and reflect orthodox theoretical considerations; they are not estimated behavioural equations. This section briefly discusses these equations and their implied responses to shocks. For illustrative purposes, this exposition pertains to the Murphy model.

The process of expectation formation is central to the performance of the macro model equations. Financial-sector expectations are assumed to be completely forward looking. In the long run, the equilibrium inflation rate is secured by assuming that the authorities target an exogenously determined money growth path. Quarterly inflationary expectations are then calculated from a weighted average of current inflation and the model's one-quarter-ahead predicted long-term equilibrium inflation rate. The equilibrium inflation rate is that rate which is consistent with the difference between money supply (nominal income) and real output growth in period \(t+40\), as derived from the steady-state version of the macro model.

1.1 Exchange rate determination

Each of the macro models employs a concept of the equilibrium real exchange rate. This is defined as that rate which achieves macroeconomic (that is, simultaneous internal and external) balance; it is calculated by a calibration of the steady-state version of the model prior to any dynamic simulation. Following any shock, adjustment back to the equilibrium rate is assumed to be complete within 40 quarters. After tying down the long-run real exchange rate, current and future changes in the real exchange rate are determined by an uncovered interest parity condition – if foreign long (10-year) interest rates are above domestic rates, the current value of the exchange rate must be below its equilibrium value.

More specifically, in the long run (\(t+40\) quarters), the interest differential collapses (either to zero or, alternatively, to some constant risk premium). Agents are assumed to be forward looking and to understand the fundamental structure of the economy and so form model-consistent (rational) estimates of the equilibrium real exchange rate. As mentioned above, this rate realises macroeconomic balance and is akin to the concept of the so-called fundamental equilibrium exchange rate (FEER), popularised by Williamson in the early 1980s.

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1 Developed by Mr. Chris Murphy; the current disaggregated Murphy model consists of 538 equations.

2 TRYM was developed between 1990 and 1993 and consists of 23 estimated equations, 3 financial market identities, 2 default response functions for monetary and fiscal policy and about 100 identities linking these key variables (Downes (1995)). Other macroeconomic models of the Australian economy include the Monash (see ORANI) model, developed by the Centre of Policy Studies and Impact Project, Monash University, Melbourne; MSG2 and G-Cubed Models developed by Prof. Warwick McKibbin, of the Australian National University Canberra. The financial sector treatment in these models is comparable.
Internal balance is interpreted, in the standard way, as achieving the underlying level of potential output which is consistent with the NAIRU. External balance is more difficult to define and In 't Veld (1991), in calculating equilibrium exchange rates for each of the G3 countries, found that his results were very sensitive to changes in this definition. The concept is intended to describe an equilibrium position in the current account; in the Australian macro models this is achieved with a stable ratio of foreign liabilities to GDP (typically stabilised at around 45 per cent, a little higher than the current level). As with any intertemporal analysis, the path to external balance depends on current assessments of the future values of variables. The part of the macroeconomic model that is critical in this exercise is the trade sector which consists of equations expressing the dependence of output and the balance of payments on demand and competitiveness (the real exchange rate). For example, the present discounted value of future terms-of-trade shocks impacts upon the current exchange rate to the extent that it moves the equilibrium exchange rate, in period $t+40$, to offset income effects on the current account and restore external balance.

The equilibrium exchange rate reflects the specification of interactions within the individual macroeconomic model. Bayoumi et al. (1994) conducted sensitivity analysis on the macroeconomic models of several industrial economies. They found that the estimated range in the calculated equilibrium exchange rates varied between 10 and 30 per cent. This degree of imprecision implies that interpretation of such an equilibrium rate is perhaps better restricted to the identification of relatively large exchange rate misalignments. Furthermore, the calculation of equilibrium real exchange rates as a basis for policy depends on an analysis of whether there are predictable shifts in the real exchange rate and the extent to which different sources of these shifts can be disentangled (for example, structural changes from long-lag dynamics). This is an exercise more appropriately undertaken in the behavioural framework outlined in Section 2.

1.2 Interest rate determination

Consistent with traditional textbook models, but ignoring the practical operation of monetary policy, the short-term interest rate in these macro models is endogenous. The authorities are assumed to target an exogenously determined growth path for money. A simple error-correcting money demand equation describes the link between the financial and real sectors of the macroeconomic model. The long-run component of this estimated money demand equation is inverted to produce a monetary policy rule. In this way, the current level of the short-run nominal interest rate is determined by medium-term changes in nominal demand relative to the money supply. By its nature, the policy rule is arbitrary and a highly simplified representation of the policy formation process; the primary function of these mechanisms is to ensure that the economy moves towards a stable growth path in the very long run. The Fisher effect is assumed complete and this delivers the real interest rate.

At the other end of the yield curve, determination of the long bond yield is analogous to the macro model's treatment of the exchange rate. Over the long run, international arbitrage ensures that (subject to a constant risk premium) domestic and foreign long-term real interest rates are equalised. In this way, aggregate demand and supply are equilibrated by adjustments in the real market.

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3 This definition recognises that the current account on external transactions is the counterpart of the capital account. The equilibrium current account represents the desired intertemporal reallocation of resources between countries and, by identifying the preferred path for the current account, also identifies the preferred path for international debt (Clark et al. (1994), p.14).

4 Strictly speaking, inverting an estimated money demand function to obtain the short-term interest rate is invalid.
exchange rate. Movements in the Australian bond yield away from the foreign rate (equilibrium) are then determined by a term structure calculation\(^5\).

1.3 Response to shocks

To better illustrate the relevant properties of the macroeconomic models, responses to a domestic monetary policy shock and a terms-of-trade shock are illustrated (Figures 1 and 2)\(^6\).

Figure 1

**Money supply shock**

Permanent 1% reduction

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\(^5\) The specification of the term structure calculation is model-dependent. In the Murphy model, the yield on a 10-year security is set equal to the expected return from holding a continuous sequence of one-quarter securities over the next 10 years. The expected returns from holding one-quarter securities are model-consistent (Murphy (1988)).

\(^6\) These results are obtained from simulations of the Murphy model. Given our understanding of the structure of TRYM, they are broadly representative of the financial sector properties of both macroeconomic models.
Firstly, a permanent 1 percentage point reduction in the exogenous money supply is effected; this can be thought of as a standard textbook monetary policy tightening. Unfortunately, as described earlier, the macro models are not set up to deal with an explicit interest rate shock. Such a simulation would involve successive manipulation of the money supply, producing "bumpy" response functions.

In the manner of forward-looking monetary models, the asset price variables "jump" instantaneously in reaction to any shock, typically exhibiting a damped oscillation back to their long-run paths. A permanent 1 percentage point contraction of the money supply raises real short-term interest rates by 0.63 of a percentage point (panel 1, Figure 1). This delivers a temporary fall-off in demand and a 1 percentage point reduction in the price level. The price fall is anticipated and agents immediately reduce their inflationary expectations by 0.14 of a percentage point.

---

7 This long-run adjustment behaviour is largely due to the lagged adjustment processes specified to describe the demand side of the models.
The nominal 10-year bond yield jumps up by 0.08 per cent in the initial quarter of the shock; through the uncovered interest parity (UIP) condition, the nominal exchange rate must depreciate by 0.08 of a percentage point per annum for the next 10 years in order to equalise domestic and foreign returns. This requires an immediate appreciation of the exchange rate. Consistent with the imposed theoretical condition of long-run money neutrality, the 1 per cent decrease in the money supply has no effect on real variables in the long run, but leaves the nominal exchange rate appreciated by 1 percentage point.

Alternatively, consider a sustained terms-of-trade shock, here effected through a permanent increase in the foreign price of exports (Figure 2).

This shock raises domestic income; given that not all of this income is spent on imports, the current account balance improves. The macro model's equilibrium exchange rate must appreciate to generate a smaller trade surplus in the long run and thereby restore external balance. As well, a proportion of the higher domestic income is spent on non-tradable goods; this places upward pressure on prices and interest rates, appreciating the exchange rate via the UIP condition. In total, the real exchange rate eventually appreciates by around 0.4 of a percentage point.

1.4 Assessment

The textbook-style impulse responses obtained from the macroeconomic models are useful baseline cases, but policymakers need to think more critically about the determinants of exchange rate and long bond yield behaviour. A number of points in particular are worth highlighting:

- Within the macroeconomic model framework, the exchange rate and long bond yield display an instantaneous "jump" response to all types of shocks. This is usually followed by a damped oscillation to (partly) unwind the initial impulse. Experience suggests that such impulse responses do not accurately capture real world dynamics.

- Inflationary expectations are also characterised as a "jump" variable; their instantaneous response to shocks occurs before any adjustment in actual inflation. This feature of the macro model approach does not line up closely with actual experience. In many cases, a change in inflationary expectations has not occurred until after actual inflation has changed.

- Macro models are designed to analyse shocks to the money supply. By contrast, policy simulations are more naturally examined in terms of changes in the short-term interest rate.

- The size of the estimated exchange rate responses to terms-of-trade shocks cannot comfortably accommodate the long-standing observed correlation between movements in the terms of trade and the Australian dollar (first documented by Blundell-Wignall and Thomas (1987)).

- The assumption of UIP, embodying risk-neutrality (or a constant risk premium), perfect capital mobility, efficiency in the foreign exchange market, and negligible transactions costs has no empirical support (Smith & Gruen (1989) for Australia; Goodhart (1988) and Hodrick (1987) for international evidence). Quite apart from the validity of the UIP assumption, which turns on the issue of unbiasedness, predictions of future exchange rates based on UIP tend to be highly inaccurate.

Therefore, the remainder of this paper proceeds to develop simpler, single-equation behavioural models of the exchange rate and long bond yield. This approach allows a richer characterisation of the distinctive behaviour of these variables in Australia.
2. **A behavioural model of the Australian real exchange rate**

2.1 **What determines the Australian real exchange rate?**

Previous empirical work (the most recent and comprehensive of which is Blundell-Wignall et al. (1993), hereafter BW) has identified three statistically significant determinants of the Australian real exchange rate:

- the terms of trade;
- net foreign liabilities (proxied by the cumulative current account deficit);
- real long-term interest differentials.

Each of these is addressed in turn. Firstly, while all three "fundamentals" have been reported as statistically significant determinants of the real TWI exchange rate over the period since the floating of the Australian dollar, only the terms of trade has consistently retained its explanatory power over a longer sample period (1973:2-1992:3). This latter result is consistent with the cross-country study of Amano and Van Norden (1995) which documents a robust relationship between the real domestic price of oil and real effective exchange rates in Germany, Japan and the USA. They interpret the real oil price as capturing exogenous terms-of-trade shocks and find these shocks to be the most important factor determining real exchange rates over the long run.

The relationship between the terms of trade and the Australian real exchange rate is striking, as shown in Figure 38. Depreciations of the real TWI occurring in 1974-1978; 1984-1986; and 1991-1993 were all associated with falls in the terms of trade (denoted by the pale grey bars in Figure 3). Similarly, the real TWI appreciated over 1987-1989 and 1994 when the terms of trade improved (highlighted by the darker grey bars).

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8 This is the terms of trade for goods and services. It seems reasonable to take the terms of trade as exogenous because Australia's share of world trade is small and it exports relatively few differentiated products. Dwyer et al. (1994) presents empirical evidence for Australia.
An exception can be identified in the early 1980s. This period coincides with a resources investment boom, promoted by the second OPEC oil-price shock and provides a good example of the role that expectations can play in determining movements in the exchange rate. The resources boom generated optimistic expectations about future improvements in the terms of trade and thereby, future income; the TWI appreciated despite little change in the prevailing terms of trade. Given that the anticipated improvements never eventuated, a correction in expectations contributed to the magnitude of the real TWI depreciation over 1985 and the first half of 1986.

Secondly, Australia experienced a rapid and sustained rise in net foreign liabilities over the 1980s (Figure 4). Increasing net foreign liabilities, as a share of wealth, require larger balance of trade surpluses to restore equilibrium. Similar to the macro model mechanism of maintaining external balance, this may require a depreciation of the real exchange rate to attract resources into the tradables sector (of course, if the real return on investment is high, the higher trade surpluses may be achieved without a real depreciation).

Thirdly, the vast majority of the literature finds that the long-term real interest differential has the most success in obtaining significant and correctly signed estimates in exchange rate equations (Gruen and Wilkinson (1991) and BW for Australia; Isard (1988) and Shafer and Loopesko (1983) for international evidence). Long-term interest differentials are often justified on the grounds that shocks to the real exchange rate can persist for long periods and this slow reversion towards equilibrium is simply more appropriately matched by a correspondingly long-term interest rate.\footnote{Empirical work generally uses the cumulated current account deficit as a proxy for net foreign liabilities because it abstracts from valuation effects.}

\footnote{Isard (1983) supports the use of long (10 year) interest rate differentials on the grounds that they are convenient to interpret. As in the Australian macro models, he assumes that the expected real exchange rate in 10 years time is the equilibrium exchange rate; in this way, the long (10 year) real interest differential (corrected for any risk premium) can be interpreted as denoting the annual rate of real depreciation/appreciation of the dollar expected by the market over the next 10 years.}
This seems curious given that the exchange rate is considered to be an important channel for transmitting changes in the policy-determined short-term interest rate to the economy. De Kock and Deleire (1994) estimate that, post-1982 in the United States, the exchange rate accounts for roughly one-third of monetary policy transmission to output, compared to a near-negligible contribution earlier. Perhaps it is the case that previous Australian studies did not have the benefit of a sufficiently long sample period, after the floating of the Australian dollar, over which to estimate their exchange rate models. At any rate, this seems to beg further investigation.

The real long-term interest differential in existing models could simply be replaced by a real short-term interest differential. As customarily measured - using 12 months ended inflation rates - real short-term interest rate differentials would reflect the prevailing stance of domestic, relative to foreign, monetary policy; but they would fail to capture any market anticipation of the future paths of short-term interest rates, inflation and growth.

Figure 5
Episodes of policy change

It is difficult to capture these forward-looking aspects in behavioural models. This seems unsatisfactory in models of the exchange rate since financial market behaviour is generally characterised by forward-looking expectations. Therefore, the novel approach taken here is to use a
measure of the relative slopes of the domestic and foreign yield curves. Estrella and Mishkin (1995) and Mishkin (1994) provide evidence that the slope of the yield curve contains information about the current and expected future stance of monetary policy. Inflationary expectations, and therefore expectations of the future path of short-term interest rates, are reflected in long bond yields. Although well-understood by policymakers, it is worthwhile digressing to illustrate this operational point further.

Figure 5 depicts two episodes of monetary policy action in Australia. Between April and December 1987 (top panel) and from December 1990 to March 1992 (bottom panel), the operational instrument of monetary policy in Australia - the nominal cash rate - was reduced by around 5 percentage points. In the first episode, in 1987, the long bond yield remained relatively unchanged (falling by a small 0.48 of a percentage point). By comparison, over the early 1990s episode, the long bond yield fell by almost 4 percentage points. At this time, some progress on inflation was already widely apparent in Australia and so market expectations for future inflation may well have moderated with the reduction in the cash rate. To the extent that this explains the fall in the long end of the yield curve, agents were not expecting short-term rates to have to rise very much in the future. Relative to the example in 1987, the slope of the yield curve remained fairly flat. By this measure, the stance of monetary policy was relatively tighter than over the 8 months to December 1987, despite equivalent movements in the nominal cash rate.

Also of interest to policymakers is the role of fiscal policy in determining exchange rate behaviour. Rarely mentioned in earlier work on the Australian exchange rate, the impact of fiscal policy can occur through two separate channels and is theoretically ambiguous:

- Firstly, the simplest Mundell-Fleming model predicts that expansionary fiscal policy causes an appreciation of the exchange rate. The intuition for this result is that increased government spending raises demand for domestic output which, in turn, induces a currency appreciation (alternatively, increased demand exerts upward pressure on interest rates which induces capital inflow and a stronger currency). The appreciated currency reduces the value of foreign demand, which restores the original level of output.

- Secondly, fiscal policy can impact upon the exchange rate through a risk premium. Fiscal expansion may be penalised by investors who perceive an increased probability of default or expect higher inflation in the future because they believe that the incentive exists for the Government to "inflate" its debt away; in order to hold Australian dollar assets, they demand a risk premium on domestic interest rates. Furthermore, it is often argued that higher government budget deficits are associated with negative sentiment on the exchange rate because they imply lower national savings and thus greater net foreign liabilities in the longer run. In this way, it is argued that the exchange rate depreciates. To the extent that the negative sentiment arises because of the overall size of net foreign liabilities, rather than their public/private composition, this effect may be partly captured, over the long run, by a cumulated current account variable.

Both the monetary and fiscal policy variables discussed above seem likely to be important, in addition to the variables identified in earlier work, for explaining movements in the Australian real exchange rate. To ascertain the empirical validity of this proposition, the BW equation, being the most recent in this literature, is tested for and appears to suffer from omitted variable bias.

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11 See also Cook and Hahn (1990) for a survey of the more recent literature and some support for the idea that parts of the yield curve are useful in forecasting interest rates; Lowe (1992) provides evidence for Australia.
Table 1 summarises the results from application of the "rainbow test", a member of the Ramsey (1969) RESET family of tests for the omission of unknown variables (Utts (1982)). The test is conducted over several post-float sample periods, when the exchange rate became a channel of transmission for monetary policy; the null hypothesis of no omitted variables is consistently rejected. The omitted variable(s) will be captured in the error process and as a consequence, the estimated coefficients in the BW equation will be both biased and inconsistent.

<table>
<thead>
<tr>
<th>BW equation</th>
<th>RESET Rainbow test</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984:1-1992:3 (BW original estimation period)</td>
<td>3.11**</td>
<td>F(18.10) 0.035</td>
</tr>
<tr>
<td>1984:1-1995:2 (Update of BW estimation period)</td>
<td>2.61**</td>
<td>F(24.16) 0.026</td>
</tr>
<tr>
<td>1985:1-1995:2 (This paper's estimation period)</td>
<td>2.02*</td>
<td>F(22.14) 0.089</td>
</tr>
</tbody>
</table>

*, ** Denote the null hypothesis of no omitted variables rejected at the 5% and 10% significance level respectively.

In an effort to address this bias, several modifications to the BW specification are made. Specifically, the terms of trade and cumulated current account deficit are retained. A yield gap differential \((YGAP)\) replaces the long-term interest differential and takes the form:

\[
YGAP = \left\{ (i_s - i_L) - (i_s - i_L) \right\}^*
\]

(1)

where:

\((i_s - i_L)\) : measures the slope of the domestic yield curve as the difference between the domestic nominal cash rate \((i_s)\) and the domestic nominal long (10-year) bond yield \((i_L)\);

\((i_s - i_L)^*\) : measures the slope of the foreign yield curve using equivalent foreign interest rates (see Appendix A for details on the construction of world interest rates and Table B.2 in Appendix B for statistical confirmation of the implied restrictions in (1)).

In addition, a role for fiscal policy is accommodated by including a measure of the change in the Commonwealth Government budget balance, expressed as a proportion of GDP (hereafter, the fiscal variable). While it would be preferable to use a cyclically-adjusted measure of the fiscal position, this was not available for Australia.

---

12 The "rainbow test" compares estimates of the variance of the regression disturbance obtained from estimation over the full post-float sample and a truncated sub-sample; if the null hypothesis is true, both variance estimates are unbiased. The test statistic is an F-statistic, adjusted for the appropriate degrees of freedom. See Kmenta (1990, pp.454-455) for a full description of the test. It should be noted that the consequences of omitting relevant explanatory variables are the same as those of using an incorrect functional form.

13 Typically, the government budget tends to be in surplus when the economy is growing strongly and vice versa. The fiscal variable was tested against domestic and foreign growth variables and measures of the output gap to eliminate the possibility that it was just proxying the economic cycle. The fiscal variable retained its explanatory power over both the shorter post-float period (1985:1-1995:2) and the longer, historical sample period (1973:4-1995:2).


2.2 The empirical results

Following the convention for time series methodology, the order of integration of the real exchange rate and its proposed explanatory variables is established (see Table B.3 in Appendix B for detailed statistics). To this end, the Augmented Dickey-Fuller (Dickey and Fuller (1981); Said and Dickey (1984)) and Elliot, Rothenberg and Stock (1992) (DF-GLS) tests of a unit root null, together with the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) test of a stationary (trend stationary) null, are employed. Confirming the results of Bleaney (1993) and Gruen and Kortian (1996), these tests imply mean reversion of the Australian real exchange rate to a slowly declining trend. Similar evidence of stationarity exists for other countries (see, for example, Phylaktis and Kassimatis (1994); Liu and He (1991); Huizinga (1987)). The integration tests also provide evidence that the terms of trade and other explanatory variables are I(0) processes.

Nevertheless, the analytical convenience of the unrestricted error correction framework is exploited to specify a behavioural model of the real Australian TWI exchange rate. The model is specified with 4 lags of each explanatory variable in the dynamics; sequential F-tests are used to derive the following parsimonious representation:

\[ \Delta rer_t = \alpha + \beta rer_{t-1} + \delta tot_{t-1} + \phi cad_{t-1} + \gamma YGAP_t + \sum_{i=0}^{1} \phi_i \left[ \Delta GDef_{GDP} \right]_{t-i} + \theta tot_t + \varepsilon_t \]  

where:

- \( rer \): log Australian real TWI exchange rate;
- \( tot \): log terms of trade;
- \( cad \): log cumulated current account deficit, expressed as a proportion of GDP (defined such that a current account deficit is a positive number);
- \( YGAP \): relative slopes of domestic and foreign yield curves as described in (1) above;
- \( \Delta GDef_{GDP} \): fiscal variable, defined as the log change in the Commonwealth Government deficit and expressed as a proportion of GDP (defined such that a budget deficit is a positive number);
- \( \varepsilon \): white noise error term;
- \( \Delta \): first difference operator.

\[ \Delta \]

14 The null hypothesis of a unit root in the ADF and DF-GLS tests may result in a type II error; series may appear to contain a unit root because the data are insufficient to provide strong evidence for rejection of that null. This is why the KPSS test, with a null of stationarity, is also applied (see Appendix B for a brief description of this test).

15 From the perspective of modelling, the essential difference between the trend-stationary and integrated model specifications is the nature of the process driving the stochastic component, and whether the series is trended.

16 Phylaktis and Kassimatis (1994), in examining real exchange rates in eight Pacific Basin countries (calculated using the unofficial black market exchange rates), find evidence for mean reversion which suggests a half-life of four quarters. Using amended variance ratio tests, Liu and He (1991) offer evidence that mean reversion is quicker in the developing Asian countries relative to industrialised countries. Huizinga (1987) employs spectral methods to analyse real exchange rates for ten major currencies vis-à-vis the US dollar. Various real bilateral rates against the US dollar and the pound sterling were found to be mean-reverting, but against the Japanese yen, the exchange rates were indistinguishable from random walks.

17 In this way, the analysis recognises that in finite samples, any trend stationary process is nearly observationally equivalent to a unit root process where shocks are substantially reversed - that is, where the errors have a moving-average component with a root near minus one (or a fat-tailed distribution for the error process). And, irrespective of the order of integration of the variables, this modelling technique remains valid.
Given the time-series properties of the data, this specification is used to distinguish different types of influences on the real exchange rate and, in this way, retains one characteristic of the macroeconomic models described in Section 1 - namely, the general framework wherein the real exchange rate - affected by speculative and cyclical factors - eventually tends towards a path determined by underlying structural factors. The macroeconomic fundamentals, identified in Section 2.1 above, set the parameters within which the exchange rate should move in the short to medium term and provide a pertinent framework from which to assess the appropriateness of policy settings.

Table 2
Real exchange rate model
Dependent variable: change in log real TWI

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>β: Speed of adjustment</td>
<td>-0.51*** (0.12)</td>
<td>-0.25*** (0.07)</td>
</tr>
<tr>
<td>δ: Terms of trade_{t-1}</td>
<td>0.46*** (0.14)</td>
<td>0.22*** (0.07)</td>
</tr>
<tr>
<td>ϕ: Cumulated current account_{t-1}</td>
<td>-0.01 (0.05)</td>
<td>-0.04** (0.02)</td>
</tr>
<tr>
<td>γ: Yield curve differential_{t-1}</td>
<td>1.10*** (0.35)</td>
<td>0.08 (0.21)</td>
</tr>
<tr>
<td>Σ ∑<em>{i=0}^{I} Fiscal</em>{t-i}</td>
<td>-4.87*** (7.15)</td>
<td>-1.36*** (5.43)</td>
</tr>
<tr>
<td>Θ: Δ Terms of trade_t</td>
<td>1.41*** (0.19)</td>
<td>0.89*** (0.16)</td>
</tr>
<tr>
<td>α: Constant</td>
<td>0.16 (0.31)</td>
<td>0.09 (0.25)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.74</td>
<td>0.35</td>
</tr>
<tr>
<td>DW</td>
<td>1.53</td>
<td>1.88</td>
</tr>
<tr>
<td>ARCH(4) test</td>
<td>1.33 [0.86]</td>
<td>3.26 [0.52]</td>
</tr>
<tr>
<td>AR(4) test</td>
<td>5.48 [0.24]</td>
<td>5.19 [0.27]</td>
</tr>
<tr>
<td>Jarque–Bera normality test</td>
<td>2.28 [0.32]</td>
<td>2.40 [0.30]</td>
</tr>
<tr>
<td>Rainbow test</td>
<td>1.08 [0.45]</td>
<td>0.84 [0.72]</td>
</tr>
</tbody>
</table>

1 This speed of adjustment implies a half life of 1 quarter; this is not unreasonable given that the real exchange rate is trend stationary.

* ** *** denote significance at the 10, 5 and 1% level respectively.

Standard errors are in round brackets, probability values are in square brackets, and the F test statistic for the joint significance of the fiscal variable is in parentheses, {}.
The model is estimated over two sample periods; three decades of data encompass two broad exchange rate regimes in which the dynamics of the real exchange rate are unlikely to be identical. With this in mind, results for the real TWI over the post-float period (1985:1-1995:2) and a longer, historical sample (1973:4-1995:2) are reported in Table 2.

Two points are worth noting immediately:

The model is estimated over two sample periods; three decades of data encompass two broad exchange rate regimes in which the dynamics of the real exchange rate are unlikely to be identical. With this in mind, results for the real TWI over the post-float period (1985:1-1995:2) and a longer, historical sample (1973:4-1995:2) are reported in Table 2.

- As expected, it is only after the floating of the Australian dollar that the exchange rate has played a role in channelling changes in real interest rates through to the broader economy.\(^\text{18}\)

- On the other hand, the cumulated current account deficit is only significant in explaining the real exchange rate over the fuller, historical sample period; this accords with its longer-run structural nature.\(^\text{21}\) Over this period, the level of Australia's net foreign liabilities is estimated to have exerted some downward pressure on the real exchange rate, but this has been of a relatively small order of magnitude; a 1 percentage point increase in net foreign liabilities to GDP, *ceteris paribus*, eventually leads to around 1/6 of a percentage point depreciation in the real exchange rate.

The remainder of this section concentrates on interpreting the results obtained from estimation of this model over the post-float period. Simple impulse response diagrams show the estimated impact of a change in each of the explanatory variables, *ceteris paribus*, on the real exchange rate.

As in Section 1.3, consider first a temporary *monetary policy* shock.

This is executed through a one percentage point (negative) steepening of the Australian yield curve relative to the foreign yield curve, maintained for 8 quarters. In response, the real exchange rate is estimated to appreciate by 2.2 percentage points; 76 per cent of the adjustment is complete after 2 quarters (Figure 6a). This gradual adjustment of the real exchange rate to a monetary policy shock is quite different to the "jump" response elicited in the macro model.

Secondly, similar to the results obtained by earlier work, a sustained one percentage point increase in the terms of trade eventually delivers a 0.9 per cent appreciation of the real Australian exchange rate (Figure 6b). This estimated response is almost double that returned by simulation of the macro model exchange rate equation in Section 1.3. While there is some uncertainty about the operation of the short-run dynamics, a literal interpretation of the behavioural model suggests that the real exchange rate could appreciate by as much as 1.4 per cent in an initial response to this shock.

---

\(^\text{18}\) The foreign exchange market is given one year after the floating of the Australian dollar in 1983:4 to overcome initial turbulence and establish its new regime; thus, estimation over the shorter sample period begins in 1985:1. If the entire period since the float is included in the estimation period (ie. 1984:1) then a direct role for monetary policy is no longer significant at the 10 per cent level.

\(^\text{19}\) The foreign exchange market is given one year after the floating of the Australian dollar in 1983:4 to overcome initial turbulence and establish its new regime; thus, estimation over the shorter sample period begins in 1985:1. If the entire period since the float is included in the estimation period (ie. 1984:1) then a direct role for monetary policy is no longer significant at the 10 per cent level.

\(^\text{20}\) It is worth noting that the relative yield gap variable outperforms (statistically) the alternative short-term real interest differential over this sample period (see Table B.4 in Appendix B for details).

\(^\text{21}\) This is the opposite of the BW result that the cumulated current account deficit is only significant over the shorter, post-float sample period and even then, that it is outperformed by a simple trend (see Table B.1 in Appendix B).
Figure 6
Real TWI exchange rate: impulse response
6a: Temporary 1% monetary policy shock – 1% steepening of domestic yield curve for 8 quarters

6b: Sustained 1% terms of trade shock

6c: Permanent 1% fiscal policy tightening – affected over 4 subsequent quarters
The magnitude of the estimated real exchange rate response to terms-of-trade shocks is something of a puzzle. Gruen and Kortian (1996) contend that this observed historical response results from inefficiency in the foreign exchange market. They demonstrate the existence of large and variable predictable excess returns to holding Australian assets over horizons of a year or more. This is interpreted as evidence of a relative scarcity of forward-looking foreign exchange market participants with an investment horizon of this length.

If this myopic behaviour does indeed prevail, participants in the foreign exchange market may not be adequately distinguishing between temporary, soon-to-be-reversed, shocks and longer, more sustained, shifts in the terms of trade. This would result in Australia's real exchange rate moving more tightly with the terms of trade than is consistent with perfectly forward-looking investor behaviour. While the smaller responses to temporary terms-of-trade shocks generated by the macro models is theoretically appealing, the presence of excess returns in the foreign exchange market undermines the predictions of UIP; this condition is the central relationship determining exchange rate outcomes in the macro models.
Finally, over the full sample period (1973:4-1995:2), the average absolute value of annual changes in fiscal policy has been in the order of 1 per cent of GDP. The largest fiscal contraction occurred in the year to June 1988 and represented almost 1.7 per cent of GDP; the largest fiscal expansion occurred in the year to June 1992, representing 2.9 per cent of GDP. Movements of this magnitude are infrequent.

Given this historical profile, the fiscal policy shock illustrated in Figure 6c is a permanent contraction of the Commonwealth Government budget deficit by 1 percentage point of GDP. The shock is engineered through four quarters of 0.25 percentage point reductions in the ratio of the deficit to GDP. As discussed in Section 2.1 above, the theoretical effect of a fiscal policy change on the real exchange rate is indeterminate. But, consistent with the prediction from a standard Mundell-Fleming model, a permanent 1 percentage point fiscal contraction is here estimated to instantly depreciate the real exchange rate by around 2 percentage points, other things being constant.

To give some idea of the model's fit, Figure 7 compares the actual behaviour of the real Australian TWI exchange rate over the post-float period, with its predicted values from this model.
The top panel of Figure 7 plots the fitted values from the model when it is estimated using the post-float data set, 1985:1-1995:2. In sample, the model fits very well.

The bottom panel of Figure 7 presents the model’s out-of-sample forecasts. These are obtained by re-estimating equation (2) using data to the December quarter of 1989 (or half the sample period). Subsequently, actual values of the exogenous variables are used to obtain one-step-ahead forecasts of the real exchange rate out to the end of the sample, 1995:2. Out-of-sample, the equation captures most of the actual movements in the real exchange rate and picks the major turning points in the early 1990s and again around the end of 1993.

It is also instructive to ascertain the model’s interpretation of historical movements in the real exchange rate. To this end, using all the data over the post-float sample period (1985:1-1995:2), the model is simulated dynamically. Sub-periods of pronounced exchange rate movement are then identified. Over each of these periods, the change in the simulated value of the real exchange rate is calculated and decomposed into the contribution attributable to movements in the terms of trade and each of the policy variables (Figure 8).

22 These contributions do not sum to 100 per cent because the contribution from the dynamic specification of the model is not included.

The rapid depreciation of the real TWI to mid-1986 is overwhelmingly attributable to the falling terms of trade. Over the first half of this sub-period, despite a relatively steeper yield curve in Australia, the effect on the real exchange rate from the declining terms of trade dominated. While it is clear that relative monetary policy movements affect the real exchange rate, their contribution is often overwhelmed by other (temporary) factors.

A rising terms of trade was responsible for 65 per cent of the predicted appreciation of the real TWI over the remainder of the 1980s. The yield differential made some smaller contribution at the beginning of this period. Fiscal policy had little effect.

Between 1990 and end-1991, the real exchange rate was relatively stable, with downward pressure from the terms of trade largely offset by expansionary fiscal policy. Possibly reflecting expectations of domestic inflationary pressure, the yield differential as well as fiscal policy made some contribution to the depreciation of the dollar over 1992-1993. Most recently, the terms of trade have, once again, appeared to dominate.

The overwhelming importance of temporary terms-of-trade shocks for Australia's real exchange rate is a documentable historical fact. Nevertheless, this result appears at odds with standard economic theory and, as discussed above, the assumption of market efficiency.

3. A behavioural model of the Australian long-term interest rate

In contrast to the volume of literature on determinants of the exchange rate, work on modelling the behaviour of the Australian long bond yield is scarce. This paper takes Orr et al. (1995) as a starting point for its research; these authors provide a succinct yet comprehensive discussion of the determinants of real long-term bond yields for a panel of seventeen OECD countries, including Australia. By using the "fundamental" variables identified by Orr et al., this section develops a time-series equation for the Australian ex ante real long bond yield. Ex ante real rates are difficult to measure because inflation expectations are largely unobservable. In this regard, the paper takes two alternative approaches.

Firstly, expectations are assumed to be adaptive (backward-looking) so that the nominal long bond yield is deflated, in the customary way, using actual past inflation rates. The parsimonious specification of this model seems dependent on an inflation risk premium variable which has little appeal within this time-series representation.

---

22 These contributions do not sum to 100 per cent because the contribution from the dynamic specification of the model is not included.
An alternative approach is posited in Section 3.2. *Forward-looking* expectations are generated by estimation of a model that endogenises shifts between a high and a low inflation regime. This methodology seems particularly apt for Australia, where successful inflation reduction policies in the early 1990s have been accompanied by a discrete shift in existing survey measures of inflationary expectations. A single equation, time-series model for the real long bond yield deflated with this unconventional forward-looking inflationary expectations series, is well-behaved.

### 3.1 The real bond yield fundamentals in brief

I begin with the principle determinants of real long bond yields. Orr et al. list these determinants as the domestic rate of return on capital, the world real long bond yield, and various risk premia. They note that these risk premia are likely to depend on:

- the perceived degree of each country's monetary policy commitment to price stability. Recognising that the expectations of market participants may follow some adaptive process, they use the existing level of inflationary expectations, conditioned on some longer-run historical performance (the average rate of inflation over the preceding 10 years). In this way, movements in bond yields relative to changes in current inflationary expectations will depend on the weight that investors attribute to Australia's relatively poorer historical inflation performance;
- the expected sustainability of government fiscal and net external debt positions. Orr et al. measure these by the ratios of government budget positions and cumulated current account deficits, respectively, to GDP;
- some undiversifiable domestic portfolio risk associated with holding bonds.

Following the time-series methodology outlined in Section 2, the real long bond yield, $(r)$, deflated, first of all, with (annualised) quarterly underlying inflation rates, is determined by an unrestricted error correction model. Tests of the order of integration of each variable are presented in Table C.1 in Appendix C. Four lags of each of the differenced "fundamental" variables, together with domestic growth, were included in the initial dynamic specification of the model; F-tests were then used to derive the parsimonious final model:

\[
\Delta r_t = \alpha \Delta r_{t-1} + \beta \left\{ \pi_{10} - E_t (\pi) \right\} + \gamma_0 \text{RetCap}_{t-1} + \sigma \Delta r_{t-2} + \phi \Delta \text{RetCap}_{t-1} + \lambda_0 \Delta \left[ E_t (\pi_{PR}) \right] + \lambda_2 \Delta \left[ E_t (\pi_{PR}) \right]_{t-2} + \theta GDef_t + \sum_{i=0} g_{t-i} + e_t
\]

where:

- $r_t$ : real Australian 10-year bond yield deflated with annualised quarterly underlying inflation rates;
- $\left\{ \pi_{10} - E_t (\pi) \right\}$ : inflation differential variable;
- RetCap : return on capital;
- GDef : Commonwealth Government Budget deficit, expressed as a proportion of GDP (a deficit is denoted as a positive number);
- $g$ : domestic GDP growth;

---

23 It may also be the case that some degree of liquidity risk exists for Australia, due to a relatively shallow bond market.

24 Annualised quarterly inflation rates are used to avoid the introduction of autocorrelation.
A one percentage point rise in the domestic return on capital in this model implies an eventual increase in the real long bond yield of about 1/3 of a percentage point; this compares to around 1/4 of a percentage point in the Orr et al. estimation. While the inflation differential variable has some appeal for estimation with panel data, its appropriateness within this time-series framework is difficult to justify. This is because, by construction, the real bond yield will often be relatively high in periods when the current (expected) rate of inflation is low; this will also be true of the inflation variable. That is, the existence of some mean reversion in inflation would generate this positive, significant coefficient.

The fit of the model is represented in the top panel of Figure 9. Out-of-sample forecasts are obtained by estimating the model to December 1991; actual values of the exogenous variables are then used to forecast the real long bond yield forward through time. The results are presented in the lower panel of Figure 9. The model predicts the fall in the real long bond yield over the early 1990s and its trough in 1993. However, it fails to anticipate the extent of the rise in the real bond yield over the course of 1994, suggesting, perhaps, that the world-wide bond market sell-off was not completely consistent with fundamentals. Despite the fact that a similar pattern was documented in most OECD
countries over 1994, the panel estimation in Orr et al. also fails to predict bond yield behaviour over this period.

Figure 9
Real long bond yield model: simulation and out-of-sample forecast

![Graph showing real bond yield model: simulation and out-of-sample forecast.]

Given the reservations with the model's (likely spurious) dependence on the inflation differential term, \((\bar{\pi}_{10} - E_{i}(\pi))\), it may be the case that the dependent variable, measured as it is, with backward-looking inflationary expectations, is not an adequate measure of the ex ante real long bond yield. The remainder of this section explores an alternative real long bond yield model that assumes inflation expectations to be forward looking.

### 3.2 Measuring inflationary expectations

The gap between nominal and indexed 10-year bond yields is often used to estimate financial market expectations of the average rate of inflation over the next 10 years. However, in Australia's case, the indexed bond market has only very recently become liquid; historically, indexed
bonds were held in concentrated parcels and were not actively traded in a secondary market at all until 1993. An alternative measure of inflationary expectations is available from the Westpac Bank and the Melbourne Institute. A random selection of 1,200 adults aged 18 and above, sampled Australia-wide, are asked to respond to a question about how much they expect prices to rise over the next twelve months; their responses are weighted to reflect population distribution. The disadvantage of this survey series is that it asks about inflationary expectations over the next 12 months - not over the next 10 years. Perhaps more importantly, the expectations of consumers might differ from those of financial market participants (Figure 10).

Figure 10
Survey measure of inflationary expectations and the nominal 10-year bond rate

This paper proposes a different approach to measuring expectations which exploits the Markov switching technique and endogenises shifts in the inflation process through time\textsuperscript{25}. In brief, this methodology allows the process of inflation to be characterised by two different regimes, the first identified by relatively high inflation; the second, by relatively low inflation. Switches between these states are based on a probabilistic process\textsuperscript{26}. Maximum likelihood estimation of the two-state model returns a probability that inflation is in one or other of these regimes. This is used to construct a probability-weighted $n$-period-ahead inflationary expectations series which, by its nature, forward-looking. Thus constructed, this series is found to be superior to its survey alternative in a model of the nominal bond yield (Section 2.3).

More specifically, inflation is specified to depend on its own past values and forward-looking measures of the output gap (itself measured by a Hodrick-Prescott filter on GDP(A)). Three forecasting methods are tried:

\textsuperscript{25} Initial work with Markov switching models was done by Hamilton (1989, 1990) with applications to business cycles. Recent work by Evans and Wachtel (1993) and Laxton, Rickets and Rose (1994) (and Simon and Tarditi (1995, mimeo) for Australia) has applied the technique to inflation with a view to examining the issue of central bank credibility. The Gauss programme used for estimation of the Markov switching model is an adaptation of that used by Hamilton (1989) and Goodwin (1993) and I thank Thomas Goodwin for generously providing me with the computer code.

\textsuperscript{26} A Markov process is one where the (fixed) probability of being in a particular state is only dependent upon what the state was last period.
First, agents are assumed to have perfect foresight so that they know the output gap existing in the period over which their inflationary forecast is relevant. In this case, the probability-weighted inflationary expectations series is a function of lagged inflation and the actual future output gap (and is denoted $E_{PF_i}(\pi_{i+n})$ for perfect foresight Markov measure):

$$E_{PF_i}(\pi_{i+n}) = \frac{1}{k} \sum_{i=0}^{n-1} \text{GAP}_{i+1}; \quad i = 0,1,2,...,n-1 \quad (4)$$

In this way, inflationary expectations over the next year ($n=4$ quarters) would be $E_{PF_i}(\pi_{i+4})$; over the next 10 years, $E_{PF_i}(\pi_{i+40})$.

Alternatively, the assumption of perfect foresight can be relaxed so that inflationary expectations are a function of lagged inflation and a mean-reverting output gap (and this measure is denoted $E_{MR_i}(\pi_{i+n})$ for mean-reverting Markov measure):

$$E_{MR_i}(\pi_{i+n}) = g\{\pi_{i-1}, \text{GAP}_{i+\ell} \}; \quad i = 0,1,2,...,n-1 \quad (5)$$

where:

$$\text{GAP}_{i+\ell} = \text{GAP}_{i-1} \left(1 - \frac{(i+1)}{n}\right)$$

In this way, $n=4$ quarters is roughly consistent with a 4 to 5 year business cycle; at any point in time, $t$, the output gap is not known (although $\text{GAP}_{i-1}$ is known), but is expected to close within 5 quarters.

Finally, since similar analysis in the literature has commonly been univariate, the output gap is excluded altogether (this worsens the fit of the model but leaves the general dynamics relatively unchanged).

Quarterly data from the past 35 years (1959:4-1995:2) are used to estimate the model parameters with maximum likelihood techniques. For convenience, only the results from estimation of the first specification, $E_{PF_i}(\pi_{i+n})$, which assumes perfect foresight of the output gap, are presented below. State 0 identifies the 1970s and 1980s as episodes of relatively high inflation in Australia and the estimated model describes underlying inflation as a persistent (but not integrated) process around a mean of 8.7 per cent. State 1 identifies the 1960s and 1990s as low inflation regimes where shocks are less persistent and inflation reverts to a mean of 3.3 per cent.

State 0: High inflation regime

$$\pi_t^0 = 0.40 + 0.8\pi_{t-1} + 0.09\text{GAP}_{t-1} + \varepsilon_t^0$$

$$(0.17) \quad (0.07) \quad (0.03)$$

$$\varepsilon_t^0 = z \cdot 1.04 \sqrt{\sigma_t^2}$$

$$p(s_t = 0|s_{t-1} = 0) = 0.989$$

$z \sim N(0,1)$ \quad $\sigma_t^2 = 0.14 + 0.47\varepsilon_{t-1}^2$

$$(0.03) \quad (0.20)$$

State 1: Low inflation regime

$$\pi_t^1 = 0.54 + 0.34\pi_{t-1} + 0.11\text{GAP}_{t-1} + \varepsilon_t^1$$

$$(0.12) \quad (0.14) \quad (0.03)$$

$$\varepsilon_t^1 = z \sqrt{\sigma_t^2}$$

$$p(s_t = 1|s_{t-1} = 1) = 0.980$$

$z \sim N(0,1)$ \quad $\sigma_t^2 = 0.14 + 0.47\varepsilon_{t-1}^2$

$$(0.03) \quad (0.20)$$
Figure 11 illustrates the probability of being in the high inflation state, 0, at each point in time. It is this series which is used to appropriately weight one-step-ahead forecasts from the inflation models of state 0 and state 1 to construct what will be referred to as the "Markov inflationary expectations series". This approach has two advantages. It explicitly incorporates the forward-looking behaviour customarily associated with financial market participants and assumed in the macro model approach. Furthermore, this method can deliver a longer-horizon measure of inflationary expectations, \( n \) periods ahead, as per (4) or (5). These \( n \)-step-ahead estimates embody more realistic, behavioural processes than the simple log linear interpolated values used in the macro models. Expectations 2 years ahead, as well as 1 year ahead, are calculated.

It is clear from Figure 12 that the behaviour of the Markov expectations series is quite distinct from that of the consumer survey measure. For exposition, only the Markov 1-year-ahead inflationary expectations, generated by agents with perfect foresight of the output gap, \( E_{PF}(\pi_{t+1}) \), are illustrated in Figure 12. The alternative, mean-reverting output gap specification and the 2-year-ahead forecasts of Markov expectations exhibit similar patterns and timing.

3.3 **Empirical results for the long bond yield equation with forward-looking inflationary expectations**

The relevance of the various Markov forward-looking expectations series, in contrast to the survey measure of consumer expectations, is examined for explaining movements in the nominal bond yield. This is achieved by estimating an unrestricted ECM of the form:

\[
i_t = \alpha \hat{E}_t(\pi_{t+4}) + \gamma Z + \theta \Delta X + \epsilon_t
\]  

(6)
where:

\[ i_t \quad : \quad \text{nominal 10-year bond yield}; \]

\[ \hat{E}_t(\pi_{t+40}) \quad : \quad \text{estimated average rate of inflation expected over the next 10 years proxied either by one of the Markov measures of inflation expectations or the consumer survey measure;} \]

\[ Z \quad : \quad \text{vector of explanatory variables for the real 10-year bond yield as described by Orr et al. (1995) and discussed in Section 3.1 above;} \]

\[ X \quad : \quad \text{vector of dynamics;} \]

\[ \varepsilon_t \quad : \quad \text{white noise error term.} \]

Four lags of each of the differenced explanatory variables were initially included in the dynamic specification of the model; F-tests were then used to derive the parsimonious final model. Table 4 summarises the results from estimation of (6) using the competing measures of \( \hat{E}_t(\pi_{t+40}) \).

**Figure 12**

**Alternative measures of inflationary expectations**

The Markov one-year-ahead inflationary expectations measure, calculated using the assumption of perfect foresight for the output gap, \( E_{PF}(\pi_{t+4}) \), was found to have the greatest explanatory power for movements in the nominal long bond yield (model #1); it clearly outperforms the survey measure (model #3). The alternative Markov measure, based on an assumption of mean-reversion in the output gap, rather than perfect foresight, but with an equivalent 1-year forecast horizon, \( E_{MR}(\pi_{t+4}) \), also performed better than the survey measure; however, in this equation, model #2, the real long foreign bond yield became insignificant. Two-year-ahead Markov expectations in models #3 and #4 were slightly less significant; the foreign long bond yield was insignificant in these equations as well.
**Table 4**

**Australian nominal long bond yield equation**

Dependent variable: $\Delta r$


<table>
<thead>
<tr>
<th>Model #</th>
<th>Measure of $E_t(\pi_{t+n})$</th>
<th>$\beta$ (t-stat)</th>
<th>$\alpha$ (t-stat)</th>
<th>$\gamma_0$ (t-stat)</th>
<th>$\gamma_1$ (t-stat)</th>
<th>$R^2$</th>
<th>(p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E_{PF}(\pi_{t+4})$</td>
<td>0.204 (4.80)</td>
<td>-0.241 (4.36)</td>
<td>0.079 (2.68)</td>
<td>0.13 (1.90)</td>
<td>0.332</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>$E_{MR}(\pi_{t+4})$</td>
<td>0.223 (4.10)</td>
<td>-0.226 (4.38)</td>
<td>0.083 (2.97)</td>
<td>-</td>
<td>0.284</td>
<td>0.93</td>
</tr>
<tr>
<td>3</td>
<td>Survey</td>
<td>0.262 (3.09)</td>
<td>-0.299 (3.47)</td>
<td>0.083 (2.31)</td>
<td>-</td>
<td>0.271</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>$E_{PF}(\pi_{t+8})$</td>
<td>0.091 (2.05)</td>
<td>-0.065 (2.95)</td>
<td>-</td>
<td>-</td>
<td>0.216</td>
<td>0.32</td>
</tr>
<tr>
<td>5</td>
<td>$E_{MR}(\pi_{t+4})$</td>
<td>0.245 (3.21)</td>
<td>-0.210 (3.61)</td>
<td>0.066 (2.33)</td>
<td>-</td>
<td>0.214</td>
<td>0.44</td>
</tr>
</tbody>
</table>

The remainder of this section concentrates on the results obtained from model #1’s specification (see Table C.3 in Appendix C for the full estimated dynamics):

\[
\Delta_i = \alpha_i_{t-1} + \beta E_{PF}(\pi_{t+4}) = \gamma_0 RetCap_{t-1} + \gamma_1 r^* + \sum_{i=0}^{2} \alpha_i \Delta r_{t-i} + \phi \Delta E_{PF}(\pi_{t+4}) + \psi g_{t-3} + \epsilon_t
\]

(7)

where:

- $i_t$ : nominal Australian 10-year bond yield;
- $E_{PF}(\pi_{t+4})$ : Markov model estimates of inflationary expectations as defined in (4) above or consumer survey measure;
- $RetCap$ : return on capital;
- $r^*$ : US real 10-year bond rate;
- $g$ : domestic GDP growth;
- $\Delta$ : difference operator;
- $\epsilon_t$ : white noise error term.

Full-sample predictions from this very simple nominal long bond yield equation fit the actual data very well (Figure 13). As in Section 3.1, out-of-sample forecasts were obtained by estimating the model to December 1991; actual values of the exogenous variables were then used to forecast the nominal long bond yield forward in time. The model anticipates the turning point in bond
yields in late 1993 as well as their subsequent pick-up over 1994, presumably because it contains the foreign bond yield (r*); the other models did not.

Figure 13
Dynamic simulation and out-of-sample forecasts

The null hypothesis in the final column of Table 4 tests whether the Fisher Hypothesis holds, such that movements in inflationary expectations are matched one-for-one by movements in the nominal interest rate. This restriction is necessary for valid reparameterisation of model #1 (equation (7)) as a real bond yield equation; the null hypothesis could not be rejected. Trivially, additional restrictions are also accepted such that this model, re-estimated as a real bond yield equation, delivers the same parameter estimates on the Z variables.

In this way, while equation (3) in Section 3.1, presented a model of the real 10-year bond yield, deflated with backward-looking expectations, equation (7) provides an alternative model which derives real yields by using a forward-looking Markov measure of expectations and has the following main features:
the Australian nominal long bond yield reacts to a change in inflationary expectations with a lag (bottom panel of Figure 14). In contrast, a permanent 1 percentage point rise in the US real long bond rate, ceteris paribus, causes the Australian real long bond yield to react instantaneously; by the second quarter after the shock, the domestic long bond yield would be around 0.54 of a percentage point higher (panel 2, Figure 14; this is larger than the 0.30 of a percentage point implied by the Orr et al. cross-section estimates for Australia).

consistent with the result obtained from estimation of equation (3), a permanent 1 percentage point improvement in the return on Australian capital raises the domestic real yield by around 1/3 of a percentage point; this response occurs more slowly than that estimated for a change in the US real rate (panel 1, Figure 14).

Further research could investigate the possibility of including elements of both forward- and backward-looking expectations within a model of Australian bond yields.

Figure 14
Bond yield responses to permanent 1% shocks to:

Return on capital

US real bond rate

Inflation expectations
Conclusion

There is no single, simple conclusion to be drawn from this research but rather, a series of points can be made.

Interest rates and exchange rates now form part of the transmission mechanism by which policy changes feed through to the broader economy. Expectations play a critical role in this mechanism, affecting both the timing and speed of transmission. Theoretical discussions of interest rate and exchange rate markets typically characterise expectations as forward looking. However, it has been difficult to model this type of behaviour within an empirical framework.

One approach has been to rely on the relevant components of full-scale, intertemporal macroeconomic models. These models embody theoretically consistent long-run properties and rational forward-looking expectations. In Australia, such exchange rate and bond yield equations are not estimated; they reflect orthodox theoretical considerations including uncovered interest parity and the term structure hypothesis. But the textbook-style results produced by these macro-models have limited relevance for practical policymaking.

Alternatively, single equation, behavioural models can be used to document the observed historical relationships in the data. These have typically assumed that expectations are formed adaptively, that is, are backward looking. The research in this paper concentrates on introducing a forward-looking element into behavioural models of the Australian real exchange rate and long bond yield.

Given that expectations play a central role in determining the responses to various shocks, the macroeconomic and behavioural model approaches are probably best distinguished by a comparison of impulse response functions. In particular, these two methodologies provide different characterisations of the behaviour of the real exchange rate. In the macro model framework, monetary policy shocks elicit an instantaneous change in the real exchange rate which is subsequently and gradually unwound. In contrast, the behavioural model does not return this instantaneous "jump" response. Instead, the real exchange rate only gradually transmits a change in monetary policy through to the broader economy so that the full impact of the policy change through this channel is felt with a lag. Despite very different adjustment paths, both models produce final responses of a similar order of magnitude.

On the other hand, about half of a sustained terms-of-trade shock is finally passed through to the real exchange rate in the macro models; this occurs through an initial jump in the exchange rate, followed by gradual adjustment towards the long run. While this result is theoretically appealing, it does not describe the actual behaviour of the Australian real exchange rate. The behavioural model estimates that the real exchange rate moves much more closely with terms-of-trade shocks, regardless of whether the shocks are temporary or sustained over very long periods. Some overshooting is estimated to occur immediately. This result is puzzling but it is consistent with the idea that agents in the foreign exchange market have only a relatively short horizon. The inherent difficulty of incorporating inefficient mechanisms into the macro model framework may be one source of the disparity between the macro model results and those recorded by the behavioural models.

Incorporating forward-looking behaviour into a bond yield equation is less straightforward. In this paper, it is achieved by explicitly modelling the formation of inflation expectations. Expectations are generated from a series of assessments about the probability of shifting between a high and a low inflation regime. This is particularly apt in Australia, since a discrete shift in inflationary expectations occurred in the early 1990s. The superior performance of the shorter horizon expectations suggests that some myopia may exist in this market as well. Further work in this area might consider whether there are roles for both forward and backward-looking elements within the model.
Appendix A: Data sources

The data for Section 2 of the paper were collected for the period from September 1973 to June 1995. The data for Section 3 were collected for the period from December 1979 to June 1995. All indexes are based to 1989/90 = 100. This Appendix lists each of the variables used in the paper together with their method of construction and original data source(s).

Real exchange rate
Index.
Reserve Bank of Australia.

Terms of trade
Index; seasonally adjusted; goods and services measure.
The terms of trade was spliced to the goods and services trend measure at September 1974.
Australian Bureau of Statistics, Catalogue 5302.0, Table 9.

Nominal gross domestic product (GDP)
Millions of A$; seasonally adjusted; income measure.
Australian Bureau of Statistics, Catalogue 5206.0.

Real gross domestic product
Average measure.
The growth variable is the quarterly growth of real GDP.
Australian Bureau of Statistics, Catalogue 5206.0.

Cumulated current account
Current account balance; millions of A$; seasonally adjusted.
The cumulated current account for each quarter is calculated as the cumulative sum of quarterly current account balances from September 1959 and taken as a proportion of annualised GDP:

\[ \sum_{j=1}^{t} \text{current account}_j / (GDP \times 4) \]

Australian Bureau of Statistics, Catalogue 5302.0, Table 3.

Net foreign liabilities
Net international investment position at end of period; millions of A$; not seasonally adjusted.
Annual data for the period June 1974 - June 1985, quarterly data afterwards; expressed as a proportion of annual GDP.
June 1974-June 1978: Reserve Bank of Australia Occasional Paper No. 8;
June 1979-June 1995: Australian Bureau of statistics Catalogue 5306.0, Table 1.

Fiscal
Commonwealth government budget balance.
The fiscal variable for the four quarters of each fiscal year is measured as the change in the annual Commonwealth government budget balance as a proportion of GDP, calculated on a quarterly basis.
Cash rate
Reserve Bank of Australia Bulletin, Table F1 and internal sources.

90-day bank bill
Reserve Bank of Australia Bulletin, Table F1 and internal sources.

10-year bond rate
Reserve Bank of Australia Bulletin, Table F2 and internal sources.

GDP in US dollars
Annual GDP for the United States, Canada and the United Kingdom, measured in millions of US dollars, are applied as weights in the construction of world variables. The UK measure of GDP is quarterly and is converted into an annual measure.

World short interest rates
The world short interest rate is calculated as the weighted arithmetic average of short interest rates (3-month Treasury bills) from the United States, Canada and the United Kingdom. Each country's GDP, measured in US dollars, are used as weights.

World long interest rates
The world long interest rate is calculated as the weighted arithmetic average of long interest rates for the above countries, with GDP in US dollars used as weights.

Real interest rates
Real interest rates for the exchange rate section are calculated by deflating the interest rate by a corresponding measure of four-quarter-ended inflation ie. \( \left( 1 + i_t / 1 + \pi_t \right) - 1 \). For the bond yield equation, US long bond yields are deflated by quarter-ended inflation.

Australia: Treasury underlying price index. Commonwealth Treasury.
Canada: Underlying price index; Datastream code: cnd20833.
Consumption deflator. Datastream code: cnipdcone.
The underlying price index is spliced to the consumption deflator at March 1986.

United Kingdom: Underlying price index. Datastream code: ukrpiy..f.
Consumption deflator. Datastream code: ukipdcone.
The underlying price index is spliced to the consumption deflator at March 1987.

Yield differential
The yield differential is calculated as the difference between the Australian and world yield curves. The yield curve for Australia is measured as the difference between the cash rate and the 10-year bond rate. The world yield curve is measured as the difference between short and long nominal world interest rates.
Inflation
Treasury underlying rate.
Commonwealth Treasury.

Return on capital
The return on capital is measured as corporate GOS divided by gross capital stock. 
Australian Bureau of Statistics catalogue 5206.0 and 5221.0.

Inflation expectations
Constructed from a Markov switching model using underlying inflation and an output gap. The output gap is calculated as the percentage deviation of nominal GDP(A) from a Hodrick-Prescott trend.

Survey
The survey variable is the Westpac/Melbourne Institute survey of consumer inflation expectations over the next four quarters.
Appendix B: The behavioural model of the Australian real exchange rate: integration tests and
diagnostics

Table B.1
Testing the Blundell-Wignall et al. (1993) equation:
cumulative current account deficit (CCAD) or trend?

<table>
<thead>
<tr>
<th>Model</th>
<th>Estimate coefficient¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCAD</td>
</tr>
<tr>
<td>Original specification - CCAD</td>
<td>-0.281 (-2.60*)</td>
</tr>
<tr>
<td>Adding a trend term</td>
<td>0.626 (1.49)</td>
</tr>
<tr>
<td>Replacing CCAD with a trend term</td>
<td></td>
</tr>
</tbody>
</table>

¹ Estimates are taken from the Bewley Transformation of an unrestricted error correction model; figures in parentheses denote t-statistics; * denotes significance at the 10% level.

Table B.2
Yield gap variable: testing the null of the validity of the implied restrictions

\[ YGAP = \gamma \left\{ (i_s - i_L) - (i_s - i_L)^* \right\} \]

<table>
<thead>
<tr>
<th>Sample period</th>
<th>Test-statistic</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985:1-1995:2</td>
<td>1.06</td>
<td>F(3,31)</td>
</tr>
<tr>
<td>1973:4-1995:2</td>
<td>0.77</td>
<td>F(3,76)</td>
</tr>
</tbody>
</table>

The DF-GLS test (Elliot et al. (1992)) is a modified version of the Augmented Dickey-Fuller (ADF) t-test, having the advantages that it exhibits superior power properties and suffers from only small size distortions in finite samples. The testing procedure involves demeaning or detrending the series using Generalised Least Squares and then running the ADF test regression using that series. The constant and time trend terms are omitted from the test regression. The t-statistic on (p-1) is then used to test for significance against the appropriate critical value. The demeaned case (DF-GLSH) is comparable to including a constant term in the ADF test; the critical values are taken from Fuller (1976) and the no-constant variant of the MacKinnon (1991) table. The detrended case (DF-GLS) is comparable to including a constant and a time trend in the ADF test; the critical values have been tabulated by Elliot et al. (1992).
### Table B.3a
**Integration tests: 1973:4–1995:2**

<table>
<thead>
<tr>
<th></th>
<th>( H_0 ): Non-stationarity</th>
<th>( H_0 ): Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Phi_3 )</td>
<td>( \tau_t )</td>
</tr>
<tr>
<td>Real exchange rate.....</td>
<td>5.48*</td>
<td>-3.31*</td>
</tr>
<tr>
<td>Terms of trade.........</td>
<td>10.6***</td>
<td>-4.07***</td>
</tr>
<tr>
<td>Prices..................</td>
<td>8.33**</td>
<td>-2.32</td>
</tr>
<tr>
<td>Current account........</td>
<td>7.44**</td>
<td>-3.86**</td>
</tr>
<tr>
<td>Debt....................</td>
<td>1.76</td>
<td>-1.87</td>
</tr>
<tr>
<td>Government deficit.....</td>
<td>6.97**</td>
<td>-3.71**</td>
</tr>
<tr>
<td>Yield gap..............</td>
<td>5.00</td>
<td>-3.15*</td>
</tr>
<tr>
<td>Yield gap*.............</td>
<td>5.01</td>
<td>-3.16*</td>
</tr>
</tbody>
</table>

*, ** and *** denote significance at the 10, 5 and 1% levels respectively. \( \Phi_3 \) refers to the likelihood ratio test of \( (\alpha, \beta, \rho) = (\alpha, 0, 1) \) in \( y_t = \alpha + \beta_1 y_{t-1} + \epsilon_t \). The critical values are from Dickey and Fuller (1981). \( \tau_t \) refers to the Augmented Dickey Fuller (ADF) "t-tests": \( \tau_t \) includes a constant and trend and \( \tau_p \) includes a constant only. The critical values are from Fuller (1976). DF-GLS and DF-GLS\( _\mu \) are a modified trend and constant versions, respectively, of the ADF tests proposed by Elliot, Rothenberg and Stock (1992). KPSS is a test proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992) which tests the null hypothesis of stationarity. A truncation lag of 8 is used for the calculation of the estimate of the error variance.

### Table B.3b
**Integration tests: 1984:1–1995:2**

<table>
<thead>
<tr>
<th></th>
<th>( H_0 ): Non-stationarity</th>
<th>( H_0 ): Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Phi_3 )</td>
<td>( \tau_t )</td>
</tr>
<tr>
<td>Real exchange rate.....</td>
<td>3.03</td>
<td>-2.36</td>
</tr>
<tr>
<td>Terms of trade.........</td>
<td>8.96***</td>
<td>-4.17***</td>
</tr>
<tr>
<td>Prices..................</td>
<td>6.67*</td>
<td>-1.99</td>
</tr>
<tr>
<td>Current account........</td>
<td>6.13*</td>
<td>-3.43*</td>
</tr>
<tr>
<td>Debt....................</td>
<td>11.35***</td>
<td>-4.74***</td>
</tr>
<tr>
<td>Government deficit.....</td>
<td>6.61**</td>
<td>-3.31*</td>
</tr>
<tr>
<td>Yield gap..............</td>
<td>6.78**</td>
<td>-3.63**</td>
</tr>
<tr>
<td>Yield gap*.............</td>
<td>2.78</td>
<td>-2.29</td>
</tr>
</tbody>
</table>

*, ** and *** denote significance at the 10, 5 and 1% levels respectively.
The KPSS (1992) test is applied in the following way. All series can be written as the sum of a trend \((\xi_t)\), a random walk \((r_t)\), and a stationary component \((\varepsilon_t)\) such that:

\[
y_t = \xi_t + r_t + \varepsilon_t
\]

where: \(r_t = r_{t-1} + u_t\)

If the series is stationary (that is, there is no random walk component), the variance of \(u_t\) will be zero. The test statistic for the null hypothesis of no unit root is an LM statistic which is a function of the estimated residuals and an estimate of the long run error variance. These residuals are either the demeaned series \((n_{mt})\) or the demeaned and detrended series \((n_{rt})\). The critical values for these tests are detailed in Kwiatkowski, Phillips, Schmidt and Shin (1992), page 166.

These tests provide evidence over the full sample period (1973:4-1995:2) that the real exchange rate, terms of trade, interest differentials, yield gaps, current account deficit, government budget balance, and relative productivity differentials are I(0), with the first two series exhibiting this stationarity around a trend. These conclusions are also supported over the shorter sample period (1984:1-1995:2).

### Table B.4
A comparison of the statistical significance of competing measures of interest rates in the real exchange rate equation

<table>
<thead>
<tr>
<th>Interest differential term</th>
<th>Estimated coefficient</th>
<th>t-statistic (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\left{ (i_t - i_{t-1}) - (i_{t-1} - i_{t-2}) \right} )</td>
<td>2.48</td>
<td>3.05 (0.00)</td>
</tr>
<tr>
<td>((r_t - r_{t-1}))</td>
<td>2.98</td>
<td>2.45 (0.02)</td>
</tr>
<tr>
<td>((r_t - r_{t-1})) as per the BW equation</td>
<td>0.16</td>
<td>0.11 (0.91)</td>
</tr>
<tr>
<td>((r_t - r_{t-1})^2)</td>
<td>3.64</td>
<td>2.39 (0.02)</td>
</tr>
</tbody>
</table>

1 The real TWI exchange rate model is specified as a function of the terms of trade, the cumulated current account deficit, an interest differential term, and a fiscal policy variable.

2 The real long interest differential is here tested in the B-W specification which expresses the real TWI as a function of the terms of trade, the cumulated current account deficit as a proportion of GDP, and this real long interest rate differential.
Figure B.1
Real exchange rate model: equation (2)
Diagnostics

CUSUM

CUSUM Squared
Figure B.2
Parameter stability tests

[Graph showing parameter stability tests with two panels, each with a y-axis labeled in percentage (%), a legend for 1 Standard deviation, and x-axis labeled from 86/87 to 94/95.]
### Appendix C: The behavioural model of Australian long bond yields: integration tests and diagnostics

#### Table C.1
Integration tests: 1979:4–1995:2

<table>
<thead>
<tr>
<th>Variable</th>
<th>$H_0$ : Non-stationarity</th>
<th>$H_0$ : Stationarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Phi_t$</td>
<td>$\tau_t$</td>
</tr>
<tr>
<td>Real 10-year bond</td>
<td>10.51***</td>
<td>-4.37***</td>
</tr>
<tr>
<td>Real US 10-year bond</td>
<td>4.38</td>
<td>-2.94</td>
</tr>
<tr>
<td>Return on capital</td>
<td>6.29**</td>
<td>-3.53**</td>
</tr>
<tr>
<td>Cash rate</td>
<td>2.47</td>
<td>-1.96</td>
</tr>
<tr>
<td>Government deficit</td>
<td>5.14</td>
<td>-3.18*</td>
</tr>
<tr>
<td>Undiversifiable risk</td>
<td>1.36</td>
<td>-1.60</td>
</tr>
<tr>
<td>Current account</td>
<td>3.54</td>
<td>-2.21</td>
</tr>
<tr>
<td>Inflation expectations</td>
<td>12.03***</td>
<td>-4.89***</td>
</tr>
<tr>
<td>$\Delta$ Inflation</td>
<td>58.8***</td>
<td>-10.80***</td>
</tr>
</tbody>
</table>

* *, ** and *** denote significance at the 10, 5 and 1% levels respectively.

$\Phi_t$ refers to the likelihood ratio test of $(\alpha, \beta, \rho) = (\alpha, 0, 1)$ in $Y_t = \alpha + \beta I_t + \rho Y_{t-1} + \epsilon_t$. The critical values are from Dickey and Fuller (1981). $\tau$ refers to the Augmented Dickey Fuller (ADF) "$t$-tests"; $\tau_\mu$ includes a constant and trend and $\tau_\mu$ includes a constant only. The critical values are from Fuller (1976). DF-GLS$_\tau$ and DF-GLS$_\mu$ are a modified trend and constant versions, respectively, of the ADF tests proposed by Elliot, Rothenberg and Stock (1992). KPSS is a test proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992) which tests the null hypothesis of stationarity. A truncation lag of 8 is used for the calculation of the estimate of the error variance.

All three tests support the stationarity of the Australian real 10-year bond rate around a constant or a trend. On the other hand, evidence for the US real long bond rate is mixed; the ADF and DF-GLS tests fail to reject the null of a unit root, but the KPSS tests fail to reject the null hypothesis that the real US long bond rate is stationary around either a mean or a trend. The return on domestic capital and inflationary expectations are both clearly stationary; the ratio of the Commonwealth government budget balance to GDP is mean stationary; the evidence for the undiversifiable risk term, "beta", is mixed.
### Table C.2

**Real long bond equation (3)**

Dependent variable: change in real bond


<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of adjustment parameter</td>
<td>-0.513***</td>
</tr>
<tr>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>Return on capital</td>
<td>0.164***</td>
</tr>
<tr>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>Inflation term ((\tilde{\pi}_t - \hat{E}_t(\pi)))</td>
<td>0.256***</td>
</tr>
<tr>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \text{Real bond}_{t-2})</td>
<td>0.369***</td>
</tr>
<tr>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \text{Return on capital}_{t-1})</td>
<td>0.431*</td>
</tr>
<tr>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \hat{E}_t(\pi))</td>
<td>-1.22***</td>
</tr>
<tr>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \hat{E}_{t-2}(\pi))</td>
<td>0.93***</td>
</tr>
<tr>
<td>(0.29)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \text{Government deficit})</td>
<td>0.89*</td>
</tr>
<tr>
<td>(0.52)</td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>0.40*</td>
</tr>
<tr>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>Growth_{t-1}</td>
<td>-0.39*</td>
</tr>
<tr>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.60</td>
</tr>
<tr>
<td>DW</td>
<td>1.76</td>
</tr>
<tr>
<td>ARCH test</td>
<td>(\chi^2)</td>
</tr>
<tr>
<td>(0.882)</td>
<td>[0.347]</td>
</tr>
<tr>
<td>AR (4) test</td>
<td>(\chi^2)</td>
</tr>
<tr>
<td>3.28</td>
<td>[0.512]</td>
</tr>
<tr>
<td>Jarque-Bera normality test</td>
<td>(\chi^2)</td>
</tr>
<tr>
<td>0.96</td>
<td>[0.618]</td>
</tr>
</tbody>
</table>

*, ** and *** denote significance at the 10, 5 and 1% level respectively.

Standard errors are in parentheses; probability values are in square brackets.
Figure C.1  
Real long bond equation (3)

Figure C.2  
Nominal bond equation (7)
Table C.3
Nominal bond equation (7)
Specification #1
Dependent variable: change in nominal bond

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal bond, j-1</td>
<td>-0.241***</td>
</tr>
<tr>
<td>Capital return, j-1</td>
<td>0.079***</td>
</tr>
<tr>
<td>US real, j-1</td>
<td>0.127*</td>
</tr>
<tr>
<td>{E_{PP}(\pi_{t+4})}_{t-1}</td>
<td>0.204***</td>
</tr>
<tr>
<td>Δ US real</td>
<td>0.268*</td>
</tr>
<tr>
<td>Δ Expectations, j-1</td>
<td>0.375***</td>
</tr>
<tr>
<td>Δ GDP, j-3</td>
<td>-0.304***</td>
</tr>
</tbody>
</table>

R² .................................................. 0.332
DW .................................................. 2.07
ARCH test \(\chi^2\) ................................ 0.416 [0.519]
AR (4) test \(\chi^2\) ................................ 3.0846 [0.544]
Jarque-Bera normality test \(\chi^2\) ........ 1.408 [0.495]

* , ** and *** denote significance at the 10, 5 and 1% level respectively.

Standard errors are in brackets, probability values are in square brackets and the F-test for the joint significance of the US real rate dynamics are in parentheses {}.

The small negative coefficient on the third lag of growth in the dynamics of equation (7) corresponds with the (roughly) 3-year cycle in bond yields in Australia.
References


Bleaney, M., 1993, "The Australian Real Exchange Rate, Terms of Trade and Primary Commodity Prices 1901-91", CREDIT Research Paper No.93/6, University of Nottingham.


Comments on paper by A. Tarditi by Robert McCauley (BIS)

This paper offers a contrast between its careful empirical results and local macroeconomic models that, as far as the financial sectors are concerned, sacrifice reality to theoretical appeal. This comment attempts to advance the interpretation of the results of the exchange rate analysis and pose a pair of questions regarding the interest rate analysis.

1. **Exchange rate**

The analysis of the real exchange rate very nicely underscores the importance of the terms of trade, the stance of monetary and fiscal policy and the accumulating net liability position.

The terms of trade effect in this analysis and in previous analyses seems too strong, and this is one puzzle of Australia's exchange rate. Another puzzle is why does the Australian dollar tend to strengthen when the US dollar appreciates and to fall when the US dollar depreciates. These two puzzles may have common roots in the portfolio behaviour of foreign investors but quite different behavioural grounds. The strong reaction of the Australian dollar to the terms of trade reflects the scarcity value of Australian assets as commodity bets. With Australia, you get industrial country risk and developing country exposure to commodities. Foreign investors buy Australian stocks, currency and even bonds when they are worried about commodity prices. Given the adverse effect of commodity prices on corporate profits and bond prices in the rest of the industrial country portfolio, Australian assets offer some insurance.

Market participants report that the Australian dollar's resonance with the dollar-mark exchange rate arises because Continental and Japanese investors tend to treat the Australian dollar, in common with its Canadian and New Zealand cousins, as a supercharged dollar. Thus, if buying the US dollar looks good, buying the Aussie dollar looks even better. Here the motive is quite different: reaching for yield means accepting risk, not avoiding it.

A question that arises not just in connection with the Australian paper but also in other papers is the theoretical underpinning of net international liabilities or assets. Does this variable measure the growing exposure of global portfolios to Australian dollar assets in a Branson portfolio balance model of exchange rates on the assumption that the current account deficit is financed entirely in Australian dollars? Or does this variable measure a country's international indebtedness, or solvency, where all the debt might be in, say, US dollars? The theoretical underpinning should be thought out so that a proper measure is selected.

2. **Long-term interest rate**

The paper analyses the real bond yield and finds it related only to the real return on capital and the difference between current and average inflation. The author is quite sensibly unsatisfied with an approach that divides the current yield by current inflation to generate the dependent variable only to enter current inflation as a regressor. She moves on to modelling the nominal bond yield, and that strikes me as a good place to start as well as to end. In such a framework, one can test whether one can cast the model as a real yield equation.

The habit of entering the real return to capital as a determinant of the real rate strikes me as both obscure and sneaky. Obscure because it is not clear how much it is a cyclic variable and how much it is picking up differences across cycles in profitability. Sneaky because high real interest rates, caused by high fiscal deficits in the conventional wisdom, may have constrained managers to show better profits. In other words, might not the real side show the effects of financial markets rather than vice-versa?
The author's experiments with a Markov model for inflation regimes are quite promising. The Markov model seems well justified by the survey expectations that show 10 percent inflation until the end of the 1980s and then drop to 5 percent in the 1990s in one step. But the claim that the Markov-derived expectations are different from and better than "inherently backward-looking survey expectations" should be explicitly demonstrated. Are the estimated mean inflation rates of 8.7 percent and 3.3 percent significantly different from 10 and 5 percent, respectively? Given the correlation of survey and Markov expectations, do the Markov expectations dominate the survey expectations if they are run head-to-head?

The author makes a pitch for so-called behavioural models over the imposition of model-consistent, perfect-foresight expectations. It is sign of danger that such a pitch is felt to be necessary. Policy could only suffer were economists at central banks to yield to their aesthetic inclination to neatness and coherence in model building and thereby miss observed regularities that cannot be derived from some economically correct model of house-trained agents.
The expectations theory: tests on French, German and American euro-rates

Eric Jondean\(^1\) and Roland Ricart\(^2\)

Introduction

The expectations theory of the term structure of interest rates (ETTS) has received a great deal of attention for several years now. The interest undoubtedly stems in part from the fairly pragmatic implementation of the theory and the scope of its proposals. A widely accepted idea is that the slope of the term structure on a given date contains information about future changes in interest rates.

The implications of the ETTS have, however, long been contested by empirical work (see Shiller [1990] for a summary). According to the problem set by Campbell and Shiller [1991], the link between the slope of the term structure and the future long-term rate has the wrong sign, and the correlation between the slope of the term structure and changes in the future short-term interest rate is not as strong as expected. This work is based almost exclusively on American data from the post-war period, but some findings are atypical of this overall statement (Mankiw and Miron [1986], Fama [1984], Mishkin [1988]). The most recent work on countries other than the United States, is more favourable to the expectations theory (Gerlach and Smets [1995], Dahlquist and Jonsson [1995], Hurn et al [1995]).

Does this mean we should accept the idea that the theory is valid for some countries and not for others? That some interest rates contain predictive information and others do not? We shall attempt to develop this point using a methodological and pragmatic approach that differs from the one usually applied. Two main approaches are developed.

The first approach is based on the implications of the apparent non-stationarity of interest rates, which has been proven for many years now. This property is implicitly taken into account in the formulation of the usual tests of the ETTS. Nevertheless, the complete dynamics of the links between interest rates should be specified in the form of an error-correction model (ECM) that incorporates the long-term link as well as the short-term dynamics (as proposed by Engle and Granger [1987]). This omission in the test of the expectations theory can lead to specification biases (Hakkio and Rush [1989]).

The second approach aims to isolate the impact on the estimates of observations made in times of monetary tension. As a general rule, experience has shown that the estimated parameters are fairly unstable. The findings of Mankiw and Miron [1986] show that coefficients are highly dependent on the period used for the estimates. In a somewhat different context, the work of Dahlquist and Jonsson [1995] leads to the same type of conclusion. This variability of parameters, which is generally linked to turmoil on money markets, shows up as a particularly important phenomenon in the empirical applications presented in this paper. In fact, a sensitivity analysis shows that just a few observations suffice to have a major impact on the estimated parameters.

Based on this dual approach, we attempted to test the ETTS using the French, German and American euro-rates between January 1975 and October 1995. The organisation of the paper is as follows: the first section presents the three usual tests of the ETTS, along with the characteristics of the results they give. In the second section, the same tests are made within the framework of an error-correction model; the third section gives details of the data used, the stationarity test results and the

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method used to evaluate the sensitivity of estimates to observations. The last section comments on the findings of the tests based on the usual specifications and those based on ECMs.

1. **The expectations hypothesis according to the usual approach**

1.1 **The expectations theory**

The ETTS is essentially based on the assumption that there is no arbitrage opportunity: two investment strategies applied at \( t \) for the same horizon must have the same expected yield. Otherwise, all of the investors would prefer the investment with the highest expected yield, thus raising the prices of the underlying securities and thereby reducing the yield. However, it is generally accepted that different yields on different transactions could reflect a premium arising from the liquidity preference, for example, or from preferred-habitat phenomena, or even from institutional restrictions. But the expectations theory postulates that this premium is constant over time (even though it may vary according to the maturities of the securities in question). In its most traditional form, the expectations theory establishes that, save the constant premium, the yield at \( t \) on an investment with a maturity of \( n \) is equal to the expected yield at \( t \) on successive investments in short-term securities with a maturity of \( m \) at \( t, t+m, \ldots, t+n-m \) (in the following \( m \) will refer to the shorter maturity or investment horizon and \( n \) to the longer maturity or investment horizon) (Shiller [1979]):

\[
r(t, t+n) = \frac{1}{n-m} \sum_{i=0}^{n-m} E_t r(t+i, t+m) + \frac{m}{n} c(m, n) \quad \text{where} \quad \frac{n}{m} \text{ is an integer} \tag{1}
\]

and where \( r(t, t+n) \) is the yield at time \( t \) of a zero-coupon bond with a maturity of \( t+n \) and \( E_t \) is the expectation conditional upon the information available at time \( t \). The premium \( c(m, n) \) may depend on \( m \) and \( n \), but it must be constant over time.

1.2 **Deriving the usual tests**

There has been an abundant literature over the last fifteen years on tests of the assumptions of the ETTS. Even though other specifications have been presented elsewhere, three main test forms can be distinguished. They stem directly from equation (1). In every case, the specifications are reformulated to show an interest rate movement on the left-hand side and a yield spread on the right-hand side. This is done to take into account that interest rates may be non-stationary.

The first equation is based on the *correlation between the expected change in the short-term rate and the spread between the forward rate and the short-term rate*:\(^3\)

\[
[E_t r(t+n-m, t+n) - r(t, t+m)] = \frac{m}{n-m} [r(t, t+n) - r(t, t+m)] + \frac{m}{n-m} [c(m, n-m) - c(m, n)] \tag{2}
\]

\[
[E_t r(t+m, t+n) - r(t, t+n)] = \frac{m}{n-m} [r(t, t+n) - r(t, t+m)] + \frac{m}{n-m} [c(m, n-m) - c(m, n)] \tag{3}
\]

\(^3\) See Box 1 for the definitions of the different types of yield.
Box 1: Different types of yield

If \( r(t,t+m) \) is the yield at \( t \) on a zero-coupon bond with a remaining maturity of \( m \), the three following types of yield can be defined (see Shiller [1990]):

- the forward rate: this is the yield at \( t \) on holding a zero-coupon bond with a maturity of \( m \) from \( t+n-m \) to \( t+n \) \((m < n)\). The forward rate can be inferred from the yield at \( t \) of a bond with a remaining maturity of \( n \) and from the yield on a bond with a remaining maturity of \( n-m \):

\[
f(t,t+n-m,t+n) = \frac{nr(t,t+n)-(n-m)r(t,t+n-m)}{m}
\]

- the holding yield: this is the yield at \( t \) from the purchase of a zero-coupon bond with a remaining maturity of \( n \) that is resold at \( t+m \) \((m < n)\). It is written as:

\[
h(t,t+m,t+n) = \frac{nr(t,t+n)-(n-m)r(t+m,t+n)}{m}
\]

- the rollover rate: this is the yield at \( t \) from successive purchases at \( t, t+m, ..., t+n-m \) of zero-coupon bonds with remaining maturities of \( m \) \((m < n)\). It is written as:

\[
h'(t,m,t+n) = \frac{m}{n} \sum_{i=0}^{n-1} r(t+im,t+im+m) \text{ where } \frac{n}{m} \text{ is an integer}
\]

The third equation is based on the correlation between the average expected variation in the future short-term rate over a long period and the slope of the term structure. It is obtained directly by subtracting the current short-term rate \( r(t,t+m) \) from both sides of equation (1):

\[
\frac{m}{n} \sum_{i=0}^{n-1} E_t[r(t+im,t+im+m)-r(t,t+m)] = \frac{1}{n}\left[r(t,t+n) - r(t,t+m)\right] c(m,n) \quad (4)
\]

The last two specifications show that an increase in the spread between long-term and short-term rates should be accompanied by a future increase of both long-term and short-term rates. The initial spread will decrease, however, should the short-term rate rise by more than the long-term rate.

In fact, the ETTS implies that when one of the specifications (2) – (4) holds for any \( m \) and any \( n \), then the other two also hold for any \( m \) and any \( n \).

1.3 The standard results

Tests of the ETTS are usually based on estimates of the specifications (2) – (4). But they require a further assumption as to how expectations are formed. In practice, these tests are based on the joint assumption that there is no arbitrage opportunity and that expectations are rational. Equations (2) – (4) are rewritten as:

\[
[r(t+n-m,t+n) - r(t,t+m)] = \alpha + \beta [f(t,t+n-m,t+n) - r(t,t+m)] \quad (5)
\]

\[
[r(t+m,t+n) - r(t,t+n)] = \alpha + \beta \frac{m}{n-m}[r(t,t+n) - r(t,t+m)] \quad (6)
\]

\[
\frac{m}{n} \sum_{i=0}^{n-1} \left[1-\frac{m}{n}\right] r(t+im,t+im+m) - r(t+im-m,t+im) = \alpha + \beta \left[r(t,t+n) - r(t,t+m)\right] \quad (7)
\]
In their "pure" form the ETTS hypotheses require that \( \alpha = 0 \) and \( \beta = 1 \) but, in empirical work, the null premium is often dropped to concentrate on parameter \( \beta \) being equal to one.

Even if one is more specifically interested in the analysis of short-term securities markets (typically for securities with a maturity of one year or less), interpreting the findings of much empirical work is a delicate matter. The findings can vary from one study to the next depending on the tests run, the segment of the yield curve examined or the period under study. Nonetheless, some robust conclusions can be highlighted.

- The first specification, which is the variation in the short-term rate as a function of the spread between the forward and short-term rates, gives results that tend to favour the ETTS when the maturities are short enough: coefficient \( \beta \) in regression (5) is generally between 0 and 1, even if equality to 1 is rejected in most cases (Fama [1984], Fama and Bliss [1987]).

- Tests based on the second specification, which is the variation in the long-term rate as a function of the slope of the term structure, are the ones least favourable to the ETTS: the estimates of coefficient \( \beta \) in regression (6) are almost always negative and significantly different from 1 (Campbell and Shiller [1991], Campbell [1995], Evans and Lewis [1994]).

- The estimates of the third specification, which is the variation in the short-term rate as a function of the slope of the term structure, tend to favour the ETTS. Even though the coefficient \( \beta \) in regression (7) is generally significantly different from 1 for the shortest maturities, it is often positive and close to one for the shortest maturities (Campbell and Shiller [1991], Campbell [1995]).

The international dimension is also important in the analysis of the results. Comparisons in recent years have shown that developments in the American financial market tends to be unfavourable to the ETTS. In a broad international comparison based on the third specification, Gerlach and Smets [1995] concluded that the term structure of euro-dollar rates is the least favourable to the ETTS, while for countries such as France, Belgium, Italy and Spain, the theory is more broadly validated. Dahlquist and Jonsson [1995], using a test based on the first specification, were unable to reject the ETTS for interest rate data from Swedish government bonds. Hurn et al [1995] also obtained favourable results from interest rates on the British interbank market, using the third specification. What transpires from the various work is that the prevailing quasi-automatic rejection of the ETTS by American data should be at least mitigated in the case of other countries.

2. The expectations theory according to the cointegration approach

Most of the empirical work done to test the ETTS recognises the problem of the non-stationarity of interest rates. Generally, the variables are made stationary (see the three specifications above), which makes it possible to make econometric estimates on the basis of stationary series. Nevertheless, aspects that are directly linked to cointegration are rarely taken into account (with the notable exception of Campbell and Shiller [1987] or Dahlquist and Jonsson [1995]). As was shown by Engle and Granger [1987], a cointegration relationship between two series leads to certain restrictions in the specification of the short-term dynamics of the series. More precisely, if two variables \( X \) and \( Y \) are integrated of order one (or \( I(1) \)) and cointegrated, there is a relationship between the levels of the two variables \( X_t = a + bY_t + \varepsilon_t \), where \( \varepsilon_t \) is a stationary (but not necessarily white noise) error term. In this case, the full dynamics of the system can be written as an ECM:

\[
A(L) \begin{pmatrix} \Delta X_t \\ \Delta Y_t \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \varepsilon_{t-1} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}
\]

(8)
where $A(L)$ is a matrix polynomial in the lag operator, and $u_t$ and $v_t$ are white noise. Engle and Granger have shown that if $X$ and $Y$ are cointegrated, then $\gamma_f$ and/or $\gamma_g$ is significant.

The argument being developed here is that when interest rates are non-stationary, the premia suggested by Shiller [1990] are logical candidates for the status of a cointegration relationship. In fact, these same premia are taken as the starting points for proposing the usual specifications of (2) – (4). The specifications can then be deduced for ECMs describing the changes in yield. At this point it is observed that the ECM specifications, which fairly naturally represent a general framework for testing the ETTS, are not necessarily compatible with the three specifications (5) – (7). These could then be seen as representations that are too specific and likely to contain specification errors.

2.1 Long-term relationships

Shiller [1990] proposed three definitions of time-independent premia based on the assumption that there are no arbitrage opportunities between different types of investment (for which the yields are defined in Box 1).

The forward premium $\varphi_f$ is defined by:

$$\varphi_f(n-m,n) = f(t, t+n-m, t+n) - E_r(t+n-m, t+n) \quad 0 < m < n$$

that is to say by the difference between the yield on a forward investment at $t$ in $n-m$ periods on a security maturing at $t+n$ and the expected yield at $t$ on an investment at time $t+n-m$ on a security maturing at $t+n$.

The holding period premium $\varphi_h$ is defined by:

$$\varphi_h(m,n) = E_r(h(t, t+m, t+n) - r(t, t+m) \quad 0 < m < n$$

that is to say by the difference between the expected yield at $t$ from buying at $t$ a security maturing at $t+n$ and selling it at $t+m$ and the yield on a spot purchase at $t$ of a security maturing at $t+m$.

The rollover premium $\varphi_r$ is defined as:

$$\varphi_r(m,n) = r(t, t+n) - E_r(h'(t, m, t+n) \quad 0 < m < n \quad \text{where } \frac{n}{m} \text{ is an integer}$$

that is to say by the difference between the expected yield at $t$ of a sequence of purchases at $t, t+m, \ldots, t+n-m$ of securities with a remaining maturity of $m$ and the yield on the spot purchase at $t$ of a security maturing at $t+n$.

The interpretation of the latter two premia is of the same type as for equations (3) and (4). This means that, ceteris paribus, a steeper slope of the term structure, stemming from a drop in short-term rates, for example, leads to a lower holding yield and thus a rise in expected long-term rates. In the same way, a steeper slope of the term structure caused by a rise in long-term rates leads to a higher rollover yield and thus a rise in expected short-term rates, all else being equal.

If interest rates are non-stationary, the assumption that premia are constant over time leads to three cointegration relationships:

$$r(t+n-m, t+n) = f(t, t+n-m, t+n) - \varphi_f(n-m,n) + \varepsilon_1(t+n-m, t+n) \quad (9)$$

$$h(t, t+m, t+n) = r(t, t+m) + \varphi_h(m,n) + \varepsilon_2(t+m, t+n) \quad (10)$$

$$h'(t, m, t+n) = r(t, t+n) - \varphi_r(m,n) + \varepsilon_3(t+n-m, t+n) \quad (11)$$

where $\varepsilon_i$, for $i = 1$ to 3, reflects investors' expectation errors.
These cointegration relationships exist independently of the assumptions made about how expectations are formed. Indeed, cointegration requires only that the errors in the equation remain stationary, which is what happens as long as expectation errors themselves are stationary. The rationality of expectations, on the other hand, implies that these errors are white noise, which leads to errors in the form of moving averages (due to overlapping data, see below). This explains why the hypothesis of the ETTS cannot be tested directly within the framework of a cointegration relationship. The non-standard properties associated with the estimators in these regressions make it impossible to test the value of the parameters or the whiteness of the residuals. This is why the tests can only be done within the framework of an error-correction model.

The errors associated with cointegration relationships (9) – (11) are defined as follows:

\[ \varepsilon_1(t + n - m, t + n) = r(t + n - m, t + n) - E_t r(t + n - m, t + n) \]  
\[ \varepsilon_2(t + m, t + n) = h(t, t + m, t + n) - E_t h(t, t + m, t + n) \]
\[ = \frac{n - m}{m} \left[ r(t + m, t + n) - E_t r(t + m, t + n) \right] \]
\[ \varepsilon_3(t + n - m, t + n) = h'(t, t + m, t + n) - E_t h'(t, t + m, t + n) \]
\[ = \sum_{i=0}^{m} \left[ r(t + im, t + im + m) - E_t r(t + im, t + im + m) \right] \]

Thus, it can be seen that the errors associated with cointegration relationships, which are expressed directly as a function of the expectation errors, refer to different dates depending on the relationship. In (9) and (10), the errors stem from expectation errors made at \( t \) about \( t+n-m \) and \( t+m \) respectively. In equation (11) on the other hand, the error refers to expectation errors made at \( t \) about \( t+m, \ldots, t+n-m \). This is an essential point in the choice of the differentiation order for the error-correction model, and also for defining the degrees of overlapping.

### 2.2 Error-correction model specifications

The existence of the cointegration relationships (9) – (11), which still has to be validated empirically, makes it possible to establish a link with the ECMs. The specification of the models is a slightly more delicate matter than is usually the case. The cointegration relationships (9) – (11) show the arbitrage between investments that are assumed to be alternatives. Yet, at time \( t \), only one of the yields is perfectly known (this is the forward rate in (9) and the zero-coupon rates in (10) and (11)), the other yield is known after respective lags of \( n-m, m \) and \( n-m \). Yet, the ECM specification, of course, implies that the error-correction term (term \( \varepsilon_{t,m} \) in (8)) is known at time \( t \). Therefore, the orders of differentiation must be compatible with the number of periods required for the error-correction term to be known at time \( t \). The specifications of the error-correction models associated with cointegration relationships (9) – (11), with no lagged terms, are respectively:

\[ [r(t + n - m, t + n) - r(t, t + m)] = a_1 [r(t, t + m) - \delta f(t - n + m, t + m) + \phi_f(n - m, n)] \]
\[ + b [f(t, t + n - m, t + n) - f(t - n + m, t + m)] + \eta_1(t + n - m) \]

\[ [h(t, t + m, t + n) - h(t - m + t, t + m)] = a_2 [h(t - m + t, t + m) - \delta r(t - m, t) - \phi_h(m, n)] \]
\[ + b [r(t, t + m) - r(t - m, t)] + \eta_2(t + m) \]
\[ h'(t,m,t+n) - h'(t-n+m,m,t+m) = a[h'(t-n+m,m,t+m) - \delta r(t-n+m,t+m) + \varphi,(m,n)] \\
+ b[r(t+t+n) - r(t-n+m,m,t+m)] + \eta_3(t+n-m) \] (17)

The choice of the number of lags in writing the error-correction term is a natural one for the first two specifications. When writing the premium (9), the expected term (the zero coupon rate for maturity \( t+n \) in \( n-m \) periods) becomes known with a lag of \( n-m \) periods. In the same way, the holding yield between \( t \) and \( t+m \) on a security maturing at \( t+n \) only becomes known at time \( t+m \). In both cases, the necessary lag is, therefore, \( n-m \) and \( m \) periods respectively. If the investment horizon is \( n \) periods in the third specification, the rollover yield for a sequence of investments at \( t, t+m, ..., t+n-m \) is fully known right from time \( t+n-m \). Therefore, \( n-m \) periods must pass before the error-correction term is likely to influence changes in the rollover yield. The lag is indeed \( n-m \) periods, as shown in (17).

The first-difference terms in these equations are stationary if the yield variables are I(1). In the same way, if the expectations theory is valid, the error-correction term, which is the first term on the right-hand side, is stationary. Thus, standard econometric techniques can be used and they give convergent estimators. However, in view of the non-standard properties of the long-term parameters ("superconsistency", Stock [1987]), \( \delta \) shows non-standard properties and, unlike \( a \) and \( b \), cannot be checked with statistical tests that are easy to use in a single-variable framework.

It is easy to check that the expectations theory implies that \(-a = \delta = b = 1\) for the three ECM equations (15) – (17).

2.3 The consistency of the ECM specifications and the standard specifications

The problem of consistency between the ECM specifications and the usual specifications was first raised by Hakkio and Rush [1989] in a test of the efficiency hypothesis on the foreign exchange market. Their purpose was to show that when a spot rate and a forward rate are cointegrated, the best framework for testing efficiency is the ECM and that, in this case, carrying out the test with the usual specification (type (5)) can lead to a specification error.

The consistency between the usual specifications and the ECM representations must be analysed to develop this point. Of course, the various specifications are all consistent when the ETTS holds. In this case \(-a = \delta = b = 1\) in (15) – (17) and \( \beta = 1 \) in (5) - (7).

On the other hand, the situation is more complex under the alternative assumption. If the specifications (5) and (15) based on the forward rates are compared, it can be seen that the usual specification is clearly included in the ECM, because the two equations are only equivalent for \( a = -b \) and \( \delta = 1 \). Imposing these constraints when estimating the usual relationships can lead to a bias in the estimate of parameter \( \beta \), which will be all the greater because of the strong correlation between the two variables (the spread between spot rates and the forward rates, which is present, and the change in the forward rate, which is missing.

As for specifications (6) and (16), based on the long-term rates, it is clear that (6) is not included in (16), as the two specifications are only equivalent when \( a = -1, b = \delta = 1, \) and \( \beta = 1 \); or in other words, when the ETTS holds. Furthermore, specifications (7) and (17) are only consistent if \( a = -1, \delta = b \) and \( \beta = 1 \). In both cases, the usual specification does not appear to be a special case of the ECM. All in all, when the yields are I(1) and cointegrated, there is indeed a specification bias for the three tests based on the usual specifications.
3. The data and their characteristics

3.1 The data

Empirical analysis was based on the euro-rates quoted in London. The sample covers the end-of-month data from January 1975 to October 1995. The data collected are the average of the bid and offered rates at the close of trading for maturities of 1, 3, 6 and 12 months. Intermediate maturities were not available for the whole period, so they were obtained using linear interpolation. This technique is admittedly imperfect, but it provides uniform data and avoids the inherent estimation problems in complex interpolation procedures (e.g. Nelson and Siegel [1987]).

Because the authors work for the Banque de France, French rates are the first to be studied. However, the close ties between France and Germany naturally led them to consider German rates as well. American rates are also examined as a reference, insofar as the market has been widely studied in the literature. The choice of the euro-rates stems from a concern for uniformity between the countries under study in order to make international comparisons of the results possible, along with comparisons of the restrictions on domestic markets. Thus, even though long-run interest-rate data is available for the French and German interbank markets, the introduction of reference rates on these markets is a fairly recent development.

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4 The PIBOR and the FIBOR were launched in 1986.
However, there are problems that come up when euro-currency rates are used. For example, the fact that foreign exchange controls remained in place in France until 1986, means that arbitrage between France's domestic interest rates and the euro-rates was imperfect. Even though this situation did not necessary prevent arbitrage between the different maturities on the euro-currency markets, it is clear that in times of foreign exchange turmoil, the existence of segmented markets would lead to serious liquidity problems.

3.2 Outliers

The segmentation between domestic rates and euro-rates seemed to merit consideration of aspects relating to the detection of outliers, especially as tests of the ETTS hypotheses are, in some cases, very sensitive to the presence of such outliers. In times of currency turmoil, disturbances can be serious enough to give rise to outliers in the rates quoted. This was the case during the very severe turmoil in France in 1981 to 1983, and during the less severe bouts in 1987 and in 1992 to 1993. The outliers can be accidental, stemming, for example, from a momentary liquidity shortage that coincide with the observation of the reference rate, or else they can stem from exceptional positions taken momentarily, for example, in the days leading up to a realignment in the European Exchange Rate Mechanism. These outliers often lead to an artificial acceptance of the ETTS hypotheses. During a currency crisis, the sharp rise in short-term rates causes the average slope of term structure of interest rates to flatten. This is then generally followed by a rapid fall in short-term rates and a return to a steeper slope. These movements can be big enough to change the outcome of estimates made on a large sample.

We used usual procedures for detecting outliers in order to deal with this problem as rigorously as possible. As we are specifically interested in the value of the coefficients in the various regression equations, we examined the DFBETAS statistics proposed by Besley et al [1980]. The principle is to compare the value of the coefficient $\beta$ in the regression $Y_t = \beta X_t + \varepsilon_t$, $t = 1,...,T$, over the whole of the period and the value of the same coefficient, when the data observed at time $t = i$ are omitted. The test statistic for each date $i$ is defined by comparing the deviation obtained between the parameters to the standard deviation of the coefficient. Krasker et al [1983] suggested comparing this series to $3/\sqrt{T}$, where $T$ is the number of observations. The variable that was used as the basis for detecting outliers is the error-correction term (associated with coefficient $a$) in each of the error-correction equations (15) – (17).

To illustrate the importance of detecting outliers, we consider equations (6) (variation in the long-term rate as a function of the slope of the term structure), and (7) (variation in the short-term rate as a function of the slope of the term structure) to show how much some observations can influence the estimated parameters. For French rates, we used three distinct samples for the estimates.

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5 The one-month Euro-franc rate stood at an average of 18% in 1981-82, while the rate on the domestic money market over the same period stood at 15%.

6 Gerlach and Smets [1995] mention this type of dependence in the results they obtained from equation (4) using monthly Belgian, Danish, French, Spanish, Irish, Italian and Swedish Euro-rates. Shiller et al [1983], using equation (2) on monthly American data from the period 1959-1982, observed that their results vis-à-vis the observations after October 1979 were highly dependent. On the other hand, Mankiw and Miron [1986], working with quarterly American data from the period 1890-1979, found that the financial crises of 1890, 1893 and 1907 did not have any effect on their findings, which were favourable to the ETTS.

7 This methodology was used in a footnote by Shiller et al [1983] to show the high degree of sensitivity of the Shiller [1981] results to the data from 1970.
The first covers the whole period, the second excludes the data observed in March 1983\(^8\) and the third excludes all of the outliers. For the German and American rates, we examined only two samples. The first covers the whole period and the second the whole period except for the outliers. The results of the estimations are given in Appendix 1.

For French rates (Table A1-1), the most disrupted periods were mainly 1981 and 1982. The sensitivity of the results is clear for the estimations of the equation based on the nearest maturities of the long-term rates (i). Thus, for \(m = 1\) month and \(n = 3\) months, the coefficient goes from 1.23 for the whole of the sample to 0.25, when March 1983 is removed and it becomes negative at \(-0.23\) after the 7 most disruptive observations are removed. In the same way, for the 3-6 month pair of maturities for the equation based on the short-term rate (ii), the parameter goes from 0.97 to 0.95 and 0.32 respectively (5 observations are removed). Finally, while the ETTS hypothesis is accepted for each of the 12 pairs of maturities in Table A1-1, when the whole sample is used for the estimate, there are only 5 left when the most disruptive observations are removed.

In the case of American rates, the data problems are less serious than with the French rates. However, some problems remain since the short-term rates were disrupted between 1979 and 1982, when the Fed changed its operating procedures. In December 1980, the euro-dollar rates even reached 21%. The removal of the most disruptive observations leads to a sweeping change in the estimation of the parameters in some cases (Table A1-3). On the other hand, the results for German rates are not very sensitive to the way outliers are dealt with, as their effects tend to offset each other (Table A1-2).

All in all, the situation observed warrants systematic detection of outliers for the econometric estimates made in the rest of this paper.

### 3.3 The statistical properties of the series

Before undertaking any analysis with an error-correction model, the degree of non-stationarity of the data used must be checked. Up until now, we have implicitly accepted that the yield was \(I(1)\) and that premia inferred from that were \(I(0)\), which makes it possible to write the error-correction models (15) – (17). In order to validate these hypotheses, we tested the zero-coupon rate processes for a unit root. These processes are the basis for defining all of the other yields. We also tested for forward, holding period and rollover premia. The stationarity of the yield spread between long and short rates in specifications (3) and (4) was also tested (see Appendix 2).

In the case of the French rates in Table A2-1, the zero-coupon rates are clearly integrated of order one and the premia are all stationary, at least up to a significance level of 5%.

The results are less clear-cut in the case of the German rates shown in Table A2-2. While the Dickey and Fuller statistics clearly point to the non-stationarity of zero-coupon rates, the ADF statistics do not make it possible to conclude systematically that the rates are integrated, particularly for the shortest and longest maturities. Yet the autoregression coefficients are very close to 1 (to the order of 0.96 – 0.98 for all maturities). In spite of the inconclusiveness of the stationarity tests, interest rate behaviour looks very similar to that of an integrated process. Furthermore, the holding and rollover premia are clearly stationary. The test also makes it possible to conclude that the forward premia are stationary, but at significance levels of 5%, or even 10% for the 9-month rate in 3 months.

The American rates in Table A2-3 also display the characteristics of the \(I(1)\) process, with autocorrelations to the order of 0.96 – 0.98. But the variations in interest rates seem highly autoregressive: the ADF statistics are only significant at a significance level of 10% for maturities of 1 to 3 months and a significance level of 5% for longer maturities. Yet, the autoregression coefficient

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\(^8\) There was particular disruption in March 1983 because of the foreign exchange crisis that led to the realignment of 21 March (-2.5% for the FRF and +5.5% for the DEM. At the end of March, the one-month Euro-franc rate still stood at 45%, and the one-year rate at 18.5%.
for each of the maturities is to the order of 0.15 for 12 lags and 0.30 for 15 lags (the number of lags needed to whiten the residuals). On the whole, the premia seem to be stationary even though the significance level can reach 10% as the maturity lengthens. This is the case for the rollover premium for a maturity of more than 8 months. However, the forward premium in 10 months seems non-stationary, with an autoregression coefficient of 0.9, when the number of lags makes it possible to whiten the residuals.

These stationarity tests indicate that in almost all of the cases, the error-correction model specification is the suitable form for testing the ETTS hypotheses.

4. The empirical results

In view of the importance we gave to dealing with outliers, it is necessary to describe the approach used. The observations are selected on the basis of error-correction models, using the method described above. For each pair of maturities, we determined which observations distorted the estimate the most. These observations were removed and the error-correction models and the usual specifications were estimated using the samples with these points removed. In general, the number of observations removed was limited and, in practice, never came to more than 4% of the sample.

The analysis is made in two steps. First we presented the estimates and the tests based on the three usual specifications, obtaining results that are similar (except for the handling of outliers) to those generally obtained in the literature. Then, we examine the estimates based on the ECMs and tested the restrictions implied by the ETTS.

Even though they are all based on the same expectations theory, the three equations are not equivalent for any investment horizons \( m \) and \( n \). The idea is that each of them can be used to examine a different aspect of the expectations. The first equation can thus be used to test the ability of the expectations theory to forecast rates for fairly short remaining maturities \( m \), but at fairly distant horizons \( n-m \). Conversely, the second equation can be used to examine the short-term change (between \( t \) and \( t+m \)) of the yield on instruments with a fairly long remaining maturity \( n \). The third equation is an intermediate version in some ways. It can be used to examine the change over a fairly long period \( n \) in the yield on securities with a fairly short remaining maturity \( m \). From this point of view, even if the theory is rejected, the contrasting results from empirical tests can help identify more clearly how investors' expectations are formed.

4.1 Estimating the usual specifications

The usual specifications (5) – (7) were estimated for the main pairs of maturities from the database on French, German and American rates (Tables 1 to 3 respectively). For each of the three specifications, the configuration of the estimates is similar to that obtained in previous work:

- For estimates based on the forward rates, the coefficients are between 0.4 and 1 for the European rates, reaching the order of 1 for French rates for investment horizons from 4 to 8 months. In the case of French rates, they are close to one when the horizon is from 4 to 8 months. For the American rates, on the other hand, the absolute values of the coefficients are smaller;

- For estimates based on variations in the long-term rate, the test yields contrasting results in the French case. The coefficients are negative when the investment horizon is 1 month but are very close to 1 when the horizon is from 3 to 6 months. On the other hand, the coefficients are often negative, in the case of German rates, or systematically negative, in the case of American rates;

9 The three specifications are equivalent in the particular case where \( n = 2m \) (Campbell and Shiller [1991]).
Table 1
Estimates of usual specifications - France

This table shows the estimation results of the specifications:

\[ r(t + n - m, t + n) - r(t, t + m) = \beta_1 \left( f(t + n - m, t + n) - r(t, t + m) \right) + \text{constant} \]  

(i)

\[ r(t + m, t + n) - r(t, t + m) = \beta_2 \frac{m}{n} \left( r(t + n) - r(t, t + m) \right) + \text{constant} \]  

(ii)

\( \frac{m}{n} \sum_{i=0}^{m-1} \left( r(t + im, t + im + m) - r(t + im - m, t + im) \right) = \beta_3 \left( r(t, t + n) - r(t, t + m) \right) + \text{constant} \)  

(iii)

<table>
<thead>
<tr>
<th>( m - n )</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_1 )</td>
<td>( R^2 )</td>
<td>p-value</td>
</tr>
<tr>
<td>1 - 2</td>
<td>0.46 (0.17)</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>1 - 3</td>
<td>0.45 (0.15)</td>
<td>6</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 4</td>
<td>0.80 (0.24)</td>
<td>12</td>
<td>39.9</td>
</tr>
<tr>
<td>1 - 5</td>
<td>1.07 (0.19)</td>
<td>29</td>
<td>72.9</td>
</tr>
<tr>
<td>1 - 6</td>
<td>0.93 (0.16)</td>
<td>29</td>
<td>63.9</td>
</tr>
<tr>
<td>1 - 7</td>
<td>0.96 (0.21)</td>
<td>24</td>
<td>84.1</td>
</tr>
<tr>
<td>1 - 8</td>
<td>0.84 (0.16)</td>
<td>24</td>
<td>31.0</td>
</tr>
<tr>
<td>1 - 9</td>
<td>0.62 (0.15)</td>
<td>17</td>
<td>0.9</td>
</tr>
<tr>
<td>1 - 10</td>
<td>0.54 (0.13)</td>
<td>15</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 11</td>
<td>0.55 (0.14)</td>
<td>13</td>
<td>0.2</td>
</tr>
<tr>
<td>1 - 12</td>
<td>0.58 (0.16)</td>
<td>15</td>
<td>0.8</td>
</tr>
<tr>
<td>3 - 6</td>
<td>0.58 (0.17)</td>
<td>9</td>
<td>1.3</td>
</tr>
<tr>
<td>3 - 9</td>
<td>0.85 (0.16)</td>
<td>23</td>
<td>34.0</td>
</tr>
<tr>
<td>3 - 12</td>
<td>0.43 (0.15)</td>
<td>9</td>
<td>0.0</td>
</tr>
<tr>
<td>6 - 12</td>
<td>0.45 (0.16)</td>
<td>10</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Notes: The estimates relate to the period 1975-95. The observations from March 1983 have been removed. The usual relationships between the \( \beta_i \) for \( n = 2m \) are not necessarily seen, because the observations removed as outliers are selected on the basis of specifications in the form of associated error-correction models. The estimate of the constant is not shown in the table. Standard deviations, shown in parentheses, are corrected for heteroscedasticity (White [1980]) and for overlapping (see Box 2). The variance-covariance matrix is estimated as suggested by Newey and West [1987]. \( R^2 \) is the \( R^2 \), in %, corrected for the number of degrees of freedom and p-value is the significance level for the test of the hypothesis \( \beta_1=1 \).
Table 2
Estimates of usual specifications - Germany

This table shows the estimation results of the specifications given in Table 1.

<table>
<thead>
<tr>
<th>m - n</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_1 )</td>
<td>( \bar{R}^2 )</td>
<td>p-value</td>
</tr>
<tr>
<td>1 - 2</td>
<td>0.95</td>
<td>(0.11)</td>
<td>18</td>
</tr>
<tr>
<td>1 - 3</td>
<td>0.63</td>
<td>(0.10)</td>
<td>13</td>
</tr>
<tr>
<td>1 - 4</td>
<td>0.60</td>
<td>(0.12)</td>
<td>13</td>
</tr>
<tr>
<td>1 - 5</td>
<td>0.54</td>
<td>(0.13)</td>
<td>11</td>
</tr>
<tr>
<td>1 - 6</td>
<td>0.47</td>
<td>(0.15)</td>
<td>9</td>
</tr>
<tr>
<td>1 - 7</td>
<td>0.51</td>
<td>(0.21)</td>
<td>7</td>
</tr>
<tr>
<td>1 - 8</td>
<td>0.51</td>
<td>(0.22)</td>
<td>8</td>
</tr>
<tr>
<td>1 - 9</td>
<td>0.55</td>
<td>(0.21)</td>
<td>10</td>
</tr>
<tr>
<td>1 - 10</td>
<td>0.59</td>
<td>(0.21)</td>
<td>12</td>
</tr>
<tr>
<td>1 - 11</td>
<td>0.51</td>
<td>(0.24)</td>
<td>9</td>
</tr>
<tr>
<td>1 - 12</td>
<td>0.52</td>
<td>(0.23)</td>
<td>10</td>
</tr>
<tr>
<td>3 - 6</td>
<td>0.44</td>
<td>(0.14)</td>
<td>6</td>
</tr>
<tr>
<td>3 - 9</td>
<td>0.41</td>
<td>(0.21)</td>
<td>5</td>
</tr>
<tr>
<td>3 - 12</td>
<td>0.54</td>
<td>(0.21)</td>
<td>9</td>
</tr>
<tr>
<td>6 - 12</td>
<td>0.27</td>
<td>(0.21)</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Same as Table 1 except that no observations have been removed.

- The best results are obtained from the specifications based on variations in the short-term rate. The estimated coefficients are always positive for all three countries and often close to 1, especially for French rates.

The three markets show substantial differences in the tests based on the \( \chi^2 \) statistic:

- For American rates, the ETTS is accepted on the 5% significance level only for the forward rate of 1 month in 7 months. If the 1% level is allowed, the ETTS hypotheses are accepted in a few more cases (e.g., in the test based on the forward rate of 1 month in 1 month and the test based on the long-term rate in 12 months for an investment over 6 months or for the short-term rate in 1 month for an investment over 8 or 9 months).
Table 3
Estimates of usual specifications - United States

This table shows the estimation results of the specifications given in Table 1.

<table>
<thead>
<tr>
<th>$m - n$</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>$R^2$</td>
<td>p-value</td>
</tr>
<tr>
<td>1 - 2</td>
<td>0.61 (0.19)</td>
<td>3</td>
<td>4.6</td>
</tr>
<tr>
<td>1 - 3</td>
<td>0.25 (0.16)</td>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 4</td>
<td>-0.00 (0.20)</td>
<td>-0</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 5</td>
<td>0.11 (0.18)</td>
<td>-0</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 6</td>
<td>0.20 (0.17)</td>
<td>-0</td>
<td>0.0</td>
</tr>
<tr>
<td>1 - 7</td>
<td>0.45 (0.28)</td>
<td>3</td>
<td>5.0</td>
</tr>
<tr>
<td>1 - 8</td>
<td>0.39 (0.23)</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>1 - 9</td>
<td>0.36 (0.19)</td>
<td>3</td>
<td>0.1</td>
</tr>
<tr>
<td>1 - 10</td>
<td>0.32 (0.18)</td>
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<td>0.0</td>
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<td>1 - 11</td>
<td>0.36 (0.19)</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>1 - 12</td>
<td>0.27 (0.21)</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3 - 6</td>
<td>-0.06 (0.19)</td>
<td>-0</td>
<td>0.0</td>
</tr>
<tr>
<td>3 - 9</td>
<td>0.32 (0.27)</td>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>3 - 12</td>
<td>0.20 (0.17)</td>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>6 - 12</td>
<td>0.04 (0.22)</td>
<td>-0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note: Same as Table 1 except that no observations have been removed.

- On the 10% significance level, the German rates only accept the ETTS exceptionally for the three specifications, when $m = 1$ month and $n = 2$ months. When the significance level is reduced to 1%, the ETTS hypotheses are accepted for short investment horizons at the long-term rate ($m = 1$ month, $n = 2$ to 4 months) and at the short-term rate ($m = 1$ month, $n = 2$ to 3 months) and long investment horizons at the short-term rate ($m = 1$ month, $n = 9$ to 12 months), as well as for long horizons for forward investments ($m = 1$ month and $n = 7$ to 12 months, or $m = 3$ months and $n = 12$ months).

- In the case of French rates, the ETTS is more widely validated, even on the 5% significance level. This is the case for the test based on the forward rate when the investment horizon is 1 month (where $n = 4$ to 8 months) or 3 months (where $n = 9$ months), for the test based on the variation in the long-term rate when the investment horizon is 3 or 6 months ($m = 3$ or 6 months), or for the test based on the variation in the short-term rate when the short investment horizon is 1 month ($m = 1$ month, $n = 2$, 4 or 5 to 12 months). Most importantly, when the
significance level is reduced to 1%, the ETTS hypotheses are accepted for practically all pairs of maturities based on the long-term rate and for the pairs where \( m = 1 \) based on the short-term rate. However, in the case of the long-term rate, the result is primarily due to the very high standard deviation of the parameter, which becomes negative in most cases where the test is based on investments of 1 month.

This gives the following validity ranking of the ETTS: exceptionally for the American rates, rarely for German rates and often for French rates.

4.2 The estimates of the specifications based on the error-correction models

We directly required the premia as error-correction terms for the estimation of the error-correction models, but did not attempt to estimate the long-term parameter \( \delta \), which was later set at 1. Two things must be considered in this light: first, the stationarity tests on the premia make it possible to conclude that the premia are stationary; second, it is impossible within our analytical framework to test the value of parameter \( \delta \) (particularly to see if it is equal to 1, which is the theoretical value inferred from the ETTS\(^{10}\)). In other words, it is impossible to test the theory if \( \delta \) has to be estimated.

We made successive estimates for each of the three premia in error-correction model form (15) – (17) for pairs of maturities that are comparable to those in the usual tests. Tables 4 to 6 show the estimates of \( a \) and \( b \), which should equal -1 and 1 respectively under the expectations hypothesis, the associated standard deviations, the corrected \( R^2 \) and finally the significance level of the test of the joint hypothesis \(-a = b = 1\).

The test based on the relationship between the variation in the short-term rate and the forward premium yields quite contradictory results. The ETTS often seems to be validated for French rates at intermediate investment horizons \((m = 1 \text{ month and } n = 4 \text{ to } 8 \text{ months, and } m = 3 \text{ months and } n = 6 \to 9 \text{ months})\). For German rates the level of the estimated coefficients is more satisfactory when \( n \) is quite high in relation to \( m \), and the ETTS is accepted when \( m = 1 \) month and \( n = 2 \) and 9 to 12 months, and when \( m = 3 \) months and \( n = 12 \) months. Finally, for the American data, the coefficients are generally very low in absolute value, and the \( b \) parameter even turns negative for \( m = 1 \) month and \( n = 4 \) to 6 months. The ETTS is validated only exceptionally for intermediate investment horizons of the forward rate \((m = 1 \text{ month and } n = 7 \text{ or } 8 \text{ months, } m = 3 \text{ months and } n = 9 \text{ months})\).

In the test based on the relationship between the variation in the holding period yield and the holding period premium, the estimated coefficients always have the right sign. The coefficients for the three countries nearly always vary between 0.5 and 1.5 in absolute value. In fact, the logic behind the test is a priori favourable to the ETTS, since unlike the preceding test, it is based on the change in interest rates in the coming months (e.g., for \( m = 1 \) months and \( n = 12 \) months, where the test is aimed at forecasting the rate in 12 months the course of the next month). Yet, even though the estimated coefficients are fairly close to the level required, the ETTS hypotheses are rejected in most cases. This is mainly due to the very precise estimates of the parameter \( a \) in the error-correction term. It can also be seen that, unlike the estimate based on the forward premium, the statistical fit is fairly good (with the \( R^2 \) ranging between 0.3 and 0.9). Finally, the ETTS is validated several times, mainly when \( m \) is fairly high. In the case of French rates, the ETTS cannot be rejected when \( m = 3 \) or 6 months (at a very broad significance level); it also holds for American rates when \( m = 3 \) months and \( n = 9 \) or 12 months and when \( m = 6 \) months, and for German rates when \( m = 6 \) months.

As in most previous empirical work, the ETTS seems more widely validated by the test based on the rollover yield, and for all pairs of maturities in the case of French rates. It is only rejected in two cases with the American rates \((m = 1 \text{ month and } n = 2 \text{ months, } m = 3 \text{ months and } n = 6)\).

\(^{10}\) Such a test is possible, but in the context of a multivariate analysis (Johansen [1988]).
months). The tests on German rates give more contradictory results, but at a significance level of 1%, the ETTS hypotheses can only be rejected in four cases \((m = 1\) month and \(n = 4\) to 6 months, and \(m = 3\) months and \(n = 6\) months). The statistical fit is again very good, especially for the German rates, where the \(R^2\) is of the order of 0.6 - 0.7 in each case). It can be seen that, even regardless of the test result, the estimated coefficients are very close to those required by the ETTS (between 0.8 and 1.1 for the French rates, between 0.6 and 1.1 for the German and American rates, with a few rare exceptions). In contrast, the two other tests take interest rate forecasts with a distant horizon (for the forward rates) or a very near horizon (for the holding period yield). The latter involves an "average" predictive power (e.g. for \(m = 1\) month and \(n = 12\) months, the aim is to forecast changes in the 1-month rate over the 12 coming months).

All in all, the ECM specification leads to following results: for all three markets, the estimates based on the short-term rate are nearly always favourable to the ETTS; those based on forward rates are less favourable than the previous ones; and those based on the long-term rates are favourable only when \(m\) is high enough.

### 4.3 Comparison of Estimates from the Usual Specifications and those from the ECMs

When the econometric results obtained from the usual specifications are compared with those from ECMs, several important points are highlighted. The specification bias and, more especially, the sign of the estimated parameters need to be considered, along with the validity of the ETTS hypotheses. At a more qualitative level, an attempt will be made the summarise the markets' ability to "anticipate" interest-rate movements.

As a rule, two types of bias can result from the usual formulations of tests of the ETTS. The first stems from the incompleteness of the relationships between the interest rates. The second can result from the fact that the same variable can be found on both the right and left-hand sides of the estimated equations. As for the omitted-variable bias, the comparison of the usual specifications and the ECM has shown that equation (2) (forward rates) is the only one included in equation (15), whereas (3) (variations in the long-term rate) and (4) (variations in the short-term rate) cannot be considered as special cases of (16) and (17). This means that a strict comparison between the models is only possible in the case of the forward rates. Tables 1–6 show several cases where the ETTS hypotheses are rejected for the estimates based on the usual specifications but accepted with a significance level of more than 10% for the ECMs: with maturity pairs (3,6) and (6,12) for the French data; (1,10) to (1,12) and (3,12) for the German data; and (3,9) for the American data. In each of these cases, a bias shows up in the usual specification. The estimate made with the equality constraint of the ECM parameters \((-a_t = b_r\) which is the same as estimating the usual specification) gives an estimated coefficient that is far from the theoretical value imposed by the ETTS, while for the estimate based on the ECMs, the ETTS hypotheses is accepted by a \(\chi^2\)-based test. On the whole the omitted variable bias is fairly small, as it affects only 7 cases out of 45.

The second source of bias can stem from the having the same variable on both the left and right-hand side of the estimated equations. This argument is put forward Campbell [1995] to justify the negative sign of the estimated coefficient in equation (3), while the parameter of equations (1) and (4) is positive. Indeed, in (3), the long-term rate is found on both sides of the equation but with opposite signs, whereas it is only found on the right-hand side of equation (4). This asymmetry could give rise to a measurement error (or a shock in expectations, which is the same thing in this case) on the long-term rate \(r(t,t+n)\) that is likely to change the sign of \(\beta\) in specification (3) and likely only to bias \(\beta\) towards 0 in specification (4). This is what is shown in Tables 1–3. On the other hand, in the ECM specification, this configuration is no longer found (in the equation with the holding yield, the rate of maturity \(t+n\) only shows up on the left-hand side, and the rate of maturity \(t+n-1\) shows up on both sides, but with the same sign) and there are no excessive differences between the coefficients estimated from the holding yield and the rollover yield (Tables 4–6) no longer exist.
Table 4

Estimates of the error-correction models - France

This table shows the estimation results of the specifications:

\[
[r(t+n+m+t+n) - r(t,n+m+t,n+m)] = a_1[r(t,t+m) - f(t+n+m,t+t+m)] + b_1[f(t+n+m,t+n) - f(t+n+m,t+m)] + \text{constant}
\]

(i)

\[
[h(t,t+m+t+n) - h(t,m,t+t+n)] = a_2[h(t-m,t+n+m) - r(t-m,t)] + b_2[r(t,t+m) - r(t-m,t)] + \text{constant}
\]

(ii)

\[
[h(t,m,t+n) - h(t-n+m,m+t+m)] = a_3[h(t-n+m,m+t+n) - r(t-n+m,t+n+m)] + b_3[r(t,t+n) - r(t-n+m,m+t+n)] + \text{constant}
\]

(iii)

<table>
<thead>
<tr>
<th>(m-n)</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_1)</td>
<td>(b_1)</td>
<td>(R^2)</td>
</tr>
<tr>
<td>1-2</td>
<td>-0.55</td>
<td>0.78</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.18)</td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>-0.46</td>
<td>0.46</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.16)</td>
<td></td>
</tr>
<tr>
<td>1-4</td>
<td>-0.80</td>
<td>0.66</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>1-5</td>
<td>-1.07</td>
<td>0.94</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>1-6</td>
<td>-0.96</td>
<td>1.01</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.19)</td>
<td></td>
</tr>
<tr>
<td>1-7</td>
<td>-1.01</td>
<td>1.18</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>1-8</td>
<td>-0.88</td>
<td>0.99</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.21)</td>
<td></td>
</tr>
<tr>
<td>1-9</td>
<td>-0.63</td>
<td>0.66</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.14)</td>
<td></td>
</tr>
<tr>
<td>1-10</td>
<td>-0.54</td>
<td>0.45</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>1-11</td>
<td>-0.56</td>
<td>0.52</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>1-12</td>
<td>-0.59</td>
<td>0.56</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.16)</td>
<td></td>
</tr>
<tr>
<td>3-6</td>
<td>-0.96</td>
<td>0.91</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>3-9</td>
<td>-1.26</td>
<td>1.43</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.28)</td>
<td></td>
</tr>
<tr>
<td>3-12</td>
<td>-0.62</td>
<td>0.53</td>
<td>16</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.16)</td>
<td></td>
</tr>
<tr>
<td>6-12</td>
<td>-0.92</td>
<td>1.02</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.27)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The estimates relate to the period 1975-95. The observations from March 1983 have been removed. The estimate of the constant is not shown in the table. Standard deviations, shown in parentheses, are corrected for heteroscedasticity (White [1980]) and for overlapping (see Box 2). The variance-covariance matrix is estimated as suggested by Newey and West [1987]. \(R^2\) is the \(R^2,\) in %, corrected for the number of degrees of freedom and \(p\)-value is the significance level for the test of the joint hypothesis \(-a_i=b_i=1.\)
Table 5
Estimates of the error-correction models - Germany

This table shows the estimation results of the specifications given in Table 4.

<table>
<thead>
<tr>
<th>m - n</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_i$</td>
<td>$b_i$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1 - 2</td>
<td>-0.83 (0.12)</td>
<td>0.99</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 3</td>
<td>-0.49 (0.09)</td>
<td>0.63</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 4</td>
<td>-0.43 (0.11)</td>
<td>0.70</td>
<td>22</td>
</tr>
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<td></td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 5</td>
<td>-0.39 (0.11)</td>
<td>0.68</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 6</td>
<td>-0.36 (0.14)</td>
<td>0.59</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 7</td>
<td>-0.45 (0.17)</td>
<td>0.73</td>
<td>18</td>
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<tr>
<td></td>
<td>(0.18)</td>
<td></td>
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</tr>
<tr>
<td>1 - 8</td>
<td>-0.48 (0.19)</td>
<td>0.75</td>
<td>17</td>
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<td></td>
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<tr>
<td>1 - 9</td>
<td>-0.57 (0.19)</td>
<td>0.84</td>
<td>20</td>
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<td>1 - 10</td>
<td>-0.65 (0.19)</td>
<td>0.91</td>
<td>23</td>
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<td></td>
<td>(0.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - 11</td>
<td>-0.56 (0.25)</td>
<td>0.84</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td></td>
<td></td>
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<tr>
<td>1 - 12</td>
<td>-0.59 (0.26)</td>
<td>0.85</td>
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<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3 - 6</td>
<td>-0.27 (0.12)</td>
<td>0.52</td>
<td>13</td>
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<tr>
<td></td>
<td>(0.11)</td>
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<td></td>
</tr>
<tr>
<td>3 - 9</td>
<td>-0.40 (0.20)</td>
<td>0.67</td>
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<tr>
<td>3 - 12</td>
<td>-0.62 (0.20)</td>
<td>0.88</td>
<td>19</td>
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<tr>
<td></td>
<td>(0.20)</td>
<td></td>
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<tr>
<td>6 - 12</td>
<td>-0.31 (0.22)</td>
<td>0.53</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
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<td></td>
</tr>
</tbody>
</table>

Note: Same as Table 4 except that the March 1983 observation was not removed.

Are the ETTS hypotheses validated more frequently when both the usual and ECM specifications are used? Table 7 recapitulates the number of times the ETTS hypotheses are validated at the 10% significance level from the results presented in Tables 1–6. The results of the test on variations in the long-term rate shows no gains on this point, even though the signs of the parameters obtained for the ECMs are correct. On the other hand, the advantage is large for the other two tests, particularly for variations in the short-term rate. For the latter test, the ETTS hypotheses are validated 14 times with the ECM for the French data and only 7 times with the usual specification. The respective results are 6 and 1 times for the German data and 12 and 0 times for the American data.
Table 6
Estimates of the error-correction Models - United States

This table shows the estimation results of the specifications given in Table 4.

<table>
<thead>
<tr>
<th>m - n</th>
<th>Forward rate (i)</th>
<th>Variation in long rate (ii)</th>
<th>Variation in short rate (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_i$</td>
<td>$b_i$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1 - 2</td>
<td>-0.55</td>
<td>0.93</td>
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<td></td>
<td>(0.18)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>1 - 3</td>
<td>-0.25</td>
<td>0.33</td>
<td>1.0</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>1 - 4</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>1 - 5</td>
<td>-0.13</td>
<td>-0.02</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>1 - 6</td>
<td>-0.20</td>
<td>-0.07</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>1 - 7</td>
<td>-0.39</td>
<td>0.19</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.37)</td>
<td></td>
</tr>
<tr>
<td>1 - 8</td>
<td>-0.36</td>
<td>0.19</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.35)</td>
<td></td>
</tr>
<tr>
<td>1 - 9</td>
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<td>0.38</td>
<td>3.0</td>
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<tr>
<td></td>
<td>(0.19)</td>
<td>(0.26)</td>
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</tr>
<tr>
<td>1 - 10</td>
<td>-0.38</td>
<td>0.47</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>1 - 11</td>
<td>-0.38</td>
<td>0.49</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.25)</td>
<td></td>
</tr>
<tr>
<td>1 - 12</td>
<td>-0.36</td>
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<td>2.0</td>
</tr>
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<td></td>
<td>(0.22)</td>
<td>(0.29)</td>
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</tr>
<tr>
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<td>(0.19)</td>
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<td></td>
<td>(0.19)</td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>6 - 12</td>
<td>-0.02</td>
<td>-0.09</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.44)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Same as Table 4 except that the March 1983 observation was not removed.

Qualitatively, the results obtained for the usual specifications with maturity pairs of (1, 2) to (1, 12) months show a clear contrast between the tests based on the forward rate and the variation in the short-term rate and the test based on the variation in the long-term rate. In the first instance, the coefficients obtained are comparable overall from one test to the next in terms of level and change. However, the coefficients for the variation in the long-term rate tend to diminish as the maturities of the rates increase. This movement is particularly visible in the American data and is partly due to differences in the nature of the tests. The tests on the forward rate and the variation in the short-term rate try to see if the markets are able to anticipate a short-term rate for investment horizons that are further and further into the future, while the test on the variation in the long-term rate assumes a fixed forecasting horizon of one month (for maturity pairs of (1,2) to (1,12)) but for an increasingly long investment horizon. The configuration that emerges from the results can be summed up as follows: for the three countries, the markets seem to have fairly satisfactory foresight of changes in short-term rates, but they are poor predictors of long-term rate movements.
Table 7
Number of times the ETTS hypotheses hold true
At the 10% significance level

<table>
<thead>
<tr>
<th>Country</th>
<th>Specification</th>
<th>Forward rate</th>
<th>Long rate</th>
<th>Short rate</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>usual</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>ECM</td>
<td>8</td>
<td>4</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>Germany</td>
<td>usual</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ECM</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>United States</td>
<td>usual</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>ECM</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: See also Tables 1–6.

The configuration of the ECM results does not establish a distinction that is as clear as the one emerging from the usual specifications. The two representations of forward rates give estimated parameters that are comparable in absolute value. In the test of the variation in the long-term rate, the estimates of the two coefficients from the ECM seem to be relatively homogeneous for the German and American data and qualitatively close to the theoretical values, unlike the estimates from the usual specification. However, the gain from the ECM specification is marginal for the French data. In the test of the variation in the short-term rate, the ECM parameters are fairly stable and closer to the theoretical values than in the usual specifications. All in all, the use of ECMs to test the ETTS hypotheses on maturity pairs (1,2) to (1,12) months makes it possible to argue that German and American operators on the euro-rates market seem to have fairly satisfactory foresight for both short-term and long-term rates, whereas, on the French market, only the short-term rates are correctly foreseen.

Conclusion

This paper presents three tests of the hypotheses of the expectations theory of the term structure of interest rates based on the usual specifications found in the literature, on the one hand, and in form of error-correction models on the other hand. The estimates based on the euro-franc, euro-Deutschemark and euro-dollar rates for the period 1975-95 produce some results.

The monetary turmoil that occurred during the estimate period has a very large impact on the estimates for the French rates. This dependence is less noticeable for American rates and negligible in the case of German rates. As a rule, the test based on the average variation in the short-term rate lead to the acceptance of the hypotheses of the expectations theory in nearly every case with an error-correction model, and more rarely with the usual specifications. The advantage of error-correction models seems less apparent in the test based on the forward rate and negligible in the test based on the long-term rate.

This illustrates the contrasts usually seen in the literature between the test based on the variation in the long-term rate and those based on the variation in the short-term rate. However, this difference is less marked in the case of error-correction models as long as the estimated coefficients have the right sign for both tests. This contrast is still surprising nonetheless. The forecast horizon for the test of the variation in the long-term rate (which is equivalent to a test of the holding yield) is in fact fairly short, compared to the forecast horizon of the test based on the variation in the short-term rate (which is equivalent to a test of the rollover yield). In the first instance, the horizon is 1, 3 and 6.
months, as opposed to 1 to 12 months in the second test. The argument that can be put forward to explain this difference is based on the fact that the forecasting errors are smoothed out for the rollover yield and therefore tend to cancel each other out.

Finally, the comparison of the results obtained for each country with the tests based on error-correction models gives rise to a typology that is somewhat different from that derived from the usual specifications. While the expectations theory often holds for the French data in both cases, the results obtained from American rates in relation to those from Germany rates are more favourable with error-correction models than with the usual specifications.
Appendix 1: Impact of outliers based on estimates of usual specifications

Table A1-1
French rates

This table shows the estimation results of the specifications:

\[
[r(t + m, t + n) - r(t, t + n)] = \beta_m \frac{m}{n-m} [r(t, t + n) - r(t, t + m)] + \text{constant} \tag{i}
\]

\[
\frac{m}{n} \sum_{i=0}^{n-1} \left[ r(t + im, t + im + m) - r(t + im - m, t + im) \right] = \beta_n [r(t, t + n) - r(t, t + m)] + \text{constant} \tag{ii}
\]

for which the samples are defined as follows:

1. All observations between January 1975 and October 1995;
2. All observations except those from March 1983;
3. All observations except those from March 1983 and the observations selected from the test of DFBETAS.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Variation in long rate (i)</th>
<th>Variation in short rate (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>$\sigma_{\beta_1}$</td>
</tr>
<tr>
<td>1-3</td>
<td>1</td>
<td>1.23</td>
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<tr>
<td></td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.23</td>
</tr>
<tr>
<td>1-6</td>
<td>1</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.19</td>
</tr>
<tr>
<td>1-12</td>
<td>1</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.41</td>
</tr>
<tr>
<td>3-6</td>
<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.18</td>
</tr>
<tr>
<td>3-12</td>
<td>1</td>
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<tr>
<td></td>
<td>2</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.81</td>
</tr>
<tr>
<td>6-12</td>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: The outliers are selected on the basis of the correction term of ECMs (16) – (17). The estimates relate to the period 1975-95. The observations from March 1983 have been systematically removed. The usual relationships between the $\beta_i$ for $n = 2m$ are not necessarily seen, because the observations removed as outliers are selected on the basis of the associated error-correction models and may be different from one test to the next. The estimate of the constant is not shown in the table. Standard deviations, shown in parentheses, are corrected for heteroscedasticity (White [1980]) and for overlapping (see Box 2). The variance-covariance matrix is estimated as suggested by Newey and West [1987]. $\bar{R}^2$ is the $R^2$, in %, corrected for the number of degrees of freedom and p-value is the significance level for the test of the hypothesis $\beta_i = 1$. 
Table A1-2

German rates

This table shows the estimation results of the specifications given in Table A1-1, using the samples:

1. All observations between January 1975 and October 1995;
2. All observations except those selected from the test of DFBETAS.

<table>
<thead>
<tr>
<th>$m-n$</th>
<th>Sample</th>
<th>Variation in long rate (i)</th>
<th>Variation in short rate (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_1$</td>
<td>$\sigma_{\beta_1}$</td>
</tr>
<tr>
<td>1 - 3</td>
<td>1</td>
<td>0.34</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.46</td>
<td>0.22</td>
</tr>
<tr>
<td>1 - 6</td>
<td>1</td>
<td>-0.23</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>1 - 12</td>
<td>1</td>
<td>-0.47</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>3 - 6</td>
<td>1</td>
<td>-0.28</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.17</td>
<td>0.26</td>
</tr>
<tr>
<td>3 - 12</td>
<td>1</td>
<td>-0.62</td>
<td>0.38</td>
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<td>0.37</td>
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<tr>
<td>6 - 12</td>
<td>1</td>
<td>-0.40</td>
<td>0.47</td>
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<tr>
<td></td>
<td>2</td>
<td>-0.35</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Note: Same as Table A1-1 except that the March 1983 observation was not removed.

Table A1-3

American rates

This table shows the estimation results of the specifications given in Table A1-1, using the samples:

1. All observations between January 1975 and October 1995;
2. All observations except those selected from the test of DFBETAS.

<table>
<thead>
<tr>
<th>$m-n$</th>
<th>Sample</th>
<th>Variation in long rate (i)</th>
<th>Variation in short rate (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_1$</td>
<td>$\sigma_{\beta_1}$</td>
</tr>
<tr>
<td>1 - 3</td>
<td>1</td>
<td>-0.46</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.36</td>
<td>0.39</td>
</tr>
<tr>
<td>1 - 6</td>
<td>1</td>
<td>-0.59</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-1.07</td>
<td>0.47</td>
</tr>
<tr>
<td>1 - 12</td>
<td>1</td>
<td>-0.71</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-1.64</td>
<td>0.59</td>
</tr>
<tr>
<td>3 - 6</td>
<td>1</td>
<td>-0.18</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.35</td>
<td>0.52</td>
</tr>
<tr>
<td>3 - 12</td>
<td>1</td>
<td>-0.56</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-1.21</td>
<td>0.74</td>
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<td>-0.46</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.47</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Note: Same as Table A1-1 except that the March 1983 observation was not removed.
Appendix 2: Stationarity tests

The estimates relate to end-of-month data from the period 1975-95. The number of lags was chosen to whiten the residuals for the ADF tests (at the 10% level for the Box-Pierce statistic. The critical values for the DF and ADF tests are 2.57 for a 10% level, 2.88 for a 5% level and 3.46 for a 1% level. The observations of French rates from March 1983 have been removed.

Table A2-1
French rates

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Zero-coupon rate</th>
<th>Variation</th>
<th>Differential from 1-month rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
<td>DF</td>
</tr>
<tr>
<td>1</td>
<td>-3.52</td>
<td>-1.83</td>
<td>-16.82</td>
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<tr>
<td>2</td>
<td>-2.86</td>
<td>-1.01</td>
<td>-15.36</td>
</tr>
<tr>
<td>3</td>
<td>-2.26</td>
<td>-0.69</td>
<td>-13.98</td>
</tr>
<tr>
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<td>-0.57</td>
<td>-13.67</td>
</tr>
<tr>
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<td>-1.81</td>
<td>-0.46</td>
<td>-13.34</td>
</tr>
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<td>-13.09</td>
</tr>
<tr>
<td>8</td>
<td>-1.50</td>
<td>-0.37</td>
<td>-13.16</td>
</tr>
<tr>
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<td>-1.46</td>
<td>-0.39</td>
<td>-13.23</td>
</tr>
<tr>
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<td>-1.41</td>
<td>-0.42</td>
<td>-13.31</td>
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<tr>
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<td>-1.38</td>
<td>-0.47</td>
<td>-13.41</td>
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<td>-0.53</td>
<td>-13.50</td>
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</table>

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<th>Holding premium</th>
<th>Rollover premium</th>
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<tr>
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<td>DF</td>
<td>ADF</td>
<td>DF</td>
</tr>
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<td>-5.14</td>
<td>-12.58</td>
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<td>-12.45</td>
</tr>
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<td>-11.95</td>
</tr>
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<td>-11.97</td>
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<td>-4.46</td>
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Table A2-2

German rates

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<th>Differential from 1-month rate</th>
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</thead>
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<tr>
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<td>Level</td>
<td>Variation</td>
</tr>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
</tr>
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<td>-2.82</td>
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<td>-2.11</td>
</tr>
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<th>Rollover premium</th>
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<td>ADF</td>
<td>DF</td>
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<td>-11.98</td>
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<td>-11.90</td>
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<td>-3.18</td>
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<td>-3.23</td>
<td>-5.47</td>
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<td>-4.03</td>
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</table>
## Table A2-3
### American rates

<table>
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<tr>
<th>Maturity</th>
<th>Zero-coupon rate</th>
<th>Differential from 1-month rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Variation</td>
</tr>
<tr>
<td></td>
<td>DF</td>
<td>ADF</td>
</tr>
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</tr>
<tr>
<td>2</td>
<td>-1.87</td>
<td>-2.47</td>
</tr>
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<td>-1.84</td>
<td>-2.40</td>
</tr>
<tr>
<td>4</td>
<td>-1.81</td>
<td>-2.34</td>
</tr>
<tr>
<td>5</td>
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References


This paper contains a very thorough test of the expectations hypothesis of the term structure of interest rates using short term Euro-market interest rates for the period 1975-95. There are several aspects of the paper which I like.

First, the use of data for several countries – France, Germany and the United States – is particularly interesting since, as noted in the paper, tests of the expectations hypothesis tend to fare better on non-US than on US data. In light of this and the fact that an overwhelming proportion of research on the term structure has focused on US data, more comparative work is warranted. Indeed, the authors find that the results for France, and to a lesser extent Germany, are less at odds with the expectations hypothesis than the results for the US.

Second, the entire short end of the yield curve (that is, maturities between one and twelve months) is considered. Since the short end is probably the most interesting part of the yield curve for monetary policy purposes, it is nice to see a full spectrum of short rates used. While there doesn't seem to be any major differences across maturities, the results appear somewhat more supportive of the expectations hypothesis when the short interest rate is the one month rate, and the long rate is the twelve months rate. One minor problem, however, is that since the authors only have access to interest rates with 1, 3, 6 and 12 months maturities, they are forced to interpolate the yields for the missing maturities. This induces measurement errors on the constructed yields. While these errors are probably not important, it would be of interest to know a bit more about how large the interpolations errors are likely to be.

Third, three implications of the expectations hypothesis are tested: loosely speaking, whether (i) the spread between forward and spot interest rates predicts changes in short rates, (ii) whether the spread between long and short rates predicts changes in long rates, and (iii) whether the spread between long and short rates predicts changes in short rates. One interesting finding is that the expectations hypothesis fares best when the third implication is tested.

Fourth, the authors use two econometric approaches. They first estimate a set of "standard" equations, which relate the dependent variable to the spread between a long and a short interest rate (or between a forward and a spot interest rate). Next they go on to estimate error-correction models. An interesting point is that while the "standard" and error-correction equations are consistent when the expectations hypothesis hold, they allow for different alternatives hypotheses. One striking finding is that the authors reject the expectations hypothesis much more frequently when the "standard" equations are estimated.

Fifth and finally, the authors demonstrate that the results are sensitive to the inclusion of a few data points, that is, there is considerable sub-sample instability in the estimates. Since there is little work on the temporal stability of term structure relationships, this finding suggests that more work on the causes of this instability is warranted. Furthermore, it suggests that the information content of the yield spreads for future short-term interest rates varies over time.
The behaviour of long-term interest rates in the FRB/US model

Sharon Kozicki, Dave Reifschneider and Peter Tinsley

Introduction

For several years now, staff at the Federal Reserve Board have been engaged in a project to redesign its primary model of the US economy. Our goal in this project has been to produce an empirical model that clearly distinguishes the formation of expectations from other adjustment processes, under the paradigm that households and firms are rational optimising agents. This project is now at an advanced stage, and this paper is in part a progress report on one facet of the modelling effort, the behaviour of bond rates.

The theoretical basis for the bond rate model is a version of the standard Expectations Hypothesis: The yield to maturity on a bond equals a weighted sum of future rationally-expected short-term interest rates, plus a risk premium that may be time-varying. To make the model operational for estimation work and forecasting, we employ a small-scale VAR system to generate expectations. The structure of the VAR is unconventional, in that it incorporates moving endpoints derived from market expectations of the long-term level of inflation and the real rate of interest. We believe that this specification has two advantages. First, it provides a more satisfactory characterisation of interest rates than conventional l(0) or l(1) formulations. Second, it allows us to distinguish between two primary forces influencing the level of long-term interest rates — a stationary element associated with the business cycle and monetary policy stabilisation, and a nonstationary component linked to long-term policy objectives.

The structure of the paper is as follows. We begin with a brief summary of the theoretical basis of the model. Next, we discuss our strategy to implement the model by using VAR-derived expectations. Here is where we discuss the drawbacks of standard VAR specifications, and introduce the concept of moving endpoints. From there, we turn to a closer look at endpoints, and consider the measurement and behaviour of long-term inflation expectations. The fourth section of the paper addresses the empirical model, and documents its behaviour and statistical properties. Finally, we conclude with a review of recent bond market developments in the US from the prospective of the model. An important theme in this discussion is the potential link between federal deficit reduction and recent declines in long-term interest rates.

1. Theory: RE models of the term structure

The theoretical basis of our bond rate model is the standard Expectations Hypothesis: The yield to maturity on a bond is equal to a weighted average value of the short-term rate (rationally) expected to prevail over the life of the bond, plus a risk premium. Depending on the capital asset pricing model or the arbitrage pricing theory used in the theoretical derivation, the risk premium may be constant or time varying (perhaps predictably so).

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1 This paper draws heavily on work by a large number of people associated with the model reestimation project of the Federal Reserve Board. The opinions expressed here do not necessarily represent those of the Board of Governors of the Federal Reserve System or the staff of the Federal Reserve System.

2 For a description of a preliminary version of the new model (called FRB/US), see: Brayton and Tinsley (1995); French, Kozicki, Mauskopf and von zur Muehlen (1995); Kennedy, Reifschneider and Schuh (1995); and Bomfim, Brayton, Tinsley and Williams (1995).
Modern asset valuation theory\(^3\) suggests that the price of a claim to a real payout of \(X_{t+n}\) in period \(t+n\) is determined by the Euler equation

\[
P_{n,t} = E_t[\frac{X_{t+n} M_{t+n}}{M_t}]
\]

where the ratio \(M_{t+n}/M_t\) is an equilibrium discount factor. In the literature, \(M\) is often functionally related to the marginal utility of consumption. Generally, it is assumed that the \(m\)-period log difference of \(M, \left(1 - L^m\right)\log M_t,\) is a stationary stochastic process, where \(L\) denotes the lag operator.

In the case of an \(n\)-period discount bond with a terminal price of one dollar, the no-arbitrage counterpart to equation 1 is

\[
P_{n,t} = E_t\left[\frac{M_{t+n}}{M_t}\right] = E_t\left[\prod_{i=1}^{n} \left(\frac{\mu_i}{\pi_i}\right)\right]
\]

where \(P^c_t\) is the consumption price level, and \(\mu_i\) and \(\pi_i\) are the period-to-period ratios, \(M_i/M_{t-1}\) and \(P^c_i/P^c_{t-1},\) or one plus the usual growth rates \((\mu_i = 1 + \mu_i).\) In models such as those developed by Rubinstein (1976) and Lucas (1978), \(\mu\) is determined by the (stochastic) growth rate of consumption or household endowments.\(^4\)

Under the assumption that \(\mu\) and \(\pi\) are lognormally distributed, the conventional rational expectations term structure can be expressed as:

\[
r_{n,t} = \Phi_n^t - \frac{1}{n} \sum_{i=1}^{n} \left[\mu_i^e - \pi_i^e\right] = \Phi_n^t + \frac{1}{n} \sum_{i=0}^{n-1} r_{t+1}^e
\]

where \(r_{n,t} = -\log P_{n,t}/n\) is the nominal yield to maturity of the \(n\)-period discount bond, \(r_{t+1}^e\) denotes the one-period nominal rate expected in period \(t+1,\) and the superscript \(e\) indicates agent expectations conditioned on information available at time \(t.\)

Although ignored for now, it will be important in later discussion to have an explicit definition of the term premium, \(\Phi_n^t.\) A compact definition is obtained by stacking the component discount factors and inflation rates into \(n \times 1\) vectors, \(\mu\) and \(\pi,\) whose distributions are normal with the \(n \times n\) variance-covariance matrices, \(V_{\mu\mu}, V_{\pi\pi},\) and \(V_{\mu\pi}^T.\) Denoting the \(n\)-element unit vector by \(1_n,\) the term premium for the \(n\)-period discount bond is:

\[
\Phi_n^t = -\frac{1}{2n} \left[1_n' V_{\mu\mu} 1_n + 1_n' V_{\pi\pi} 1_n + 2 1_n' V_{\mu\pi} 1_n\right]
\]

If variance-covariance matrices are stable over time, then \(\Phi_n^t\) is a constant. Otherwise the risk premium fluctuates in ways that may be correlated with changes in macroeconomic conditions.

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\(^3\) Historical contributions include Rubinstein (1976), Breeden (1979), and Cox, Ingersoll, and Ross (1985a).

\(^4\) In asset pricing models, it is frequently assumed that the utility function of the representative agent is characterised by constant relative risk aversion, i.e., \(U(c) = c^\gamma/\gamma.\) This functional form implies that log differences of the pricing kernel \(M\) are proportional to the expected growth rate of log endowments or consumption.
In the case of coupon bonds, the analogue to equation 5 developed by Shiller (1979) is

\[ R_t^{(n)} = \Phi_n + \frac{1 - B}{1 - B^n} \sum_{i=0}^{n-1} B^i r_{t+i}^e \]  

(5)

where the constant discount rate, \( B = \frac{1}{1 + \overline{R}} \), is determined by the sample average \( \overline{R} \) of the yield to maturity on a coupon bond, \( R_t^{(n)} \). Equation 5 is the standard version of the (rational) Expectations Hypothesis, under the assumption that the risk premium \( \Phi_n \) is a constant.\(^5\) Note that the risk premium \( \Phi'_n \) in equation 3 is not equal to \( \Phi_n \) in equation 5, owing to the different risk characteristics of discount and coupon bonds. However, the risk premium for an \( n \)-period coupon bond is a function of the same underlying variances and covariances that determine the premiums on 2-through-\( n \) period discount bonds.

2. Implementation strategy: modelling expectations

The theoretical model developed in the previous section is incomplete. The expected path of future short-term interest rates is unobserved, and the size of the risk premium is not known a priori. If equation 5 is to be used in estimation, forecasting or policy simulation exercises, it must be augmented with equations that explain how \( r_{t+i}^e \) evolves over time and responds to changes in the macroeconomic environment.

Because the equations used to determine bond rates are part of a general RE model of the economy, it is theoretically possible to use the full FRB/US model to derive expectations for use in estimation and forecasting. Such an approach is in principle preferable, because it would ensure that expectations conform with the behaviour of the overall system. A full-information maximum likelihood estimation procedure has been used by Leeper and Sims (1994) in their estimation of a small-scale macro model of the US economy. Unfortunately, the FRB/US model is too large to make this approach computationally feasible, at least at present. In addition to estimation and forecasting, the FRB/US model is also used in policy analysis, for which it is straightforward to use Fair-Taylor or other algorithms to compute model-consistent RE solutions. This approach – an example of which is discussed in Section 4 – is now often used at the Federal Reserve Board to answer such questions as the likely macroeconomic impact of changes in fiscal and monetary policy.

As an alternative for estimation and forecasting, we have made the assumption that agents’ expectations can be characterised by a small-scale VAR forecasting system. This system is used to generate all expectations used in the standard version of the FRB/US model, not just those associated with the prediction of bond yields.\(^6\) The core portion of this system, which includes the short-term interest rate, inflation, and the output gap, can be expressed as:

\[ z_{t+i}^e = H^{i+1}z_{t-1} \]  

(6)

where \( H \) denotes the companion matrix of the first-order representation of the VAR model, and the vector \( z_{t-1} \) is a column stack of the relevant lagged values of the VAR model. \( z_{t-1} \) thus summarises

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\(^5\) Although, as Shiller notes, there is no inherent reason why \( \Phi_n \) necessarily is constant over time.

\(^6\) Other important expectational variables used in the model include a variety of present value calculations (for household income and corporate profits), the future average rate of inflation, and the average level of resource utilisation expected to prevail in the near term.
the information set of agents. Substituting the predictions of equation 6 into the definition of the RE term structure in equation 5 provides a tractable linear formulation of the term structure:

$$R_t^{(n)} = \frac{1 - B}{1 - B^n} \left[ \sum_{i=0}^{n-1} (BH)^i H z_{t-1} \right] + \Phi_n$$ (5b)

which in turn simplifies to

$$R_t^{(n)} = \frac{1 - B}{1 - B^n} \mathbf{1}_r \left[ I - BH \right]^{-1} \left[ I - (BH)^n \right] H z_{t-1} + \Phi_n$$ (7)

where $\mathbf{1}_r$ is a column selector vector that contains a one to identify the position of the one-period rate $r_t$ in the information vector $z_t$, and zeroes elsewhere.

Because the annualised discount factor $B$ used in equation 7 equals 0.92, "distant" forecasts of $r_{t+i} = H_t z_{t-1}$ receive a relatively large weight in the calculation of $R_t^{(n)}$. For example, 50 percent of the value of a ten-year coupon bond is associated with the expected level of short-term interest rates after the first two years of the bond, and 20 percent after the first five years. The weight given to out-year forecasts means that the low-frequency characteristics of the VAR model are critical to the predicted behaviour of bond yields. However, this aspect of VAR specification is not usually given a great deal of attention by modellers, probably because VARs are typically regarded as short-run forecasting models.

To illustrate the importance of endpoint assumptions for bond rate forecasts, consider the three panels of Figure 1. All three panels display RE constructions of the 10-year bond rate using the formula in equation 7. To simplify the exposition for the moment, the forecast model is restricted to a simple $m$-order autoregression in the federal funds rate, which is selected as the effective one-period (monthly) rate. (In the empirical section below, we will return to a more complicated VAR system.) The models differ only in their characterisation of the long-run endpoint of the funds rate which, hereafter, we denote as $r_t^\infty = \lim_{t \to \infty} r_t^f$.

### 2.1 A stationary I(0) format

The top panel of Figure 1 illustrates bond rate predictions from a model in which the short rate is a stationary stochastic process - a common assumption in the finance literature.\(^7\) In discrete time, the format is

$$\Delta r_t = \alpha_0 + \gamma r_{t-1} + A(L) \Delta r_{t-1} + \varepsilon_t$$ (8)

where $A(L)$ denotes a finite polynomial in the lag operator, $L$. Estimates of the parameters of equation 8 are displayed in the first column of Table 1, where $A(L)$ is a fourth-order polynomial and the sample is the 34-year span starting in 1960. One-month-ahead predictions of the 10-year Treasury bond rate are constructed by recasting equation 8 into companion form and substituting this into equation 7. Predictions of this autoregressive funds rate model display a tendency to lie above (below) the historical bond rate when the latter is below (above) its sample mean. This is because predictions of long-horizon instruments are eventually dominated by the limit or endpoint of the forward funds rate.

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\(^7\) For example, see Cox, Ingersoll, and Ross (1985b).
Figure 1
RE predictions of the 10-year bond rate

stationary model of funds rate

difference-stationary model of funds rate

moving endpoint model of funds rate

- historical bond rate
- predicted bond rate
forecasts and, in the case of the stationary funds rate model, the funds rate endpoint is a constant, \( r^\infty = -\alpha_0 / \gamma \), which in large samples is the sample mean.\(^8\)

A more intuitive view of the role of the constant endpoint is obtained by rewriting equation 8 as

\[
\tilde{r}_{t+i} = (1 + \gamma) \tilde{r}_{t-1} + A(L) \Delta \tilde{r}_{t-1}
\]

where \( \tilde{r}_t \) denotes the current displacement of the funds rate from the endpoint, \( \tilde{r}_t = r_t - r^\infty \). The fading impact of the initial displacement on forecasts of forward funds rates can be gauged by the mean lag of an initial shock. According to the parameter estimates in the first column of Table 1, the mean lag of a displacement shock is 40.4 months.\(^9\) In other words, the predicted forward rates have reached the neighbourhood of the funds rate endpoint by the fourth or fifth year of the forecast horizon, implying that the constant \( r^\infty \) is a good approximation of the average expected funds rate in the second five years of the 10-year bond rate.

Table 1

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2.2 A nonstationary I(1) format

In contrast to the stationary model of a representative short-term interest rate that is generally assumed in finance, a number of recent studies of the term structure in macrofinance, such as Campbell and Shiller (1987) and Mougoue (1992), are predicated on the assumption that all nominal interest rates are I(1). Indeed, because the format of equation 8 is the same as that required for an augmented Dickey-Fuller (ADF) test of stationarity, the first column of Table 1 indicates that the t-statistic associated with \( \gamma \) is below the critical value (2.57 for a p-value of 10%) that would be required to reject the hypothesis that the funds rate contains a unit root. The second column of Table 1 contains the estimated parameters of the differenced funds rate model.

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\(^8\) Bond rate predictions in Figure 1 are adjusted for the difference between the sample mean of the funds rate and that of the bond rate.

\(^9\) In this case the mean lag equals \(- (1 + \gamma - A(1)) / \gamma \).
\[ \Delta r_t = \alpha_0 + A(L) \Delta r_{t-1} + \epsilon_t \]  

(10)

where the coefficient of the lagged level of the funds rate is restricted to zero. As indicated by the standard error of estimate (SEE) shown in the second column of Table 1, the deterioration in fit due to this restriction is negligible.

Bond rate predictions from the first-difference model of the funds rate are displayed in the second panel of Figure 1. These predictions differ markedly from those in the first panel and mirror rather closely the movements of the historical funds rate level, exceeding the 10-year bond rate in the early 1980s and remaining largely below the bond rate since the mid-1980s. In contrast to the bond rate forecasts in the first panel – which appear to be excessively damped relative to the movements of the historical bond rate – the bond rate forecasts in the second panel appear to be, if anything, too responsive to the last measured position of the funds rate.

The reason for the higher sensitivity of the bond rate predictions in the second panel to recent levels of the funds rate is that the unit root in equation 10 induces nonstationarity in the endpoint, as well as in the funds rate. Because the characteristic roots of \( A(L) \) are stable, the forecasts of forward rate changes will approach a limit which is an \((m+1)\)-order moving average of the funds rate. Then, summing over the forward rate changes and taking the limit indicates that the endpoint of the forward rate forecasts is also an \((m+1)\)-order moving average of the funds rate,

\[ r_t^\infty = r_{t-1} + \sum_{i=1}^{m} w_i \Delta r_{t-i}. \]

Thus, the endpoint is fixed in any given forecast period but will closely track the funds rate over time.

### 2.3. A moving endpoint format

So far, the discussion of endpoints indicates that both I(0) and I(1) formats are associated with undesirable low-frequency properties. In the I(0) case, the assumption of a fixed endpoint yields bond rate predictions that are too stable. By contrast, tying the system's endpoint to the current level of short-term interest rates produces forecasts that are too volatile. These results suggest that better predictive performance might be had from models that incorporate time-varying endpoints which are not too closely tied to current economic conditions.

To this end, we now consider the approach pursued by Kozicki and Tinsley (1995a), in which the forecast system is extended to include the effects of an explicit moving endpoint, \( r_t^\infty \).

\[ E_t \Delta r_t = \alpha_0 + \gamma \left( r_{t-1}^\infty - r_{t-1} \right) + A(L) \Delta r_{t-1} \]

\[ E_t r_{t-1}^\infty = r_{t-1}^\infty \]  

(11)

Note that there are now two equations, the first describing the evolution of funds rate forecasts over the forecast horizon that begins in period \( t \), and the second indicating that the endpoint forecast is fixed over the forecast horizon. It is important to observe also that (11) is not a closed system because the second equation is silent about the actual evolution of the interest rate endpoint over the historical sample. Indeed, as discussed later, the estimated endpoint appears to be a nonstationary process, indicating that its conditional moments have shifted over the historical sample. The second equation in (11) indicates only that the conditional expectation of the endpoint is fixed over the horizon of forecasts originating at time \( t \).

Unlike most areas of macroeconomics, where agents' perceptions of the relevant transversality conditions or endpoints associated with Euler equation descriptions of optimal intertemporal behaviour are not observable, agents' current forecasts of the nominal rate endpoint are readily available from the observed term structure of nominal rates. One such measure, similar to that employed by Kozicki (1995), is the average of the forward rates from \( t + m \) to \( t + m' \), for \( m' > m \):
\[
\tilde{r}_t^{\text{oo}} = \frac{D_m r_{t+m} - D_m r_{t+m}}{D_{t+m} - D_m}
\]  

(12)

where the duration associated with an \( m \)-period coupon bond is estimated by \( D_m = \left(1 - B^m\right)/(1 - B) \).\(^{10}\)

In the estimate of the funds rate equation in (11), displayed in the third column of Table 1, the endpoint regressor series, \( r_t^{\text{oo}} \), is a concatenation of monthly endpoint constructions based on the forward rates between the 10-year and the 30-year Treasury bonds.\(^{11}\) Note that estimated characteristics of the moving-endpoint funds rate equation in the third column are very similar to those of the stationary rate equation shown in the first column of Table 1. This is because standard reporting statistics, such as \( R^2 \), are based on the one-step-ahead forecast properties of fitted equations and are relatively insensitive to assumptions about long-horizon endpoints. By contrast, bond rate predictions are long-horizon forecasts and the bond rates generated by the moving endpoint forecast system, shown in the third panel of Figure 1, track the historical 10-year bond rate much more closely than do the predictions in the first panel.

3. A closer look at endpoints

As one might suspect from the preceding analysis, the moving endpoint for the nominal short-term interest rate provides the lion’s share of motion in the predicted bond rate. In fact, the squared correlation between \( r_t^{\text{oo}} \) and the historical 10-year bond rate indicates that the moving endpoint alone explains about 85% of the sample variation in the level of the 10-year bond rate. Of course, a consequence of the open design of (11) is that we must provide a plausible model of the economic determinants of \( r_t^{\text{oo}} \). Furthermore, because the forecasting system used to generate expectations of short-term interest rates in FRB/US is not autoregressive, but instead is a VAR model that includes inflation in the information set, we must also consider the related question of how a moving endpoint for this variable can be measured and explained.

An obvious place to start the analysis is the standard Fisherian decomposition of a nominal interest rate between the expected real rate and expected inflation. In the current context, the nominal rate endpoint can be partitioned into an expected real rate endpoint and an expected inflation endpoint:

\[
r_t^{\text{oo}} = \rho_t^{\text{oo}} + \pi_t^{\text{oo}}
\]  

(13)

Unfortunately, given the absence of indexed bonds in the United States, let alone an indexed term structure, both components – the expected real rate endpoint, \( \rho_t^{\text{oo}} \), and the inflation rate endpoint, \( \pi_t^{\text{oo}} \) – are unobserved.\(^{12}\)

---

\(^{10}\) See the discussion in Shiller, Campbell, and Schoenholtz (1983) and Shiller (1990).

\(^{11}\) Prior to February 1977, estimates of the constant-maturity 30-year Treasury bond rate are not published and these observations were replaced by estimates of the constant-maturity 20-year rate. Recent work by Mark Fisher and Christian Gilles of the FRB staff on estimates of the daily term structure after the mid-1980s suggests that published estimates of the 20-year constant-maturity Treasury rate may contain significant measurement errors.

\(^{12}\) Throughout this discussion, we define the expected real rate component by \( \rho_t^{\text{oo}} = \eta_t - \pi_t^{\text{oo}} \). As will be apparent, this definition of the real rate includes an assortment of term premium components, including those associated with the uncertainty of expected inflation.
Survey evidence on expected inflation promises a way out of this conundrum, but for the United States such data are almost exclusively concerned with short-term expectations. However, there are two notable exceptions:

1) A survey of market participants conducted in the 1980s by Richard Hoey, an economist at Drexel Burnham Lambert, which asked for forecasts of inflation over a ten-year forecast horizon. The survey also distinguished between inflation expectations for the first and second five-year subperiods of the forecast period. Although this survey has been discontinued, a contiguous quarterly series of long-term expected inflation can be assembled for the span 1981 Q1 through 1991 Q1.13

2) A quarterly survey of professional forecasters conducted since late 1980 by the Federal Reserve Bank of Philadelphia, which queries participants for the expected average rate of inflation over the next ten years.

In principle, either survey could be used to decompose $r^{\pi}$ into its real and inflation components, at least over some portion of history. On the face of it, the Hoey survey is preferable to the Philadelphia survey, both because it contains information on expected inflation 5-to-10 years ahead, and because its participants are drawn from the investment community.14

Unfortunately, neither survey by itself is adequate to solve the $r^{\pi}$ decomposition problem, because we need a long time series for $\pi_t^{\pi}$ to estimate the VAR model. Therefore, we consider two indirect ways of estimating the inflation endpoint:15

- a regression decomposition of the nominal rate endpoint; and
- a learning model that extracts shifts in expected inflation from actual inflation.

In the first approach, the Hoey survey results are used directly in the analysis. Survey evidence is also useful for the other method, as it provides a check on the plausibility of our results.

13 A missing observation in 1990 Q1 is estimated by linear interpolation.

14 In practice, results from the two surveys are quite similar for the 10-year expectation – perhaps because a large portion of Philadelphia survey respondents are economists who work for financial institutions, and a substantial portion of Hoey respondents were professional forecasters.

15 In addition to these two methods, we also investigated an unobserved components decomposition $r^{\pi}$. Under this approach, it is assumed that the unobserved real rate endpoint is stationary. Following the procedure developed by Harvey (1985) and Clark (1987), the inflation endpoint is then identified by assigning to it all the nonstationary movements in the nominal interest rate endpoint. Use is made of survey data in parameter identification by fitting the model to the Hoey survey over the 1981-1991 subsample period. Unfortunately, the measure of endpoint inflation produced by this method - shown in the bottom portion of Figure 2 - had the drawback of being highly sensitive to high-frequency movements in $r^{\pi}$. Furthermore, the measure had a tendency to be negative during the initial periods of the sample.
Figure 2
Decompositions of nominal rate endpoints

nonstationary inflation and nonstationary real rate endpoints

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- dashed: nominal rate endpoints
- dashed-dotted: inflation endpoints
- solid: expected inflation 5-10 years ahead (Hoey)

random walk inflation and stationary real rate endpoints

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- dashed: nominal rate endpoints
- dashed-dotted: inflation endpoints
- solid: expected inflation 5-10 years ahead (Hoey)
3.1 Regression decompositions of the nominal rate endpoint

To begin, consider the first equation in Table 2, which shows the results of the simple regression of $r_t^\pi$ on the Hoey estimate of long-term expected inflation, $\pi_{ht}^\pi$. One interesting feature of this regression is that the coefficient of expected inflation is 1.44 and significantly greater than one. One possible explanation is that many holdings of US Treasuries are subject to taxation of earnings. Under this interpretation, the coefficient is $\frac{1}{1-t_x}$, where $t_x$ is the marginal tax rate. The value of the coefficient in Table 2 suggests a marginal tax rate around 0.31. Using flow of funds historical estimates of sectoral holdings of Treasury securities, Kozicki and Tinsley (1995b) estimate that the effective tax rate on Treasury securities faced by domestic households and businesses has fallen from around 0.39 in 1960 to around 0.21 in 1993. Of course, it is difficult to be definitive about the effect of tax rates (marginal or otherwise) on pre-tax bond yields, given that a large number of market participants – e.g., pension funds, foreign investors – pay no taxes.

Another plausible interpretation is that some element of the real rate endpoint may be related to the level of the nominal interest rate endpoint. To develop this approach, rewrite the generic Euler equation 2 for a nominal return as

$$1 = E_r(\mu^\prime r^\prime) = E(\mu^\prime)E_r(r^\prime) + v_{\mu^\prime,r^\prime}$$  \hspace{1cm} (14)
where, as earlier, the primes now indicate the discount factor and gross return \( r' = 1 + r \), and \( V_{\mu,r'} \) denotes the covariance. Condition 14 can be used to define a risk free rate, \( r_f \), whose correlation with the discount factor is zero, and a market portfolio yield, \( r_m \), whose correlation with the discount factor is one. Using these alternative yields, equation 14 (an arbitrage condition) can be restated as the standard expression for portfolio valuation of the return to an arbitrary asset, \( r \):

\[
- r_f = \lambda_m V_{r,r_m}
\]

where \( \lambda_m \) is the market price of risk, \( \lambda_m = (r_m - r_f) / V_{r,r_m} \). Equation 15 indicates that the expected real return includes the risk premium defined by the covariance between the asset yield and the return on the market portfolio.

Although a time series of the estimated return to the aggregate market portfolio is not easily constructed, it may be noted that the required covariance in equation 15 is equal to the product of the return standard deviations with the correlation between the asset and portfolio returns, \( V_{r,r_m} = \rho_{r,r_m} \sigma_r \sigma_{r_m} \). Under the assumption of a constant correlation, some earlier studies of the term structure, such as Shiller, Campbell, and Schoenholtz (1983), use a moving standard deviation of the asset return, \( r \), to capture time variation in the term premia of bonds.

More recent work in finance has suggested that the standard deviation of interest rates is a function of the level of interest rates. For example, Chan, Karolyi, Longstaff and Sanders (1992) estimate a class of autoregressive interest rate models similar to that in equation 8 above, with the additional specification that the standard deviation of the residual innovation is proportional to \( r \). In theoretical models of the term structure, \( \nu \) has ranged from 0 to 1.5. The CKLS paper reports \( \nu = 1.5 \) in regressions using the monthly Treasury bill rate. Our own experience is that point estimates of \( \nu \) can vary all over the map with different sample spans. However, the CKLS paper points out that the high-end 1.5 estimate has the considerable advantage that it can fit samples that contain the marked shift in 1979 of monetary policy without requiring additional dummy variables to capture any remaining rate volatility effects of the policy shift.

The second and third equations in Table 2 provide a regression approximation of a similar analysis for the nominal rate endpoint, \( r^n \). The second equation describes a standard autoregressive model of the nominal rate endpoint, and the third equation summarises the results of a regression of the log absolute value of the autoregressive residual on the log level of the endpoint. Similar to the CKLS finding, the elasticity of endpoint volatility with respect to the endpoint level is 1.53 and significantly greater than one.

Although there is no necessary reason to expect an interest rate level indicator of rate volatility to adequately characterise the risk premium of the endpoint, the fourth equation of Table 2 presents an estimate of the regression

\[
\log\left( r^n - \pi_{h,t} \right) = \alpha_0 + \alpha_1 \log\left( r^n \right)
\]

Surprisingly, the estimated elasticity in the fourth equation, \( \alpha_1 = 1.51 \), is remarkably similar to the elasticity estimate in the third equation, although in this instance the estimate is not significantly different from one. In any event, equation 16 is the basis for our first decomposition of the nominal rate endpoint, \( r^n \), into real and inflation endpoints. The resulting nominal rate and

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16 A motivation in theoretical term structure models for a variance that is heteroskedastic in the level of the interest rate is to prevent Jensen inequality terms, the negative variance terms in equation 4's definition of the term premium, from predicting negative nominal rates.
inflation rate endpoints are plotted in the first panel of Figure 2. Note that the decompositions before 1981 and after 1991 are outside of the 10-year sample that is available for the Hoey estimate of expected long-term inflation. The low-frequency motion of the inflation endpoint identified by the log regression is not unreasonable, rising over the sample until the early 1980s and largely falling thereafter. However, the inflation endpoint is also relatively responsive to high-frequency movement in the nominal rate endpoint, such as the recent increase in 1994.

3.2 An agent learning model for shifts in expected inflation

Our second decomposition procedure is motivated, in part, by the observation that an \( \text{l}(1) \) description of inflation is problematic, if the real rate is assumed to be stationary: Under these conditions the inflation risk premium embedded in \( p_t \) is unbounded in the limit.\(^{17}\) Beyond this logical difficulty, \( \text{l}(1) \) characterisations of inflation and nominal interest rates – prevalent in the macrofinance literature – also would seem to be of questionable empirical relevance: After all, such a characterisation is only one subset of the general class of nonstationary time series,\(^{18}\) and it is well-known that standard tests for unit roots have low power against other descriptions of nonstationarity, such as the episodic shifts analysed in Perron (1989).\(^{19}\)

In particular, we now consider a model in which the nonstationarity of nominal interest rates is due (at least in part) to episodic shifts in the expected level of inflation. Following the analysis in Kozicki and Tinsley (1995a), consider two distributions of information between private and public agents. In the first case, information is symmetric and the inflation endpoint perceived by agents, \( \pi_t^{\omega} \), is identical to the long-run inflation target of monetary policy, \( \pi^* \). Typically, central bankers of developed economies are cautious and slow to change either operational policies or strategic objectives. This suggests that policy changes are episodic with frequencies that are more appropriately measured in half-decades rather than months. This inference seems to be consistent with the small number of policy regimes typically identified in postwar analyses of US policy, such as Huizinga and Mishkin (1986).

In the second case, information is asymmetric. The long-run policy objective for inflation is not known (or believed), and private agents must infer that a shift in \( \pi^* \) has occurred by examining observable consequences of policy. A simple example of the latter is the following changepoint analysis of an autoregressive model of inflation:

\[
\Delta \pi_t = \alpha_0 + \sum_k \alpha_k \delta_k + \gamma \pi_{t-1} + A(L) \Delta \pi_{t-1} + \alpha_t
\]

\(^{17}\) As noted in equation 4, the term premium of nominal bond rates is decreasing in the variance of expected inflation rates. This variance term does not vanish even for risk-neutral representative agents (when the covariance between the marginal utility of consumption and inflation is zero). Because the variance of an \( \text{l}(1) \) process grows without bound over the forecast horizon, the variance of the inflation endpoint is also unbounded. But because the real rate contains the term premium for the variance of uncertain inflation, the real rate endpoint must fall without bound.

\(^{18}\) The shifting-regime model in Hamilton (1989) is an example of nonstationary behaviour that need not exhibit unit roots.

\(^{19}\) A nonstationary series is one whose moments are not constant over time. Changes in moments may be continuous and persistent, with expected absolute values that are predictable and small relative to current levels, as in most real aggregates identified as \( \text{l}(1) \) in macroeconomics, Nelson and Plosser (1982). Alternatively, changes may be infrequent, unpredictable in both sign and size, often large relative to the last observed level, and persistent only in the conditional sense that the next change cannot be predicted – characteristics that seem to describe well the long-term behaviour of inflation and nominal interest rates.
where each $\delta_k$ is a dummy variable that switches on in period $t + k$, $(k = k_1, k_2, \ldots)$. Both the size, $\alpha_k$, and timing, $t + k$, of changepoints are unknown to private agents. Analogous to the endpoint construction for the stationary autoregressive model discussed earlier, the perceived endpoint in period $\tau$ is

$$\bar{z}_\tau = \alpha_0 + \sum_{i+k < t} \alpha_k / \gamma$$

which includes all changepoints recognised by agents as of period $t$. If changepoints are assigned to each period, the endpoint is $I(1)$.

The reliability of a detected changepoint varies inversely to the number of observations since the last changepoint. In the autoregressive changepoint problem discussed in Kozicki and Tinsley (1995a, 1995b), agents may choose among eight minimum recognition lags, ranging from one year (12 months) to eight years (96 months). The top panel of Figure 3 displays the inflation rate changepoints selected by agents using a minimum recognition lag of eight years. The thin solid line is the concatenation of actual changepoints in the inflation endpoint detected by these agents, using 1% critical values. The dashed line is the associated concatenation of virtual endpoints, allowing for the recognition lag between the month of the actual shift in the estimated inflation endpoint and the month when the shift was detected. The virtual recognition lag, the distance in months between the actual shift date and the virtual shift date, can sometimes greatly exceed the minimum recognition lag. However, this discrepancy is small, generally, for agents with lengthy minimum recognition lags, as in the case shown.

After estimating a series of concatenated virtual changepoints for each class of agents, the frequency distribution of agents is estimated by projecting the nominal rate endpoint, $\bar{r}^\infty$, onto the full set of virtual changepoint series. The projection also yields the mean inflation endpoint series, $\bar{\pi}^\infty$. The endpoints of both the nominal interest rates and inflation rates are plotted in the second panel of Figure 3, along with the Hoey estimate of expected long-term inflation. The remarkable feature of this figure is that the estimated inflation endpoint is aligned very closely with available Hoey estimates, even though no information in the latter survey was available to any of the learning agents (in contrast to the regression-based decomposition, which incorporates survey information in the estimation procedure). Another important feature of this model is that the estimated mean recognition lag exceeds five years, indicating that it can take years for agents to recognise shifts in the long-term inflation objective of policy.

### 3.3 Multiple indicators

Among the two alternative methods for constructing historical measures of the inflation endpoint, we find the agent learning model the most promising. The learning model is also attractive because it provides a procedure for updating the inflation endpoint in forecasting and policy analysis. However, survey information provides alternative measures for the post-1980 period that are arguably superior to any constructed proxy, on the grounds that the surveys are direct measures of expected inflation for at least a subset of economic agents.

For this reason, we have chosen a splicing methodology that uses survey measures of $\bar{\pi}^\infty$, where available, and predictions from the learning model elsewhere, in constructing a historical endpoint series for use in estimation and forecasting. A challenge under this approach is how best to model the inflation endpoint in forecasting and in policy simulations: As a static variable invariant to transitory shocks to the system, or as an expectation that responds dynamically to changes in the
Figure 3
Shifted inflation endpoints

minimum recognition lag of 8 years

historical inflation
final changepoints
virtual changepoints

nominal rate and shifted inflation endpoints

nominal rate endpoints
shifted inflation prediction
shifted inflation endpoints
expected inflation 5-10 years ahead (Hoey)
macroenvironment? At this stage we are evaluating two solutions to this problem. The first is simply to assume that innovations in surveyed expectations evolve according to the predictions of the agent learning model. The second solution is to model the innovations in survey expectations empirically, by regressing changes in surveyed expectations on VAR equation innovations. Preliminary results suggest that VAR innovations – a measure of incoming news available to agents – can explain a significant portion of the historical path of expected long-run inflation.

4. The empirical model

As noted earlier, the actual procedure used to approximate interest rate expectations in the FRB/US model generalises the autoregressive funds rate model of equation 11 to a VAR system that incorporates moving endpoints for its nonstationary components. Forecasts from this system are then used, via the formulas presented in equations 6 and 7, to construct estimates of the bond rate, under the assumption that the risk premium, $\Phi_n$, is constant. The latter is approximated by the sample mean spread between the bond rate and the weighted sum of the projected short-term interest rates.

4.1 Specification and estimation of the VAR model

Since Sims (1980), a voluminous literature on VARs has arisen. It is filled with debates, both theoretical and empirical, about the proper specification of a macroeconomic VAR in terms of number and type variables to include. While there are a number of potentially important variables eligible for inclusion in our VAR model, to date we have restricted our work to simple specifications that include some version of the three basic variables used by Sims: (1) a measure of real economic activity; (2) a measure of price; and (3) a measure of monetary activity. Although our experience with parsimonious three-variable systems has been satisfactory, we leave open the possibility of adding other variables in the future.\footnote{Some variables under consideration for an expanded VAR include: commodity price inflation; oil prices; the exchange rate; and fiscal policy variables (e.g. the deficit-to-GDP ratio).}

Based on a review of the VAR literature, the earlier discussion of moving endpoints, and a fairly extensive empirical investigation, we settled on the following specification of the VAR:

- **Inflation ($\pi_t$)** – defined as the rate of change in the chain-weight price index for personal consumption in the National Income and Product Accounts. We selected this price inflation measure primarily because it was the measure least susceptible to the "price puzzle" – the positive impulse response in inflation following a positive interest rate shock – in a three-variable system.

- **Output gap ($y_t$)** – defined as the log of real business (excluding farm and housing) output minus the log of a measure of potential output. The potential output series is consistent with the aggregate production function of the full FRB-US model. In particular, the potential output calculation assumes that the labour market is in equilibrium (unemployment equals the NAIRU), and that aggregate labour productivity is a function of the capital-labour ratio and exogenous technical progress. The latter is proxied by a split time trend to capture the post-1973 slowdown in productivity growth.
• Interest rate ($r_t$) – defined as the federal funds rate (effective annual rate basis). We use the funds rate for two primary reasons. First, it tends to outperform most other measures of monetary activity in VARs such as monetary aggregates and interest rate spreads according to conventional statistical criteria – see, for example, Bernanke and Blinder (1992) and Sims (1988). Second, it is often cited as the most reliable indicator of the stance of monetary policy.

For the two non-stationary components of the VAR system – inflation and the nominal funds rate – we control for moving endpoints using $\pi_t^m$ and $r_t^m$. The nominal interest rate endpoint is derived as described earlier from the observed term structure. As our measure of the inflation endpoint, we use a spliced estimate based on predictions from the agent learning model before 1981, and survey-based measures of expected long-run inflation thereafter. By construction the third variable of the VAR, the output gap, is stationary with an endpoint fixed at zero.

The system of equations to be estimated can be written in compact form as

$$\tilde{z}_t - z_t^m = A(\tilde{z}_{t-1} - z_{t-1}^m)$$

where $\tilde{z}_t$ is a vector denoting the primary variables of the system, $z_t^m$ is a vector of the corresponding moving endpoints (zero in the case of $y_t$), and $A$ denotes a lag operator matrix. The order of $A$ – four lags of each variable – was selected based on standard information criteria tests. Variations in the lag length from four to eight periods do not substantially change the properties of the VAR. Tests for structural breaks in theVAR, which is estimated over the period 1960 Q1 to 1994 Q4, tend to suggest a relatively stable system. Estimates of equation 19 are readily incorporated into the reduced-form expression for the bond rate (equation 7) by noting that $z_t = [\tilde{z}_t, z_t^m]$ and

$$H = \begin{bmatrix} A & -A \\ 0 & I \end{bmatrix}$$

The structure of $H$ ensures that forecasts of future $\tilde{z}_{t+i}$ made at time $t$ are based on the most recent available estimate of the endpoints, $z_t^m$. But if the VAR system is treated as a closed system and is used to generate future values of the endpoints (as we do next), it also implies that the endpoints follow a random walk. When endpoints are modelled in this naive fashion, VAR-based forecasts of nominal interest rates are very insensitive to how endpoint inflation is measured. However, if the behaviour of the endpoints is modelled in a more sophisticated manner – for example, by linking its behaviour to the overall FRB/US model – then the decomposition of the nominal rate endpoint can matter to the dynamic behaviour of the bond rate. We return to this issue in Section 5, where we consider some rudimentary models of the real interest rate endpoint.

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21 To be specific, the survey portion of the spliced estimate equals the Hoey survey for 10-year ahead inflation expectations for the period 1981 Q1 to 1991 Q1, and the Philadelphia survey thereafter.

22 However, there are signs of instability in the funds rate equation, which fails a Chow test for a break around 1980.

23 For example, replacing the spliced measure with the learning model or regression-based estimates of endpoint inflation has little effect on the properties of the estimated system. As regards the bond rate, the key variable in the expectations generating system is $r_t^m$, not its decomposition.
4.2 Dynamic properties of the VAR and the bond rate models

Figures 4 and 5 show the impulse response functions for the estimated VAR model – dashed lines are one standard error bands – under the assumption that the two moving endpoints follow a random walk. We ordered the VAR variables as follows: $\pi_t$, $\gamma_t$, $r_t$, $\pi_t^\infty$ and $r_t^\infty$. The funds rate is ordered after $\pi_t$ and $\gamma_t$ on the argument that the monetary authority can respond to contemporaneous shocks to output and inflation, but that current activity does not react instantaneously to changes in short-term interest rates. With the funds rate third, the system is not particularly sensitive to alternative orderings of inflation and output. Endpoints are ordered last because they summarise agents expectations of the future, conditioned on all available information.

Figure 4
Impulse response functions of the VAR model
Figure 4 shows responses to innovations in $\pi_t$, $\gamma_t$, and $\alpha$. As these variables are ordered prior to $\pi_t^\infty$ and $\alpha^\infty$, no shocks to the endpoint are introduced. Thus, these impulse responses, while persistent, are stationary. Hence, shocks to the system gradually fade away and variables return to their original equilibrium (the zero line) in about 5 years.

Figure 5 shows the impulse response functions for output, inflation and the funds rate to innovations in the endpoints. Because of the random-walk nature of the endpoint equations, innovations to endpoints result in permanent shifts. The upper three panels show the effects of a unit change in endpoint inflation, under the assumption that $\rho_{\pi}^\infty$ is unaffected — implying that the nominal interest rate endpoint moves one-for-one with $\pi_t^\infty$. Both inflation and the funds rate exhibit overshooting, but eventually move one-for-one with the shift in their expected steady-state levels. By contrast, output initially expands following the shock to $\pi_t^\infty$, but after five years returns to equilibrium. Response patterns are roughly the same for a shock to $\alpha^\infty$ alone (i.e., a permanent shock to the real rate), except that in this case the initial inflation surge fades away after a few years.

**Figure 5**

*Responses to innovations in endpoints*

Upper panel: innovation in the inflation endpoint; lower panel: innovation in real rate endpoint

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Figure 6 illustrates the implications of the behaviour of the VAR for the 10-year yield on Treasury bonds. As indicated by the solid lines in the upper panel, a unit innovation in the output gap yields an immediate jump in long-term interest rates of about 25 basis points. However, the rise is transitory and quickly fades away, in contrast to the response of the funds rate (the dashed line), which continues to build for a year or so before peaking at about a percentage point. Bond yields and the funds rate respond in a similar, but more muted, pattern to an innovation in inflation. But short and long-term rates behave quite differently following a shock to the funds rate. Because the VAR projects funds rate innovations to die out quickly, bond yields hardly respond to changes in short-term interest rates that are not driven by shocks to output or inflation.
Figure 6
VAR-generated response of interest rates to innovations in output, inflation and the funds rate
Solid: 10-year Treasury yield; dashed: funds rate

Response to an Innovation in the Output Gap

Response to an Innovation in Inflation

Response to an Innovation in the Funds Rate
Figure 7
VAR-generated response of interest rates to innovations in endpoint inflation and the real rate
Solid: 10-year Treasury yield; dashed: funds rate

Response to an Innovation in Endpoint Inflation

Response to an Innovation in the Endpoint Real Rate
Figure 8
VAR-based predictions of the yield on 10-year Treasury bonds
to innovations in endpoint inflation and the real rate

Yields, Actual and Predicted
(dash -- predictions corrected for serially correlated errors)

Prediction Errors
(dash -- after correction for serial correlation)
Although the model predicts that long-term rates are less responsive to transitory shocks than short-term rates, the situation is reversed for permanent shocks that alter the expected endpoint levels of inflation and the real interest rate. As shown in Figure 7, bond yields respond almost instantaneously (and fully) to a permanent shift in either endpoint. The funds rate, however, takes a full two years to respond as much as the bond rate to a shift in long-term expectations. Accordingly, positive endpoint shocks are associated with increases in the slope of the term structure, while positive shocks to output and inflation yield a less steep slope. Innovations in the funds rate are also associated with a decline in the slope of the yield curve, unless they are associated with changed expectations concerning the long-run target level of inflation.

4.3 Statistical evaluation of the bond rate model

The upper panel of Figure 8 compares the historical yield on 10-year Treasury bonds to that predicted by the VAR model, under the assumption that the term premium $\Phi_n$ is constant. The bottom panel displays the corresponding prediction errors. For both panels, results are plotted for two different predicted paths of the bond rate. The first path (the dotted line in the upper panel) represents the weighted sum of future short-term interest rates projected at each point in time, adjusted for the average 1965-1995 difference between the 10-year rate and the weighted sum. The second series (the dashed line in the upper panel) is the same as the first, except that account is taken of serial correlation in the prediction errors. Thus, the second series can be regarded as the one-step ahead prediction of the bond rate.\(^{24}\)

As can be seen, both series do a reasonable job of capturing the overall historical path of the bond rate. Nonetheless, the lower panel demonstrates that the model makes significant prediction errors. However, after controlling for residual serial correlation, the model's in-sample tracking performance is competitive with that obtained from other models of the term structure. The standard error of our bond rate model is 38 basis points after correcting for residual serial correlation, as compared to 46 basis points for an equation styled after that used in the Federal Reserve's old MPS model, estimated over the same sample period.\(^{25}\)

Aside from its magnitude, residual correlation in the bond model's errors is problematic because it indicates a violation of the underlying assumptions of the model: If the risk premium $\Phi_n$ is constant and expectations are rational, the prediction errors of the model should be white noise. However, a simple regression of the prediction error on its own lag yields an estimated first-order autoregressive parameter of 0.83. More formally, the hypothesis that the prediction errors do not display $n$-th order serial correlation can easily be rejected at the 5 percent level for $n$ equal to 1, 4 or 12, using the Lagrange multiplier test developed by Breusch (1978) and Godfrey (1978).

Furthermore, the model fails a test of the rational expectations overidentifying restrictions imposed on the bond rate model. Following the procedure suggested by Hansen (1982), the unadjusted bond rate errors are regressed on the elements of the information set used to construct $\sum_{t=0}^{n-1} B_t^e f_{t+1}$ – that is to say, current and lagged observations on the funds rate, inflation, and the output gap, plus current observations on the two endpoints.\(^{26}\) The value of $R^2$ from this regression, multiplied by the number of observations, is distributed $\chi^2$ with $k-1$ degrees of freedom, where $k$

\(^{24}\) Strictly speaking this statement is incorrect, because the information set includes contemporaneous observations on the funds rate, inflation, the output gap and the endpoints, and lagged information on the bond rate equation's errors.

\(^{25}\) The MPS-style equation is an error-correction model in which the change in the effective 10-year bond rate is regressed on lagged bond rate changes, the lagged level of the bond-funds rate spread, and current and lagged changes in the effective funds rate.

\(^{26}\) The number of lags allowed in the regression equals the number used in the construction of the bond rate.
equals the number of regressors. The p-value from this test is less than 0.01. Examination of the estimated coefficients from this regression reveals that the bond model underestimates the sensitivity of long-term interest rates to changes in the funds rate, and overestimates its sensitivity to movements in output or inflation.\textsuperscript{27}

One must be cautious in interpreting the failure of the model to pass the tests for serial correlation and overidentifying RE restrictions. For example, one could interpret the evidence as a rejection of the rational expectations hypothesis. However, the test results may reflect an information set for the VAR model that is too restrictive – and in fact, preliminary work with expanded VARs does suggest that other variables, such as oil prices, are important. Furthermore, all the tests are conducted under a joint hypothesis of rational expectations and a constant term premium. As suggested by the finance literature, the correlation of bond model errors with macroeconomic factors and lagged errors may simply be the result of risk premiums that vary over time in a predictable fashion. We believe it is more fruitful from a modelling prospective to allow for this possibility, than to abandon the assumption of rationality.

As noted at the beginning of Section 3, a drawback of basing expectations on a small-scale VAR system is that, when embedded in a larger model such as FRB/US, the simulated behaviour of the VAR system is likely to be inconsistent with that of the broader system – a problem that does not affect model-consistent expectations. But this drawback is not necessarily a serious problem, if the moving endpoints in the VAR are consistent with those of the full model,\textsuperscript{28} and the impulse response patterns of the two system are broadly similar. To minimise this problem, simulations of FRB/US are always designed to ensure endpoint consistency. However, gauging the similarity of impulse responses is more difficult. One approach to this problem is to compare the behaviour of the VAR system estimated using historical data, with a VAR estimated on a synthetic dataset derived from stochastic simulations of the FRB/US model itself. As discussed by Bomfim, Brayton, Tinsley and William (1995), the impulse responses generated by the two approaches are fairly similar. This result suggests that the expectations generated by the small-scale VAR are effectively rational within the context of the FRB/US model, since they are consistent with the behaviour of the overall system.

5. Explaining recent bond market behaviour

As a final test of our model, we consider what light it can shed on recent developments in the US bond market. As shown in the upper panel of Figure 9, long-term interest rates have fallen roughly 2\% percentage points over the past five years. The decline has not been steady, but was marked by a major back-up in rates during 1994 that has since been reversed. Short-term rates experienced a similar decline on balance over the entire 1990-1995 period, but the co-movement between long and short rates has not been stable. In particular, the funds rate did not display anything like the 1992-1995 bond rate cycle. Furthermore, the typical lagging behaviour of long-term interest rates, which causes the yield curve to steepen (flatten) during periods of falling (rising) short-term interest rates, disappeared at times – most notably in the first months following the February 1994 change in monetary policy, when bond rates rose twice as much as the funds rate.

\textsuperscript{27} Because the bond rate prediction errors are serially correlated, the overidentifying restrictions test we carry out may not be appropriate. However, it is worth noting that even if the test is run with the errors corrected for serial correlation, the explanatory power of the VAR information set in the LM regression is still very high – $R^2 = 0.43$.

\textsuperscript{28} It should be noted that using model-consistent expectations does not allow one to avoid issues concerning the specification of endpoint conditions. As in the case of the VAR-based expectations, it is necessary to be specific about the determinants of terminal conditions – e.g., the inflation goals of the central bank and the target level of government indebtedness.
Figure 9
Recent movements in US interest rates to innovations in endpoint inflation and the real rate
Solid: 10-year Treasury yield; dashed: funds rate

Treasury Bond Yields and the Federal Funds Rate

Treasury Bond Yields and the Nominal Rate Endpoint
Figure 10
Decomposition of recent bond rate movements to innovations in endpoint inflation and the real rate
Solid: 10-year Treasury yield; dashed: funds rate

Expected Endpoints for Inflation and the Real Interest Rate

10-Year Treasury Yields, Actual and Predicted
(predictions conditioned on observed elements of the VAR)
How well can the model account for this complicated pattern of interest rate movements? We begin by considering the accompanying behaviour of the nominal interest rate endpoint, illustrated by the dashed line in the bottom panel of Figure 9. As can be seen, market expectations were relatively stable through 1992. During the first 9 months of 1993, however, there was a steep decline in \( r_t \) of about 2 percentage points. Much of this decline was reversed over the next 12 months, but the nominal rate endpoint has since returned to its mid-1993 level.

As shown by the solid line in the top panel of Figure 10, essentially none of the 1993-1995 gyrations in \( r_t \) can be attributed to a change in expected long-term inflation prospects – although a gradual fall in \( \pi_t \) can account for a substantial portion of the overall decline in the nominal rate endpoint since 1990.\(^{29}\) Rather, recent fluctuations in the bond rate reflect changed perceptions concerning the long-term level of real interest rates. This interpretation is the opposite of that reached by Campbell (1995), who attributes the bulk of the 1994 back-up in bond yields to a rise in inflation expectations. His conclusion appears to be largely based on an a priori assumption that the real rate is stationary and relatively constant. However, Campbell's interpretation is not necessarily at odds with ours, because his definition of the real rate may exclude risk premia. If so, the rise in our measure of the real rate endpoint, which includes a risk premium, could theoretically be attributed to market-perceived changes in the variance of inflation or in its covariance with other factors.

The lower panel of Figure 10 compares the actual bond rate path (solid line) to that predicted by the model (dotted line), conditioned on the actual path of output, inflation, the funds rate, and the two endpoints. Here, bond rate predictions are not corrected for residual serial correlation (as they were in the dashed line in the upper panel of Figure 8), but are instead defined as $\Phi_n + \frac{1-B}{1-B^2} \sum_{i=0}^{n-1} B^i r_{t+i}$. As it is, even uncorrected predictions do a good job of capturing the overall movement in bond rates since 1990. The panel also displays predictions (dashed line) that control for the economic structure of the bond equation errors, i.e., that incorporate an estimate of the dependence of model errors on the VAR information set.\(^{30}\) The similarity between the two predicted bond rate series indicates that the correlation between model errors and economic conditions is not that quantitatively important, at least over the last few years.

As one might suspect from the upper panel of Figure 10, the key to explaining the recent cycle in bond rates hinges on the behaviour of the expected real interest rate endpoint. Unfortunately, we have just begun to develop an empirical model of \( \pi_t \). However, it is instructive to consider some preliminary results.

As noted in the prior section, the VAR system incorporates naive random-walk forecasting equations for the endpoints. The upper panel of Figure 11 compares the actual path of the real rate endpoint (here plotted at a quarterly frequency) to that which would have been predicted in 1989 Q4 using the VAR rule (dotted line). Conditioning on a constant value for \( \pi_t \), but on the observed values of the other elements of the VAR, yields the projection of the bond rate illustrated in the bottom panel of Figure 10 by the dotted line. As suspected, under these conditioning assumptions the model does not predict a pronounced bond rate cycle.

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\(^{29}\) The upper panel of Figure 10 displays monthly data. A monthly series for expected inflation is constructed from quarterly survey data via cubic-spline interpolation.

\(^{30}\) Specifically, bond model errors are regressed on the elements of the VAR, and then predictions of the errors are made conditional on the observed path of the VAR variables.
Figure 11
Effect on bond rate predictions of alternative forecasts of the real rate endpoint
Solid: 10-year Treasury yield; dashed: funds rate

Real Rate Endpoint, Actual and Predicted

10-Year Treasury Yields, Actual and Predicted
(predictions conditioned on observed inflation endpoint)
Nor does the model do so if an attempt is made to control for the historical correlation of the real endpoint with aggregate macroeconomic conditions. Regressions of $p_f^\infty$ on current and lagged values of the other variables of the VAR (plus lags of itself) suggest a statistically significant link between the endpoint and changes in current economic conditions—particularly the funds rate. However, conditioning on this link does not appear to be that important from an economic standpoint. As indicated by the dashed line in the upper panel of Figure 11, a projection of the real rate endpoint based on this crude model differs only modestly from a random-walk projection.\textsuperscript{31} This difference changes the projected bond rate (dashed line, lower panel) only marginally.

An alternative approach to this problem is to link forecasts of $p_f^\infty$ to the behaviour of the larger-scale FRB/US model. In simulation work with the full model under VAR-based expectations, our current practice is to employ an ad hoc adjustment equation that forces the expected real rate endpoint to converge slowly to the real funds rate generated by the overall model. Because the real interest rate produced in simulations of this sort is generated by a monetary policy reaction function that targets a specific rate of long-run inflation (using nominal short-term interest rates as an instrument), this practice is equivalent to forcing $p_f^\infty$ to converge to the overall model's steady-state real interest rate.\textsuperscript{32} In the context of the FRB/US model, this implies that $p_f^\infty$ is a function of a large number of variables, including such fiscal variables as the level of government indebtedness, the mix of taxes and transfers, and marginal tax rates.\textsuperscript{33} Unfortunately, although the overall macro model provides a framework for tying down $p_f^\infty$ in policy simulations, it cannot be used directly in estimation and forecasting. However, it can provide useful guidance in the specification of an empirical structural model of the real rate endpoint.

An example of such guidance is provided by work-in-progress on the link between real interest rates and government budget deficits. In non-Ricardian large-country open-economy models (such as FRB/US), a key theoretical determinant of the equilibrium real interest rate is the steady-state ratio of the government budget deficit to GDP: The deficit ratio determines the debt-to-GDP ratio, which in turn influences the private saving rate. The neoclassical growth model which is at the core of FRB/US suggests that a sustained 1 percentage point rise in the deficit ratio should boost the equilibrium real interest rate by ¼ to ½ percentage point.\textsuperscript{34} This model-generated estimate is in line with the historical correlation between real interest rates and the deficit-to-GDP ratio. As shown in the upper panel of Figure 12, annual averages of the cyclically-adjusted budget deficit and $p_f^\infty$ tend to move together on a contemporaneous basis.

\textsuperscript{31} The 1990-1995 predictions of the VAR-information model are conditioned on observations of $p_f^\infty$ through 1989 Q4 only, but on actual post-1989 observations for the other elements of the VAR.

\textsuperscript{32} Strictly speaking, real rate convergence also depends on $x_f^\infty$ converging to the policy target rate of inflation. In simulation this condition is met via updating rules similar to those implicit in the learning agent model, or through ad hoc adjustment equations.

\textsuperscript{33} In the FRB/US model, the primary channel through which fiscal policy influences the equilibrium real interest rate is household wealth, owing to the fact that consumers are non-Ricardian. However, the mix of taxes and transfer payments also matters, since the propensity to spend out of the present value of transfer income is higher than that for after-tax labour or property wealth, implying that changes in the tax/transfer mix influence the aggregate saving rate, and thus the real interest rate. Finally, changes in marginal income tax rates influence the desired stock of capital per worker, and thus the aggregate productivity level. Changes in the latter influence the steady-state real interest rate, because the real rate is defined as that which equilibrates aggregate demand and supply in the long run.

\textsuperscript{34} The range of estimates is produced by simulating FRB/US under different assumptions about the responsiveness of foreign interest rates to a rise in domestic rates; the less responsive are foreign rates, the greater is the sensitivity of the domestic real rate to a change in government saving.
Figure 12
Relationship between the real interest rate endpoint and the cyclically adjusted federal budget deficit
Deficit expressed as a ratio to GDP

Contemporaneous Deficit Ratio

3-year Ahead Average Deficit Ratio
However, the two series are more closely correlated if the real rate endpoint is plotted against a forward moving average of the deficit ratio over the near future, as evidenced by the lower panel of Figure 12, which illustrates the relationship for a 3-year-ahead moving average.\textsuperscript{35} That the fit is tighter for the average future deficit ratio is not surprising, given that $\rho_t^w$ is an estimate of the level of real rates expected to prevail five or ten years into the future.

### Table 3

**Stationarity and regression tests for the real interest rate endpoint and the deficit-to-GDP ratio**

Sample period 1965 Q1 to 1992 Q4

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real interest rate endpoint based on:</td>
<td></td>
</tr>
<tr>
<td>Learning model inflation measure</td>
<td>-3.11</td>
</tr>
<tr>
<td>Spliced inflation measure</td>
<td>-2.51</td>
</tr>
<tr>
<td>Federal budget deficit-to-GDP ratio:</td>
<td></td>
</tr>
<tr>
<td>Actual</td>
<td>-1.90</td>
</tr>
<tr>
<td>Cyclically adjusted</td>
<td>-3.11</td>
</tr>
</tbody>
</table>

#### Regression tests

\[
\rho_t^w = \alpha_0 + \alpha_i def_t + \sum_{j=1}^{8} \left[ \beta_j \Delta \rho_{t-j} + \omega_j \Delta def_{t-j} \right]
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated $\alpha_i$</th>
<th>ADF statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual deficit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real interest rate endpoint based on:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Learning model inflation measure</td>
<td>0.47</td>
<td>-3.74</td>
</tr>
<tr>
<td>Spliced inflation measure</td>
<td>0.64</td>
<td>-4.00</td>
</tr>
<tr>
<td>Cyclically-adjusted deficit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real interest rate endpoint based on:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Learning model inflation measure</td>
<td>0.42</td>
<td>-2.06</td>
</tr>
<tr>
<td>Spliced inflation measure</td>
<td>0.67</td>
<td>-2.82</td>
</tr>
</tbody>
</table>

\textsuperscript{35} Forward-moving average estimates for the most recent period incorporate CBO projections of the federal deficit for 1996 to 1998.
Of course, pictorial evidence such as Figure 12 is subject to many criticisms. But regression analysis suggests the degree of co-movement is remarkably similar to that suggested by the steady-state model. As shown in the lower portion of Table 3, regressions of $p_t^\infty$ on the deficit-to-GDP ratio ($def_t$), plus leads and lags of changes in $p_t^\infty$ and $def_t$, suggest that a 1 percentage point permanent increase in the ratio raises the real rate endpoint by 42 to 67 basis points. This result, which is estimated with a high degree of precision, holds whether or not the deficit is measured on an actual or cyclically-adjusted basis.

Figure 13
Medium-term effect of the congressional budget proposals on output, inflation and interest rates, based on RE simulations of the new FRB model of the US economy
Solid: standard response; dashed: myopic consumers

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36 For example, the correlation would be more persuasive if it were based on real-time publicly-available projections of the deficit, not actual deficit outcomes. This proposition can be tested using forecasts of the deficit prepared and published by the Congressional Budget Office since the 1970s; we hope to conduct these tests in the near future. In addition, there is the issue of the direction of causality, given that the correlation may partly reflect the link between current interest rates and future government interest expense. However, this channel could at most account for only a small part of the correlation given the size of the net interest share of the budget.

37 It might be objected that the high t-statistics shown in Table 3 are an artefact of a possible trend in $p_t^\infty$ and $def_t$, given that both series appear to be only borderline stationary. If it was thought that both series are I(1), the regression results would be interpreted as a cointegration tests. Under this assumption, ADF statistics for the residuals from the regressions indicate that the actual deficit ratio is cointegrated with the real rate endpoint, but not the cyclically-adjusted deficit. However, because we don't view either series as I(1), we regard the ADF test results as moot.
These results suggest that fiscal policy may have been a possible cause of the recent cycle in the real rate endpoint. Indeed, 1993 saw enactment of a major fiscal package that significantly reduced projected budget deficits – the same year that the real endpoint fell 1½ percentage points. Similarly, 1995 saw both a large fall in $\rho_r^m$ and legislative action that has greatly increased the odds that the budget will be in approximate balance around the turn of the century. Experiments with FRB/US concerning the effects of the Congressional budget proposals currently under debate suggest that the magnitude of the proposed changes, if fully credible, are sufficient to explain almost all of the decline in the real endpoint experienced since late last year. As shown in Figure 13, simulations of the full model under model-consistent expectations indicate that the proposed budget savings would lower real interest rates in the long run by about 1¾ percentage points. The model would generate a similar decline in real rates from passage of the 1993 budget agreement, given that its savings are comparable to those currently being proposed.

Although changes in the stance of fiscal policy can help explain why bond rates fell sharply in 1993 and 1995, fiscal policy cannot account for the 1994 back-up in rates. For this episode, we must look for some other cause. One possibility is that the market perceived a shock to aggregate demand that would be highly persistent, and so raise the expected long-term level of real interest rates. In fact, during this time there was a very large upward revision to the expected level of future real output. As shown in Table 4, column 1, the 1994 Blue Chip consensus forecast of current year real GDP growth (4th quarter to 4th quarter) rose almost a percentage point between January and December. When coupled with the accompanying 1994 revision to 1995 growth (column 2), these revisions imply an upward revision in the projected level of output in 1995 Q4 of 1.1 percentage points. Simulations of FRB/US indicate that a shock of this magnitude to ex ante aggregate demand, if sustained, would raise steady-state real interest rates by 70 basis points – about half as much as the observed change in the real rate endpoint. Of course, it is difficult to say what sort of aggregate demand shock would display such persistence. Furthermore, inspection of Table 4 reveals that Blue Chip forecast revisions are poorly correlated with movements in $r_i^m$.

Table 4
Revision to Blue Chip consensus forecasts of real GDP and movements in the real interest rate endpoint

<table>
<thead>
<tr>
<th>Year</th>
<th>Current year growth</th>
<th>Next year's growth</th>
<th>Next year's Q4 level</th>
<th>Q4 to Q4 change in real rate endpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>0.1</td>
<td>0.4</td>
<td>0.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>1994</td>
<td>0.9</td>
<td>0.2</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>1993</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-1.6</td>
</tr>
<tr>
<td>1992</td>
<td>0.4</td>
<td>-0.3</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>1991</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>1990</td>
<td>-0.7</td>
<td>-1.9</td>
<td>-2.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>

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Conclusion

In this paper, we have presented a model of the bond rate that in some ways is quite traditional: As suggested by conventional rational expectation theories of the term structure, bond yields are modelled as a weighted sum of future expected short-term interest rates (plus a constant risk premium), with expectations derived from a VAR forecasting system. Where the model differs from standard practice is in its use of moving endpoints in the design of the VAR to help account for the observed nonstationarity of nominal interest rates and inflation. These moving endpoints, which denote investors' expectations on the long-term level of nominal rates and inflation, provide a means to decompose bond rate movements into two components – a stationary element associated with the business cycle and monetary policy stabilisation, and a nonstationary portion linked to longer-term monetary and fiscal policy goals.

VAR models that incorporate moving endpoints (derived from the term structure and surveys of inflation) provide more sensible predictions of the historical path of long-term interest rates than do models that assume interest rates to be stationary or I(0). In terms of goodness of fit, our approach also compares favourable with atheoretic error-correction models of the term structure, if allowance is made for serially-correlated movements in the term premium. However, our work in this area is still at a preliminary stage. In particular, more remains to be done to explore the empirical relationship between policy and other determinants of expected long-run inflation and the real rate of interest.
References


Comments on paper by S. Kozicki, D. Reifschneider and P. Tinsley by G. Sutton (BIS)

The goal of this very interesting paper is to develop a model of long-term interest rate behaviour which is firmly grounded in economic theory and usable for policy analysis. This is a difficult task and the authors deserve credit for their clever efforts in this direction, breaking new ground close to the frontier.

The theoretical basis of the model is the expectations hypothesis of the term structure. Therefore, a key component of the model is a mechanism for generating expectations of future short-term interest rates. It is assumed that expectations are consistent with forecasts from a small scale VAR. The VAR is somewhat unconventional because it includes exogenous variables which influence the time path of the endogenous variables of the VAR.

The authors refer to the exogenous variables in the VAR as "moving endpoints". It is hoped that these moving endpoints contain information about the evolution of the endogenous variables in the VAR beyond that incorporated in their past behaviour. For example, one of the moving endpoints, the "nominal interest rate endpoint", is an average of forward interest rates. The conjecture is that this variable contains information about the future course of short-term interest rates above what is contained in the past history of short-term rates and the other endogenous VAR variables.

There are good reasons to believe that this is indeed the case. For many countries, the slope of the term structure contains information relevant for predicting the future course of short-term interest rates. The nominal interest rate endpoint – an average of forward interest rates – appears to be a useful variable for exploiting this information. Perhaps not surprisingly, empirical evidence reported in the paper supports the view that the nominal interest rate endpoint contains useful information about the future course of short-term interest rates.

The model is used to explain the recent behaviour of the US bond market. The conclusion of the exercise is that the bond market cycle of 1993-95 cannot be attributed to shifts in inflation expectations. Instead, these recent fluctuations in long-term interest rates reflect changes in the long-run level of real interest rates or in risk premia.

I will bring my comments to an end by raising several issues. First, the conclusion that the nominal interest rate endpoint contains information useful for forecasting the future course of short-term interest rates above what is contained in the past history of short-term rates is based on an examination of in-sample forecast performance. It would be interesting to test this hypothesis on the basis of out-of-sample forecast performance. In particular, it would be useful to compare the authors preferred model of short-rate dynamics, which includes the use of a "nominal interest rate endpoint", with a very parsimonious alternative on the basis of out-of-sample forecasts.

Second, I am less optimistic than the authors that the serial correlation of the model's errors can be explained by time variation in risk premia. A paper that looks exactly at this issue is the recent study by Hardouvelis. He concludes that time variation in term premia is not an adequate explanation of deviations of ten-year government bond yields from the predictions of the expectations theory of the term structure, at least for the US market.

Serial correlation of the model's errors is most likely the result of the failure of the VAR to adequately capture market participants' expectations of future short-term interest rates. As the authors point out, a potential solution to this problem is to include more variables in the VAR. But there is a tradeoff here. As more variables are included in the VAR, more parameters are estimated and forecast performance may well deteriorate. Therefore, it might be useful to place additional restrictions on the VAR, perhaps through the use of Bayesian estimation, in order to reduce the number of estimated parameters with the goal of improving out-of-sample forecast performance.
The determination of interest rates and the exchange rate
in the Bank of Canada's Quarterly Projection Model

Agathe Côté and Tiff Macklem

Introduction

In August 1993, the Quarterly Projection Model or QPM as it is known, replaced RDXF as the main model used by the staff of the Bank of Canada for economic projections and policy analysis. In this paper, our modest goal is to outline the determination of interest rates and the exchange rate in QPM, with particular emphasis on the interaction between these two key prices and the outcomes for real and nominal variables in the model. In doing so we also highlight some areas for future work.

The traditional approach to empirical macro modelling has been to estimate the individual equations for all the endogenous variables in the system, and then to combine these to form a macro model. Typically, empirical interest rate and exchange rate equations and the predictive power of these equations were a key ingredient in the model. Experience has shown, however, that the predictive power tends to deteriorate as new data are added outside the estimation period. This result is not very surprising and reflects a number of problems, including: simultaneity, the dangers of over-fitting, changes in policy regimes, and other structural shifts. Of particular relevance in the current context was the announcement of explicit inflation targets in Canada in February 1991. This marked a change in regime that should affect the way agents are forming their expectations, and therefore, the behaviour of interest rates and exchange rates.

In constructing QPM, the Bank staff broke from the past practice of building a model by using single-equation econometric techniques. QPM is not an estimated model; it is calibrated. Calibration has the important advantage that it allows the model builder to put more weight on features of the data that are thought to reflect "deep" structure, while putting relatively less weight on historical correlations that have more to do with shocks over history and the policy regime in place at the time. As monetary policy has increasingly emphasised medium to long-run objectives, the importance of a theoretically consistent tool for policy analysis has been reinforced.

The modelling of interest rates and the exchange rate in QPM reflects the model's overall objective - to meld the rigorous theoretical structure necessary for modern policy analysis with the practical requirements of a model designed to support economic projections. Interest rate and exchange rate determination in the model combines the essential elements of mainstream economic theory with a healthy respect for the short-run features of the data. The result is a model with a well-defined long-run equilibrium that produces dynamics that both converge on the long-run solution and replicate the stylised facts as captured by the short-run correlation structure of the data. Thus, although it is not estimated, we nevertheless consider QPM to be very much an empirical macro model.

In outlining how interest rates and the exchange rate are modelled in QPM, we begin in Section 2 with a brief overview of the model. This is followed in Section 3 by a description of the interest rate sector. Section 4 discusses the real and nominal exchange rates. With this as background, Section 5 illustrates the behaviour of interest rates and exchange rates under two policy shocks: a monetary disinflation shock and a fiscal shock that permanently raises the level of government indebtedness. We conclude by discussing some extensions to the base case model that would incorporate a richer description of interest rate and exchange rate behaviour.
1. Overview of QPM

In comparison to most other models used for similar purposes, QPM is relatively small. This reflects a conscious decision to abstract from the micro-sectoral details of the Canadian economy in order to focus on the core macro linkages in a theoretically consistent framework that takes full account of long-run budget constraints.

At the heart of the QPM is a steady-state model (see Black, Laxton, Rose and Tetlow 1994). The steady-state model describes the determinants of the long-term choices made by profit-maximising firms and overlapping generations of consumers, given the policy settings of the fiscal and monetary authorities, all in the context of an open economy with important relationships with the rest of the world. The economic behaviour of these agents, given their long-run budget constraints, and the market-clearing conditions of an open economy determine the long-run equilibrium or steady state to which the dynamic model converges.

The dynamic model has several important features\(^1\). First, agents in QPM are forward-looking. In particular, they act based on intertemporal optimisation, conditioned by expectations that are forward-looking, albeit not fully model-consistent. The evolution of expectations plays a key role in the overall dynamic response to shocks. In addition, adjustment of both quantities and prices is presumed to be costly, so there are also "intrinsic" elements to the model's dynamic properties. These include labour market contracts, the fixed costs associated with investment, and so on.

Second, the model provides a complete and consistent solution for all stocks and flows. When a shock affects the level of a stock, this often creates the necessity for cycles in flow variables, which can be an important contributor to overall dynamics.

Third, monetary policy is conducted using a forward-looking policy rule that calls for the monetary authority to adjust its policy instrument in such a way as to bring expectations into line with the targeted inflation rate. The instrument of monetary policy in QPM is the short-term interest rate, which has its influence on spending through the slope of the yield curve. Movements in the short-term nominal interest rate also affect the nominal exchange rate, and hence import prices and inflation, through an uncovered interest parity condition. Inflation is influenced directly by the state of excess demand and by expectations about future inflation.

Finally, fiscal policy in QPM, like monetary policy, is characterised by a set of objectives that are consistent with achieving a sustainable equilibrium. In particular, the fiscal authority picks a target level of government expenditures on goods and services as a proportion of output, and a target debt-to-GDP ratio. Taxes net of transfers and the deficit adjust to achieve these targets.

With this overview as background, we turn now to the main focus of this paper - modelling interest rates and the exchange rate.

2. Interest rates

2.1 The yield curve and monetary stance

Canada is a relatively small economy with highly internationally integrated financial markets. As a result, over the longer term, real interest rates in Canada are largely determined in world markets. However, over the shorter term, domestic monetary policy exerts an important influence on real interest rates. Monetary actions affect short-term interest rates most directly, and these effects

\(^1\) For an overview of the QPM system, see Poloz, Rose and Tetlow (1994). See also Hunt, O'Reilly and Tetlow (1995) for a discussion of the model's simulation properties.
reverberate up the term structure and over to the exchange rate, all of which impact on aggregate spending and ultimately inflation.

The determination of interest rates in QPM reflects this characterisation of the transmission mechanism. Real interest rates in QPM are pinned to world real rates in the long run up to an exogenously specified risk premium. In the short run, monetary actions can affect real rates because prices are slow to adjust. The instrument of monetary policy in the model is the short-term nominal interest rate, and monetary actions are transmitted to real activity through the impact of changes in the short rate on the slope of the yield curve. Formally, the link between the yield curve and real activity in the model arises because consumer expenditures, housing and inventories (which are aggregated together) are a function of the yield spread - the short-term interest rate less the long rate.

The use of the yield spread as the key variable through which monetary policy is transmitted to real activity in QPM reflects two main considerations. First, it reflects the view that the yield spread provides a better indicator of the stance of monetary policy relative to the underlying momentum in the economy than do short-term real interest rates alone. Second, the use of the yield spread provides a parsimonious way of capturing the effects of the full term structure of interest rates on aggregate spending.

QPM is used for economic projections, and an important challenge in the projection exercise is to interpret the underlying shocks in the economy that are producing the incoming data. In this context, the yield spread has the attractive feature that it helps in isolating monetary influences on real interest rates. Movements in both long and short rates reflect fluctuations in the equilibrium or natural real interest rate (as determined by productivity and thrift in the world economy). Changes in the short rate also reflect changes in the stance of monetary policy, while long rates are relatively immune to changes in monetary conditions; thus to a large extent the yield spread serves to isolate the monetary component of changes in real interest rates.

For the monetary authority, there is useful information in long rates on the credibility of monetary policy which can serve as a useful guide to the changes in short-term interest rates that are required to control inflation. In the typical interest rate cycle, the long rate will initially rise with short rates when the central bank tightens monetary conditions to combat inflation, since, initially, credibility will tend to be low. As the central bank reveals its determination to reverse the rise in inflation, the long rate may begin to fall. This serves as a signal to the monetary authority that it can ease off a little on short rates. Measuring monetary stance in terms of the spread is thus a convenient way to summarise this relationship between policy actions and their credibility.

The yield spread also captures an intertemporal aspect of consumers' expenditure decisions. In particular, the spread provides information on the expected path of interest rates and this may influence the timing of expenditures and thus the dynamics of aggregate demand. For example, a consumer who is considering purchasing a car or a house may be enticed to do so sooner as opposed to later if the long rate is considerably above the short rate indicating that short rates are expected to rise in the future. Conversely, faced with an inverted yield curve, the consumer is likely to postpone major expenditures on the expectation that the cost of financing is going to fall.

2.2 Modelling short and long rates

The interest rate sector in QPM comprises three main equations: a monetary policy rule, an equation for the representative long-term interest rate (10-year and over government of Canada bond rate) and an identity that describes the yield spread. One can think of the latter as solving for the short-term interest rate (the 90-day commercial paper rate), although in actual simulations, the yield curve, long and short-term interest rates are all determined simultaneously.

The approach that was used to calibrate the interest rate sector provides a good illustration of the general principles that were followed in the construction of QPM. In particular, it
shows how one can combine traditional empirical methods with the prediction of theoretical models to seek to exploit the advantages of both approaches. This flexible approach has a strong appeal when building a model that is designed as both a projection and a research tool.

2.2.1 A forward-looking policy rule

In a forward-looking model, the role of the monetary authority is to provide a nominal anchor for expectations. Because inflation expectations depend, at least in part, on future monetary policy, a policy rule needs to be specified in terms of an attainable objective. Without an endogenous policy response to economic developments, agents do not have enough information to form their expectations, and nominal values become undefined (in other words, the model does not solve). An endogenous policy rule or reaction function is therefore an essential part of QPM. The rule specifies a path for the monetary policy variable in order to achieve an intermediate or final policy objective.

As discussed above, in QPM, the policy variable that is used is the yield spread. More specifically, the reaction function determines a path for the yield spread gap. The yield spread is defined as the difference between three-month and ten-year interest rates (50 basis points in steady state), while the yield spread gap is the difference between the actual yield spread and its equilibrium value. The reaction is specified in terms of the ultimate policy objective, that is to control inflation at some target level. In the simulations presented in the following sections, the target is assumed to be 2 per cent, the mid-point of the current official inflation target bands in Canada.

Because it takes time for monetary policy actions to have their effect on aggregate demand and inflation, monetary authorities are forced to look ahead when setting a path for their instrument. In the model, this is achieved with an explicit forward-looking policy rule. The policy instrument depends on the model’s predictions of inflation in future periods. The base case reaction function used has the following form:

\[
yieldgap_t = \alpha_1 \left( \sum_{k=6}^{2} \left( \frac{1}{2} \left( \hat{P}_{t+k} - \hat{P}_{t+k}^{\text{targ}} \right) \right) \right) + \alpha_2 \yieldgap_{t-1}
\]

where \( \yieldgap_t = \left( i_t^s - i_t^l \right) - \left( i_t^s - i_t^l \right)^{ss} \) is the deviation of inflation from its targeted rate, \( i_t^s \) and \( i_t^l \) are the short and long-term interest rates respectively, and \( ss \) denotes a steady-state value. As shown, the yield spread gap is a function of the deviation of inflation (based on the CPI excluding food and energy) from its target six to seven quarters in the future. The reaction function also includes a lagged dependent variable to smooth the movement of the policy instrument. If a shock tends to push inflation above (below) its targeted level in six to seven quarters, the authority increases (decreases) its instrument, the 90-day commercial paper rate, so as to achieve a level for the yield spread which will result in aggregate demand conditions that will bring inflation back towards its target.

Although this reaction function is an ad hoc rule, in the sense that it is not derived from an optimal control problem, the choice of parameters and the degree of forward-lookingness were not chosen arbitrarily. The six-to-seven quarters horizon is a good approximation of the sort of horizon over which monetary policy has a meaningful effect on trend inflation. Trying to hit an inflation target over a very short period of time would imply considerable volatility in interest rates (and the exchange rate), leading to an instrument instability problem. Even though the reaction function does not allow for secondary objectives other than smoothing of the policy instrument, the magnitude of the parameter, which is linked to the degree of inflation drift that the authorities are ready to accept, was also chosen by taking into consideration that the authorities may not be completely indifferent to the path of other macroeconomic variables.

The use of a forward-looking rule implies that the monetary authority has knowledge of the origin and nature of the shocks. In a model in which private agents are assumed to be (at least
partly) forward-looking, it would be hard to argue that the authorities should not be characterised by
the same behaviour. Because the world is plagued with uncertainty, it may nevertheless be very
difficult for the authorities to extract information from volatile economic data. To take this into
account, one can easily entertain shocks in QPM from which a more muted policy response is
assumed. The important point to stress is that the model response to any shock depends importantly
on the specification of the policy rule. In other words, in QPM, the policy regime matters.

2.2.2 Long-term interest rate

Given that the yield spread is the main monetary variable in QPM, the determination of
the long-term interest rate plays a key role in the transmission mechanism. In previous models used at
the Bank, the long-term interest rate was determined by a distributed lag of Canadian short-term rates
and US long and short-term interest rates. This equation fit the historical data very well, as Canadian
and US long-term interest rates have been strongly correlated over the post-war period. However,
there is a presumption that this strong correlation reflects the fact that the two countries have generally
faced similar shocks and have responded to them in similar ways. If one wants to consider scenarios
with different inflation paths in the two countries, more structure needs to be added to the model.

The standard theory used in most policy simulation models is the expectations theory of
the term structure, according to which the yield on the long-term bond should equal a weighted
average of the current and expected future short-term interest rates, up to a term premium. However,
when combined with pure model-consistent expectations, this theory is unable to replicate the
historical behaviour of longer-term interest rates. The existence of time-varying term premia, the
unpredictability of short-term rates, and the lack of credibility of macroeconomic policies are among
the reasons that have been offered for explaining this apparent failure. Whatever the source of the
failure, it would not seem appropriate to rely exclusively on this theory in a model designed in part for
forecasting purposes.

For this reason, the QPM equation is constructed as a combination of both the
expectations model and a reduced-form model, as follows:

\[ i_t^l = \beta_1 \left\{ \sum_{k=0}^{39} \gamma^k i_{t+k} + (1 - \theta) i_t^s + \text{term premium} + (1 - \beta_1) \gamma_l \left[ (\hat{p}_{US} - \hat{p}_{US}^e + \text{risk}_l) + (1 - \gamma_1) i_t^s \right] \right\} \]

where \( \hat{p}_{US}^e \) denotes the expected rate of inflation (based on the GDP deflator). The first
part represents the expectations theory. As in other sectors of the model, expectations are expressed as
a weighted combination of the model-consistent and extrapolative solutions. However, in this case,
the extrapolative solution does not contain any lags. Moreover, it is represented by the
contemporaneous short-term rate only, based on the view that financial markets respond quickly to
new information. The second part is built from the traditional estimated equation. The Canadian rate
is a function of the US long-term rate adjusted by the inflation expectations differential between the
two countries and a risk premium, \( \text{risk}_l \). The contemporaneous short-term rate is included with a
fairly large coefficient in order to mimic the historical sensitivity of long-term rates to movements in
short-term rates.

The weight currently assigned to the reduced-form portion is, at 75 per cent, quite high.
It could be reduced if, over time, more support develops for the expectations model, for instance, as a
result of the adoption of clear policy targets by the authorities. The country risk premium and the term
premium are currently exogenous. In steady-state, the long-term interest rate simply equals the short-
term interest rate plus the term premium.
3. Real and nominal exchange rates

In addition to affecting interest rates, monetary policy in QPM influences the exchange rate with important effects on trade. The exchange rate also responds to external shocks, such as changes in world commodity prices, providing an important shock absorber through which the Canadian economy digests changes in external conditions. Over the longer term, the real exchange rate is the key relative price in the model that re-equilibrates the economy. Since real domestic interest rates are pinned to world rates in the long run, it is the real exchange rate that must ultimately adjust to bring aggregate demand in line with aggregate supply.

As a key relative price in the model, the real exchange rate is one of the "most endogenous" variables in the system in the sense that its determination reflects the simultaneous solution of all the essential elements of the model - monetary conditions, real allocations, prices and inflation, and international arbitrage. As a result, there is no single equation or even a small group of equations in the model that can be meaningfully described as determining the exchange rate. Having said this, there are some key building blocks in the determination of the exchange rate in QPM.

In the short run, the two key relationships influencing the nominal and real exchange rates are an interest parity condition and aggregate price adjustment. The interest parity condition requires investors in Canadian dollar assets to be compensated for expected changes in the value of the Canadian dollar:

\[ i_t = i^*_t + (s_t - s^*) + \text{risk} \]  

(3)

where \((s_t - s^*)\) is the expected change in the nominal exchange rate (at annual rates), and \(\text{risk}\) is an exogenous country risk premium (set to 40 basis points in the steady state). The exchange rate is defined as the Canadian dollar price of foreign exchange, where foreign exchange is the trade-weighted basket of currencies of the rest of the G-7 countries (hereafter we will call this the G-6). Since 80 per cent of Canada's trade is with the United States, \(s_t\) is typically quite similar to the Canada-US exchange rate. The real exchange rate is defined as the nominal rate adjusted for relative prices:

\[ e_t = \frac{s_t P_t^*}{P_t} \]  

(4)

where \(e_t\) is the real exchange rate, and \(P_t\) and \(P_t^*\) are the domestic and foreign price levels respectively (measured by the GDP deflator). Parallel to the definition of the real exchange rate, the foreign price level is the trade-weighted price level in the G-6. Since the foreign price level is taken to be exogenous, the link between real and nominal exchange rates depends on the behaviour of domestic prices.

An important feature of exchange rate data is that nominal and real exchange rates tend to move together (see Mussa, 1986). This is captured in the model by sluggish adjustment of the aggregate price level. For example, a rise in domestic interest rates will result in an appreciation of the Canadian dollar vis-à-vis the G-6 currencies that is large enough to generate an expected depreciation in the future so as to satisfy the interest parity arbitrage condition (3). Since domestic price adjustments are gradual, due both to sluggish adjustment of expectations and to rigidities such as nominal contracts, the nominal exchange rate appreciation also results in a real appreciation from (4).

In the short run, the real exchange rate is, therefore, largely determined by the behaviour of the nominal rate together with the pace of price adjustments. Looking beyond the short run when price adjustments have caught up with nominal exchange rate changes, the real exchange rate adjusts so that the trade flows in the model will sustain the real equilibrium. More specifically, in the long run the real exchange rate adjusts to produce the trade balance surplus that is required to sustain the desired level of net foreign assets.
In the model, consumers hold three types of assets: the national capital stock, the consolidated federal, provincial and municipal government debt, and net foreign assets. The optimal level of physical capital is chosen by firms, and is essentially determined by the world real interest rate and the rate of labour-embodied technological progress. The level of government debt relative to GDP is chosen by the government, so this leaves net foreign assets as the residual component of non-human wealth through which consumers can adjust the level of wealth to be consistent with their desired flow of consumption expenditures. Overlapping generations of consumers accumulate non-human wealth so as to maximise the discounted present value of the utility of consumption over their expected lifetimes.

A permanent supply shock will, in general, alter the consumers optimal level of wealth and consumption, and this in turn will show up as a change in the level of net foreign assets. To sustain this level of net foreign assets, the trade balance of the economy must be consistent with the flows of interest payments on the outstanding stock of net foreign assets. This is achieved by the adjustment of the real exchange rate. For example, since Canada is assumed to face a downward sloping world demand curve for its exports, a real depreciation will raise exports, thereby improving the trade balance.

4. Two policy shocks

Perhaps the easiest way to understand how interest rates and the exchange rate are determined in QPM is to see the model in action. Below we consider two different policy shocks: a monetary disinflation shock, and a fiscal shock that permanently raises the level of government indebtedness. The disinflation shock highlights the transmission of monetary actions from interest rates to the exchange rate, and on to real behaviour and inflation. Since there is no long-run trade-off in the model between inflation and output, the disinflation shock does not influence the real economy in the long run.

4.1 Disinflation shock

4.1.1 Base QPM

Figure 1 shows the results of a permanent one percentage point reduction in the rate of inflation targeted by the monetary authority. The solid lines show the response for the base calibration of QPM.

In order to reduce inflation, the authorities tighten monetary conditions. Short-term interest rates increase by almost 100 basis points on average during the first year. Long-term rates, however, increase only marginally in the first two quarters and start falling below control by year-end, as expectations of lower inflation develop rapidly. The resulting rise in the yield spread reduces consumption, and investment spending also falls due to the decline in expected output and the rising cost of capital.

The rise in short rates also results very quickly in an appreciation of the real value of the Canadian dollar which peaks at 0.9 per cent above its starting point. This rise in the value of the Canadian dollar leads to a marked decline in exports. In the very short run there is also a small rise in imports, so the trade balance deteriorates initially, thereby contributing to the emergence of excess supply. Beyond the very short run, imports also decline sharply despite the exchange rate appreciation, as consumption falls off in response to higher interest rates and declining employment and personal incomes. As a result, consistent with the stylised facts, the trade balance turns positive as the downturn gains momentum, and this contributes to the more rapid recovery of aggregate demand (as measured by the output gap) as compared to consumption.

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Figure 1
Permanent 1 percentage point reduction in inflation target
Time in years

---

**Short-term interest rate**
(% pt. deviations from control)

---

**Long-term interest rate**
(% pt. deviations from control)

---

**Yield curve gap**
(level)

---

**Real exchange rate**
(% deviations from control)
+ = depreciation

---

**Output**
(% deviations from control)

---

**Employment**
(% deviations from control)

---
Figure 1 (cont.)
Permanent 1 percentage point reduction in inflation target
Time in years

- Consumption
  (% deviations from control)

- Exports
  (% deviations from control)

- Imports
  (% deviations from control)

- Trade balance
  (deviation from control)

- Core inflation
  (level)

- Nominal exchange rate
  (% deviations from control)
  + = depreciation
The maximum effect on aggregate demand is felt in the third year, when the output gap averages 1.1 per cent. It takes almost five years for inflation to reach its new 1 per cent target level. Note that by then, interest rates have overshot their long-run equilibrium level. This is required to curtail the building of disinflationary momentum. The total output foregone in reducing inflation by 1 percentage point from steady state is about 3.0 per cent of one year's output.

As the economy settles into long-run equilibrium, all real variables in the model return to their initial steady-state levels, while inflation remains permanently lower. Since foreign inflation is unchanged this results in an on-going appreciation of the nominal exchange rate at a rate of 1.0 per cent per year, while the real exchange rate returns to its initial steady-state level.

4.1.2 More forward-looking behaviour

The base version of QPM puts a considerable weight on the backward-looking component in price and wage expectations. If one were to assume, instead, that expectations are to a large extent forward-looking, the costs of disinflating would appear extremely low, as can be seen by the dashed lines in Figure 1. In this alternative scenario, we increased the weight on the forward-looking component from 30 per cent to 70 per cent in the price equations and from 10 per cent to 50 per cent in the marginal cost and nominal wage equations.

Long-term rates fall significantly (60 basis points) on impact. This allows short-term rates to fall as well, but to a lesser extent, such that the yield spread tightens by about 30 basis points on average during the first year of the simulation. This small tightening is sufficient to bring inflation down to 1 per cent after three years. The cumulative output loss in this scenario amounts to only 0.6 per cent of one year's output. This small output loss reflects both the considerably smaller decline in consumption together with the rise in exports. The latter reflects the fact that the fall in short rates produces a small depreciation of the real and nominal exchange rates.

It is hard to believe that the monetary authority could, in fact, engineer a reduction in inflation without having to raise short-term interest rates or the value of the Canadian currency. The above results reflect the fact that in a deterministic model, forward-looking expectations become equivalent to perfect foresight. Assigning a significant weight to the extrapolative solution provides a source of propagation and allows one to produce a dynamic behaviour for the economy that seems to replicate the properties of the data fairly well. Although one might argue that agents are sophisticated enough and that their behaviour should not be represented by naive autoregressive assumptions, relying only on model-consistent expectations in a model without costs of uncertainty would produce results that are not judged to be reasonable.

4.1.3 Less credibility for lower inflation

With regards to long-term interest rates, one might in fact argue that the speed at which expectations are revised is too rapid in the core version of the model. The experience of the last few years would seem to suggest that it takes a long time for financial markets to revise their expectations for long-term inflation. Agents may have faith in the ability of the central bank to achieve its current inflation targets but, nevertheless, assign a certain probability to an outcome where the authorities will have to revert back to a high inflation regime, for instance to mitigate fiscal problems.

To illustrate how the expectations behaviour in the bond market could affect the outcome of a monetary shock, we modified the long-term interest rate equation such that the expected inflation differential does not play any role during the first five years of the simulation. This is equivalent to assuming that inflation expectations in Canada do not diverge from those in the United States over this period. Thereafter, the expected inflation differential term is phased-in rapidly, over a period of five quarters.
Figure 2
Permanent 1 percentage point reduction in inflation target
Time in years

- Short-term interest rate (% pt. deviations from control)
- Long-term interest rate (% pt. deviations from control)
- Yield curve gap (level)
- Real exchange rate (% deviations from control)
- Output (% deviations from control)
- Consumption (% deviations from control)
The results of the disinflation shock under this assumption for the long-term interest rate behaviour are shown by the dashed lines in Figure 2 - the solid line continues to be the base QPM response. Initially, the results are very similar to what was obtained with the base version of the model, since the latter places a higher weight on the autoregressive solution in the price expectations equation. By the third year of the simulation, long-term rates are about 50 basis points higher relative to the base-case scenario, and this difference persists for about three years. The cost of servicing the external debt is therefore higher and the negative effect on the current account balance and net foreign asset position is larger than in the base case. As a result, consumption remains below control for longer despite the bigger easing in monetary conditions. There is also more cycling in real output and inflation.

4.2 A government debt shock

The disinflation shock has no long-run effects on real variables in the model. By contrast, since QPM is non-Ricardian, a change in the level of government debt (relative to GDP) does alter the economy's long-run equilibrium and result in permanent changes in consumption and the real exchange rate. The government debt shock also illustrates the short-run implications for interest rates when fiscal policy alters the level of aggregate demand and monetary policy must respond to achieve the inflation target.
Figure 3
Dynamic effects of a debt increase starting from the steady state
Time in years

Debt-to-GDP ratio
(level)

Direct tax rate
(level)

Deficit-to-GDP ratio
(level)

Consumption
(% deviations from control)

Trade
(% deviations from control)

Foreign liabilities-to-GDP ratio
(level)
Figure 3 (cont.)
Dynamic effects of a debt increase starting from the steady state
Time in years

Excess Demand
(level)

Employment
(% deviations from control)

Short-term interest rate
(level)

Long-term interest rate
(level)

Real exchange rate
(% deviations from control)

Core inflation
(level)
In QPM, government debt has real effects primarily for two related reasons. First, economic growth is fuelled by the birth (or immigration) of new consumers. Current consumers, therefore, act knowing that they will not be responsible for the full tax burden of servicing the debt, since some portion of this burden will automatically be assumed by future generations. Consumers also act knowing that they are mortal and may, therefore, not be around to pay even a reduced share of future taxes associated with current deficits. Thus, changes in government debt levels alter the real choices of households. Second, in the context of an open economy, there are consequences for net indebtedness to foreigners, which have an impact on the real exchange rate and this in turn feeds back to the level of output.

Figure 3 depicts the dynamic effects of a rise in the debt-to-GDP ratio from 50 to 70 per cent that is brought about by a cut in direct taxes. As shown, the tax cut is temporary, since in the new steady state with more government debt, taxes will be higher to support the higher debt service. However, current generations expect that some of the burden of higher future taxes will fall on future generations. As a result, for current generations the present value of the rise in disposable income during the temporary period with lower taxes is greater than the present value of the fall in disposable income thereafter. Households, therefore, increase their consumption in the short run rather than saving all the additional disposable income they receive. This rise in consumption increases imports and reduces exports as more output is absorbed domestically. The trade balance, therefore, deteriorates, and since Canada is a net international debtor, this results in a rise in net foreign liabilities. With more foreign liabilities, the steady-state trade surplus must be larger to cover the additional interest payments on the foreign debt. This requires a larger proportion of output to go to foreigners, leaving less for domestic consumption. Thus, while consumption rises in the short run, the impact of the higher level of foreign indebtedness is a permanently lower level of consumption thereafter.

### Table 1

#### Steady-state effects of a rise in government debt

<table>
<thead>
<tr>
<th></th>
<th>Base QPM</th>
<th>MRT extension</th>
<th>Country risk premium extension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shock to control (% change )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>-0.29</td>
<td>-0.38</td>
<td>-0.48</td>
</tr>
<tr>
<td>Consumption</td>
<td>-1.54</td>
<td>-2.01</td>
<td>-0.99</td>
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<tr>
<td>Exports</td>
<td>0.78</td>
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<td>0.08</td>
</tr>
<tr>
<td>Imports</td>
<td>-1.26</td>
<td>-1.65</td>
<td>-0.83</td>
</tr>
<tr>
<td>Capital stock</td>
<td>-0.84</td>
<td>-1.09</td>
<td>-1.39</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.90</td>
<td>1.17</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Level as a percentage of GDP</strong></td>
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<td></td>
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<tr>
<td>Government debt [50.00]</td>
<td>70.00</td>
<td>70.00</td>
<td>70.00</td>
</tr>
<tr>
<td>Net foreign liabilities [40.00]</td>
<td>61.11</td>
<td>61.41</td>
<td>47.30</td>
</tr>
<tr>
<td>Government deficit [2.21]</td>
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<td>3.10</td>
<td>3.10</td>
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<tr>
<td>Current account [-1.77]</td>
<td>-2.70</td>
<td>-2.72</td>
<td>-2.09</td>
</tr>
<tr>
<td>Interest payments to foreigners [2.67]</td>
<td>4.06</td>
<td>4.32</td>
<td>3.27</td>
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<tr>
<td>Taxes net of transfers [6.12]</td>
<td>6.84</td>
<td>7.13</td>
<td>7.05</td>
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<tr>
<td><strong>Changes in rates in basis points</strong></td>
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<tr>
<td>Risk premium</td>
<td>0</td>
<td>34&lt;sup&gt;2&lt;/sup&gt;</td>
<td>21&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>User cost of capital</td>
<td>17</td>
<td>22</td>
<td>28</td>
</tr>
</tbody>
</table>

<sup>1</sup> The figures in square brackets are the initial steady-state levels.

<sup>2</sup> This risk premium is applied only to the rate on government debt and net foreign liabilities.

<sup>3</sup> This risk premium is applied to the interest rates on government debt, net foreign assets and both long and short-term private domestic borrowing rates.
The short-run behaviour of interest rates and the exchange rate largely reflects the temporary rise in aggregate demand associated with the fiscal expansion together with the monetary reaction to this fiscal shock. Since the simulation starts from conditions of full economic capacity, the stimulus from fiscal policy pushes up inflation, which is inconsistent with the monetary objective of maintaining inflation at 2 per cent. The monetary authority, therefore, raises short rates, and via the interest parity condition, this leads to an appreciation of the Canadian dollar in the short run. This monetary reaction serves to dampen the consumption boom but, given the lags associated with the effects of monetary policy, inflation nevertheless rises to a peak of 4 per cent before turning the corner.

As the inflationary pressures abate in the medium term, short rates decline, eventually returning to their initial steady-state level. With the decline in interest rates, the real exchange rate depreciates but, unlike the real long and short-term interest rates, it does not simply return to its initial steady-state level. Rather in the long run, the real exchange rate must depreciate in order to stimulate exports and reduce imports enough to generate the larger trade balance surplus that is required to sustain the higher level of net foreign liabilities. The resulting real depreciation raises the cost of capital in Canada, since about 70 per cent of our machinery and equipment is imported. The impact of the long-run real depreciation is, therefore, to lower the steady-state capital stock, and thus output.

These long-run effects of the debt shock are summarised in the first column of results in Table 1. As shown, the real exchange rate depreciation is 0.9 per cent, which results in a relatively small decline in output of 0.3 per cent. With both lower output and higher foreign liabilities, consumption declines by 1.5 per cent.

5. Extending the base model: endogenous risk premiums

QPM has evolved since it went into regular use in August 1993, and it will continue to evolve as we learn how to improve it or add new features that might be particularly important for some issues. Looking ahead, one area that warrants more attention and is particularly relevant to interest rates and the exchange rate is the determination of the risk premiums in the model. In the base model, the risk premiums on different assets within Canada as well as the country risk premium for Canadian dollar-denominated assets are taken to be exogenous. However, the recent experience of Canada, as well as some other industrialised countries with high and rising levels of government debt, suggests that risk premiums can change substantially. By deferring taxes into the future, government deficits create uncertainty about how the government will ultimately deal with its obligations, the price of this uncertainty is a premium.

In a recent study, Macklem, Rose and Tetlow (1995) examine the implications of rising levels of government indebtedness using an extended version of QPM that considers one aspect of this risk premium issue. Based on the evidence reported by Alesina et al. (1993) for 12 OECD countries, they incorporate an endogenous risk premium in QPM which is applied to interest rates on government debt and net foreign assets. They find that while the risk premium itself is relatively modest, endogenising the risk premium has the effect of magnifying the long-run effects of higher government debts. The second column of results in Table 1 incorporates the risk-premium effect considered by Macklem, Rose and Tetlow (MRT) and illustrates their finding in the context of the debt shock considered above. Note, in particular, that the addition of the risk-premium effect increases the long-run decline in consumption from 1.5 per cent in the base model to 2.0 per cent. In addition, the real depreciation associated with the rise in debt is also slightly larger - 1.2 per cent in the extended model as compared to 0.9 in the base model.

In MRT's analysis the effects of government indebtedness are confined to interest rates on government debt and foreign borrowing - private domestic borrowing rates are not affected by the levels of government debt. Casual observation suggests, however, that larger debts, by increasing
aggregate uncertainty, spill over to private borrowing rates and the exchange rate, although such effects have proven difficult to isolate with any precision.

In a preliminary effort to capture this type of effect in QPM, we extended the base model by specifying the country risk premium as a positive function of the government budget deficit and the current account deficit based on the pooled-time-series evidence for 17 OECD countries reported in Orr, Edey and Kennedy (1995). More specifically, the country risk premium in the model (which appears in the interest parity equation (3) among others) is assumed to increase by 17 basis points per percentage point increase in the government deficit-to-GDP ratio, and another 17 basis points per percentage point increase in the current account-to-GDP ratio. With this characterisation of the risk premium, all rates on Canadian-dollar assets are affected by changes in the level of government indebtedness, both directly due to the link between the deficit and the debt, and indirectly through the effects of government debt on net foreign liabilities and thus the current account. The final column of Table 1 reconsiders the effects of raising the level of government debt from 50 to 70 percent of GDP with this country-risk-premium effect.

Interestingly, this extension to the base model produces somewhat counterintuitive results. In broad terms, the effects of raising the level of government debt relative to GDP are similar to those discussed above, but surprisingly the long-run fall in consumption, as well as the real exchange rate depreciation, are now both smaller than in the base model with exogenous risk premiums. In particular, consumption now falls only 1 per cent as compared to a drop of 1.5 per cent in base QPM, and the real depreciation is 0.3 per cent as compared to 0.9 per cent in base QPM.

The reason is that endogenising the country risk premium in this way imposes a type of market discipline on behaviour, and the endogenous response of consumers to this discipline results in a better long-run equilibrium. The government debt shock raises the country risk premium and thus real interest rates (by 21 basis points in the steady state). Consumers, realising that higher debt levels mean higher interest rates, decide to consume less and save more, other things equal. As a result, whereas net foreign liabilities rise almost one-for-one with government debt in the base model, they now rise by considerably less than one-for-one - in the base model net foreign liabilities rise from 40 to 61 per cent of GDP, as compared to an increase from 40 to only 47 per cent of GDP in the extended model. With a smaller rise in net foreign liabilities, the long-run decline in consumption is smaller, as is the real depreciation that is required to produce the trade balance surplus that is consistent with the new level of interest payments to foreigners.

Note also that, as expected, adding the country risk premium channel to the model does increase the steady-state decline in output associated with the rise in government debt - from 0.3 per cent in the base model to 0.5 per cent with the country risk premium. This reflects the fact that the increase in the country risk premium feeds into the cost of capital, therefore reducing the optimal capital stock, and thus output. However, the additional increase in the cost of capital as a result of the country risk premium is quite small (only 11 basis points). This is because part of this effect is offset by the smaller rise in the price of imported capital that is associated with the smaller real exchange rate depreciation.

More generally, this experiment points out the discipline of a general equilibrium model. Since economic agents respond to incentives, raising interest rates increases saving. Moreover, for the current calibration of the model, this "savings effect" outweighs the "cost-of-capital effect". Output is, therefore, lower, but consumption is higher (relative to the exogenous risk premium case) because saving is higher. Whether, in practice, this saving effect is, in fact, bigger than the cost-of-capital effect is an empirical question. Casual empiricism suggests, however, that changes in risk premiums magnify the effects of government indebtedness, at least for some time, before inducing a response in behaviour that will mitigate the costs of larger government debts. In future work we plan to explore further how best to model risk premiums.
Conclusion

To conclude, the determination of interest rates and exchange rates in QPM combines several relatively simple concepts - international arbitrage, expectations theory, and sluggish price adjustments - together with some respect for the empirical evidence when implementing these features in the model. Our experience with QPM suggests that these relatively simple ingredients, when combined with forward-looking behaviour, complete stock-flow consistency, and endogenous reaction functions for both monetary and fiscal policy, can produce very rich dynamics for interest rates and the exchange rate. The main message that comes out of the model is that interest rate and exchange rate changes reflect the interplay between aggregate demand and supply, and the monetary and fiscal reactions to the implied real and nominal outcomes.

Like any model, QPM makes some important simplifications. Looking ahead, on-going work with the model will continue to explore areas in which the model may be usefully enriched. At the same time, the desire to improve different parts of the model must be balanced against the need to retain the usefulness of the model as a tool for economic projections and policy analysis. This objective argues for simplicity and transparency.
References


Introduction

This excellent paper describes the financial sector of the Bank of Canada's model and explains the role of the model in the formulation of monetary policy. Essentially, the model is used to compute paths for the yield curve and short-term interest rates, which the Bank can influence, that would allow the Bank to hit its inflation target.

These comments will focus on the specification of the monetary policy reaction function in the model and some of the possible limitations inherent in using such a model for policy formulation. Also, some proposals for future work that address these limitations will be provided. First, however, it is useful to review the specification of the financial sector.

1. The financial sector and the specification of the monetary policy reaction function

This sector contains three basic equations: (1) An open interest parity equation; (2) an equation based on the expectations theory of the yield curve augmented to include the U.S. long-term interest rate and the Canadian-U.S. inflation differential—variables that have been found to be empirically important determinants of Canadian long-term interest rates; (3) a monetary policy reaction function specifying an inflation rate of 2 percent as the inflation target for monetary policy. In this reaction function, the yield curve, the spread between the 3-month rate and the long-term rate, serves as the operational target for monetary policy. This operational target is influenced by the central bank through adjustments in the short-term interest rate.

This last equation is an essential feature of any model with forward looking expectations since financial markets react to anticipated future monetary policy. Without a monetary policy reaction function, nominal magnitudes would not be defined and the model could not be solved.

An appealing feature of this specification is that the yield curve plays a central role, performing several functions simultaneously: (1) it is an important link in the transmission of monetary policy changes to the real economy; (2) it serves an indicator of monetary stance; (3) it is the operational target for monetary policy in the monetary policy reaction function.

2. Relevance of the reaction function in the model to monetary policy in Canada

From the modelling point of view, use of a reaction function in which the yield curve is the operational target is attractive. However, this reaction function differs from that actually used by the Bank of Canada. As is well known, the Bank uses the monetary conditions index - which combines the short-term rate (the call money rate) and the exchange rate into an index - as its operational target and principle indicator of monetary stance.

The principle limitation of the yield curve-based reaction function is that in practice the yield curve is unlikely to be a completely reliable indicator of monetary stance, a consideration that limits its usefulness as an operational target. The reason is that it is not possible to know whether shifts in the yield curve due to changes in long-term interest rates are the result of changes in expected inflation, the real interest rate, or the risk premium. Consequently, it is not possible to unambiguously infer the stance of policy from the yield curve. This implies that if the yield curve serves as the operational target in a monetary policy reaction function, as in the Canadian model, it could give misleading signals to policy makers. For example, suppose the risk premium dropped,
lowering long rates. If this were incorrectly interpreted as a drop in expected inflation this could lead the central bank to ease when it would not be appropriate to do so.

This limitation of the yield curve-based reaction function raises the question of whether use of this reaction function rather than one based on the MCI significantly reduces the usefulness of the model as a guide to policy? The answer depends on the policy issue that is being analysed in the model simulations. For exogenous policy shifts, such as the deterministic simulations of a reduction in the inflation target and an expansionary fiscal policy reported in the paper, the choice of reaction function is unlikely to have a significant effect. In these simulations long-term interest rates are completely endogenous and adjust to the policy change according to the dynamics of the model. The problem of interpretation noted above does not arise because movements in long-term interest rates are almost entirely due to the behaviour of expected inflation.

3. Factors limiting the use of the Bank of Canada model for policy analysis

There are, however, monetary policy issues likely to be of relevance to policy makers which would be hard to analyse using the model containing the yield curve reaction function. These arise from the fact that monetary policy operates in an environment of uncertainty where it has to respond to different stochastic shocks. These shocks can be shocks to the exchange rate or to long-term interest rates. In practice, a large proportion of the Bank of Canada's monetary policy actions probably are in response to such shocks so the capacity to simulate the effects of these shocks and the policy response would be quite useful. In particular, two problems are likely to arise if the Bank of Canada's model were used to simulate these shocks.

First, reliance on a yield curve-based reaction function could lead to an inappropriate monetary policy response in the case of a temporary shock to the term premium in long-term interest rates. Specifically, monetary policy would treat the resulting increase in long term interest rates as if it were a rise in expected inflation and tighten sharply, while the optimal response would be, at most, a slight tightening. One solution to this problem would be to make a corresponding temporary adjustment in the steady state value of the yield gap in the reaction function. However, such an adjustment would imply that the central bank knows the source of the rise in long-term interest rates which, as noted above, is unlikely to be the case in practice. It is worth noting that this problem would not arise with an MCI based reaction function.

Second, there can be significant differences in macroeconomic performance depending upon whether an MCI or yield curve reaction function is used in the case of shocks to the exchange rate (which can be interpreted as shocks to the risk premium in the open interest parity equation). Under an MCI reaction function, the central bank responds immediately to offset shocks to the exchange rate before they have much impact on the economy. In contrast, macroeconomic performance is likely to be quite different under a yield curve based reaction function. In this case, shocks are allowed to have their full impact on the economy, and the monetary policy response occurs only once the shock has its impact on the yield curve.

4. Deterministic versus stochastic simulation methods

The above argument suggested that the choice of reaction function can have a significant impact on the macroeconomic performance predicted by the model. It would be extremely useful to have some measure of the importance of this choice. A related question, likely to be of importance to monetary policy makers at the Bank of Canada, is which reaction function yields the best performance in terms of output and inflation? This question reflects the stochastic environment in which policy makers actually operate where frequent shocks to the economy make it impossible for the Bank to hit its inflation targets exactly. In this environment, it is important to know which policy reaction
function will enable the Bank to hold the inflation rate close to its target (or within the target band) at the lowest cost in terms of output variance.

These issues cannot be addressed using deterministic simulations, such as those reported in the Bank of Canada paper, which can only calculate the effects of one-time shifts in exogenous variables. However, stochastic simulation of the model can be used to compare policy rules since it is possible to calculate the variance of output and inflation under the two alternative policy rules for a given distribution for the exogenous shocks to the model. This information should make it possible to rank policy rules and calculate how much of a difference the choice of policy rule makes to macroeconomic performance.

Stochastic simulations are difficult to implement in forward looking models, largely due to the computational cost. However, they represent an obvious avenue for future research. The Bank of Canada model is already quite useful for policy analysis and formulation. However, the set of policy issues that can be analysed is limited by the deterministic simulation method used. Stochastic simulation of the model would make it possible to analyse additional issues and would represent a significant step towards a more realistic representation of the stochastic environment in which policy makers actually operate.
Determination of asset prices in the Bank of Japan Macroeconometric Model

Tsutomu Watanabe and Haruhiro Matsuura

Introduction

The purpose of this paper is twofold: (i) to describe the asset price equations in the Bank of Japan Macroeconometric Model (hereafter BOJMOD); (ii) to evaluate these equations by focusing on the role of market participants' expectations in the determination of asset prices.

The paper is organised as follows. Section 1 is an introduction to the asset price equations of the BOJMOD. It provides a brief description of the determination of long-term interest rates, foreign exchange rates and stock prices. Section 2 will study the dynamic property of the BOJMOD's asset price equations by computing the responses of asset prices to various types of shock. To overcome the Lucas-critique with regard to simulation analysis based on macroeconometric models, this section will adopt the methodology proposed by Sargent (1977), Sargent and Sims (1977) and Mishkin (1979). Section 3 evaluates the asset price determination mechanism of the BOJMOD. Our particular interest is in whether the backward-looking expectations adopted in the BOJMOD is appropriate to describe the way market participants' expectations are formed in Tokyo markets. For this purpose, we will compute the response assuming that asset pricing is based on forward-looking expectations and compare this "theoretical" response to that obtained in Section 2. The final section concludes the paper.

1. Asset price equations of the BOJMOD

1.1 Long-term interest rate

The long-term interest rate equation is based on the expectation hypothesis of the term structure of interest rates, which is represented by the second term of the equation in Table 1. The interest rate on a long-term bond equals an average of short term interest rates that market participants expect over the life of the long-term bond; i.e.,

\[ i_t = (1 - \beta) E_t \left( i_s + \beta^2 i_{s+1} + \beta^3 i_{s+2} + \beta^4 i_{s+3} + \cdots \right) \]

where \( i \) and \( i_s \) stand for long-term and short-term nominal interest rate, respectively, \( \beta \) represents the discount factor and \( E \) stands for the expectations operator.

An important issue is how expectations of future short rates are formed. In the BOJMOD, the assumption of adaptive expectations is adopted and, reflecting this, the lagged values of CD rates, denoted by RCD, are included on the right-hand side of the equation in Table 1. The parameters corresponding to them are estimated by the Shiller lag procedure.
Table 1
Long-term interest rate equation

\[
\ln(RBND_t) = \alpha_0 + \sum_{j=0}^{3} \alpha_{1j} \ln(RCD_{t-j}) + \alpha_2(GDS_t/GDPN_t) + \alpha_3 DEL(1, OCR_t) \\
+ \alpha_4 \left( \ln \left( [USBND_t + \text{SUM}(0.3, GR(4, PGDP_t) - GR(4, USP_t)/4)] \right) \right) \\
+ \alpha_5 \text{SUM}(0.2, GR(1, IBDR_t))/3
\]

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\alpha_{1j}$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>S.E.</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.019</td>
<td>0.261 (8.98)</td>
<td>0.013</td>
<td>0.001</td>
<td>0.178</td>
<td>0.061</td>
<td>0.005</td>
<td>0.84</td>
<td>0.85</td>
<td>82.3</td>
</tr>
<tr>
<td>(3.81)</td>
<td></td>
<td>(1.92)</td>
<td>(0.96)</td>
<td>(2.54)</td>
<td>(2.89)</td>
<td></td>
<td></td>
<td></td>
<td>94.1</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are t-statistics. $\alpha_{1j}$ (j=0,1,2,3) are estimated by the Shiller lag procedure.

Interest rates in percentages (i) are entered as $1 + i/100$ and inflation rates in percentages (j) as $1 + j/100$.

$DEL(\cdot, \cdot)$, $GR(\cdot, \cdot)$, and $SUM(\cdot, \cdot, \cdot)$ are defined as:

\[
DEL(n, X) \equiv X_t - X_{t-n}; \quad GR(n, X) \equiv X_t - X_{t-n} - 1; \quad SUM(n, m, X) \equiv \sum_{i=n}^{m} X_{t-i-n}.
\]

$RBND$ Japanese long-term government bond yield.

$RCD$ Average interest rate on certificates of deposit.

$GDS$ Government debt, proxied by accumulated annual budget deficits, in billions of yen.

$GDPN$ Nominal GDP, in billions of yen.

$OCR$ Index of capacity utilisation of manufacturing industries, 1990=100.

$USBND$ U.S. government bond yield. 30-year constant maturities.

$PGDP$ Japanese GDP deflator, 1985=100.

$USP$ US GDP deflator, 1987=100.

$IBDR$ Nominal yen-dollar exchange rate.

The other terms of the equation are interpreted as follows: the third term, the ratio of government debt to nominal GDP, represents the supply side of the long-term bond markets; the fourth term, the difference in the capacity utilisation index, is a proxy for the expectation of inflation; the fifth term represents interest rate arbitrage with U.S. long-term interest rates; the sixth term captures the expected change in the nominal exchange rate.

The performance of the equation is shown in Figure 1.
1.2 Yen-dollar exchange rate

The exchange rate equation is a variation of the so called portfolio-balance models. To make the explanation simpler, let us start with uncovered interest parity:

\[ s_t = E_t(s_{t+1}) - (i_t - i_t^*) \]  

where \( s \) is the logarithm of the nominal exchange rate (yen/dollar) and superscript * stands for foreign variables. A simple modification of (2) yields:

\[ q_t = E_t(q_{t+1}) - (r_t - r_t^*) \]  

where \( q \) is the logarithm of the real exchange rate and \( r \) represents the real interest rate.

It is assumed that expectations of the real exchange rate at \( t+1 \) are formed as:

\[ E_t(q_{t+1}) = \mu f_t + (1 - \mu) q_t \]  

where \( f_t \) is the factor representing fundamentals and \( \mu \) is a parameter satisfying \( \mu \in [0,1] \). In words, market participants believe that the real exchange rate approaches the fundamental value at the adjustment speed of \( \mu \). Note that market participants' expectations are backward-looking.
Substituting (4) into (3), we get:

\[ q_t = f_t - \mu^{-1}(r_t - r^*) \]  

(5)

This is the basis for the specification listed in Table 2, although the precise specification adopted here differs from equation (5) in the following respects. First, the factor representing fundamentals, \( f_t \), is assumed to be invariant over time. This is based on the idea that the nominal exchange rate tends to converge to the purchasing power parity and, therefore, the real exchange rate converges to a constant value. The constant term of the equation represents the constant value to which the real exchange rate converges. Second, the risk premium associated with exchange rate risks is included as the third term of the equation. Based on the idea that the risk premium would increase with the Japanese private sector's net external asset denominated in dollars, the cumulative current account surplus of Japan, denoted by \( RPJ \), is included on the right-hand side of the equation.\(^2\) Also, following the argument of Fukao (1983, 1987) that the yen/dollar rate tends to comove with the

| Table 2 |

| Yen-dollar exchange rate |

\[
\ln(RDR_t) = \alpha_0 + \alpha_1 (RBNDR_t - USBNDR\,10_t)/100 + \alpha_2 (0.0030RPJ_t + 0.0017RPE_t)/ITI_t 
\]

<table>
<thead>
<tr>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( S.E. )</th>
<th>( \bar{R}^2 )</th>
<th>( D.W. )</th>
<th>( \text{Sample} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.461 ( (309.7) )</td>
<td>-2.923 ( (-4.62) )</td>
<td>-1.373 ( (-15.55) )</td>
<td>0.091</td>
<td>0.79</td>
<td>0.44</td>
<td>76.2</td>
</tr>
<tr>
<td>( \text{Note:} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>93.4</td>
</tr>
<tr>
<td>Numbers in parentheses are t-statistics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{RDR} )</td>
<td>Real yen-dollar exchange rate index, 1985=100. Spot rates on the Tokyo interbank market deflated by GDP deflators of the United States and Japan.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{RBNDR} )</td>
<td>Japanese real long-term interest rate. Defined as the Japanese government bond yield minus the annual growth rate of the Japanese GDP deflators.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{USBNDR10} )</td>
<td>US real long-term interest rate. Defined as the US government bond yield minus the annual growth rate of the US GDP deflator.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{RPJ} )</td>
<td>Cumulative current account surplus of Japan minus direct investments, in millions of dollars and starting in 1974Q1.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{RPE} )</td>
<td>Cumulative current account surplus minus direct investments of Germany, France, Italy and the Netherlands, in millions of dollars and starting in 1974Q1.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{ITI} )</td>
<td>Index of nominal GDP of Japan, the United States, Germany, France, Italy and the United Kingdom, 1975Q1=100.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^2\) The net external position of the private sector includes direct investments. These, however, should be subtracted because they are irrelevant to the risk premium. Taking this into consideration, \( RPJ \) in Table 2 is defined as the cumulative current account surplus minus direct investments. The same applies to \( RPE \).

\(^3\) It is assumed here that the cumulative current balance could be used as a proxy for the private sector's net external asset. One might argue that this assumption is too strong because (i) a non-negligible part of the cumulative current balance is owned by the public sector, including the government and the central bank, and (ii) the investment behaviour of the public sector could differ from that of the private sector. We, in part, agree to this argument, but still believe that the assumption could be justified in some cases. For example, the assumption is justified in the Ricardian world where the private sector "internalises" the budget constraint of the public sector. Also, the intervention policy of the public sector is sometimes characterised by profit maximisation, particularly in small countries in Asia and Europe. If this is the case, it would be safe to use the cumulative current balance as a proxy for the private sector's external assets. See Fukao (1987) for more on this issue.
DM/dollar rate, the cumulative current account surplus of the major European countries, denoted by RPE, is included.\(^4,5\)

Figure 2 shows the performance of the estimated equation. Roughly speaking, the depreciation of the yen in the first half of the 1980's is explained by the widening of interest rate differentials between U.S. and Japan; the rapid depreciation of the yen since the Plaza agreement is explained by an increase in the accumulated current account surplus.

**Figure 2**

**Real yen-dollar rate: actual and fitted**

\[1985 = 100\]

---

1.3 **Stock price**

The stock price equation is based on the idea that the stock price is determined by the present discounted value of future profits. As shown in Table 3, the Nikkei Average is explained by the current profits of all industries and the long-term interest rate. Reflecting the assumption that the expectation of future profits is formed in an adaptive way, the lagged values of profits are included in the right-hand-side of the equation. The parameters associated with the lagged values are estimated by the Shiller lag procedure.

\(^4\) The third term of the exchange rate equation is a linear combination of RPJ and RPE. The weight given to RPJ (0.0030) is the variance of the yen/dollar rate; the weight to RPE (0.0017) is the covariance between the yen/dollar rate and the DM/dollar rate. See Fukao (1983, 1987) for more details.

\(^5\) Several interesting things can be read from the estimation result. First, the fundamental value is 4.461, which corresponds to the level of the real exchange rate just before the Plaza agreement. Second, the estimated value for $\mu$ ($\mu = 1/\alpha_x$ by definition) is 1/2.923. This means that market participants expect the real exchange rate to approach the fundamental value at the rate of 1/2.923 per quarter.
Table 3

Stock price equation

\[ \ln(SPI_t) = \alpha_0 + \sum_{j=0}^{2} \alpha_{1j} \ln(PROF_{t-j}) + \alpha_2 \ln(1/RBND_t) \]

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\alpha_{1j}$</th>
<th>$\alpha_2$</th>
<th>S.E.</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.918</td>
<td>0.838 (9.04)</td>
<td>15.143</td>
<td>0.103</td>
<td>0.92</td>
<td>0.969</td>
<td>84.1</td>
</tr>
<tr>
<td>(-5.43)</td>
<td>0.448 (11.18)</td>
<td>(9.36)</td>
<td></td>
<td></td>
<td></td>
<td>94.3</td>
</tr>
<tr>
<td></td>
<td>0.032 (0.40)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum \alpha_{1j}$</td>
<td>1.318</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are t-statistics. $\alpha_{1j}$ ($j=0,1,2$) are estimated by the Shiller lag procedure. $RBND$ entered as in Table 1.

$SPI$: Nikkei average, in yen.

$PROF$: Current profits of all industries reported in Financial Statements of Incorporated Businesses, in 100 million yen.

$RBND$: Long-term government bond yield.

Figure 3

Stock prices: actual and fitted

Yen

- Fitted
- Actual
The performance of the stock equation is shown in Figure 3. An interesting thing to note is that almost all fluctuations during the "bubble" period are explained by profits and long-term interest rates. One might say that something must be wrong with the estimation procedure because fluctuations caused by a speculative bubble should not be explained by fundamentals such as profits and interest rates. It is not easy and beyond the scope of this paper, to determine whether fluctuations in stock prices in the late 1980's were due to a speculative bubble or not. But, the estimated result as it is, can be interpreted as follows. As shown in Table 3, the estimated coefficient of current profit is well above one; the sum of $\alpha_{1j}$ equals to 1.318. This means that, when profit increases by 1%, the expectation of future profits increases by 1.318%, thereby pushing up the stock prices by 1.318%. This is how expectations were formed in the Tokyo stock market; one may call this a speculative bubble.

2. Impulse response function of asset prices

2.1 Innovation-simulation technique

The Lucas-critique of conventional simulation analysis might apply to the BOJMOD in which expectations mechanisms are adaptive, or backward-looking as described in the previous section. That is, we cannot rule out the possibility that the parameters of the BOJMOD would shift when a simulation shock is added.

There are two alternative ways to overcome the Lucas-critique. The first one is to modify the BOJMOD by incorporating forward-looking expectations and estimating structural, or deep parameters. The second one is to modify the simulation analysis without changing the structure of the BOJMOD itself. In this section, following the second method, we will study the dynamic response of asset prices to various types of shock.

The methodology we will adopt in this section is the so-called "innovation-simulation technique" proposed by Sargent (1977), Sargent and Sims (1977) and Mishkin (1979). The basic idea of this methodology is as follows.\(^6\) Suppose the structural equations of a macroeconometric model are characterised by:

\[
A(L) y_t + B(L) x_t = e_t
\]

with $y$ a vector of endogenous variables; $x$ a vector of exogenous variables; $e$ a vector of disturbances and $A(L)$ and $B(L)$ matrices of polynomials in the lag operator $L$. Also, suppose that the time-series process of the exogenous variables is described by:

\[
C(L) x_t = u_t
\]

where $u$ is a vector of disturbances. Usually, parameters of $A(L)$, $B(L)$ and $C(L)$ are estimated using a set of historical data.

The standard simulation based on the macroeconometric model computes the deviation of the endogenous variables from the baseline when a shock is added to some of exogenous variables. A problem with this procedure is that when an arbitrary dynamic path of the exogenous variables is chosen for a simulation analysis, there is no guarantee that the path is consistent with equation (7). Put differently, a researcher alter the parameters of $C(L)$ when he chooses an arbitrary path of the exogenous variables. This is a serious problem because the parameters of $A(L)$ and $B(L)$ are not determined.

\(^6\) The following explanation is based on Mishkin (1979).
invariant to changes in the parameters of $C(L)$. It is particularly so when the macroeconometric model adopts backward-looking expectations. The moment a researcher chooses an arbitrary path of the exogenous variables, the parameters of $A(L)$ and $B(L)$ would change; therefore, simulation results based on the estimated parameters could be quite misleading.

A remedy to this problem is to conduct a simulation analysis taking equation (7) into consideration. More specifically, we can generate a dynamic path of the exogenous variables which is consistent with the parameters of $C(L)$ by the following procedure: (i) add an innovation to the disturbance term of (7); (ii) compute the dynamic path of the exogenous variables using (7).\(^7\)

The simulation procedure in this section is as follows. First, we estimate an ARIMA model for an exogenous variable (e.g. the short-term money market rate) to which a shock is given. Second, we calculate the dynamic response of the exogenous variable to an innovation. Third, we put the response into the BOJMOD as a dynamic shock. Finally, we calculate the response overtime of the BOJMOD's endogenous variables to the shock.\(^8\)

### 2.2 Response to an innovation in the call rate

The first experiment is to study the dynamic response of asset prices to an innovation in the call rate. Using the identification procedure based on the AIC, we estimate an ARIMA model of the call rate. As shown in the note to Figure 4, ARIMA(1,0,1) is chosen and estimated over 1982Q1 to 1994Q1 period by the maximum likelihood method.

The upper-left panel shows the dynamic behaviour of the call rate when an innovation of 1 percent is added to the first quarter. It is observed that the call rate goes up further to 1.4 percent in the second quarter because of the existence of a MA term, and then decays gradually over time.

Based on this movement of the call rate, the response of the asset prices is computed using the BOJMOD, which is shown in the other three panels of Figure 4. The first impact of an innovation on the call rate appears on the long-term interest rate.\(^9\) As shown in the upper-right panel, the long-term interest rate jumps up by 0.2 percent in the first quarter and then goes up for three quarters to 0.58 percent, followed by a gradual decay. In response to this, the yen measured by the real yen/dollar rate appreciates 2 percent for the first three years and then gradually returns to the base line: the stock price declines 13 percent for the first two years and returns to the base line.

---

7 In other words, innovation-simulation technique reduces the risk of misleading stimulation results by deliberately specifying the type of simulation shocks that a macroeconometric model with backward-looking expectations is able to handle. This imply a limitation on the innovation-simulation technique: i.e., since the dynamic path of the exogenous variable is specified by the time-series process of the exogenous variables, researchers are not allowed to arbitrarily choose the type of simulation shock. For example, even when a researcher wants to study the impact of a temporary shock for an exogenous variable, he is not allowed to do if the exogenous variable contains stochastic trend.

8 One of the problems in this procedures is that the feedback from the BOJMOD to the ARIMA model is completely ignored. It might be interesting to see how the results of the experiments in this section would change if the interaction between the two models is taken into consideration.

9 In the BOJMOD, a change in the call rate affects the CD rate contemporaneously, and then the long-term interest rate.
2.3 Response to an innovation in the current balance/nominal GDP ratio

In the BOJMOD, the impact of an innovation in the current account surplus appears on the real side of the economy through net exports and on the monetary side of the economy through the exchange rate. Since the focus of this paper is on the asset price determination, the impact on the real side of the economy is neglected in this experiment.10

An original shock is given only to the third term of the exchange rate equation. But we do not rule out the possibility that changes in the exchange rate would affect the real side of the economy through net exports.

---

10 An original shock is given only to the third term of the exchange rate equation. But we do not rule out the possibility that changes in the exchange rate would affect the real side of the economy through net exports.
As shown in the note to Figure 5, the risk premium term of the exchange rate equation, \((0.003RPJ+0.002RPE)/IT1\), follows ARIMA(1,1,3). Roughly speaking, this means that the ratio of the current account surplus to nominal GDP follows ARIMA(1,0,3). The dynamic behaviour of the current account surplus/GDP ratio caused by an innovation of 1 percent is depicted in the upper-left panel. The ratio increases during the first four quarters up to 1.4 percent and then decays rapidly.

**Figure 5**

Response of asset prices to an innovation in the ratio of the current balance to nominal GDP

(a) Ratio of current balance to nominal GDP

(b) Long-term interest rate

(c) Exchange rate

(d) Stock price

Note: Response of asset prices is calculated based on the following ARIMA model of the ratio of accumulated current balance to nominal GDP estimated over the period 1975Q2 to 1994Q4. Numbers in parentheses are t-statistics.

\[
(1-0.82L)(1-L)((0.003RPJ_t + 0.002RPE_t)/IT1) - 0.005 = (1+0.21L + 0.385L^2 + 0.432L^3)\epsilon_t
\]

\((-11.38)\quad (-1.43)\quad (1.89)\quad (3.89)\quad (4.06)\)

\(\sigma^2 = 0.00001\quad AIC = 666.4\)

*RPJ* Cumulative current account surplus of Japan minus direct investments, in millions of dollars and starting in 1974Q1.

*RPE* Cumulative current account surplus minus direct investments of Germany, France, Italy and the Netherlands, in millions of dollars and starting in 1974Q1.

*IT1* Index of nominal GDP of Japan, the United States, Germany, France, Italy and the United Kingdom, 1975Q1=100.
Since the risk premium is a function of the ratio of the cumulative current balance to nominal GDP, what governs the behaviour of the exchange rate is the integral of the deviation of the current account/nominal GDP ratio from the base line, which monotonically increases with time. As shown in the lower-left panel, the exchange rate gradually appreciates during the first 18 months and then stabilises.

In response to this, the long-term interest rate first goes down and then returns to the base line. Meanwhile, the stock price goes up slightly during the first seven quarters responding to lower long-term interest rates. In the eighth quarter it starts to decline, reflecting the deterioration of corporate profits caused by the appreciation of the yen.

2.4 Response to an innovation in the budget deficit to nominal GDP ratio

The impact of an innovation in the budget deficit appears on the real side of the economy through the government's net saving and on the monetary side of the economy through the long-term interest rate. As in the previous experiment, the impact on the real side of the economy is neglected in this experiment.

As shown in the note to Figure 6, the government debt to nominal GDP ratio follows ARIMA(0,1,1). Combined with the finding that the estimated parameter of the MA1 term almost equals unity (0.9998), this means that the ratio of government debt to nominal GDP is characterised by a white noise process with a deterministic trend.11 The dynamic behaviour of the budget deficit to nominal GDP ratio, which is depicted in the upper-left panel, indicates that the shock is transitory in the sense that it has no effects in and after the second quarter.

The response of the long-term interest rate, which is depicted in the upper-right panel, shows that the long-term rate jumps up immediately in the first quarter and then stabilises. Responding to this, both the exchange rate and the stock price show a discrete jump in the first quarter and a gradual change in the subsequent quarters.

11 As shown in the note to Figure 6, the ratio of government debt to nominal GDP ratio has a deterministic trend which decreases at the rate of 0.22 percent per quarter.
Figure 6
Response of asset prices to an innovation in the ratio of the budget deficit to nominal GDP

(a) Ratio of budget deficit to nominal GDP

(b) Long-term interest rate

(c) Exchange rate

(d) Stock price

Note: Response of asset prices is calculated based on the following ARIMA model of the ratio of accumulated budget deficit to nominal GDP estimated over the period 1980Q1 to 1989Q4. Numbers in parentheses are t-statistics.

$$ (1 - L) \left( \frac{GDS_t}{GDPN_t} \right) + 0.0022 = (1 - 0.9998L) \eta_t $$

$$ \hat{\sigma}^2 = 0.00003 \quad \text{AIC} = 281.8 $$

$GDS/GDPN$ Ratio of the accumulated budget deficit to nominal GDP.

2.5 Response to an innovation in the Japan-US interest rate differential

The final experiment is to study the dynamic response of asset prices to an innovation in the Japan-US interest rate differential. As shown in the note to Figure 7, the interest rate differential follows the ARIMA(1,0,2) process. When an innovation of 1 percent is added to the process, the interest rate differential reaches a peak in the first quarter and then quickly decays (see the upper-left panel).
This dynamic shock affects first the exchange rate. As shown in the lower-left panel, the yen/dollar rate appreciates 2.7 percent in the first quarter and then returns to the base line. This leads to a discrete decline in the long-term interest rate and subsequently to a discrete rise in the stock price.

Figure 7
Response of asset prices to an innovation in the Japan-US interest rate differential

(a) Japan-U.S. interest rate differential

(b) Long-term interest rate

(c) Exchange rate

(d) Stock price

Note: Response of asset prices is calculated based on the following ARIMA model of the Japan-US interest rate differential estimated over the period 1981Q1 to 1995Q2. Numbers in parentheses are t-statistics.

\[ (1 - 0.587L)(RBNDR_t - USBNDR10_t)/100 + 0.630 = (1 + 0.183L + 0.334L^2)\epsilon_t \]

\[ (-3.64) \quad (1.99) \quad (1.99) \quad (2.22) \]

\[ \hat{\sigma}^2 = 0.46 \quad \text{AIC} = 130.1 \]

*RBNDR*  Japanese real long-term interest rate. Defined as the Japanese government bond yield minus the annual growth rate of the Japanese GDP deflator.

*USBNDR10*  US real long-term interest rate. Defined as the US government bond yield minus the annual growth rate of the US GDP deflator.
3. **Backward-looking versus forward-looking expectations**

The experiment we will conduct in this section is to compute the responses of asset prices to the same shock as in Section 2 but under the assumption of forward-looking expectations and to compare them with those obtained in Section 2.

Among the simulations, we have chosen two experiments: (i) the response of long-term interest rate to an innovation in the call rate; (ii) the response of the yen/dollar rate to an innovation in the Japan-US interest rate differential.\(^{12}\)

### 3.1 Response of the long-term interest rate to an innovation in the call rate

To compute the response of the long-term interest rate to an innovation in the call rate under the assumption of forward-looking expectations, suppose market participants have the estimated time-series model for the call rate. That is, market participants know that the call rate follows ARIMA(1,0,1) and have the estimated parameters shown in the note to Figure 4. Based on this knowledge, they are able to compute the dynamic behaviour of the call rate depicted in the upper-left panel. Those future values of the call rate are put into equation (1) to calculate the long-term interest rate in each quarter.\(^{13}\) Panel (c) of Figure 8 depicts the response of the long-term interest rate computed in this way.

Comparing panel (c) with panel (a), we find the following. First, the shape of response functions looks similar: a quick response to the shock and then a gradual decay.

Second, the magnitude of the response is almost the same: the peak of the response is 0.5 to 0.6 percent and the response in the 20th quarter is 0.1 to 0.2 percent. These similarities could be interpreted as additional evidence that the simulation in Section 3 based on innovation-simulation technique has not yielded misleading results. This point becomes clearer when we compare the above results with panel (b) which shows the result obtained when a sustained shock of 1 percent is added to the call rate (this case is called "standard simulation technique"). One of the differences between panels (b) and (a) as well as (c) is that the long-term interest rate continues to rise over the 20 quarters. This seems to be an inevitable consequence of the two assumptions: (i) expectations are backward-looking; (ii) shocks are permanent. But the problem here is that the combination of the two assumptions is not realistic in the following sense. If market participants are in some sense rational, they will, at some point, recognise that the shock is permanent and will stop using the mechanical backward-looking expectation process. At the moment market participants start to expect the future short rates on a forward-looking basis, the long-term interest rate would show a discrete jump. In this sense the result obtained by "standard simulation technique" is misleading.

The third thing we should note is the speed at which the long-term interest rate rises in response to an innovation in the call rate. In panel (c), the reaction takes place immediately: the long-term interest rate jumps up in the first quarter and starts to decay in the second quarter. This is consistent with the market efficiency hypothesis. On the other hand, it takes 4 quarters to reach the peak in panel (a). Obviously, this is not consistent with the market efficiency hypothesis but the length of 4 quarters could be justified as the time required for market participants to recognise that the shock is a permanent one. Given the absence of empirical evidence on the speed of learning, it is next to impossible to say which one is a right reaction to the shock.

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\(^{12}\) The reason for choosing these two experiment is that simulation results under the assumption of backward expectations are easy to compute. See footnote 14 for the evaluation of other experiments.

\(^{13}\) \(\beta\) is set at 0.97 (the discount rate is 3 percent).
Figure 8
Backward-looking versus forward-looking:
response of the long-term interest rate to an innovation in the call rate

(a) innovation-simulation technique

(b) standard simulation technique

(c) forward-looking expectations

Note: See the text for details on the calculation of impulse response functions.
3.2 Response of the yen/dollar rate to an innovation in the interest rate differential

The response of the yen/dollar rate to an innovation in the interest rate differential under the assumption of forward-looking expectations is computed as follows. First, it is assumed that market participants have the estimated time-series model for the interest rate differential; i.e. they know that the interest rate differential follows ARIMA(1,0,2) and have the estimated value shown in the note to Figure 7. Based on this knowledge, they are able to compute the dynamic behaviour of the interest rate differential plotted in the upper-left panel.

The next thing we should do is to compute the expectation of the real exchange rate, $E_t(q_{t+1})$ of equation (3), without relying on equation (4). As equation (3) indicates, we need $q_{t+1}$ to compute $q_t$, and $q_{t+2}$ to compute $q_{t+1}$ and so on; therefore, in principle, we have to know $q_\infty$ to compute $q_1$. But the convenient fact in this experiment is that since the interest rate differential reaches zero on the 14th quarter and stays zero in the subsequent quarters, the exchange rate is also zero on and after the 14th quarter. This means that the expectation of the real exchange rate for the 14th quarter formed on the 13th quarter, $E^{\infty}_1(q_{14})$, is zero. Since we know $E^{\infty}_1(q_{14})$ and $r_{13} - r_{13}^\ast$, we can now compute $q_{13}$. Applying the same backward-induction methodology, we can compute $q_{12}$, $q_{11}$, $q_{10}$, ..., and finally $q_1$. Panel (c) of Figure 9 shows the response of the yen/dollar rate computed in this way.

A casual comparison between panels (c) and (a) reveals the following. First, the shape of response functions is surprisingly similar: both panels show a discrete jump in the first quarter and decays in and after the second quarter. A minor difference is that panel (a) shows a slight appreciation of the yen from the second to the third quarter while panel (c) shows a consistent depreciation in and after the second quarter. Second, the magnitude of the response is almost the same: the peak of the response is 3 to 4 percent. Again, these two findings could be interpreted as evidence that the combination of the BOJMOD and innovation-simulation technique works well. Meanwhile, panel (b) which plots the response obtained when a sustained shock of 1 percent is added to the interest rate differential seems to show another misleading result.\(^{14}\)

\(^{14}\) In this section, we have evaluated the results of the two experiments conducted in Section 2 by comparing them with the results obtained under the assumption of forward looking expectations. Let us briefly evaluate the remaining two experiments based on the responding speed of asset prices to shocks. Responses of the exchange rate to an innovation in the current balance to nominal GDP ratio is very slow and it is not until the 18th quarter that the exchange rate starts to stabilise (Figure 5). This is not a reasonable response of market participants: if they are in some sense rational, the exchange rate should show a discrete jump. As for the response of the long-term interest rate to an innovation in the budget deficit to nominal GDP ratio (see Figure 6, the computed response is quick so that the result is consistent with the hypothesis of efficient markets.
Backward-looking versus forward-looking: response of the yen/dollar rate to an innovation in the Japan-US interest rate differential

(a) innovation-simulation technique

(b) standard simulation technique

(c) forward-looking expectations

Note: See the text for details on the calculation of impulse response functions.
Conclusion

The asset price equations of the BOJMOD adopt backward-looking expectations. As Lucas pointed out in his celebrated paper in 1976, simulation analysis using this type of macroeconometric model could yield misleading results because parameters would shift when a shock is added. There are two alternative ways to overcome the Lucas-critique. The first is to modify the BOJMOD by incorporating forward-looking expectations and estimating structural, or deep parameters; the second is to modify the way simulation analysis is conducted without changing the structure of the BOJMOD itself. In this paper we have chosen the second way to study the dynamic response of asset prices to various types of shock.\footnote{The innovation-simulation technique is a useful tool but has some limitations. For example, when we want to study the consequences of unprecedented policy changes, this technique will not work. All that we can do by this technique is to study the impact of various shocks which have taken place more than several times in the past. If we really want to know the effect of unprecedented shocks, we need to estimate structural parameters.}

The methodology we have adopted in this paper is the so-called "innovation-simulation technique" proposed by Sargent, Sims and Mishkin. The basic idea of this methodology is straightforward: (i) the parameters of conventional macroeconometric models are not independent of the process generating an exogenous variable to which a simulation shock is added; (ii) the reason for the shift of parameters is that conventional simulation methodology gives a shock to the model which is not consistent with the process generating the exogenous variable; (iii) therefore, if we add a shock consistent with the process generating the exogenous variable, parameters will not shift.

Using this methodology, we have computed the dynamic responses of asset prices to various types of shock to find that: (i) the responses of asset prices, overall, do not contradict the hypothesis of efficient markets; (ii) the responses of asset prices computed in this way are very similar to those computed under the assumption of forward-looking expectations.

Overall, our experiments show that the innovation-simulation technique is a useful tool to study the dynamic property of conventional macroeconometric models like the BOJMOD.
References


Comments on paper by T. Watanabe and H. Matsuura by R. McCauley (BIS)

This very useful paper points to a puzzle regarding the power of monetary policy over the bond market in Japan; raises a question regarding how to operationalise a portfolio-balance model; and perhaps sounds a warning that a fundamental model should not perform too well in the presence of asset inflation and deflation.

1. Long-term interest rates

The estimated equation says that the long-term interest rate is firmly tied to short-term bank rates in the current and previous quarter, to the government debt, to a combination of the US bond yield, Japanese and US inflation, and to the exchange rate, all with expected signs.

The first question is which Japanese long-term government bond yield is being explained, since the Japanese bond market shows an extremely strong benchmark effect. In particular, the benchmark bond can trade as much as 50 basis points away from adjoining non-benchmark bonds at times like mid-1987, and its yield volatility is much higher than nonbenchmark bonds of much shorter maturity.

The very close relation of short-term to long-term interest rates may be related to the long-term puzzle of why, over the cycle, Japan's money and bond markets show one of the flattest yield curves in the world. Of course, this question may seem beside the point at a time when the Japanese yield curve is quite steep. One way of putting the question is why does Bank of Japan policy have such a large and fast effect on long-term interest rates? As Inoue, Ishida and Shirakawa put it with considerable understatement at the Autumn Economists' meeting here last month, "[bond market] investors are quite sensitive to the expected future course of short-term rates."

Would the equation perform better if it included money market rates a little less tied to current overnight rates and rather more indicative of future overnight rates? Including forward money market rates from Euroyen futures or implied in the structure of cash bank deposit rates, one could test for whether the an appreciation of the yen works directly to lower the expected price level and thereby to lower bond yields, or whether it works indirectly through market participants' expectations of easier Bank of Japan policy. Moreover, one wonders whether the use of a CD rate as the representative short-term rate - which is quite an understandable choice - yields forecasting errors when, as now, a gap opens between Treasury bills and Japanese bank rates (which is related to the so-called Japan premium).

This reader did not understand why US long rates, Japanese and US inflation are all combined into one variable. The variable seems to be based on the notion that investors compare real yields on Treasuries against real yields on JGBs. This investment strategy makes sense in a world in which the yen-dollar exchange rate moves to offset inflation differentials. It is hard to imagine that the wild swings in the purchasing power of the yen in relation to the dollar have left many investors inclined to bet on real yield differentials.

The fiscal factor seems to neglect the ownership of the debt. One wonders whether various government trust-fund purchases/holdings of government bonds should be excluded. At high frequencies, at least, the market seems to react to reports that a trust fund is to buy or to sell government bonds.
2. Yen-dollar exchange rate

The model relates the real yen-dollar exchange rate to the difference between real long-term interest rates in Japan and the US and to the accumulated current account surplus of Japan and Europe, weighted more or less 2 to 1. Again, given the large swings in purchasing power parity, and all the evidence of a persistent gap between the internal and external value of the yen, it is strange to start with the notion that the market is itching to get back to a fundamental purchasing power rate and is temporarily dragged away by first interest rate differentials and then by an accumulating international asset position.

The inclusion of Japan's net asset position is fairly common in models of the yen-dollar exchange rate but this model distinguishes itself in excluding direct investment. Whether direct investment into Japan is excluded does not much matter empirically, but the rationale - that direct investment is "irrelevant to the risk premium" - would bear elaboration.

More importantly, the same rationale might well require an exclusion of the build-up of Bank of Japan reserves, and perhaps also the foreign assets of the postal life insurance system, Japan Exlm and other such government holdings. When, as in 1995, the accumulation of foreign assets by the Bank of Japan and others recycles the bulk of the current account surplus, this alternative measure would grow much more slowly than the accumulated current account balance. Although there has been some public discussion in Japan of the accumulated losses on official dollar holdings, it is hard to believe that private investors see through the government balance sheet and perceive as their own the exchange risk on official holdings of dollars. Would the exchange rate really remain unaffected by the Japanese authorities' liquidating their international reserves?

3. Equity prices

The model relates the log of the level of the Nikkei average to the log of corporate earnings and to the log of the inverse of the long-term government bond yield. At an econometricians' meeting, one can expect the comments to focus on whether the regression in levels with lags should be expected to produce reliable results. Instead, consider the choice of variables and sample period.

If the stock market discounts future earnings, one can relate the capitalisation to total earnings of listed companies or a share price index to earnings per share. The model relates the index to total profits, an acceptable approximation if share issuance were negligible. In fact, in the late 1980s Japanese corporations issued new shares into a booming market, so that total profits grew significantly faster than earnings per share. If earnings per share were used the coefficients on corporate earnings would be still higher.

The response of share prices to corporate earnings is very strong, a case of Hicksian elastic expectations. As it stands the sample period seems almost designed to model the bubble in share prices. Did Japanese share prices show such elasticity to earnings before 1985? Or would an equation estimated over an earlier period have shown share prices cutting loose from fundamentals in the mid- to late-1980s?

The answer to the question regarding the pre-sample performance of the equity equation is of more than econometric interest. As head of the Economic Planning Agency, Yoshitomi could specify the year in which the agency's land price equation broke down. One of the reasons for maintaining a central bank model for an important asset price is to inform judgements of whether the price has lost contact with the fundamentals, that is, to provide an early warning. Chart 11 in the Inoue, Ishida and Shirakawa paper tells the tale: a reasonable present value calculation shows that the Nikkei lost touch with the underlying earnings growth in the late 1980s.
Expectations and monetary policy transmission: the determination of the exchange rate and long-term interest rates in the Banca d'Italia's quarterly econometric model

Eugenio Gaiotti and Sergio Nicoletti-Altimari

Introduction

In this paper, we address the issue of the determination of the exchange rate and of the long-term interest rates in the Banca d'Italia quarterly model (BIQM), by introducing an explicit expectation formation mechanism. The interplay between exchange rate expectations and inflation expectations contributes to the endogenous determination of asset prices, together with international factors, like currency market volatility and foreign long-term rates.

The results we present are part of a research work still in progress. To a large extent, they reflect the changes that took place in the Italian and international economy in the last few years, which strengthened the role of expectations in the transmission of monetary policy and in the determination of asset prices. Structural changes of foremost importance took place in the currency market and in domestic securities markets (the lifting of controls on international capital movements, the development of a deep and efficient market for long-term securities, the floating of the exchange rate).

These changes have had two consequences: a shift in the relative importance of different channels of monetary policy transmission, with an increased emphasis on the "expectations channel"; and a larger sensitivity of domestic variables to developments in the expectational climate on international markets. To assess these effects and evaluate their quantitative importance, an investigation on the determination of expectations about the exchange rate, future inflation and interest rates is needed.

The paper represents a first attempt to do so. First, mechanisms of endogenous determination of exchange rate expectations and inflation expectations are introduced: exchange rate expectations are of foremost importance in determining the actual behaviour of the spot exchange rate and the impact of monetary policy on currency markets; while inflation expectations exert important effects in the wage-setting block of the model and contribute to determining the real interest rate, the ex-ante evaluation of real wealth relevant for consumption choices and the ex-ante real cost of capital relevant for investment decisions. Second, a new forward-looking determination of long-term interest rates that links them to inflation expectations and to international factors is analysed and introduced in the model. The relative role of domestic short-term rates and of yields on foreign market in determining long-term rates is tested. The role of monetary policy, if it can affect exchange rate and inflation expectations, is significantly altered.

The paper is organised as follows. Section 1 presents a brief review of the different methods used to model expectations in the BIQM and the research still in progress. Section 2 addresses the problem of endogenising exchange rate expectations and their role in determining the exchange rate. Section 3 presents an estimate for the determination of long-term interest rates that

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1 Banca d'Italia, Research Department. We are indebted to F. Altissimo, L. Buttiglione, K. Tsatsaronis and I. Visco for useful comments and suggestions.

2 The structure of the quarterly model is described in Galli, Terlizzese, and Visco (1989) and Terlizzese (1995). Its long-run behaviour is consistent with a neo-classical model with exogenous growth. In the short run a number of adjustment processes governs the dynamics; the most important reflect the putty-clay nature of capital, the stickiness of prices and wages, the possibility that expectations differ from realised values and the corresponding revisions of both plans and expectations.
links domestic long yields to domestic short rates and inflation expectations, to foreign yields and to volatility in the currency market. Section 4 presents the main results on the formation of inflation expectations. Finally, in Section 5, the working of the whole model under the estimated mechanism of expectation formation is exemplified by means of a simulation exercise of the effects of monetary policy. Results are compared with those obtained from alternative schemes such as rational expectations or purely adaptive-regressive mechanisms. The effects of an increase in uncertainty in the currency markets are also studied.

1. The modelling of expectations formation in the BIQM

For most of the profession, both for theoretical and empirical purposes, it is customary to assume that expectations are rational\(^3\). The advantage of this hypothesis, it is argued, lies in its relative “neutrality” with respect to the structure of the model whose results would then be independent from arbitrary assumptions for the expectations formation mechanism. The latter conclusion, however, is not warranted. First, it is not granted even in a context of rational expectations (e.g. in the presence of self-fulfilling expectations and rational bubbles) and the possibility, in many rational expectations models, of multiple equilibria, poses serious problems of selecting the equilibrium in a non-arbitrary way.\(^4\) Secondly, the extreme informational requirements of the REH are not to the credit of the absence of arbitrariness. More generally, in order to assess the arbitrariness of an assumption, the latter has to be tested.

Research work is being conducted on the Bank of Italy quarterly model to assess the implications and the relative merits of different mechanisms of expectations formation. The approach we follow in this paper is based on the use of direct observations from survey data on expectations; on the one hand, this makes it possible to assess the validity of the REH\(^5\) and, on the other, to directly estimate alternative models of expectations formation.\(^6\)

This approach is implemented using a survey conducted quarterly by Forum - Mondo Economico since 1957 on a group of Italian experts, belonging to different sectors (finance, commerce, production and academics).\(^7\) In general, it is assumed that agents know (or think they know) the reduced form of the relevant model and the values of its parameters. The parameters of the expectation formation mechanism are then estimated using the direct observations on expectations, with particular attention to the specification of the reduced-form model used by agents.

A second approach that is being investigated which is worth mentioning although we do not present it here, is to solve the model by assuming that expectations are formed under the “bounded rationality” hypothesis.\(^8\) The hypothesis is that agents know the reduced form of the relevant model but do not know, or are uncertain about, the value of its parameters and use some reasonable rule to estimate them. The estimated parameters, therefore, change through time and, if expectations enter the behavioural equations, the parameters of the structural model will also be time-varying. When the

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3 The rational expectations hypothesis, REH, of Muth (1961).

4 This problem is particularly serious, for example, in the case of endogenising the exchange rate by assuming rational expectations and an uncovered interest parity condition in capital markets; see the next section.

5 That could alternatively be verified only indirectly and conditionally on the chosen behavioural model - i.e. testing cross equation parameters restrictions.

6 It also allows us direct verification whether the expectations formation mechanism is invariant to regime changes (the Lucas critique) and to explore the way in which the mechanism is eventually revised.

7 The main characteristics of this survey are described in Visco (1984).

8 See Marcet and Sargent (1989), Sargent (1994) and Evans and Honkapohja (1995), among others. For a first implementation of this hypothesis in a large scale econometric model see Hall and Garratt (1994).
estimates converge to a stable solution, a rational expectation equilibrium is found; however neither convergence nor stability of the equilibrium are granted, as the results will in general depend on both the chosen expectation rule and the behavioural model.  

2. The modelling of expectations and the determination of the exchange rate

The role of expectations in determining the lira spot exchange rate increased after 1987-1990, when the removal of restrictions on international capital movements was completed; and after 1992, with the exit of the lira from the ERM of the EMS. In the last few years, the fluctuations of the exchange rate were mostly linked to shifts in expectations, originating either from domestic factors or from international shocks.

In the period during which the lira participated in the Exchange Rate Mechanism, both the stability of expectations and the presence of controls on capital movements limited, in the short run, the scope for a fully market-based determination of the spot exchange rate. Control of the exchange rate by the monetary authorities was obtained, in the short run, by intervention in the currency market and in the longer run by adjusting interest rates to the level necessary to avoid reserve outflows. In econometric modelling, the exchange rate was usually considered exogenous and determined by the monetary authorities.

The endogenisation of the exchange rate is based on an uncovered interest parity condition (UIP) of the form:

\[ S_{t+1} = S_t + r_t + r^*_t + \rho_t \]  

where \( S_{t+1} \) represents the logarithm of the exchange rate expected in period \( t \) for the period \( (t+1) \), \( S_t \) is the logarithm of the spot exchange rate, \( r_t \) and \( r^*_t \) are the domestic and the foreign interest rates over the same time span and \( \rho_t \) a time varying risk premium. For given interest rates and risk premium, the exchange rate is determined once an expectation formation mechanism is specified.

A standard way to close the model is to impose rational expectations; this is the approach followed and discussed in Nicoletti et al. (1995). However, the assumption requires imposing a terminal condition for the exchange rate in (1), which implies a high degree of arbitrariness. The alternative approach used in this paper builds on the work of Altissimo et al. (1995) and estimates an

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9 A first attempt to solve the BIQM under the hypothesis that agents revise their expectations on inflation and on the exchange rate using a Kalman filter rule is contained in Altissimo et al. (1995), where the convergence of the model and the consequences of this hypothesis for the transmission channels of monetary policy are studied.

10 See Gressani, Guiso and Visco (1988) for interaction of exchange rate and interest rate policy in the EMS period. They give a rationalisation of the monetary policy transmission mechanism prevailing in those years. The transmission of monetary policy to domestic prices occurred mainly through the exchange rate; interest rates were then adjusted in order to make the exchange rate target sustainable in terms of the current account. The working of this mechanism was, to some extent, conditional on imperfect capital mobility.

11 Three months euro-deposit rates are used in the BIQM.

12 An alternative approach is to directly estimate a reduced form for the exchange rate and can be obtained by substituting the expectation equation into the UIP condition. Parigi and Prati (1993) follow this approach for the EMS period; they find that the exchange rate appreciates in response to an increase in the interest differential and in the long run is affected by relative prices.

13 As it is well known, the presence of a forward unit root in (1) implies that the terminal condition has always the same effect on the solution of the model, no matter how far in time it is imposed, and completely determines the evolution of the system, as well as the effects of policy changes. See, for example, Fisher et al. (1992).
Figure 1
The lira-DM exchange rate
1a: Forecasts and actual values

Note: All variables are in logs.
equation for \( s_{t+1/4} \) using direct observations from the *Forum - Mondo Economico* survey discussed above, and then uses equation (1) to determine the spot exchange rate.\(^{14}\)

The survey-based expected lira-DM exchange rate, compared with actual values, and the implied forecast errors are reported in Figure 1. The latter clearly increased and became more volatile after the exit of the lira from the ERM of the EMS. Tests of unbiasedness of these expectations on the exchange rate were performed by Altissimo et al. (1995);\(^{15}\) according to their results, the presence of a systematic forecast error could not be rejected, and a closer look indicates a tendency to overestimate up to 1990 and to underestimate after 1992.\(^{16}\)

For our purposes, i.e. in order to use survey-based data in the framework of equation (1), it is relevant to test whether the uncovered interest parity condition is actually satisfied for the expectations of the survey participants. Since (1) is an identity, it actually amounts to testing that the risk premium term is not correlated with the other variables on the RHS of (1) and that it is not too volatile (or it is a stable function of some variables). If this is not the case, changes in interest rates would be reflected in changes in the risk premium rather than in expected depreciation. As it is well known from the literature on the subject, starting from the work of Froot and Frankel (1988),\(^{17}\) the UIP condition was usually rejected when tested in conjunction with the hypothesis of rational expectations, while the results have been more favourable when survey data were used.

We tested the UIP condition by regressing the survey-based, three-month ahead expected depreciation of the lira-DM exchange rate\(^{18}\) on both the domestic and the German three month interest rates.\(^{19}\) We tested the UIP jointly with the assumption of a white noise (plus a constant risk premium \( \rho \)). The results (rows I and II in Table 1) show that the UIP condition is not rejected: the coefficients on the domestic and foreign yield are not significantly different from 1 and -1 respectively, while the constant term is not significantly different from 0. However, some autocorrelation in the residuals suggests that some systematic behaviour of the risk premium may be present; we re-estimated the equation introducing some very simple modelling of this term, using, as a proxy, the coefficient of variation of the exchange rate in the period (both current and lagged).\(^{20}\) This variable proved to be significant and its introduction improved the fit, while retaining the basic result (rows III and IV in Table 1). We can conclude that the standard link between expected depreciation and the interest

---

\(^{14}\) The survey collects data on expectations on the Lira-Dollar and Lira-Deutsche Mark exchange rate quarterly since 1981. Both one-quarter and two-quarter ahead expectations are available. The survey is not, however, homogeneous through time. Up to the second quarter of 1990 only qualitative data are available. Both the direction (appreciation or depreciation) and the intensity (little or much) of the expected movement of the exchange rate were asked. Afterwards, point expectations were collected and the consensus forecast is constructed as an arithmetic mean of all survey participants after deletion of outliers. To have a continuous series of expectations the method of converting of qualitative expectations proposed by Carlson and Parkin (1975) was employed. For a survey of the possible methodologies and the associated problems see Visco (1984) and Pesaran (1989).

\(^{15}\) For the methodology to be used to test for unbiasedness in the presence of non-stationary series, see Giorgianni (1995a). See also the works of Frankel and Froot (1987) and Froot and Frankel (1988) for an empirical application to surveys on exchange rates expectations.

\(^{16}\) These tests, however, can not be considered as conclusive since the systematic error in the first period might well be due to the process of converting the qualitative data and the more recent period is too short to give a precise answer.

\(^{17}\) For a comprehensive survey on the subject, see Takagi (1990).

\(^{18}\) Since the survey is collected during the last month of each quarter, in computing expected depreciation we used the average spot rate over the same period. The interest rates on the right hand side refer to the same interval.

\(^{19}\) A risk premium correlated with the yields on the RHS in (1) would bias the estimated coefficients away from 1 and -1. This testing procedure is a more general version of the one used in Froot and Frankel (1988), who regress the expected depreciation on the forward exchange rate premium; the two approaches coincide when the restriction of equal coefficients on the domestic and foreign interest rate is imposed.

\(^{20}\) In this estimate, the coefficient of variation is measured over daily observation in the last month of each quarter.
differential seems to hold for Italy. Part of the variability of the time varying risk-premium, as measured using the observed expectations, can be explained by the volatility of the exchange rate; however, the residual variability is still quite high, on average 0.8 percent per quarter.

Table 1
Tests of the uncovered interest parity condition
Dependent variable: expected depreciation

<table>
<thead>
<tr>
<th></th>
<th>Const.</th>
<th>(\gamma_{\text{lira}})</th>
<th>(\gamma_{\text{DM}})</th>
<th>(\sigma_t)</th>
<th>(\sigma_{t-1})</th>
<th>Corr. R²</th>
<th>SEE</th>
<th>DW</th>
<th>Restrictions test</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.....</td>
<td>-0.002</td>
<td>1.25</td>
<td>-1.64</td>
<td>-</td>
<td>-</td>
<td>0.47</td>
<td>0.09</td>
<td>1.4</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.3)</td>
<td>(4.1)</td>
<td>(-5.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II....</td>
<td>-0.005</td>
<td>1.0</td>
<td>-1.0</td>
<td>-</td>
<td>-</td>
<td>0.44</td>
<td>0.09</td>
<td>1.2</td>
<td>12.0%</td>
</tr>
<tr>
<td></td>
<td>(3.7)</td>
<td>(res.)</td>
<td>(res.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III...</td>
<td>-0.008</td>
<td>1.4</td>
<td>-1.4</td>
<td>-0.49</td>
<td>-0.37</td>
<td>0.59</td>
<td>0.08</td>
<td>1.96</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.9)</td>
<td>(res.)</td>
<td>(res.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV....</td>
<td>-0.002</td>
<td>1.0</td>
<td>1.0</td>
<td>-0.46</td>
<td>-0.42</td>
<td>0.57</td>
<td>0.09</td>
<td>1.8</td>
<td>17.4%</td>
</tr>
<tr>
<td></td>
<td>(0.9)</td>
<td>(res.)</td>
<td>(res.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Interestingly enough, the recursive estimation of the UIP over the 1982-1994 period, shown in Figure 2, seems to suggest that the restrictions are accepted much more significantly in the floating (post-June 1992) period. Although, in principle, the move to floating rates could have increased the volatility of both expectations and the premium, rendering the estimation of the UIP more troublesome, this did not seem to have happened.

The deviations from the UIP resulting from equation (1) are shown in Figure 3. In the years after 1992 the premium based on survey data is constantly positive; according to our estimates, the higher mean level of volatility after the move to floating rates lead to an increase in the risk premium of about 3 percentage points. The residual component is also positive in this period; it may be due to a systematic bias in the timing of the observations, although other factors may be present. In the same figure the risk-premium resulting from the assumption of perfect foresight is also plotted: the latter is much more volatile.

In estimating the expectation formation equation, we started from a general specification of the kind:

\[
s_{t+1} = c + \sum_{i=1}^{P} \alpha_i s_{t+i-1} + \sum_{i=0}^{P} \beta_i s_{t-i} + \sum_{i=0}^{P} \gamma_i x_{t-i} + \mu_i
\]

where, besides lagged values of the expected and spot exchange rate, other variables \((x_i)\) in the information set of the agents\(^{22}\) are allowed to affect the formation of expectations of the lira-DM exchange rate. The interest rate differential, the relative price of exports \((pp)\), the change in official reserves relative to GDP \((VP)\) and the change in the dollar-DM exchange rate \((d\text{mus})\) were initially

---

\(^{21}\) For the limited purpose of this paper, we do not address the issue of the "fundamental" shocks underlying exchange volatility, which would be needed to give a full theoretical explanation of the risk premium. See Fornari, Monticelli and Tristani (1995). Further research is in progress on this topic. For a study of the determinants of the risk-premium in Italy using a different survey on exchange rate expectations, see Giorgianni (1995b).

\(^{22}\) The timing of the variables entering the information set of the agents when forming the expectations is crucial. It must be remembered that expectations are taken during the last month of the quarter \(t\) when forecasting the quarter \((t+1)\). Two strategies have been followed when estimating the equation: excluding the last month of the quarter from the variables in the RHS of the equation or using all the information of the quarter \(t\) but instrumenting with variables dated at \((t-1)\). Only the results of the latter procedure are reported in this paper. Results using the former are only marginally different. If no information for the quarter \(t\) was used, however, the fit of the estimate decreases.
Figure 2
Uncovered interest parity: recursive estimates of the coefficients

Domestic interest rate

Foreign interest rate

Exchange rate volatility
included in $x_t$. $\mu_t$ represents a stochastic error. This specification is sufficiently general to encompass adaptive, extrapolative or regressive schemes of expectation formation. In sample, we obtained the following specification:

$$
(s_{t+1/4} - s_{t/4}) = c + \beta_1(s_{t/4} - s_t) + \beta_2(s_{t-4} - PP_{t-4}) + \beta_3(s_{t-1} - s_{t-1}^*) + \mu_t
$$

where $PP_t$ represents the logarithm of the ratio of prices of tradables in the two countries. The results of the estimates are presented in Table 2. $s_t$ was instrumented using its past values.

Figure 3
Implicit risk premium in the UIP
Percentage points on annual basis

The specification indicates a very strong adaptive behaviour: more than three quarters of the deviation of the exchange rate from its forecasted value are incorporated in next period's expectation. The coefficient of the PPP is of the expected sign but not significant. The long-run convergence of the expected exchange rate to the PPP appears to be very slow; the regressive component of short-run expectations is, at best, very weak.

A short-run adaptive behaviour of expectations is a rather common result in the literature on survey-based exchange rate expectations. In this literature (e.g. Frankel and Froot, 1987 and Froot and Frankel, 1988) only expectations over longer horizons exhibit a "regressive" behaviour, i.e. the tendency to return to some nominal value. For practical purposes, this increases the persistence of shocks that affect the spot exchange rate.

The interest rate differential has a positive effect on the expected exchange rate, which partially compensates its effect on the spot exchange rate via the UIP; in the framework of this model, this means that a spot appreciation due to an increase in the current differential is only partially translated into expected rates.

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24 The current exchange rate must be conveniently instrumented, in order to avoid simultaneity with the UIP above.
Table 2

<table>
<thead>
<tr>
<th>Exchange rate expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrumental variables estimates</td>
</tr>
<tr>
<td>Sample: 1981.3 - 1994.4</td>
</tr>
</tbody>
</table>

Dependent variable: \( \left( s_{t+1/4} - s_{t/4} \right) \)

<table>
<thead>
<tr>
<th></th>
<th>Estimate 1</th>
<th>Estimate 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.346</td>
<td>0.379</td>
</tr>
<tr>
<td>( s_{t/4} - s_t )</td>
<td>-0.714</td>
<td>-0.663</td>
</tr>
<tr>
<td>( r_{t-1} - r^*_t )</td>
<td>0.596</td>
<td>0.534</td>
</tr>
<tr>
<td>( s_{t-4} - pp_{t-4} )</td>
<td>-0.052</td>
<td>-0.057</td>
</tr>
<tr>
<td>( v_{pp_{t-1}} )</td>
<td>-0.001</td>
<td>-0.055</td>
</tr>
<tr>
<td>( dmus_t )</td>
<td>-0.041</td>
<td>-0.061</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Estimate 1</th>
<th>Estimate 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>D.W.</td>
<td>1.92</td>
<td>1.77</td>
</tr>
<tr>
<td>S.D. dependent variable</td>
<td>0.0237</td>
<td>0.0237</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.0119</td>
<td>0.0124</td>
</tr>
<tr>
<td>Serial correlation ( \chi^2(4) )</td>
<td>1.87 (0.76)</td>
<td>2.21 (0.70)</td>
</tr>
<tr>
<td>Normality ( \chi^2(2) )</td>
<td>2.07 (0.15)</td>
<td>3.47 (0.176)</td>
</tr>
<tr>
<td>Heteroscedasticity ( \chi^2(1) )</td>
<td>0.291 (0.58)</td>
<td>2.48 (0.12)</td>
</tr>
<tr>
<td>Functional form ( \chi^2(1) )</td>
<td>2.07 (0.15)</td>
<td>3.05 (0.08)</td>
</tr>
</tbody>
</table>

Note: White's consistent t-statistics in parentheses.

Solving the system for the exchange rate we get:

\[
s_t - s_{t-1} = k + \frac{\beta_2}{(1 + \beta_1)}(s_{t-4} - pp_{t-4}) + f(\Delta r_t, \Delta r_{t-1}, \rho_t, \rho_{t-1})
\]

where \( \Delta r_t = r_t - r^*_t \) and \( f \) is a linear function of the exogenous variables. The dynamics of the exchange rate is, therefore, determined by the evolution of the exogenous variables \( \Delta r_t \) and \( \rho_t \) and the cointegrating vector \( (s_{t-4} - pp_{t-4}) \). The estimates imply a very slow adjustment of the exchange rate to the relative price ratio.
Equations (3) and (1) give some insight into some of the basic features of the determinants of expected and spot exchange rate. Some fundamental issues, however, still remain unanswered on econometric grounds and will require further investigation. Regarding expectations, the unexplained component is large, particularly so in the first half of 1995, suggesting that other factors may be present. Regarding the determination of the risk premium, the estimated link with exchange rate volatility is a first step, but it does not explain its fundamental determinants. In a general equilibrium model, the risk premium (as well as the exchange rate volatility) would be determined by variances and covariances of the various shocks hitting the economy, on the real side, on the monetary side, on the fiscal side (for an attempt along these lines, see Fornari et al., 1995). Although it is difficult to assess it econometrically, in the Italian case the issue of the link between fiscal imbalances, expectations on fiscal policy, inflation and the exchange rate will have to be addressed to further understand the nature of the disturbances to the exchange rate. Anecdotal evidence based on higher frequency data suggests that, in 1995, "news" regarding the domestic fiscal situation was a key determinant of exchange rate fluctuations, although it is hardly measured by some simple indicator, like the debt/GDP or deficit/GDP ratios.

All in all, the above results indicate that:

- the uncovered interest parity condition is a useful tool in modelling the determination of the exchange rate, even in the post-1992 period;
- the risk premium on short-term Italian interest rates is positively correlated with the volatility in the exchange rate market. It was constantly positive after 1992. Its unexplained component is nonetheless large;
- the effect of changes in the interest rate differential on the spot rate has the expected sign;
- in the short run, the strong adaptive characteristics of the estimated equation for exchange rate expectations tend to amplify the effect of a shock on the spot rate and to increase its persistence.

3. Long-term interest rates

In recent years, the determination of long-term interest rates in Italy was significantly affected by the growth of a large and efficient securities market, that took place mostly in the first part of the 90s.25 The speed of adjustment of market rates and their reaction to shifts in expectations increased substantially.

Some facts about the behaviour of long-term rates in the 1992-1995 period are shown in Figure 4. The relation between the domestic financial markets and the currency market strengthened: the short-term movements of bond yields and those of the exchange rate were clearly positively correlated; the same correlation showed up between the exchange rate and international interest differentials. On the contrary, movements in the interest rates directly controlled by monetary policy were often not reflected in bond yields (a positive correlation can be observed in 1993, while in 1995, as short rates increased, bond yields followed a decreasing trend).

25 On the primary market, the practice of setting a floor-price (a maximum yield) at the auctions for long term securities was abandoned in 1992, leaving the market free to determine the yields. The screen-based market for State securities (MTS) was established in 1988; new maturities for long-term securities were introduced in the following years (7, 10 and 30 years BTPs, respectively in 1990, 1991, 1993); futures markets on BTPs were created in 1991 in London and Paris; a domestic futures market started operating in 1992. In the same period, as a consequence of the full liberalisation of international capital movements completed in 1990, non-resident investors entered the market. For a description and institutional details, see Passacantando (1995).
Recent research for other EU countries (Fell, 1995) suggests that the relative importance of movements in short term rates, on the one hand, and of foreign yields, on the other, in determining domestic bond yields changed in the last decade: in the second part of the eighties and in the nineties, international linkages between bond markets increased, while the effect of policy rates on the term structure became less direct, possibly due to the different responses of inflation expectations.

The approach followed in previous versions of the model (e.g., Nicoletti et al., 1995) is based on the expectation theory of the term structure, according to which the yield of an \( m \)-period bond is given by:

\[
R_t(m) = \frac{1}{m} \sum_{j=0}^{m-1} r_t^{i+j} + \phi_m
\]

(5)

---

26 (5) holds for discount bonds. The general relation also includes terms for duration. For a survey, see Shiller (1990).
where \( r^e \) are expected one-period rates and \( \phi \) is a term premium. Under the assumption that expected real rates and the inflation rate follow an autoregressive process, the long rate is modelled as a distributed lag of past short rates and inflation rates (Modigliani-Shiller, 1973); the shape of the lag structure may be used to test assumptions on the autoregressive process used to forecast interest rates. This way to model long yields, however, implies a constant effect of policy rate changes on long-term rates, which seems at odds with some of the stylised facts above. Moreover, the expectations hypothesis in (5) abstracts from international linkages between presented bond and currency markets.

An improvement is possible by explicitly modelling inflation expectations, using survey data to estimate them, and introducing an effect of foreign bond yields; the latter may either represent a short-run effect or, more fundamentally, derived from the tendency of real yields to converge in the long run.

Under the expectations hypothesis, the following long run condition must hold:

\[
R(m) = r + \phi_m
\]

while a real interest parity, RIP (that holds if both the uncovered interest parity and ex-ante purchasing power parity hold in the long run), would imply:

\[
R(m) = R^*(m) + \pi - \pi^* + \omega
\]

where \( \pi \) is the long run expected inflation rate, \( \omega \) is a real exchange rate premium and an asterisk denotes foreign variables.

Conditions (6) and (7) may both hold in equilibrium.\(^{27}\) We estimated a model for the long rate that admits both (6) and (7) as equilibrium solutions. In Tables 3 and 4, the return on fixed income long-term bonds (TBTP)\(^{28}\) is regressed on the yield on long-term German securities (TBUND), the 3-month interbank rate on the domestic market (TIB3) and on proxies for unobservable variables as expected domestic and foreign inflation and the risk premia. The premia are modelled using currency market volatility (the coefficient of variation of daily observation in each month, EXCVOL)\(^{29}\) and, as a fiscal variable, the debt/GDP ratio. German long-run inflation was approximated with an interpolation of past realised inflation, following the approach in Jahnke (1995).

A relevant issue is how to model long-term inflation expectations. Unfortunately, the Forum-ME survey only reports short-term (one or two quarter ahead) forecasts, not long-term expectations. In some macro-models, some econometric techniques to estimate long-term expectations have been used; however, they usually make use of some - at least partial - survey evidence to perform the estimation; so, for instance, for the U.S. and (Tarditi, 1995 and Kozicki, Reifschneider and Tinsley, 1995) for Australia for the United States. Our approach is to assume that expected long-term inflation is a function of both past inflation rates (INFL) and current short-term survey-based inflation expectations (EXPINFL); through the latter variable some forward-looking elements are introduced.

---

\(^{27}\) Fell (1995) tests the impact of both short-term rates and foreign rates on long yields for a number of EU countries (Italy is not included) by estimating autoregressions that include both equilibrium conditions.

\(^{28}\) As pointed out in Nicoletti et al. (1995), the data series on BTP yields is not homogeneous through the whole period. Only since 1988, with the opening of the screen-based market, are data on constant maturity medium and long term bonds available; before this date, the existing series is a weighted average of one to ten year bonds quoted in the stock market, whose average maturity varies over time. In the estimation, we used the yield on 9 to 10 year bonds on the screen based market since after they were available (1990), and the "average" data series before this year.

\(^{29}\) Implied volatility in currency options prices may be considered a better measure of market opinions than actual volatility; however, such data are available for the lira/DM exchange rate only since 1994. The implied volatility, however, seems to follow actual volatility (with a lag) very closely.
Simultaneity problems may arise when using contemporaneous short-term rates on the RHS of the equation, since shocks to expected inflation or risk premia may produce both an increase in long rates and a policy reaction. The contemporaneous short rate was, therefore, instrumented using its past values and changes in the German three-month rate.

The estimates (including the current value and one lag of each variable (Table 3)) indicate that expected inflation and foreign rates do significantly affect the long-term rate, both dynamically (this is shown by the F-tests on the exclusion of all lags on each variable in the last column) and in equilibrium (the test on the sum of the coefficients of each variable is reported in the third column). In particular, expected inflation outperforms past actual inflation, which is no longer significant when the former is included among the regressors. The short term rate is significant, but only marginally. Expected German inflation is not statistically significant, although it has the right sign and dimension. The tests for the coefficient on currency market volatility and the debt/GDP ratio suffer from collinearity between these two variables in the sample period; after the selection procedure, the first proved to be significant.

We tested the restrictions derived from both (6) and (7) and imposed them in the final specification\(^{30}\) (Table 4): all the interest rates on the right hand side are homogeneous of degree one; the sum of the coefficients on foreign and domestic inflation is zero; the steady-state coefficient on inflation is one.

\(^{30}\) Selection proceeded from general to specific, according to the methodology in Hendry (1989).
### Table 4
#### Long-term interest rate (reduced model)
Instrumental variables estimates
Sample: 1985.2 - 1994.4
Dependent variable: TBTP

<table>
<thead>
<tr>
<th></th>
<th>Unrestricted</th>
<th>Restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 - Constant</td>
<td>-1.48</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(1.68)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>C2 - TBTP(-1)</td>
<td>0.47</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(8.52)</td>
</tr>
<tr>
<td>C3 - TBUND</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(5.04)</td>
<td>(4.81)</td>
</tr>
<tr>
<td>C4 - TBUND(-1)</td>
<td>-0.49</td>
<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>(-2.42)</td>
<td>(2.62)</td>
</tr>
<tr>
<td>C5 - EXPINFL</td>
<td>-0.32</td>
<td>-0.33</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(5.87)</td>
</tr>
<tr>
<td>C6 - TIB3Q</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(2.17)</td>
<td>(1.59)</td>
</tr>
<tr>
<td>C7 - INFLG(-1)</td>
<td>0.06</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(5.87)</td>
</tr>
<tr>
<td>C8 - EXCVOL</td>
<td>61.9</td>
<td>84.1</td>
</tr>
<tr>
<td></td>
<td>(2.76)</td>
<td>(4.64)</td>
</tr>
</tbody>
</table>

Corrected R2: 0.94 0.93
SEE: 0.38 0.39
DW: 1.78 1.77
Autocorrelation: F(1.28) = 0.07 (0.80)
Normality: χ²(2) = 1.07 (0.59)
Heteroscedasticity: χ²(1) = 0.03 (0.85)

Tests on restrictions:
- C2+C3+C4+C6=1  F(1.30) = 1.36 (0.25)
- C2+C5+C6=1  F(1.30) = 2.49 (0.12)
- C5+C7=0  F(1.30) = 0.0004 (0.98)
- Test on joint restrictions: F(3.30) = 1.12 (0.36)

The final specification in Table 4 may be rewritten as:

\[
\Delta TBTP = -0.03 + 0.83 \Delta TBUND + 0.15 \Delta TIB3Q + 0.33 \Delta EXPINFL \\
- 0.11(TBTP_{t-1} - TIB3Q_{t-1}) \\
- 0.33(TBTP_{t-1} - TBUND_{t-1} - EXPINFL_{t-1} + INFLG_{t-1}) \\
+ 84.1 EXCVOL_{t-1}
\]

The long-run solution of the equation is a combination of the expectations hypothesis (6) and of the RIP (7), with a risk premium correlated with exchange rate volatility. The coefficient on RIP is both larger in value and more significant in statistical terms; this seems to suggest a proportionately bigger impact of foreign interest rates in determining domestic yields. Since exchange

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31 Only short-term expectations enter the equation above; however, if one assumes that long-term expectations follow a partial adjustment process on short-term expectations, it is straightforward to show that their lagged level enters the RIP term in brackets with unit coefficient, and that their change enters the equation with coefficient 0.33/α (where α is the partial adjustment coefficient).
rate variability increased dramatically after the exit of the lira from the ERM of the EMS in 1992, its inclusion in the equation has the effect of permanently increasing, given other factors, the interest rate differential (the estimated effect is about one and a half percentage points).

The short-run behaviour of long yields is driven by expected inflation, that has an immediate impact on long rates of around 0.3, and by the German long rate, whose impact is about 0.8.

The equation directly links the domestic yields to foreign asset markets, to expected inflation and to the uncertainty on the currency market; correspondingly, the direct effect of current short rates on long rates is much lower. These results are similar to those obtained for other EU countries, mentioned above. A stronger, indirect effect of changes in policy rates on long yields is transmitted via inflation expectations (determined in the model along the lines discussed in section 4 below); depending on this effect, an increase in policy rates does not necessarily imply a rise in bond yields. Although no direct effect of the exchange rate on long rates is included in the equation, in a simulation of the whole model, a shock to the risk premium does generate common movements in the two variables.

In this formulation, the final effect of short-rate movements on long yields depends on the effect on inflation expectations. To close the model, one needs to specify the expectations formation mechanism.

4. Monetary policy and inflation expectations

Expected consumption price inflation, as collected by the Forum-Mondo Economico survey, actual inflation and forecast errors are shown in Figure 5.

Previous work has shown that the inflation forecasts are systematically biased and inefficient during the periods of high and volatile inflation (from 1973 to the mid-eighties) while forecast errors are very small (even if, statistically, unbiasedness is rejected by the data) and not correlated with available information during the periods of low and relative stable inflation. Purely extrapolative and/or regressive models of price expectations have little explanatory power.

An equation describing the expectation formation mechanism was recently estimated in Nicoletti-Altimari (1995), to which we refer for a more detailed analysis. It is there assumed that to forecast inflation, the agents use the variables included in the reduced form of the price-wage block of the BIQM, namely the rate of change of the effective exchange rate \( \dot{e} \); the deviation of the capacity utilisation rate from its "normal" value \( \{CPU - CPU\} \); the unemployment rate \( U \); the foreign inflation rate, \( \pi^* \) (the rate of change of average prices of manufactured goods of fourteen competitors of Italy, weighted using Italian imports shares); the rate of change of energy prices \( \dot{pe} \). To ascertain the possibility of an autonomous effect of monetary policy on inflation expectations the official discount rate \( r \) is included in the above list.

The parameters of the equation estimated with OLS are not stable over time when using the test on the constancy of parameters proposed by Granger and Terasvirta (1993) and Lin and Terasvirta (1994). Using the technique proposed by the same authors, the degree of nonlinearity of the parameters is assessed and modelled using smooth transition functions. The final estimates are reported in Table 5. All parameters have the expected sign. One sixth of the previous period's forecast error is incorporated in the revision of inflation expectations. Important effects on expectations are

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32 A thorough analysis of direct observations on inflation expectations in Italy collected in the Forum-Mondo Economico survey is contained in Visco (1984 and 1987) and, for the more recent period, in Nicoletti-Altimari (1995).
Figure 5
Actual and expected inflation
Consumption prices

Forecast errors
exerted by the exchange rate, the unemployment rate, the capacity utilisation rate and the foreign inflation rate.

Two sources of instability of the estimated coefficients were detected. The first one, captured by the transition function $LN$ (Figure 6), "transfers" the model from a specification that does not satisfy the necessary condition for rational expectations in a hypothetical long-run equilibrium to one that does$^{33}$; we interpreted it as learning. In the seventies, the economic agents were continuously surprised by innovations in the inflationary process (the two oil shocks in 1974 and in 1979, the introduction of a formal indexation mechanism in 1976), which most likely slowed the speed of the learning process. According to this interpretation, a fast convergence of the learning process is observed afterwards.

The second change in parameters, modelled by the transition function $MP$, signals the emergence of a positive impact of monetary policy, measured by changes in official rates, on inflation expectations; according to the estimates, this effect was not present before the end of 1984. Most likely, this reflects the passage from direct to indirect instruments of monetary policy (completed in 1983). Moreover, since the early eighties, inflation became the primary concern of monetary policy, and movements in official rates signalled the determination to defend the EMS parity, viewed as the main instrument to keep inflation under control$^{34}$.

Table 5
Inflation expectations
Non-linear least squares estimates
Sample period: 1971.2 - 1995.1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient (t-statistic)</th>
<th>Coefficient (t-statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1} - \pi_{t-3}$</td>
<td>$= 1.351LN - 0.485(\pi_{t-2} - \pi_{t-3}LN) + 0.152(\pi_{t-1} - \pi_{t-3})$</td>
<td>$(3.84)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 0.019\Delta \pi_{t-2} + 0.053\Delta(CPU_{t-2} - CPU) - 0.215\Delta U_{t-2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ 0.041\Delta \pi_{t-2} + 0.005\Delta \pi_{t-2} - 0.107(\Delta r_{t-1}MP)$</td>
</tr>
<tr>
<td>$LN$</td>
<td>$= \exp(-0.004(t-29.365)^2)$</td>
<td>$(-2.89)$</td>
</tr>
<tr>
<td>$MP$</td>
<td>$= 1 - \frac{1}{1 + \exp(-0.97(t-59.487))}$</td>
<td>$(6.84)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$= 0.51$</td>
<td>$\sigma_e = 0.260$</td>
</tr>
</tbody>
</table>

Note: White's t-statistics in parentheses.

Autocorrelation (1-4) : F(4.80) = 1.333 (0.265)
Heteroscedasticity : $\chi^2(1) = 5.486$ (0.019)
Normality : $\chi^2(1) = 0.439$ (0.802)
Functional Form : F(3.80) = 1.140 (0.338)

$^{33}$ Essentially the condition of cointegration of actual and expected inflation, with cointegrating vector (1, -1).

$^{34}$ See Angeloni and Gaiotti (1990).
The estimated impact of monetary policy on expectations is substantial: an increase in the official discount rate of 100 basis points decreases inflation expectations by about 0.4 percentage points on an annual basis.

The equation remains stable for the period after 1992, notwithstanding the changes that took place in the exchange rate regime and in the labour market.35

5. Simulations

In this section, we present simulations of the model under the estimated expectation mechanism as described above (hereafter EE, estimated expectations), comparing it with a “benchmark” version, where inflation expectations are modelled by a simple adaptive scheme, monetary policy has no effect on the spot exchange rate and interest rates expectations are backward-looking (hereafter BC, benchmark case); we also perform a simulation based on the rational expectations hypothesis for inflation, the exchange rate and forward rates (hereafter RE, rational expectations).

In the first exercise we analyse the consequences of a monetary policy shock under the three expectations formation schemes described above; both the risk premium and the volatility of the exchange rate are kept constant in this case. In the second exercise we analyse the effects of increased uncertainty in the currency markets by shocking both the volatility and the risk premium by an amount that takes into account the results obtained in Section 2 on the relation between them; this second case is analysed only under the EE expectations mechanism.

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35 Some evidence of overprediction is actually evident in the survey for those years: agents may have been excessively prudent with respect to those innovations and not incorporated them fully in the model. From the estimates with recursive least squares it appears however that some of the coefficients, mostly those linked to foreign shocks, have decreased somewhat after 1992. An attempt was made to introduce a third transition function for those coefficients related to the change of the degree of indexation of wages to inflation. This attempt was, however, unsuccessful.
5.1 A monetary policy shock

The first exercise consists of an increase in the policy-controlled interest rates (the overnight rate and the discount rate in the BIQM) of one hundred basis points, sustained for one year. In the RE case, a return of the nominal exchange rate to the baseline value at the end of the simulation period was assumed as a terminal condition, in line with Nicoletti et al. (1995).

The effect on real activity is very similar in all cases (Figure 7a). In the EE case, the decrease in GDP is, however, slightly stronger and longer-lasting; this reflects mainly the different behaviour of the exchange rate and long-term interest rates in the different scenarios.

Differences are substantial in the response of consumption prices (Figure 7b). In the BC version prices are virtually unaffected. There is, in fact, no direct link in the BIQM from monetary policy to prices, aside from the exchange rate: prices slowly adjust through a mark-up over average costs, where the latter are a function of unit labour costs. Since productivity in this mechanism is expressed as a long distributed lag of past productivity (due to the putty-clay nature of capital) and changes slowly, unit labour cost in the short run, mainly reflect changes in nominal wages. The latter are, however, very small, since employment and unemployment move slowly (as a consequence of both labour hoarding and the slow adjustment towards equilibrium) and the backward-looking inflation expectations do not move at all.

In the EE and RE scenarios, the dynamics of prices are very different. The initial effect is stronger under RE; the decrease in prices under EE builds up more slowly, but it is eventually stronger and more persistent, about 0.6 percent below the baseline.

The price behaviour mainly reflects the different responses of the exchange rate (Figure 7c) to the policy shock and, to a smaller extent, the different responses of inflation expectations. In the RE case, the typical overshooting pattern for the exchange rate is observed: given the (exogenous) terminal condition, the exchange rate has to appreciate in order to generate expectations of a depreciation equal to the difference in the interest rates. After an appreciation of one percent in the first period the exchange rate returns smoothly to the baseline value in the following two years. Under EE, the exchange rate keeps appreciating during the whole period of the shock as a result of the interplay between the adaptive expectation formation and the working of the UIP; afterwards, the convergence to the PPP starts to operate (Figure 7d). However, since the response of prices to the exchange rate is much faster than the response of the exchange rate to prices, the PPP tends to be reestablished at a lower level of both prices and the exchange rate. The downward movement of inflation expectations after the policy shock, on the other hand, pushes down wages, reinforcing the disinflationary process in the economy.

The behaviour of long term interest rates is shown in Figure 7e. In the EE case, the impact of monetary policy on long rates is low, and amounts to only few basis points: this reflects the pattern of inflation expectations, that adjust immediately to the increase in official rates and then keep decreasing, following the actual trend in prices. By lowering inflation expectations, the monetary tightening can leave long-term rates almost unaffected. After the third period the pattern of the long rates under EE coincides with that under RE; after six periods all three cases are very similar. The lower effect on long-term rates under EE implies lower net interest payments on public debt (Figure 7f); however, this shows up only after the second year of simulation, given the average maturity of the Italian public debt.

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36 Initial conditions for the simulation are those of the first quarter of 1993.
37 Here we will focus essentially on the differences of results under the different expectational schemes. For a complete description of the transmission channels of monetary policy in the BIQM the reference is Nicoletti et al. (1995).
38 Particularly the response of export prices that are the ones relevant in our specification. The downward movement of domestic prices is the result of both the decrease of prices of imported goods and raw materials and of the loss of competitiveness of domestic producers which narrows the mark-up.
Figure 7

Effects of a one-year increase in the policy-controlled interest rate

- Endogenization of observed expectations
- Rational expectations
- Adaptive expectations and fixed exchange rate

GDP
(Percentage deviations from the baseline)

Figure 7a

Consumption Prices
(Percentage deviations from the baseline)

Figure 7b

LIRA-DM
(Percentage deviations from the baseline)

Figure 7c
Effects of a one-year increase in the policy-controlled interest rate

**Figure 7d**

*Purchasing Power Parity*

(Percentage deviations from the baseline)

**Figure 7e**

*Ten years bond rate*

(Absolute differences from the baseline)

**Figure 7f**

*Net interest Payments on Public Debt*

(Percentage deviations from the baseline)
As far as forecast errors are concerned (Figures 7g and 7h), it is seen that errors are not white noise under EE (persistence); they reproduce the historical behaviour of expectation errors (see Figures 1a and 4 above). Expectations errors are bigger under EE than under BC; it must, however, be considered that in the latter case the underlying price profile is much less volatile than in the first case.

5.2 A shock to the risk premium

In a second exercise, limited to the EE case, the effects of an increase in uncertainty in the currency market, represented by an increase in the monthly coefficient of variation of the lira/DM exchange rate, were simulated. In performing the exercise, the coefficient of variation was shocked by an amount corresponding to a 1 percent increase of the risk premium in the UIP (annual basis), as estimated in Section 2.
Figure 8
Effects of an increased uncertainty in the currency markets

GDP
(percentage deviations from the baseline)

Consumption Prices
(percentage deviations from the baseline)

LIRA-DM
(percentage deviations from the baseline)

Purchasing Power Parity
(percentage deviations from the baseline)

Ten years Bond-Rate
(absolute differences from the baseline)

Net Interest payments on Public Debt
(percentage deviations from the baseline)

Inflation Forecast Errors
(absolute differences from the baseline)

Exchange Rate Forecast Errors
(percentage deviations from the baseline)
The results are shown in Figure 8 (in evaluating them, it must be kept in mind that they are conditional on the assumption of no reaction of the policy rates to the depreciation in the exchange rate and to the increase in inflation). The exchange rate depreciates by almost three percent during the first year of simulation; afterwards, it slowly returns towards its new equilibrium. The real exchange rate initially depreciates, by up to 1.5 percent; subsequently, it returns to the baseline by the end of the simulation period; as prices increase after the exchange rate shock, this happens at a higher level of both prices and the nominal exchange rate. The increased risk premium, on the other hand, exerts an upward pressure on the long-term interest rate, as described in Section 3: the yield on ten-year bonds increases by 60 basis points by the beginning of the second year, declining steadily afterwards. As a result, a co-movement of the exchange rate and long-term interest rates is observed. If not contrasted by a monetary policy action, a higher level of inflation (by 0.4, 0.7, 0.4 percent in the first three years, and 0.1 afterwards) is observed through the whole simulation period. In the first year, the real exchange rate depreciation generates a stronger GDP growth, up to above 0.3 percentage points. The level of real activity tends to go back to that of the baseline simulation in the following years, as the gain in competitiveness starts shrinking.

Conclusion

The analysis in this paper is still tentative. However, some conclusions may be drawn.

The study of survey data on exchange rate and inflation expectations suggests that the exchange rate in the short run is characterised by a strong adaptive behaviour; it is also affected by a risk premium correlated with currency market volatility. Long-run interest rates react to changes in inflation expectations; they are also strongly affected by foreign yields and volatility on the currency market. A monetary policy tightening has a significant effect on inflation expectations; it affects the exchange rate through the UIP condition.

Monetary policy transmission

A more careful modelling of expectations may substantially alter the way monetary policy works its way through the economy in the macroeconomic model of the Banca d'Italia. Two effects were examined in this paper. The first, and by far the more important, effect deals with the endogenisation of the exchange rate; to the extent that an increase in interest rates is not compensated by an increase in the risk premium or by depreciating expected exchange rate, it can induce a spot appreciation. An adaptive expectation formation can then in the short run generate a virtuous circle of exchange appreciation and lower inflation.

The second effect deals with the impact of a monetary tightening on inflation expectations and, consequently, on long-term rates. The evidence we present shows that, under proper conditions, it is not unrealistic to imagine that an increase in short rates diminishes inflation expectations and leaves long rates unaffected (what has been defined the “dream of a central banker”). However, beyond the framework of the model, the occurrence of this possibility depends on a number of conditions to ensure that the monetary policy announcement is perceived as credible by the market.

A third effect, not discussed here although to some extent connected with the former two, will have to be addressed to assess the effectiveness of monetary policy through the expectations channel. It deals with the interaction between monetary and fiscal policy in determining expectations of debt sustainability and their feedback on long-term inflation expectations and the exchange rate; the issue of the need for coordinating monetary and fiscal policy in pursuing exchange rate and price stability is connected to this effect. The positive effect of monetary policy on inflation expectations described above may be seen as a first indication that a tight monetary policy is perceived as inducing
also more fiscal discipline, and that the “unpleasant monetarist arithmetic” does not hold; the issue, however, deserves further research.

**Effect of uncertainty on the currency market on domestic monetary conditions**

Domestic monetary conditions are seen to be dependent on the uncertainty originating in the currency market, via its effect on both the spot exchange rate and long-term yields. If not counteracted by a monetary tightening, an increase in the volatility results in an exchange rate depreciation, higher inflation and higher long-term rates

**Effect of different mechanisms of expectation formation**

As far as the comparison of the effects of different expectation mechanisms are concerned, RE and EE give rather similar results, although expectations under EE are not unbiased. A major difference, however, lies in the absence of an exogenous terminal condition for the nominal exchange rate, hence for prices, under EE. For given policy rates, this opens the possibility of a price-exchange rate spiral, that increases the effect on prices of both monetary policy and external shocks.

All in all, the endogenous determination of the exchange rate and long term rates makes monetary policy more effective, and it opens the possibility of an exchange-price virtuous circle. However, it makes the economy more vulnerable to external shocks. Changes in the risk premium may adversely affect long rates and the exchange rate; if not offset by monetary policy, they have permanent effects on prices.

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39 Using a VAR approach for Italy and Ireland (two high-debt countries), Lane and Prati (1995) recently found that a monetary restriction both reduces inflation and induces an improvement in the primary balance in the long-run, concluding that the “unpleasant monetarist arithmetic” does not hold.
References


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The theorist who studies the modelling of financial asset prices is immediately confronted with the difficult question of how do economic agents form their expectations about future market developments. For the econometrician working on the same topic the problem is even more complex as he or she must also assign numerical values to something that is not directly measurable. The treatment of expectations in macroeconomics has been a contentious issue for more than thirty years now. The parallel history of attempts to incorporate explicit expectational assumptions in the building of empirical macroeconomic models (of any scale) has been at least as contentious.

One approach to deal with this issue has been to offer some rationale why expectation formation might obey a fixed general pattern or *ad hoc* rule and subsequently impose it on the empirical model before estimation. The problems with the internal consistency of the estimated model were pointed out by Robert Lucas in his famous critique. An alternative way was to bypass explicit estimation by using some clever trick which would allow to essentially sweep the problem under the carpet; and I tend to think of rational expectations assumptions in this way. Using the strong orthogonality restrictions suggested by rational expectations the empirical analyst can usually substitute the unobservable expectations component with some ex-post observable or "rational" quantity. This is not meant to minimise the contribution of rational expectations to economics. Quite the opposite, by posing the simple question: "If expectations are not rational then what are they?" rational expectations theorists have done the profession a big favour by enforcing a greater degree of intellectual discipline and internal consistency in both theoretical and empirical study.

The authors of this paper have taken yet a third route by proposing to ask the market participants directly about their expectations. They did so through the Forum Mondo-Economico, and then incorporated these expectations into the large scale Quarterly Model of the Banca d'Italia (QM). This is an interesting approach and one that this reader would encourage as we stand to learn a lot about the economic process from this kind of exercise. We economists are certainly guilty of projecting the assumptions that make our theoretical constructions elegant on human behaviour. Tests like the one at hand will either make us feel better about this practice, if we can indeed reconcile the existing theory with actual expectations, or force us to look in a different direction.

Having said that, we should also recognise the fact that survey data do not represent the magic solution of the problem of quantification of expectations. There are some obvious, and maybe some not so obvious, pitfalls in taking these data as representing what we call in our models "expectations". The main issues to be resolved have to do with the representativeness of these measures and of the characteristics of the market participants' expectations as forecasts of future developments. First there is the question of measurement. The number of different answers you will get if you ask the question of "what is inflation going to be one year from now?" will be bounded above only by the number of people you approach. This does not fit well the representative agent paradigm and therefore some way of condensing the information from the sample of answers to a single figure is necessary. What is the best way of extracting such a representative measure of expectations is very much an open question, and not much research has been done on this issue.

The second question has to do with the forecasting properties of survey measures of expectations. Are they accurate predictors of future realisations? Are they efficient? There is no reason why these forecasts would have to be "rational" in this sense, but it is important to subject them to the same tests we do put other statistical predictors, and examine whether they pass. In case that they fail the tests it is also interesting to investigate why. The paper goes some way in addressing this issue of validation in an indirect way when it tests the Uncovered Interest Parity condition as a modelling device for the exchange rate using the Mondo-Economico survey measures. The result is that expectations of the future path of the exchange rate are in line with this "arbitrage" restriction. This is quite encouraging as it complies with a notion of rationality that many would find uncontroversial, but I feel that further evaluation is necessary before one can feel comfortable using these data in econometric models. To cite an example of alternative tests I found the comparison in
the Federal Reserve Board paper of the long-run model implied equilibrium with the survey expectations for the long-run inflation rate quite instructive.

Gaiotti and Nicoletti-Altimari examine the determination of the Lira/DM exchange rate and the interest rate on the long term Italian government bond yields. To accomplish this they employ one equation which determines the price of the respective asset and another which explains the expectation formation process. Subsequently they tie these equations together as a block to the QM and simulate the response of the economy to monetary policy and uncertainty shocks. I will briefly discuss each building block separately.

First, regarding the exchange rate block, my main problem is about the expectation formation equation. It is not clear to me why the particular specification was used, and in particular why was the PPP term included in the final specification given that it is not imposed on the exchange rate equation and that it is not statistically significant. The only interpretation I can give to this fact is that its inclusion is an indirect way of bringing the exchange rate block in line with the structure of the rest of the Quarterly Model. But in that case shouldn't there be an explicit accounting for PPP in the main exchange rate equation?

Another point which also applies to the bond rate equations regards the particular choice of volatility measure as a proxy for exchange rate risk. The DM/lira volatility may not be the best way of capturing the relative risk of Lira denominated assets compared to those denominated in DM because it cannot differentiate between the two currencies in terms of relative variability. High variability of the lira/DM exchange rate can be associated with either currency being relatively stable with respect to the rest of the world and the other being volatile. Consequently the sign of the risk premium is not clearly determined, at least in theory. Although one could argue that historically the DM has been the "anchor" currency and there have not been periods when the lira was the more stable member of the pair, a measure like the spread of the volatilities of the two currencies with respect to a third one (e.g. USD, CHF) would be a preferable alternative.

As far as the long rate block is concerned, I would have to raise the obvious concern with the fact that the survey measure of inflationary expectations refers to an interval significantly shorter than the maturity of the assets. This is a clear example of the possible problems involved in the incorporation of direct observation measures of expectations in statistical models, and one that will require serious attention before we can use them more extensively. We basically need some evidence to validate the assumption made by the authors that the one-period ahead expectations are good proxies for the expected inflation ten years into the future. This implies that we need to extrapolate these one-year ahead expectations somehow; assuming that they are constant is a way of doing this extrapolation, but it will require validation.

The inclusion of the foreign bond yield in the equation is not uncontroversial but does not surprise me. I believe that cross-country correlation of the long rate process may not necessarily be compatible with the closed economy expectations hypothesis of the term structure. My concern has to do with the inclusion of contemporaneous foreign rates in view of the simulation exercise included in the paper. I am not familiar with the QM model but I suspect that does not include the German long interest rate as an endogenous variable. This raises question of what is the assumed path of the German rate when the simulations were performed? And if in fact it is the interest rate spread that the researchers are interested in would it not be better to model it explicitly in the first place?

Regarding the simulation exercise I would not have much to say other than it provides a useful tool to perform a joint evaluation of the usefulness of the particular survey measure of expectations and the way they were incorporated in the Banca d'Italia Quarterly model. The discrepancies of the results from the different expectational assumptions need to be studied very carefully in order to assess the validity of these assumptions, and in this respect the paper is a step in the right direction which needs to followed up by more extensive research. Macroeconomic models represent an excellent framework for evaluating the usefulness of the information found in these surveys, and central bank economists which have access to such tools have a comparative advantage in performing those tests. I certainly hope that we can see more work on this subject.
The yield curve as a predictor of recessions in the United States and Europe

Arturo Estrella and Frederic S. Mishkin

Introduction

Economists often use complex mathematical models to forecast the path of the US economy and the likelihood of recession. But simpler indicators such as interest rates, stock price indexes, and monetary aggregates also contain information about future economic activity. In this paper, we examine the usefulness of one such indicator - the yield curve, that is, the spread between long and short-term interest rates.

Our analysis differs in two important respects from earlier studies of the predictive power of financial variables. First, we focus simply on the ability of these variables to forecast recessions rather than on their success in producing quantitative measures of future economic activity. We believe this is a useful approach because signs of an oncoming recession are always of concern to policymakers and market participants. Second, instead of focusing solely on in-sample performance, we also focus on out-of-sample performance, that is, accuracy in predictions for quarters beyond the period over which the model is estimated. This is particularly important because out-of-sample performance provides a much truer picture of how well an indicator will do when it is actually used in a real world forecasting exercise.

1. Why consider the yield curve?

For several reasons, the steepness of the yield curve should be an excellent indicator of a possible future recession. First, current monetary policy has a significant influence on the yield curve spread and hence on real activity over the next several quarters. A rise in the short rate would tend to flatten the yield curve as well as slow real growth in the near term. Although this relationship is very likely part of the story, it is not the whole story. Expectations of future inflation and real interest rates contained in the yield curve spread seem to play an important additional role in the prediction of future activity. The yield curve spread variable we examine here corresponds to a forward interest rate applicable from three months to ten years into the future. As explained in Mishkin (1990a, 1990b), this rate can be decomposed into expected real interest rate and expected inflation components, each of which may be helpful in forecasting real growth. The expected real rate may be associated with expectations of future monetary policy. Moreover, because inflation tends to be

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2 Stock and Watson (1989, 1992) and Watson (1991) also focus on predicting recessions. Boldin (1994), in an alternative approach, models recessions using a regime-switching formulation. In a recent paper, Reinhart and Reinhart (1996), using very different methods than in this paper, find that the best predictors of recession in Canada are the US and Canadian term structure spread, a conclusion that is similar to the one found in this paper.

3 The analysis in Estrella and Hardouvelis (1990,1991) and Estrella and Mishkin (1995) suggests why the yield curve contains information beyond that related to monetary policy.
positively related to activity, the expected inflation component may be informative about future real growth.

2. **Estimating the probability of recession**

To assess how well each indicator variable predicts recessions, we use the so-called probit model, in which the probability of being in a recession is directly related to a specific explanatory variable such as the yield curve spread.\(^4\) To see how the model works, consider the results of one of the most successful models in the article which estimates the probability of being in a recession four quarters in the future in the United States as a function of the current value of the yield spread between the ten-year Treasury note and the three-month Treasury bill. (The model is estimated using data from the first quarter of 1960 to the first quarter of 1995.) Table 1 shows the values of this yield curve spread that correspond to estimated probabilities of a US recession four quarters in the future. As the table indicates, the estimated probability of a recession four quarters ahead estimated from this model is 10 percent when the spread averages 0.76 percentage points over the quarter, 50 percent when the spread averages -0.82 percentage points, and 90 percent when the spread averages -2.40 percentage points.

![Table 1](image)

**Table 1**

*Estimated US recession probabilities for probit model using the yield curve spread*

<table>
<thead>
<tr>
<th>Recession probability (percent)</th>
<th>Value of spread (percentage points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.21</td>
</tr>
<tr>
<td>10</td>
<td>0.76</td>
</tr>
<tr>
<td>15</td>
<td>0.46</td>
</tr>
<tr>
<td>20</td>
<td>0.22</td>
</tr>
<tr>
<td>25</td>
<td>0.02</td>
</tr>
<tr>
<td>30</td>
<td>-0.17</td>
</tr>
<tr>
<td>40</td>
<td>-0.50</td>
</tr>
<tr>
<td>50</td>
<td>-0.82</td>
</tr>
<tr>
<td>60</td>
<td>-1.13</td>
</tr>
<tr>
<td>70</td>
<td>-1.46</td>
</tr>
<tr>
<td>80</td>
<td>-1.85</td>
</tr>
<tr>
<td>90</td>
<td>-2.40</td>
</tr>
</tbody>
</table>

Note: The yield curve spread is defined as the spread between the interest rate on ten-year US Treasury note and the three-month US Treasury bill.

The usefulness of the model can be illustrated through the following examples. Consider that in the third quarter of 1994, the spread in the United States averaged 2.74 percentage points. The corresponding predicted probability of recession in the third quarter of 1995 was only 0.2 percent, and indeed, a recession did not materialise. In contrast, the spread averaged -2.18 percentage points in the first quarter of 1981, implying a probability of recession of 86.5 percent four quarters later. As

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\(^4\) For a technical discussion of this model and how it is estimated, see Estrella and Mishkin (1996). The economy is designated as "in recession" starting with the first quarter after a business cycle peak and continuing through the trough quarter. The peak and trough dates are the standard ones issued by the National Bureau of Economic Research (NBER) and used in most business cycle analysis. These dates are not without controversy, however, because the NBER methodology makes implicit assumptions in arriving at these dates.
predicted, the first quarter of 1982 was in fact designated a recession quarter by the National Bureau of Economic Research (NBER).

3. Results for the United States

Although the yield curve has advantages as a predictor of future economic events, several other variables have been widely used to forecast the path of the economy. Among financial variables, stock prices have received much attention. Finance theory suggests that stock prices are determined by expectations about future dividend streams, which in turn are related to the future state of the economy. Among macroeconomic variables, the Commerce Department's (now the Conference Board's) index of leading economic indicators appears to have an established performance record in predicting real economic activity. Nevertheless, its record has not always been subjected to careful comparison tests. In addition, because this index has often been revised after the fact to improve its performance, its success could be overstated. An alternative index of leading indicators, developed in Stock and Watson (1989), appears to perform better than the Commerce Department's index of leading economic indicators. In the discussion below, we compare the predictive power of these three variables for US recessions with that of the yield curve.5

Using the probit model estimates, we can compare how well the yield curve forecasts US recessions with that of the New York Stock Exchange (NYSE) stock price index, the Commerce Department's index of leading economic indicators, and the Stock-Watson index. Charts 1-8 plot the forecasted probabilities of a recession in the United States for one, two, four, and six quarters in the future and the actual periods of recession (shaded in the charts).6 To understand how to read these charts, consider the forecast for the fourth quarter of 1990, which is the first quarter after the peak of the business cycle and is thus at the start of the last shaded recession region on the charts. In Chart 1, which shows the forecast for the fourth quarter of 1990, which is the first quarter after the peak of the business cycle and is thus at the start of the last shaded recession region on the charts. In Chart 1, which shows the forecast for the fourth quarter of 1990, the probability of recession from the probit model using the yield curve spread variable (SPREAD) forecasted in the third quarter of 1990 for the fourth quarter of 1990 is 13 percent. Similarly, in Chart 7, which shows forecasts six quarters ahead, the forecasted probability of recession for the fourth quarter of 1990 - 22 percent - is generated from a model using the yield curve spread as of the second quarter of 1989.

5 In Estrella and Mishkin (1996), we have examined in detail the predictive ability of these and other variables, including interest rates by themselves, other stock market indexes, interest rate spreads, monetary aggregates (both nominal and real), the component series of the index of leading economic indicators, and an additional experimental index of leading indicators developed in Stock and Watson (1992). Of all the variables, the four singled out in this article have the best ability to predict recessions.

6 Note that the forecasts in these charts are true out-of-sample results which have been obtained in the following way: First, a given model is estimated using past data starting with the first quarter of 1959 up to a particular date, say the first quarter of 1970. Then these estimates are used to form the forecasts, say four quarters ahead. In this case, the projection would apply to the first quarter of 1971. After adding one more quarter to the estimation period, the procedure is repeated. That is, data up to the second quarter of 1970 are used to make a forecast for the second quarter of 1971. In this way, the procedure mimics what a forecaster would have predicted with the information available at any point in the past.
Chart 3
Probability of recession in the United States, two quarters ahead

Chart 4
Probability of recession in the United States, two quarters ahead
Chart 7
Probability of recession in the United States, six quarters ahead

Chart 8
Probability of recession in the United States, six quarters ahead
In assessing these charts, we must also understand that even a probability of recession that is considerably less than one can be a strong signal of recession. Because in any given quarter the probability of recession is quite low, a forecasted probability of, say, 50 percent is going to be quite unusual. Indeed, the successful forecasting model described in the table yields probabilities of recession that are typically below 10 percent in nonrecession (unshaded) periods (Chart 5). Thus, even a probability of recession of 25 percent - the figure forecast for the fourth quarter of 1990 from data on the yield curve spread one year earlier - was a relatively strong signal in the fourth quarter of 1989 that a recession might come one year in the future.

The charts invite two basic conclusions about the performance of our four variables: 7

- Although all the variables examined have some forecasting ability one quarter ahead, the leading economic indicator indices, particularly the Stock-Watson index, produce the best forecasts over this horizon.

- In predicting recessions two or more quarters in the future, the yield curve dominates the other variables, and this dominance increases as the forecast horizon grows.

Let's look in more detail at the probability forecasts in Charts 1-8. Charts 1 and 2 show that the indexes of leading economic indicators typically outperform the yield curve spread and the NYSE stock price index for forecasts one quarter ahead. For the 1973-75, 1980, and 1981-82 recessions, both indices of leading economic indicators, and particularly the Stock-Watson index, are quite accurate, outperforming the yield curve spread and the NYSE stock price index with a high predicted probability during the recession periods. However, despite excellent performance in these earlier recessions, the Commerce Department indicator provides several incorrect signals in the 1982-90 boom period and the Stock-Watson index completely misses the most recent recession in 1990-91. 8 Although the financial variables - the yield curve spread and the NYSE stock price index - are not quite as accurate as the leading economic indicators in predicting the 1973-75, 1980, and 1981-82 recessions, they do provide a somewhat clearer signal of an imminent recession in 1990.

As the forecasting horizon lengthens to two quarters ahead and beyond, the performance of the NYSE stock price index and the leading economic indicator indexes deteriorates substantially (Charts 3-8). Indeed, at a six quarter horizon, the probabilities estimated using the three indexes are essentially flat, indicating that these variables have no ability to forecast recessions. By contrast, the performance of the yield curve spread improves considerably as the forecast horizon lengthens to two and four quarters. The estimated probabilities of recession for 1973-75, 1980, and 1981-82 based on the yield curve spread are substantially higher than at the one-quarter horizon, and the signal for the 1981-82 recession no longer comes too early (compare Charts 3 and 5 with Chart 1).

Furthermore, in contrast to the other variables, the yield curve spread does give a relatively strong signal in forecasting the 1990-91 recession four quarters ahead. Although the forecasted probability is lower than in previous recessions, it does reach 25 percent (Chart 5). There are two reasons why the signal for this recession may have been weaker than for the earlier recessions. First, restrictive monetary policy probably induced the 1973-75, 1980-81, and 1981-82 recessions, but played a much smaller role in the 1990-91 recession. Because the tightening of monetary policy also affects the yield curve, we would expect the signal to be more pronounced at such times. Second, the

7 Note that all conclusions drawn from looking at the charts are confirmed by more precise statistical measures of out-of-sample fit in Estrella and Mishkin (1996).

8 These results have already been noted in very useful postmortem analyses by Watson (1991) and Stock and Watson (1992).
amount of variation in the yield curve spread has changed over time and was much less in the 1990s than in the early 1980s, making a strong signal for the 1990-91 recession difficult to obtain.\footnote{Another potential explanation is that the 1990-91 recession was relatively mild and so a weaker signal might be expected. However, as shown in Estrella and Hardouvelis (1991), the yield curve spread also provides much weaker signals for recessions in the 1950s, even though they were not mild. Furthermore, the signal for the 1969-70 recession is strong, although the recession itself was mild. Thus the severity of the recessions does not seem to be associated with the strength of the signal from the yield curve.}

When we look at how well the yield curve spread forecasts recessions six quarters in the future (Chart 7), we see that the performance deteriorates from the four-quarter-ahead predictions. Nonetheless, unlike the other variables considered, the yield curve spread continues to have some ability to forecast recessions six quarters ahead.

4. Results for Europe

Given the results for the United States which indicate that the yield curve spread has its best forecasting performance four quarters ahead, we examine how well the domestic yield curve spreads for France, Germany, Italy and the United Kingdom perform in predicting recessions in these countries four quarters in the future.\footnote{The yield curve spreads for each country are comparable to those for the United States. For France the yield curve spread is the interest rate on long term public and semi-public sector bonds, secondary market minus the 3-month Paris interbank offer rate; for Germany, the interest rate on 10-year, federal public bonds, secondary market, minus 3-month loan rate; for Italy, the interest rate on Treasury bonds, net of tax, secondary market, minus the interest rate on 3-month ordinary Treasury bills, gross of tax; and for the United Kingdom, the interest rate on 10-year, medium dated, government stocks, minus the 91-day Treasury bill, average allotment rate. Bernard and Gerlach (1995) conduct a similar exercise for France, Germany and the United Kingdom and also find that foreign yield curve spreads have additional explanatory power in forecasting recessions in some cases.} For each of these countries, Chart 9 provides the forecasted probabilities of recessions four quarters in the future together with the actual periods of recession in shaded areas.\footnote{For the European countries, the economy is designated as "in recession" starting with the first quarter after a business cycle peak and continuing through the trough quarter. The peak and trough dates for each of these countries are from the following sources suggested in Bernard and Gerlach (1995): France, Allard (1994, p. 28, Table 2); Germany, Deutsche Bundesbank (1995, p. 86); Italy, Center for International Business Cycle Research, Columbia University; United Kingdom, Central Statistical Office (1995, p. T76).} Since we cannot estimate models and then perform out-of-sample forecasts with less than ten years of data and our sample for the European countries only starts in 1974, Chart 9 differs from the previous charts in that it only shows out-of-sample forecasts beginning in 1985. Because there is thus a very short sample period for the out-of-sample forecasts, the chart also provides in-sample estimates of the recession probabilities. To ensure comparability, the US results are also reported using the same sample period as for the European countries.

As we can see in Chart 9, the out-of-sample forecasts are generally quite close to those of the in-sample forecasts; thus it is reasonable to look at the in-sample results in addition to the out-of-sample results to assess the yield curve's forecasting performance in these countries. The yield curve spread seems to have some ability to forecast recessions in all these countries and formal statistical measures confirm this. Particularly striking are the results for Germany which indicate that the German yield curve spread has been an accurate forecaster of German recessions; as in the United States, forecasted probabilities of recession are low during nonrecession periods and the probabilities reached during recession periods are even higher than in the United States. The results for the United Kingdom are also quite good, but are not quite as strong as in the United States or Germany. Peaks in the forecasted probabilities are more prone to be late and estimated recession probabilities are often fairly high in nonrecession periods. The results for France and Italy are weaker than in the other
Chart 9
Probability of recessions with yield curve spread, four quarters ahead

In-Sample, 1974-1994 ———
Out-of-Sample, 1985-1995 - - - -
countries. The differences between probabilities in recession and nonrecession periods for France and Italy are less than in the other countries and there are more false signals of recession when recession probabilities rise above one-half during nonrecession periods. These weaker results are not too surprising because there may be substantial measurement error in recession dates for these countries, as is evidenced by disagreements about the appropriate recession dates for European countries.  

Conclusion

This article has examined the performance of the yield curve spread and several other financial and macroeconomic variables in predicting future recessions. The results obtained from a model using the yield curve spread are encouraging and suggest that the yield curve spread can have a useful role in macroeconomic prediction, particularly with longer lead times. Because forecasters and policymakers care more about longer term forecasts, the fact that the yield curve strongly outperforms other variables at longer forecasting horizons makes its use as a tool in the forecaster’s toolbox even more compelling.

With the existence of large-scale macroeconometric models and with the judicious predictions of knowledgeable market observers, why should we care about the predictive ability of the yield curve? There is no question that judgmental and macroeconometric forecasts are quite helpful. Nevertheless, the yield curve can usefully supplement large econometric models and other forecasts for three reasons. First, forecasting with the yield curve has the distinct advantage of being quick and simple. With a glance at long-term bond and three-month bill rates on the computer screen, anyone can compute a probability forecast of recession almost instantaneously by using a table such as ours. Second, a simple financial indicator such as the yield curve can be used to double-check both econometric and judgmental predictions by flagging a problem with the results of more involved approaches. On the one hand, if forecasts from an econometric model and the yield curve agree, confidence in the model’s results can be enhanced. On the other hand, if the yield curve indicator gives a different signal, it may be worthwhile to review the assumptions and relationships that led to the prediction. Third, using the yield curve to forecast with the framework outlined here provides a forecasted probability of a future recession, a probability that is of interest in its own right.

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12 For example, the Center for International Business Cycle Research (CIBCR) at Columbia University has quite different recession dates for France than the Allard (1994) source we use here. European economists have indicated to us that the Allard (1994) dates are more accurate than the CIBCR dates and we do find better results with the Allard (1994) dates.
Notes to Charts 1-8

Source: Authors' calculations

Notes: The probabilities in these figures are derived from out-of-sample forecasts, one, two, four and six quarters ahead. For example, the forecasted probabilities in charts 1 and 2 are for one quarter ahead - that is, the probability shown is a forecast for the contemporaneous quarter, using data from 1 quarter earlier - while for charts 7 and 8, the forecasted probabilities are for six quarters ahead. SPREAD denotes the forecasts from the model using the yield curve spread (the difference between the interest rate on ten-year Treasury bonds and on three-month Treasury bills, both on a bond-equivalent basis) as the explanatory variable. NYSE denotes the results from the model using the quarterly percentage change in the NYSE stock price index as the explanatory variable. LEAD denotes the forecasts from the model using the quarterly percentage change in the Commerce Department's index of leading indicators as the explanatory variable, while Stock-Watson denotes the forecasts using the quarterly percentage change in the Stock-Watson (1989) leading economic indicator index. Shaded areas designate "recessions" starting with the first quarter after a business cycle peak and continuing through the trough quarter. The peak and trough dates are the standard ones issued by the National Bureau of Economic Research.

Notes to Chart 9

The probabilities in these figures are for forecasts four quarters ahead using a probit model with the yield curve spread (as defined in footnote 10) as an explanatory variable; that is, the probability shown is a forecast for the contemporaneous quarter, using data from four quarters earlier. The economy is designated as "in recession" starting with the first quarter after a business cycle peak and continuing through the trough quarter. The peak and trough dates for each of these countries are from the following sources: France, Allard (1994, p. 28, Table 2); Germany, Deutsche Bundesbank (1995, p. 86); Italy, Center for International Business Cycle Research, Columbia University; United Kingdom, Central Statistical Office (1995, p. T76).
References


Comments on paper by A. Estrella and F. S. Mishkin by Stefan Gerlach (BIS)

During the last few years there has been considerable increase in central bank interest in the term structure of interest rate as an indicator for monetary policy purposes. There are essentially five factors that explains this interest:

- First, the slope of the term structure has been shown in a number of studies, using data for different time periods and countries, to contain considerable information about the future path of short term interest rates, inflation rates and real economic growth.

- Second, since interest rates are essentially instantaneously observed, the term structure provides immediate information about changes in financial market participants' expectations about the future path of the economy. This is particularly important in conditions of large discrete changes in economic policy, such as the announcement of a new fiscal plan, or a change in the exchange rate regime.

- Third, interest rate data are not subject to data revisions. The common problem of forecasting changes in economic conditions on the basis of preliminary estimates - that are likely to be reversed to an unknown extent in an unknown direction - of macroeconomic data is therefore avoided.

- Fourth, since yields are observed on financial instruments that may have long maturities - 10 years or even more - it is possible to provide estimates of financial market participants' expectations for long time horizons. Such information is difficult to come by in other ways.

- Fifth, expectations of the future embodied in interest rates constitute large bets by market participants about the future path of the economy. Needless to say, market participants have very good reasons for trying to get those guesses rights. This is not necessarily the case for answers to surveys of market expectations.

The paper by Estrella and Mishkin demonstrates, as the authors have done together and with other co-authors elsewhere, that the term-structure of interest rates contains information that is useful in predicting the likelihood of a future recession in the United States and in four European countries. While the findings reported in the paper are of considerable interest to monetary policy makers, I have two concerns with the paper.

First, the results presented stem from estimated probit models, which can be thought of as a (non-linear) regressions in which the dependent variable takes the value of 1 if a recession occurred and 0 otherwise. One aspect of this approach is the fact that it uses the data inefficiently. To see this, note that instead of using, say, the growth rate of GDP as the dependent variable, the observations are grouped into low growth (recession) and high-growth (non-recession) quarters. Thus, much of the "detail" in the data is disregarded. Since forecasts of the future real growth rates can be used to construct estimates of the likelihood of a recession, the authors' argument that "...recessions are always of concern to policy makers and market participants..." does not explain their choice of technique, however persuasive it may be.

Second, while the authors demonstrate that the slope of the yield curve contains information about future real economic conditions, they do not explain why it does so. The authors indicate that there are two possible answers. First, the relationship may be due to expectations: financial market participants that expect a recession to come may quite naturally expect inflation to subside, and short-term interest rates to fall in response to a relaxation of monetary policy. If this is

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13 Probit techniques are typically used when it is difficult to quantify the dependent variable, or when it can only take two values. This is not the case here.
the case, the relationship could very well shift if it was used in the conduct of policy. Second, the relationship may be *causal*: a negatively sloped yield curve may be a reflection of tight monetary policy which in turn will lead to a slow-down in economic activity in the future. In this case the relationship is structural and can be used for policy purposes. In particular, the slope of the yield curve can be used as a measure of how tight policy is. Since it is likely that the correlation between the current slope of the yield curve and future real economic conditions is due to both expectation and causal factors, it would seem desirable to have a clearer sense of their relative importance before the relationship is exploited for policy purposes.
A comparison of alternative monetary policy rules in the Federal Reserve Board's Multi-Country Model

Andrew Levin

Introduction

In recent years, monetary policy rules have received increasing attention from academic economists and policymakers. Most analysts do not believe that the underlying structure of industrialised economies is sufficiently well understood to obtain an optimal policy rule that could be used to fine-tune an economy in every conceivable situation. Instead, monetary policy rules have primarily been designed to serve as an approximate gauge in determining an interest rate target consistent with stable inflation and sustainable real economic growth. The announcement of such a rule may also enhance the public's understanding of current monetary policy actions, and thereby strengthen the overall credibility of the central bank.

This note reviews recent modifications of the Federal Reserve Board's Multi-Country Model (FRB/MCM) that have facilitated the comparison of alternative monetary policy rules under model-consistent expectations (often referred to as forward-looking or "rational" expectations) as well as under VAR-based expectations (also referred to as backward-looking or "adaptive" expectations). These modifications have mainly involved renormalisation of equations for the term structure of interest rates, the determination of overlapping wage contracts, and uncovered interest parity in the foreign exchange market.

The updated FRB/MCM has been used to evaluate three specific monetary policy rules, each of which prescribes a short-term interest rate target based on the current output deviation from potential and either (a) the current price level deviation from a specified target path; or (b) the current inflation deviation from a specified target rate. Dynamic simulations of the global model in response to US aggregate supply and demand shocks generally confirm the favourable properties of a policy rule considered by Henderson and McKibbin (1993). By targeting inflation rather than the price level, the H-M rule generates greater output stability and similar inflation stability compared with a policy rule based on nominal GDP targets. By prescribing larger interest rate adjustments in response to the current output gap and current inflation deviation from target, the H-M rule generates more stable economic activity and inflation compared with the monetary policy rule analysed by Taylor (1993). Similar simulation experiments for Germany and Japan do not yield such clear-cut differences between the alternative monetary policy rules, and highlight the importance of assumptions about expectation formation.

1 Joe Gagnon, Jaime Marquez and Ralph Tryon were primarily responsible for the initial formulation of the FRB/MCM; its current development has been undertaken by Shaghil Ahmed, John Rogers and the author with the invaluable research assistance of Asim Hussain, Jonathan Otting and Sebastian Thomas. Simulations are performed in Troll 1.02, using innovative multi-tasking solution procedures developed by Jon Faust and Ralph Tryon. The VAR-based expectations algorithm was developed by David Bowman and Jonathan Otting. This research has also benefited greatly from the comments and suggestions of Dale Henderson, Will Melick, Dave Reifsneider, Volker Wieland and participants in the December 1995 Econometrician's Conference at the Bank for International Settlements. Finally, the views expressed in this note are those of the author and should not be interpreted as representing the views of the Federal Reserve Board of Governors or other members of its staff.

2 See, for example, Bryant, Hooper and Mann (1993).

3 In a previous paper, Tryon (1994) analysed the monetary transmission properties of the FRB/MCM.
The remainder of this paper is organised as follows: Section 1 outlines the general features of the FRB/MCM, with particular emphasis on the treatment of expectations. Section 2 outlines recent modifications of the FRB/MCM that have facilitated the analysis of inflation targets under model-consistent expectations. Section 3 analyses the essential properties of the three monetary policy rules under consideration. Section 4 reports the results of dynamic simulations of the FRB/MCM, and compares the performance of these monetary policy rules in response to country-specific fiscal and productivity shocks. Finally, Section 5 indicates several issues to be considered in future work on the FRB/MCM.

1. General features of the FRB/MCM

1.1 Country coverage

The FRB/MCM is a dynamic global economic model with nearly 1400 equations. As in the Federal Reserve Board's previous multi-country model, the FRB/MCM is comprised of twelve country/regional sectors. Each of the Group of Seven industrial economies (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States) is represented by about 35 behavioural equations and 100 accounting identities. The specification of these equations is fairly similar for all seven countries/sectors; the differences are mainly with respect to the estimated regression coefficients and bilateral trade weights. Three other sectors - Mexico, the newly industrialising economies (NIEs), and other OECD economies (ROECD) - are modelled on a more aggregated and stylised basis, with about 20 behavioural equations and 75 accounting identities each. Finally, a total of about 45 equations are used to represent the behaviour of OPEC members and of other developing and transition economies (ROW).

1.2 Long-run properties

Like its immediate predecessor, the MX-3 model, the FRB/MCM is designed to exhibit long-run stability and balanced growth, similar to that of a standard neoclassical growth model. As discussed further below, these long-run properties are particularly important in performing simulations with model-consistent expectations.

Thus, each consumption equation incorporates error-correction mechanisms to ensure that the level of consumption (in natural logarithms) is cointegrated with disposable income and the real interest rate. In other words, conditional on the long-term real interest rate, the savings rate is stationary (perhaps around exogenous trends related to demographics or other factors). Similarly, imports are cointegrated with domestic absorption and the real exchange rate, while exports are cointegrated with foreign absorption and the real exchange rate.

The long-run stability of the FRB/MCM is also facilitated by explicitly incorporating stock-flow relationships for physical capital and present-discounted-value constraints for government and net external debt. Thus, private investment exhibits short-run accelerator-type effects in response to output fluctuations. In the longer run, however, the investment rate adjusts to equate the marginal product of capital to its real rate of return. This adjustment effectively serves as an error-correction mechanism, ensuring that the level of investment and the capital stock are each cointegrated (in natural logarithms) with gross output and with the long-term real interest rate (net of depreciation).

Long-run fiscal solvency is maintained by an endogenous tax rate reaction function, which adjusts the income or sales tax rate when the nominal government debt/GDP ratio deviates

from a specified target. In the FRB/MCM, government expenditures and tax revenues are subject to
cyclical movements as well as exogenous shocks. Since budget deficit fluctuations affect the stock of
debt and hence subsequent interest payments, the tax rate adjustment must be sufficiently large to
prevent an explosive path of government debt. Thus, given an appropriate specification of the tax rate
reaction function, the model ensures that the stock of government debt is cointegrated (in natural
logarithms) with nominal GDP.

Finally, changes in the net external debt/GDP ratio lead to corresponding movements in
the sovereign risk premium. Thus, through uncovered interest parity, a deterioration of the current
account induces an increase in the domestic real interest rate and/or a depreciation of the real
exchange rate. A reasonable degree of sovereign risk premium adjustment ensures that improved net
exports of goods and non-factor services will outweigh the higher net factor payments resulting from
the initial increase in external debt, and thereby prevents an explosive path for the current account and
net external debt.

1.3 The role of expectations

The explicit treatment of expectations has played an important role in the formulation of
the FRB/MCM. In all sectors except OPEC and ROW, expected values of future variables directly
influence the determination of interest rates, consumption and investment expenditures, the aggregate
wage rate, and the nominal exchange rate. First, the long-term nominal interest rate and long-term
expected inflation rate are each determined as geometric weighted averages of future short rates.
Second, consumption, residential investment, and business fixed investment each depend on the ex
ante long-term real interest rate (the long-term nominal interest rate less expected inflation), while
business and petroleum inventory investment each depend on the ex ante short-term real interest rate.
Third, the aggregate nominal wage rate is defined in terms of the current and past values of
overlapping four-quarter wage contracts, where each wage contract depends on expected future
aggregate wages and expected deviations of unemployment from its natural rate. Finally, each
bilateral nominal exchange rate (local currency/$US) is determined by uncovered interest parity; i.e.,
the expected rate of depreciation depends on the current bilateral interest rate differential, adjusted by
the endogenously determined sovereign risk premium described above.

The FRB/MCM can be simulated under two alternative assumptions about expectations
formation: VAR-based expectations (referred to as backward-looking or "adaptive" expectations),
and model-consistent expectations (also referred to as forward-looking or "rational" expectations).
Since assumptions about expectations formation can have important implications for the simulation
results, it is useful to review the implementation of these assumptions in some detail.

1.4 VAR-based expectations formation

The implementation of adaptive expectations formation in the FRB/MCM closely
parallels the approach followed in the FRB/US quarterly model (cf. Kozicki, Reifschneider, and
Tinsley 1995). In particular, regression equations have been estimated for each of the Group of Three
economies (Germany, Japan, and the United States) using historical data on the output gap (i.e., the
deviation of real GDP from potential), the GDP price deflator, the short-term Treasury bill rate, and
the average wage rate. The current output gap and the current price inflation rate are each regressed
on up to eight quarters of lagged output gaps, inflation rates, and interest rates; and the wage inflation
rate is regressed on its own lags as well as lags of the other three variables.

For a given simulation experiment, a monetary policy rule must also be specified, in
which the short-term interest rate is determined as a linear function of the current output gap and the
price inflation rate; e.g., the rule analysed by Taylor (1993), or the rule considered by Henderson and
McKibbin (1993). The interest rate reaction function is combined with the reduced-form output gap and price inflation equations to create a three-variable VAR model. For any forecasting horizon $N \geq 0$, the VAR model can be evaluated recursively to obtain a forecasting equation for each variable, in which the $N$-step-ahead forecast is expressed in terms of the current and lagged values of all three variables. An algorithm developed by David Bowman and Jonathan Otting is used to compute the geometric weighted average of these forecasts over all horizons, yielding reduced-form equations for the long-term nominal interest rate and long-term expected inflation in terms of the current and lagged values of the output gap, inflation rate, and short-term interest rate.

In each period of a dynamic simulation, current and lagged variables are used to evaluate each reduced-form equation and obtain new expectations of future variables. For example, the reduced-form price inflation equation is used to determine short-term expected inflation, which is needed to calculate the ex ante short-term real interest rate for each of the inventory investment equations. The reduced-form equations for the long-term interest rate and long-term expected inflation are used to calculate the ex ante long-term real interest rate, which enters the consumption, fixed investment, and uncovered interest parity equations. Finally, the aggregate wage rate is determined directly from the reduced-form wage equation.

1.5 Model-consistent expectations formation

For each dynamic simulation of the FRB/MCM, model-consistent expectations are implemented by obtaining the perfect foresight solution path for all endogenous variables. To understand how this solution is obtained, it is useful to define the set of “expectations variables” as those endogenous variables whose expected future value enters into one or more equations in the model. The solution algorithm requires the long-run stability of all expectations variables: i.e., after a shock occurs, each expectations variable must eventually return to the baseline (or to some other known steady-state value). In this case, the baseline or steady-state values can serve as terminal conditions for the expectations variables at some date sufficiently far into the future.

Thus, the perfect foresight solution algorithm determines the paths of all endogenous variables over the simulation period, using prespecified values for the terminal conditions as well as for the initial conditions and the exogenous variables.

For example, suppose that one wishes to evaluate the effects of an exogenous change in government spending over the period 1996, Q1 to 1999, Q4. If the model is reasonably stable, one might expect that all variables would return to baseline within about 25 years. Thus, the use of model-consistent expectations would typically require a dynamic simulation over the period 1996, Q1 through 2025, Q4. In this case, the required initial conditions would be the pre-1996, Q1 values of all lagged variables in the model, which can be specified using historical data and/or an extrapolated baseline. The required terminal conditions would be the post-2025, Q4 values of all expectations variables in the model, which would be specified based on the long-run properties of these variables.

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5 The solution algorithm also permits the interest rate reaction function to include lags of the interest rate, output gap, price inflation, and wage inflation, but this possibility has not yet been investigated.

6 It is straightforward to determine whether a given simulation horizon is appropriate by simulating the model over a longer period and checking the extent to which the simulation results differ from those obtained using the shorter horizon (cf. Fair and Taylor 1983).
2. Recent modifications of the FRB/MCM

2.1 Implications of inflation rate targets

In most multi-country models, such as the IMF's Multimod and the Taylor MCM, each country's monetary policy rule has incorporated a target path for the price level, thereby ensuring that prices eventually return to the target path after a shock. Since the real wage and real exchange rate are stationary in these models, price level targets ensure that consistent terminal conditions can also be specified for the nominal wage rate and the nominal exchange rate. Given these terminal conditions, one of the standard solution algorithms can be used to perform simulations with model-consistent expectations.

Nevertheless, it is useful to be able to consider other monetary policy rules in which the price level is non-stationary. For example, Taylor (1993) analysed a policy rule that adjusts the short-term interest rate in response to deviations of current inflation from a specified target rate, and considered the extent to which this rule provides a reasonably accurate description of the Federal Reserve Board's actions over the past decade. Henderson and McKibbin (1993) utilised small-scale theoretical models and the large-scale MSG-2 model to analyse the properties of a wide range of monetary policy rules based on inflation rate targets.

Under these types of monetary policy rules, a shock to the model can induce a permanent change in the price level: the central bank's target inflation rate determines the long-run slope of the price path, but the specific level of the path is dependent on the initial conditions and the particular shock(s) hitting the economy. Thus, different initial conditions or shocks generate price paths which eventually become parallel to the baseline path.

2.2 Renormalised equations

To facilitate the analysis of inflation target rules under model-consistent expectations, the behavioural equations in the FRB/MCM have been renormalised so that expected future levels of nominal variables do not enter the model. This renormalisation ensures that all expectations variables in the FRB/MCM are stationary under both price level targets and inflation rate targets. Simulations involving permanent changes in the inflation target can also be performed, since the appropriate terminal conditions can be easily derived from the steady-state inflation target.

The key feature of this renormalisation is that the behavioural properties of the model have not been changed: under a price level target, the output of the model is identical to that of the previous version of the FRB/MCM. An alternative approach would be to express all nominal variables in terms of rates of change (as in the model of Fuhrer and Moore 1995), but such a model would ignore key long-run relationships between the levels of the nominal variables, and would not retain the properties of the earlier version of the FRB/MCM. In particular, a model expressed only in terms of nominal price inflation, wage inflation, and exchange rate depreciation would not ensure a stationary path for the real wage or the real exchange rate.\footnote{This outcome corresponds exactly to the excluded-variable bias that results from estimating a vector autoregression in first-differences when some of the variables are actually cointegrated in levels (cf. Granger 1981; Engle and Granger 1987).}

As seen in the Appendix, the renormalisation of the FRB/MCM has mainly involved the modification of three behavioural equations and the addition of several identities to the model. After defining the one-period inflation rate (DPABS), the long-term expected inflation rate (DPEXP) can easily be expressed in terms of stationary variables, using essentially the same term structure formula as for the long-term interest rate.
The aggregate nominal wage rate (W) can still be expressed in terms of current and lagged four-quarter wage contracts (WX). However, each wage contract depends on expected wage levels and unemployment over the life of the contract. By defining the absorption price-adjusted real wage and the contract wage/aggregate wage differential and then rearranging terms, the contract wage equation can be expressed solely in terms of stationary variables: the contract wage/aggregate wage differential (XDW) depends on the expected short-term price inflation rate (DPABS), expected real wages (WDPABS), and expected unemployment deviations from the natural rate (UN - UNNAT).

As noted in Section 1, each bilateral nominal exchange rate is determined by uncovered interest parity, subject to an endogenously determined sovereign risk premium. By defining the bilateral CPI-adjusted real exchange rate (RER) and then rearranging terms, each uncovered interest parity equation can be renormalised to express expected bilateral real exchange rate changes \((\log(RER) - \log(RER(1)))\) as a function of the differential between the US short-term real interest rate \((URS - UDPABS(1))\) and the domestic short-term real interest rate \((RS - DPABS(1))\).  

2.3 Perfect foresight solution algorithm

The choice of solution algorithm has a large impact on the computational resources required to simulate the renormalised FRB/MCM. Using version 1.02 of TROLL, either the Fair-Taylor algorithm or the new stacked Newton algorithm can be used to obtain a solution for an individual country model. However, the Fair-Taylor algorithm converges much more slowly in simulating price level target rules for the renormalised country models, compared with the previous versions of these models. In this case, although the price level returns to the baseline path after a shock, the Fair-Taylor algorithm must determine this result through numerous iterations, rather than by using the price levels as terminal conditions as in the earlier version of the FRB/MCM. In contrast, the rapid convergence rate of the stacked Newton algorithm does not appear to be sensitive to the renormalisation of the model or to the choice of monetary policy rule.

At this point, the stacked Newton algorithm cannot be used to simulate the entire FRB/MCM due to memory constraints. However, the global model can be solved very efficiently (in less than 5 minutes of CPU time) by an iterative procedure, using multi-tasking on a midsize Unix workstation with four processors. Starting with an initial guess for the solution path, each country model is solved using the stacked Newton algorithm, and the output is stored to disk, where it can be accessed by each of the other country models in the next iterative step. A main control program ensures that all individual country model solutions have been obtained prior to initiating the next iteration.

In contrast, the Fair-Taylor algorithm appears to be a highly computationally intensive and somewhat unreliable method of solving the current FRB/MCM. Lack of convergence occurs if an insufficient number of Fair-Taylor iterations are applied to each country model prior to sharing the output with other country models, whereas performing a large number of Fair-Taylor iterations at each step can require several hours of CPU time, even using the multi-tasking procedure.

---

8 In light of the econometrics literature (cf. previous footnote), the real wage (WDPABS), contract wage/aggregate wage differential (XDW), and the real exchange rate (RER) may be viewed as error correction terms which reflect three cointegrating relationships among the absorption price, nominal aggregate wage, nominal contract wage, and nominal bilateral exchange rate. In the FRB/MCM, the non-stationarity of all four variables is explained by a single integrated common factor that results from the monetary policy rule.
3. Alternative monetary policy rules

3.1 Nominal GDP target

A large literature has considered the properties of a monetary policy rule which adjusts the short-term interest rate in response to deviations of nominal GDP from a specified target path:

\[ i = \bar{r} + \pi^* + \alpha (PGAP + YGAP) \]  (1)

where \( i \) indicates the nominal short-term interest rate, \( \bar{r} \) the equilibrium real short-term interest rate, \( \pi^* \) the target inflation rate for the domestic absorption price deflator, \( PGAP \) the current deviation of the absorption price deflator from its target path, and \( YGAP \) the current deviation of real GDP from potential (all variables are expressed in terms of percentage points). The sum \( (PGAP + YGAP) \) indicates the deviation of nominal GDP from target, so that the monetary policy parameter \( \alpha \) can be interpreted as the partial elasticity of the short-term interest rate in response to nominal GDP deviations from target.\(^9\) In the simulations reported here, the parameter \( \alpha \) is set equal to 2.

The equilibrium real rate is defined as the real short-term interest rate at which the inflation rate remains constant and output remains at potential. Thus, when the price level is on target, expected inflation is at the target rate \( \pi^* \), and real GDP is at potential, the nominal GDP rule yields an \textit{ex ante} real interest rate, \( i - \pi^* \), equal to the equilibrium real rate, \( \bar{r} \).

It is important to note that the nominal GDP rule generates a trend-stationary path for the price level. For example, if nominal GDP rises one percent above target (due to a higher price level and/or an increase in output above potential), then the nominal interest rate is raised by \( \alpha \) percentage points, thereby putting downward pressure on economic activity and prices until nominal GDP returns to its target path. If the price level is above target and real GDP is below potential, then nominal GDP can still be on target, so that the nominal interest rate remains unchanged. In this case, however, the output gap corresponds to a relatively high unemployment rate, which depresses wage and price inflation. Thus, the nominal GDP rule implies a unique equilibrium path in which the price level is on target and real GDP is at potential.

To illustrate this feature of the nominal GDP rule, it is useful to consider the case where \( \pi^* = 0 \), so that the aggregate price level is stable around a constant level. As the parameter \( \alpha \) becomes arbitrarily large, the nominal GDP rule functions somewhat like a gold standard, except that this rule targets the price of a basket of goods and services rather than a single commodity. For smaller values of \( \alpha \), the nominal GDP rule also permits temporary price deviations from target, whereas the price of gold is essentially constant under the gold standard.

3.2 Taylor's rule

Taylor (1993) analysed the properties of the following monetary policy rule, which adjusts the short-term interest rate based on deviations of inflation from its target rate and on deviations of output from potential:

\[ i = \bar{r} + \pi^* + 1.5INFGAP + 0.5YGAP \]  (2)

where \( INFGAP \) is defined as the deviation of current inflation from its target rate, \( \pi - \pi^* \). Taylor calculated the US federal funds rate implied by this rule using \( \bar{r} = 2 \) and \( \pi^* = 2 \), and found that the

\(^9\) If \( PGAP \) were computed using the GDP price deflator, then \( (PGAP + YGAP) \) would equal the deviation of nominal GDP from its target path. In the simulations reported here, however, all monetary policy rules are expressed in terms of the domestic absorption price deflator, so that the relationship is only approximate.
implied interest rate followed a path quite similar to that of the actual federal funds rate over the period 1983-92.

If both current and expected inflation are at the target rate, and output is at potential, then Taylor's rule implies that the *ex ante* real interest rate is at the equilibrium rate, \( r^* \), yielding steady inflation and sustainable real GDP growth. If current inflation exceeds the target rate by one percentage point, Taylor's rule prescribes a 1.5 percentage point increase in the nominal interest rate, which will typically raise the *ex ante* short-term real interest rate by about 50 basis points. (The exact increase in the *ex ante* real interest rate depends on short-term expected inflation, but this is typically quite close to the current inflation rate.) The increase in the real interest rate dampens economic activity, thereby depressing employment and placing downward pressure on wages and prices until inflation returns to its target rate.

Taylor's rule also indicates that the federal funds rate should be adjusted in response to deviations of output from potential. When economic activity is relatively weak, this component of Taylor's rule reflects the effect of an interest rate cut in stimulating economic activity. However, this component also serves to reduce fluctuations in the inflation rate: when output exceeds potential, raising the nominal interest rate can help avoid an overheated economy and the associated upward pressure on wages and prices.

In contrast to the trend-stationary price path generated by a nominal GDP target, Taylor's rule induces a non-stationary price level, sometimes referred to as "price level drift". Thus, a zero inflation target is not the same as a constant price level target: when \( \pi^* = 0 \), the price level follows a random walk under Taylor's rule, whereas the nominal GDP rule induces long-run price stability. On the other hand, maintaining a price level target can be expected to involve greater costs in terms of output volatility compared with maintaining an inflation target.

These considerations can be illustrated by considering the monetary policy response to a one-time positive price level disturbance that leaves aggregate demand unchanged. In this case, if the *ex ante* real interest rate remains at its equilibrium value, then output stays at potential, and the inflation rate stays on target (apart from the deviation during the period of the shock). Thus, apart from an initial blip, Taylor's rule maintains a relatively constant nominal interest rate, and permits a permanent increase in the aggregate price level. In contrast, the nominal GDP rule prescribes an interest rate hike that depresses aggregate demand and places downward pressure on wages and prices until the aggregate price level falls back to its target path.

### 3.3 The H-M rule

Henderson and McKibbin (1993) studied the performance of a wide range of monetary policy rules in response to various shocks, and found that the following rule performed quite well in generating stable inflation and sustainable real growth:

\[
i = \tilde{r} + \pi^* + 2\text{INF GAP} + 2\text{YGAP}
\]

The H-M rule and Taylor's rule have the same functional form, and prescribe fairly similar interest rate adjustments in response to inflation deviations from target. However, the H-M rule prescribes a much stronger interest rate adjustment in response to the current output gap. In principle, an excessively strong interest rate adjustment in response to output deviations could lead to oscillating or even explosive outcomes; i.e., real GDP continually overshooting its potential level in response to interest rate changes. In most macroeconomic models, however, real GDP exhibits a relatively high degree of inertia, so that a higher partial interest rate elasticity with respect to output deviations can be expected to generate greater output stability.
Thus, the key question is whether the H-M rule obtains greater output stability at the cost of substantially higher inflation volatility compared with Taylor's rule. The possibility of a highly favourable output-inflation volatility trade off is less surprising if one views the current output gap as a proxy for near-term inflationary pressures which are not yet reflected in the current inflation rate. Given a sufficient degree of nominal inertia, changes in aggregate demand will tend to have strong initial effects on output and employment, leading to subsequent pressure on wages and prices. Thus, by promptly adjusting the nominal interest rate, it might be possible to offset the aggregate demand shock, and thereby stabilise both economic activity and inflation.

These considerations raise the possibility that Taylor's rule could be dominated by another monetary policy rule possessing the same functional form but with different INFGAP and YGAP coefficients; i.e., a different rule (possibly even the H-M rule) might yield both lower output volatility and lower inflation volatility compared with Taylor's rule. Evaluating this possibility requires the analysis of macroeconomic model simulations like the ones performed by Henderson and McKibbin (1993) and those reported in the following section of this paper.

4. FRB/MCM simulation results

4.1 Simulation design

Using the renormalised equations discussed in Section 3, simulations of the FRB/MCM can be used to evaluate the properties of alternative monetary policy rules under either model-consistent or VAR-based expectations. This section analyses simulation experiments in which one of the Group of Three economies (the United States, Germany, and Japan) experiences a temporary unanticipated shock to either aggregate demand or aggregate supply. The aggregate demand shock consists of an exogenous change in real government purchases of goods and services, which rise 5 percent above baseline during 1996 and 1997, and then gradually return to baseline by the end of 1999. The aggregate supply shock consists of an exogenous change in total factor productivity, which rises 0.5 percent above baseline during 1996 and 1997 and then returns to its baseline path at the beginning of 1998.

The country experiencing the exogenous shock follows one of the three interest rate rules described in Section 4, while the monetary policy rules of all other countries remain unchanged. In particular, a hybrid interest rate rule is followed by Italy, the United Kingdom, and the two other Group of Three economies (i.e., the two which have not experienced the exogenous government spending or total factor productivity shock). This hybrid rule has the same functional form as Taylor's rule and the H-M rule, with an INFGAP coefficient of 2 and a YGAP coefficient of 1. In all simulations, Mexico, the NIEs, OPEC, and ROW maintain a fixed exchange rate with respect to the US dollar; while France and other OECD economies maintain a constant exchange rate with respect to the German mark.

Every simulation is performed over the period 1996 through 2022, and the results are examined to verify that all economic variables have returned sufficiently close to the baseline by the end of the simulation period. The simulation results are shown in the attached table and charts. Only the first ten years of each simulation are shown to make the output more readable. The results for all price and expenditure variables are reported in terms of the relative deviation from baseline, in percentage points (indicated by the % symbol); while results for other variables such as the interest rate, inflation rate, and tax rate are reported in terms of the absolute deviation from baseline, in percentage points (indicated by the +/- symbol).

---

10 The output-inflation volatility trade off was originally discussed by Taylor (1980) based on the properties of a small macroeconomic model with forward-looking staggered wage contracts.
Table 1a
Country-specific aggregate demand shocks
Temporary 5% change in government spending

<table>
<thead>
<tr>
<th>Country</th>
<th>Expectations formation</th>
<th>Output volatility¹</th>
<th>Inflation volatility²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Nominal GDP target</td>
<td>Taylor's rule</td>
</tr>
<tr>
<td>United States</td>
<td>Model-consistent</td>
<td>0.108</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>0.145</td>
<td>0.204</td>
</tr>
<tr>
<td>Germany</td>
<td>Model-consistent</td>
<td>0.263</td>
<td>0.663</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>1.573</td>
<td>0.386</td>
</tr>
<tr>
<td>Japan</td>
<td>Model-consistent</td>
<td>0.167</td>
<td>0.388</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>0.380</td>
<td>0.528</td>
</tr>
</tbody>
</table>

1 Measured as the standard deviation of the real GDP gap over the first 40 quarters of the simulation experiment.
2 Measured as the standard deviation of the absorption price inflation rate over the first 40 quarters of the simulation experiment.

Table 1b
Country-specific aggregate supply shocks
Temporary 0.5% change in total factor productivity

<table>
<thead>
<tr>
<th>Country</th>
<th>Expectations formation</th>
<th>Output volatility¹</th>
<th>Inflation volatility²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Nominal GDP target</td>
<td>Taylor's rule</td>
</tr>
<tr>
<td>United States</td>
<td>Model-consistent</td>
<td>0.722</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>0.762</td>
<td>0.790</td>
</tr>
<tr>
<td>Germany</td>
<td>Model-consistent</td>
<td>0.602</td>
<td>0.474</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>1.153</td>
<td>0.504</td>
</tr>
<tr>
<td>Japan</td>
<td>Model-consistent</td>
<td>0.386</td>
<td>0.440</td>
</tr>
<tr>
<td></td>
<td>VAR-based</td>
<td>0.361</td>
<td>0.399</td>
</tr>
</tbody>
</table>

4.2 Results for US country-specific shocks

Chart 1 provides detailed simulation results for US macroeconomic variables in response to a temporary US aggregate demand shock, under the assumption of model-consistent expectations. As seen in the upper-right panel of Chart 1a, the shock consists of an exogenous increase in real government purchases above baseline during 1996 through 1999. The upper-left panel of Chart 1a shows the response of US real GDP under each of the alternative monetary policy rules. Under all three rules, output rises above baseline during the first several years, and subsequently falls below baseline near the end of the fiscal expansion.
However, real GDP exhibits less volatility under the H-M rule compared with either the nominal GDP rule or Taylor's rule. Under the H-M rule, output initially rises about 0.3 percent above baseline, and then returns fairly smoothly to baseline over the next four years. The nominal GDP rule generates a similar initial output response, but then causes output to fall below baseline for several years to ensure that the price level returns to its target path. Finally, output exhibits a larger initial increase of about 0.4 percent under Taylor's rule, and then displays very persistent cyclical behaviour during its return to baseline over the next decade.

The remainder of Chart 1a shows the response of other components of aggregate demand. Consumption expenditures increase modestly during the first several years and then return toward baseline, mainly due to changes in disposable income. Investment exhibits a sharp initial rise under Taylor's monetary policy rule, due to the strong accelerator effect. The nominal GDP rule and the H-M rule generate larger increases in the long-term real interest rate compared with Taylor's rule. Thus, under the nominal GDP and H-M rules, investment does not exhibit any initial increase, and falls below baseline almost two years earlier than under Taylor's rule. The H-M rule also generates a stronger initial real exchange rate appreciation compared with the nominal GDP rule and Taylor's rule, and thereby induces a larger initial contraction of real exports. Imports expand rapidly during the first several years under all three rules, due to higher domestic demand as well as the real exchange rate appreciation.

The top-left panel of Chart 1b shows how the US short-term interest rate is adjusted under each of the three monetary policy rules. The price level and inflation rate respond slowly to the shock, due to nominal wage and price inertia. Thus, the initial interest rate adjustments prescribed by all three rules mainly reflect the rapid rise of real GDP above potential. Due to the use of a much higher $YGAP$ coefficient, the nominal GDP and H-M rules prescribe an immediate 75 basis point increase in the federal funds rate, nearly twice the adjustment prescribed by Taylor's rule. As seen in the lower-right panel of Chart 1b, these short-term interest rate movements cause the long-term real interest rate to jump about 20 basis points above baseline under the nominal GDP and H-M rules, compared with about 10 basis points above baseline under Taylor's rule. As noted above, these higher real interest rates dampen investment and net exports during the first several years, thereby offsetting a substantial fraction of the aggregate demand stimulus associated with higher government expenditures.

As seen in the top two panels of Chart 1c, the nominal GDP rule succeeds in keeping the aggregate price level relatively close to its target path, whereas the H-M rule and Taylor's rule both permit the price level to deviate permanently from baseline. Under all three monetary policy rules, the CPI-adjusted real exchange rate gradually moves back toward baseline; i.e., long-run purchasing power parity holds in this case. However, since Germany, Italy, Japan, and the United Kingdom are following independent monetary policies based on inflation targets rather than price level targets, the trade-weighted foreign price level does not return to baseline. Thus, as shown in the left centre panel of Chart 1c, the trade-weighted value of the dollar deviates permanently from baseline, even when the United States follows a nominal GDP rule. Due to persistent current account deficits, the ratio of net external debt to nominal GDP rises by about one percent under all three monetary policy rules. Thus, the sovereign risk premium on US securities increases by about 5 basis points by 2005, thereby contributing to slightly higher US real interest rates.

Finally, Charts 1d and 1e give additional details on US fiscal and aggregate supply variables, and provide further insight into the long-term error-correction mechanisms built into the FRB/MCM. For example, the exogenous increase in government spending causes the budget deficit and the stock of government debt to rise above baseline. In response, the tax rate reaction function generates an increase in the personal income tax rate of 0.3 to 0.7 percent, thereby gradually pushing the government debt/GDP ratio back toward its target value.
Chart 1a
US aggregate demand shock: model-consistent expectations
US aggregate demand variables

Gross Domestic Product (%)  
Real Government Purchases (%)

Real Consumption Expenditures (%)  
Real Investment (%)

Real Exports (%)  
Real Imports (%)

Alternative U.S. Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dot-Dashed: Nominal GDP Target
Chart 1b
US aggregate demand shock: model-consistent expectations
US interest rates and expected inflation

**Short-term Interest Rate (+/-)**

**Long-term Interest Rate (+/-)**

**Short-term Expected Inflation (+/-)**

**Long-term Expected Inflation (+/-)**

**Short-term Real Interest Rate (+/-)**

**Long-term Real Interest Rate (+/-)**

**Alternative U.S. Monetary Policy Rules**
- Solid: Taylor's Rule
- Dashed: Henderson-McKibbin Rule
- Dot-Dashed: Nominal GDP Target
Chart 1c
US aggregate demand shock: model-consistent expectations
US prices, exchange rates and current account

Alternative U.S. Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dot-Dashed: Nominal GDP Target
Chart 1d

US aggregate demand shock: model-consistent expectations
US fiscal variables

- Government Deficit / Nominal GDP (%)
- Government Debt / Nominal GDP (%)
- Nominal Government Expenditures (%)
- Interest Paid on Debt (%)
- Nominal Government Receipts (%)
- Tax Rate (%)

Alternative U.S. Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dot-Dashed: Nominal GDP Target
Chart 1e
US aggregate demand shock: model-consistent expectations
US aggregate supply variables

Alternative U.S. Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dot-Dashed: Nominal GDP Target
Chart 2
US aggregate demand shock: temporary 5% change in government spending

Model-Consistent Expectations

U.S. Real GDP

U.S. Absorption Price Inflation

VAR-Based Expectations

U.S. Real GDP

U.S Absorption Price Inflation

Alternative U.S. Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
Chart 3

US aggregate supply shock: temporary 1% change in production

Model-Consistent Expectations

U.S. Real GDP

U.S. Absorption Price Inflation

VAR-Based Expectations

U.S. Real GDP

U.S. Absorption Price Inflation

Alternative U.S. Monetary Policy Rules

Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
Chart 2 reports simulation results for US output and inflation in response to a US aggregate demand shock under alternative assumptions about expectations formation. The upper panels of Chart 2 reproduce the results from Chart 1 for the case of model-consistent expectations, while the lower panels of Chart 2 report simulation results for the case of VAR-based expectations. The three alternative monetary policy rules have roughly similar features under both assumptions about expectations formation. The most striking difference is that the nominal GDP and H-M rules induce substantial initial volatility under VAR-based expectations, but not under model-consistent expectations; whereas Taylor's rule generates fairly similar paths for the inflation rate under both expectations assumptions.

Chart 3 reports the response of US output and inflation to a temporary US aggregate supply shock under alternative monetary policy rules and alternative assumptions about expectations formation. As indicated above, this shock consists of an exogenous 0.5 percent increase in US total factor productivity (TFP) during 1996 and 1997. All three interest rate rules have fairly similar implications during the first two years of high productivity: real GDP is initially below potential, which generates downward pressure on wages and prices until aggregate demand rises to the level of potential output. After TFP returns to its baseline path, aggregate demand suddenly exceeds potential output, generating positive inflationary pressure. At this point, the interest rate hikes prescribed by the nominal GDP and H-M rules are large enough to push aggregate demand back toward baseline fairly smoothly, whereas Taylor's rule yields a much longer and more cyclical adjustment path for both output and inflation.

4.3 Other country-specific shocks

Charts 4 and 5 report simulation results for German output and inflation in response to temporary shocks to German government spending and total factor productivity, respectively. Charts 6 and 7 report simulation results for Japanese output and inflation in response to the corresponding Japanese shocks.

Under the assumption of model-consistent expectations, the H-M rule yields greater output and inflation stability than Taylor's rule, regardless of the type of aggregate shock. Compared with Taylor's rule, the H-M rule prescribes a larger initial increase in the short-term interest rate, which dampens investment and net exports and thereby partly offsets the aggregate demand stimulus of the change in government spending.

Under VAR-based expectations, the comparison is less clear-cut: for the German aggregate demand shock, Taylor's rule yields greater output and inflation stability than the H-M rule; for the German aggregate supply shock, there is an output-inflation volatility trade off in choosing between the two rules; and for both Japanese shocks, the two rules generate fairly similar output and inflation behaviour. These results highlight the importance of the expectations formation mechanism in evaluating alternative monetary policy rules.

Finally, in contrast to the US results, both the German and Japanese simulation experiments indicate that the nominal GDP rule tends to provides greater output and inflation stability than either the H-M rule or Taylor's rule. This finding suggests that a price level target may be superior to an inflation rate target for economies in which international trade comprises a relatively high fraction of real GDP.
Chart 4
German aggregate demand shock: temporary 5% change in government spending

Model-Consistent Expectations

![Graph of German Real GDP and German Absorption Price Inflation with Model-Consistent Expectations]

VAR-Based Expectations

![Graph of German Real GDP and German Absorption Price Inflation with VAR-Based Expectations]

Alternative German Monetary Policy Rules
Solid: Taylor's Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
Chart 5
German aggregate supply: temporary 1% change in production

Model-Consistent Expectations

VAR-Based Expectations

Alternative German Monetary Policy Rules
Solid: Taylor’s Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
Chart 6
Japanese aggregate demand shock: temporary 5% change in government spending

Model-Consistent Expectations

Japanese Real GDP

Japanese Absorption Price Inflation

VAR-Based Expectations

Japanese Real GDP

Japanese Absorption Price Inflation

Alternative Japanese Monetary Policy Rules
Solid: Taylor’s Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
Chart 7
Japanese aggregate supply shock: temporary 1% change in production

Model-Consistent Expectations

VAR-Based Expectations

Alternative Japanese Monetary Policy Rules
Solid: Taylor’s Rule
Dashed: Henderson-McKibbin Rule
Dotted: Nominal GDP Target
5. Directions for future research

The FRB/MCM simulations of US aggregate demand and aggregate supply shocks generally confirm the favourable properties of the monetary policy rule considered by Henderson and McKibbin (1993). By targeting inflation rather than the price level, the H-M rule generates greater output stability and similar inflation stability compared with a policy rule based on nominal GDP targets. By prescribing larger interest rate adjustments in response to the current output gap and current inflation deviation from target, the H-M rule generates more stable economic activity and inflation compared with Taylor’s (1993) rule.

Based on the German and Japanese simulation experiments, the choice of an appropriate monetary policy is less clear-cut. Under model-consistent expectations, the H-M rule provides greater output and inflation stability than Taylor’s rule, just as in the US simulations. Under VAR-based expectations, however, neither rule clearly dominates the other. Furthermore, the nominal GDP target appears to generate a lower degree of output and inflation volatility than either rule based on an inflation rate target. These results highlight the crucial role of assumptions about how economic agents’ expectations are formed - an issue that is not very well understood and that deserves further investigation.

A number of prospective modifications of the FRB/MCM could also have significant implications for the performance of alternative monetary policy rules. First, the current version of the FRB/MCM incorporates Taylor’s (1980) overlapping contract structure, which yields substantial persistence in the nominal wage level but not necessarily in the wage inflation rate. Thus, it will be useful to consider alternative formulations that yield a higher degree of inflationary inertia; e.g., the contracting structure considered by Fuhrer and Moore (1995). Second, in the current version of the FRB/MCM, consumption and investment are sensitive to the ex ante real interest rate and to current and lagged disposable income or aggregate demand. In future research, it will be useful to consider specifications in which expected future changes in real GDP also influence the current levels of consumption and investment. Third, empirical research is already underway to provide estimated values for a much larger number of FRB/MCM parameters, which will tend to generate larger differences in the macroeconomic behaviour of the various sectors of the FRB/MCM. Finally, Federal Reserve staff are in the process of constructing a joint FRB model, which combines the new quarterly domestic model (FRB/US) with the foreign sectors of the FRB/MCM. Since the FRB/US model incorporates a number of innovative modelling features, it will be highly informative to investigate the properties of alternative monetary policy rules using the joint FRB model.
Appendix: Renormalised forward-looking equations in the FRB/MCM

1. Definitions of variables

RS  Short-term nominal interest rate (annual rate).
RL  Long-term nominal interest rate (annual rate).
PABS Domestic absorption price deflator.
DPABS Short-term inflation rate (annual rate).
DPEXP Long-term expected inflation (annual rate).
W   Aggregate nominal wage rate (annual rate).
RW  Aggregate real wage (adjusted by absorption price deflator).
X   Nominal contract wage rate (annual rate).
XW  Contract/aggregate wage differential.
UNDEV Unemployment deviation from natural rate.
ER  Nominal exchange rate (local currency/US$).
URS US short-term interest rate (annual rate).
UPABS US domestic absorption price deflator.
RER Bilateral domestic/US real exchange rate (adjusted by absorption prices).
NXDEBT Net external debt (in US$).
GDPPOTV Nominal potential GDP (in local currency).
ERR.* Exogenously determined residual for equation*.
2. Identities

Domestic absorption inflation rate
\[ DPABS = 400 \left[ \log(PABS) - \log(PABS(-1)) \right] \]

Aggregate nominal wage rate
\[ \log(W) = 0.25 \left[ \log(X) + \log(X(-1)) + \log(X(-2)) + \log(X(-3)) \right] \]

Real wage rate
\[ RW = 100 \left[ \log(W) - \log(PABS) \right] \]

Contract wage/aggregate wage differential
\[ XW = 100 \left[ \log(X) - \log(W) \right] \]

Real exchange rate
\[ RER = ER \times \frac{UPABS}{PABS} \]

3. Behavioural equations

Long-term interest rate (term structure)
\[ RL = 0.05 + 0.05 \times RS + 0.95 \times RL(+1) + ERR.RL \]

Long-term expected inflation
\[ DPEXP = 0.05 \times DPABS(+1) + 0.95 \times DPEXP(+1) + ERR.DPEXP \]

Contract wage determination
\[ XW = -0.75 \times RW + 0.25 \times RW(+1) + 0.25 \times RW(+2) + 0.25 \times RW(+3) \]
\[ + 0.1875 \times DPABS(+1) + 0.125 \times DPABS(+2) + 0.0625 \times DPABS(+3) \]
\[ - 0.005 \left[ \text{UNDEV} + \text{UNDEV}(+1) + \text{UNDEV}(+2) + \text{UNDEV}(+3) \right] \]
\[ + ERR.XW \]

Uncovered interest parity
\[ \log(RER) = \log(RER(+1)) + \frac{[URS - UDPABS(+1) + RS - DPABS(+1)]}{400} \]
\[ + 0.01 \times ER \times \frac{NXDEBT}{GDPPOTV} + ERR.RER \]
References


Comments on paper by A. Levin by C. Borio (BIS)

The technical core of A. Levin's paper is the development of an algorithm for models simulated under model-consistent expectations permitting the evaluation of policy rules under which the price level is non-stationary. Taylor's rule, prescribing changes in the short-term rate partly in response to deviations of inflation from its target path, is a popular example of such a reaction function. This the paper does very successfully, making a significant contribution to the existing literature.

The paper then goes on to evaluate the performance of three alternative policy rules (a nominal GDP target as well as Taylor's and Henderson-McKibbin's inflation-cum-output gap targets) in response to two types of shock (an aggregate demand and an aggregate supply shock) for three countries (the United States, Germany and Japan) using the Federal Reserve Board's multi-country model. The evaluation is carried out assuming, alternatively, model-consistent and adaptive (VAR-based) expectations. At least regarding the United States, the author appears to come down in favour of the Henderson-McKibbin rule. For Germany and Japan, the choice is said to depend partly on the expectations formation mechanism assumed. Under adaptive expectations, a nominal income target rule is argued to be preferable.

My remarks will not deal with the technical part of the paper. Rather, they will pay particular attention to its evaluation of policy rules and its possible implications for policy. My first set of comments will take the framework underlying the paper as given and make a few suggestions regarding avenues for improvement and issues that would deserve closer examination. I shall then broaden the horizon a bit and have something to say about the usefulness of the underlying framework itself as a guide to policy making.

The basic framework employed goes back to Poole's (1970) seminal paper on the comparison between interest rate and monetary targets in a simple IS-LM model based on a quadratic "objective" function for the final goal, in that case output. That paper has spawned an enormous literature of increasing degree of sophistication. This piece of work can be thought of as one distant offspring.

Let's take for granted for the moment that the relevant criterion for assessing performance is the volatility of inflation and output from baseline. When no rule is dominant two ingredients necessary to rank rules are:

(a) a representative "objective" function, stating how variability in the goals should be traded off, and

(b) an idea of the historical and likely future evolution of the shocks. Neither of these elements, however, is still present in the paper.

Point (b) would ultimately lead to stochastic simulations, a conceptually appealing but probably computationally overwhelming exercise. Less demandingly, the author could provide some idea of the historical distribution of the shocks. Point (a) could best be addressed by finding out the cut-off parameters that would tip the balance in favour of one type of rule relative to another. Some independent criterion would then be needed to assess how "reasonable" such a cut-off could be given the assumed costs of variability in each of the goals.\(^\text{11}\)

How far do these objections affect the basic findings of the paper? They do not impinge so much on the choice between Taylor's and Henderson-McKibbin's rules, since the latter tends to dominate in pair-wise comparisons. They do, however, have an impact on the relative ranking of the

\(^{11}\) Of course, steps (a) and (b) could not deal with the issue of model uncertainty, in this case resulting from alternative assumptions about the expectations formation mechanism.
nominal GDP target rule (see Table 1), which in fact receives little prominence in the author's conclusions. An analysis of the kind suggested would help to clarify if such an emphasis is indeed justified within the confines of the assumed framework.

A second issue concerns the range of policy rules considered. Two kinds of rules can be distinguished in the literature. The first are "simple rules". In this case the premium is on rules that are not only feasible in practice but also robust across models when the policy maker has little basis for choosing between them ("model uncertainty"). The second are "optimal rules" given the model and the distribution of the shocks. Poole's "combination policy" is one such example. The reaction functions considered in the paper are clearly "simple" ones. Yet this leaves unanswered several teasing questions. If the source of the superiority of the Henderson-McKibbin's rule over Taylor's is the greater weight on the output gap, possibly as a forward-looking indication of inflation, why not raising it still further? Is there not a case, that is, for seeing what parameter values of the simple rules would do better? What is the reason for considering those rules rather than others? This is indeed not such a gratuitous observation, given that at least one country in the sample, Germany, has operated, and is avowedly continuing to operate, a monetary target rule. In fact, to the extent that the nominal GDP target is its closest equivalent out of the reaction functions considered, the authorities may find it surprising that it should perform comparatively poorly. This, of course, raises the issue of the extent to which it may be appropriate or helpful to run such experiments for different countries assuming a similar structures and disregarding country-specific factors.

A third issue concerns the interpretation of the results. A very interesting, possibly surprising, finding of the paper is that the assumption regarding expectation formation mechanisms appears to have little impact on the ranking of policy rules in two out of three countries, viz. the United States and Japan. By contrast, it could have significant implications for Germany, depending on the nature of the shocks. A further, less clear, finding is that the most appropriate policy rule may differ between countries. The curious reader would clearly like to know more about the possible reasons for such differences in order to be able to draw more general and robust conclusions. The paper contains little by way of elucidation.

Turning next to the usefulness of the basic framework assumed, I would like to mention here only one neglected issue. This is that, contrary to what it is implicitly assumed in the paper, in most real world circumstances asymmetries are important for policy. These asymmetries relate to the "cost function", trading off different goals, as well as to the feasible policy rules. Let me elaborate briefly on each.

Quadratic cost functions focusing on the variance of various goal variables have several nice mathematical properties, not least linear reaction functions. Yet I wonder whether policy makers' objectives should not be better captured by asymmetric functions. In normal circumstances, overriding concerns about the risks of losing control over inflation would result in asymmetric policy rules, as the authorities would be more tolerant of inflation rates below original targets paths than they

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12 For clarity, it would be useful if the table was complemented by explicit rankings for each row (country and expectational assumption) and goal (output and inflation variability).

13 The conclusion that for Germany "the nominal GDP target appears to generate a lower degree of output and inflation volatility than either rule based on an inflation rate target" does not appear to correspond to what is shown in Table 1. According to this table, the nominal GDP target rule actually yields the highest output and inflation volatility under adaptive expectations. While generally performing much better under model-consistent expectations, it still yields the highest output volatility in the case of aggregate supply shocks.

14 See eg. BIS (1995).

15 It is not easy to draw such a clear cut inference from Table 1.

16 In fact, the inference that openness favours nominal GDP targeting does not seem warranted. Even assuming that a rigorous ranking would indicate that such a strategy was clearly superior in Japan and Germany but not in the United States, Japan is hardly more open than the United States.
would be of errors in the opposite direction. The authorities, that is, would be prepared to take advantage of any opportunities that arose in order to reduce inflation. An opposite situation could emerge when inflation is already very low. In this case the risk of deflation could tip the balance in favour of greater acceptability of inflation outcomes above original targets. This asymmetry can arise either because the costs of deflation are perceived as larger than those of (moderate) inflation, or because of the possibly more limited room for manoeuvre at very low inflation rates: interest rates may not be able to fall enough to counteract the deflationary effects of shocks, say, to the exchange rate. The recent Japanese experience might well be viewed in this light.

Similar considerations can easily apply to the range of feasible policy rules. Limited credibility can act as a powerful constraint on the range of options available to the authorities. This is clearly illustrated by the Canadian experience. A long history of comparatively high inflation exacerbated by weak public finances, high foreign debt and by an at times uncertain political climate have made it harder for the authorities to ease than to tighten, as the markets have exhibited limited tolerance for easing moves that they perceive as unjustified (Zelmer (1995)). The point here is that credibility and communication issues, so central to policy making, are assumed away in the framework of the paper.

Let me end with a final remark on the role of the output gap in the formulation of monetary policy. The well-known problem for central banks is how to control an economic magnitude (inflation) that responds with an uncertain and long lag to policy. Monetary targets, for a time, had been perceived as a useful compass to guide the authorities' actions. Nowadays, that compass has effectively been lost, at least for most central banks. From this perspective, the paper raises two additional teasing questions. Looking back, given the dominance of the Henderson-McKibbin rule over Taylor's, which appears to be a good approximation to actual Fed policy in the past, I wonder whether the author would like to draw the counterfactual implication that the Fed should de facto have placed even greater weight on the output gap in its decisions. Looking ahead, would he also like to argue that the output gap should perhaps be put on the pedestal from which monetary targets have so embarrassingly been dislodged? I would be surprised if the search for the Holy Grail had ended, for this is search that is bound to fail.

References


BIS, 1995, Financial structure and the monetary policy transmission mechanism, Basle, March.


17 Canada was originally included in the paper.