Comovements of Canadian, UK and US bond yields

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Introduction

A striking aspect of the behaviour of interest rates in the most recent past has been the large increases in long rates across a number of countries following the monetary tightening by the US Federal Reserve in early 1994. For central banks, the question is how to interpret these movements in long rates. Two potential explanations immediately suggest themselves. The first is that the increases in long rates were caused by upward revisions in market participants' expectations of price inflation. The other potential explanation is that the global upturn in long rates was instead primarily caused by changes in term premia that were positively correlated across markets. Clearly, these explanations of the recent global increase in long-term interest rates have different policy implications.

A popular model of interest rate determination often used to interpret changes in the slope of a country's term structure is the expectations theory of the term structure. The well-known intuition underlying the expectations theory is that the investment strategy of rolling over a sequence of short-term bonds is an alternative to holding a long-term bond. According to the expectations theory, the expected rates of return on these alternative investment strategies differ by a constant term (risk) premium. This implies that long-term interest rates equal a weighted average of expected short-term interest rates plus a constant term premium. Therefore, changes in term premia are ruled out by assumption. Within the context of the expectations theory, the degree of comovement in long rates between countries is determined by comovements in expectations of future short rates.[1]

This paper compares historical comovements of ten-year government bond yields in Canada, the United Kingdom and the United States with the theoretical predictions of the expectations theory of the term structure. An alternative hypothesis entertained concerning the evolution of long rates is excess comovement between countries. Interest rates in two countries display excess comovement if, when one rate is relatively high (low), the other rate is on average too high (low) relative to the predictions of the expectations theory. The outcome of Shiller's (1989) test indicates the presence of excess comovement of ten-year bond yields between all three countries.

Excess comovement of long-term bond yields has important implications for interest rate determination: firstly, that deviations of long rates from the predictions of the expectations theory of the term structure contain an international component; secondly, that global changes in long-term interest rates need not signal shifts in market participants' expectations of price inflation.[2]

The remainder of the paper is organised as follows. Section 1 begins by presenting a linear model of the expectations theory of the term structure as it applies to the joint behaviour of long rates in a group of countries. This basic framework is then extended to allow for the possibility that

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1 Of course, the degree of comovement in expectations of future short rates between countries depends on many factors, including, for instance, exchange rate policies.

2 In a recent study, Hardouvelis (1994) concludes that ten-year government bond yields in Canada, the United Kingdom and the United States deviate from the predictions of the expectations theory. For US long-term interest rates, this conclusion was also reached by Shiller (1979), Shiller, Campbell and Shoenholtz (1983) and Campbell and Shiller (1984). Nevertheless, there is an important element of truth to the expectations theory of the term structure as a model of the relationship between long and short rates. Employing the methodology of Campbell and Shiller (1987), Hardouvelis (1994) shows that the slope of the term structure is an empirically relevant indicator of the future evolution of short rates in many countries.

long rates deviate from the predictions of the model. The implications of the expectations theory for the joint behaviour of long rates within a group of countries are derived in this extended framework and Shiller's (1989) test for excess comovement within a group of asset prices is discussed. Section 2 presents empirical results from application of the test to ten-year government bond yields in Canada, the United Kingdom and the United States. The outcome of Shiller's test indicates the presence of excess comovement of ten-year bond yields between all three countries. Section 3 concludes the paper.

1. The expectations theory and comovements in bond yields

According to the expectations theory of the term structure, long-term interest rates are the sum of a weighted average of expected short-term interest rates and a constant risk premium. This section presents Shiller's (1989) test for excess comovement of asset prices in the context of the expectations theory of the term structure. A concept related to excess comovement is excess volatility of an asset price. In order to relate the concepts of excess volatility and excess comovement (a precise definition of these terms will be given below), an analogous test for excess volatility of an asset price series is presented first.

Let R_{it} denote the yield to maturity on an n-period bond in country i at time t. The expectations theory of the term structure can be formally expressed as

$$R_{it} = \theta_i + \sum_{j=0}^{n-1} w_{ij} E_t r_{i,t+j}, \tag{1.1}$$

where θ_i is a constant term (risk) premium, r_{it} is the one-period rate of interest in country i from time t to t+1 and E_t is the expectations operator given all publicly available information at time t. The $\{w_{ij}\}$ are weights that are determined by the duration of the long-term bond. For a pure discount bond, $w_{ij} = 1/n$. For coupon bonds, the weights (which will be discussed in more detail below) decline monotonically and sum to one.[3] It is useful in what follows to define R_{it}^*, the *perfect foresight* long rate:

$$R_{it}^* = \theta_i + \sum_{j=0}^{n-1} w_{ij} r_{i,t+j} \tag{1.2}$$

With this notation, the expectations theory of the term structure can be expressed as

$$R_{it} = E_t R_{it}^* \tag{1.3}$$

In order to derive restrictions on comovements of bond yields between countries implied by the expectations theory, it is necessary to simultaneously model long rates in a group of countries. To this end, let $R_t \equiv (R_{1t}, R_{2t}, ..., R_{kt})'$ denote the vector of time t long rates in k countries. The expectations theory of the term structure can be expressed for this group of countries as

$$R_t = E_t R_t^*, \tag{1.4}$$

where $R_t^* \equiv (R_{1t}^*, R_{2t}^*, ..., R_{kt}^*)'$.

Let $$U_t \equiv R_t^* - R_t \tag{1.5}$$

denote the $k \times 1$ vector of discrepancies between perfect foresight long rates and R_t.[4] Note that U_{it} positive (negative) corresponds to R_{it} less (greater) than the perfect foresight long rate R_{it}^*. The expectations theory of the term structure imposes restrictions on the random vector U_t. In particular,

3 This linear representation of the expectations theory of the term structure is discussed in more detail in Shiller, Campbell and Schoenholtz (1983).

4 Of course, the realisation of the random vector U_t is not known at time t.

the theory implies that U_t is a mean zero random vector that is unforecastable given information publicly available at time t. Under the assumption that the value of R_t is known at time t, the expectations theory requires that $Cov(U_t, R_t) = 0$, where $Cov(\cdot,\cdot)$ denotes unconditional covariance.[5]

Of course, long rates may deviate from the values predicted by the expectations theory of the term structure. To allow for this possibility, reinterpret R_t as the vector of "theoretical" long rates predicted by the expectations theory, i.e. let R_t be defined by (1.4). To allow for the possibility that long rates deviate from the predictions of the expectations theory, let $R_t^o = (R_{1t}^o, R_{2t}^o, \ldots, R_{kt}^o)'$ denote the k x 1 vector of actual time t long rates. Let

$$\varepsilon_t \equiv R_t^o - R_t \,. \tag{1.6}$$

The k x 1 vector ε_t represents the discrepancies between actual long rates at time t and the values predicted by the expectations theory of the term structure. Below, bond yield volatilities and comovements are related to the elements of the covariance matrix $\Omega \equiv E(\varepsilon_t \varepsilon_t')$.

In order to derive the implications of the structure of Ω for bond yield volatilities and comovements, it is necessary to make assumptions concerning the properties of ε_t. In what follows, it is assumed that ε_t is an iid random vector with zero mean and finite variance. It is also assumed that the process $\{\varepsilon_t\}$ evolves independently from short rates in the k countries. This last condition implies that ε_t is uncorrelated with R_t^*, R_t and U_t. These assumptions are sufficient to relate bond yield volatilities and comovements to the structure of Ω.

In order to derive the implications of the structure of Ω for bond yield volatilities and comovements, note that the relationship between perfect foresight long rates and R_t^o is

$$R_t^* = R_t^o + U_t^o, \tag{1.7}$$

where $U_t^o \equiv U_t - \varepsilon_t$. Applying the unconditional variance operator to both sides of relation (1.7) gives

$$Var\,(R_t^*) = Var(R_t^o) + Var(U_t^o) + Cov(R_t^o, U_t^o) + Cov(U_t^o, R_t^o), \tag{1.8}$$

where $Var(\cdot)$ denotes unconditional variance. Restrictions that the expectations theory imposes on the joint behaviour of long-term interest rates are easily derived from relation (1.8).

By substituting for U_t^o and R_t^o, it follows that

$$Cov(U_t^o, R_t^o) = Cov(U_t - \varepsilon_t, R_t + \varepsilon_t)\,.$$

If long rates are exactly determined by the expectations theory, then $\varepsilon_t \equiv 0$. Under the maintained assumption that $Cov(U_t, R_t) = 0$ it follows from (1.8) that

$$Var\,(R_t^*) = Var(R_t^o) + Var(U_t^o)\,. \tag{1.9}$$

The diagonal elements of the matrix relation (1.9) take the form

$$Var\,(R_{it}^*) = Var(R_{it}^o) + Var(U_{it}^o). \tag{1.10}$$

Expression (1.10) is the first important restriction that the expectations theory of the term structure imposes on bond yield volatilities. A violation of (1.10) of the form

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5 $Cov(A,B) \equiv E\{(A - EA)(B - EB)'\}$, where E is the unconditional expectations operator. In the theoretical discussion of the implications of the expectations theory of the term structure, it is assumed that the required covariances and variances exist. Issues related to non-stationarity of the interest rate series will be addressed in the next section.

$$Var\,(R_{it}^{*}) < Var(R_{it}^{o}) + Var(U_{it}^{o}) \tag{1.11}$$

will be referred to as *excess volatility* of the long rate in country *i*. It is easily verified that excess volatility of the long rate in country *i* is equivalent to a negative (unconditional) covariance between U_{it}^{o} and R_{it}^{o}, which implies that when the long rate in country *i* is high, it is typically too high, relative to the fundamental value R_{it}^{*}.[6]

The expectations theory of the term structure also places restrictions on the comovements of bond yields between countries. The off-diagonal ($i \neq j$) elements of expression (1.8) are of the form

$$Cov(R_{it}^{*},\, R_{it}^{*}) = Cov(R_{it}^{o}, R_{jt}^{o}) + Cov(U_{it}^{o}, U_{jt}^{o}) + Cov(R_{it}^{o}, U_{jt}^{o}) + Cov(U_{it}^{o}, R_{jt}^{0}).$$

If the expectations theory holds, then $Cov(R_{it}^{o}, U_{jt}^{o}) = 0$ and $Cov(U_{it}^{o}, R_{jt}^{o}) = 0$, since $\varepsilon_t \equiv 0$ and $Cov(U_t, R_t) = 0$. Therefore, the expectations theory implies that

$$Cov(R_{it}^{*},\, R_{jt}^{*}) = Cov(R_{it}^{o}, R_{jt}^{o}) + Cov(U_{it}^{o}, U_{jt}^{o}). \tag{1.12}$$

A violation of restriction (1.12) of the form

$$Cov(R_{it}^{*},\, R_{jt}^{*}) < Cov(R_{it}^{o}, R_{jt}^{o}) + Cov(U_{it}^{o}, U_{jt}^{o})$$

will be referred to as a case of *excess comovement* of bond yields between countries *i* and *j*. Clearly, the case of excess comovement is equivalent to

$$Cov(R_{it}^{o}, U_{jt}^{o}) + Cov(U_{it}^{o}, R_{jt}^{o}) < 0. \tag{1.13}$$

This represents a violation of the expectations theory of the term structure, because the theory implies that both covariances in the inequality (1.13) are zero.

Excess comovement of bond yields implies that there exists a negative correlation between the deviation of the long rate in one country from its fundamental value and the level of the long rate in another country. In other words, when the long rate in one country is high, the long rate in the other country is on average too high relative to the predictions of the expectations theory. It is easily verified from (1.13) that excess comovement of bond yields between countries *i* and *j* is equivalent to $Cov(\varepsilon_{it}, \varepsilon_{jt}) > 0$. Thus, it is clear that the comovements of bond yields between countries may be consistent with the predictions of the expectations theory even if bond yields display excess volatility. This case corresponds to a diagonal covariance matrix Ω.

2. Joint behaviour of government yields in Canada, the United Kingdom and the United States

This section applies the tests for excess volatility and excess comovement presented in the previous section to interest rate data for Canada, the United Kingdom and the United States. These countries are the focus of the present analysis because they are the three G-7 countries for which the interest rate series are available for the longest time span. The time series analysed are post-war quarterly data on three-month and ten-year government bond yields. The sample begins in 1961 Q1. Following Hardouvelis (1994), only data up to 1992 Q2 are studied, so the results will be comparable with his.

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6 Another indicator of excess volatility is $Var(R_{it}^{*}) < Var(R_{it}^{o})$. It is easily verified that, although not a necessary condition, this inequality implies that U_{it}^{o} and R_{it}^{o} are negatively correlated.

The tests for excess volatility and excess comovement examine the behaviour of perfect foresight bond yields. In order to construct perfect foresight bond yields, the weights $\{w_{ij}\}$ and risk premium θ_i in equation (1.2) must be specified. Following Shiller, Campbell and Schoenholtz (1983), set

$$w_{ij} = g_i^j(1-g_i)/(1-g_i^n),$$

where $g_i \equiv 1/(1+\bar{R}_i^o)$ and $\bar{R}_i^o$ is the mean n-period rate over the sample period. Perfect foresight bond yields in Canada, the United Kingdom and the United States are constructed under the assumption that risk premia are constant across countries. Perfect foresight bond yields are constructed for the cases $\theta = 0$, $\theta = 1$ and $\theta = 2$. The outcome of the tests for excess volatility and excess comovement are identical for all three cases, so only the results for the case $\theta = 1$ are discussed below. Recall that the time t perfect foresight ten-year bond yield is a function of short rates up to quarter t+39. Accordingly, perfect foresight bond yields are constructed over the period 1961 Q1-1982 Q3.

The tests for excess volatility and excess comovement rely on an examination of the unconditional moments of the vector time series under review. The existence of unconditional moments requires the vector time series to be stationary. If short rates are non-stationary in levels, then the expectations theory of the term structure implies that long rates will also be non-stationary in levels. In this case, it is necessary to transform the processes $\{R_{it}^o\!: i=1,...,k\}$ and $\{R_{it}^*\!: i=1,...,k\}$ before applying the tests presented in the previous section.

Two issues arise when deciding on transformations of the processes $\{R_{it}^o\!: i=1,...,k\}$ and $\{R_{it}^*\!: i=1,...,k\}$. First, the transformed processes must be stationary. Second, good transformations should not induce volatility in the transformed processes, so as to maximise the power of the tests for excess volatility and excess comovement. With these goals in mind, the raw data are transformed by deflating by a distributed lag of long rates. In particular, define the transformed processes $\{\tilde{R}_{it}^o\!: i=1,...,k\}$ and $\{\tilde{R}_{it}^*\!: i=1,...,k\}$ by $\tilde{R}_{it}^o \equiv z_{it}R_{it}^o$ and $\tilde{R}_{it}^* \equiv z_{it}R_{it}^*$, where $z_{it}^{-1} \equiv (R_{i,t-20}^o + R_{i,t-19}^o + ... + R_{i,t-1}^o)/20$. The tests for excess volatility and excess comovement presented in the preceding section are applied to the transformed time series $\{\tilde{R}_t^*\}$, $\{\tilde{R}_t^o\}$ and $\{\tilde{U}_t^o\}$, where $\tilde{U}_t^o \equiv \tilde{R}_{it}^* - \tilde{R}_t^o$, over the time period 1966 Q1-1982 Q3.

The table below presents the sample analogues of the variance matrices $Var(\tilde{R}_t^*)$, $Var(\tilde{R}_t^o)$ and $Var(\tilde{U}_t^o)$ for the period 1966 Q1-1982 Q3. Recall that the expectations theory of the term structure places restrictions on these matrices according to relation (1.9). In particular, under the expectations theory, the elements of the top matrix equal the sum of the corresponding elements of the lower two matrices.

The restriction that the expectations theory imposes on bond yield volatility in a single domestic market, given by relation (1.10), relates to the diagonal elements of the matrices. The top left element of each matrix concerns the US market in isolation. The estimated variance matrices are consistent with the view that ten-year bond yields in the United States are too volatile to accord with the expectations theory of the term structure (1.9 < 1.6 + 3.5). Likewise, the estimated variance matrices are consistent with the view that ten-year bond yields in Canada and the United Kingdom are also excessively volatile (for Canada 2.2 < 1.3 + 3.2; for the United Kingdom 1.6 < 1.9 + 2.0).

The restrictions that the expectations theory imposes on bond yield comovements between countries are given by relation (1.12). The estimated variance matrices indicate the presence of excess comovement of ten-year bond yields between the United States and Canada

(1.2 < 1.3 + 2.7). The variance matrices also indicate the presence of excess comovement between US and UK ten-year bond yields (0.7 < 0.3 + 1.6) and between Canadian and UK ten-year bond yields (-0.3 < 0.5 + 0.7).

Table 1
Variance matrices

$$Var\begin{bmatrix}\tilde{R}_{US}\,^* \\ \tilde{R}_{CA}\,^* \\ \tilde{R}_{UK}\,^*\end{bmatrix} = \begin{bmatrix}1.9 & & \\ 1.2 & 2.2 & \\ 0.7 & -0.3 & 1.6\end{bmatrix}$$

$$Var\begin{bmatrix}\tilde{R}_{US}\,^o \\ \tilde{R}_{CA}\,^o \\ \tilde{R}_{UK}\,^o\end{bmatrix} = \begin{bmatrix}1.6 & & \\ 1.3 & 1.3 & \\ 0.3 & 0.5 & 1.9\end{bmatrix}$$

$$Var\begin{bmatrix}\tilde{U}_{US} \\ \tilde{U}_{CA} \\ \tilde{U}_{UK}\end{bmatrix} = \begin{bmatrix}3.5 & & \\ 2.7 & 3.2 & \\ 1.6 & 0.7 & 2.0\end{bmatrix}$$

Conclusions

This paper compares historical comovements of ten-year government bond yields in Canada, the United Kingdom and the United States with the predictions of the expectations theory of the term structure. The empirical evidence indicates the presence of excess comovement of ten-year bond yields between all three countries.

Excess comovement of long-term bond yields has important implications for interest rate determination. First, it implies that deviations of long rates from the predictions of the expectations theory of the term structure contain an international component. Second, excess comovement between Canadian, UK and US bond yields implies that common movements in long rates in these countries need not signal shifts in market participants' expectations of price inflation.

Clearly, the present analysis must be extended to cover a broader cross-section of countries before any conclusions regarding global interest rate movements can be reached. Nevertheless, the present results suggest that an international component of time variation in term premia may be an important factor in interest rate determination at the long end of the term structure.

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