Bond market volatility in Germany: evidence, causes and identification

Dietrich Domanski and Holger Neuhaus

Introduction

Volatility in financial markets has become an important point of discussion and concern among central bankers. In monetary policy terms, periods of higher volatility are important in two respects: on the one hand, sharply fluctuating market prices may blur interest rate policy measures by the central bank (or intensify them in a way which is undesirable). On the other hand, greater interest rate uncertainty may influence non-banks' portfolio decisions and thus complicate monetary targeting. Moreover, if price swings occur as fundamental misalignments, the bursting of a speculative price bubble could face the central bank with a demand to perform a compensatory function, which would make it more difficult to comply with its primary mandate of safeguarding monetary stability.

Volatility is often seen as an inevitable consequence of the rapid structural change in financial markets and is therefore sometimes described as "unavoidable". Against this background, the paper addresses three questions:

- Is there empirical evidence for an increase in financial market volatility in Germany?
- Can financial innovation and structural change in the German financial markets help to explain the pattern of volatility?
- How can the central bank detect market uncertainty?

The focus of the paper is on the bond market, which is the largest German securities market. As of mid-1995, outstanding bonds amounted to DM 2,819 billion (market value) compared with DM 718 billion for the market value of exchange-traded shares. In addition, the bond market is of particular importance for financing conditions within the German economy, because a large proportion of long-term credit rates are linked to bond yields. Therefore, day-to-day volatility of bond yields will be scrutinised in the following analysis.

1. Volatility in the German bond market

1.1 Measuring volatility

When analysing volatility, the first problem is to find a suitable definition and measurement concept. Generally, volatility can be interpreted as the variability of an economic variable during a given period of time. In a narrow sense - as it is used in this paper - it comprises short-term (day-to-day or even intraday) price fluctuations. If, by contrast, price movements appear as

1 The views expressed in this paper are those of the authors, and do not necessarily reflect those of the Deutsche Bundesbank.

2 Such a situation occurred in the first half of 1994, when monetary capital formation in Germany was outstandingly weak, partly due to high bond market volatility. See Deutsche Bundesbank (1995), p. 75.

3 According to the available (incomplete) information, about 53% of bank credit is granted at relatively rigid rates in Germany; see Bank for International Settlements (1994a), p. 139.
significant (and persistent) deviations from the longer-term fundamental equilibrium, they are often referred to as misalignment.\(^4\) It seems obvious that the appropriate definition of volatility crucially hinges on the point in question. While financial market stability may be jeopardised by very short-term - and maybe unique - price swings, the effectiveness of interest rate policy or its impact on portfolio decisions is more related to the price fluctuations normally prevailing in the markets.

In this respect, the question also arises as to whether total realised price fluctuations or only unexpected volatility should be considered. In order to analyse the impact of market structure and dynamics on volatility, it seems appropriate to start with \textit{ex post} short-term price movements which are in total the result of market participants' behaviour and institutional arrangements. By contrast, in the final section, where the focus is on expected price fluctuations, we describe a method of ascertaining uncertainty from market prices.

One straightforward method of measuring \textit{ex post} volatility is to calculate the range between the highest and the lowest values of a given time series over a specific time horizon. The measure thus defined is easy to interpret and indicates by how much the price or rate in question has changed over the predetermined period. To show more clearly periods of market stress, it is also possible to record the largest day-to-day jump (in absolute terms) that occurs in a chosen period. To get a representative gauge of the price fluctuations \textit{usually} prevailing in the market, on the other hand, it is advisable to calculate, in addition, another measure that incorporates the values of all observations within a given time interval. An indicator matching this requirement is the standard deviation, which reveals by how much a variable is fluctuating around its mean.

All these measures (range, jump and standard deviation) may be subject to the same disadvantages, as they may be influenced by the level of the variable, the volatility of which they are supposed to describe. The easiest way to cope with this difficulty is simply to scale the above-mentioned measures by the average value the variable in question displays over the respective period. This procedure yields three more measurement concepts. One is a scaled or relative range, the others are the coefficient of variation and the relative jump.

Additionally, a seventh means of detecting volatility is to be employed which is capable of tracking how changes in the value of a variable are spread around an average rate of change rather than a level. This can be done by computing the standard deviation of the daily percentage changes in a variable. Actually, financial analysts and traders usually rely on this measure. Therefore, this measure will henceforth be labelled "financial volatility".\(^5\)

For a volatility analysis based on yields, it is important to bear in mind that the relationship between the price of a fixed-income security and its yield is convex. Therefore, the results can materially deviate from those based on price data. If bond prices move by the same amount (expressed as a percentage) at different yield levels, the impact on the yield - and, in turn, on volatility - may be significantly different.\(^6\) However, this effect, which should be most pronounced for absolute volatility measures, is of minor empirical relevance for the German bond market.

\subsection{1.2 Stylised facts on volatility in the German bond market}

Since 1982, the German bond market has experienced two "bull" periods (see Chart 1a) with yields falling by 4.5 percentage points (1982-87) and 3.5 percentage points (1991-94), respectively. The "bear" bond markets from 1987-90 and in 1994 were accompanied by increases in

\(^4\) See, for example, Frenkel and Goldstein (1988).

\(^5\) For details see, for example, Cox and Rubinstein (1985).

\(^6\) A numerical example: if the price of a 6% bond with ten years to maturity is at 80.00, which coincides with a yield of 9.13%, a price variation of +5% would change the yield by 70 basis points. At a price of 120.00 and a yield of 3.58%, the same price variation would have an impact of 63 basis points.
Chart 1a

Bond yields and volatility - I

1 Yield on public bonds outstanding. — 2 Monthly standard deviation.
Chart 1b
Bond yields and volatility - II

1 Yield on public bonds outstanding. 2 Annualised standard deviation of the last 20 day-to-day yield changes.
yields of 3.5 percentage points and almost 2 percentage points. The longer-term movement of volatility is shown by one absolute measure (standard deviation) and one relative measure (financial volatility).

A visual inspection of the volatility calculated as a monthly standard deviation shows no clear pattern of price fluctuations over time. Most remarkable are the exceptional peaks in early 1990 and late 1992, reflecting the reassessment of economic prospects in the wake of the announcement of German monetary union and the ERM crisis. Periods with a persistently higher volatility are 1982 and 1994. This underlines the fact that price movements measured by standard deviations also mirror the changing trend in the bond market and a reassessment of economic fundamentals.

Financial volatility, by contrast, only takes into account fluctuations around the trend. The volatility pattern shown by this measure differs significantly from the one above (see Chart 1b). Episodes with highest instability are the crash in the stock market in 1987 and the bond market turmoil in 1994. German monetary union, the ERM crisis and the 1982 period of rapid disinflation only appear as events of slightly higher volatility. An interesting finding is that volatility seems to peak in early "bear" markets. This may support the view that, at the very end of a "bull" market, extrapolative expectations play an important role, causing a shock (in terms of high volatility) after the market has reached its turning-point.

Taken together, a comparison of the volatility patterns shown by both measures reveals large differences. However, as pointed out earlier, this does not mean that one indicator is the "better" volatility measure. The standard deviation gives useful information about large absolute price swings, which may cause important behavioural adjustments in the financial as well as in the real sector (for example through the impact on banks' profitability). Financial market volatility more clearly gives an indication of "market noise".

1.3 Has volatility increased?

Neither of the patterns of volatility shown in Charts 1a and 1b gives a clear indication of increasing volatility in the German bond market. Given the phenomenon of more pronounced volatility in "bear" markets, one should be particularly cautious in interpreting price fluctuations in the relative short period of 1994 as clear evidence of persistently higher volatility. In order to test whether price fluctuations have changed significantly over a longer time, we used a standard large-sample test for equality of means.7

For the reasons outlined above, the absolute volatility measures largely reflect the behaviour of the yield level, therefore showing higher volatility in the 1990s against the period of low yields in the second half of the 1980s. The relative (not level-dependent) indicators exhibit no statistically significant increase in short-term price fluctuations over time, with the exception of financial volatility and relative jumps. Taken together, there is no clear evidence of significantly higher volatility.

2. Market structure and volatility

Against this background, there does not yet seem to be an obvious link between realised volatility and factors which are often blamed for causing volatility. Some of the most prominent influences in this respect are the greatly increased amount of tradable assets, the evolution of derivative markets which permit the taking of positions at low transaction costs, and thus make the

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7 As most of the volatility measures introduced above are measured using daily data over some predetermined period, we arbitrarily chose to calculate monthly values for all variables. This choice should ensure that the sample sizes of the three sub-samples in Table 1 are large enough to allow for a rigorous testing whether volatility has - on average - changed over time.
Table 1

Bond yields: level, non-scaled and relative volatility measures

<table>
<thead>
<tr>
<th>Period</th>
<th>Level</th>
<th>Non-scaled</th>
<th>Relative</th>
<th>Financial volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Range</td>
<td>Standard deviation</td>
<td>Jump</td>
</tr>
<tr>
<td>Jan. 1990-</td>
<td>7.56%</td>
<td>0.282</td>
<td>0.087</td>
<td>0.099</td>
</tr>
<tr>
<td>Sept. 1995</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>versus</td>
<td>Jan. 1979-</td>
<td>7.52%</td>
<td>0.303</td>
<td>0.094</td>
</tr>
<tr>
<td>Dec. 1989</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 1984-</td>
<td>6.59% ~</td>
<td>0.238 ~</td>
<td>0.072 ~</td>
<td>0.075 ~</td>
</tr>
<tr>
<td>Dec. 1989</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: ++/+ (-/-): the respective values are significantly larger (smaller) than those of the period from January 1990 to September 1995 at an error level of 1%/5%.

markets more sensitive to new information, or the increasing importance of institutional investors.\(^8\) Indeed, the overall activity in the bond market, as measured by the turnover on the stock exchange, provides just as little definite explanation for the pattern of volatility as the introduction of derivatives on German government bonds in September 1988 and April 1989.\(^9\)

Nevertheless, the question remains if the potential for volatility and therefore the risk of large price swings have increased owing to these factors. This section analyses whether the structural characteristics of the German bond market can help to reveal sources of "realised" day-to-day volatility and provide any information on "potential" volatility.

2.1 Structural features of the German bond market

The German bond market can be separated into the bank bond and the public bond segment, with the former accounting for 54% or DM 1,499 billion (nominal value) of total bonds outstanding and the latter for 45% (or DM 1,251 billion) as of mid-1995. The bank bond segment is far less liquid than the public bond sector. Firstly, it is dominated by a profusion of relatively small issues. Only 4% of the amount outstanding can be assigned to issues with a volume of DM 1 billion or more, compared with 88% in the public bond sector. Secondly, no index for bank bonds which could have served as a benchmark for institutional investors' portfolios existed until April 1995. Lastly, as a consequence of the aforementioned factors, there has been no trade in futures on bank bonds up to now.

Given the fact that the bank bond market is less liquid than the public bond market, one might expect higher volatility of bank bond yields, as the same transaction would tend to cause

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\(^8\) An example of the ambiguity of these influences is financial derivatives. Under normal market conditions, derivatives can be expected to enhance liquidity in the market for the underlying asset and to facilitate arbitrage; see Bank for International Settlements (1994b), p. 17. Thus, derivatives can help to smooth price movements and in turn even reduce price volatility. For the mixed empirical evidence on the effect of derivatives on volatility see, for example, Cronin (1993) or Robinson (1993).

\(^9\) The BUND future contract was introduced at the Liffe on 29th September 1988 while trade in options on these contracts began on 20th April 1989. Simple regression analysis with a dummy variable for the period beginning in September 1988 and April 1989, respectively, does not provide any significant explanation for bond market volatility.
smaller price movements in the more liquid market. However, the day-to-day price fluctuations in public bonds are consistently higher than those in banks' issues (see Chart 2).\(^\text{10}\)

The volatility spread was close to zero in the early 1980s and widened in the period from 1984 to 1987. It fluctuated between 2.5 percentage points and 5 percentage points until the end of 1993, exceeding the 5 percentage point mark in some periods of market stress. In 1994 the volatility spread between public bonds and bank bonds reached a new peak.

![Volatility in the bank bonds and the public bonds segment](chart2.png)

A general explanation of the lower volatility of bank bond yields is that the transactions taking place in the bank bond market are different from those in the public bond sector, or - more precisely - that the portfolio behaviour of market participants differs; aware of the (relative) liquidity constraints, investors might prefer bank bonds for more long-term investments (e.g. "buy and hold" strategies) while public bonds are employed for shorter-term investment strategies. Significant differences in the type of bondholder in each market segment can be seen as an indication that such factors might be of particular importance.

### 2.2 Volatility and foreign activity in the German bond market

Indeed, in the German bond market, the separation on the supply side is mirrored by a sharp contrast in the type of bond holder. While bank bonds are mainly held by domestic investors, foreigners play a predominant role as buyers of public bonds (see Table 2).

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\(^{10}\) For the analysis in this section, financial volatility is employed as a measurement concept.
Table 2

Foreign activity in the German bond market

<table>
<thead>
<tr>
<th>Period</th>
<th>Bank bonds</th>
<th>Public bonds</th>
<th>Gross transactions in public bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM billion</td>
<td>% of total amount outstanding</td>
<td>DM billion</td>
</tr>
<tr>
<td>1980</td>
<td>8</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>1985</td>
<td>27</td>
<td>4</td>
<td>52</td>
</tr>
<tr>
<td>1990</td>
<td>29</td>
<td>3</td>
<td>184</td>
</tr>
<tr>
<td>1991</td>
<td>40</td>
<td>4</td>
<td>231</td>
</tr>
<tr>
<td>1992</td>
<td>101</td>
<td>4</td>
<td>306</td>
</tr>
<tr>
<td>1993</td>
<td>181</td>
<td>13</td>
<td>476</td>
</tr>
<tr>
<td>1994</td>
<td>204</td>
<td>14</td>
<td>488</td>
</tr>
</tbody>
</table>

1 End of year, nominal values. 2 Cumulative transactions over year, transaction values.

The amount of public bonds outstanding held by foreign investors grew from 11% at the end of 1980 to 40% as of end-1994, with a peak of 44% at the end of 1993. By contrast, foreign investment in bank bonds only represents 14% of the amount outstanding. The gross volume of transactions in bonds underlines the outstanding role of the public bond segment for non-resident investors. In 1994, public debt securities accounted for 94% of their purchases and sales of German bonds.

The important role of foreign investors in the public bond segment and the higher volatility observable there suggest that volatility in the German bond market might be a phenomenon related to foreign activity. In that case, a changing weight of non-residents' transactions in public bonds should be systematically associated with fluctuations in the volatility spread between public bonds and bank bonds.

The relevant measure for non-residents' activity with regard to price fluctuations is their market share. As an indication of foreigners' market share over time, their gross transactions in German public bonds are related to the turnover in public bonds as reported by the German stock exchanges. Since these turnover figures only cover a limited share of the trade in public bonds, the ratio of foreign transactions to stock exchange turnover may temporarily be greater than unity. On the assumption that the relationship between trade on the exchange and trade not included in these figures has not changed dramatically over time, this ratio should provide a fairly reliable indication of foreign investors' market share.

Visual inspection of the volatility spread and foreigners' market share shows a relatively close correspondence between the two variables. An increase in the estimated proportion of foreign investors is accompanied by a rise in the volatility spread and vice versa (see Chart 3). The differing

11 However, even this figure overstates foreigners' "genuine" holdings of German bank bonds. It partly reflects the effects of the withholding tax on interest payments introduced in 1993. In the wake of this tax reform, funds were to a large extent shifted abroad by domestic private investors to the affiliates of German banks in Luxembourg. These funds were, in turn, re-invested in the German bond market, in particular in bank bonds. For details see Deutsche Bundesbank (1994).

12 In all transactions, both the buying and the selling side are counted. Trading among brokers is generally included as well. Non-local securities transactions and the direct interbank transactions which are fed into the stock exchange computer are likewise recorded.
relationship of both variables during the ERM crisis period from September 1992 to August 1993 is striking. The massive capital inflows into the German bond market caused a sharp decline in bond yields but were only accompanied by a slightly larger volatility spread. A possible explanation may be that foreign investors' enormous demand for (public) bonds induced unusually large portfolio adjustments of domestic investors, leading to a simultaneous rise of volatility in the bank bond market.

A more formal analysis (OLS estimation) of the relationship between foreign activity and volatility supports these findings (see Table 3). In the first equation (SPREAD I), the volatility spread in the period \( t \) (\( \text{VOLSp}_t \)) is explained by the lagged endogenous variable (\( \text{VOLSp}_{t-1} \)) and contemporaneous foreigners' market share (\( \text{FMS}_t \)).

\( \text{FMS} \) is significant with a positive coefficient, suggesting that an increase in non-residents' market share is accompanied by an increase in volatility. The spread-reducing effect of foreign bond purchases during the ERM crisis is confirmed by the results of equation SPREAD II, where a dummy variable (\( \text{DUM} \)) with a value of 1 for September 1992 to August 1993 is introduced.

The impact of foreigners' activity in the German public bond market together with the volatility spread between "domestic" bank bonds and the more "international" public bonds support the view that international investors' activity and volatility in the public bond segment are closely interrelated. This can be seen as an indication that the group of market participants represented by the statistical aggregate "foreigners" behaves differently from domestic investors as a whole. This, in turn, raises the question of how such different dealing on the part of the various investor groups can be explained.
Table 3

Volatility spread in the bond market and foreigners' market share

<table>
<thead>
<tr>
<th>Variable</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOLSp&lt;sub&gt;t+1&lt;/sub&gt;</td>
<td>FMS&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>SPREAD I</td>
<td>0.617</td>
</tr>
<tr>
<td></td>
<td>(9.96)**</td>
</tr>
<tr>
<td>SPREAD II</td>
<td>0.576</td>
</tr>
<tr>
<td></td>
<td>(8.91)**</td>
</tr>
</tbody>
</table>

Note: **/* = significant at a 1% / 5% error level. Sample: January 1982 - September 1995.

2.3 Role of foreign and domestic institutional investors

A possible key to the explanation of behavioural differences between foreigners and residents lies in the role of institutional investors. Institutions' activities can be a source of higher price volatility in financial markets. Generally, the price effect of new information can be amplified if it causes simultaneous and parallel portfolio adjustments. In the case of institutional investors especially the principal-agent problems arising from the delegation of investment decisions to professional fund managers may cause such "herding" behaviour. Mechanisms possibly inducing higher volatility are, for example, regular performance checks against the markets or - more generally - portfolio managers' fear of deviating from market opinion, particularly in periods of high uncertainty. Both can provide incentives to imitate others' behaviour or even to follow "noisy" signals so as to avoid an underperformance relative to the market. Moreover, specific portfolio management strategies (such as portfolio insurance or stop-loss orders) may cause positive feedback effects.

Leaving aside the problem of whether resident and non-resident institutional investors' behaviour differs significantly, the extreme divergence in the importance of both groups alone would in itself give rise to a different effect on volatility. Although there is no detailed statistical information on who is behind the foreign banks holding almost all securities owned by non-residents, it seems plausible to equate them with institutional investors. Among other factors, relatively high transaction and information costs for cross-border transactions may explain the predominant role of institutional investors able to take advantage of economies of scale in international investment. This theoretical argument is supported by the fact that the vast bulk of foreign transactions in the German bond market is with the United Kingdom - that is, London, from where most of the investment activity of mutual funds and insurance companies in European markets is managed.

Among residents, institutional investors (narrowly defined as funds of investment companies and insurance companies) play only a minor role as measured by their share in holdings of public bonds outstanding (see Table 4). In addition, one-third of these bonds is held by domestic banks. One of the main reasons for the traditionally minor role of institutional investors in Germany, by international standards, is the contributions-financed social security system. Furthermore, company pension schemes are largely funded by provisions for pensions within the company, rather than by investments of funds in the capital markets.

Whether the behaviour of resident and non-resident institutional investors differs significantly is difficult to assess. Figures which allow an evaluation of German institutional investors' gross market activity or give insight into their trading strategies are not available. There are only qualitative indications pointing in the direction of a more "conservative" attitude regarding fund management. Legal bindings for insurance companies and investment funds, which limit the scope for

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13 For an overview of potentially destabilising behaviour patterns of institutional investors, see Davis (1995).
using derivative instruments and therefore for the "leveraging" of portfolios, are such an example. High leverage of foreign investors' bond holdings may have contributed to the sharp increase in volatility in February 1994, exerting pressure to liquidate positions in an environment of falling bond prices.

A second fact pointing towards a different behaviour on the part of resident institutional investors is that savers' demand is focused more on the long-term return of investments. The offering premia on fund units of German investment funds, which are traditionally relatively high and discourage short-term investments in such instruments, may be seen as an indication for this. In turn, investment funds usually do not face large short-run swings in fund unit holdings by private households which could trigger large portfolio shifts and increase price fluctuations.

### 2.4 Conclusion

Institutional investors' activity in the German bond market seems to have a significant impact on market volatility. Against this background, an increase in the importance of resident institutional investors in future and the ongoing process of international diversification of portfolios might further increase the potential for price fluctuations. Therefore, the central bank could face periods of higher day-to-day volatility in the bond market more often in future. However, it does not seem to be the existence of institutions, but rather the prevalence of specific potentially destabilising investment strategies and trading techniques, that may ultimately lead to higher actual volatility.

### 3. Means of detecting expected price fluctuations in the bond market

#### 3.1 Implied volatilities

The most common approach is to employ market players' methods of pricing options on bond futures, as these are standardised and more liquid than bond options. The standard model for pricing the former derivatives is based on the approach developed by Black and Scholes (1973), and enables traders to calculate an option's value by a formula that requires very little input. All inputs but one are

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easily obtainable, such as the appropriate risk-free short-term interest rate, the current value of the underlying asset of the option, its strike price and its time to maturity. The single missing variable is the expected volatility of the underlying asset during the remaining maturity of the option. Knowing the price of an option, the formula that market participants use to derive this value, and all but one input, it is possible to compute the value of the single unknown. As this estimate of the expected volatility is implicitly contained in the option price, it is also referred to as the implied volatility. Its values are expressed as an annualised percentage figure but, owing to the finite remaining maturity of the option, they only incorporate the expectations over the time remaining until the derivative expires.

However, implied volatilities can be used for more than "just" revealing the market expectations of future price fluctuations. If we assume that these expectations are rational, we could use them as an actual predictor of future volatility. We assess predictive accuracy in two stages. The first is to establish whether they correctly predict the direction of movement of future volatility and then - as a second stage - to see whether the implied volatilities could be used as a reliable (exact) proxy for future volatility. In the following section we report the results for the first test only since we find that with respect to the second step there is little to be gained from quantifying the actual outcomes.\footnote{For a detailed description of the tests applied, see Neuhaus (1995).} We proceed as follows: whenever implied volatility exceeds the current (historical) volatility, we interpret this as a predicted increase and vice versa.\footnote{This approach is in line with the approach of Feinstein (1989).} With rational expectations and in the absence of risk premia in option prices, market players should not be wrong systematically.

This proposition was tested for options on the German BUND future, which is the future on German long-term government bonds, for the period lasting from June 1989 to November 1994 (22 observations). Indeed, using call options whose strike prices are equal to or slightly higher than the current BUND future price,\footnote{Beckers (1981) was the first to propose to make use of at-the-money options only.} it can be shown that, with 60, 40 and 20 trading days left until the option expires, the forecasts were very reliable. For example, with 40 days to maturity, more than 90% of the forecasts were correct (see Table 5).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Forecast horizon & Forecasts/results & Overall & Increase & Decrease \\
\hline
60 days & Correct forecasts & 19 & 12 & 7 \\
& Out of & 22 & 14 & 8 \\
40 days & Correct forecasts & 20 & 12 & 8 \\
& Out of & 22 & 12 & 10 \\
20 days & Correct forecasts & 14 & 8 & 6 \\
& Out of & 22 & 12 & 10 \\
\hline
\end{tabular}
\end{table}

But, when trying to use the implied volatilities to generate quantitative volatility forecasts, the results are less than satisfying. This may be due to several factors. One is that, especially in the financial markets, news arrives with such a high frequency that expectations may be quickly outdated, and thus not reliable for precise quantitative forecasts.\footnote{This does not mean that recovering the expectations may not be useful for a market analysis.} Another explanation is that the assumptions the Black-Scholes model imposes are too strong. Two of these assumptions are that daily returns are normally distributed, implying that future prices or rates are log-normally distributed, and that no jumps in prices may occur. However, many market participants do not believe in the normality
assumption, and correct for possible jumps in prices or use a non-normal distribution. Thus, it could be advisable not to impose a specific probability density function, but rather to recover the probabilities "the market" attaches to specific events, like the future rate or price trading below or above certain values or within a given range.

3.2 Implied probability distributions

This is the second way to obtain the information contained in option premiums and to shed light on the dispersion of market expectations. It is based on the most general way of pricing an option. A risk-neutral economic agent would be ready to pay as much for an option as the discounted pay-off the option is expected to generate. For a call option that is margined, the discounting has to be omitted (because the writer of the option only receives the premium at the time the option expires) and its premium \( C \) is calculated in the following way:

\[
C = \int_{-\infty}^{+\infty} w(F_T) \max(0; F_T - K) dF_T.
\]  

\( F_T \) is the price of the underlying asset on the expiry day \( T \); \( K \) is the option's strike price and \( w \) is the probability density function that market participants believe reliably describes the behaviour of the underlying asset. If either this function or the probability distribution were known, the likelihood the market attaches to the underlying asset being above or below a certain value, say the strike price of the option, could be computed. To back out the probability distribution, one simply has to calculate the first order derivative of \( C \) with respect to \( K \).

\[
C_K = -\int_{k}^{+\infty} w(F_T)dF_T.
\]  

This is equivalent to

\[
-C_K = p(F_T \geq K).
\]  

A drawback of this approach is that it assumes a variable \( C \) that is continuous in \( K \). However, as only a finite number of options is traded within each maturity class, \( C \) is a discrete variable. Since it is not possible artificially to generate the missing call premia without imposing assumptions on the structure of the probability distribution, it is advisable simply to approximate \( C_K \) by the first order difference quotient, making use of the fact that options - especially those traded on exchanges - usually exhibit a constant difference \( \Delta K \) between the different strike prices. Thus, the probability of the underlying asset exceeding the strike price \( K_i \) can be approximated by:

\[
p(F_T \geq K_i) = \frac{C_{i-1} - C_{i+1}}{2 \cdot \Delta K}.
\]  

Using as many \( K_i \)'s as are available, it is possible to generate the empirical or implied probability distribution (ipd). With the help of this ipd, a lot of useful information can be recovered.

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19 See, for example, Cookson (1993) or Gemmill (1993), p. 113.

20 Moreover, the marging also allows American style options to be used for the procedure outlined below as the likelihood of early exercises is close to zero.

21 The index \( i \) is equal to one for the most expensive option (i.e. the one with the lowest strike price).

22 The first approach to recover probabilities implied by option prices was developed by Breeden and Litzenberger (1978). However, their method deviates from the one presented here. For a more extensive discussion of the ipd approach see Neuhaus (1995).
For example, as the ipd allows calculation of the implied probabilities, it is possible to check whether the probability density functions that market participants associate with the underlying asset exhibit special features like fat tails or even multi-modality. The latter probably occurs when market participants believe that two or more scenarios are likely with different consequences for the underlying asset's price or rate.

To monitor the expectations over time, it is also possible to summarise the information contained in the ipd with only a few variables. The summary statistics should contain the expected value of the variable and an indicator that reveals the dispersion of the expectations. Preferably, the dispersion parameter should describe the true distribution as accurately as possible. Hence, it should not be a symmetrical parameter as with the standard deviation. A superior approach is to exploit the fact that the ipd is known and calculate a confidence interval. The level of the percentiles and the distance between them indicate what the market expectations are and how large their dispersion is.

An example of this method is shown below for options on the Liffe BUND future. In this case, the quartiles were calculated, which in turn define the 50% confidence interval. Thus, as the future is believed to move above or below this range with a probability of 25%, these thresholds specify a range that represents "mainstream" expectations. If the purpose of the monitoring is to detect what the market believes the maximum movement of the underlying asset could be, the quartiles could easily be replaced by larger percentiles. The expected future value itself is easily determined, because for futures it coincides with their respective current price.

Chart 4 illustrates what information is revealed by these indicators for the period lasting from the beginning of February to the end of March 1994. The shaded area represents the confidence interval, the width of which is explicitly displayed underneath. The solid line within the shaded area is the (risk-neutral) expected value of the future.

As Chart 4 shows, neither the Federal Reserve's tightening of its monetary policy at the beginning of February nor the Bundesbank's interest rate cut on 17th February had a significant impact on the uncertainty in the German bond market. However, when on 2nd March 1994 the unexpectedly high annualised rate of change of the money stock M3 was published, exceeding the target range of 4-6% by nearly 15 percentage points, the spread between the 75 and the 25% threshold rose dramatically by more than 100 basis points to a level of 4.8. Since the consequences of the news for both interest rate decisions and the outlook for inflation were not clear to market participants, the uncertainty in the market did not decline to the level that prevailed prior to the data release. Hence market participants would not rule out the possibility that larger price movements might occur in the future.

Finding an increase in the dispersion of expectations, as manifested in a widening of the confidence interval, is similar to detecting a rise in expected volatility. However, for most monetary policy purposes, applying the probability distribution implied by option premiums exhibits some advantages over implied volatilities. The main reasons are:

- In contrast to the way in which the Black-Scholes implied volatility is computed, the implied probability distribution and the confidence interval neither impose a probability distribution on the price or the rate of the underlying asset nor on the diffusion process that the price or the rate of the underlying asset may follow. Hence, market analysts are much more likely to back out market expectations by employing the distribution-free method to calculate implied probabilities. It also allows for possible detection of "fat tails" or multi-modality.

- In contrast to the Black-Scholes implied volatility, the width of the confidence interval is not (necessarily) a symmetrical dispersion parameter and is thus more accurate.

- In contrast to the Black-Scholes implied volatilities, the confidence interval gives an immediate and intuitive understanding of the extent to which expectations are dispersed, since the boundaries are known.
However, for some purposes, such as a cross-country comparison of the uncertainty prevailing in financial markets, implied volatilities may produce results which are both more readily available and more comprehensive. Furthermore, since implied volatilities are annualised figures, they are not as dependent on the remaining time to maturity of the option as is the width of the confidence interval, which is bound to decrease as the residual maturity grows shorter.

By applying the above methods and by recovering the information contained in the prices of derivatives, and especially in options, central banks have a means of evaluating by how much market players expect prices to fluctuate.

Conclusions

Although the German market for debt securities experienced a period of historically high volatility during the bond market turbulences in 1994, there is no clear evidence that day-to-day volatility has increased in a longer perspective. Looking at possible reasons for short-term price fluctuations, structural features of the German bond market and the volatility pattern in different market segments support the view that institutional investors' behaviour has contributed to price instability to a significant extent. Given a further increasing role of non-resident as well as resident institutional investors in the German market, monetary policy may face a growing potential for short-term price fluctuations in the future.

Against this background, it is important for the central bank to detect market uncertainty as early and precisely as possible. By recovering the information contained in the prices of derivatives, and especially in options, central banks have a means of evaluating by how much, and within which range, market players expect prices to fluctuate.
References


