Measuring international volatility spillovers

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1. Introduction

Since the early 1980s, there has been a trend to globalisation in international financial markets as investors' portfolios have become more internationally diversified.[2] Perhaps as a result of this, volatility has been closely synchronised across national bond markets since the mid-1980s (Borio and McCauley (1996)). Moreover, we have witnessed increased comovements of interest rates across national markets that do not seem to be fully explicable by fundamental domestic determinants. As an example, we have the increase in long-term rates in Europe and Japan following the Fed's tightening of monetary policy in early 1994 in spite of the fact that these economies were considerably weaker than the US economy at that time.[3]

The observations made above raise (at least) two important questions. First, where do the shocks originate – in the large bond markets or in the small markets themselves – and how are they transmitted from one market to another? Interest rates could move either in response to global events that affect all national markets or they could be primarily driven by local events. Second, do markets move together at exactly those times when investors do not want them to, i.e. when volatility is high? If this were the case, the benefits of international portfolio diversification would be reduced.

In this paper we consider the transmission of shocks from the United States and German bond markets to the Swedish bond market during the recent 1993–98 period of floating exchange rates. More specifically, we characterise the behaviour of conditional variances and covariances on German, Swedish and US bonds. The emphasis is on volatility spillovers from two larger markets – Germany and the United States – to a small market – Sweden. Furthermore, we address the question of whether markets become more correlated during periods of high volatility. Another question is how persistent are these correlations and what is of importance for longer horizon correlations?

It is well established that financial data, at least when sampled monthly or more frequently, exhibit time-varying second moments. The parsimonious autoregressive conditional volatility (ARCH) modelling seems to be able to capture this variation in second moments.[4] Univariate time-series models of asset returns have emphasised stylised facts in the form of volatility clustering and the persistence in volatility. More recently, there have also been studies employing multivariate ARCH models.[5] Unlike these studies, our focus will not only be on the conditional variances of returns, but also on the conditional covariation of returns. In addition, most previous studies have been concerned

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[2] See, for example, Davis (1991), Frankel (1992), Mussa and Goldstein (1993) and Tesar and Werner (1995).

[3] There is also anecdotal evidence that, at least in the short term, bond traders react to developments in the major markets rather than to the fundamentals in the country of relevance.

[4] See Bollerslev et al. (1992) for a survey of the empirical ARCH literature. An extensive review of the methodology is given by Bollerslev et al. (1994).

[5] Engle et al. (1990) employ factor ARCH models in their exploration of common features in returns on US bonds with different maturities. Ng et al. (1992) investigate US stock returns in a similar setting. Furthermore, Diebold and Nerlove (1989) analyse exchange rate movements in this context. The interaction between volatility and the transmission of volatility across international stock markets is considered by Hamao et al. (1990), Engle et al. (1990), Lin et al. (1994), and King et al. (1994), among others.

with equity markets rather than fixed income markets. For portfolio managers, worldwide bond markets should be of equal or greater importance.

Often, the number of parameters in multivariate models increases dramatically with the dimension of the system, making them intractable. It is therefore necessary to use a specification that reduces the number of parameters to estimate, but is still flexible enough to capture the dynamics and features of the data. In this paper, we use an ARCH model which allows local as well as global influences. The idea behind the parameterisation is that a few common economic factors determine asset prices. This is implemented on both covariances between returns and on the structure for the expected returns. Hence, this kind of model places restrictions – consistent with the basic idea – on the conditional means and conditional variances for the returns.

The paper is organised as follows. In Section 2, we give a brief description of the data and a preliminary analysis which serves as a motivation for the paper. Our empirical model is formulated in Section 3. The results from the estimation of the basic model together with diagnostics are presented in Section 4. A characterisation of conditional variances as well as the derived conditional correlations is given in Section 5. Finally, we offer some concluding comments in Section 6.

2. Data and some preliminary analysis

2.1 Data

We consider holding returns for the period 20 October 1993 – 22 December 1998 on three national bond markets: Germany, the United States and Sweden. Ideally, we would like to consider more countries but our parsimonious parameterisation, described in the next section, only allow for a small set of countries. The choice of the larger markets, Germany and the United States, is meant to capture the common global economic factors affecting returns.

The bonds have a maturity of 10 years. The sample is weekly and the holding period is one week. The holding returns for the 10-year bonds are computed from constructed zero coupon bond rates obtained using Svensson's (1995) extension of the Nelson and Siegel (1987) model. The rates are collected at around European market closing time every Tuesday, or Wednesday if Tuesday is a holiday. The source is the *Sveriges Riksbank*.

Figure 1. Swedish, German and US 10-year interest rates

Figure 2a. Weekly returns on Swedish 10-year bonds

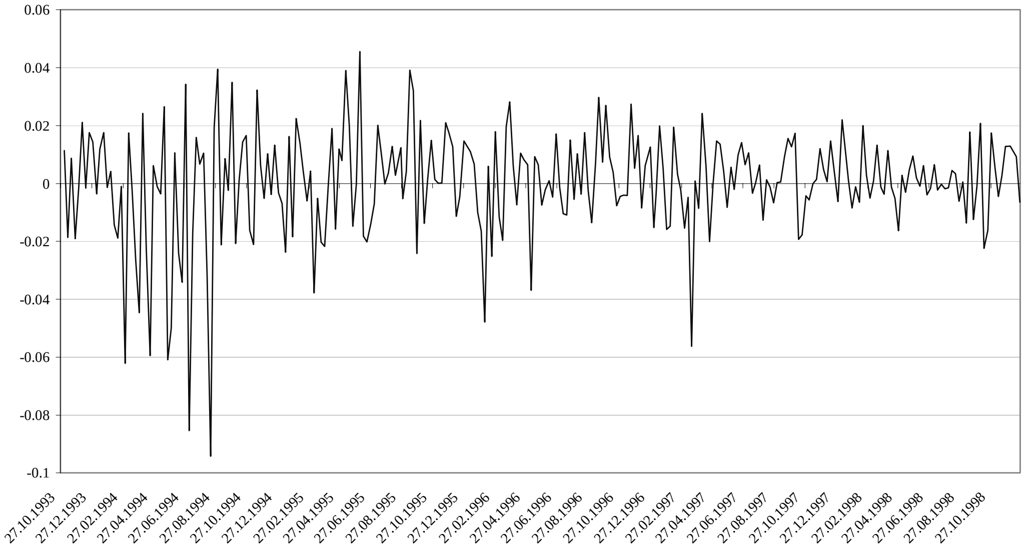


Figure 2b. Weekly returns on German 10-year bonds

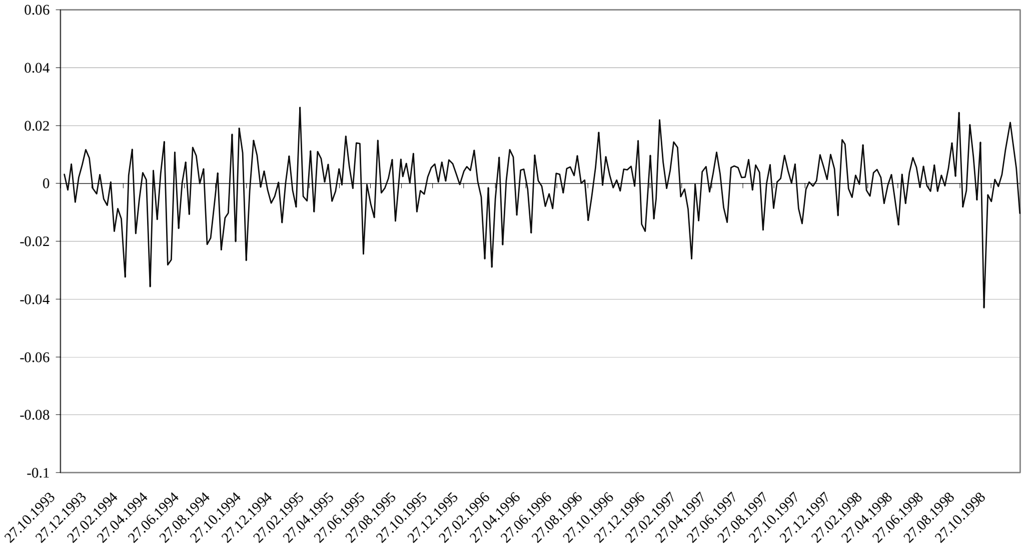
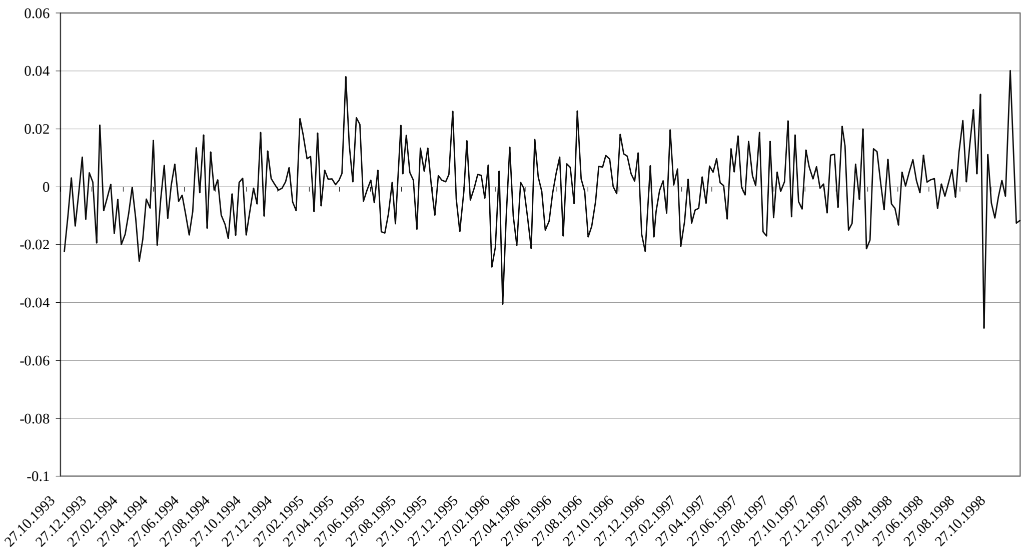


Figure 2c. Weekly returns on US 10-year bonds



Holding returns are constructed by taking the difference between the logarithm of the prices of the constructed 10-year zero coupon bonds and the10-year-minus-one-week bonds one week later. We use local returns. That is, the returns are not converted to a specific currency, but can rather be seen as (perfectly) hedged returns. We are thus abstracting from influences due to variations in exchange rates.

The 10-year interest rates on Swedish, German and US bonds are shown in Figure 1. The Swedish 10-year interest rate has clearly converged toward the lower paths followed by the German and US interest rates. During the turbulence in early 1994, there was a strong increase in the Swedish 10-year rate, which then remained at about 4 percentage points above the German and US rates for most of 1994–95. But as Swedish economic policy gained credibility among investors, the interest rate gradually declined. During most of 1998, the Swedish rate was close to the German rate and lower than the US rate.

In Figures 2a–c, we depict the weekly returns on 10-year bonds in Sweden, Germany and the United States. We use the same scales to facilitate comparisons. It is evident that the Swedish returns are more volatile than the German and US returns, especially during the earlier part of the sample. It is more difficult to see whether volatility is positively correlated across markets. We will study this question more carefully below.

2.2 Some preliminary results

This section serves to illustrate some of the interdependencies among the three national bond markets. We are primarily interested in whether, and if so to what extent, volatility in one market spills over and contributes to volatility in another market. For investors, interested in diversifying the risk in their portfolios internationally, it is of importance to know how correlations vary over time. It is especially important that markets are not more correlated in times of high volatility, since this would mitigate the benefits of diversification.

In Figures 3a–b, we look at how the volatility of interest rates and returns in the three markets has evolved over time. We do this by computing rolling standard deviations using 20 weeks of data for each observation. There seems to be some positive covariation among the series depicted in Figures 3a–b. There also seems to be a downward trend in volatility in the Swedish bond market during this period.

To see if we can detect if and when there is a shift in volatility, in Figures 4–6 we present the cumulative sum of squares, C_k, and the centred cumulative sum of squares, $\sqrt{T/2}D_k$, of the return series for the three markets. These measures are defined as follows:

(1) $$C_k = \sum_{t=1}^{k} \varepsilon_t^2$$

where ε_t is the residual from a regression of the return on a constant and lagged return, and

(2) $$D_k = \frac{C_k}{C_T} - \frac{k}{T}$$

for $k = 1,...,T$, with $D_0 = D_T = 0$. The reason for de-meaning and filtering out autocorrelation in the return series is that this makes possible correct statistical inference regarding $\sqrt{T/2}D_k$. This quantity should oscillate around zero for series with homogeneous variance. If the maximum absolute value at date k, $\max_k\left|\sqrt{T/2}D_k\right|$, exceeds the critical boundary value of 1.3, we can with 95% confidence say that there has been a shift in variance at date k.[6] We have marked these boundaries in Figures 4–6. A peak indicates a downward shift and a trough an upward shift in volatility.

[6] The critical values for different sample sizes are tabulated in Inclán and Tiao (1994).

Figure 3a. Rolling standard deviations: 10-year interest rates

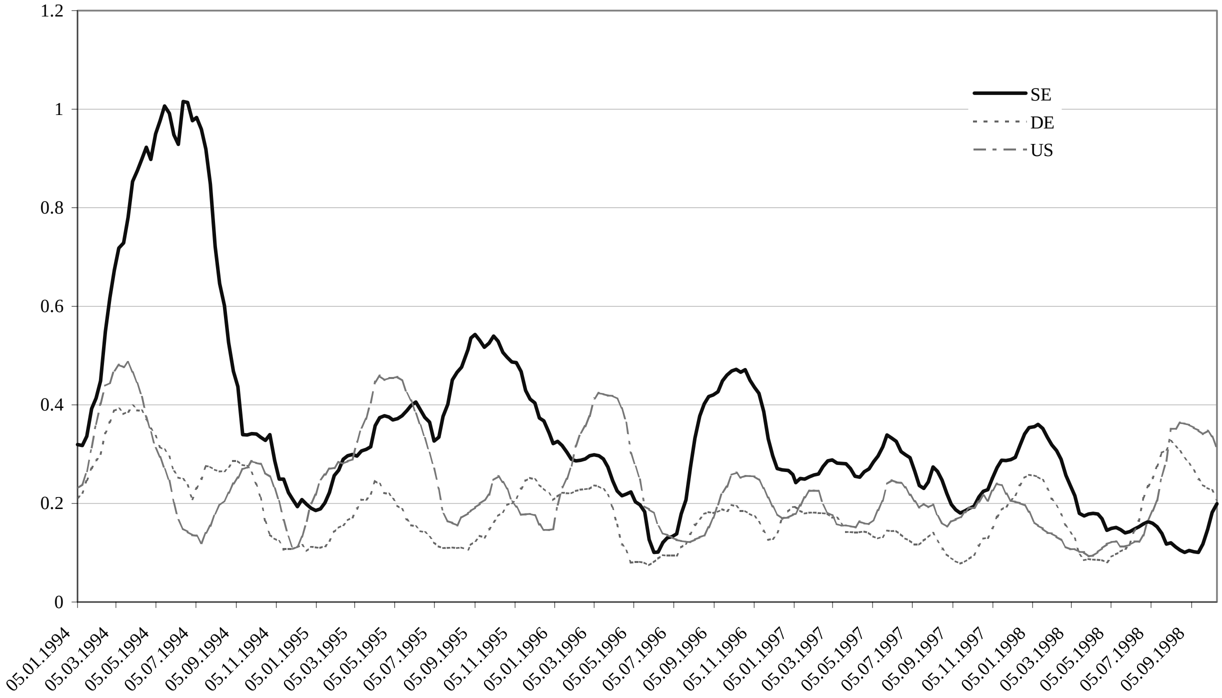
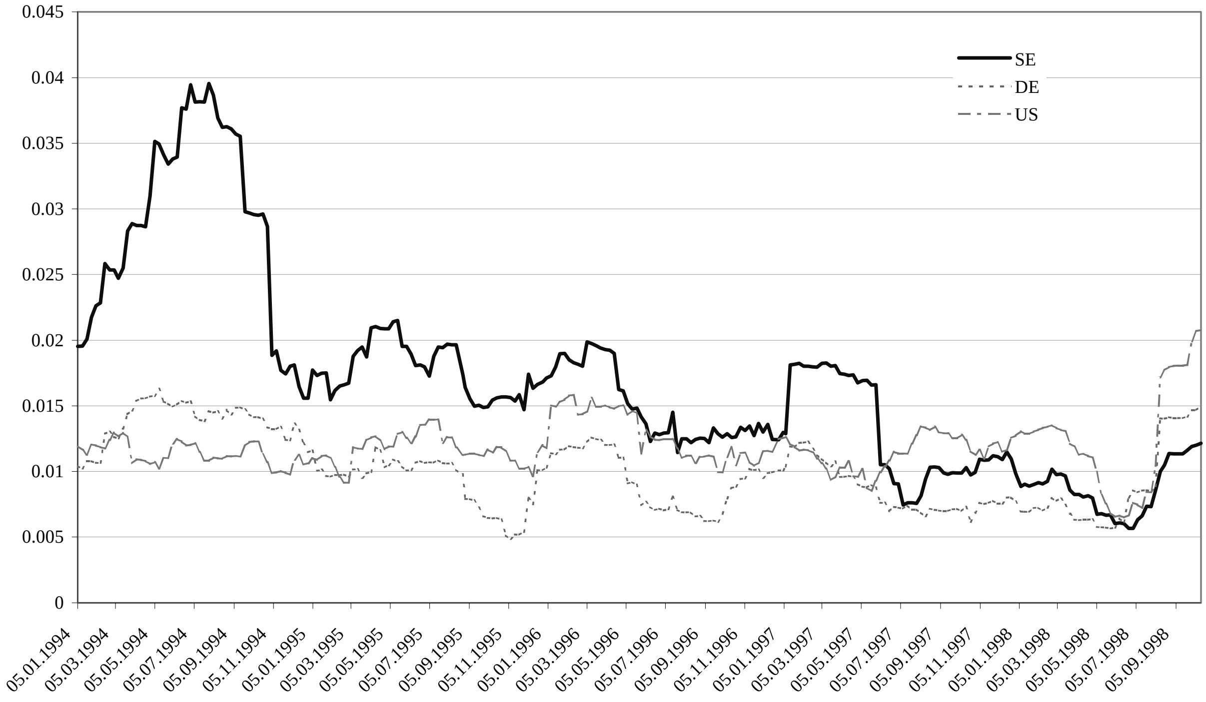


Figure 3b. Rolling standard deviations: weekly returns on 10-year bonds



Inclán and Tiao (1994) present an algorithm, based on the centred cumulative sum of squares, for detecting all the shifts in variance that have occurred in a time series. First, the maximum trough or peak is found, which then represents a candidate breakpoint. In the Swedish series, we find a maximum significant departure on 16 November 1994 (Figure 4b). Then, the series before and after this point are investigated (Figure 4c–d) in order to find candidates for the first and last breakpoints

Figure 4a. Cumulative sum of squares: Sweden

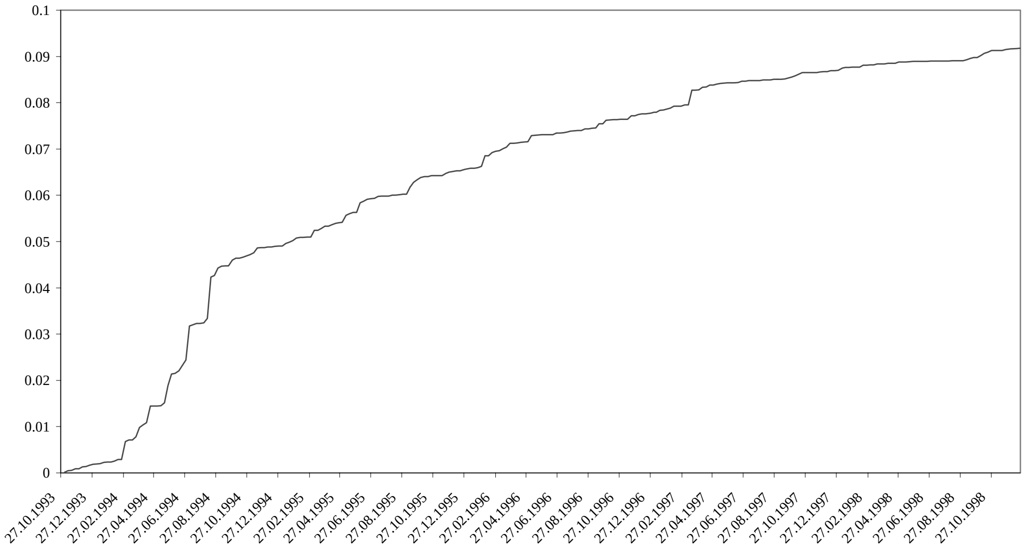


Figure 4b. Centred CSS: Sweden

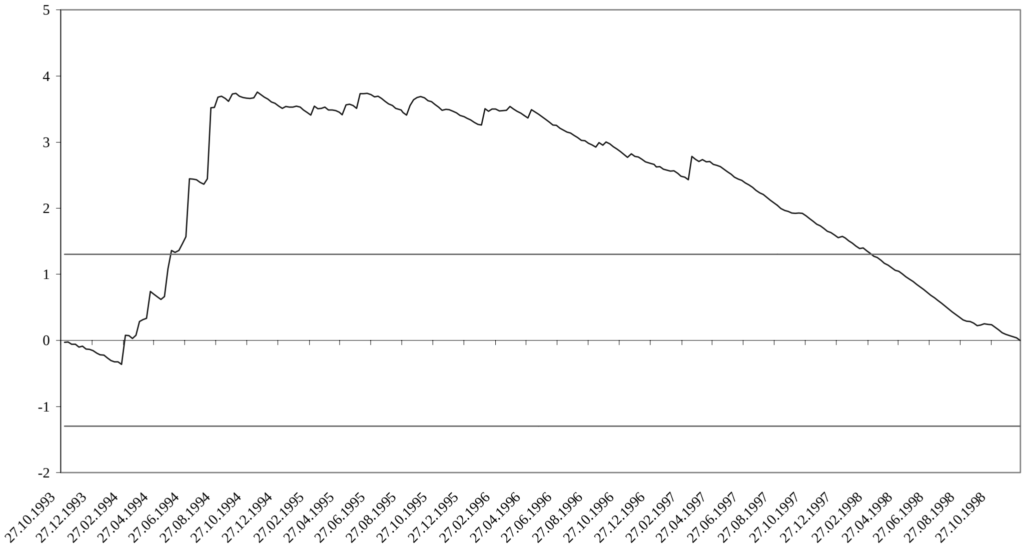


Figure 4c. Centred CSS: Sweden 3 Nov 1993 - 16 Nov 1994

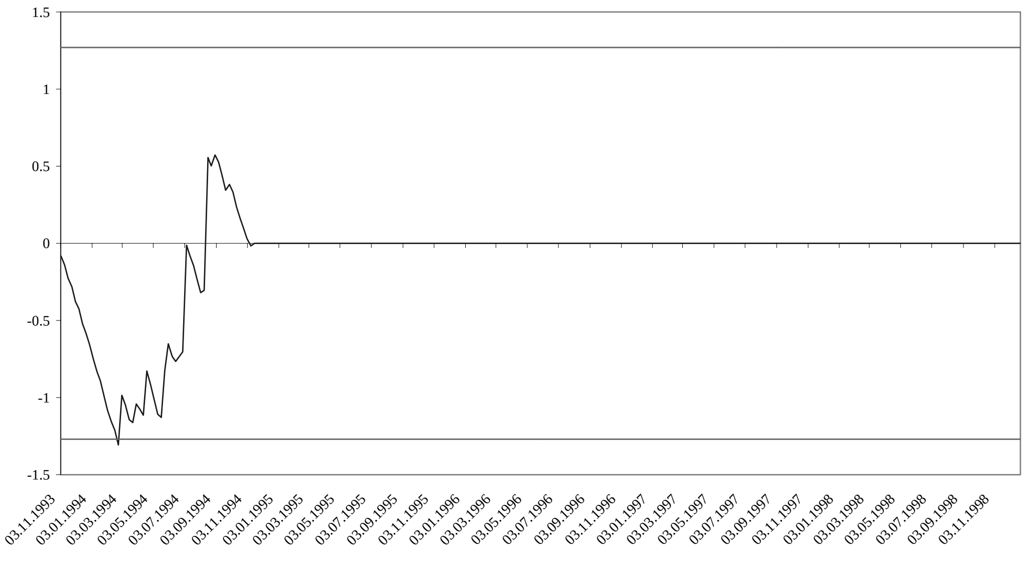


Figure 4d. Centred CSS: Sweden 23 Nov 1994 - 22 Dec 1998

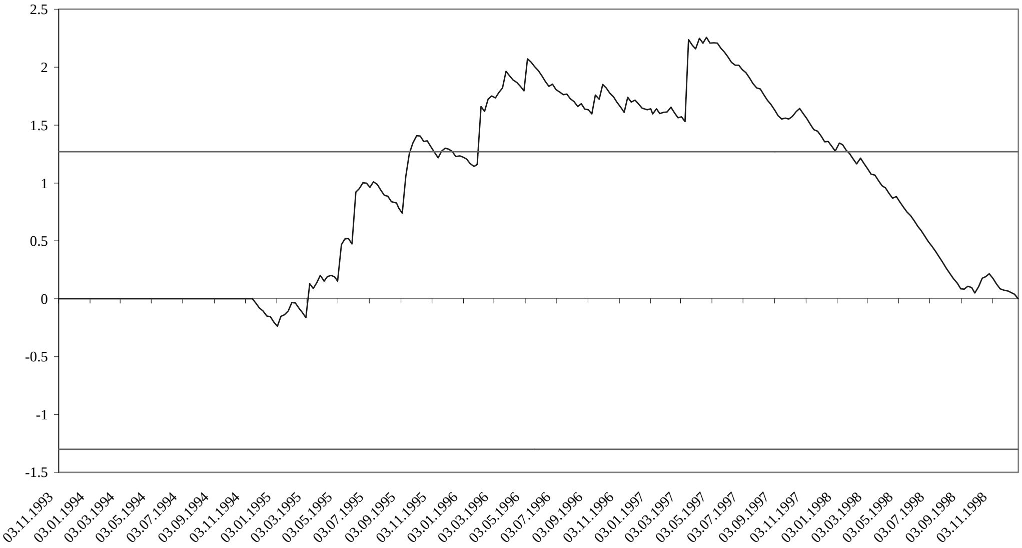


Figure 4e. Centred CSS: Sweden 29 April 1997 - 22 Dec 1998

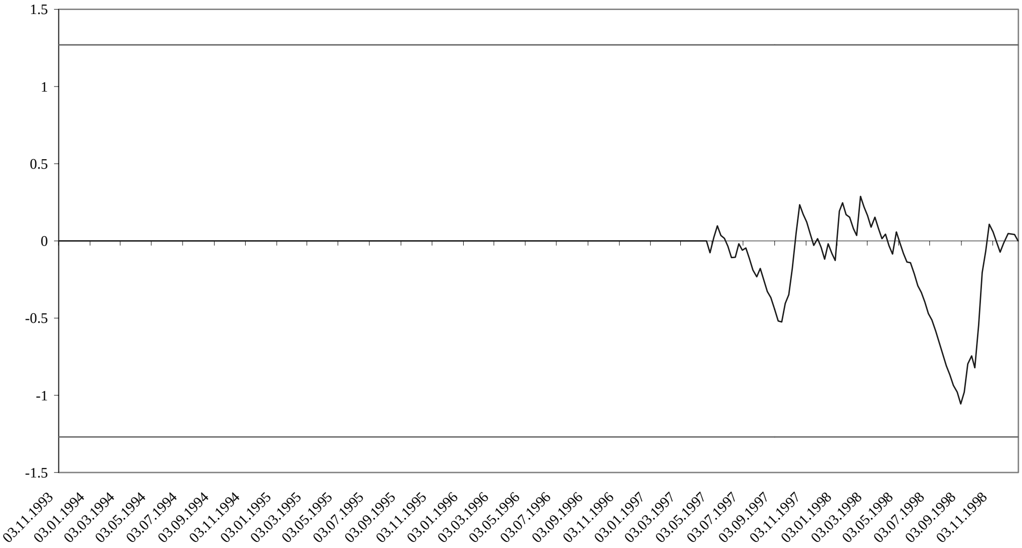


Figure 4f. Centred CSS: Sweden 2 March 1994 - 22 April 1997

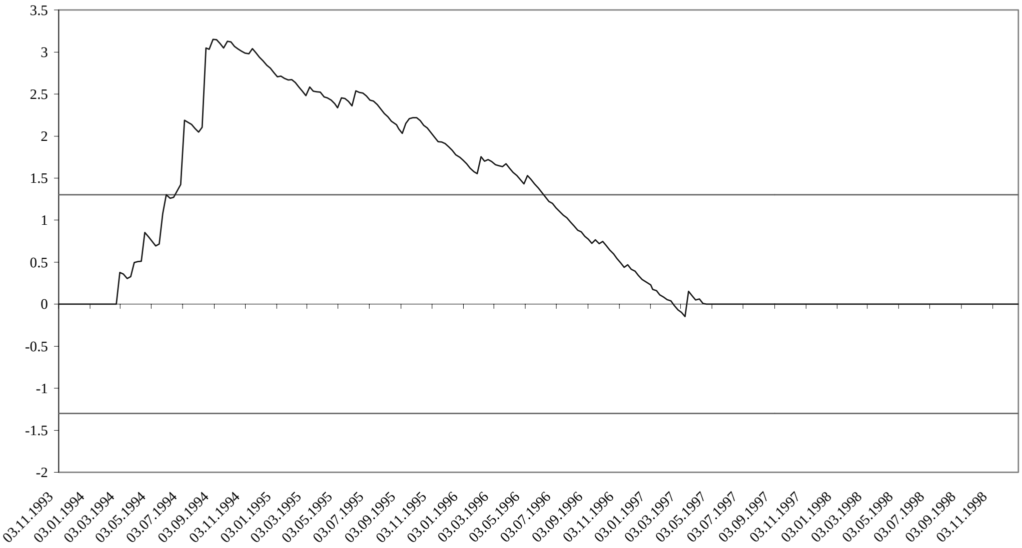


Figure 4g. Centred CSS: Sweden 2 March 1994 - 31 Aug 1994

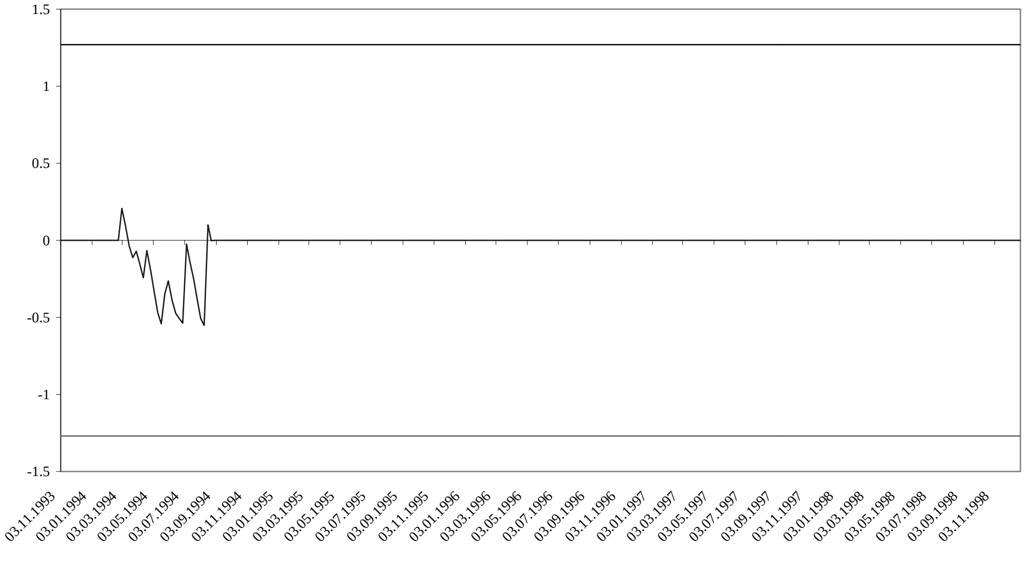


Figure 4h. Centred CSS: Sweden 7 Sept 1994 - 22 April 1997

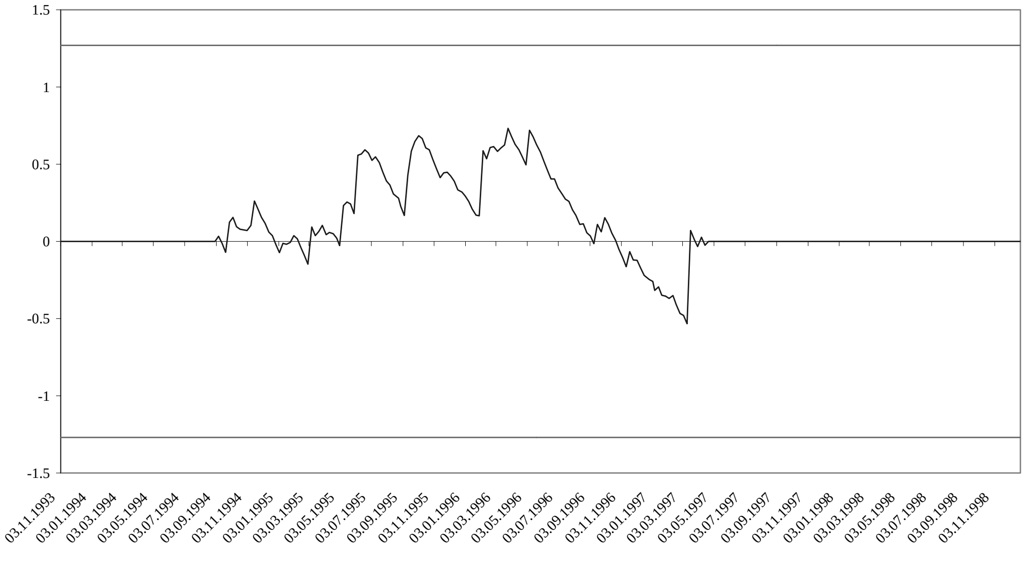


Figure 5a. Cumulative sum of squares: Germany

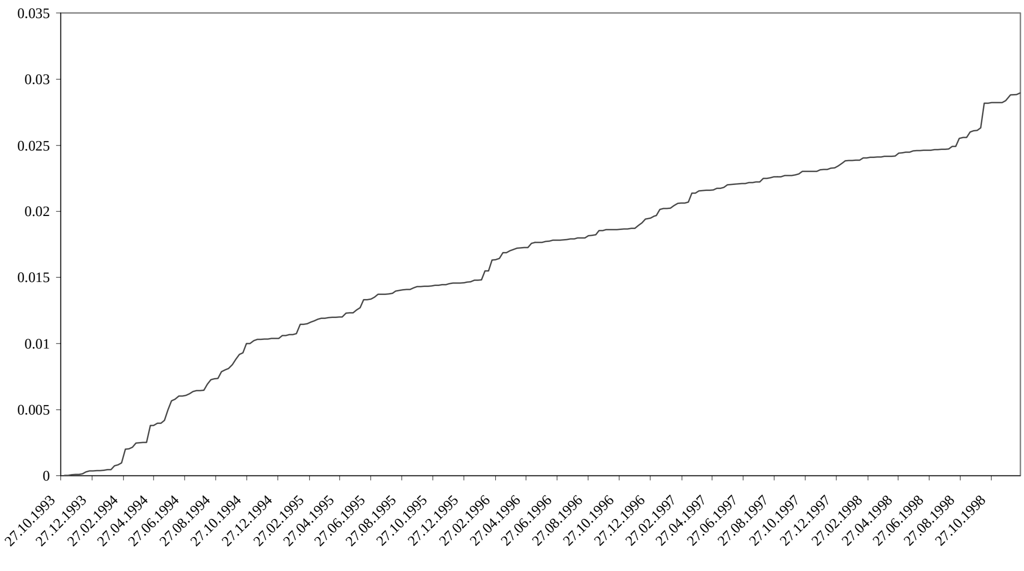


Figure 5b. Centred cumulative sum of squares: Germany

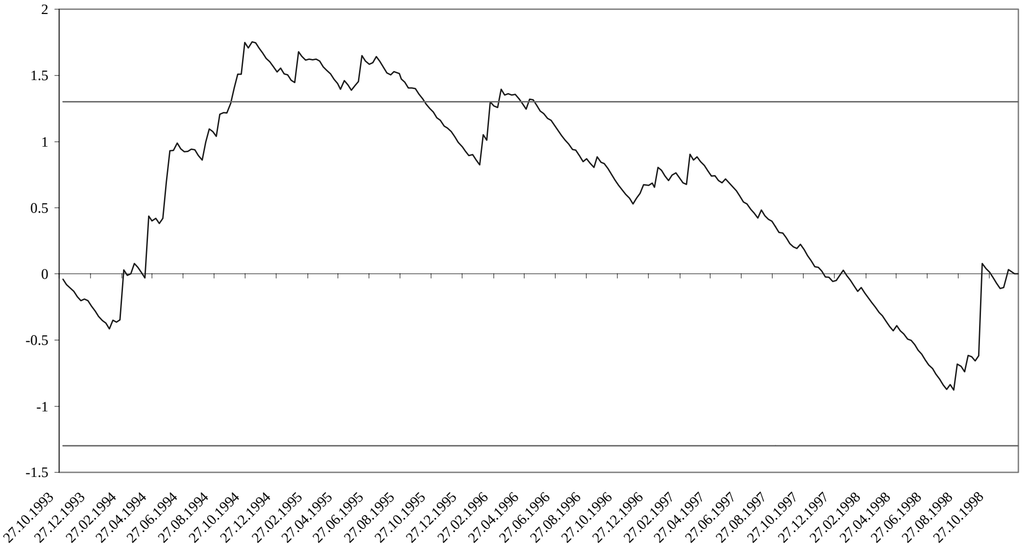


Figure 5c. Centred CSS: Germany 3 Nov 1993 - 26 Oct 1994

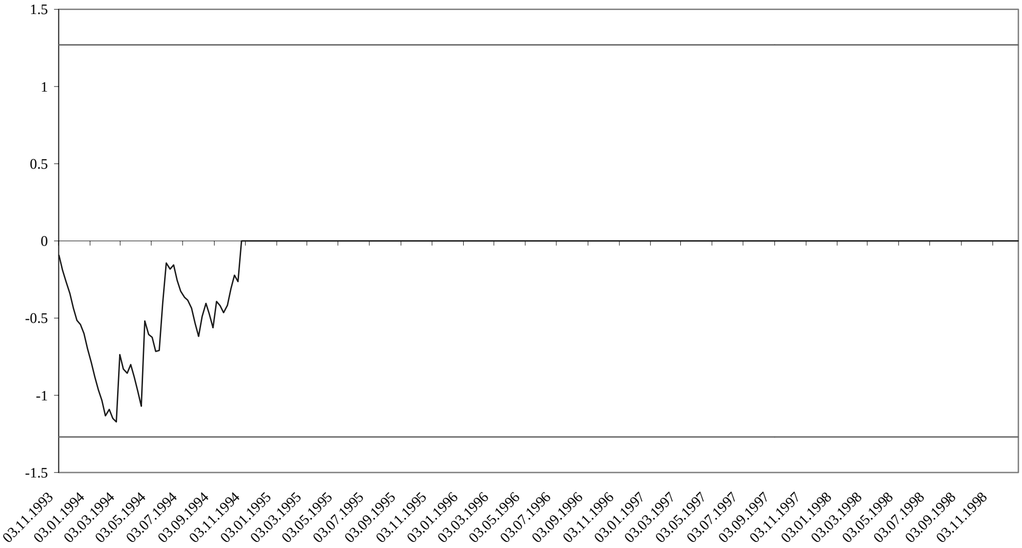


Figure 5d. Centred CSS: Germany 2 Nov 1994 - 22 Dec 1998

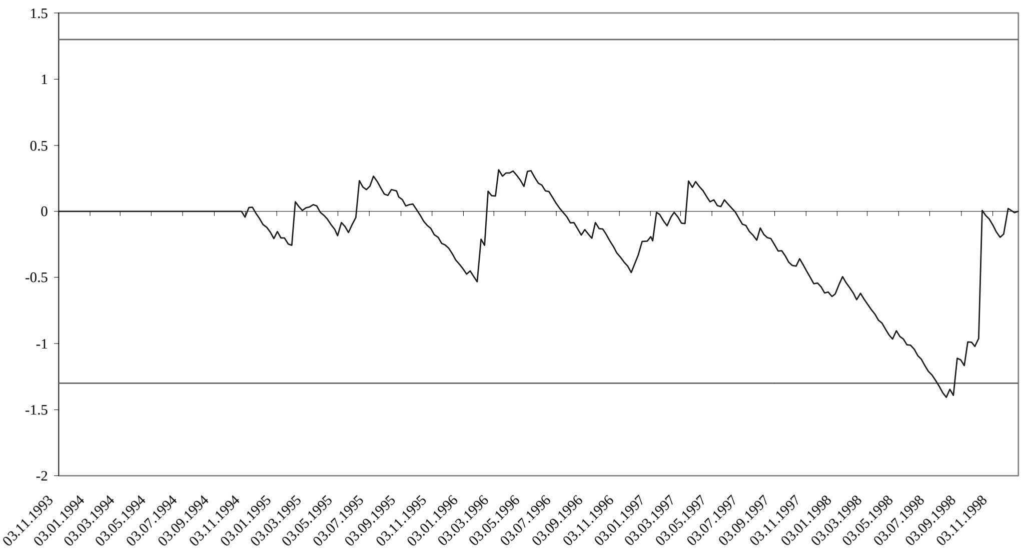


Figure 5e. Centred CCS: Germany 2 Nov 1994 - 18 Aug 1998

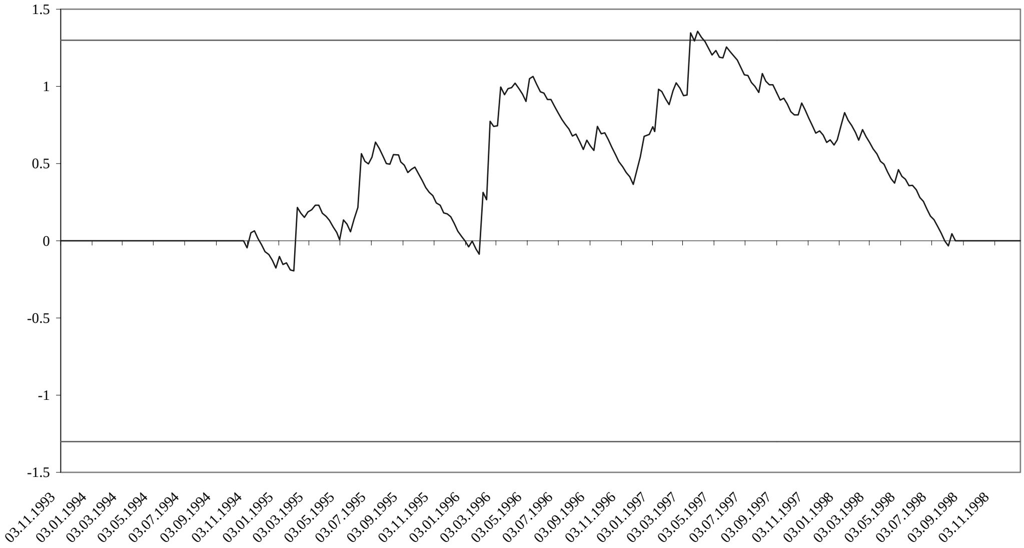


Figure 5f. Centred CSS: Germany 2 Nov 1994 - 1 April 1997

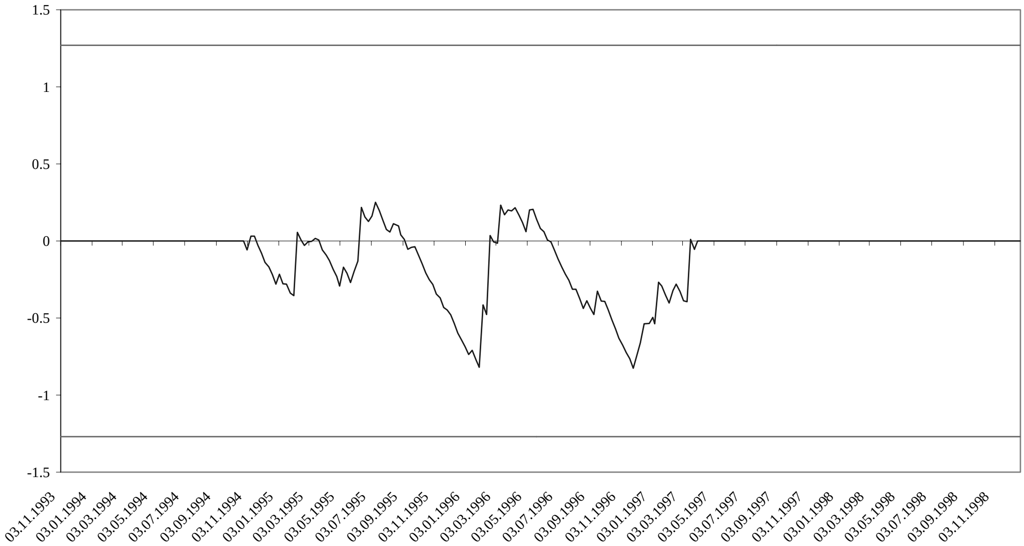


Figure 5g. Centred CSS: Germany 8 April 1997 - 18 Aug 1998

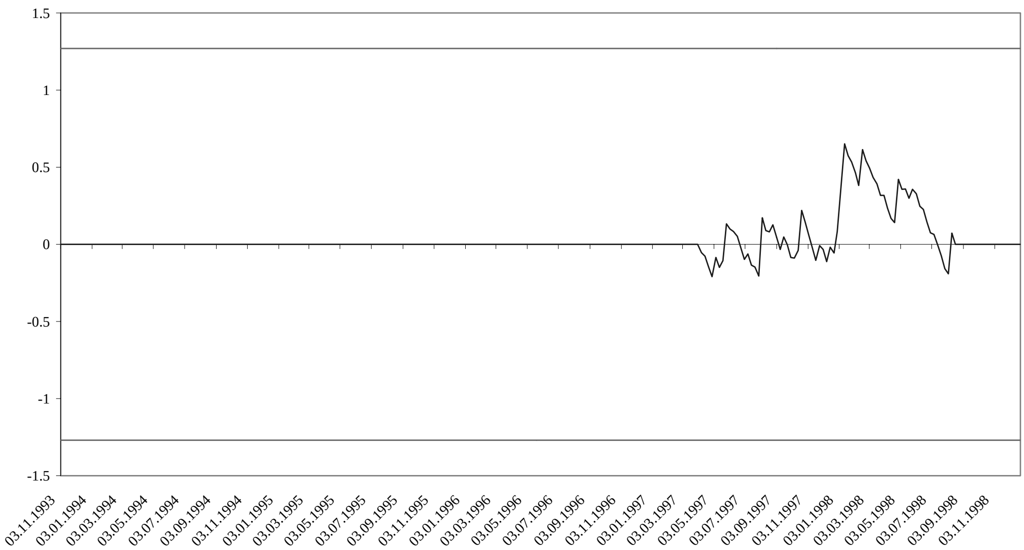


Figure 6a. Cumulative sum of squares: US

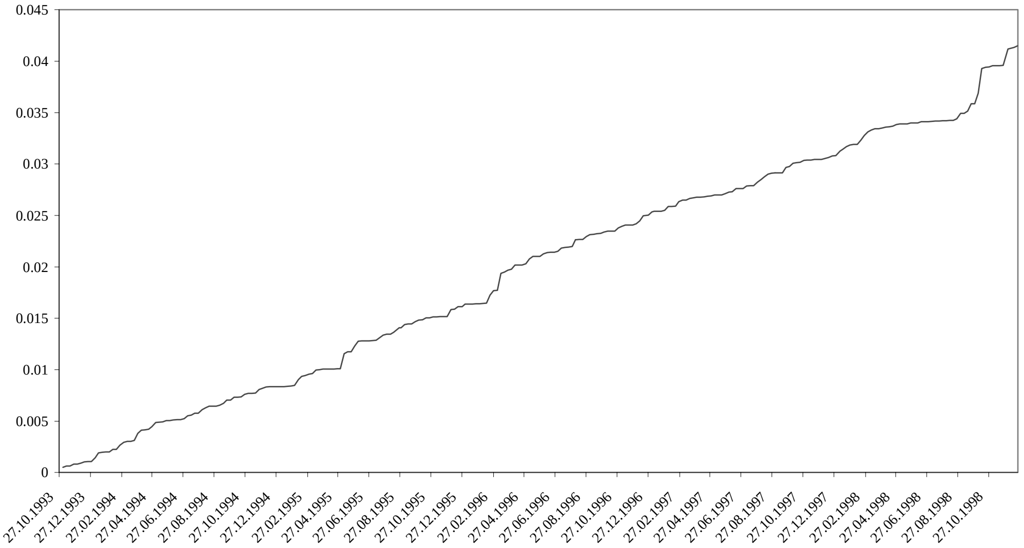


Figure 6b. Centred cumulative sum of squares: US

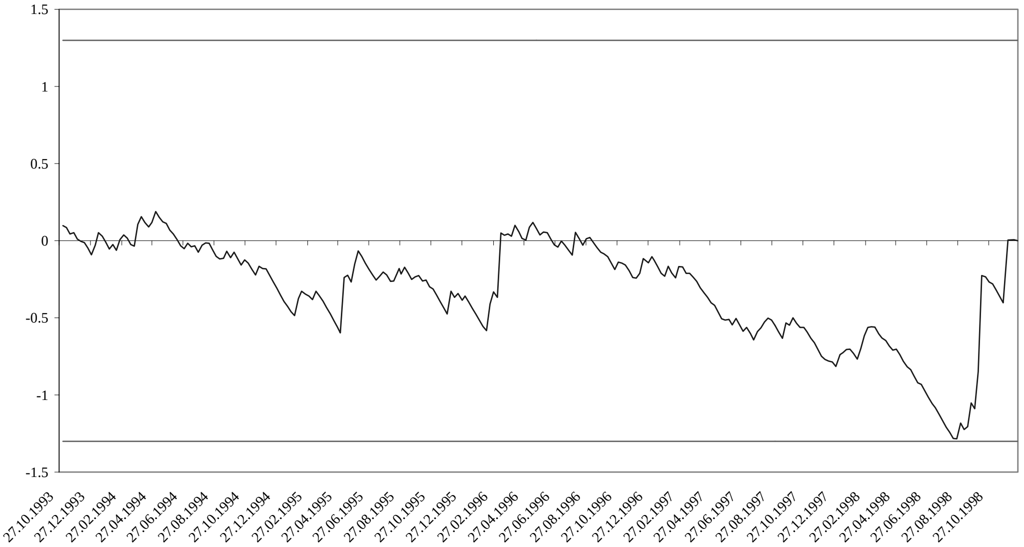
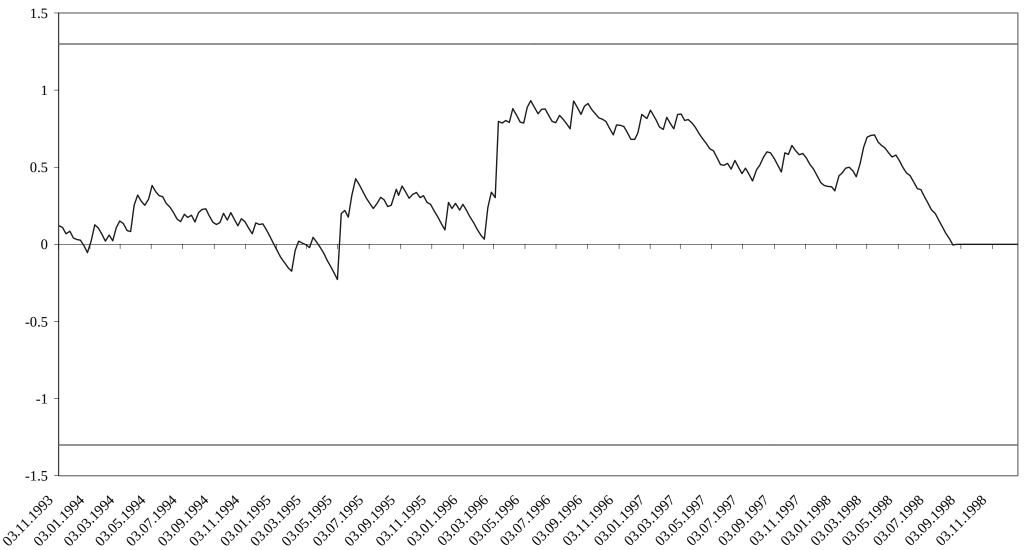


Figure 6c. Centred CSS: US 3 Nov 1993 - 25 Aug 1998



(23 February 1994 and 22 April 1997). We take 23 February 1994 as the first breakpoint, since there are too few observations to allow us to find a possible earlier breakpoint. In Figure 4e, it is confirmed that 22 April 1997 is the last breakpoint, since there are no violations of the 95% confidence intervals. The analysis is then repeated for the new series starting at the first breakpoint and ending at the last breakpoint. We iterate in this manner until all the breakpoints in the series are found. The same procedure is followed in investigating the German and US series in Figures 5 and 6 respectively. We detect a total of seven breaks in the three series: three (or possibly four) in the Swedish series, three in the German series, and one in the US series (see Table 1).

Using the Inclán and Tiao algorithm, the first shift we detect is an upward shift in Swedish volatility on 23 February 1994. This is followed by a downward shift later the same year, on 31 August. The third shift occurs in the German market on 26 October 1994, when volatility shifts down. In April 1997, there is another downward shift in volatility in the German market in the first week of the month (1 April), followed by a downward shift in the Swedish market on 22 April. There is an upward shift in volatility in the German market on 18 August 1998, immediately followed by an upward shift in the US market on 25 August (at the 80% confidence level, there is also a shift in the Swedish market on 1 September). These shifts took place in the wake of the Russian debt crisis. We note that some of the shifts in volatility occur at around the same time in at least two of the markets studied, while other shifts are market-specific.

Table 1
Weeks with a shift in volatility

Sweden	Germany	United States
23 February 1994 *(up)*		
31 August 1994 *(down)*		
	26 October 1994 *(down)*	
22 April 1997 *(down)*	4 April 1997 *(down)*	
1 September 1998 *(up)*	18 August 1998 *(up)*	25 August 1998 *(up)*

In addition to investigating concurrent shifts in volatility, it is also of interest to look at more short-term synchrony. Let us first focus on contemporaneous large interest rate shocks in the three markets. In Table 2, we report large changes in interest rates defined as greater than 30 basis points in absolute value, that is, if the change in interest rates during a week has been larger than 30 basis points in at least one of the markets we enter the changes for all markets in that week in Table 2. However, for clarity of exposition we do not report changes of less than 15 basis points. For the same reason, changes greater than 30 basis points are in boldface. There is only one instance when a change in one of the large markets is not associated with a large change in the smaller Swedish market (3 December 1998). In most cases, a large change in the Swedish interest rate is contemporaneous with a large change in the US or German rate, or both. But this is not always the case, which suggests that domestic factors also play an important role. The most striking example of this is 11 August 1994, when the Swedish Riksbank announced an increase in its repo rate. This was the first rate increase following a period of easing and came as something of a surprise to the market, resulting in the 102 basis point increase in the 10-year interest rate in the following week.

Another way to investigate short-term synchrony is to use the same technique as above to compute rolling correlations among the series. These are shown in Figures 7a–b. During most of the period, it looks as if the Swedish market is more highly correlated with the German than with the US market. We also note that the correlations vary a great deal over time, which makes it more difficult to distinguish possible trends in the correlations. But with the exception of the period of high correlation between the Swedish and German markets in 1994, the correlations seem to have trended upwards during the period.

To investigate the question of a possible interaction between volatility and correlations, we sort rolling 20-week return volatility from the lowest to the highest. Then, the correlations of the returns are ordered correspondingly (so that the dates are matched). Finally, the averages of the correlations for each quartile are computed. The results are shown in Figures 8a–c. The overall impression from these figures is that the correlation between the returns on a large and small bond market is decreasing in the volatility of the small market but increasing in the volatility of the large market. In Figure 8a, the correlation between Swedish and US returns is negatively related to the volatility in the Swedish market. The relation regarding the correlation between Swedish and German returns is not as clear. When the German market is the reference market the negative relation is weaker, as seen in Figure 8b. The evidence presented in Figure 8c tells us that volatility in the US market is positively related to the correlations with returns in the other two (smaller) markets.

The above preliminary results warrant a further assessment of the time variation in second moments. We therefore continue our analysis with the estimation of a multivariate ARCH model, which is potentially able to accommodate the features documented above.

Table 2
Large changes in interest rates

Week	Sweden	Germany	United States
2 March 1994	**66**	**32**	
30 March 1994	**45**		24
6 April 1994	**–30**		
20 April 1994	**62**	**38**	
18 May 1994	**–30**	–15	
25 May 1994	**67**	**31**	
1 June 1994	**49**	24	
22 June 1994	**35**		
29 June 1994	**–41**		
6 July 1994	**96**		16
10 August 1994	**35**	23	19
17 August 1994	**102**	17	–17
31 August 1994	**–43**		
28 September 1994	**–40**	–20	
16 November 1994	**–38**		
8 March 1995	**41**		
9 May 1995	**–43**	–18	**–40**
6 June 1995	**–51**	–14	–18
12 September 1995	**–44**		
19 September 1995	**–33**		
6 February 1996	**50**	26	
20 February 1996	27	**30**	**31**
12 March 1996	20	23	**45**
26 March 1996	**–30**		–15
7 May 1996	**39**	17	20
17 September 1996	**–33**	–19	
19 November 1996	**–31**		
18 March 1997	**59**	26	
6 October 1998	–24	–17	**–34**
13 October 1998	23	**46**	**57**
3 December 1998		–22	**–43**

Note: A large change is defined as a change greater than 30 basis points in absolute value. Changes smaller than 15 basis points are not shown in the table.

Figure 7a. Rolling correlations: 10-year interest rates

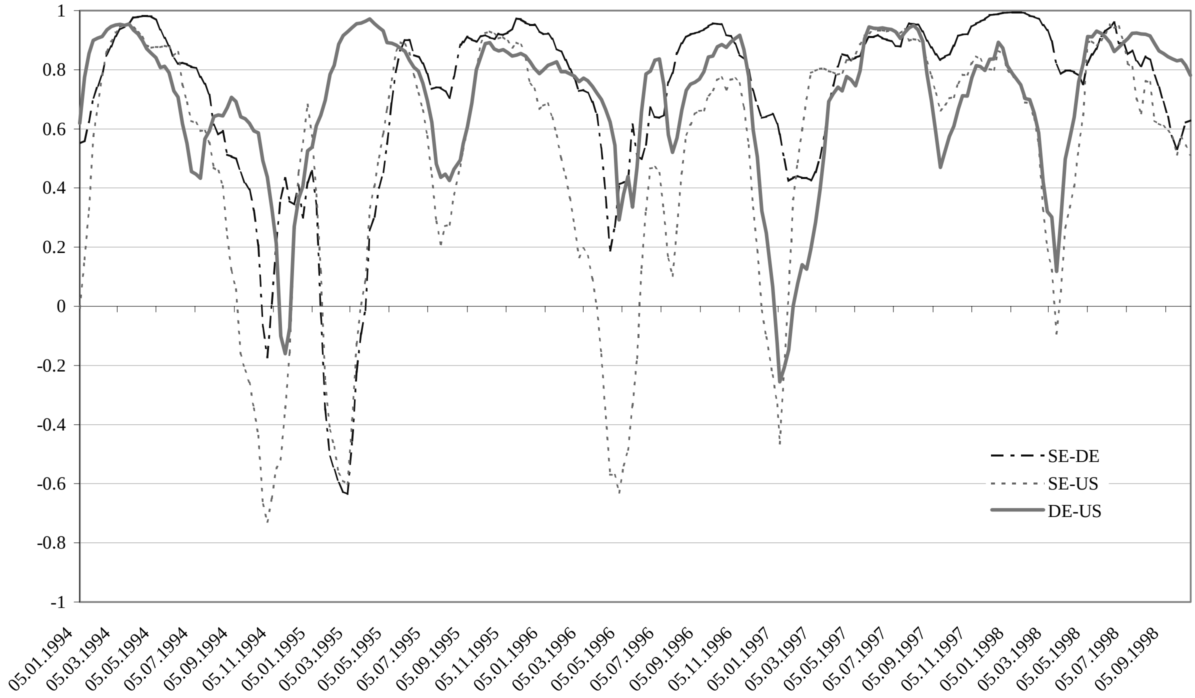


Figure 7b. Rolling correlations: weekly returns on 10-year bonds

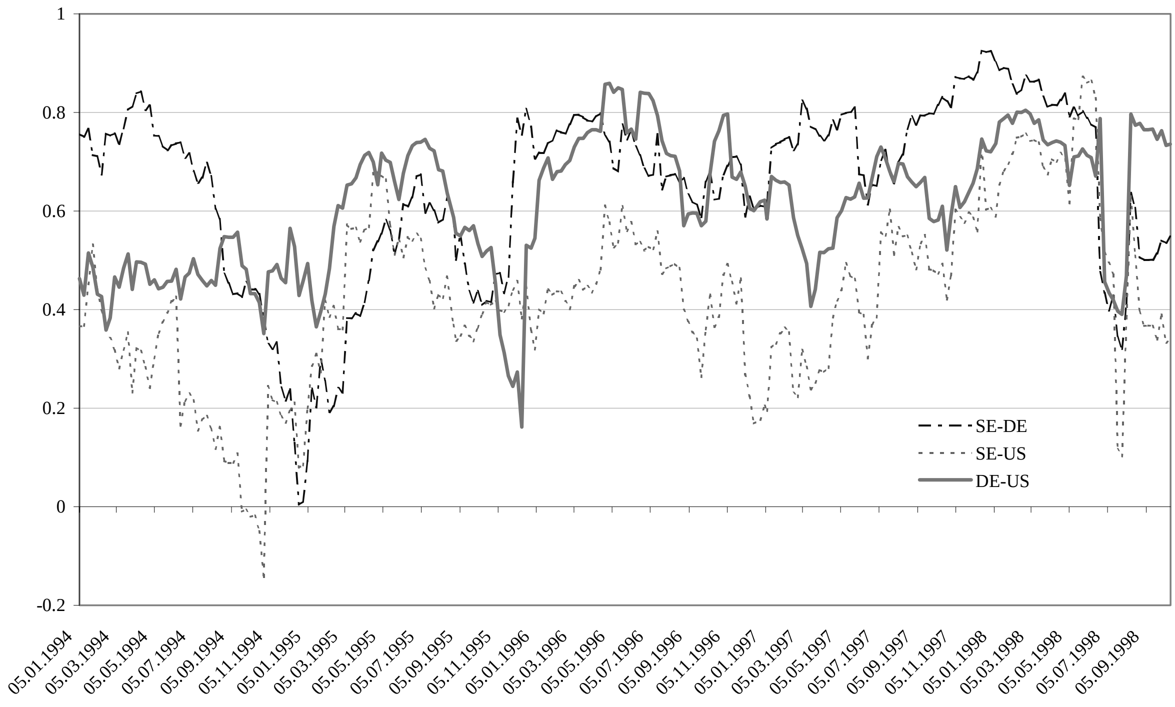


Figure 8a. Average correlations sorted on Swedish volatility

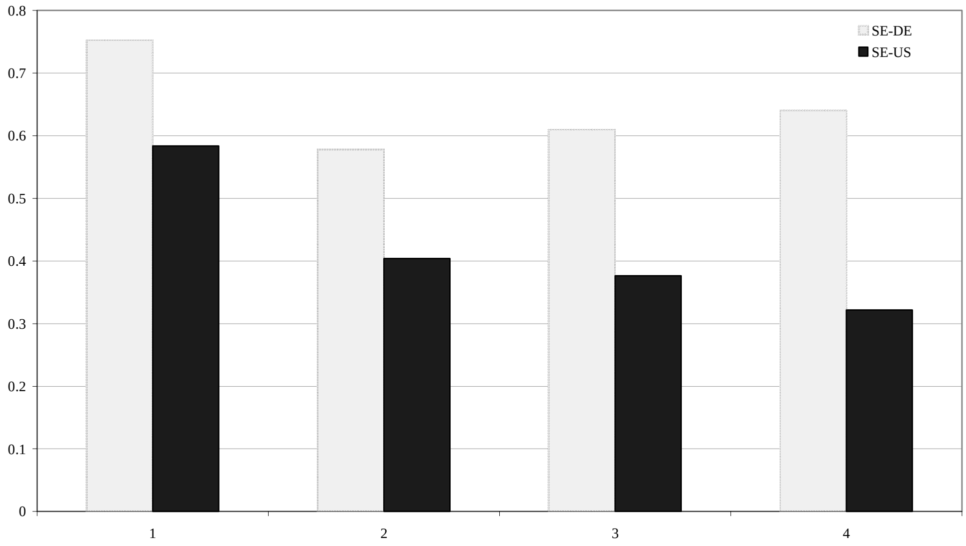


Figure 8b. Average correlations sorted on German volatility

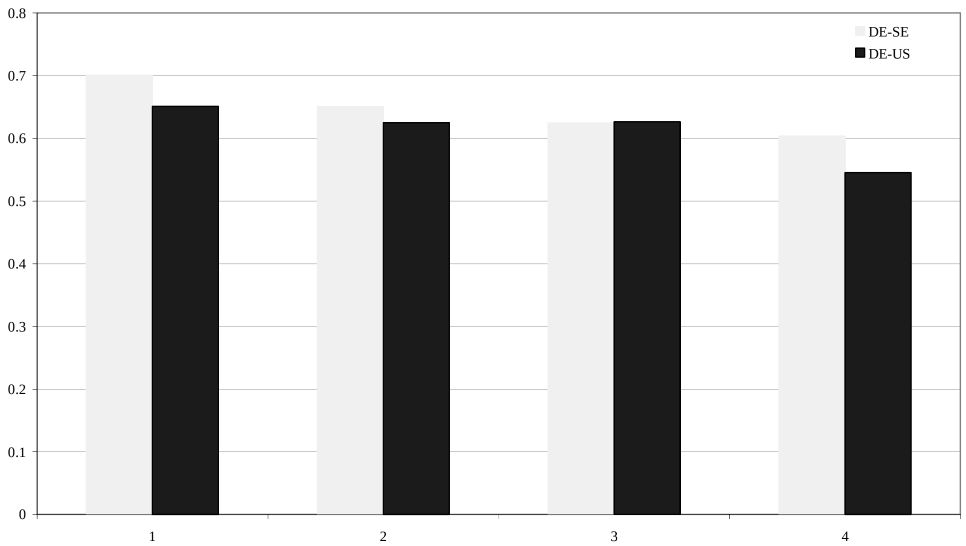
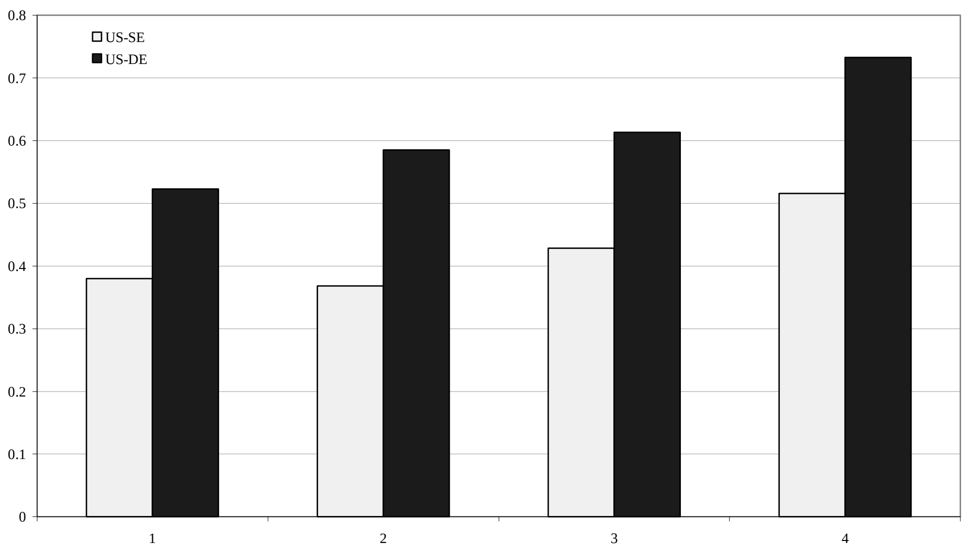


Figure 8c. Average correlations sorted on US volatility



3. The empirical model

We need a model that is flexible enough to capture the time-varying nature of the interdependencies between the bond markets, as indicated above. A multivariate ARCH model should be able to capture the observed patterns to some extent.[7]

3.1 Conditional means

A straightforward linear model will be used for the conditional means. Let R_t^i denote the return on the ith bond market, where $i = DE$ (Germany), US (United States) or SE (Sweden). We consider the following moving average (MA) specification for Germany

(3) $$R_t^{DE} = \theta_0^{DE} + \varepsilon_t^{DE} + \theta_1^{DE}\varepsilon_{t-1}^{DE} + \theta_2^{DE}\varepsilon_{t-1}^{US}$$

and for the United States

(4) $$R_t^{US} = \theta_0^{US} + \varepsilon_t^{US} + \theta_1^{US}\varepsilon_{t-1}^{DE} + \theta_2^{US}\varepsilon_{t-1}^{US}$$

This specification is consistent with the idea that price spillovers do not necessarily have to be related to volatility spillovers or the fact that shocks in the two markets are correlated. For Swedish returns, the model is formulated such that price shocks from all markets could be of importance,

(5) $$R_t^{SE} = \theta_0^{SE} + \varepsilon_t^{SE} + \theta_1^{SE}\varepsilon_{t-1}^{DE} + \theta_2^{SE}\varepsilon_{t-1}^{US} + \theta_3^{SE}\varepsilon_{t-1}^{SE}$$

In the next section, we will discuss how to model the error terms, allowing for time variation, and the link to the conditional second moments in the above pricing equations.

3.2 Conditional variances

We will use a model suggested by Engle and Kroner (1995), known as the BEKK model. This model is a special case of the more general VEC representation of a multivariate generalised ARCH model. The advantage of the BEKK model for conditional covariances is that it guarantees positive definite conditional covariance matrices under weak conditions. Moreover, compared with other models it uses few parameters, but still allows for conditional correlations and is able to capture potential cross-volatility interactions as well. For tractability, the model is here restricted to be of order (1,1). The specification below allows us to simultaneously model conditional variances, covariances and correlations.

Let $\varepsilon_t = \left[\varepsilon_t^{DE}\varepsilon_t^{US}\varepsilon_t^{SE}\right]'$ denote a combined error term from the conditional mean specifications. In the general BEKK model we consider, the dynamic for the conditional covariance matrix is given by

(6) $$H_t \equiv E\left(\varepsilon_t\varepsilon_t' \middle| I_{t-1}\right) = \Omega'\Omega + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B$$

where Ω is a 3 x 3 upper triangular matrix of parameters, and A and B are 3 x 3 matrices of parameters. We can impose some additional restrictions on the model. We make the innocuous assumption that there is no spillover from the small Swedish bond market to the larger US and German markets. This results in the following parameterisation of the A and B matrices

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

7 We started by estimating univariate GARCH models for each market separately allowing for shifts in variance, in agreement with the findings reported in the previous section. However, the shift coefficients turned out not to be statistically significant in the GARCH framework.

Note that the general model can be written in a vectorised form – a vech representation. Let vech(.) denote the vectorisation operator which stacks the elements on and below the diagonal of a matrix (i.e. the unique elements in a covariance matrix). The BEKK model can then be written as

(7) $$\mathrm{vech}(H_t) = \tilde{\Omega} + \tilde{A}\mathrm{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \tilde{B}\mathrm{vech}(H_{t-1})$$

where $\tilde{\Omega}$, $\tilde{A}$ and $\tilde{B}$ are matrices which are expressed in terms of the Ω, A and B matrices (see the Appendix). Bollerslev and Engle (1993) show that this system is covariance stationary if and only if the eigenvalue for $\tilde{A}+\tilde{B}$ with the maximum norm is strictly less than one.

3.3 Estimation

The combined error term is assumed to be conditionally multivariate normal distributed, that is,

(8) $$\varepsilon_t | I_{t-1} \sim N(0, H_t)$$

Under the assumption of conditional normality, the log-likelihood function for observation t can be expressed as

(9) $$L_t(\phi) = -\frac{3}{2}\ln(2\pi) - \frac{1}{2}\ln|H_t(\phi)| - \frac{1}{2}\varepsilon_t(\phi)' H_t(\phi)^{-1}\varepsilon_t(\phi)$$

where ϕ denotes a combined parameter vector. The log-likelihood function over the sample (t = 1, 2, …, T) is thus

(10) $$L(\phi) = \sum_{t=1}^{T} L_t(\phi)$$

and the parameter vector is given by

(11) $$\phi = \left[\phi^i_j, \omega_{ij}, a_{ij}, b_{ij}\right]'$$

where the ϕ^i_js are parameters in the conditional means, and ω_{ij}, a_{ij} and b_{ij} are typical elements on the Ω, A and B matrices, describing the conditional variances.

The maximum likelihood estimator of the parameters can be found by numerically maximising the likelihood. The estimation is done in a recursive fashion, where initial values of the conditional variances are set equal to the unconditional variances. The assumption of conditional normality is not always appropriate. It has, however, been shown that the so-called quasi maximum likelihood estimator is asymptotically normally distributed and consistent if the mean and variance functions are correctly specified. Robust standard errors can thus be calculated as in White (1982).

4. Baseline estimation

4.1 Basic model

We commence with an investigation of potential price spillovers. The estimated mean equations for the weekly returns on the 10-year bonds are

(12) $$R_t^{SE} = \underset{(0.0008)}{0.0034} + \varepsilon_t^{SE} + \underset{(0.1366)}{0.0318}\varepsilon_{t-1}^{DE} - \underset{(0.1054)}{0.0085}\varepsilon_{t-1}^{US} - \underset{(0.0590)}{0.0191}\varepsilon_{t-1}^{SE}$$

(13) $$R_t^{DE} = \underset{(0.0005)}{0.0022} + \varepsilon_t^{DE} - \underset{(0.0865)}{0.0375}\varepsilon_{t-1}^{DE} + -\underset{(0.0569)}{0.0652}\varepsilon_{t-1}^{US}$$

(14) $$R_t^{US} = \underset{(0.0006)}{0.0014} + \varepsilon_t^{US} - \underset{(0.0865)}{0.0043}\varepsilon_{t-1}^{DE} - \underset{(0.0684)}{0.1725}\varepsilon_{t-1}^{US}$$

with standard errors within parentheses below the estimated coefficients. It is clear that there are no price spillovers from the US bond market to either the German or Swedish markets. There is also no significant price spillover from the German market to the Swedish market.

Volatility spillovers will be discussed here in terms of the vech specification,

(15) $$\mathrm{vech}(H_t) = \tilde{\Omega} + \tilde{A}\mathrm{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \tilde{B}\mathrm{vech}(H_{t-1}) + \tilde{C}\mathrm{vech}(\eta_{t-1}\eta'_{t-1})$$

which facilitates interpretation of the parameters. The estimated parameters are reported below, with standard errors within parentheses below the estimated coefficients:

$$\tilde{A}\mathrm{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) =$$

$$\begin{bmatrix} \underset{(0.0614)}{0.1638} & \underset{(0.1692)}{-0.0542} & \underset{(0.1093)}{-0.3463} & \underset{(0.0267)}{0.0045} & \underset{(0.1616)}{0.0573} & \underset{(0.0982)}{0.1830} \\ 0 & \underset{(0.0414)}{0.1273} & \underset{(0.0228)}{-0.0984} & \underset{(0.0623)}{-0.0211} & \underset{(0.0830)}{-0.1183} & \underset{(0.0443)}{0.1040} \\ 0 & \underset{(0.0495)}{0.0771} & \underset{(0.0325)}{-0.0948} & \underset{(0.0397)}{-0.0128} & \underset{(0.0705)}{-0.0658} & \underset{(0.0497)}{0.1003} \\ 0 & 0 & 0 & \underset{(0.0560)}{0.0989} & \underset{(0.0683)}{-0.1529} & \underset{(0.0256)}{0.0591} \\ 0 & 0 & 0 & \underset{(0.0439)}{0.0599} & \underset{(0.0690)}{-0.1200} & \underset{(0.0313)}{0.0570} \\ 0 & 0 & 0 & \underset{(0.0411)}{0.0363} & \underset{(0.0755)}{-0.0893} & \underset{(0.0430)}{0.0549} \end{bmatrix} \begin{bmatrix} \left(\varepsilon_{t-1}^{SE}\right)^2 \\ \varepsilon_{t-1}^{SE}\varepsilon_{t-1}^{DE} \\ \varepsilon_{t-1}^{SE}\varepsilon_{t-1}^{US} \\ \left(\varepsilon_{t-1}^{DE}\right)^2 \\ \varepsilon_{t-1}^{DE}\varepsilon_{t-1}^{US} \\ \left(\varepsilon_{t-1}^{US}\right)^2 \end{bmatrix}$$

$$\tilde{B}\mathrm{vech}(H_{t-1}) =$$

$$\begin{bmatrix} 0.8362 & \underset{(0.1335)}{0.0184} & \underset{(0.0858)}{0.0253} & \underset{(0.0015)}{0.0001} & \underset{(0.0016)}{0.0003} & \underset{(0.0013)}{0.0002} \\ 0 & \underset{(0.0340)}{0.8503} & \underset{(0.0318)}{0.0381} & \underset{(0.0679)}{0.0094} & \underset{(0.0417)}{0.0133} & \underset{(0.0022)}{0.0006} \\ 0 & \underset{(0.0315)}{-0.0256} & \underset{(\ )}{0.8890} & \underset{(0.0020)}{-0.0003} & \underset{(0.0719)}{0.0094} & \underset{(0.0456)}{0.0134} \\ 0 & 0 & 0 & \underset{(0.0692)}{0.8646} & \underset{(0.0619)}{0.0746} & \underset{(0.0029)}{0.0017} \\ 0 & 0 & 0 & \underset{(0.0317)}{-0.0260} & \underset{(0.0377)}{0.0928} & \underset{(0.0338)}{0.0406} \\ 0 & 0 & 0 & \underset{(0.0019)}{0.0008} & \underset{(0.0671)}{-0.0544} & \underset{(0.0430)}{0.9451} \end{bmatrix} \begin{bmatrix} \left(h_{t-1}^{SE}\right)^2 \\ h_{t-1}^{SE,DE} \\ h_{t-1}^{SE,US} \\ h_{t-1}^{DE} \\ h_{t-1}^{DE,US} \\ h_{t-1}^{US} \end{bmatrix}$$

We have restricted the parameters $\tilde{a}_{ii} + \tilde{b}_{ii}$ to sum to one for *i = SE, US*. The unrestricted estimations suggested that we might adopt these restrictions of integrated processes in the case of the Swedish and

Table 3
Diagnostic tests on the standardised residuals from the BEKK model

Test	Sweden	Germany	United States
Ljung-Box level	22.086 (0.228)	27.421 (0.071)	37.330 (0.005)
Ljung-Box square	18.087 (0.450)	18.248 (0.439)	19.256 (0.376)
Ljung-Box absolute	21.128 (0.273)	9.036 (0.959)	24.245 (0.147)
Skewness	–0.6932 (0.000)	–0.4242 (0.005)	0.0741 (0.622)
Kurtosis	4.7586 (0.000)	4.3696 (0.000)	3.7078 (0.019)
Bera-Jarque	56.207 (0.000)	29.093 (0.000)	5.861 (0.053)

Note: Probability values are in parentheses.

US conditional variances. We will discuss the results in the next section of the paper, but first we take a brief look at some diagnostics.

4.2 Diagnostics

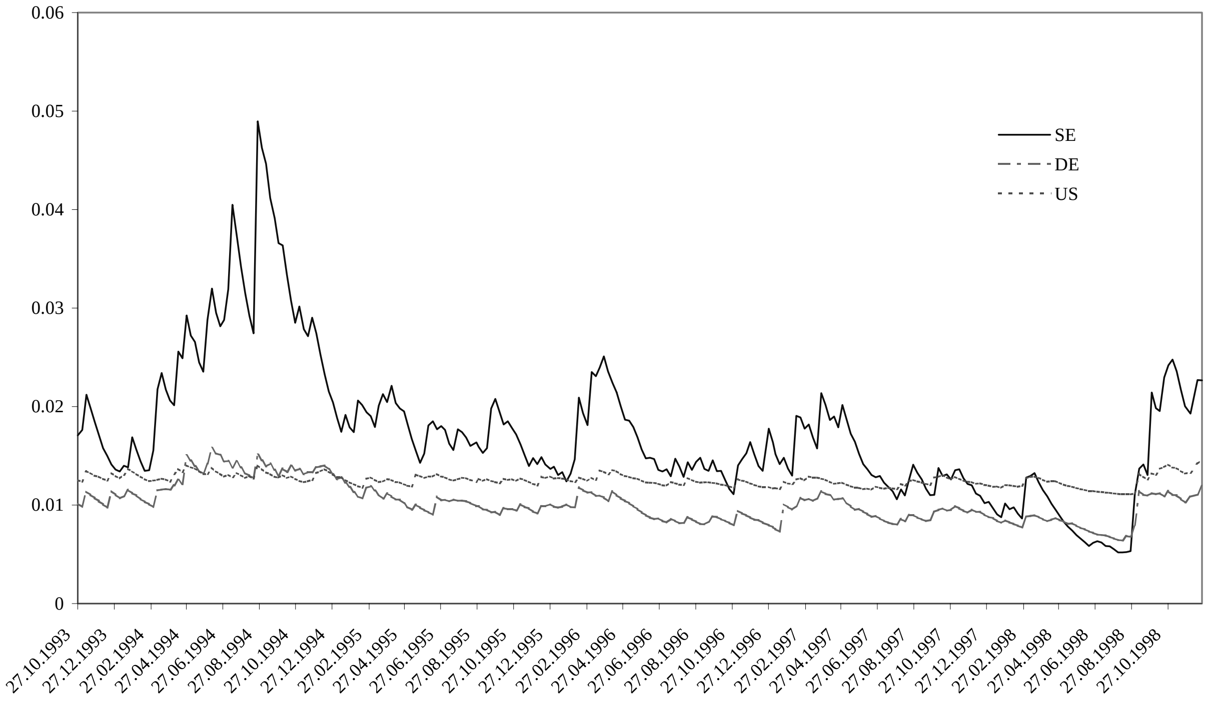
In Table 3, we present some diagnostic tests performed on the standardised residuals from the BEKK model of the previous section. The Ljung-Box tests applied to the residuals (level) show that we have not been entirely successful in allowing for serial correlation. Only the Swedish residuals are satisfactory. However, the same type of test performed on the squared residuals and the absolute value of the residuals does not indicate any remaining heteroscedasticity. The Swedish and German series are skewed and all series are characterised by excess kurtosis.

5. Empirical results

5.1 A characterisation of conditional variances

We have already noted above that there is a high level of persistence in the conditional variances, which even allowed us to restrict the conditional variances of the Swedish and US rates to be integrated GARCH processes. Figure 9 shows the conditional standard deviations in the three markets derived from the BEKK model. How does this analysis compare with the preliminary results presented above? The downward trend in Swedish volatility that we noted in Figure 3a is not as pronounced here. If it were not for the high volatility in 1994, we would be hard pressed to see any trend at all. The analysis of shifts in volatility presented in Section 3 is to a great extent confirmed, but is not in complete agreement with Figure 9. For example, the downward shift found in April 1997 in the German market is not apparent in the figure. In addition, the shift in Swedish volatility in connection with the 1998 crisis was not significant at conventional levels in the breakpoint analysis, while in Figure 9 the Swedish market is the hardest hit of the three. It thus seems that the multivariate model, which models volatility more carefully and takes interactions between the markets into account, gives results somewhat different from the simpler univariate analyses.

Figure 9. Conditional standard deviations



There are two features of the volatility series in Figure 9 that stand out. The first is that the larger the market is, the more stable the conditional standard deviations seem to be. Volatility in the US market is remarkably stable over time. But stable does not necessarily mean lower, since German volatility is actually lower than US volatility (except during 1994). A second interesting feature is that there seems to be a positive comovement between volatility in the three markets. The most dramatic example of this is of course the upward shift in volatility in all three markets during the 1998 crisis. But even during more tranquil times, volatilities seem to move together. We will take a closer look at conditional correlations below.

5.2 Transmission of volatility

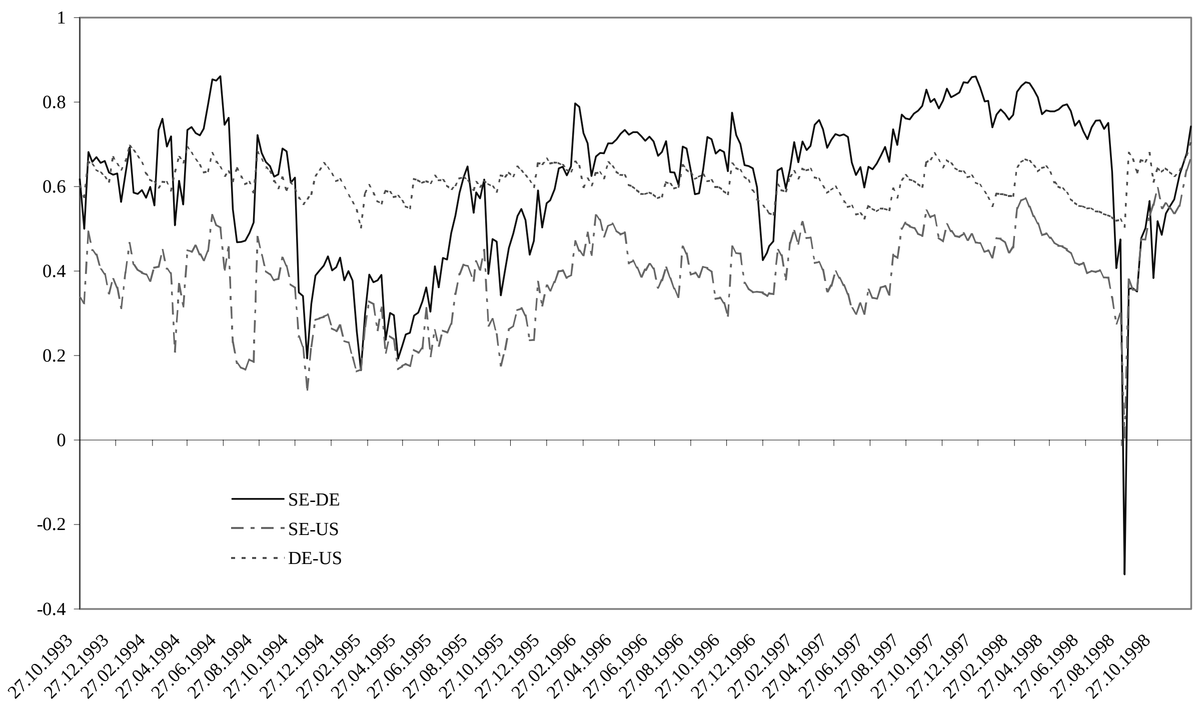
Let us now concentrate on the estimated parameters of the $\tilde{A}$ matrix and look for evidence of transmission of volatility shocks between the markets. Somewhat surprisingly, there does not seem to be any transmission of volatility from the German bond market to the smaller Swedish market. The point estimate of $\tilde{a}_{14}$ is positive (0.0045), but the standard error is quite large (0.0267). However, there is evidence of transmission of volatility from the US market to the Swedish and German bond markets. The impact on the Swedish market is greater than on the German market, with point estimates of 0.183 and 0.059 respectively.

Another finding is that shocks to US volatility have a positive impact not only on Swedish and German volatility but also on the conditional covariances between the three markets. German volatility shocks have no significant effect on conditional covariances between the markets, while shocks to Swedish volatility have no effect on the conditional covariances by assumption.

5.3 Conditional correlations

There is a high level of persistence not only in the conditional variances but also in the conditional covariances. The sums of the diagonal parameters, $\tilde{a}_{ii} + \tilde{b}_{ii}$, are all above 0.7. Using the conditional variances and covariances, we have computed the conditional correlations as

Figure 10. Conditional correlations



$$\rho_t^{ij} = \frac{h_t^{ij}}{\sqrt{h_t^i}\sqrt{h_t^j}} \tag{16}$$

for $i, j \in \{SE,DE,US\}$. These are shown in Figure 10. We can see that the conditional correlation between the two largest markets is the most stable. The correlation between the US and German returns fluctuates within a narrow band around 0.6. It is also the case that the correlation between the Swedish and US markets is more stable than that between the Swedish and German markets. Thus, it seems that the larger the markets considered, the more stable the correlation between them will be. We also note that the Swedish and German markets are more highly correlated than the Swedish and US markets. Both of these correlations fluctuate quite a bit. The correlations with the Swedish market also drop drastically during the 1998 crisis, while the correlation between the US and German markets actually increases.

6. Conclusion

The extent to which interest rates in a small national bond market are determined by domestic relative to foreign factors is of considerable interest from an investment as well as from a monetary policy perspective. We have considered the transmission of shocks from the US and German bond markets to the Swedish market during the recent period of floating exchange rates. We found evidence for the importance of both local and global factors for the small Swedish bond market. We commenced with some preliminary analyses that served to motivate the use of a multivariate GARCH model of the BEKK type. This approach enabled us to model time-varying conditional variances and covariances, which allows a great deal of flexibility in modelling the interaction between the three markets.

Several interesting facts emerged from the estimated multivariate model. The conditional variances in the German and US markets were found to be more stable over time than the conditional variance in the Swedish market. The conditional correlation between the large markets was also found to be quite stable over time, around 0.6, while the Swedish market’s correlations with these two markets were much more volatile (around 0.6 with the German market and 0.4 with the US market). We found no evidence that volatility shocks in the German bond market have any significant impact on the volatility in the Swedish bond market the next week. However, there is clear evidence of transmission of volatility shocks from the US market to the Swedish and German markets from one week to the next. Thus, it appears that news from Germany is more rapidly incorporated into Swedish bond prices than news from the United States.

The more general conclusions that can be drawn from this paper are somewhat tentative because of the limited number of countries investigated. Our main conclusion is that both local and global factors play important roles for a small bond market. We also conclude that larger markets are connected with more stable second moments, that is, conditional variances are more stable, although not necessarily lower, over time in a large market compared with a smaller market. In addition, the larger the markets investigated, the more stable the conditional correlations. It would be of great interest to see how these general conclusions hold up for other sets of countries.

Appendix

Vech representation

Let vec(.) and vech(.) denote the standard vectorisation operators. That is, let L_n denote the elimination matrix, defined so that for a square matrix M, vech(M) = $L_n$vec(M). Furthermore, let D_n denote the duplication matrix, defined so that for any symmetric matrix N, vec(N) = $D_n$vech(N). The dynamics can first be rewritten in vec form as

$$\text{(17)} \qquad \operatorname{vec}(H_t) = \Omega^* + A^* \operatorname{vec}(\varepsilon_{t-1}\varepsilon'_{t-1}) + B^* \operatorname{vec}(H_{t-1})$$

where $\Omega^* = \operatorname{vec}(\Omega'\Omega)$, $A^* = A' \otimes A'$ and $B^* = B' \otimes B'$. In vech form, the dynamics can then be expressed as

$$\text{(18)} \qquad \operatorname{vech}(H_t) = \tilde{\Omega} + \tilde{A} \operatorname{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \tilde{B} \operatorname{vech}(H_{t-1})$$

where $\tilde{\Omega} = \operatorname{vech}(\Omega'\Omega)$, $\tilde{A} = L_n A^* D_n = L_n A'_n \otimes A' D_n$ and $\tilde{B} = L_n B^* D_n = L_n B' \otimes B' D_n$.

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