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Credit Ratings Transition in Structured Finance (+)

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Abstract

This paper explore credit ratings migration in structured finance and compare ratings mobility across different sectors/products and traditional corporate finance, utilizing the ratings transition data available in the public domain. A convenient scalar metric (or mobility index), developed by Jafry and Schuermann (2004), allows us to compare ratings transition across sectors/products. Differences in ratings transition are tested for statistical significance; we rely on an analytical approach based on the large sample theory, e.g. an application of the Central Limit Theorem to the spectrum of "random matrices", to obtain confidence intervals for the matrix eigenvalues of ratings transition which characterize our adopted mobility metric.

Testing for statistically significant differences in ratings transition contributes to shed light on the "consistency" of ratings, i.e. the degree of overall credit quality comparability across sectors, regions and products implied by the ratings process. Our analysis suggests that ratings transition displays significant differences across structured finance sectors and corporates. All in all, ratings appear to be, on average, more "stable" in structured than in corporate finance over the short/medium term. However, the estimated long-run pattern of ratings migration suggests the presence of more persistent changes (e.g. greater volatility) for structured finance products than traditional corporate bonds, as structured finance invariant (long-run) ratings distribution concentrates more heavily on the lowest grades. Although the average pattern of ratings mobility in structured finance stays relatively constant over time period and across geographical area, such pattern masks important heterogeneity across sectors: CDOs and some ABS products exhibit much greater ratings mobility than other structured products or traditional corporate bonds.

A strong path dependency pattern ("ratings momentum") is detected in structured finance transition data. The pattern of ratings mobility increases substantially for structured finance, if conditional (on a downgrade or upgrade) transitions are considered. Structured finance invariant ratings distributions tend to settle to lowest grades for conditional downgrading (or highest ones for upgrading) in a relatively short period of time. Such pattern differs substantially from unconditional ratings mobility and point to rather different developments in the ratings universe, if ratings transition are conditioned upon past ratings change. Unlike in traditional corporate finance, upgrading and downgrading path dependence in structured finance appear to be broadly symmetric, although the upgrading momentum seems relatively stronger than the downgrading one.

Under certain assumptions, our computed mobility indices can perform reasonably well as *proxy* for credit risk measures that are derived from rating migration probabilities in standard risk-management applications. Greater ratings mobility is associated with increasing credit risk. Thus, differences in the pattern of ratings mobility across sectors/products could be translated into credit risk variations, in so far as rating attributes maintain a reasonably constant (over time) and uniform (across sectors) relationship with the underlying credit risk dynamics. It is still an open (empirical) question whether such "stable" relationship characterizes structured finance ratings system, as it appears to be broadly the case for traditional corporate bond finance. In light of the surfacing differences in structured finance ratings mobility highlighted in this paper, further investigation in this area may provide new insights on the changing relationship between credit risk and ratings systems.

1. Introduction

Bond ratings produced by agencies like Moody's, Standard&Poor's and Fitch provide financial market participants with informed opinions, of a standardized nature, on the likelihood that bond issues will be serviced in an orderly manner. The importance of ratings as a source of information to investors has increased in recent years, as bond markets have grown more international and come to include a wider range of obligors, products and structures. Ratings have also acquired new roles with the New Basel Accord, as supervisory authorities have made regulatory requirements for financial institutions contingent on ratings[1]. Bond market developments in recent years have transformed traditional portfolio management techniques. More efforts and resources are today devoted to rigorous (credit) risk-return assessment; credit risk management tools have become increasingly more sophisticated, especially with the continued focus on mark-to-market activity and the development of complex structured finance products - such as collateralized debt obligations (CDOs) - credit default swaps (CDS), and other complex derivative and portfolio management products.

Over time, credit products are liable to move from one rating category to another. As a result, more emphasis has been placed on understanding not just default risk but rating transition risk, i.e. the changes in credit quality assessed by rating agencies when news affecting an obligor's credit quality is revealed; this is also referred to as credit rating migration. Also, ratings volatility and default risk are likely to change across the ratings spectrum, e.g. increasing with each consecutive movement down the ratings scale, in particular, when moving from investment grade to speculative grade.

Rating transition frequencies (probabilities), as they characterize the expected changes in credit quality of obligors, are a useful input for estimating loss distribution, preparing credit scenario analysis and computing Value-at-Risk (VaR) measures. For this reason credit migration data have become cardinal inputs to many risk management applications. Ratings transition frequencies based on historical data are computed for specified time periods; under certain assumptions, they can be used for estimating historical transition probabilities. As far as pricing of credit sensitive financial instrument is concerned, for example in valuing derivatives that depend on credit rating changes such as a CDS, risk-neutral rather than historical transition probabilities would be required. To obtain a risk-neutral CTM, the historical probabilities can be modified in ways that match default probabilities implied by bond/stock market prices[2].

This paper analyzes credit migration rates in SF markets using publicly available credit ratings information; difference with traditional corporate finance (CF) ratings transition are highlighted.

[1] See Basel Committee on Banking Supervision,(2003).
[2] See Jarrow, Lando and Turnbull (1997).

We focus on the evidence provided by Credit Transition Matrices (CTMs), as estimated by the leading Credit Rating Agencies (CRAs). We elaborate on the issue of designing a proper statistical framework for testing differences in credit rating transition in SF and traditional CF. We develop a testing procedure to assess the statistical significance of the differences between migration matrices as represented by the "mobility" metric suggested in Jafry and Schuermann (2004). Such metric, designed to give a measure of rating "migration" propensity, also performs quite well as a proxy for credit risk measures, such as expected loss (EL), VaR and expected shortfall (ES). Based on hypothetical simulated credit portfolios, correlation coefficients between the mobility index and estimated VaR and ES figures can reach 70 percent or more (e.g., Truck, 2004). Indices of rating migration can offer further insights into the pattern of rating activity across SF sectors, including the asset-backed securities (ABS), residential mortgage-backed securities (RMBS), commercial mortgage-backed securities (CMBS), and CDO sectors.

Differences in ratings transition can indeed be subjected to rigorous tests of statistical significance; we rely on an analytical approach from large sample theory, e.g. the theory of random matrices, to obtain confidence intervals for the spectrum of ratings transition, which characterizes our mobility "metric". Such tests can also shed light on the consistency of ratings, i.e. the degree of overall credit quality comparability across sectors, regions and products. The leading CRAs have acknowledged historical differences in the meaning of their ratings across broad market sectors (CF, SF, sovereign and public finance). Some ratings transition studies by the CRAs show that, in general, more credit erosion occurred in the CF market than in the SF market over the past decade. According to Fitch (2004), for example, the weighted average downgrade-to-upgrade ratio for the 1990s was 1.7 to one for CF, but only 0.3 to one for SF products.

The dynamics of upgrade/downgrade leads us to examine another related aspect of migration analysis, namely rating momentum. It is widely documented that the evolution of ratings displays different types of non-Markovian behavior[3]. Not only do there seem to be cyclical components but there is also evidence that the time spent in a given state and the direction from which the current rating was reached affects the distribution of the next rating move. Even if this might be consistent with stated objectives of the rating agencies (e.g. "through -the-cycle-rating"), it is still of interest to quantify these effects characterizing the rating process, as they are critically important for the forecasts one may wish to associate with ratings.

In measuring rating momentum, as one of the main type of non-Markovian properties of rating systems, we try to determine whether corporate and structured finance ratings exhibit "path dependency", e.g. whether any one year’s rating action has any impact on the following year’s rating movement. In other words, whether the frequency of upgrades in any year is conditional on prior upgrades, and likewise whether the frequency of downgrades in any given year is conditional on prior downgrades.

Downgrading and upgrading action is likely to embody a sizeable systematic component related to business cycle movements, as economy activity and profitability expand and contract over time. Conditioning CMTs on these states is an important area of current research (e.g. Gonzales *et al.*, 2004, for a survey), as rating migrations can provide a conceptual framework for linking credit quality to underlying macroeconomic conditions. The extent to which ratings depend on business cycle variables is analyzed in, for example, Kavvathas (2000), Nickell, Perraudin, and Varotto (2000) and Bangia *et al.* (2002). Nickell, Perraudin, and Varotto (2000) use an ordered probit analysis of rating transitions to investigate sector and business cycle effects. The same technique is used in Blume, Lim, and MacKinlay (1998) to investigate whether rating practices has changed during the 1990s. Their analysis indicates that ratings have become more 'conservative' in the sense of being inclined to assign a lower rating. While we do not pursue the business cycle dimension in this paper, it is a further promising direction of research to investigate the response of SF ratings transition to changing macroeconomic conditions. The empirical evidence currently available suggests that such response be very significant for traditional corporates[4].

The paper proceeds as follows. In section 2 we introduce the main concepts and issues tied to the notion of credit rating migration. In section 3 a general overview about ratings transition in SF is provided; section 4 discusses some methodological issues arising in statistical testing for differences in ratings transitions as pursued in the Nera (2003) study, the first attempt (to my knowledge) in this field. Section 5 introduces recently developed ratings mobility indicators and suggests a simple statistical testing procedure for distinguishing empirical estimates. Section 6 examines the estimated mobility indicators for structured and traditional corporate finance. Section 7 provides some concluding remarks and a tentative suggestion for future research.

2. Credit Rating Systems and Transition Matrices.

Rating systems have become increasingly important for credit risk management in financial institutions and markets. They serve not only as a tool for the internal risk management but also play an important role in the Basel II accord proposals for banks' capital adequacy requirements. The key purpose of rating systems is to provide a simple classification of default risk of bond

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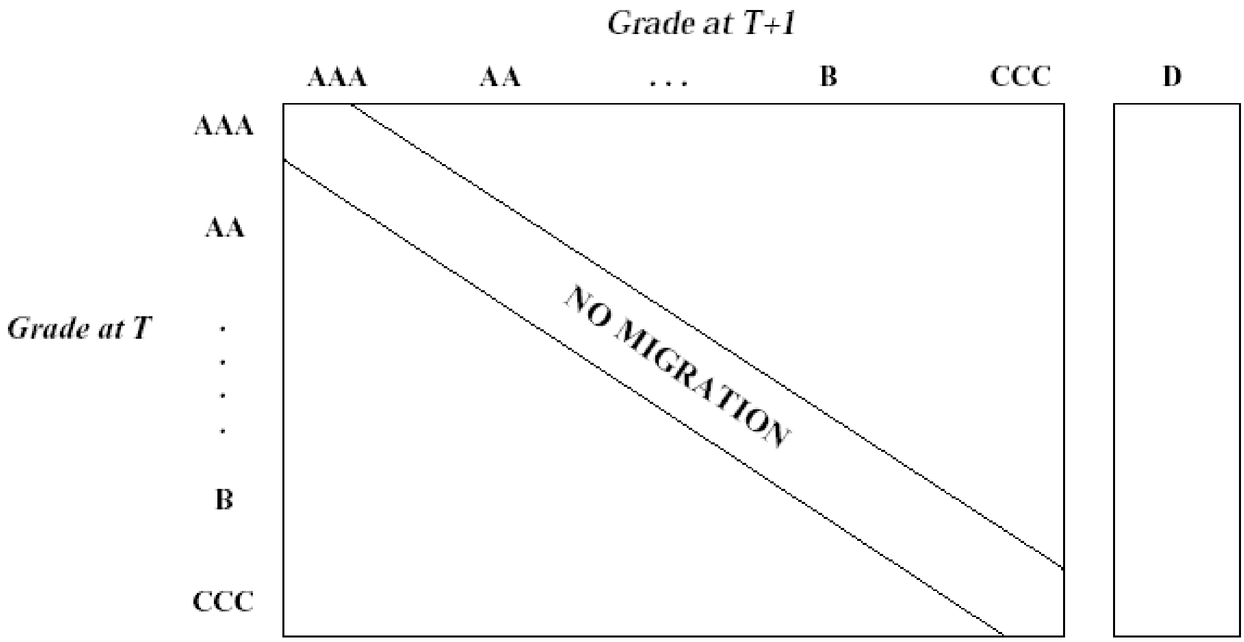
[3] The first literature to analyze non- Markovian behavior are Altman and Kao (1992b), Altman and Kao (1992a), Altman and Kao (1992c) and Lucas and Lonski (1992).

[4] See Bangia *et al*. (2002).

issuers, counterparties, borrowers etc. A desirable feature of a rating system is of course that it is successful in ordering firms so that default rates are higher for lower rated firms.

Credit Transition (Migration) Matrices characterize the past changes in credit quality of obligors (typically, firms in traditional CF or pools of assets, in SF). Such matrices are cardinal inputs to many risk management applications, including portfolio risk assessment, modeling the term structure of credit risk premia, and the pricing of credit derivatives. Also, in the New Basel Accord capital requirements are driven in part by rating migration. In such applications their accurate estimates are critical. The increased reliance on rating systems as risk measurement devices has further increased the need for focusing on statistical analysis and validation methodology for rating systems. While the formal definitions of ratings by the major agencies do not formally employ a probability, or an interval of probabilities, in their definition of the various categories, any use of the ratings for risk management and capital allocation will have to assign default probabilities to each rating category and to probabilities of transition between non-default categories.

CTMs, which are constructed from the ratings data, are a description of the probabilities, conditional upon a given grade at time T, of being in any of the various grades at T+1. It thus fully describes the probability distribution of grades at T+1 given the grade at T. For a scale with 7 grades, one seek to estimate a matrix of 49 (=7x7) unique elements. A conceptual rendition can be given as



Rating transition data enable us to meaningfully estimate probabilities of transitions; there are many statistical issues, of course, in estimating such probabilities. Theoretically, transition matrices can be estimated for any desired transition horizon; application purposes suggest different time frame. However in practice only annual transition matrices are typically used, as shorter transition horizons have yet to be published by the rating agencies; longer horizons (up to 5 years) are also available.

Perhaps the simplest use of a transition matrix is for the valuation of a bond or loan portfolio. Consider a credit asset rated today, say BBB, the value of that credit one year hence will depend, among other things, on the probability that it will remain BBB, migrate to a better or worse credit grade, or even default at year end. This can typically range from an increase in value of 1-2 percent in case of upgrade to a decline in value of 30-50 percent in case of default.

Typically, the transition matrices exhibit higher default risk and higher migration volatility for lower quality grades; more specifically default likelihood increases exponentially with decreasing grade. A characteristic of all rating transition matrices is the high probability load on the diagonal: obligors are most likely to maintain their current rating. Considering the rating transition probability distribution of an obligor given its initial rating, the second largest probabilities are usually in direct neighborhood to the diagonal. In general, the further away a cell is from the diagonal, the smaller is the likelihood of such an occurrence. This rule has frequently been addressed as (row) monotonicity property. It may sometimes be violated, as with increasing transition horizon the default rates becomes more prominent since default is an absorbing state.

3. Reported Evidence about Rating Transitions in Structured Finance.

Leading CRAs reports have recently investigated ratings transition in the SF sector[5]. While there are some differences in the evidence reported, SF ratings have been less volatile than corporate ratings overall, even though the average number of notches changed per rating action has historically been higher in structured finance than in corporate finance. Descriptive statistics from different SF sectors reveal certain similarity in their average rating volatility, but the upgrade and downgrade rates have been different across these sectors. A summary of these results, based on the longest sample analyzed by Moody's (2203) is reported below:

- SF ratings have changed much less frequently than have CF ratings, to date. As a result, structured ratings have been less volatile than corporate ratings overall, even though the average number of notches changed per rating action has historically been higher in SF than in CF.

[5] See, for example, Moody's (2003) and Fitch (2003).

- Average annual and multi-year migration rates from higher rating categories to ratings of Caa or below (on the Moody's scale) - i.e. those with the highest loss expectations - are similar in the SF and corporate sectors.

- Average annual upgrade rates and downgrade rates have been roughly equal for structured finance securities, in contrast to the corporate sector where average annual downgrade rates have historically exceeded upgrade rates.

- SF ratings, like CF ratings, experience strong positive path dependency, or rating change momentum. Securities undergoing rating changes in one year are much more likely to sustain further changes in the same direction in the following year, in comparison to securities that experience no rating change or a change in the opposite direction.

- Rating changes across tranches of the same deal are strongly interdependent. When one tranche undergoes a rating change, about 70 percent of all other tranches within the same deal undergo a rating change in the same direction in the same year and virtually none experiences a rating change in the opposite direction.

- Across SF sectors, average rating volatility is very similar. However, some sectors such as CMBS and RMBS have had higher average upgrade rates and lower average downgrade rates than other sectors; while sectors such as CDOs and ABS have had higher downgrade rates and lower upgrade rates.

- The CDO sector has been strongly hit in 2002 by credit deterioration; it experienced an extremely high downgrade rate and a very low upgrade rate, driven primarily by an extraordinarily high rate of downgrades and defaults in corporate bonds that formed the underlying pools in collateralized bond obligations (CBOs).

- Although the international SF sector is relatively small with most of its growth occurring in recent years, the available data to date indicates that ratings in this sector are also highly stable and show similar rating transition properties to those observed in the U.S. structured finance market.

In comparing SF to CF ratings transition descriptive statistics, Moody's report found that:

- SF ratings have been less likely to change than CF ratings, over both one-year and multiyear horizons.

- When structured securities have experienced rating changes, their downgrade-to-upgrade ratio has been lower than that of corporates.

- When there are rating changes in SF, they have been of larger magnitude on average than those of corporates.

- Overall, SF ratings have been more stable than CF ratings, as the effect of structured securities' lower frequency of rating changes has more than outweighed their greater propensity for large rating changes when rating changes do occur.

- For speculative-grade securities, migration rates to the lowest rating categories (Caa or below, on the Moody's scale) have been much lower for structured finance than for corporate ratings, over both short and long horizons.

- For investment-grade securities, migration rates to the lowest rating categories have been higher for structured finance than for corporate ratings, over both short and long horizons.

- Like corporate securities, SF securities undergoing rating changes in one year are much more likely to sustain further changes in the same direction in the following year (rating momentum), in comparison to securities that experience no rating change or a change in the opposite direction.

Many factors may contribute to the observed differences between SF and CF ratings transition experiences. Some of these factors include differences in the: (i) sectoral rating distributions; (ii) macroeconomic drivers of risk in the corporate sector and in the consumer and mortgage finance sectors; (iii) the nature of pooled and idiosyncratic risks; (iv) the concentration risk associated with some originators and servicers. Suggested explanations for these findings that can be summarized as follows:

- SF securities may have experienced more stability than CF ratings because Aaa-rated securities are more common and Aaa ratings tend to be particularly stable. At the beginning of 2003, about one third of all structured securities carried Aaa ratings, compared to merely 1.5% of all corporate ratings.

- The relative rating stability in SF may also be explained in part by the numerous instances in which originators or servicers of structured securities have supported troubled transactions, something typically not considered in rating assignments in structured finance, and thereby averted rating transitions on the affected securities.

- The higher propensity of investment-grade structured securities (relative to corporate investment-grade securities) to transition into the lowest rating category may be explained by the SF sector’s large concentration in exposure to high yield bonds issued in the late 1990s.

- Asset pools originated or serviced by weak corporate entities are often structured with high levels of credit enhancement that make rating changes less likely to respond to modest changes in pool performance, but nonetheless may require sharp rating downgrades if the servicer’s credit quality deteriorates or the servicer (or originator) commits fraud. This may partially explain the stability and larger size of rating actions in structured finance than in CF.

- There are differences in the evolutions of credit risk between the structured and corporate sectors. In the corporate sector, as an issuer's credit circumstances change, ratings often change gradually, as more is learned over time about how management is reacting to the changed circumstances. In SF, however, the underlying asset pool is often fixed so that as pool performance begins to deviate from expected performance, there is generally little likelihood that this deviation will be reversed over time, and therefore less reason to take a gradual approach to rating changes. This helps explains why, when changed, SF ratings tend to move in larger steps than CF ratings if rating adjustments are required.

Differences across SF sectors and traditional CF appear to be important enough to warrant further investigation. Comparing CTMs across sectors within a proper statistical framework can shed further light on the reported differences; however, establishing their significance would also require adding statistical testing to indicators derived from descriptive statistics.

4. NERA's Study Comparing CTMs: Methodological Issues.

NERA (2003) is the first attempt (to my knowledge) at testing in a comprehensive way differences in ratings transitions across bond market sectors and changes over time. This goal is pursued by setting up a statistical framework aimed at comparing CTMs. Nera's study performs a number of statistical tests in order to determine the stability of one-year credit transition matrices over time. In its report (NERA, 2003), NERA relies extensively on a χ^2 (chi-squared) type of test of statistical significance for the purpose of testing the differences between credit transition matrices. In this instance, it adjusts the standard set-up for such statistical tests to accommodate for the singularity of the row covariance matrices Φ_i. As a result, the tests are carried out using the generalized inverse $\Phi_{i(g)}$, so that the test statistic becomes

$$N \lim_{N\to\infty} [(n_i / N)] (p^*_i - p_i)' \, \Phi_{i(g)}(p^*_i - p_i) \to \chi^2_{\mathrm{rank}(\Phi i)} \, , \tag{1}$$

Where N is the number of credit transition observations, n_i are the number of securities rated i at the beginning of the period, p_i (and p^*_i) is the vector of conditional probabilities of ratings at the end of the period conditional on a security being rated i at the beginning of the period. No reference is given in the NERA's study about the claimed statistical property of a measure like (1), although the indicator suggested by Tsiatis (1980), improving upon the standard Pearson-type chi-square non parametric test for goodness-of-fit and independence/homogeneity in contingency tables, may come close to the framework adopted in the NERA's study.

The model from which these tests are derived assumes that, conditionally on being rated i at the beginning of the period, the probability of migrating to another rating at the end of the period is given by the multinomial distribution. The distribution of the test employed above simply relies on the limiting distribution of such probability estimates properly normalized. If the data-set employed in the analysis is large, estimates can be very “efficient” (e.g. have extremely small standard errors). However, in the modeling of the rating process it is assumed that the rating actions are independent, namely NERA’s study does not model correlations between credit transitions (e.g., over time, over the business cycle, within industry group). It is not clear how well these assumptions correspond with the workings of credit market and the rating process. To the extent that these assumptions do not hold exactly, or nearly so, the cogency with which one can press NERA’s results is correspondingly diminished.

We now focus on some of the details of NERA's statistical testing; the null hypothesis subject to testing in the NERA's study is

$$H_0: p^*_i = p_i \quad i=1,k$$

(2)

The study is silent on the grouping (aggregation) of all ratings (CTM rows, i=1,2,..,k, corresponding to the beginning-of-period ratings) in one single statistics for the entire CTM, perhaps (asymptotically) χ^2-distributed, as reported in the tables summarizing the results of the statistical tests (exhibits VI.2-VI.18). One possible interpretation of the reported summary statistics would be to consider the following non-parametric statistics

$$(p^* - p)' \Phi_g (p^* - p)$$

(3)

where $p^*=[p^*_1, p^*_2,\ldots, p^*_k]'$, $p=[p_1, p_2,\ldots, p_k]'$ and Φ_g is the generalized inverse of the covariance matrix Φ with generic elements

$$\Phi_{i,j} = COV(X^*_{i,r} - X_{i,r} ; X^*_{j,s} - X_{j,s}) = - N [p_{i,r}p_{j,s} - p_{i,r}p^*_{j,s} - p^*_{i,s}p_{j,r} + p^*_{i,r}p^*_{j,s}]$$

i,j,r,s=1,k

(4)

where:

$$p_{i,r}=Prob(X_{i,r}),\ p_{j,s}=Prob(X_{j,s}),\ p^*_{j,,r} = Prob(X_{j,r}^*),\ p^*_{i,s} = Prob(X_{i,s}^*)$$

i,j,r,s=1,k

(5)

with X_i X^*_i indicating the vector of Bernoulli random variables, with multinomial distribution, $M_i(k,\ p_{i,j}: j=1,k)$ and $M_i(k,\ p^*_{i,j}: j=1,k)$, which are modeling the number of end-of-period rating assignment conditional on a beginning-of-period rating, i. Unfortunately we are not in a position to replicate the testing performed in the NERA's study, as the matrices published in it only in part cover the range of data needed for the testing. In particular, the main impediment is the lack of data related to single-year rating transition matrices (only CTM obtained by averaging multi-year annual transition are actually published). More importantly, perhaps, NERA's study does not compare traditional corporate bond finance with structured finance CTMs.

Nera's study points out that because of the large data-set employed in their analysis, they have very “efficient” estimators (i.e. estimates have extremely small standard errors). This claim is probably relevant for the hypothesis testing across CTMs provided that statistics (3) has the asymptotic properties stated in the study (e.g. convergence to a χ^2-distribution). The problem, however, is that such convergence is known for standard Person-type measure of goodness-of-fit, which are based on the difference between actual and expected frequency in grouped data. It is not clear whether this approach can be extended to the testing of the difference between two samples of actual frequencies.

In the next paragraph we outline an alternative framework that should enable us to test differences across CTMs using readily available rating transition data and taking into account measurement errors in ratings transition estimates. Such framework for comparing CTMs is based on a new ratings transition mobility measure that is more risk-sensitive than traditional ones, e.g. greater ratings mobility is associated with increasing credit risk.

5. Criteria for Evaluating Transition Matrices Performance.

Perhaps surprisingly, there has been so far little work (to my knowledge, at least) in establishing formal statistical testing for the comparison of different CTMs across sectors and products. In the statistical literature the metrics applied to transition matrices for general Markov chains, which measure the amount of migration (mobility), are sometimes called mobility indices. Shorrocks (1978), looking at income mobility, propose indices for Markov matrices using eigenvalues and determinants, a line of inquiry extended in Geweke, Marshall and Zarkin (1986). They present a set of criteria by which the performance of a proposed metric (for arbitrary transition matrices) should be judged.

Jafry and Schuermann (2003) and (2004) develop a convenient scalar metric (or mobility index) suitable to comparing CTMs across sectors and investigating CTM changes over time. Their mobility indicator captures the overall dynamic size of a given matrix and contains sufficient information to facilitate meaningful comparisons between different credit migration matrices. They also propose a distribution discrimination criterion which has a metrics sensitive to the distribution of off-diagonal probability "mass". This is important since far migrations have different economic meaning than near migrations; also such sensitivity turns out to be particularly relevant in using CTMs estimates for valuation and risk management purposes. The most obvious example is migration to the "default state" (typically the last column of the transition matrix) which clearly has a different impact than migration of just one grade down (i.e. one off the diagonal). CTMs are said to be diagonally dominant, meaning that most of the probability mass resides along the diagonal; most of the time there is no migration. Also, the mobility index is shown to be closely related to the average probability of migration (i.e. the average off-diagonal "mass") of the CTM. This property provides an intuitively appealing "calibration" for the metric. According to the simulation performed by Truck (2004), the mobility metric also performs quite well as a credit risk proxy for

EL, VaR and ES measure. Their correlation coefficients with the mobility index vary between 0.60 and 0.65 for the short-term horizon (1-year) and 0.73 for the longer horizons (3-year)[6].

Needless to say, transition matrices are estimated with error and estimation noise (uncertainty) is an issue that need to be properly addressed. For example, NERA's study is somewhat elusive on this issue, as it does not give specifics about the treatment of parameters uncertainty or, more precisely, how covariance matrices, Φ_i , are estimated, in order to account for the presence of noise in the estimated cells of the transition matrices.

Measurement error should be acknowledged in assessing the statistical significance of differences among transition matrices; proper statistical testing should be conducted to filter out noise in gauging cell-by-cell differences between transition matrices. Bangia *et al.* (2002) found that the diagonal elements are estimated with high precision; the further one moves away from the diagonal, the lower the degree of estimation precision. This happens because of the endemic low number of observations for far-off diagonal elements. To obviate such critical limitation Christensen and Lando (2002) develop bootstrap methods to estimates confidence sets for transition probabilities (focusing on the default probability in particular), which are superior to traditional multinomial estimates. However, such bootstrap methods tend to be cumbersome to apply as they rely on Montecarlo simulations of rating histories.

5.1 Measuring Ratings Mobility using CTM Eigenvalues.

To gauge the difference between transition matrices, say P^a and P^b, we adopt Jafry and Schuermann (2004) methodology. They suggest the following metric:

$$m_{SVD}(\tilde{P}^a, \tilde{P}^b) = M_{SVD}(\tilde{P}^a) - M_{SVD}(\tilde{P}^b)$$

and with

$$M_{SVD}(\tilde{P}) = \frac{\sum_{i=1}^{N} \sqrt{\theta_i(\tilde{P}'\tilde{P})}}{N}, \qquad \text{where:} \quad \tilde{P} \equiv P - I \tag{6}$$

$\tilde{P}$ is referred to as the Mobility Matrix (MM), as it is obtained by subtracting the identity matrix, which corresponds to a static migration matrix, from the original raw transition matrix, P, containing the transition probability elements, $P_{i,j}$, e.g. the conditional probability of being in rating j "tomorrow" given that today's rating is reckoned at i; and $\theta_i(\tilde{P}'\tilde{P})$, $\mathrm{i} = 1, \mathrm{N}$ indicate the set of

[6] See Truck (2004) for more details and some examples.

eigenvalues associated with the product matrix, $\tilde{P}'\tilde{P}$. Such metric satisfies two desirable criteria regarding the appropriate responsive to the likelihood of state migration:

- monotonicity (greater off-diagonal elements of P imply a greater value of the statistics);
- distribution discrimination (sensitivity to off-diagonal concentrations of migration probabilities);

An intuitive interpretation can be attributed to $M_{SVD}(\tilde{P})$ in terms of "average migration rate", as it would yield exactly the average probability of transition if such probability were constant across all possible states. Such intuitively appealing “calibration” for the magnitude of the metric has also is independent of N, the dimension (number of rating categories) of the migration matrix.

Jafry and Schuermann (2004) also suggest a methodology to take into account estimation uncertainty and measurement errors in the transition matrices. In the absence of any theory on the asymptotic properties of estimates $m_{SVD}(\tilde{P}^a, \tilde{P}^b)$, a resampling technique of bootstrapping is a reasonable (and feasible) alternative. However such technique requires the creation of a bootstrap sample by sampling with replacement form the *original* underlying dataset of ratings transition. Such information may not be easily available or manageable for that matter. As an alternative, in Appendix A1 a methodology for establishing a confidence interval (with probability one) for CTMs and MMs eigenvalues is provided, based on recent developments in random matrix theory and techniques. Such confidence range can be estimated without resampling from ratings transition micro-data, as it relies on aggregate ratings transition information already in the public domain. As a result, a computationally simple framework for testing the differences in the estimated mobility index, $m_{SVD}(\tilde{P}^a, \tilde{P}^b)$, is made available.

5.2 Introducing the Markov Properties of Ratings Transition.

Eigenvalues (eigenvectors) analysis can also help to assess whether ratings dynamics are Markovian, as many of the recently developed credit risk models assume the credit migration process to be Markovian; more precisely, the distribution of default time is modeled via a discrete time, time-homogeneous finite state space Markov chain. All transition matrices have a trivial eigenvalue of unity; this eigenvalue also has the highest magnitude and stems from the symmetry in the matrix (all rows add up to unity since all transition probabilities sum to one). The remaining set of eigenvalues of the transition matrix have magnitudes smaller than unity. A transition matrix can be taken to the s-th power by simply splitting the matrix into its eigenvalues and eigenvectors and taking the eigenvalues to the s-th power while leaving the eigenvectors unchanged. This property can easily be obtained by exploiting the spectral decomposition of the transition matrix, P:

$$P = S_P \Theta_P S_P^{-1}$$

(7)

where Θ_P represents the diagonal matrix containing the eigenvalues of P, and S_P contains the corresponding eigenvectors (one per column).

For transition matrices to follow a Markov chain process, two conditions have to be met:

- the eigenvalues of transition matrices for increasing time horizons need to decay exponentially. In other words, if all the eigenvalues, $\Theta_{P,k}(T)$, of (empirical) transition matrices of varying time horizon, T, were ranked in order of their magnitude, one should observe a linear relationship between $Log[\Theta_{P,k}(T)]$ and the transition horizon T, for each k.
- the set of eigenvectors for each transition matrix need to be identical for all transition time horizons.

Needless to say time independence and homogeneity are two strong assumptions (properties) for a transition process. A Markov chain process allow us to implement a procedure to determine the "long-run" (unconditional) credit rating distribution, $\overline{P}$, by using the (column) eigenvector, s_P^1, associated with the largest eigenvalue of P' (e.g., one), as P' is a column stochastic matrix (e.g. each column adds up to one; P is of course row-stochastic)

$$\overline{P} = \left[\left(s_P^1 * i\right)/\left(i * s_P^1\right)\right]' \qquad \mathrm{i} = [1,1,...1]$$

(8)

where each row of $\overline{P}$, equals

$$\overline{p} = \left[s_P^1 / \left(i * s_P^1\right)\right]'$$

(8')

contains the unique limiting rating probability distribution

$$\underset{T \to \infty}{Lim} P^T = \overline{P}$$

(9)

irrespective of the initial rating distribution[7]. Also, $\overline{p}$ equals the long-run (average) proportion of time spent in each rating category and defines the unique invariant ratings distribution as

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[7] See Ross (2003), theorem 4.1, pp. 200-201. Los (2001), pp. 156-157, gives an example of a limiting distribution calculation, based on S&P estimated CF average annual CTM, where all credit ends up in default with certainty, whatever the initial distribution. However, such outcome is to be expected irrespective of the specific ratings transition data, since the default state is assumed to act as an absorbing state.

$$\overline{p}P = \overline{p}$$

(9')

Any irreducible and aperiodic Markov chain has exactly one stationary (invariant) distribution to which any arbitrary initial distribution converges[8]. One can provide further clues about the time taken to decay towards the final distribution, $\overline{p}$, by recalling the time evolution equation for the state distribution probability of a standard (time homogenous) Markov chain process with transition matrix, P (cf. Haggstrom, 2002, th. 2.1, p. 11)

$$p(T) = p(T-1)P$$

(10)

and noting that after τ iterations, starting at date T=0 from a given initial distribution, p(0), the ratings distribution at time τ satisfies

$$p(\tau) = p(0)S_P(\Theta_P)^{\tau} S_P^{-1} = \overline{p} + c_2 s_P^2 (\theta_{P,2})^{\tau} + \textit{other faster}\ \text{decaying}\ \textit{terms}$$

(10')

regardless of the initial distribution. The rate at which the system decays towards $\overline{p}$ is governed by the slowest-decaying term or the 2nd largest eigenvalue of P, denoted $\theta_{P,2}$. Hence we can define a measure of the time taken to decay to, say, within 10% of the steady state distribution, given by

$$\tau_{10\%} = \frac{\ln(10\%)}{\ln(\theta_{P,2})}$$

(11)

In economic terms, as we shall see, the convergence rate may take in practice quite a long time (decades or so), especially when we consider that records of credit ratings are available only over a few decades. Moreover, the validity of the standard, homogeneous time-invariant, Markov properties become more questionable over such long time horizon. In addition, the interpretation of the steady state distribution is problematic, when the “Default” rating category is included in the CTM and treated as an absorbing state, implying that any firm which has reached this state can never return to another credit rating. An important technical consequence of the inclusion of such absorbing state is that the steady-state distribution solution, $\overline{p}$ (i.e. the first eigenvector of the transpose of P), is identically equal to the absorbing row of P. In other words, for a general migration matrix which exhibits non-trivial probabilities of default (i.e. with some non-zero

[8] Cf. Haggstrom (2002), theorems 5.2-5.3, pp. 34-37.

elements in the absorbing column), the invariant probability distribution, $\bar{p}$, will always settle to the default state. Given sufficient time, all obligors will eventually sink to the default state. Even in this case, however, the terms of the eigenvector corresponding to the 2nd largest eigenvalue continue to provide useful insights on the dynamics of ratings migration as modeled in equation (10'). Specifically, they indicate the long-term (asymptotic) distribution of obligors not ending in default and thus the direction of rating convergence of the surviving population.

The information conveyed by the invariant distribution, $\bar{p}$, is virtually zero if the CTM includes the default state as an absorbing state. Such peculiar distribution structure with all obligors in default state would only result from an almost trivial implication, stemming from the idealized linear, time invariant assumptions inherent in the simple Markov model with an absorbing state. In reality the economy (and hence the migration matrix) will change on time-scales far shorter than required to reach the idealized default steady-state prescribed by an assumed constant migration matrix. Also, there are exceptions to the default trapping, as firms re-emerge from bankruptcy and obtain a credit rating on a debt instrument. Even if only few exceptions where duly reflected in the transition matrix, the relaxation of the absorbing state hypothesis for the default category can have a drastic impact on the structure of the invariant distribution, $\bar{p}$. Thus, the implications on $\bar{p}$ of default as an absorbing state would not be robust to small measurement errors in the ratings transition data. In our mobility estimates we have therefore avoided the inclusion of a distinct default category in the migration matrix, even when transition to default frequencies were, in certain cases, available[9].

5.3 Migration Risk and Path-Dependency: (Non) Markovian Transitions.

Many of the recently developed credit risk models assume the credit migration process to be Markovian. The most common ways of testing the Markovian property of a matrix is through eigenvalue and eigenvector analysis. As said, all transition matrices have a trivial eigenvalue of unity; the remaining set of eigenvalues of the transition matrix have magnitudes smaller than unity. A transition matrix can be taken to the kth power by simply splitting the matrix into its eigenvalues and eigenvectors and taking the eigenvalues to the kth power while leaving the eigenvectors unchanged (cf. eq. 7). Hence, their values can be used to compare CTMs evaluated at different time horizons to assess ratings dynamics and analyze path dependence. Transition matrices with varying transition horizons follow a Markov chain, if their eigenvalues decay exponentially for increasing time horizons and the set of eigenvectors stays constant.

[9] Data in the public domain on SF ratings transitions do not often specify a separate default category. See section 6 for more details on SF data availability.

In a first-order Markov chain process next period's distribution is only dependent on the present state and not on any developments in the past. In other words, transitions have only a one-period memory. Tests reported above about the structure of eigenvalues/eigenvectors over different horizon are not sufficient to cover all the implications regarding the first-order Markov behavior of the transition matrices, a key assumption in many applications in credit risk. Path dependence is a clear violation of the one-period memory Markov behavior, as it presupposes that prior rating changes carry predictive power for the direction of future rating changes. The basic hypothesis is that issuers that have experienced prior downgrading are prone to further downgrading, while issuers that have been upgraded before are less frequently downgraded. The momentum is captured through the directional movements in the path period, defined for simplicity to be one year, i.e. specific transition matrices are computed for issuers that have been upgraded, remained unchanged, or have been downgraded in the previous period.

6. Measuring Ratings Mobility across SF and Traditional CF Sectors.

In this section we analyze CTMs, estimated by Moody's for the period 1983-2002 (cf. Moody's, 2003, reported in tables 1 and 2) and Fitch (cf. Fitch, 2003 and 2004, table 3) for the period 1990-2003, in SF and traditional CF. Rating categories are those reported by the Agencies data; unlike other studies, we do not append a distinct column with default frequency data to the migration matrix. Moody's data do not provide separate information on SF default frequencies; instead, transitions to default state are aggregated to the lowest non-default rating category. Such choice is likely to dampen somewhat estimated ratings mobility, as the mobility from the lowest non-default category and the default state is suppressed. However, as discussed in the previous section, it allows more plausible estimates of the invariant (long-run) distribution of ratings. As a robustness check we have also computed (but not reported) mobility indicators based on CTMs with a distinct default rating category using Fitch data; differences with reported estimates are relatively small, except of course for the invariant long-run ratings distribution. Also, we have computed the same indicators for a finer grading scale without finding major changes in our estimated mobility indices.

Transition matrices are generally published and applied without rating modifiers, as this format has emerged as an industry standard. Therefore, we exclude the rating modifiers in the course of this paper. Consequently, for example, we consider Aa+ and Aa ratings as Aa ratings. This methodology reduces the range of grades from 17 to 7 rating categories, which normally ensures sufficient sample sizes for all rating categories. As in many other transition ratings study, we also must deal with transitions to "not rated" or withdrawn rating (WR) status. Transitions to

WR may be due to any of several reasons, including expiration of the debt, calling of the debt, failure to pay the requisite fee to the rating agency, etc. Unfortunately, however, the details of individual transitions to WR are not known. In particular, it is not known whether any given transition to WR is “benign” or “bad.” Bad transitions to WR occur, for example, when a deterioration of credit quality known only to the issuer (debtor) leads the issuer to decide to bypass an agency rating. Hence, there is more than one method for removing WR’s as a transition class. We adopt a method, which has emerged as an industry standard, that treats transitions to WR status as non-information. The probability of transitions to WR is distributed among all rating states in proportion to their values.

Using the mobility indicators illustrated in section 5, we highlight differences in ratings transition across SF products and traditional CF. Results are summarized in tables 4-6 for Moody's data and tables 7-8 for Fitch data. In addition to the information implied by the standard unconditional (average) rating transitions, we also shed light on the empirical relevance of momentum effects implied by rating changes.

According to our computed mobility index (table 4), all SF CTM is on average less "mobile" (volatile) than traditional all CF ratings; the average probability of rating change equals 8.38% in structured finance vs. 13.24% in traditional corporate finance. Such difference is also statistically significant as it remains broadly unchanged if mobility indices adjusted for sampling errors in the empirically measured CTMs are considered (see Appendix A1 for more details); more specifically, they are reckoned at 7.58% and 12.82%, respectively. The "adjusted" mobility index should be taken as a "conservative" measure of mobility, because they are computed under the assumption of probability-one type of confidence range for the CTM eigenvalues compared to significance levels normally set at 95% (or 99%) in standard statistical inference.

The final values of the limiting (unconditional) distribution of ratings differs substantially across structured and corporate finance. Irrespective of the initial rating distribution, more than 80 percent of the SF ratings would end up in junk/default rate status (e.g. Caa rating or below) in the long-run, compared to only 25 percent in traditional corporate finance. Also, for investment grade securities (As rating grades) the difference for the stationary distribution appears to be large: in the long-run less than 15 percent of the distribution of all structured finance is concentrated on the As rating grades compared to 24 percent for traditional corporate ratings. A pronounced skewness towards non-investment grade in SF limiting distribution is confirmed by a set of Kolgomorov-Smirnov (KS) tests; according to KS test the estimated structured finance limiting distribution cannot be statistically distinguished (at 95 percent significance level; KS statistics equal to 0.522) from a junk/default one-point distribution where the entire probability mass is concentrated on the junk/default rating status. Symmetrically, we can strongly reject the hypothesis that the SF limiting distribution coincide with the uniform distribution (KS statistics equals 1.745, significant at 99 percent level). Exactly the opposite applies to the corporate finance limiting distribution: we cannot reject the uniform distribution hypothesis (KS statistics equal to 0.594), while we strongly reject the junk/default distribution hypothesis (KS statistics equal to 1.995, significant at 99.9 percent level).

In comparing the limiting distribution across types of finance we should keep in mind there is a huge difference in the expected time to convergence: 881 year for SF, compared to only 65 year in traditional CF. However, such difference narrows significantly (actually it is reversed in sign as well) when we consider the estimated "minimum-time" to reach the neighborhood of the steady-state (e.g. 37 year for SF vs. 57 for CF). As documented by the much lower estimates for the "minimum time" to convergence, ratings mobility in the SF sector is estimated with less precision compared to that estimated for traditional CF. As a result of their timing being actually more similar, long-run movements towards lower rating grades are therefore likely to be more extreme in SF than in traditional CF.

At first sight, considerations regarding long-run ratings volatility, time-to-convergence and final distributions might appear to be largely irrelevant for any practical purpose, as the maturity at issue of any security normally would not go beyond, say, 10 or 20-year. However, this argument would be missing an important point, if accounting for uncertainty in ratings transition estimates were to be ignored, as it can drastically shorten the estimated time to convergence. Also, the general pattern of ratings mobility describes the sector as a whole and not the vicissitude of a specific bond issue. Of course, the particular mix of bond issues and expirations at any point in time may contribute much more to changing current ratings distributions than the "endogenous" trend of migration towards the invariant distribution implied by current transition probabilities. As, fluctuations in ratings transition from one period to the next would not alter much the long-run distribution and the time to convergence, the implied trend towards the invariant ratings distribution maintains some relevance.

The pattern of rating mobility in the all SF sector appears to be fairly stable across time period. In table 5 we report the same SF CTM rating indicators computed for the recent sub-period, 1995-2002; very little, if anything at all, has changed in these years compared to the whole sample period. We also compute the rating indicators for the US structured finance, which does not appear to differ significantly from the all structured finance estimates, with the exception grade regarding the limiting distribution which has a less pronounced skewness toward junk/default rating (70 percent) and a larger weight for the invest grade (overall A rating grades, over 24 percent) compared to all structured finance. Such values correspond closely to the estimates found for traditional corporate finance.

The average pattern of rating mobility in SF masks important heterogeneity across sectors (table 6). Rating mobility appears to be much higher for CDOs and ABS products (15.84% and 12.53% respectively). What is more striking, however, is the shape of the limiting distribution, which turns out to be entirely concentrated on the junk/default rating grade. The estimated expected time to convergence towards the final distribution is relatively fast for CDOs (54 year), while it is definitely much slower for ABS (668 year). Broadly, a pattern similar to the CDOs sector is found for the miscellaneous category of "other structured finance products", which includes credit derivative securities and structured notes). CMBS and RMBS sectors show a very different mobility pattern in their rating changes. In the CMBS sector the estimated mobility index is very low, at 5.27%. Also its limiting distribution has 67 percent in the highest Aaa rating and 88 percent in the investment grade category (As rating grades). Thus, the CMBS sector stands apart from all other structured finance products as well as traditional corporate instruments. The RMBS sector has similar low ratings volatility; its mobility index, reckoned at 7.6 percent is comparatively low. CBS and RMBS mobility indices are well below ABS and CDOs "noise-adjusted" mobility measures (9.65% and 15.84%, respectively). Thus RMBS products are on average significantly different from ABS and CDOs products. About 57 percent of RMBS products limiting distribution is concentrated on the highest Aaa grades; however, 33 percent of its limiting distribution ends up in the junk/default category; expected time-to-convergence is very slow (1383 year; however the corresponding minimum time is much shorter, at 38 year).

We have also investigated the rating momentum hypothesis. Such hypothesis is tested by constructing distinct ratings transition matrices according to past rating changes, downgrading or upgrading. Visual CTMs inspection often suggests different pattern of mobility, as most downgrade probabilities for the down-momentum matrix are larger than the corresponding values in the unconditional matrix. The exact opposite is true for the up-momentum matrix, which exhibits smaller downgrade probabilities than the unconditional matrix. The upgrade probabilities of the up-momentum matrix for below investment grade classes are higher than for the unconditional matrix, while for investment grade categories they are lower. Overall, reduced upgrading and downgrade probabilities for the up-momentum matrix lead to an increased probability mass on the diagonal, while the down-momentum matrix displays the exact opposite.

We use ratings transition data computed by Fitch’s rating migration research, covering the period from 1991–2002/3. As far as SF finance is concerned such data refer to US based ratings, compared to the global reaching for corporates ratings, as covered by Fitch analysts. The estimated conditional CTMs reveal that both CF and SF ratings exhibited rating path dependency over the 1991–2002 period. At almost every rating level, a rating upgraded in one year showed a higher probability of being upgraded than downgraded the following year. In addition, both positive and negative rating momenta were generally more evident when moving down the ratings scale. Rating path dependency appears to be stronger in SF than in traditional CF, even though unconditional (average) ratings mobility is very similar for SF and CF finance in the sample period covered by Fitch data.. Such similarity in ratings mobility is in contrast with the estimates presented above based on Moody's data; this difference may be due in part to the shorter Fitch sample period as well as different ratings coverage. More specifically, SF average (unconditional) rating mobility is much greater for the Fitch sample, at 12.69%, than in Moody's ratings transition data; this conclusion is unchanged when "adjusted" mobility measures are considered. Such high level of average ratings mobility matches the corresponding CF mobility estimates, at 11.68%; this latter figure is close to the mobility measure computed on Moody's data for corporates. The long-run ratings distribution based on Fitch data, unlike Moody's one, is skewed towards the highest grades: 73% of the ratings would be in the AAA category and only 18 percent in the C or below. Similar to estimates based on Moody's data, the time to convergence would however be very long indeed (595 year). Perhaps not surprisingly, global CF transition ratings based on Fitch data display a mobility pattern very similar to that estimated on Moody's data. Average mobility rate frequency stands at 11.68%, close to Moody's based estimates, and the ratings distribution are remarkably similar, with a large concentration on BBB and below investment-grade categories.

The pattern of ratings mobility increases substantially for SF, if conditional CTM are considered. Conditional on a downgrade or upgrade, the estimated mobility index shoots up from below 13 to over 40 percent. The final distribution would be entirely concentrated into its extreme tail, C rating or below for downgrading or AA (or above) rating for upgrading, in a relatively short period of time (22 or 5 year, respectively). Such estimates differ substantially from unconditional mobility measures and point to rather different developments in the rating universe, when conditioning upon a previous rating change is considered. There is a high degree of symmetry between upgrading and downgrading path dependence in SF, although the upgrading momentum appears to be slightly stronger than the downgrading momentum. The percentage of SF upgrading in the Fitch based sample is about four times the incidence of downgrading, which is as low as less than 3 percent of the rated universe; this is in contrast with developments in the CF, where the incidence of downgrading, at just over 12 percent, is larger than the percentage of upgrading (just over 8 percent; cf. Fitch, 2004).

Global corporate ratings exhibits ratings path dependency as well, but at far less pronounced levels. Conditional on a previous downgrading, ratings mobility increases to almost 23 percent, but only by two percentage points, to below 14 percent, conditional upon upgrading. CF conditional long-run distributions becomes more skewed, compared to the unconditional estimates, although to a lesser extent than the drastic changes estimated for the SF sector.

6.1 Implications of CTM Implied Ratings Mobility Index for Risk Analysis.

According to our estimated rating mobility index all SF is on average less "mobile" (volatile), in the short run, than traditional all CF. However, the long-run rating volatility differ substantially between structured and corporate finance, as a large fraction of the SF ratings ends up in junk/default rate status, compared to only one quarter for traditional CF. Within the investment grade category (As rating grades) the difference in the final distributions is also quite large. Thus, SF would appear to be more volatile (e.g. riskier) over the long-run than traditional finance, as the sliding towards lower rating grades is estimated to be more pronounced. Skewness towards non-investment grade for SF limiting distribution is confirmed by the KS tests.

In assessing long-term risk by comparing limiting distributions across credit products, we should not neglect the fact that there may be large differences in the time taken to converging towards the final ratings distribution. For example, long time-to-convergence for the average SF product reflects the relatively low intensity of (annual) mobility among rating classes, compared to the level estimated for traditional corporate finance. Hence while short-run risk for SF is probably lower on average than traditional CF, credit risk measures for the SF sector are likely to be greater than traditional CF for longer term investment horizons. The long duration implied by the estimated time-to-convergence leaves the short-run dynamics of rating mobility with a large weight in any plausible credit risk assessment.

The pattern of rating mobility in all SF sector appears to be relatively stable across time period and geographical area. However, such pattern estimated for the SF sector as a whole masks important heterogeneity across sectors, as ratings mobility appears to be much higher for CDOs and ABS products. Also, CDO and ABS sectors limiting distribution are entirely concentrated on the junk/default rating grades. CDOs and ABS differ substantially as far as the estimated expected time to convergence to the final (long-run) distribution is concerned, which is ten times faster for CDOs (54 year) than ABS products. On this basis CDOs tranches are definitely much more risky, on average, than ABS finance.

SF appears to be characterized by a fairly strong ratings momentum, more pronounced than traditional CF. Conditional upon a rating change, ratings mobility increases substantially more in the SF sector than in traditional CF. The most striking finding regarding such effect is the impact of ratings downgrading, which is powerful enough in the SF sector to concentrate in full the long-run distribution on the lowest rating grades (junk/default grade). This is not the case for the corporate sector. There are also extreme differences in average default rates conditional upon previous rating actions. The down momentum average default rate is nearly five times as large as the unconditional one, whereas the up momentum average default rate is less than one-fifth of the unconditional expectation. Thus, the default probability is most sensitive to a prior downgrading history.

The presence of momentum effect might have a significant impact on the risk and value of credit sensitive securities. Financial instrument with an asymmetric payoff structure, such as options on the yield spread or embedded options created by the tranching mechanism in SF, should be affected by both the increased negative drift for issuers with a down momentum and the increased volatility of migration in general.

7. Concluding Remarks

Utilizing transition data published by CRAs, ratings migration behavior in structured and traditional corporate finance is explored. Our investigation suggests that ratings transition displays significant differences across SF sectors and in relation to traditional CF. Ratings appear to be, on average, more "stable" in structured than in corporate finance. However, the pattern of migration for SF products implies much more long-run persistency and therefore greater long-run volatility, as the steady state ratings distribution concentrates more heavily on its lowest grades tail. While the average pattern of ratings mobility in SF appears to be fairly stable across time period and geographical area, such average masks important heterogeneity across sectors. CDOs and ABS ratings mobility appear to be much higher than in other SF sectors or traditional CF. Such differences persist when controlled for measurement errors and noise in the sampled data by suitable statistical testing.

A strong path dependency (ratings momentum) effect is detected in the data. The pattern of ratings mobility increases substantially for SF, if conditional (on a downgrade or upgrade) CTMs are considered. The long-run ratings distributions tend to concentrate into the tails, C rating or below for downgrading or AA rating (or above) for upgrading, in a relatively short period of time. Such estimates differ substantially from unconditional mobility measures and point to rather different developments in the rating universe, when conditioning upon a previous rating change is considered. According to our estimated mobility index, upgrading and downgrading path dependence in SF appear to be fairly symmetric, although the upgrading momentum appears to be slightly stronger than the downgrading momentum.

Under certain assumptions, our computed mobility indices can perform reasonably well as *proxy* for credit risk measures that are derived from rating migration probabilities in standard risk-management applications. Greater ratings mobility is associated with increasing credit risk. Thus, differences in the pattern of rating mobility across sectors/products can be translated into credit risk variations, in so far as rating attributes maintain a reasonably constant (over time) and uniform (across sectors) relationship with the underlying credit risk dynamics. Whether this relationship is weak, strong or changing over time and across sectors we cannot tell immediately from the pattern of measured ratings mobility. However, part of the difference in ratings mobility may well reflect the dynamic adjustment process towards preserving (or strengthening) the link to credit risk. Such process is likely to raise the issue of ratings consistency, cross-sectionally and over time, with measures of priced credit risk, e.g. bond market spread or stock market implied distance-to-default measures. A number of studies have looked at the relationship between ratings and bond/stock prices for traditional CF and have found some inconsistencies. However, Perraudin and Taylor (2003) conclude that allowing for non credit-risk determinants of spreads and for dynamic adjustments, the discrepancies between ratings and corporate bond pricing can be largely accounted for. No similar study (at least that I am aware of) has so far tackled the "ratings-consistency" issue in SF. It is still an open (empirical) question whether "consistency" characterizes SF ratings system, as it appears to be broadly the case for traditional corporate bonds finance. In light of the surfacing differences in SF ratings mobility highlighted in this paper, further investigation in this area may provide new insights on the changing relationship between credit risk and ratings systems.

Appendix

A1. Comparing CTM and the statistical properties of random matrices

One interesting application of the concerns the spectral properties of `random matrices'. The theory of Random Matrices has made enormous progress during the past thirty years, with many applications in physical sciences and elsewhere (cf. Bouchaud and Potters, 1999). More recently, it has been suggested that random matrices might also play an important role in finance as correlation analysis and CTM evaluation are possible area of applications. It is therefore appropriate to give a cursory discussion of some salient properties of random matrices. The simplest set of random matrices is one where all elements of a matrix H are Independently and Identically Distributed (IID) random variables, with the only constraint that the matrix be symmetrical ($H_{i,j} = H_{j,i}$). One interesting result is that in the limit of very large matrices, the distribution of its eigenvalues has universal properties, which are to a large extent independent of the distribution of the elements of the matrix. This is actually the consequence of a central limit theorem applied to the spectral decomposition of a random matrix.

Let us introduce first some notations. The matrix H is a square, (NxN) symmetric matrix. We assume that the H_{ij}'s are IID random variables, of mean, μ, and variance, σ^2, which are properly scaled so that the quadratic mean is equal to

$$\left\langle H_{i,j}^2 \right\rangle = \frac{\sigma^2 + \mu^2}{N}$$

(A1-1)

This scaling with the matrix dimension, N, can be understood as follows: when the matrix H acts on a certain vector, each component of the image vector is a sum of N random variables. In order to keep the image vector (and thus the corresponding eigenvalue) finite when N → ∞, one should scale the elements of the matrix with the factor 1/√N.

One can get a(n) (asymptotic) density function, f(θ), for the H matrix eigenvalues, θ, as[10]

$$f(\theta) = \frac{1}{2\pi\left(\sigma^2 + \mu^2\right)} \sqrt{4\left(\sigma^2 + \mu^2\right) - \theta^2} \quad \text{for } |\theta| \le 2\sqrt{\left(\sigma^2 + \mu^2\right)}$$

(A1-2)

and zero elsewhere.

In the particular case where the eigenvalues of a matrix C are simply obtained from those of H by taking the square root of them, e.g.

$$\theta_C = \sqrt{\theta_H}$$

(A1-3)

the density of eigenvalues of C can easily be obtained from:

$$f_H(\theta_H) d\theta_H = f_C(\theta_C) d\theta_C$$

(A1-4)

if one assumes that the elements of H are random variables,

Replacing (A1-2)-(A1-3) into (A1-4) leads to the expression for the density of C eigenvalues,

$$f_C(\theta_C) = \frac{\theta_C^2}{\pi\left(\sigma^2 + \mu^2\right)} \sqrt{\frac{4\left(\sigma^2 + \mu^2\right)}{\theta_C^2} - \theta_C^2} \quad \text{for } \theta_C^2 \le 2\sqrt{\sigma^2 + \mu^2}$$

(A1-5)

and zero elsewhere.

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[10] Cf. Bouchaud and Potters, (1999), sect. 1.8, pp. 44-48, for details.

Note that all the above results are valid, strictly speaking for large N (in the limit, as N $\rightarrow \infty$).

Formula (A1-5) can give a strict quantitative test for deciding whether a particular eigenvalue of a mobility matrix, $\theta_i\left(\tilde{P}'\tilde{P}\right)$, reflects a genuine "mobility" signal present in the data or just the spurious noise transmitted by measurement errors of a finite data set. We can restrict the attention only to those eigenvalues that, with probability one, give a sure signal of "mobility" and therefore are distinct from zero at the highest possible confidence level, e.g.

$$\theta_i\left(\tilde{P}'\tilde{P}\right) \geq 2\sqrt{\sigma^2 + \mu^2}$$

(A1-6)

or, for the square root eigenvalues entering into the mobility indicator, $M_{SVD}\left(\tilde{P}'\tilde{P}\right)$, in equation (6):

$$\sqrt{\theta_i\left(\tilde{P}'\tilde{P}\right)} \geq \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}}$$

(A1-7)

with

$$H = \tilde{P}'\tilde{P}$$

(A1-8)

We can therefore compute a "conservative" mobility indicator by setting to zero (e.g. the case of absence of mobility) those eigenvalues that would not fulfil the signal-to-noise constraint (A1-6)

$$M_{SVD}\left(\tilde{P}'\tilde{P}\right)_{Conservative} = \frac{\sum_{i=1}^{k} I\left[\theta_i\left(\tilde{P}'\tilde{P}\right)\right]\sqrt{\theta_i\left(\tilde{P}'\tilde{P}\right)}}{k}$$

$$\text{with: } I[.] = 1 \text{ if } \sqrt{\theta_i\left(\tilde{P}'\tilde{P}\right)} \geq \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}} \;\; ; \;\; I[.] = 0 \text{ if } \theta_i\left(\tilde{P}'\tilde{P}\right) < \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}}$$

(A1-9)

We can also define a cut-off level for the 2nd largest eigenvalue of the CTM, P, as its eigenvalues $\theta_i(P)$ are related to those of the (squared) mobility matrix, $\theta_i\left(\tilde{P}'\tilde{P}\right)$. We set the probability-one confidence range according to the constraint (A1-7). When the following inequality is fulfilled

$$\left[1 - \theta_i(P)\right] \leq \sqrt{\theta_i\left(\tilde{P}'\tilde{P}\right)}$$

(A1-10)

we can plough constraint (A1-7) into (A1-10) to get a "conservative" threshold, $\hat{\theta}(P)$,

$$1 - \hat{\theta}(P) = \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}}$$

(A1-11)

for $1\text{-}\theta_i(P)$. Such threshold is at least as large as the (unknown) true one, as it is valid for a greater random variable. As a result, it can function as a lower bound for $\theta_i(P)$, making sure that when all eigenvalues but the largest of P fulfil

$$\theta_i(P) \le \hat{\theta}(P) = 1 - \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}} \quad \forall \mathrm{i} \neq 1 \tag{A1-12}$$

they would be statistically significant with probability one. We use the threshold $1-\hat{\theta}(P)$, defined in equation (A1-11), to report in tables 4-8 the confidence range, with probability one,

$$\left[0\,,\ \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}}\right] \tag{A1-13}$$

selecting the (root-squared) eigenvalues of the mobility matrix defining the "conservative" mobility index in equation (A1-9). In addition, constraint (A1-11) serves to identify the 2nd largest eigenvalue of P that would be statistically significant, e.g. reflecting a genuine mobility pattern in the data. Such "statistically minimum" 2nd largest eigenvalue, $\hat{\theta}(P)$, replaces the actual 2nd largest eigenvalue, $\theta_{P,2}$, in equation (11) for the computation of the time to convergence indicator, reported in tables 4-8 as "minimum time" to convergence to the invariant long-run ratings distribution. Such "minimum time" statistics is reported only if the threshold, $\hat{\theta}(P)$, is actually "informative", namely when

$$\theta_{P,2} > \hat{\theta}(P) = 1 - \left(2\sqrt{\sigma^2 + \mu^2}\right)^{\frac{1}{2}} \tag{A1-13}$$

e.g. the actual 2nd largest eigenvalue cannot be statistically distinguished (with probability one) from the largest eigenvalue which is equal to one. In this case, the uncertainty surrounding ratings transition data is large enough to constraint the 2nd largest eigenvalue estimate.

A2. Comparing Rating Probability Distributions: a Non Parametric Test.

The Kolmogorov-Smirnov (KS) test can be used to quantify the degree to which a given probability (frequency) distribution differs from uniformity (or any distribution for that matter) at a prespecified statistical significance level. This test is a basic and well-understood tool of nonparametric statistics (see Siegel and Castellan, 1988 for details). In the one-sample version of this test we can determine the significance with which a sample of data differs from a given baseline frequency distribution—in this case the uniform distribution. The KS test works on the distributional characteristics of the maximum distance between the cumulative density functions of the sample and the baseline. This maximum distance is known as the D-statistic:

$$D_k = \sup_{-\infty<x<\infty} \left| F_k(x) - F^*(x) \right|$$

(A2-1)

where $F_k(x)$ is the sample distribution function (k is the sample size) to be tested against F*(x), a continuous distribution function which is hypothesised in the null of a given type. If the null hypothesis H_0 is true, then the distribution function of $k^{1/2}D_k$ will always converge to the following distribution function

$$\underset{k\to\infty}{Lim} \Pr ob[k^{1/2}D_k \le d] = H(d) \equiv 1 - 2\sum_{i=1}^{\infty}(-1)^{i-1}e^{-2(id)^2} \quad ,\forall \mathrm{d} > 0$$

(A2-2)

The value of H(d) are routinely tabulated and can be used, jointly with the level of significance, α, to construct a KS test which rejects H_0 when

$$k^{1/2}D_k > d_\alpha \quad , H(d_\alpha) = \alpha$$

(A2-3)

Table 1: *Moody's Transition Matrices*:

ALL STRUCTURED FINANCE ANNUAL RATING TRANSITION MATRIX, 1983-2002

All Structured Finance Annual Rating Transition Matrix, 1983-2002								
	TO:							
FROM:	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	89.92%	0.81%	0.12%	0.04%	0.00%	0.00%	0.03%	9.08%
Aa	5.15%	86.34%	2.15%	0.60%	0.08%	0.03%	0.06%	5.59%
A	1.05%	2.54%	86.90%	1.69%	0.48%	0.06%	0.17%	7.10%
Baa	0.51%	0.63%	2.17%	86.93%	3.68%	1.21%	1.04%	3.83%
Ba	0.14%	0.06%	0.75%	3.84%	83.09%	3.11%	5.26%	3.75%
B	0.00%	0.05%	0.05%	0.44%	0.82%	85.54%	9.26%	3.83%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.15%	0.30%	86.72%	12.84%

All Corporate Finance Annual Rating Transition Matrix, 1983-2002									
	TO:								
FROM:	Aaa	Aa	A	Baa	Ba	B	Caa or below	Default	WR
Aaa	86.09%	8.79%	0.96%	0.00%	0.00%	0.00%	0.00%	0.00%	4.16%
Aa	0.76%	86.18%	8.69%	0.36%	0.09%	0.02%	0.00%	0.03%	3.87%
A	0.04%	2.43%	86.97%	5.54%	0.67%	0.21%	0.02%	0.02%	4.10%
Baa	0.05%	0.27%	5.63%	82.40%	4.97%	1.06%	0.17%	0.22%	5.24%
Ba	0.01%	0.03%	0.56%	5.02%	75.31%	8.22%	0.68%	1.36%	8.81%
B	0.01%	0.05%	0.21%	0.56%	5.67%	74.40%	3.84%	6.86%	8.40%
Caa or below	0.00%	0.00%	0.00%	0.91%	2.31%	5.87%	54.44%	26.29%	10.18%

ALL STRUCTURED FINANCE ANNUAL RATING TRANSITION MATRIX, 1995-2002

	TO:							
FROM:	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	89.72%	0.61%	0.12%	0.04%	0.00%	0.00%	0.03%	9.47%
Aa	5.12%	85.76%	1.71%	0.62%	0.11%	0.02%	0.07%	6.59%
A	1.08%	2.23%	87.05%	1.38%	0.50%	0.07%	0.18%	7.51%
Baa	0.54%	0.63%	2.20%	86.85%	3.54%	1.21%	1.05%	3.96%
Ba	0.09%	0.06%	0.77%	3.88%	83.02%	3.11%	5.36%	3.71%
B	0.00%	0.06%	0.06%	0.45%	0.84%	85.71%	9.13%	3.75%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.15%	0.30%	86.99%	12.56%

U.S. STRUCTURED FINANCE ANNUAL RATING TRANSITION MATRIX, 1983-2002

	TO:							
FROM:	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	89.81%	0.63%	0.09%	0.03%	0.00%	0.00%	0.03%	9.40%
Aa	5.65%	86.30%	1.92%	0.52%	0.06%	0.03%	0.06%	5.47%
A	1.14%	2.24%	87.15%	1.54%	0.46%	0.07%	0.16%	7.25%
Baa	0.60%	0.69%	2.26%	87.38%	3.47%	1.02%	0.84%	3.74%
Ba	0.16%	0.07%	0.85%	4.17%	83.47%	2.93%	4.89%	3.46%
B	0.00%	0.06%	0.06%	0.48%	0.91%	86.67%	7.93%	3.88%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.16%	0.32%	86.58%	12.94%

Table 2: *Moody's Transition Matrices:*

Annual Rating Transition Matrices in Structured Finance Sectors, 1983-2002

ABS, 1983-2002

	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	87.21%	0.75%	0.03%	0.01%	0.00%	0.00%	0.07%	11.93%
Aa	2.36%	88.74%	1.67%	0.62%	0.07%	0.00%	0.22%	6.32%
A	0.58%	0.97%	87.96%	1.06%	0.58%	0.07%	0.09%	8.68%
Baa	0.56%	0.44%	0.92%	85.74%	6.25%	1.16%	0.68%	4.25%
Ba	0.27%	0.13%	0.40%	7.09%	71.12%	5.35%	11.10%	4.55%
B	0.00%	0.00%	0.00%	0.57%	0.00%	73.14%	21.71%	4.57%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	77.98%	22.02%

CDO, 1991-2002

	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	89.12%	2.89%	0.87%	0.48%	0.10%	0.00%	0.00%	6.54%
Aa	0.56%	85.27%	5.25%	3.35%	1.00%	0.22%	0.11%	4.24%
A	0.15%	0.90%	82.36%	5.53%	1.79%	0.30%	0.90%	8.07%
Baa	0.00%	0.08%	0.49%	81.97%	5.87%	3.51%	3.18%	4.89%
Ba	0.00%	0.00%	0.00%	1.34%	79.43%	5.69%	11.04%	2.51%
B	0.00%	0.00%	0.00%	0.00%	0.00%	66.52%	29.91%	3.57%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	96.43%	3.57%

CMBS, 1988-2002

	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	88.67%	1.43%	0.00%	0.00%	0.00%	0.00%	0.00%	9.90%
Aa	5.11%	85.67%	0.66%	0.16%	0.00%	0.16%	0.08%	8.15%
A	1.36%	3.05%	88.52%	1.36%	0.00%	0.00%	0.00%	5.70%
Baa	0.59%	1.19%	2.90%	89.06%	1.38%	0.26%	0.20%	4.41%
Ba	0.00%	0.00%	0.57%	2.41%	90.77%	1.70%	0.43%	4.12%
B	0.00%	0.00%	0.16%	0.63%	1.73%	91.81%	2.36%	3.31%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.00%	2.50%	90.00%	7.50%

RMBS, 1983-2002

	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	92.49%	0.45%	0.13%	0.03%	0.00%	0.00%	0.00%	6.90%
Aa	7.05%	85.91%	2.05%	0.40%	0.00%	0.00%	0.00%	4.58%
A	1.98%	4.09%	87.49%	2.06%	0.20%	0.03%	0.20%	3.95%
Baa	0.65%	0.74%	3.27%	89.44%	1.83%	0.83%	0.86%	2.39%
Ba	0.21%	0.07%	1.37%	3.97%	87.48%	1.71%	2.39%	2.80%
B	0.00%	0.13%	0.00%	0.40%	0.53%	88.25%	6.41%	4.27%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.24%	0.00%	87.08%	12.68%

OTHERS, 1990-2002

	Aaa	Aa	A	Baa	Ba	B	Caa or below	WR
Aaa	83.36%	3.80%	0.15%	0.00%	0.00%	0.00%	0.00%	12.70%
Aa	0.29%	84.37%	3.76%	0.00%	0.00%	0.00%	0.00%	11.58%
A	0.00%	11.75%	69.50%	1.75%	1.00%	0.00%	0.25%	15.75%
Baa	0.00%	0.00%	4.29%	67.86%	8.57%	1.43%	2.14%	15.71%
Ba	0.00%	0.00%	0.00%	2.38%	75.00%	1.19%	2.38%	19.05%
B	0.00%	0.00%	0.00%	0.00%	0.00%	95.35%	2.33%	2.33%
Caa or below	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	85.71%	14.29%

Note: In this exhibit, each sector uses all of its available rating observations.

Table 3: *Fitch Transition Matrices:*

Structure Finance

Average Annual U.S. Structured Finance Transition Matrix: 1991–2003*

(%)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC'–'C'	'D'	Total
'AAA'	99.24	0.45	0.19	0.07	0.02	0.01	0.03	0.00	100.00
'AA'	12.11	86.57	0.75	0.33	0.14	0.07	0.03	0.01	100.00
'A'	3.99	7.70	85.62	1.68	0.48	0.29	0.21	0.03	100.00
'BBB'	1.07	3.67	5.97	84.95	1.77	1.50	0.95	0.12	100.00
'BB'	0.25	1.09	3.66	7.50	82.70	2.14	2.14	0.51	100.00
'B'	0.09	0.13	0.69	2.69	6.31	83.41	5.48	1.20	100.00
'CCC'–'C'	0.00	0.00	0.00	0.00	0.24	0.35	83.08	16.33	100.00

*Includes asset-backed, collateralized debt obligation, residential mortgage-backed, and commercial mortgage-backed securities.

U.S. Structured Finance Conditional Upgrade Average Annual Transition Matrix

(%, 1991–2002)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC–C'	'D'
'AAA'	99.66	0.34	0.00	0.00	0.00	0.00	0.00	0.00
'AA'	35.84	64.00	0.16	0.00	0.00	0.00	0.00	0.00
'A'	9.09	35.68	55.23	0.00	0.00	0.00	0.00	0.00
'BBB'	2.26	11.06	30.65	56.03	0.00	0.00	0.00	0.00
'BB'	1.50	0.75	10.86	27.34	59.18	0.37	0.00	0.00
'B'	2.70	1.35	0.00	6.76	37.84	50.00	1.35	0.00

Note: Rating movements examined across the broad rating categories.

U.S. Structured Finance Conditional Downgrade Average Annual Transition Matrix

(%, 1991–2002)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC–C'	'D'
'AA'	0.00	89.04	2.74	4.11	1.37	2.74	0.00	0.00
'A'	0.00	12.50	52.78	22.22	6.94	2.78	2.78	0.00
'BBB'	1.09	0.00	1.09	58.70	13.04	7.61	15.22	3.26
'BB'	0.00	0.00	0.00	0.00	40.40	27.27	28.28	4.04
'B'	0.00	0.00	0.00	0.00	0.45	60.63	25.79	13.12
'CCC–C'	0.00	0.00	0.00	0.00	0.00	0.00	62.44	37.56

Note: Rating movements examined across the broad rating categories.

Table 3 (continue): *Fitch Transition Matrices:*

Corporate Finance

Average Annual Global Corporate Transition Matrix: 1990–2003

(%)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC'–'C'	'D'	Total
'AAA'	96.54	3.31	0.14	0.00	0.00	0.00	0.00	0.00	100.00
'AA'	0.09	90.99	8.47	0.40	0.03	0.03	0.00	0.00	100.00
'A'	0.03	2.50	91.78	5.29	0.24	0.02	0.10	0.05	100.00
'BBB'	0.00	0.25	4.85	89.26	3.97	0.87	0.40	0.40	100.00
'BB'	0.07	0.13	0.20	7.33	79.39	8.06	2.71	2.11	100.00
'B'	0.00	0.00	0.00	0.51	8.09	83.83	5.01	2.57	100.00
'CCC'–'C'	0.00	0.00	0.00	0.44	0.00	10.62	58.85	30.09	100.00

*Includes issuers worldwide of all Fitch publicly rated corporate long-term debt.

Global Corporate Finance Conditional Upgrade Average Annual Transition Matrix

(%, 1991–2002)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC–C'	'D'
'AAA'	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
'AA'	0.00	97.26	2.74	0.00	0.00	0.00	0.00	0.00
'A'	0.00	3.12	95.84	1.04	0.00	0.00	0.00	0.00
'BBB'	0.00	0.78	13.57	83.72	1.16	0.39	0.00	0.39
'BB'	0.00	0.00	1.18	16.47	71.76	8.24	1.18	1.18
'B'	0.00	0.00	0.00	0.00	21.88	71.88	6.25	0.00

Note: Rating movements examined across the broad rating categories.

Global Corporate Finance Conditional Downgrade Average Annual Transition Matrix

(%, 1991–2002)

	'AAA'	'AA'	'A'	'BBB'	'BB'	'B'	'CCC–C'	'D'
'AA'	0.00	80.23	19.21	0.56	0.00	0.00	0.00	0.00
'A'	0.00	2.37	87.93	9.05	0.22	0.00	0.22	0.22
'BBB'	0.00	0.00	4.90	84.15	8.36	0.86	0.29	1.44
'BB'	0.72	0.00	0.00	5.76	64.03	13.67	6.47	9.35
'B'	0.00	0.00	0.00	0.00	18.56	64.95	10.31	6.19
'CCC–C'	0.00	0.00	0.00	1.54	3.08	1.54	35.38	58.46

Note: Rating movements examined across the broad rating categories.

Table 4

MOODY'S Data:	Ratings Grade						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa or below**
All Structured Finance Annual CTM: 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.111	0.017	0.015	0.012	0.015	0.028	0.802
Time to Convergence to Invariant Ratings Distribution	[minimum time: 37 year ; expected time: 881 year]						
CTM Ordered Eigenvalues	1.000	0.997	0.959	0.919	0.895	0.883	0.837
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.171	0.152	0.113	0.094	0.054	0.003	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0838 , [0 ; 0.061], 0.0758						
All Corporate Finance Annual CTM: 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.006	0.055	0.178	0.173	0.162	0.180	0.246
Time to Convergence to Invariant Ratings Distribution	[minimum time: 57 year ; expected time: 65 year]						
CTM Ordered Eigenvalues	1.000	0.966	0.911	0.875	0.836	0.799	0.727
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.278	0.206	0.178	0.143	0.092	0.030	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1324 , [0 ; 0.0396] , 0.1282						
Long-Run Rating Distribution: KS Difference Tests	Significance Level [95%: 1.36; 99%: 1.65; 99.9%=1.95]						
Strucured vs. Corporate Finance: KS Statistics	1,041						
Corporate Finance vs. Uniform: KS Statistics	0,594						
Structured Finance vs. Uniform: KS Statistics	1.745 (**)						
Corporate Finance vs. Junk-Default: KS Statistics	1.995 (***)						
Structured Finance vs. Junk-Default: KS Statistics	0,522						

Table 5

MOODY'S Data:	***Ratings Grade***						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa or below**
All Structured Finance Annual CTM: 1995-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.135	0.016	0.015	0.011	0.014	0.028	0.781
Time to Convergence to Invariant Ratings Distribution	[minimum time: 37 year ; expected time: 1002 year]						
CTM Ordered Eigenvalues	1.000	0.997	0.961	0.920	0.903	0.884	0.837
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.171	0.150	0.106	0.094	0.052	0.002	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0823 , [0 ; 0.0598] , 0.0745						
US Structured Finance Annual CTM: 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.203	0.022	0.019	0.014	0.015	0.030	0.697
Time to Convergence to Invariant Ratings Distribution	[minimum time: 40 year ; expected time: 1089 year]						
CTM Ordered Eigenvalues	1.000	0.998	0.961	0.923	0.898	0.893	0.839
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.167	0.134	0.113	0.091	0.052	0.002	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0800 [0 ; 0.0554] , 0.0723						

Table 6

MOODY'S Data:	***Ratings Grade***						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa or below**
Structured Finance Annual CTM: ABS 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Time to Convergence to Invariant Ratings Distribution	[minimum time: 10 year ; expected time: 668 year]						
CTM Ordered Eigenvalues	1.000	0.996	0.971	0.937	0.921	0.763	0.720
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.383	0.292	0.097	0.066	0.034	0.004	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1253 [0 ; 0.2076] , 0.0965						
Structured Finance Annual CTM: CDO 1991-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Time to Convergence to Invariant Ratings Distribution	[minimum time: 53 year ; expected time: 54 year]						
CTM Ordered Eigenvalues	1.000	0.958	0.919	0.876	0.864	0.800	0.690
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.441	0.226	0.166	0.143	0.088	0.045	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1584 [0 ; 0.0429] 0.1584						
Structured Finance Annual CTM: CMBS 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.671	0.176	0.034	0.021	0.018	0.036	0.044
Time to Convergence to Invariant Ratings Distribution	[minimum time: 129 year ; expected time: 351 year]						
CTM Ordered Eigenvalues	1.000	0.993	0.967	0.948	0.927	0.914	0.907
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.097	0.091	0.075	0.060	0.038	0.008	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0527 [0 ; 0.0176] 0.0516						
Structured Finance Annual CTM: RMBS 1983-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.569	0.040	0.027	0.017	0.012	0.005	0.331
Time to Convergence to Invariant Ratings Distribution	[minimum time: 57 year ; expected time: 1383 year]						
CTM Ordered Eigenvalues	1.000	0.998	0.958	0.924	0.911	0.880	0.868
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.139	0.127	0.107	0.010	0.059	0.001	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0760 [0 ; 0.0393] 0.0759						
Structured Finance Annual CTM: OTHERS 1990-2002							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Time to Convergence to Invariant Ratings Distribution	[minimum time: 38 year ; expected time: 439 year]						
CTM Ordered Eigenvalues	1.000	0.995	0.976	0.953	0.947	0.811	0.760
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.257	0.209	0.064	0.055	0.035	0.006	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.0893 [0 ; 0.0587] , 0.0757						

Table 7

FITCH Data: US All Structured Finance Annual CTM: 1990-2003	***Ratings Grade***						
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC–C or below**
Unconditional (Average) Ratings Transition							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.734	0.039	0.019	0.012	0.008	0.007	0.181
Time to Convergence to Invariant Ratings Distribution	expected time : 595 year						
CTM Ordered Eigenvalues	1.000	0.996	0.921	0.863	0.837	0.800	0.800
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.223	0.198	0.193	0.166	0.105	0.004	0.000
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1269 [0 ; 0.00292] , 0.1269						
Conditional upon Downgrading	**AA or above**	**A**	**BBB**	**BB**	**B**	**CCC–C or below**	
CTM Implied Invariant (Long- Run) Ratings Distribution	0.00	0.00	0.00	0.00	0.00	0.00	1.000
Time to Convergence to Invariant Ratings Distribution	expected time : 22 year						
CTM Ordered Eigenvalues	1.000	0.902	0.612	0.607	0.496	0.398	
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.734	0.596	0.552	0.409	0.112	0.000	
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.4004 [0 ; 0.0093] 0.4004						
Conditional upon Upgrading	**AAA**	**AA**	**A**	**BBB**	**BB**	**B or below**	
CTM Implied Invariant (Long- Run) Ratings Distribution	0.991	0.009	0.000	0.000	0.000	0.000	
Time to Convergence to Invariant Ratings Distribution	expected time : 5 year						
CTM Ordered Eigenvalues	1.000	0.643	0.607	0.560	0.546	0.498	
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.693	0.659	0.583	0.442	0.290	0.000	
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.4444 [0 ; 0.0001] 0.4444						

Table 8

FITCH Data: Global Corporate Finance Annual CTM: 1990-2003	***Ratings Grade***						
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC–C or below**
Unconditional (Average) Ratings Transition							
CTM Implied Invariant (Long- Run) Ratings Distribution	0.006	0.059	0.178	0.195	0.125	0.218	0.220
Time to Convergence to Invariant Ratings Distribution	expected time: 82 year						
CTM Ordered Eigenvalues	1.000	0.972	0.964	0.902	0.850	0.798	0.721
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.289	0.202	0.159	0.102	0.043	0.023	0.00
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1168 [0 ; 0.0033742] 0.1168						
Conditional upon Downgrading	**AA or above**	**A**	**BBB**	**BB**	**B**	**CCC–C or below**	
CTM Implied Invariant (Long- Run) Ratings Distribution	0.014	0.082	0.146	0.121	0.076	0.561	
Time to Convergence to Invariant Ratings Distribution	expected time: 39 year						
CTM Ordered Eigenvalues	1.000	0.942	0.823	0.774	0.733	0.479	
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.524	0.319	0.297	0.181	0.053	0.000	
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.2290 [0 ; 0.0062] 0.2290						
Conditional upon Upgrading	**AAA**	**AA**	**A**	**BBB**	**BB**	**B or below**	
CTM Implied Invariant (Long- Run) Ratings Distribution	1.00	0.00	0.00	0.00	0.00	0.00	
Time to Convergence to Invariant Ratings Distribution	expected time: 3.45664D+15 year						
CTM Ordered Eigenvalues	1.000	1.000	0.946	0.917	0.811	0.593	
Mobility Matrix Ordered Eigenvalues (Square-Root)	0.443	0.230	0.112	0.046	0.00	0.00	
Mobility Index Indicators [eigenvectors c.r. with prob.=1]	0.1385 , [0 ; 0.0024] , 0.1385						

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