Transparency and liquidity in securities markets*

Takashi Ui

Institute for Monetary and Economic Studies
Bank of Japan

(E-mail: takashi.ui@boj.or.jp)

Abstract

This paper provides a framework which deals with various types of transparency concerning the composition of order flow. Using this framework, we study the relationship between transparency and price volatility as a measure of liquidity. We derive conditions under which increasing transparency reduces price volatility, demonstrating that increased transparency does not always imply less volatility.

* The views expressed herein are those of the author. They do not reflect those of the Bank of Japan.

1. Introduction

Market transparency is defined as "the ability of market participants to observe the information on the trading process" by O'Hara (1995). Transparency has many dimensions because a market has many kinds of participants and many types of information.

In this paper, we focus on a special type of transparency that concerns the composition of order flow, especially liquidity-motivated order flow. We should note that even it still has multiple dimensions.

Madhavan (1996) considered a market in which all traders observe the entire liquidity-motivated order flow. It is transparent in one sense. Röell (1990) considered a market in which broker-dealers trade based on private information regarding order flow by their liquidity-motivated customers. It is also transparent in another sense.

It should be noted that, when we consider transparency concerning the composition of order flow, we must pay attention to how much of, what part of, and by whom the order flow is observed. These features' distinctions have rarely been theoretically discussed in the literature.

Extending the models of Kyle (1989), Röell (1990) and Madhavan (1996), this paper provides a model in which a part of order flow is disclosed to the public, part of it is observed by a part of traders, and part of it is not observed by anyone.

Using the model, we study the relationship between transparency and price volatility as a measure of liquidity.[1] More precisely, we investigate the optimal level of transparency that minimises price volatility. We consider two types of transparency. One is transparency for public information. This concerns the situation in which all traders commonly observe the same order flow, which is similar to that in Madhavan (1996).[2] The other is transparency for private information. This concerns the situation in which different traders observe different and independent order flow, which is similar to that in Röell (1990).

According to the main results, we know the following. In the case of transparency for public information, when the variance of order flow is large enough and the market is not transparent, increasing transparency reduces price volatility. Too much transparency, however, increases price volatility. In the case of transparency for private information, when the variance of order flow is large enough, increasing transparency reduces price volatility, and the most transparent markets enjoy the least price volatility.

This paper is organised as follows. Section 2 introduces the model. Section 3 shows the existence of linear symmetric equilibria of the model, which we concentrate on. Section 4 studies transparency for public information. Section 5 studies transparency for private information. Section 6 concludes the paper.

2. A model

In our model, a single risky asset is traded in an auction market. The liquidation value of the asset is denoted by $\tilde{v}$ which is a random variable normally distributed with mean *0* and variance τ_v^{-1}. The realised value of $\tilde{v}$ is donated by v. In the remainder of the paper, a variable with a tilde denotes a random variable and that without a tilde denotes its realised value.

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[1] Price volatility suggests the degree of market transparency, at least indirectly, though it may not necessarily be a direct measure of market liquidity.

[2] Madhavan (1996) assumed that all order flow is observed by traders. Our model only allows for part of it to be observed. We are interested in the amount of order flow that minimises price volatility.

There are N informed traders, each of whom is indexed by $n = 1,\ldots,N$. Trader n receives a private information signal concerning $\tilde{v}$, which is a random variable $\tilde{i}_n = \tilde{v} + \tilde{\varepsilon}_n$. $\tilde{\varepsilon}_1,\ldots,\tilde{\varepsilon}_n$ are independently, identically, and normally distributed with mean 0 and variance τ_e^{-1}.

Noise traders' order in aggregate, denoted by $\tilde{Z}$, is normally distributed with mean 0 and variance σ_Z^2. We assume that there are random variables $\tilde{z}_0, \tilde{z}_1,\ldots,\tilde{z}_N, \tilde{z}_{N+1}$ such that

$$\tilde{Z} = \sum_{k=0}^{N+1} \tilde{z}_k$$

where $\tilde{z}_k$ is independently and normally distributed with mean 0 and variance σ_k^2 for $k = 0,\ldots, N+1$. Thus, $\sigma_Z^2 = \sum_{k=0}^{N+1} \sigma_k^2$. We also assume that $\sigma_1^2 = \sigma_k^2$ for $k = 1,\ldots, N$.

$\tilde{z}_0$ is a part of $\tilde{Z}$ which is publicly observed by everyone. For $n = 1,\ldots, N, \tilde{z}_n$ is a part of $\tilde{Z}$ which is only by trader n partly observed. $\tilde{z}_{N+1}$ is a part of $\tilde{Z}$ which is not observed by anyone. Thus, it can be interpreted that larger σ_0^2/σ_Z^2 and σ_1^2/σ_Z^2, or smaller $\sigma_{N+1}^2/\sigma_Z^2$, imply more transparency. σ_0^2/σ_Z^2 concerns transparency for public information. σ_1^2/σ_Z^2 concerns transparency for private information.

The models of Kyle (1989), Madhavan (1996) and Röell (1990) can be considered as special cases of the above model in the following sense. Kyle (1989) studied markets with $\sigma_0^2/\sigma_Z^2 = 0$ and $\sigma_1^2/\sigma_Z^2 = 0$. Madhavan (1996) studied markets with $\sigma_0^2/\sigma_Z^2 = 1$ and $\sigma_1^2/\sigma_Z^2 = 0$. Röell (1990) studied markets with $\sigma_1^2/\sigma_Z^2 = 1/N$ and $\sigma_0^2/\sigma_Z^2 = 0$. We investigate the optimal degree of transparency by considering intermediate cases and conducting comparative statics of price volatility with respect to σ_0^2 or σ_1^2 for given σ_Z^2.

After receiving signals i_n, z_0 and z_n, trader n creates a demand schedule $X_n(\cdot; i_n, z_0, z_n)$. The vector of demand schedules are denoted by $X = (X_1,\ldots, X_N)$. The market clearing price $\tilde{p}$ is determined by the following quotation.

$$\sum_{n=1}^{N} X_n(\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n) + \tilde{Z} = 0.$$

To emphasise the dependence on X, we denote the market clearing price and the quantity traded by trader n as $\tilde{p} = \tilde{p}(X)$ and $\tilde{x}_n = \tilde{x}_n(X)$, respectively.

Each informed trader has exponential utility with risk aversion coefficient ρ. Each informed trader has a non-stochastic initial endowment, which is normalised to zero. Thus, the utility function of trader n can be written as $u_n(\pi) = -\exp(-\rho\pi)$ where $\pi = (v - p)x_n$ and x_n is the quantity traded.

Definition 1

The Bayesian-Nash equilibrium of the game is a vector of strategies X *such that*

$$Eu_n\left(\left(\tilde{v}-\tilde{p}(X)\right)\tilde{x}_n(X)\right) \geq Eu_n\left(\left(\tilde{v}-\tilde{p}\left(X\middle|X_n'\right)\right)\tilde{x}_n\left(X\middle|X_n'\right)\right)$$

for any demand schedule X_n' *and n* = 1,…, *N.*

3. Symmetric linear equilibria

Following Kyle (1989), we focus on a symmetric linear equilibrium, in which X_n is an identical linear function of p, i_n, z_0 and z_n, for *n* = 1,…, *N*. We write the equilibrium strategy X_n as

$$X_n(p; i_n, z_0, z_n) = \beta i_n - \gamma p + \delta_0 z_0 + \delta_1 z_n \tag{1}$$

Theorem 1

There exists a symmetric linear equilibrium if $N > 3$.*The parameters in (1) are determined by the following equations:*

$$\beta = \frac{(1-\varphi)\tau_e}{(N-1)^{-1}\gamma^{-1}\tau + \rho}, \tag{2}$$

$$\gamma = \frac{\beta N^{-1}\gamma^{-1}\tau - \varphi\tau_e}{\beta N^{-1}\gamma^{-1}\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)}, \tag{3}$$

$$\delta_0 = -\frac{\varphi\tau_e(1+N\delta_0)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)}, \tag{4}$$

$$\delta_1 = -\frac{\varphi\tau_e(1+\delta_1)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)}, \tag{5}$$

where

$$\varphi = \frac{1}{1+(N-1)^{-1}g\beta^{-2}\tau_e},$$

$$\tau = \tau_v + \tau_e + \varphi(N-1)\tau_e,$$

$$g = (N-1)(1+\delta_1)^2\sigma_1^2 + \sigma_Z^2 - \sigma_0^2.$$

The proof, which is based upon the technique developed by Kyle (1989), is provided in the appendix.

In the remainder of this paper, we assume that $\tau_v = 0$. This implies that informed traders have a prior distribution for $\tilde{v}$ that is least informative.[3] This assumption simplifies our analysis due to the following lemma.

[3] In Bayesian statistics, it is often suggested to use the least informative priors.

Lemma 1 *If* $\tau_v = 0$ *then* $\gamma = \beta$.

We consider the relationship between transparency and a variance of the market clearing price conditional on $\tilde{v}$, which is given by the following lemma.

Lemma 2 *If* $\tau_v = 0$ *then*

$$\sigma^2_{\tilde{p}|\tilde{v}} = V(\tilde{p}|\tilde{v}) = N^{-1}\tau_e^{-1} + N^{-2}\beta^{-2}\left[(1+N\delta_0)^2\sigma_0^2 + (1+\delta_1)^2 N\sigma_1^2 + \sigma^2_{N+1}\right].$$

4. Transparency for public signals

This section studies the case where $0 \le \sigma_0^2/\sigma_Z^2 \le 1$ and $\sigma_1^2/\sigma_Z^2 = 0$. For $n = 1,..., N$, z_n is (almost) always equal to 0 and conveys no information. In other words, every trader receives a public signal z_0, but has no private information concerning $\tilde{Z}$.

Theorem 2

Suppose N *is large. For given* $\sigma_Z^2 > 0$ *, there exists unique* $\sigma_0^{*2}(\sigma_Z^2) \in [0, \sigma_Z^2)$ *such that*

$$0 \le \sigma_0^2 \le \sigma_0^{*2}(\sigma_Z^2) \Rightarrow \frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \sigma_0^2} \le 0$$

and

$$\sigma_0^{*2}(\sigma_Z^2) \le \sigma_0^2 \le \sigma_Z^2 \Rightarrow \frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \sigma_0^2} \ge 0 .$$

In addition, there exists σ_Z^{*2} *such that* $\sigma_0^{*2}(\sigma_Z^2) > 0$ *if* $\sigma_Z^2 > \sigma_Z^{*2}$ *and* $\sigma_0^{*2}(\sigma_Z^2) = 0$ *if* $\sigma_Z^2 \le \sigma_Z^{*2}$.

The proof is provided in the appendix.

Suppose $\sigma_Z^2 > \sigma_Z^{*2}$. In this case, increasing transparency, σ_0^2, reduces price volatility, as far as $\sigma_0^2 \le \sigma_0^{*2}(\sigma_Z^2)$. Increasing σ_0^2 more than $\sigma_0^{*2}(\sigma_Z^2)$, however, increases price volatility. Thus, $\sigma_0^{*2}(\sigma_Z^2)$ provides the optimal level of the transparency. Note that $\sigma_0^{*2}(\sigma_Z^2) < \sigma_Z^2$.

Suppose $\sigma_Z^2 \le \sigma_Z^{*2}$. In this case, increasing transparency always increases the price volatility.

5. Transparency for private signals

This section examines the cases where $0 \le N\sigma_1^2/\sigma_Z^2 \le 1$ and $\sigma_0^2/\sigma_Z^2 = 0$. z_0 is (almost) always equal to 0 and conveys no information. In other words, trader n knows his private information z_n, but does not have any public information concerning $\tilde{Z}$.

Theorem 3

Suppose N *is large. For given* $\sigma_Z^2 > 0$ *, there exists unique* $\sigma_1^{*2}\left(\sigma_Z^2\right) \in \left[0, \sigma_Z^2 / N\right]$ *such that*

$$0 \le \sigma_1^2 \le \sigma_1^{*2}\left(\sigma_Z^2\right) \Rightarrow \frac{\partial \sigma_{\tilde{p}|\tilde{v}}^2}{\partial \sigma_1^2} \le 0$$

and

$$\sigma_1^{*2}\left(\sigma_Z^2\right) \le \sigma_1^2 \le \sigma_Z^2 / N \Rightarrow \frac{\partial \sigma_{\tilde{p}|\tilde{v}}^2}{\partial \sigma_1^2} \ge 0 .$$

In addition, there exist σ_Z^{**2} *and* σ_Z^{***2} *such that* $\sigma_1^{*2}\left(\sigma_Z^2\right) = \sigma_Z^2 / N$ *if* $\sigma_Z^2 \ge \sigma_Z^{***2}, 0 < \sigma_1^{*2}\left(\sigma_Z^2\right) < \sigma_Z^2 / N$ *if* $\sigma_Z^{**2} < \sigma_Z^2 < \sigma_Z^{***2}$*, and* $\sigma_1^{*2}\left(\sigma_Z^2\right) = 0$ *if* $\sigma_Z^2 \le \sigma_Z^{**2}$.

The proof is provided in the appendix.

Suppose $\sigma_Z^2 \ge \sigma_Z^{***2}$. In this case, increasing transparency, σ_1^2 , always reduces price volatility.

Suppose $\sigma_Z^{**2} < \sigma_Z^2 < \sigma_Z^{***2}$. In this case, increasing transparency reduces price volatility as far as $\sigma_1^2 \le \sigma_1^{*2}\left(\sigma_Z^2\right)$. Increasing σ_1^2 more than $\sigma_1^{*2}\left(\sigma_Z^2\right)$, however, increases price volatility. Thus $\sigma_1^{*2}\left(\sigma_Z^2\right)$ provides the optimal level of transparency. Note that $\sigma_1^{*2}\left(\sigma_Z^2\right) < \sigma_Z^2 / N$.

Suppose $\sigma_Z^2 \le \sigma_Z^{*2}$. In this case, increasing transparency always increases price volatility.

6. Concluding remarks

This paper has provided a framework which can deal with various types of transparency concerning the composition of order flow. Using this framework, we study the relationship between transparency and price volatility. We derive some conditions under which increasing transparency reduces price volatility, demonstrating that increased transparency does not always imply less price volatility.

Possible topics for future research would be to obtain $\sigma_0^2, \sigma_1^2, ..., \sigma_N^2$ that minimises price volatility without any restrictions as those in our theorems, and to study the relationship between $\sigma_0^2, \sigma_1^2, ..., \sigma_N^2$ and other measures of market liquidity or welfare of traders.

Appendix

Proof of Theorem 1

Let χ_k be the equilibrium strategy where

$$\chi_k = \beta i_k - \gamma p + \delta_0 z_0 + \delta_1 z_k$$

for $k = 1,\ldots N$. For the market clearing price $\tilde{p}$, we write $\tilde{x}_{n=}\chi_n\left(\tilde{p};\tilde{i}_k,\tilde{z}_0,\tilde{z}_n\right)$. Then

$$\tilde{x}_n + \beta\sum_{k\neq n}\tilde{i}_k - (N-1)\gamma\,\tilde{p} + (N-1)\delta_0\tilde{z}_0 + \delta_1\sum_{k\neq n}\tilde{z}_k + \tilde{Z} = 0\,.$$

Solving for $\tilde{p}$,

$$\tilde{p} = \tilde{p}_n + \lambda\tilde{x}_n \tag{6}$$

where

$$\lambda = (N-1)^{-1}\gamma^{-1}$$

$$\tilde{p}_n = \lambda\left[\beta\sum_{k\neq n}\tilde{i}_k + (N-1)\delta_0\tilde{z}_0 + \delta_1\sum_{k\neq n}\tilde{z}_k + \tilde{Z}\right].$$

Note that $\tilde{x}_n$ is the optimal amount of trade conditional on $\tilde{i}_n,\tilde{z}_{0,}\tilde{z}_n,$ and $\tilde{p}$. Thus, $\tilde{x}_n$ maximises $E\left[u_n((\tilde{v}-\tilde{p})\,\tilde{x}_n)\,\middle|\,\tilde{p},\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]$. However, due to (6), $(\tilde{x}_n,\tilde{p})$ uniquely determines $\tilde{p}_n$. Also, $(\tilde{x}_n,\tilde{p}_n)$ uniquely determines $\tilde{p}$. This implies that $\tilde{x}_n$ is the optimal amount of trade conditional on $\tilde{i}_n,\tilde{z}_{0,}\tilde{z}_n,$ and $\tilde{p}_n$. Thus $\tilde{x}_n$ maximises $E\left[u_n((\tilde{v}-\tilde{p})\tilde{x}_n)\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]$.

When u_n is exponential utility with risk aversion coefficient ρ , it is known that maximising $E\left[u_n((\tilde{v}-\tilde{p})\tilde{x}_n)\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]$ is equivalent to maximising

$$E\left[(\tilde{v}-\tilde{p})\tilde{x}_n\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right] - \frac{\rho}{2}V\left[(\tilde{v}-\tilde{p})\tilde{x}_n\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]. \tag{7}$$

Rewriting (7)

$$E\left[\tilde{v}\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]\tilde{x}_n - (\tilde{p}_n + \lambda\tilde{x}_n)\tilde{x}_n - \frac{\rho}{2}V\left[\tilde{v}\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]\tilde{x}_n^2\,. \tag{8}$$

The first order condition for maximisation of (8) with respect to $\tilde{x}_n$ is

$$E\left[\tilde{v}\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right] - \tilde{p}_n - 2\lambda\tilde{x}_n - p\tau^{-1}\tilde{x}_n = 0 \tag{9}$$

where $\tau = 1/V\left[\tilde{v}\middle|\tilde{p}_n,\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]$. The second order condition is

$$-\left(2\lambda + \rho\tau^{-1}\right) < 0\,.$$

Because $\tilde{p}_n = \tilde{p} - \lambda\tilde{x}_n$ and $E\left[\tilde{v}\middle|\tilde{p}_n, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right] = E\left[\tilde{v}\middle|\tilde{p}_n, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right]$, (9) is rewritten as

$E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right] - \tilde{p} - \lambda\tilde{x}_n - \rho\tau^{-1}\tilde{x}_n = 0$.

Solving this for $\tilde{x}_n$

$$\tilde{x}_n = \frac{E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right] - \tilde{p}}{\lambda + \rho\tau^{-1}} = \frac{E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right] - \tilde{p}}{(N-1)^{-1}\gamma^{-1} + \rho\tau^{-1}} \tag{10}$$

Note that all the random variables, $\tilde{v}, \tilde{p}, \tilde{i}_n, \tilde{z}_{o,}\tilde{z}_n$ are jointly normal. Thus, τ is a constant and $E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right]$ is linear with respect to $\tilde{p}, \tilde{i}_n, \tilde{z}_{o,}\tilde{z}_n$. This implies that $\chi_n(p; i_k, z_0, z_n)$ is, in fact, a linear function.

The next step is to derive $\beta, \gamma, \delta_0, \delta_1$. In order to do so, we calculate $E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right]$ and $\tau = 1/V\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right]$ assuming that $\chi_k(p; i_k, z_0, z_n) = \beta i_k - \gamma p + \delta_0 z_0 + \delta_1 z_k$ for $k = 1,..., N$.

The market clearing condition is

$$-N\gamma\tilde{p} + \beta\sum_{k=1}^{N}\tilde{i}_k + N\delta_0\tilde{z}_0 + \delta_1\sum_{k=1}^{N}\tilde{z}_k + \tilde{Z}$$

$$= -N\gamma\tilde{p} + \beta\sum_{k=1}^{N}\tilde{i}_k + (1 + N\delta_0)\tilde{z}_0 + (1+\delta_1)\sum_{k=1}^{N}\tilde{z}_k + \tilde{z}_{N+1}$$

$$= -N\gamma\tilde{p} + \beta\sum_{k\neq n}\tilde{i}_k + (1+\delta_1)\sum_{k\neq n}\tilde{z}_k + \tilde{z}_{N+1} + \beta\tilde{i}_n + (1+N\delta_0)\tilde{z}_0 + (1+\delta_1)\tilde{z}_N = 0.$$

Thus

$$[(N-1)\beta]^{-1}\left[N\gamma\,\tilde{p} - \beta\tilde{i}_n - (1+N\delta_0)\tilde{z}_0 - (1+\delta_1)\tilde{z}_n\right]$$

$$= \tilde{v} + (N-1)^{-1}\sum_{k\neq n}\tilde{e}_k + [(N-1)\beta]^{-1}\left[(1+\delta_1)\sum_{k\neq n}\tilde{z}_k + \tilde{z}_{N+1}\right]$$

Let $\tilde{\mu}_n$ be the left hand side of this equation. Note the following:

- $(\tilde{p}; \tilde{i}_n, \tilde{z}_0, \tilde{z}_n)$ is statistically equivalent to $(\tilde{\mu}_n, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n)$.
- $(\tilde{z}_0, \tilde{z}_n)$ is independent of $\tilde{v}$ and of $(\tilde{\mu}_n, \tilde{i}_n)$.

This implies that $E\left[\tilde{v}\middle|\tilde{p}, \tilde{i}_n, \tilde{z}_0, \tilde{z}_n\right] = E\left[\tilde{v}\middle|\tilde{\mu}_n, \tilde{i}_n\right]$.

$(\tilde{v}, \tilde{\mu}_n, \tilde{i}_n)$ is normally distributed with mean vector

$E[(\tilde{v}, \tilde{\mu}_n, \tilde{i}_n)] = (0,0,0)$

and covariance matrix

$$V\left[\left(\tilde{v},\tilde{\mu}_n,\tilde{i}_n\right)\right]=\begin{pmatrix}\sigma_v^2 & \sigma_v^2 & \sigma_v^2\\ \sigma_v^2 & \sigma_v^2+(N-1)^{-1}\tau_e^{-1}+(N-1)^{-2}g\beta^{-2} & \sigma_v^2\\ \sigma_v^2 & \sigma_v^2 & \sigma_v^2+\tau_e^{-1}\end{pmatrix}$$

where $g=(N-1)(1+\delta_1)^2\sigma_1^2+\sigma_{N+1}^2$. These directly provide $E\left[\tilde{v}\middle|\tilde{p},\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]$ and τ.

The result of calculation is:

$$\tau=\tau_v+\tau_e+\frac{1}{(N-1)^{-1}\tau_e^{-1}+(N-1)^{-2}g\beta^{-2}}=\tau_v+\tau_e+\varphi(N-1)\tau_e \tag{11}$$

where

$$\varphi=\frac{1}{(N-1)^{-1}\tau_e^{-1}+(N-1)^{-2}g\beta^{-2}}(N-1)^{-1}\tau_e^{-1}=\frac{(N-1)\tau_e^{-1}}{(N-1)\tau_e^{-1}+g\beta^{-2}}$$

and

$$\begin{aligned}
&E\left[\tilde{v}\middle|\tilde{p},\tilde{i}_n,\tilde{z}_0,\tilde{z}_n\right]\\
&=\frac{\tau_e}{\tau}\tilde{i}_n+\frac{\varphi(N-1)\tau_e}{\tau}\mu_n\\
&=\frac{\tau_e}{\tau}\tilde{i}_n+\frac{\varphi(N-1)\tau_e}{\tau}\left[(N-1)\beta\right]^{-1}\left[N\gamma\tilde{p}-\beta\tilde{i}_n-(1+N\delta_0)\tilde{z}_0-(1+\delta_1)\tilde{z}_N\right]\\
&=\frac{(1-\varphi)\tau_e}{\tau}\tilde{i}_n+\frac{\varphi\tau_e}{\beta\tau}\left[N\gamma\tilde{p}-(1+N\delta_0)\tilde{z}_0-(1+\delta_1)\tilde{z}_N\right].
\end{aligned} \tag{12}$$

Plugging (11) and (12) into (10), we have

$$\begin{aligned}
\tilde{x}_n&=\frac{E\left[\tilde{v}\middle|\tilde{p},\tilde{i}_n,\tilde{z}_0,\tilde{z}_N\right]-\tilde{p}}{(N-1)^{-1}\gamma^{-1}+\rho\tau^{-1}}\\
&=\frac{\frac{(1-\varphi)\tau_e}{\tau}\tilde{i}_n+\frac{\varphi\tau_e}{\beta\tau}\left[N\gamma\tilde{p}-(1+N\delta_0)\tilde{z}_0-(1+\delta_1)\tilde{z}_n\right]-\tilde{p}}{(N-1)^{-1}\gamma^{-1}+\rho\tau^{-1}}\\
&=\frac{(1-\varphi)\tau_e}{(N-1)^{-1}\gamma^{-1}\tau+\rho}\tilde{i}_n-\frac{\beta N^{-1}\gamma^{-1}\tau-\varphi\tau_e}{\beta N^{-1}\gamma^{-1}\left((N-1)^{-1}\gamma^{-1}\tau+\rho\right)}\tilde{p}\\
&\qquad-\frac{\varphi\tau_e(1+N\delta_0)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau+\rho\right)}\tilde{z}_0-\frac{\varphi\tau_e(1+\delta_1)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau+\rho\right)}\tilde{z}n\,.
\end{aligned}$$

In a symmetric equilibrium, $\tilde{x}_n = \beta\tilde{i}_n - \gamma\tilde{p} + \delta_0\tilde{z}_0 + \delta_1\tilde{z}_n$. Thus, we have

$$\beta = \frac{(1-\varphi)\tau_e}{(N-1)^{-1}\gamma^{-1}\tau + \rho}, \tag{13}$$

$$\gamma = \frac{\beta N^{-1}\gamma^{-1}\tau - \varphi\tau_e}{\beta N^{-1}\gamma^{-1}\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)} \tag{14}$$

$$\delta_0 = -\frac{-\varphi\tau_e(1+N\delta_0)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)} \tag{15}$$

$$\delta_1 = -\frac{-\varphi\tau_e(1+\delta_1)}{\beta\left((N-1)^{-1}\gamma^{-1}\tau + \rho\right)}. \tag{16}$$

Let $\alpha \equiv 1+\delta_1$. Due to (13) and (16),

$$\alpha = 1-\varphi = 1 - \frac{(N-1)\tau_e^{-1}}{(N-1)\tau_e^{-1} + g\beta^{-2}}$$

$$= \frac{g\beta^{-2}}{(N-1)\tau_e^{-1} + g\beta^{-2}}$$

$$= \frac{\left[(N-1)\alpha^2\sigma_1^2 + \sigma_{N+1}^2\right]\beta^{-2}}{(N-1)\tau_e^{-1} + \left[(N-1)\alpha^2\sigma_1^2 + \sigma_{N+1}^2\right]\beta^{-2}}.$$

Thus

$$f_1(\alpha) \equiv (N-1)\sigma_1^2\alpha^3 - (N-1)\sigma_1^2\alpha^2 + \left[(N-1)\tau_e^{-1}\beta^2 + \sigma_{N+1}^2\right]\alpha - \sigma_{N+1}^2 = 0. \tag{17}$$

$f_1(\alpha) = 0$ has a solution $\alpha^*(\beta) \in [0,1]$ because $f_1(0) < 0$ and $f_1(1) > 0$. If $f_1(\alpha) = 0$ has multiple solutions in $[0,1]$, let $\alpha^*(\beta)$ be the largest one satisfying $f'(\alpha^*(\beta)) > 0$. Then, because

$$\frac{\partial f(\alpha^*(\beta))}{\partial\beta} = f'(\alpha^*(\beta))\alpha^{*\prime}(\beta) + 2(N-1)\tau_e^{-1}\beta\alpha = 0,$$

$\alpha^{*\prime}(\beta) < 0$.

Due to (13) and (14),

$$f_2(\beta) \equiv (N-1)\tau_e^{-1}\beta^3 + N\rho^{-1}\beta^2 + g\beta - g\frac{N-2}{N-1}\tau_e\rho^{-1}$$

$$= (N-1)\tau_e^{-1}\beta^3 + N\rho^{-1}\beta^2$$

$$+ \left[(N-1)\alpha^2\sigma_1^2 + \sigma_{N+1}^2\right]\beta - \left[(N-1)\alpha^2\sigma_1^2 + \sigma_{N+1}^2\right]\frac{N-2}{N-1}\tau_e\rho^{-1} = 0. \tag{18}$$

$f(\beta)=0$ has a unique solution $\beta^*(\alpha)\in\left[0,\frac{N-2}{N-1}\tau_e\rho^{-1}\right]$ because $f_2(0)<0$,

$f_2\left(\frac{N-2}{N-1}\tau_e\rho^{-1}\right)>0$, and $f_2'(\beta)>0$ for any $\beta>0$.

Due to Brower's fixed point theorem, the map $(\alpha,\beta)\rightarrow(\alpha^*(\beta),\beta^*(\alpha))$ has a fixed point. This fixed point provides the values of β and δ_1, which then determines γ by (14).

Due to (13) and (15),

$$(1+N\delta_0)=\frac{g\beta^{-2}}{N(N-1)\tau_e^{-1}+g\beta^{-2}}. \tag{19}$$

This and the values of β and γ determine δ_0.

Proof of Lemma 1

Plugging $\tau=\tau_v+\tau_e+(N-1)\varphi\tau_e=\tau_e+(N-1)\varphi\tau_e$ into (13) and (14), we can show that the right hand side of (13) and (14) are the same.

Proof of Lemma 2

Due to the market clearing condition and $\beta=\gamma$,

$$\begin{aligned}\tilde{p}&=\frac{1}{N\beta}\left[\beta\sum_{k=1}^{n}\tilde{i}_k+N\delta_0\tilde{z}_0+\delta_1\sum_{k=1}^{n}\tilde{z}_k+\tilde{Z}\right]\\&=\frac{1}{N}\sum_{k=1}^{n}\tilde{i}_k+\frac{1}{N\beta}\left[(1+N\delta_0)\tilde{z}_0+(1+\delta_1)\sum_{k=1}^{n}\tilde{z}_k+\tilde{z}_{N+1}\right]\\&=\tilde{v}+\frac{1}{N}\sum_{k=1}^{n}\tilde{e}_k+\frac{1}{N\beta}\left[(1+N\delta_0)\tilde{z}_0+(1+\delta_1)\sum_{k=1}^{n}\tilde{z}_k+\tilde{z}_{N+1}\right]\end{aligned}$$

Thus,

$$\sigma^2_{\tilde{p}|\tilde{v}}=V(\tilde{p}|\tilde{v})=N^{-1}\tau_e^{-1}+N^{-2}\beta^{-2}\left[(1+N\delta_0)^2\sigma_0^2+(1+\delta_1)^2N\sigma_1^2+\sigma_{N+1}^2\right].$$

Proof of Theorem 2

When $\sigma_1^2=0$,

$$g=(N-1)(1+\delta_1)^2\sigma_1^2+\sigma_{N+1}^2=\sigma_{N+1}^2=\sigma_Z^2-\sigma_0^2$$

and

$$\begin{aligned}\sigma^2_{\tilde{p}|\tilde{v}}&=N^{-1}\tau_e^{-1}+N^{-2}\beta^{-2}\left[(1+N\delta_0)^2\sigma_0^2+\sigma_{N+1}^2\right]\\&=N^{-1}\tau_e^{-1}+N^{-2}\left[(1+N\delta_0)^2(\sigma_Z^2-g)\beta^{-2}+g\beta^{-2}\right].\end{aligned}$$

Note that β is determined only by

$$f_2(\beta) = (N-1)\tau_e^{-1}\beta^3 + N\rho^{-1}\beta^2 + g\beta - g\frac{N-2}{N-1}\tau_e\rho^{-1}$$

which is independent of δ_1. Rewriting this:

$$g = \frac{(N-1)\tau_e^{-1}\beta^3 + N\rho^{-1}\beta^2}{\frac{N-2}{N-1}\tau_e\rho^{-1} - \beta}.$$

It is straightforward to see that

$$\frac{\partial g}{\partial \beta} > 0$$

for any $\beta \in \left(0, \frac{N-2}{N-1}\tau_e\rho^{-1}\right)$. We also know that

$$\frac{\partial g}{\partial \sigma_0^2} = -1 < 0.$$

This leads to

$$\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \sigma_0^2} = \frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta}\frac{\partial \beta}{\partial g}\frac{\partial g}{\partial \sigma_0^2} = -\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta} / \frac{\partial g}{\partial \beta}.$$

Thus the sign for $\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \sigma_0^2}$ is the opposite of the sign for $\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta}$. We evaluate $\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta}$ instead of $\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \sigma_0^2}$.

Here we introduce new variables ω and t such that $\beta = \omega\tau_e(N-1)^{-1}\rho^{-1}$ and $\sigma_Z^2 = t\tau_e\rho^{-2}$.

Then this leads to the calculation result:

$$\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta} = N^{-2}\frac{\partial}{\partial \beta}\left[(1+N\delta_0)^2(\sigma_Z^2 - g)\beta^{-2} + g\beta^{-2}\right] = A/B$$

where

$$\begin{aligned}
A = & -2(-1+N)^3\beta^3\rho^2((-3+N)N^2\beta - \sigma_Z^2\rho) + 2(-1+N)^2 N\beta^2\rho(N^2(1+N)\beta + 3\sigma_Z^2\rho)\tau_e \\
& + 2\sigma_Z^2(-1+N)N^2\beta\rho\tau_e^2 - 2\sigma_Z^2 N^3\tau_e{}^3 \\
= & \frac{2\tau_e{}^4}{(-1+N)\rho^2}\left[-1((-1+N)N^3 t) + (-1+N)N^2 t\omega\right. \\
& \left. + 3(-1+N)Nt\omega^2 + (N^3 + N^4 - t + Nt)\omega^3 + (3-N)N^2\omega^4\right]
\end{aligned}$$

and

$$B = (-1+N)^2 N^2 \beta^3 (-(N-1)\beta\rho + N\tau_e)^3$$

$$= \frac{N^2 (N-\omega)^3 \omega^3 \tau_e^6}{(N-1)\rho^3}.$$

$B > 0$ because $\omega < N-2$. Thus, the sign for $\dfrac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta}$ is the same as that of A.

Consider

$$\xi(\omega,t) = \mathrm{A} \div \frac{2\tau_e^4}{(-1+N)\rho^2}$$

$$= -((-1+N)N^3 t) + (-1+N)N^2 t\omega$$
$$+ 3(-1+N)Nt\omega^2 + (N^3 + N^4 - t + Nt)\omega^3 + (3-N)N^2\omega^4.$$

Because

$\xi(0,t) = -(N-1)N^3 t < 0$,

$\xi(N-2,t) = 6(-2+N)^3(-1+N)N^2 + 4(-1+N)^2(2+(-4+N)N)t > 0,$

$\dfrac{\partial^2 \xi(0,t)}{\partial \omega^2} = 6(-1+N)Nt > 0$,

$\dfrac{\partial \xi^4(2,t)}{\partial \omega^4} = -24(-3+N)N^2 < 0$,

There exists unique $\omega^*(t) \in (O, N-2)$ such that $\xi(\omega,t) > 0$ if $\omega \in (\omega^*(t), N-2)$ and $\xi(\omega,t) < 0$ if $\omega \in (0, \omega^*(t))$.

Let

$$t^*(\omega) \equiv g\tau_e^{-1}\rho^2 = \frac{\omega^2 (N+\omega)}{(-1+N)(-2+N-\omega)}.$$

Then we know the following:

- If $t > t^*(\omega^*(t))$, or $\sigma_Z^2 > t^*\left(\omega^*\left(\sigma_Z^2 \tau_e^{-1} \rho^2\right)\right)\tau_e \rho^{-2}$, then $\sigma_0^{*2} = \sigma_Z^2 - t^*\left(\omega^*\left(\sigma_Z^2 \tau_e^{-1}\rho^2\right)\right)\tau_e\rho^{-2}$.
- If $t < t^*(\omega^*(t))$, or $\sigma_Z^2 < t^*\left(\omega^*\left(\sigma_Z^2 \tau_e^{-1}\rho^2\right)\right)\tau_e\rho^{-2}$, then $\sigma_0^{*2} = 0$.

When $\sigma_1^2 = 0$ and thus $\sigma_Z^2 = g, \omega$ takes the largest value, and $t = t^*(\omega)$. In this case,

$$\xi(\omega, t^*(\omega)) = \frac{\omega^2}{-2+N-\omega}\eta(\omega)$$

where

$$\eta(\omega) = -N^4 + (-2+N)N^3(1+N)\omega - 2(-1+N)^2 N^2\omega^2 + N(4+(-3+N)N)\omega^3 + \omega^4 .$$

Because

$$\eta(0) = -N^4 < 0 ,$$

$$\eta(N-2) = 8(-1+N)^2(2+(-4+N)N) > 0 ,$$

$$\eta'(0) = (-2+N)N^3(1+N) > 0 ,$$

$$\eta'(N-2) = -2(-2+N)(-1+N)(8+3(-4+N)N) < 0 ,$$

there exists $\omega*$ such that, $\eta(\omega) > 0$ and $\xi(\omega, t*(\omega)) > 0$ for any $\omega \in (\omega*, N-2)$ and $\eta(\omega) < 0$ and $\xi(\omega, t*(\omega)) < 0$ for any $\omega \in (0, \omega*)$. This implies that $\sigma*_Z^2 = t*(\omega*)\tau_e\rho^{-2}$.

Proof of Theorem 3

When $\sigma_0^2 = 0$,

$$g = (N-1)(1+\delta_1)^2\sigma_1^2 + \sigma_{N+1}^2 = (N-1)\alpha^2\sigma_1^2 + \sigma_Z^2 - N\sigma_1^2 = \left[(N-1)\alpha^2 - N\right]\sigma_1^2 + \sigma_Z^2$$

Solving for σ_1^2,

$$\sigma_1^2 = \frac{\sigma_Z^2 - g}{N-(N-1)\alpha^2} .$$

Then

$$\frac{\partial\sigma_1^2}{\partial\beta} = \frac{-\dfrac{\partial g}{\partial\beta} - (\sigma_Z^2 - g)\left[-2(N-1)\alpha\dfrac{\partial\alpha}{\partial\beta}\right]}{(N-(N-1)\alpha^2)^2} < 0$$

because $\dfrac{\partial g}{\partial\beta} > 0$ and $\dfrac{\partial\alpha}{\partial\beta} < 0$. This implies that the sign for $\dfrac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\sigma_1^2}$ is the opposite of the sign for $\dfrac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\beta}$ because

$$\frac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\sigma_1^2} = \frac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\beta} / \frac{\partial\sigma_1^2}{\partial\beta} .$$

We evaluate $\dfrac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\beta}$ instead of $\dfrac{\partial\sigma_{\tilde{p}|\tilde{v}}^2}{\partial\sigma_1^2}$.

Note that

$$\sigma^2_{\tilde{p}/\tilde{v}} = N^{-1}\tau_e^{-1} + N^{-2}\beta^{-2}\left[(1+\delta_1)^2 N\sigma_1^2 + \sigma_{N+1}^2\right]$$

$$= N^{-1}\tau_e^{-1} + N^{-2}\beta^{-2}\left[(1+\delta_1)^2 \sigma_1^2 + g\right].$$

This leads to the calculation result:

$$\frac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta} = N^{-2}\frac{\partial}{\partial \beta}\left[(1+\delta_1)^2\sigma_1^2\beta^{-2} + g\beta^{-2}\right] = A/B$$

where

$$\mathrm{A} = -3N^4 t + t\omega^4 + N^3\left(4t + 9\omega^3\right) + N\left(4t\omega^3 + \omega^5\right) + 2N^2\omega\left(\omega^2(-2+3\omega) + t(2+3\omega)\right)$$

and

$$B = \frac{N^2\omega^3\left(-3N^2 + \omega^2 + 2N(2+\omega)\right)^2\tau^2}{2(-1+N)^2\rho}.$$

$B > 0$. The sign of $\dfrac{\partial \sigma^2_{\tilde{p}|\tilde{v}}}{\partial \beta}$ is the same as that of A.

Consider

$$\xi(\omega,t) = \mathrm{A}$$

$$= 4N^3 t - 3N^4 t + 4N^2 t\omega + 6N^2 t\omega^2 + \left(-4N^2 + 9N^3 + 4Nt\right)\omega^3 + \left(6N^2 + t\right)\omega^4 + N\omega^5.$$

Because $\xi(0,t) < 0, \xi(N-2,t) > 0, \dfrac{\partial \xi(\omega,t)}{\partial \omega} > 0,$ there exists unique $\omega^*(t) \in (0, N-2)$ such that $\xi(\omega,t) > 0$ if $\omega \in (\omega^*(t), N-2)$ and $\xi(\omega,t) < 0$ if $\omega \in (0, \omega^*(t))$.

Note that

$$\frac{d\xi(\omega^*(t),t)}{dt} = \frac{\partial \xi(\omega^*(t),t)}{\partial \omega}\frac{\partial \omega^*(t)}{\partial t} + \frac{\partial \xi(\omega^*(t),t)}{\partial t} = 0.$$

Thus

$$\frac{\partial \omega^*(t)}{\partial t} = -\frac{\partial \xi(\omega^*(t),t)}{\partial t} \Big/ \frac{\partial \xi(\omega^*(t),t)}{\partial \omega}.$$

A simple calculation shows that $\dfrac{\partial \xi(\omega^*,(t),t)}{\partial t} < 0$ and $\dfrac{\partial \xi(\omega^*,(t),t)}{\partial \omega} > 0$. Thus, $\dfrac{\partial \omega^*(t)}{\partial t} > 0$.

Then we know the following:

- If $t < t^*(\omega^*(t))$, or $\sigma_Z^2 < t^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)\right)\tau_e \rho^{-2}$, then $\sigma_1^{*2}\left(\sigma_Z^2\right) = 0$.
- If $t > t^*(\omega^*(t))$ and

$$N \frac{\sigma_Z^2 - t^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)\right)\tau_e \rho^{-2}}{N - (N-1)\alpha^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)(N-1)\rho\tau^{-1}\right)^2} < \sigma_Z^2,$$

then

$$\sigma_1^{*2}\left(\sigma_Z^2\right) = \frac{\sigma_Z^2 - t^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)\right)\tau_e \rho^{-2}}{N - (N-1)\alpha^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)(N-1)\rho\tau^{-1}\right)^2} < \sigma_Z^2 / N.$$

- If $t > t^*(\omega^*(t))$ and

$$N \frac{\sigma_Z^2 - t^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)\right)\tau_e \rho^{-2}}{N - (N-1)\alpha^*\left(\omega^*\left(\sigma_Z^2 \tau_e{}^{-1} \rho^2\right)(N-1)\rho\tau^{-1}\right)^2} \geq \sigma_Z^2,$$

then $\sigma_1^{*2}\left(\sigma_Z^2\right) = \sigma_Z^2 / N$.

Note that

$$0 \leq N\sigma_1^2 = N \frac{\sigma_Z^2 - g}{N - (N-1)\alpha^2} \leq \sigma_Z^2.$$

Thus

$$\frac{N-1}{N}\alpha^2 \sigma_Z^2 \leq g.$$

We can calculate that

$$\alpha = \frac{N+\omega}{2(N-1)}.$$

Therefore,

$$\frac{N-1}{N}\left(\frac{N+\omega}{2(N-1)}\right)^2 \sigma_Z^2 = \frac{(N+\omega)^2}{4N(N-1)}\sigma_Z^2 \leq g$$

and

$$g \leq \sigma_Z^2 \leq \frac{4N(N-1)}{(N+\omega)^2} g.$$

Rewriting this shows that

$$t^*(\omega) \leq t \leq \frac{4N(N-1)}{(N+\omega)^2} t^*(\omega).$$

When $\sigma_1^2 = 0$, ω takes its largest value, and $t = t^*(\omega)$. In this case,

$$\xi(\omega, t^*(\omega)) = \frac{\omega^2}{(-2+N-\omega)(N-1)}\eta_1(\omega)$$

where

$$\eta_1(\omega) = 4N^4 - 3N^5 + (8N^3 - 3N^4)\omega + (4N^2 + 6N^3)\omega^2 + (6N^2 + 9N^3)\omega^3 + (5N + 6N^2)\omega^4 + (1+N)\omega^5 .$$

A simple calculation shows that $\eta_1(0) < 0, \eta_1(N-2) > 0, \eta_1''(\omega) > 0$. This implies that there exists ω_1^* such that, $\eta_1(\omega) > 0$ and $\xi(\omega, t*(\omega)) > 0$ for any $\omega \in (\omega_1^*, N-2)$ and $\eta_1(\omega) < 0$ and $\xi(\omega, t*(\omega)) < 0$ for any $\omega \in (0, \omega_1^*)$. This implies that $\sigma_Z^{**2} = t*(\omega_1^*)\tau_e \rho^{-2}$.

When $\sigma_1^2 = \sigma_Z^2 / N, \omega$ takes its smallest value, and $t = \frac{4N(N-1)}{(N+\omega)^2} t^*(\omega)$. In this case,

$$\xi(\omega, t*(\omega)) = \frac{N\omega^2}{-2+N-\omega}\eta_2(\omega)$$

where

$$\eta^2(\omega) = 16N^2 - 12N^3 + (8N - 10N^2 + 9N^3)\omega + (4N - 3N^2)\omega^2 + (2-5N)\omega^3 - \omega^4 .$$

A simple calculation shows that $\eta_2(0) < 0, \eta_2(N-2) > 0, \eta_2''(\omega) < 0$. This implies that there exists ω_2^* such that, $\eta_2(\omega) > 0$ and $\xi(\omega, t*(\omega)) > 0$ for any $\omega \in (\omega_2^*, N-2)$ and $\eta_2(\omega) < 0$ and $\xi(\omega, t*(\omega)) < 0$ for any $\omega \in (0, \omega_2^*)$. This implies that $\sigma_Z^{***2} = t*(\omega_2^*)\tau_e \rho^{-2}$.

References

Kyle, A (1989): “Informed Speculation with Imperfect competition,” *Review of Economic Studies*, 56, 317-356.

Madhavan, A (1996): “Security Prices and Transparency,” *Journal of Financial Intermediation*, 5, 255-283.

O’Hara, M (1995): *Market Microstructure Theory*, Cambridge, MA: Blackwell.

Röell, A (1990): “Dual-Capacity Trading and the Quality of the Market,” *Journal of Financial Intermediation*, 1, 105-124.