On the costs of deflation: a consumption-based approach

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Abstract

We are seeking to understand the costs of deflation. To that end, we assess the extent to which deflationary risks have surged in a selected group of European economies. Specifically, we develop a simple consumption-based asset pricing model and, based on it, we estimate a(n) (in)deflation risk premium. We find that our aggregate risk premium is correlated with a systemic financial stress indicator. The absolute values of their (time averaged) risk premia and their indices of financial development are also correlated, which is in line with what our model predicts. In addition, we estimate panel data regressions to assess the extent to which the interaction between changes in prices and nominal debts are incorporated in the risk premium. We generally find that debt-deflation terms are statistically significant. Moreover, the magnitude of the coefficients associated with the debt-deflation terms tends to be greater than those associated with inflation. This suggests that deflationary costs are comparatively greater than inflationary ones, which again is in line with our model. We rationalise this cost asymmetry with the presence of a credit constraint during deflationary episodes.

Keywords: Consumption-based asset pricing, inflation, deflation, inflation risk premium, deflation risk premium, eurozone

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The population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicated a high marginal propensity to spend [...]. Typically their indebtedness is rationed by lenders, not just because of market imperfection but because the borrower has greater optimism about his own prospects [...], than the lender regards as objectively and prudently justified". (Tobin (1980))

1. Introduction

Over the past few years, deflation has been observed in a number of European economies. While deflation may evoke negative recollections associated with specific historical episodes, it is not necessarily a negative element for an economy. In fact, deflation has historically taken place under different economic environments, and the negative relationship between deflation and output observed during the Great Depression is not always present (see Borio et al (2015)).[2] However, we believe that one should be concerned about the conditions under which deflation is more likely to entail economic costs.

Against this backdrop, we argue that key to understanding the costs of deflation involves a consideration of the level and type of debt as well as of the development of financial markets. In effect, changes in the price level lead to variations in the real resources needed to fulfil nominal debt contracts. From the point of view of an economy as a whole, changes in the price level might have distributional and wealth effects. Of course, as financial markets develop, agents are in a better position to protect themselves against shocks, including those associated with the price level.[3] In the context of the European debt crisis and its aftermath, we think that it is worth examining some of these elements to better understand the potential costs of deflation.

Naturally, there are other factors that may shape the costs of (in)deflation. For example, price setting mechanisms à la Calvo (1993) can lead to a deviation of output from its trend, bringing about welfare costs. The period since the onset of the Great Financial Crisis (GFC) of 2008–2009 offers a highly relevant example. When nominal interest rates are at or near the zero lower bound, and deflation sets in, then real interest rates increase, adversely affecting aggregate demand. While we acknowledge that such effects might be relevant, *inter alia*, to the economic costs of deflation, we do not explore them here.

In our model, inflation is favourable to debtors, who are subject to a credit constraint. Conversely, it is unfavourable to creditors, who are not as financially constrained. On the other hand, deflation is unfavourable to debtors and favourable to creditors. Moreover, in general debtors tend to have a higher marginal propensity to spend, as underscored by the epigraphic quote from Tobin. There is then an asymmetry in the degree of financial restriction, which implies that deflation – compared with inflation – can lead to larger economic costs. In a deflationary episode,

2 Buiter (2004) analyses the relationship between deflation and output, and tries to identify changes in the price of a good. In effect, a decrease in the price of a good might be associated with a decrease in demand for it, and thus a reduction in its quantity. Alternatively it might be associated with an increase in its supply and, thus, an increase in its quantity.

3 Our model is related to the debt-deflation theory first proposed by Fisher (1933).

creditors might be more concerned about debtors' ability to pay back their loans. We find empirical evidence in line with this argument.

To assess the possible effects of changes in the price level, we think it relevant to look beyond the level of debt to the type of debt held. For instance, from the point of view of an economy as a whole, changes in the price level in the presence of domestic debt could mostly lead to a redistribution of wealth while variations in the price level under external debt could mainly result in a change in overall wealth, everything else constant.[4]

Of course, other factors such as currency denomination, maturity and duration could be relevant to assess the possible effects of debt. We focus here on nominal debt. In one of our main exercises we consider economies from the eurozone, which have issued debt denominated mostly in euros. However, we overlook the maturity and duration of that debt, as our model only considers one period debt.

In this context, the distinction between expected and unexpected inflation is pertinent as well. Essentially, any nominal debt contract comprises an implicit expected inflation rate. Hence, deviations of the realised from the expected inflation rate have an incidence on who benefits and who is adversely affected. In this paper, we do not analyse such a distinction explicitly.[5] It is, however, an important topic in its own right (see Cecchetti and Schoenholtz (2015)).

(In)deflation risk has been measured using several methodologies. A common one uses expected inflation, nominal interest rates and real interest rates. The risk premium is defined as the nominal interest rate minus the real interest rate plus expected inflation in period t, and a common horizon n, ie $y_{t,n} - (r_{t,n} + E_t(\pi_{t,n}))$.[6] Since the bondholder is unsure about his return in real terms, the expected nominal return is adjusted by the premium.[7]

This is an appealing approach that has been implemented elsewhere (see García and Werner (2010)). Yet, it relies on having a measure of expected inflation and, importantly, real interest rates derived from index-linked bonds.[8] Generally, the latter are not available for all economies because the associated bond markets are either illiquid or non-existent.

Another possibility is to obtain inflation density data that are implicitly derived from derivatives contracts that have inflation as their underlying rate (see Fleckenstein et al (2013)). Of course, such data may not always be available or representative because derivatives markets can be, likewise, either illiquid or non-existent. However, if data are available, this approach has as the advantage that one can obtain a complete inflation density.

In sum, these two methods can work well if data are available, which is not necessarily the case for most individual economies.

4 For instance, to assess net changes in creditors' wealth one should also consider variations in the real exchange rate.

5 Since the debt we consider is exogenous, the agent takes it as given.

6 Two additional comments are in order. First, expected inflation is conditional on information known at time t. Second, the interest rates are those associated with zero-coupon bonds.

7 A possible extension of this approach considers liquidity premia for nominal and real bonds.

8 In addition, index-linked bonds do not strictly yield a real return since the price index used in determining their return is published with a time lag.

Of course, investing in nominal bonds involves other possible risks such as credit and liquidity risks. Although such risks are important, we focus solely on (in)deflation risk in this paper.

The literature has explored various aspects of the inflation risk premium. For instance, Söderlind (2011) analyses the evolution of US break-even inflation from 1997 to mid-2008, using survey data on inflation uncertainty and proxies for the liquidity premium. He highlights differences in the dynamics of expected inflation, of the inflation risk premium and of the liquidity premium.

Christensen et al (2012) construct probability forecasts for episodes of price deflation using the yields on US Treasury Inflation Protected Securities (TIPS). They identify two deflation episodes during the past 15 years: a mild one after the 2001 recession and a harsher one in 2008. On a related note, Hördahl and Tristiani (2014) use a joint macroeconomic and term structure model to estimate the inflation risk premium and inflation expectations in the United States and the euro area. They document that after 2004, inflation risk premia in those economies have demonstrated similar dynamics. Taken jointly, the results of these two papers are broadly in line with our first findings. We identify two main deflationary episodes in the euro area, which is a result similar to that of Christensen et al (2012).

However, we use a standard consumption-based model. In it, the risk premium depends on the extent to which holding a nominal bond will be conducive to smoothing the consumption path through time and across-states of nature. Hence, (in)deflationary risks are framed in terms of their relation to aggregate consumption growth. Of course, inflation and consumption data are in general readily available for most economies, even if the frequencies at which they are available are lower than for financial data.

Anticipating our results, we first find that the correlation between our aggregate risk premium and a systemic financial stress indicator is negative, and that the absolute values of their (time averaged) risk premia and their financial development indices have a negative correlation. Those results are in line with our model. In it, we consider the number of Arrow-Debreu assets with respect to the number of states of nature as a measure of financial development. Thus, the more assets an agent has access to, the better her hedging capabilities, and hence a smaller risk premium. Second, the economic costs of deflation, as measured by the (in)deflation risk premium, seem to be proportionally greater than those of inflation. In our model, a financial constraint might be binding during deflationary episodes, increasing the magnitude of the premium, as we explain in more detail below. This result is consistent with that of some of the literature (including Fleckenstein et al (2013)).[9]

The rest of the paper is divided into four sections. The second section explains the simple model we developed to frame our analysis. The third section describes the data and discusses the results of preliminary exercises. The fourth one provides an interpretation of the panel regression estimates. The last section offers some concluding remarks.

[9] Fleckenstein et al (2013) study deflation risk. Their first main result is that the market price of the economic tail risk of deflation is very similarly to other types of tail risk such as catastrophic insurance losses. Moreover, inflation tail risk bears only a relatively small risk premium. Their second main result is that deflation risk is linked to measures of financial tail risk such as swap spreads, corporate credit spreads and the pricing of super senior tranches of debt.

2. The model

We start with a standard endowment economy model (Lucas (1978)) with two periods, t and $t+1$. We assume, in turn, three possible financial market structures:

i) A market with the full range of Arrow-Debreu securities (ie a complete market);

ii) A market with a proper subset of all the Arrow-Debreu securities, denoted by I (ie an incomplete market);[10] and

iii) A market with a proper subset of all the Arrow-Debreu securities, denoted by I, in addition to a credit constraint (ie an incomplete market).

In ii) and iii), we think of I as fixed but arbitrary.

Those three structures, accordingly, lead to three different kinds of budgets constraint in period t:

$$C_t = W_t - D_t/P_t + \sum_{s=1}^{S} \alpha(s)\, B_t(s)\,/P_t;$$

$$C_t = W_t - D_t/P_t + \sum_{s \in I} \alpha(s)\, B_t(s)\,/P_t;$$

$$C_t = W_t - D_t/P_t + \sum_{s \in I} \alpha(s)\, B_t(s)\,/P_t \text{ and}$$

$$max_s(\alpha(s) B_t(s)/P_t) \leq \theta E_t[W_{t+1} - (D_{t+1}/P_{t+1})].$$

The respective budget constraints in period $t+1$ for each state s' are:

$$C_{t+1}(s') = W_{t+1}(s') - D_{t+1}/P_{t+1}(s') - \sum_{s=1}^{S} \alpha(s)\, 1_{t+1}(s)\,/P_{t+1}(s);$$

$$C_{t+1}(s') = W_{t+1}(s') - D_{t+1}/P_{t+1}(s') - \sum_{s \in I} \alpha(s)\, 1_{t+1}(s)\,/P_{t+1}(s); \text{ and}$$

$$C_{t+1}(s') = W_{t+1}(s') - D_{t+1}/P_{t+1}(s') - \sum_{s \in I} \alpha(s)\, 1_{t+1}(s)\,/P_{t+1}(s);$$

where C_t is the agent's consumption, $C_{t+1}(s')$ is its consumption in state s', W_t is its endowment, $W_{t+1}(s')$ is its endowment in state s', D_t is an exogenous nominal debt if positive (and credit if negative), P_t is the price level, $\alpha(s)$ is the number of Arrow-Debreu securities s the agent buys or sells, and $B_t(s)$ is the price of such securities, with $s = 1,2,3,\dots,S$.[11] The indicator function $1_{t+1}(s')$ equals one in state s', and zero in all other states. In short, the Arrow-Debreu security s' costs $B_t(s')$ and pays one if state s' occurs and zero in other states. The price level, P_{t+1}, is an exogenous random variable.[12] The agent's asset holdings are given by $\boldsymbol{\alpha}^{\boldsymbol{T}} \equiv (\alpha(1), \alpha(2), \alpha(3), \dots, \alpha(S))^T$, where the superscript T denotes the transpose in the complete markets case. Similarly, $\widehat{\boldsymbol{\alpha}}$ is a vector representing portfolio holdings in the incomplete markets cases. Thus, its dimension is strictly smaller than S. Also, θ is a pledgeability parameter, as we explain in greater detail below.

All variables are in real terms, except for debts and Arrow-Debreu prices. Accordingly, we have divided them by their corresponding price levels, depending on

10 By a proper set, we mean that I is such that $I \subset \{e_1, e_2, \dots, e_S\}$ but $I \neq \{e_1, e_2, \dots, e_S\}$, where the elements e_s denote Arrow-Debreu securities, and S is the total number of states of nature.

11 A positive $\alpha(s)$ means that the agent is selling security s, ie borrowing; and a negative one that it is buying it, ie saving.

12 The singleton ω_{t+1} is an element of the finite sample space Ω with cardinality S. The subscript s is associated with a unique element in Ω, where $s = 1,2,3,\dots,S$. We assume a probability space (Ω, F, P), where F, the σ-algebra, is given by 2^{Ω}. We focus on the singleton elements in F, denoted by $\{\omega_1\}$, $\{\omega_2\}$, ..., $\{\omega_S\}$ and associate each of them to the corresponding Arrow-Debreu security $\{s_1, s_2, \dots, s_S\}$.

their periods and states. Hence, the agent maximises the following utility function with respect to $\boldsymbol{\alpha}$ in the complete markets case, and with respect to $\widehat{\boldsymbol{\alpha}}$ in the incomplete markets cases:

$$u(C_t) + \beta E_t(u(C_{t+1}))$$

subject to the respective budget constraint i); ii); or iii). The subjective discount factor β is $0 < \beta < 1$, and E_t is the expectation conditional on information known at time t.

Having posited the main elements of our model, we analyse, in turn, the covariance between consumption growth and inflation for each market structure. This covariance has a direct relationship with the (in)deflation risk premium, as we examine in more detail later.

2.1 Complete markets

Under complete markets, we have the following Euler equations, obtained from the first order condition with respect to each $\alpha(s)$, for $s = 1,2,3,\dots,S$.

$E_t[\beta(u'(C_{t+1})/u'(C_t))1_{t+1}/P_{t+1}] = B_t(s)/P_t$ for $s = 1,2,3,\dots,S$.

We will show that the covariance between consumption growth and inflation is zero if all securities have actuarially fair prices.[13] To see this, consider the Euler equation of a given Arrow-Debreu security s.

$\beta(u'(C_{t+1}(s))/u'(C_t))q(s)/P_{t+1}(s) = B_t(s)/P_t,$

where the probability of state s has been denoted by $q(s)$. Since we assume that all securities have actuarially fair prices, it follows that $u'(C_{t+1}(s))/u'(C_t) = 1$ and, thus, $C_{t+1}(s) - C_t = 0$. Hence, we have that $C_{t+1}(s) = C_{t+1}(s')$ for all $s \neq s'$, and $Cov_t(\Delta c_{t+1}(\boldsymbol{\alpha}^{CM}), \pi_{t+1}) = 0$, where we have that $\Delta c_{t+1} = \log(C_{t+1}/C_t)$, $\pi_{t+1} = \log(P_{t+1}) - \log(P_t)$, and $\boldsymbol{\alpha}^{CM}$ is the vector that solves the respective optimisation problem. The superscript CM stands for complete markets.

2.2 Incomplete markets

Consider the market structure with a proper subset of all Arrow-Debreu securities. As first order conditions we have:

$\beta(u'(C_{t+1})/u'(C_t))q(s)/P_{t+1}(s) = B_t(s)/P_t$ for all $s \in I$.

Similarly, assuming that all securities prices are actuarially fair, we obtain $u'(C_{t+1}(s))/u'(C_t) = 1$ and, thus, $C_{t+1}(s) - C_t = 0$ for all $s \in I$. Hence, $C_{t+1}(s) = C_{t+1}(s')$ for all $s \neq s'$ and $s, s' \in I$. Nonetheless, for those $s \notin I$, it is generally not the case that $C_{t+1}(s) - C_t = 0$, which implies that:

$0 = Cov_t(\Delta c_{t+1}(\boldsymbol{\alpha}^{CM}), \pi_{t+1}) \leq |Cov_t(\Delta c_{t+1}(\widehat{\boldsymbol{\alpha}}^{IM}), \pi_{t+1})|,$

where $\widehat{\boldsymbol{\alpha}}^{IM}$ is the portfolio that solves the optimisation problem under incomplete markets. The superscript *IM* stands for incomplete markets.

13 In a two period model, a security has an actuarially fair price if it is equal to its expected discounted payoff. Thus, in the context of our model and in the case of an Arrow-Debreu security, this happens if $(\beta/P_{t+1}(s))q(s) = B_t(s)/P_t$, where $q(s)$ denotes the probability of state s occurring.

2.3 Credit constraint

Our credit constraint is motivated by the creditor's concern that the debtor might not have the capacity or the incentives to honour its debts. In our model, such concern is heightened under deflationary episodes and a high debt level, as they would call for the agent to obtain more real resources to repay its debts. Specifically, we add to the agent's problem the following exogenous credit constraint in period t:

$$max_s(\alpha(s)B_t(s)/P_t) \leq \theta E_t[W_{t+1} - (D_{t+1}/P_{t+1})]$$

In short, the debtor's maximum debt at time t is bounded by a fraction of its expected wealth in period $t+1$. Since the borrower knows that the lender will not grant it additional resources if the inequality is binding, it considers it to be part of its own constraint.

Under a deflationary environment (ie a higher value of $E_t[P_t/P_{t+1}]$), and under a higher debt D_{t+1}, the upper bound becomes tighter. On the other hand, a larger endowment W_{t+1} provides the agent with financial slack.

We interpret θ as a pledgeability parameter, with $0 \leq \theta \leq 1$. We think of pledgeability as the fraction of expected net endowment that is automatically directed to the creditor. In practice, it is determined by several factors including the information possessed by the creditor and the capacity to enforce contracts. This notion of pledgeability is based on Diamond et al (2016).[14]

Having pledgeability concerns, the creditor is not willing to lend more than a fraction of the borrower´s expected net endowment. Hence, if θ is small, it reflects a higher concern, tightening the constraint. On the other hand, a sufficiently large θ might lead to an unbinding constraint. There is also a possibility that the agent´s portfolio positions are sufficiently small for the constraint not to be binding.

In addition, the credit constraint is similar to the one posited by Aiyagari and Gertler (1999), although it exhibits two important differences. First, the constraint does not depend on any variable determined by the model, making it exogenous. Second, it depends on an expected value and, thus, might not hold ex-post. Overall, we assume that θ is sufficiently small for the credit constraint to be effectively binding for at least one $s' \in I$. The first order conditions for this problem are:

$\beta(u'(C_{t+1})/u'(C_t))q(s)/P_{t+1}(s) \leq B_t(s)/P_t$ for all $s \in I$; and,

$\beta(u'(C_{t+1})/u'(C_t))q(s')/P_{t+1}(s') < B_t(s')/P_t$ possibly for one or more $s' \in I$ for which $(\alpha(s')B_t(s')/P_t) = \theta E_t[W_{t+1} - (D_{t+1}/P_{t+1})]$.

It follows that $|Cov_t(\Delta c_{t+1}(\widehat{\alpha}^{IM}), \pi_{t+1})| \leq |Cov_t(\Delta c_{t+1}(\widehat{\alpha}^{CC}), \pi_{t+1})|$, where $\widehat{\alpha}^{CC}$ is the portfolio that solves the optimisation problem under incomplete markets and the presence of the credit constraint. The superscript CC stands for credit constraint. Hence, in this case, the agent generally does not borrow as many resources as it would like: ie $\widehat{\alpha}^{CC} \leq \widehat{\alpha}^{IM}$.

To gain an intuitive understanding of the inequality involving the covariance terms, consider the following remarks. Any portfolio that is feasible under problem iii) is also feasible under problem ii). Thus, given the assumption of actuarially fair prices,

[14] Naturally, our context and theirs is quite different. Ours refers to aggregate debt/credit in an economy while theirs refers to corporate debt/credit.

the agent will insure itself in every possible state, ie $C_{t+1}(s) - C_t = 0$. In particular, for every state it insures for in problem iii), it will also do so in problem ii).

Accordingly, if the credit constraint binds, then the following inequality generally holds $|Cov_t(\Delta c_{t+1}(\boldsymbol{\alpha}^{IM}), \pi_{t+1})| \leq |Cov_t(\Delta c_{t+1}(\boldsymbol{\alpha}^{CC}), \pi_{t+1})|$. This is the case because the credit constraint will not allow the debtor to insure itself fully for one or more states. However, if the credit constraint does not bind, then we would obtain equality of the covariances.

Intuitively, if there is one or more states of nature against which the agent cannot insure, then it probably will not be able to smooth its consumption as much as it would like, thus increasing the covariance between consumption growth and inflation.

What is more, if there are further states of nature against which the debtor cannot insure itself because of a binding credit constraint (in our case, a binding pledgeability factor), then even less consumption smoothing would take place, further increasing the covariance between consumption growth and inflation.

More generally, based on the budget constraint, the sign of $Cov_t(\Delta c_{t+1}, \pi_{t+1})$, depends on at least the following three factors.

i) The covariance between inflation and the endowment;

ii) Whether the agent has debt or credit (ie the sign of D); and

iii) The extent to which the agent is able to hedge its consumption growth through the Arrow-Debreu securities.

Some additional comments are in order. First, in the model, a greater covariance's magnitude can be the product of a tighter credit constraint or of less developed financial markets. Although the empirical identification of the relative importance of these two elements is a significant problem, and we recognise the presence of both, we do not intend to determine their relative weight.

Second, the model does not distinguish between different types of debt. However, as mentioned, these are relevant for the kind of economic costs one would observe given changes in the price level. Thus, we estimate separate data panel regressions using different types of debt in each case, as we explain in more detail later.

Third, the sign of the covariance between output and inflation varies (see Plosser (2003) and Borio et al (2015)). Of course, this also depends on the time frequency considered (Walsh (2010)).[15] While our model allows for any sign, we do not take a stand on which sign to expect, and focus instead on how the interaction between changes in prices and debt, and financial development might affect the risk premium.

[15] For instance, at business cycle frequencies, one can think that if output is negatively correlated with inflation, aggregate supply shocks are dominant. On the other hand, if output is positively correlated with inflation, then aggregate demand shocks predominate. There is a large consensus that their correlation, at a low frequency, is close to zero (see McCandless and Weber (1995)).

3. The (in)deflation risk premium

In this section, we derive the (in)deflation risk premium and show that the covariance between consumption growth and inflation has a direct relationship with it. To see this, consider that one can also obtain the Euler equations with respect to one-period real and nominal bonds:[16]

$$\beta E_t[u'(C_{t+1})/u'(C_t)\,] = R_t$$

$$\beta E_t[u'(C_{t+1})/u'(C_t)(1/P_{t+1})] = S_t/P_t$$

where R_t denotes the price of a real bond which pays one unit of consumption in period $t+1$, and S_t is the price of a nominal bond that pays one unit of money in period $t+1$. As a result, we introduce the following definitions for the one-period real (r_t) and nominal (i_t) interest rates:

$$\beta E_t[\,u'(C_{t+1})/u'(C_t)] \;= R_t \equiv \exp(-r_t)$$

$$\beta E_t[(u'(C_{t+1})/u'(C_t))(P_t/P_{t+1})] \;= S_t \equiv \exp(-i_t)$$

Moreover, assuming a constant relative risk-aversion (CRRA) utility function $U(C_t) = (C_{t+1}^{1-\gamma})/(1-\gamma)$ would yield:

$$\beta E_t[\exp(-\gamma\Delta c_{t+1})\,] = \exp(-r_t) \tag{1}$$

$$\beta E_t[\exp(-\gamma\Delta c_{t+1} - \pi_{t+1})] = \exp(-i_t), \tag{2}$$

where γ is the coefficient of relative risk aversion. We express (2) as follows:[17]

$$\beta cov_t[\exp(-\gamma\Delta c_{t+1}), \exp(-\pi_{t+1})] + \beta E_t[\exp(-\gamma\Delta c_{t+1})]E_t[\exp(-\pi_{t+1})] = \exp(-i_t)$$

We combine this last expression with (1), to obtain:

$$\beta cov_t[\exp(-\gamma\Delta c_{t+1}), \exp(-\pi_{t+1})] + \exp(-r_t)\, E_t[\exp(-\pi_{t+1})] = \exp(-i_t) \tag{3}$$

One could interpret (3) as a generalisation of the Fisher equation.[18, 19] Moreover, using the Taylor series of the exponential function around zero, and assuming that higher order terms and cross-terms in the second component are negligible, we can obtain a simplified version of (3): $-\beta\gamma cov_t[\Delta c_{t+1}, \pi_{t+1}] + r_t + E_t\pi_{t+1} = i_t$.

Hence, the term $-\beta\gamma cov_t[\Delta c_{t+1}, \pi_{t+1}]$ can be understood as an (in)deflation risk premium.[20] In the appendix, we obtain an exact expression for the case when consumption growth and inflation follow a joint normal distribution. In the empirical exercises, we use the latter.

To gain some intuition, consider the following remarks. First, the debt is seen as a stock variable already held by the agent. On the other hand, the nominal bond

[16] Real bonds include the already discussed TIPS in the United States and index-linked gilts in the United Kingdom.

[17] For this step, we have used the well-known equality $cov(X,Y) = E(XY) - E(X)E(Y)$.

[18] The original Fisher equation is: $(1+r_t)(1+E_t\pi_{t+1}) = (1+i_t)$, ie it assumes that there is no (in)deflation risk premium.

[19] One could use a more general utility function and the relationship would still hold.

[20] Moreover, since $E_t\pi_{t+1}$, the expected inflation conditional on the information at time t, is known at t, we can rewrite the risk premium as $-\beta\gamma cov_t[\Delta c_{t+1}, \pi_{t+1} - E_t\pi_{t+1}]$, naturally interpreting $\pi_{t+1} - E_t\pi_{t+1}$ as an inflation surprise or shock.

pricing decision takes place at the margin. Pricing will depend on the extent to which holding the nominal bond will allow for consumption to be smoothed.

Second, we assume incomplete markets that, in our model, provides us a covariance between consumption growth and inflation different from zero. Moreover, the credit constraint gives us the asymmetry in inflation and deflation costs. Consider then two general settings. First setting, suppose that a lower (higher) inflation is associated with lower (higher) consumption growth, i.e., the covariance term is positive. If deflation (inflation) takes place then her consumption would typically decrease (increase). In general, as a direct effect, more deflation (inflation) will increase (decrease) the real return of a nominal bond. This effect is favorable to her consumption-smoothing motive because she will be getting a higher (lower) real return from the nominal bond when consumption growth is lower (higher). Thus, in equilibrium, she is willing to hold a nominal bond albeit it would have a negative premium.[21]

Second setting, suppose that a higher (lower) inflation is associated with lower (higher) consumption growth, i.e., the covariance term is negative. If deflation (inflation) takes place then her consumption would usually increase (decrease). In general, as a direct effect, more deflation (inflation) will increase (decrease) the real return of a nominal bond. This effect is unfavorable to her consumption-smoothing motive since she will be getting a higher (lower) real return from the nominal bond when consumption growth is higher (lower). Thus, in equilibrium, she is willing to hold the nominal bond with a positive premium.[22]

Thus, in our model, one could consider four cases. The first and second cases occur if the agent holds nominal debt, respectively, under the first and second settings. Clearly, the interaction of debt with (in)deflation contributes towards the level of the covariance being greater. On the other hand, the third and fourth cases take place if the agent is a creditor in nominal contracts, in turn, under the first and second settings. The interaction of credit with (in)deflation would contribute towards the level of the covariance being smaller. Our motivation for assuming that the agent holds nominal debt results from the present juncture in the eurozone. For that reason, in our empirical exercises, the first and second cases are the relevant ones.

Additionally, if the agent is more risk-averse, ie, if γ is larger, the risk premium increases. Moreover, under a CRRA utility function, the intertemporal elasticity of substitution is equal to γ^{-1}. Thus, a lower intertemporal elasticity of substitution, which implies that the agent is less willing to substitute consumption across time, leads to a higher premium, as the agent has to be compensated.

Finally, consider the agent's subjective discount factor β, which affects the premium positively. As the agent cares more about its future consumption, the

21 A more familiar example of this phenomenon is car insurance. The car owner is willing to pay a premium above its actuarially fair price since the insurance will pay her in the state of nature when she needs resources, i.e., when a crash takes place. She buys insurance although, in expected value, the agent will lose money. In effect, the insurance premium is not actuarially fair.

22 Fleckenstein et al's (2013) explanation is different from ours. They motivate the difference in the risk premium' sign based on the price level cyclicality or counter-cyclicality with output. In the former case, most of the output variation is due to aggregate demand shocks, leading to a positive covariance. In the latter, most of its variation is due to aggregate supply shocks, implying a negative covariance.

larger β is, the more it needs to be compensated, bearing in mind that changes in the price level distort intertemporal consumption.

4. Data and estimation

For the estimation of the risk premium, there are, at least, three important issues to consider. First, evidently, the consumption and inflation indices have to be associated with the same consumption basket.

Second, inflation targets are commonly formulated in terms of the annual percentage change of a specific price index. In the case of the eurozone, it is the Harmonised Index of Consumption Prices (HICP). This index is widely known, frequently referred to and extensively used. Thus, to measure changes in the price level, we always use the HICP.

Third, an agent obtains her or his utility from the services, the nondurables goods and, importantly, the *portion* of durables goods consumed in a given period. Ideally, one should make a distinction between the durables goods bought, which is what the data generally measure, and the *portion* of the durables goods consumed in a given period.[23] Distinguishing such concepts explicitly would entail a separate model. Thus, while we still use general consumption indices, we also estimate our risk premium with an index that excludes durable goods (index iv)).

Overall, we estimate risk premia using the following consumption and price indices: [24]

i) Final consumption expenditure (FCE);[25]

ii) Final consumption expenditure of households (Total); (a subset of i));

iii) Final consumption expenditure of households (HFCE); (a subset of ii)); and

iv) Semi-durable goods, non-durable goods and services (a subset of ii)).

In the initial exercises, we use the risk premia estimated with i) since that index has the broadest coverage across all the economies in our data set. However, in the case of the panel regressions, we estimate the risk premia using the ii), iii) and iv) indices for the reasons just explained and to maintain comparability across such regressions.[26]

[23] For similar reasons, the income elasticity of demand for durable goods tends to be greater than that for nondurable goods.

[24] The interested reader is referred to the European System of Accounts (ESA (2010)), page 70, for further details. The respective codes in the Eurostat database are P3, P31, P31_S14 and P312N.

[25] This final consumption expenditure index "consists of expenditure incurred by resident institutional units on goods or services that are used for the direct satisfaction of individual needs or wants or the collective needs of members of the community".

[26] The HFCE essentially measures the same consumption basket as the HICP, except for the coverage of expenditure for housing by homeowners It is the closest to the HICP. See: https://www.ecb.europa.eu/stats/prices/hicp/html/index.en.html and http://ec.europa.eu/eurostat/statistics-explained/index.php/HICP_methodology for further methodological details.

We estimate consumption growth and inflation in year-on-year terms, which addresses seasonal effects.[27] The frequency of all the time series used is quarterly. In this context, one is generally restricted by the lower frequency of the consumption series used, as other series are commonly available at higher frequencies. In addition, consumption data are available with a longer lag compared with the other variables. Hence, the quarterly frequency of the data, together with the model's set up, associates our risk premium with a three-month horizon.

Concretely, the estimation of the (in)deflation risk premium entails the covariance of consumption growth and inflation, conditional on the information available in period t, eg $cov_t(\Delta c_{t+1}, \pi_{t+1})$. To estimate it, we use the following expression:

$$cov_t(\Delta c_{t+1}, \pi_{t+1}) = \left((k+2)^{-1} \sum_{i=t-k}^{t+1} (\Delta c_i - E\Delta c_t)(\Delta \pi_i - E\pi_t)\right),$$

where $E\Delta c_t = (k+2)^{-1} \sum_{j=t-k}^{t+1} \Delta c_j$, and $\mathrm{E}\pi_t = (k+2)^{-1} \sum_{j=t-k}^{t+1} \pi_j$. Specifically, we take $k = 2$, ie the last four observations, which is equivalent to a year. This captures in a simple way the most recent changes in the covariance.[28]

For the relative coefficients of risk aversion, we use estimates from Gandelman and Hernández-Murillo (2014). Moreover, for those economies that are not considered in their paper, we simply take the average for the economies intersecting our database and theirs. We note that the risk aversion coefficient has implications for the magnitude of the risk premium but not for its dynamics.[29]

To determine a value for the subjective discount factor, we assume that the steady state real interest rate has a value of 2.0% a year. Since in the steady state $C_{t+1} = C_t$, based on equation (1), we have that $\beta = \exp(-0.02/4)$. This implies an estimate for β of 0.995. This value is below the 3.0% used in, for example, Schmitt-Grohé and Uribe (2007). Yet, it accounts for the secular reduction in the level of real interest rates in recent years. Similarly, in the case of the CRRA utility function, the subjective discount factor affects the magnitude of the risk premium but not its dynamics.[30]

We use three types of debt. As mentioned, such distinction is important to assess the possible effects of changes in the price level. Consider then each type in general. First, both residents and non-residents can hold total external debt, and residents owe it. Second, residents and non-residents can hold total government debt, and the government owes it. Third, both residents and non-residents may hold total domestic

27 We also estimated the quarter-on-quarter seasonally adjusted consumption growth and inflation. Their dynamics were not particularly different from those of the year-on-year estimates. We used the latter to sidestep any seasonality adjustment procedures.

28 One could use an explicit model of consumption growth and inflation such as a state-space model, and then estimate a covariance term based on it.

29 We could have used higher relative coefficients of risk aversion as is sometimes done in the literature to account for the variability of the returns on assets. However, we are interested in documenting the asymmetry in the costs of deflation and inflation, rather than their absolute values.

30 In a representative agent model, one can consider consumption growth per capita. Thus, accounting for population growth is potentially relevant. In our case, a drawback of using population data is that they are not available at a quarterly frequency for some economies in our database. Thus, we use as a working assumption that population growth is constant and equal to zero. This is not an innocuous assumption at a low frequency (see Juselius and Takáts (2015)); however, the period covered in our estimation does not surpass 15 years and comprises quarterly data.

debt. We use each type separately as the empirical counterpart to the debt term in the model.

The economies in our dataset are shown in Tables 1–5. The periods depend on the specific series and the economy in question. Some are available starting in earlier quarters. Yet, for the estimations, we have used a common starting point: Q1 2001 but the ending quarters of the time series depend on the specific economy.[31] We do this in order to have, as much as possible, a balanced panel data set. Some economies lack certain time series and are thus excluded from the respective panel regression.[32] These are indicated by a dash in Tables 1–5.

Next, we consider the main statistics of each time series. First, in almost all the economies in our database there have been deflationary periods (last column of Table 1).

Second, risk premia are both positive and negative, reflecting their time-varying nature. Moreover, all economies, except for Denmark, Iceland and the UK, have experienced negative values (last column of Table 2), potentially reflecting the presence of deflation risks.

On a related note, Greece, Ireland, Portugal and Spain present an average negative premium, possibly indicating the need for real exchange depreciation through deflation, given the lack of independent exchange rate policy at the individual country level.

Third, debt levels with respect to GDP are, in essentially all cases, sizeable (Tables 3–5).

Next, we have some additional comments on the statistics of deflation. To that end, consider a(n) in(deflation) data point $\pi_{i,t}$, which we associate with an economy i and a quarter t, in our database. First, we have that 91 data points (out of 2,186) presented deflation, accounting for 4.16% of the total. Second, on average, an economy has had 3.4 periods (out of an average of 55.3 periods) of deflation. In other words, typically, an economy has seen deflation 6% of the time. The standard deviation of this last statistic is 7%. Third, clearly, a given economy may very well face the probability of a deflation episode without actually experimenting one.

[31] As of this paper's date, the complete set of times series we have used had not been completely updated on the Eurostat website.

[32] Bulgaria and Romania are dropped altogether from the analysis as their risk premia are unconceivably large, suggesting that the associated series are probably not stationary.

Inflation statistics (HICP)

Table 1

	Mean	Std. D.	Kurt.	Skew.	Max	Min
Austria	2.05	0.89	3.51	0.17	4.05	-0.31
Belgium	2.06	1.26	4.43	0.29	5.75	-1.04
Croatia +	2.69	1.49	3.74	0.75	7.30	-0.07
Cyprus	2.24	1.57	3.22	-0.01	6.35	-1.25
Czech Republic	2.22	1.76	3.54	0.89	7.14	-0.62
Denmark	1.83	0.95	3.22	0.23	4.54	0.17
Estonia	4.02	2.81	3.92	0.53	11.47	-1.87
Finland +	1.93	1.09	2.91	0.35	4.72	-0.44
France	1.79	0.85	3.93	-0.30	3.98	-0.57
Germany +	1.68	0.79	3.08	-0.10	3.37	-0.46
Greece	2.63	1.84	3.01	-0.90	5.66	-1.82
Hungary	4.85	2.34	3.26	-0.02	10.46	-0.52
Iceland +	5.64	4.84	4.86	1.52	21.03	0.35
Ireland +	1.91	2.01	2.89	-0.69	5.08	-3.01
Italy	2.22	0.97	2.82	-0.37	4.01	-0.08
Luxembourg +	2.56	1.31	3.25	-0.47	5.34	-0.98
Malta	2.32	1.28	2.61	-0.10	4.98	-0.58
Netherlands +	2.13	1.26	3.42	0.78	5.30	-0.03
Norway	1.72	1.11	2.97	0.14	4.83	-0.40
Poland	2.65	1.60	2.04	0.08	6.18	-0.24
Portugal +	2.28	1.58	2.86	-0.77	5.13	-1.80
Slovakia	3.62	2.59	2.47	0.52	9.37	-0.16
Slovenia +	3.58	2.39	2.68	0.77	9.67	-0.10
Spain	2.58	1.40	3.36	-0.94	5.06	-0.95
Sweden	1.59	0.97	3.19	0.65	4.18	-0.35
Switzerland	0.42	1.01	3.38	0.67	2.84	-1.24
United Kingdom	2.33	1.07	3.25	0.78	5.25	0.63
Average	2.50	1.59	3.25	0.16	6.41	-0.66

Notes: year-on-year quarterly observations. Sample periods: Q1 2001 to Q2 2014 (indicated by a +) or Q3 2014, depending on the economy.

Source: Own estimations with data from Eurostat.

Risk premium statistics

Table 2

	Mean	Std. D.	Kurt.	Skew.	Max	Min
Austria	0.12	0.26	6.26	-0.41	0.80	-0.86
Belgium	-0.06	0.77	13.41	-2.77	1.18	-3.76
Croatia +	0.01	0.14	12.97	-2.31	0.35	-0.65
Cyprus	-0.79	1.91	14.51	-3.25	1.31	-9.86
Czech Republic	0.29	0.76	11.23	2.83	3.39	-0.64
Denmark	0.04	0.20	7.93	1.59	0.86	-0.41
Estonia	-0.23	1.15	8.04	-2.29	1.13	-4.83
Finland	0.01	0.18	10.60	-0.94	0.60	-0.76
France	0.07	0.35	7.22	1.09	1.24	-0.80
Germany +	0.00	0.07	8.20	1.31	0.30	-0.15
Greece	-0.59	2.48	9.22	-2.41	3.03	-10.74
Hungary	0.47	1.81	5.27	1.13	6.61	-3.03
Iceland +	4.14	7.20	12.37	2.90	37.92	-0.58
Ireland +	-0.04	0.21	10.21	-2.66	0.19	-0.91
Italy	0.03	0.21	6.50	0.33	0.69	-0.66
Luxembourg +	0.16	0.60	9.56	1.78	2.86	-1.10
Malta	0.93	1.37	4.01	1.03	4.79	-1.56
Netherlands +	0.00	0.00	4.46	0.04	0.00	-0.01
Norway	0.17	0.82	4.22	1.16	2.77	-1.12
Poland	-0.01	0.10	3.11	-0.11	0.23	-0.26
Portugal +	-0.16	0.89	5.55	-1.29	2.05	-2.84
Slovakia	0.72	1.91	16.83	2.93	10.96	-2.95
Slovenia +	-0.08	0.53	11.39	-2.22	0.97	-2.52
Spain	-0.09	0.54	13.48	-2.85	0.79	-2.43
Sweden	0.18	0.26	3.89	1.11	0.96	-0.28
Switzerland	0.22	0.31	4.39	1.07	1.22	-0.49
United Kingdom	0.02	0.37	6.95	-0.68	0.99	-1.28
Average	0.20	0.94	8.58	-0.14	3.27	-2.05

Notes: Sample periods: Q1 2001 to Q1 2014 (indicated by a +) or Q2 2014. d. Estimations based on equations (3), (4) and (6).

Sources: Own estimations with data from Eurostat, and Gandelman and Hernández-Murillo (2014).

Total government debt as a percentage of GDP, statistics — Table 3

	Mean	Std. D.	Max	Min
Austria	-	-	-	-
Belgium	120.63	8.87	136.88	101.05
Croatia	42.26	15.92	77.86	28.16
Cyprus	-	-	-	-
Czech Republic	25.97	9.82	42.46	11.08
Denmark	62.58	8.93	72.48	40.02
Estonia	3.34	1.76	7.33	1.02
Finland	53.34	8.82	71.74	36.02
France	92.25	14.85	123.61	73.64
Germany	71.23	8.75	86.73	57.39
Greece	135.59	22.49	192.32	112.73
Hungary	68.21	7.46	78.25	55.23
Iceland	-	-	-	-
Ireland	58.83	35.89	122.71	24.81
Italy	111.15	11.01	144.11	97.33
Luxembourg	12.24	7.05	27.67	5.60
Malta	66.23	4.22	75.44	57.39
Netherlands	61.83	9.48	79.36	47.31
Norway	48.60	8.08	61.14	32.51
Poland +	44.45	3.60	50.47	35.06
Portugal	80.82	27.97	139.23	48.37
Slovakia	37.80	8.68	54.95	23.30
Slovenia	31.24	13.21	68.06	18.64
Spain	71.63	26.81	141.01	43.95
Sweden	51.27	6.67	65.21	41.01
Switzerland	21.38	3.44	26.12	16.79
United Kingdom	60.23	23.73	100.76	38.32
Average	59.71	12.40	85.25	43.61

Notes: Sample periods: Q1 2001 to Q2 2014 (indicated by a +) or Q3 2014, depending on the economy.

Source: Haver Analytics.

Total domestic debt as percentage of GDP, statistics — Table 4

	Mean	Std. D.	Max	Min
Austria	-	-	-	-
Belgium	418.91	48.61	494.79	360.77
Croatia	-	-	-	-
Cyprus	-	-	-	-
Czech Republic	242.93	1.27	244.74	241.25
Denmark	521.72	86.56	627.80	384.19
Estonia	-	-	-	-
Finland	275.74	39.95	336.43	225.25
France	395.11	54.25	476.42	326.79
Germany	327.04	12.73	358.05	296.65
Greece	260.51	57.91	385.72	188.84
Hungary	271.65	54.33	338.95	187.73
Iceland	-	-	-	-
Ireland	1,683.27	572.81	2,347.57	719.99
Italy	335.21	42.06	399.13	273.72
Luxembourg +	2,786.48	1,101.57	4,871.34	1,489.88
Malta	-	-	-	-
Netherlands	801.47	13.68	820.63	780.01
Norway	368.32	39.21	443.62	299.91
Poland +	179.01	23.03	212.76	146.49
Portugal	333.15	66.20	430.74	234.88
Slovakia	199.27	5.27	205.99	191.22
Slovenia	292.84	4.61	299.48	285.86
Spain	384.15	78.71	484.13	257.96
Sweden	409.21	51.31	482.29	341.82
Switzerland	-	-	-	-
United Kingdom	444.14	70.83	541.06	335.00
Average	546.51	121.25	740.08	378.41

Notes: Sample periods: Q1 2001 to Q2 2014 (indicated by a +) or Q3 2014, depending on the economy.

Source: Haver Analytics.

Total gross external debt as percentage of GDP, statistics — Table 5

	Mean	Std. D.	Max	Min
Austria	177.61	26.66	211.89	128.22
Belgium	266.75	33.48	349.46	204.42
Croatia	81.28	20.84	109.77	48.05
Cyprus	522.92	95.20	784.07	350.93
Czech Republic	43.14	12.28	64.39	28.11
Denmark	168.05	16.75	191.26	134.85
Estonia	89.61	23.49	132.12	49.30
Finland	142.37	47.12	235.72	93.80
France	189.35	10.19	203.78	165.88
Germany	140.10	14.59	167.68	111.69
Greece	160.38	48.90	234.14	87.70
Hungary	100.58	33.72	149.48	52.34
Iceland	500.26	312.34	982.48	102.26
Ireland	815.50	235.65	1,121.80	375.02
Italy	105.69	13.40	123.09	78.89
Luxembourg	3,897.52	955.66	5,745.15	2,698.82
Malta	762.57	371.73	1,143.63	226.13
Netherlands	505.93	16.07	530.42	470.97
Norway	129.63	21.30	173.28	91.71
Poland	53.33	11.75	72.40	35.93
Portugal	193.58	33.42	238.22	128.24
Slovakia	62.10	13.85	88.87	41.52
Slovenia	85.56	29.18	124.67	39.69
Spain	134.97	28.97	168.32	81.64
Sweden	175.39	23.25	204.73	122.96
Switzerland	220.06	23.22	284.33	185.13
United Kingdom	322.34	57.06	414.76	232.78
Average	372.10	93.71	527.77	235.81

Notes: Sample periods: Q1 2001 to Q3 2014.

Source: Haver Analytics.

5. Panel data regression model

We explore the extent to which some of the factors we have thus far considered are priced in by the risk premium. In this context, it is worth revising some of our previous results. First, incomplete markets imply that the risk premium is different from zero. Second, in the presence of debt, more deflation contributes towards the risk premium being smaller. Third, the presence of the credit constraint implies that the effect deflation has on the risk premium is greater than that of inflation. Fourth, better financial development implies a smaller risk premium's magnitude. Thus, a key feature we want to assess is the extent to which the pricing implications of inflation and deflation differ..

Hence, we posit the following data panel regression:

$$RP_{i,t+1} = \gamma_0 + \gamma_1 RP_{i,t} + \gamma_2 \Delta GDP_{i,t} + \gamma_3 \left(P_{i,t}\right)^+ + \gamma_4 \left(P_{i,t}\right)^- + \gamma_5 \Delta D_{i,t} +$$

$$\gamma_6 (D_{i,t}/P_{i,t} - D_{i,t})^+ + \gamma_7 (D_{i,t}/P_{i,t} - D_{i,t})^- + u_{i,t}, \quad (5)$$

where $RP_{i,t}$ is the (in)deflation risk premium, $GDP_{i,t}$ is the gross domestic product, $P_{i,t}$ is a price index such that $\pi_{i,t} = \log(P_{i,t}) - \log(1)$ is the rate of inflation or deflation, $D_{i,t}$ stands for the debt, $(P_{i,t})^+ = \max\{P_{i,t}, 1\}$ and $(P_{i,t})^- = \min\{P_{i,t}, 1\}$. Similarly, we define $(D_{i,t}/P_{i,t} - D_{i,t})^+ = \max\{D_{i,t}/P_{i,t} - D_{i,t}, 0\}$ and $(D_{i,t}/P_{i,t} - D_{i,t})^- = \min\{D_{i,t}/P_{i,t} - D_{i,t}, 0\}$. Finally, $u_{i,t}$ is the error term of economy i in quarter t with a fixed effects model.

Thus, we construct specific variables making the distinction between the effects that inflation and deflation might have as they interact with debt. In particular, the term $D_{i,t}/P_{i,t} - D_{i,t}$ is the real change in the value of debt due to changes in the price level, recall that $\pi_{i,t} = \log(P_{i,t})$. Furthermore, $(D_{i,t}/P_{i,t} - D_{i,t})^+$ measures the real change in the value of debt under the presence of deflation and $(D_{i,t}/P_{i,t} - D_{i,t})^-$ does so under the presence of inflation. Note that our price index $P_{i,t}$ is less than one under deflation and more than one under inflation.

Before proceeding to our results, it is worth mentioning four additional points. First, for the panel estimation we have only considered economies from the eurozone. This is so since they have issued most of their debt in euros, allowing us to focus on changes in debt due to variations in the price level.

Second, we have lagged the explanatory variables to address the possible presence of endogeneity. In effect, it is plausible that a change in the (in)deflation risk premium is contemporaneously associated with some of the explanatory variables, prominently, inflation.

Third, we use fixed effects to account for the different levels of financial development across our sample of economies, along with other unobserved heterogeneity.[33] Thus, we are tacitly assuming that financial development is a slow moving variable. It is worthwhile mentioning that financial development indices are only available at a yearly frequency and for a limited number of economies.

Fourth, for the panel data, we use the moving average of adjacent observations of the risk premia. This allow us to focus on their low to medium frequencies,

33 In our model, again, we think of financial development as the number of Arrow-Debreu assets available and, thus, a reflection of the agent's capabilities to hedge changes in the price level.

discarding high frequency changes and, in tandem, attenuating possible measurement errors in these variables.

6. Estimates and discussion

6.1 Initial estimates

As a prelude to our discussion of the panel data regression model's estimates, we document two relationships in this subsection.[34]

i) A correlation between the time series of the aggregated inflation risk premium, built using the (FCE) consumption and inflation series of the EU-28 countries in Eurostat, with a systemic stress index of the financial system (Composite Indicator of Systemic Stress (CISS)); and

ii) A correlation between the absolute value of the countries' time average risk premium and their financial development indices.

With respect to the first relationship, in general, periods during which the risk premium is negative are associated with increments in the CISS (Figure 1). On the other hand, periods during which the risk premium is positive, financial stress tends to diminish. In fact, regressing the risk premium against the CISS leads to a statistically significant negative slope coefficient and a value of 0.32 for R^2.

In our model, as explained earlier, the contribution towards the covariance between inflation and consumption growth due to an agent's debt holding is positive and, hence, negative for the premium. We think of the premium as a compensation of the extent to which the holding of nominal bond leads to a smoother consumption path, which is largely determined by the interaction between changes in the price level and nominal debt.

Moreover, a higher level of inflation might unbind the credit restriction, allowing the agent to secure consumption in more states of nature and enjoy smoother consumption growth, thus leading to a smaller premium in magnitude. In contrast, higher deflation might make the credit restriction bind, constraining consumption growth in more states of nature, increasing consumption variability and, hence, making the premium larger in magnitude. This is consistent with the property that the premium's magnitude tends to be greater during deflationary episodes than during inflationary ones. In addition, the plot allows us to see that this result is driven by the second part of the sample period, mostly during the European debt crisis and its aftermath (Figure 1).

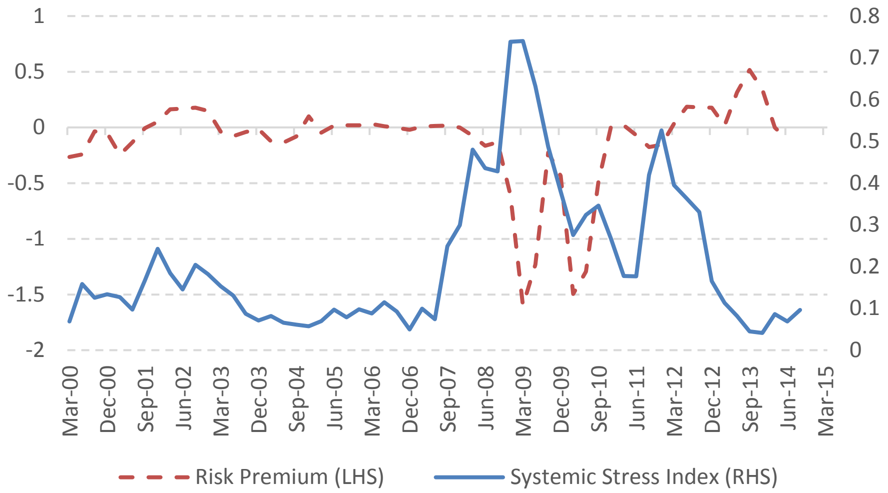
Second, for this exercise, the economies considered are restricted to those for which individual country indices of the Financial Development Index 2014 of the World Economic Forum (WEF) are available, namely for: Austria, Belgium, Czech Republic, Denmark, Finland, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Poland, Portugal, Sweden and Switzerland. We use 2014, since it is the last year of our estimation sample.

[34] For reasons explained in the main text, for these estimations we have used the FCE index.

We plot the absolute value of the average of each risk premium time series (sample period Q1 2000–Q2 2014) with the financial development indices from 2014 (Figure 2). We have an R^2 of 0.35 but if the two data points at the top (the Czech Republic and Hungary) are excluded from the estimation, the R^2 decreases to 0.2.

Inflation risk premium for the EU-28 countries and Composite Systemic Stress indicator (CISS)

Figure 1

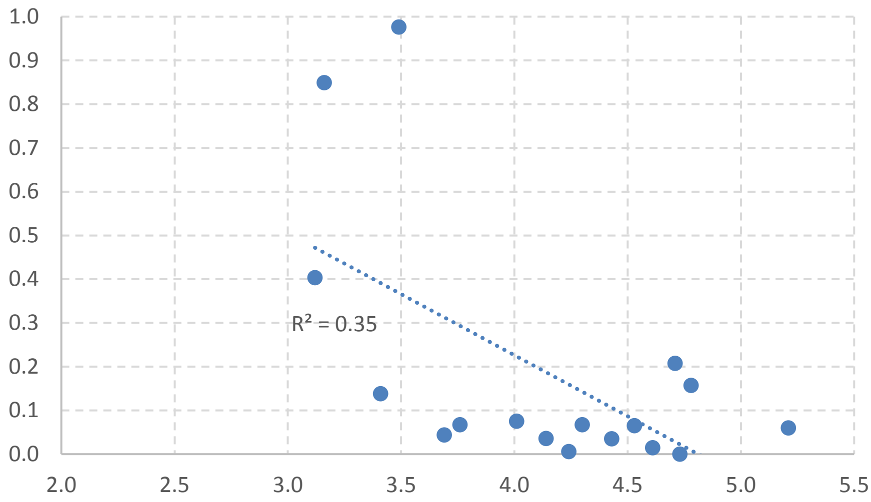


Sources: Own estimations with data from Eurostat and Gandelman and Hernández-Murillo (2014) Composite Indicator of Systemic Stress (CISS) (Holló et al (2012)).

Evidently, the risk premia could also depend on other factors. Nonetheless, our statistics are indicative of the importance played by the level of financial development in determining the magnitude of the (in)deflation risk premium. This is in line with our model insofar as financial development is captured by the number of Arrow-Debreu securities with respect to the total number of states of nature and the actuarially fair pricing assumption made.

Absolute values of the average risk premia and financial development indices for a set of economies

Figure 2



Notes: the average risk premia and financial development indices variables are represented on the y-axis and x-axis, respectively.

Sources: Own estimations with data from Eurostat, and Gandelman and Hernández-Murillo (2014). The financial development index used is the World Economic Forum's Financial Development Index (2014).

6.2 Panel data regression estimates

In this subsection, we present the panel data regressions' estimates. We consider separate regressions, depending on the type of debt that is included as part of the explanatory variables, as well as on the series with which the (in)deflation risk premium has been constructed, for reasons previously explained. Thus, we estimate several versions of the data panel model (5) in which the type of debt is varied as follows: total government debt ($GD_{i,t}$), total domestic debt ($DD_{i,t}$) and total gross external debt ($ED_{i,t}$). Respectively, we replace each for $D_{i,t}$ in (5).[35]

Similarly, we use the indices ii) final consumption expenditure of households, (total); iii) final consumption expenditure of households; and iv) semi-durable goods, non-durable goods and services to construct the risk premia.

A few comments are in order. First, we limit our panel data estimation to the following eurozone economies, namely Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia and Spain.[36] As explained earlier, this is because the bulk of their debts are denominated in euros.[37]

[35] In our model, the agent has, in particular, to pay back debt in the second period. A possible interpretation is that a closer empirical analogue is debt service, a datum that is generally not available. Thus, the levels of debt are used as proxies. This is a reasonable approximation provided that we can think of the debt being paid as a perpetual security, ie $D = P/r$ where r is a constant interest rate and P the periodic payment.

[36] Latvia and Lithuania just recently joined the eurozone in 2014 and 2015, respectively. Thus, they have not been included in the panel regressions.

[37] See www.ecb.europa.eu.

Second, the coefficients we are mostly interested in are γ_6; which are those associated with the terms $(D_{i,t}/P_{i,t} - D_{i,t})^+$, as highlighted in grey in Table 6.

When there is deflation $P_{i,t} < 1$ then $D_{i,t}/P_{i,t} - D_{i,t} > 0$. Based on our model, we would expect to obtain a negative coefficient. In effect, in our model, with debt, an increase in deflation is associated with lower consumption and vice versa. Thus, one would expect the debt-deflation term to be associated with a coefficient such that it contributes *negatively* towards the premium. In tandem, an increase in the level of debt would increase the effects deflation would have on the premium.

We note that, in accordance with the model, six of the nine regressions have a negative and statistically significant coefficient and two have a negative coefficient (although not statistically insignificant). We only observe a positive and statistically significant coefficient in one case.

Third, other coefficients of interest are γ_7; associated with the terms $(D_{i,t}/P_{i,t} - D_{i,t})^-$. When there is inflation $P_{i,t} > 1$, then we would have $D_{i,t}/P_{i,t} - D_{i,t} < 0$. Based on our model, under debt, we would expect to observe negative coefficients for these terms. In effect, an increase in inflation is associated with more consumption and vice versa. Thus, the debt-inflation term should contribute *positively* towards the risk premium.

In addition, out of the three cases for which we obtain a statistically significant coefficient, all are negative. The other six coefficients are not statistically significant. In the same vein, out of the three cases for which the coefficient is statistically significant, we observe that their magnitudes are less than that of their counterpart coefficients associated with the term $(D_{i,t}/P_{i,t} - D_{i,t})^+$.

Note two features. First, a fewer number of coefficients associated with the debt-inflation terms discussed above are significant than that of the debt-deflation terms. Second, the latter have greater magnitudes. Moreover, consider the six cases that have statistically significant coefficients associated with the debt-deflation term and a consistent negative sign. For those cases, we test whether in each debt-deflation coefficient is equal to the corresponding debt-inflation one. We find that, in only one case, the test fails to reject the null hypothesis of equal coefficients. Both results point to the asymmetric costs between inflation and deflation.

In short, the coefficients' signs associated with the (in)deflation-debt terms are consistent with the consumption smoothing motive. Indeed, when holding debt, buying a nominal bond during deflationary episodes is in general favourable to consumption smoothing. This means that the debt holder does not need to be compensated, which decreases the premium. On the other hand, buying a nominal bond during inflationary episodes is in general unfavourable to consumption smoothing and, hence, the investor would need to be compensated, increasing the premium. Moreover, the differences in the coefficients' magnitudes are an implication of the credit constraint. In sum, during deflationary episodes, in the presence of debt, those that are more financially constrained happen to be the more adversely affected.

A comment about measurement errors is required. As known, measurement error biases the coefficients' estimates; ie the so-called attenuation bias.[38] Of course, macroeconomic variables probably also contain measurement errors. Thus, albeit we have used the rolling-over sample covariance, and have taken the moving average of

[38] The presence of measurement errors introduces biases in the estimated coefficients towards zero.

adjacent observations, as mentioned, the fact that our coefficients are statistically significant is encouraging.

Fourth, the five significant coefficients associated with the inflation term $(P_{i,t})^{+}$ have positive signs. Although the model is silent regarding their sign, we find it intuitive that the presence of inflation leads directly to a higher premium. Conversely, the six significant coefficients associated with the deflation term $(P_{i,t})^{-}$ are negative. Hence, the presence of deflation reduces the magnitude of the risk premium.

Fifth, the differences in output, when significant, have positive coefficients. This is in line with the interpretation that a greater output provides slack to our credit constraint, since its right-hand side would increase. However, rather than interpreting this result, we think of the output change as a control variable.

7. Final remarks

We have presented evidence suggesting that deflation, under the presence of debt, might lead to economic costs. Such costs result from the presence of incomplete markets and credit constraints. Of course, other factors might also play a role in determining those costs.

In addition, our empirical results are indicative of a pricing in of deflation risks in terms of consumption growth. Importantly, they point towards deflation being relatively more costly than inflation.

This brings us to a number of general points. First, we think that one should be concerned about the conditions under which deflation might bring about economic costs. In our simple model, changes in the price level distort wealth intertemporally and, in tandem, affect the (in)deflation risk compensation for a nominal bondholder. A more developed financial system seems to mitigate such costs.

Second, a strand of the literature has assessed the costs of (in)deflation in terms of its relationship to output. On a related note, the literature measuring the costs of business cycles has focused on doing so in terms of consumption (directly as in Lucas (1991) or indirectly as in Alvarez and Jermann (2004)).[39] Similarly, we think that the assessment of the possible costs of deflation should also explore the impact of deflation in terms of consumption, measured directly or indirectly, as we have done.[40] Naturally, this approach comes with specific challenges and data requirements.

Third, in the periods and in the economies that we considered, we documented that deflationary episodes had been brief. In this context, one cannot robustly quantify econometrically the potential costs of deflation. Nonetheless, as we underscored, a given economy might face the probability of a deflationary episode, possibly reflected in a negative risk premium, without actually undergoing one.

Fourth, from a historical perspective, one could argue that as financial markets have developed, the potential costs of deflation have generally diminished. This is not to say, however, that one can ignore the possibility that such costs could surge if a dislocation in financial markets took place. Such a dislocation would effectively

[39] Alvarez and Jermann (2004) use asset prices to measure the costs of business cycles.

[40] In fact, in the derivation of some monetary models, one substitutes consumption for output.

disrupt their development, as the Great Financial Crisis and the European debt crisis have illustrated.

Fifth, central bankers seem to exhibit a particular dislike of deflation and, in many of the economies we have considered, have made significant efforts to avoid a deflationary scenario. We think that their distaste for deflation is not unfounded.

Finally, traditional consumption-based asset pricing models are not as popular in empirical work as they used to be.[41] For example, Hansen and Singleton (1983) documented some of their limitations. One such limitation relates to the implied magnitude of excess asset returns. Yet, our focus has been on the costs of deflation relative to inflation. We could have also used, for example, a more general utility function, and an explicit model for the consumption and inflation processes.[42] We leave such analysis for future research.

41 By traditional we mean consumption models that use utility functions that are time-separable.

42 One could consider, for example, the use of an Epstein and Zin (1989) utility function. It would lead to additional variability in the stochastic discount factor (given that the counterpart term to $\beta\exp(-\gamma\Delta c_{t+1})$ would be more variable) and possibly to a greater risk premium.

Panel regression estimates

Table 6

VARIABLES	Model 1 RP ii)	Model 2 RP ii)	Model 3 RP ii)	Model 4 RP iii)	Model 5 RP iii)	Model 6 RP iii)	Model 7 RP iv)	Model 8 RP iv)	Model 9 RP iv)
Risk Premium$_t$	0.616***	0.592***	0.574***	0.589***	0.509***	0.538***	0.593***	0.577***	0.566***
	(0.0327)	(0.0329)	(0.0420)	(0.0313)	(0.0295)	(0.0372)	(0.0317)	(0.0310)	(0.0374)
Δ GDP$_t$	0.0136	-0.0366	0.0449	0.0991***	0.109***	0.202***	0.0271	-0.0452	0.0586*
	(0.0224)	(0.0342)	(0.0348)	(0.0375)	(0.0335)	(0.0531)	(0.0227)	(0.0294)	(0.0306)
P^+	5.899*	2.684	15.35***	5.099	-0.810	10.47**	5.613*	1.870	8.361*
	(3.050)	(2.974)	(5.547)	(3.191)	(3.450)	(4.740)	(3.162)	(2.832)	(4.630)
P^-	-48.21**	-34.72	-77.53**	-90.96***	5.929	-163.1***	-41.43**	-24.08	-45.89***
	(24.37)	(25.91)	(33.70)	(25.58)	(29.63)	(30.98)	(16.63)	(15.45)	(16.60)
ED_t-ED_{t-1}	5.86e-05**			-1.09e-05**			-6.64e-06		
	(2.82e-05)			(5.33e-06)			(2.87e-05)		
$(ED_t /P_t – ED_t)^-$	-0.000345			2.44e-06			-0.000301		
	(0.000676)			(3.15e-05)			(0.000691)		
$(ED_t /P_t – ED_t)^+$	-0.00795**			-0.000377*			-0.00221		
	(0.00375)			(0.000194)			(0.00304)		
GD_t-GD_{t-1}		0.000972*			-0.000154*			0.00109**	
		(0.000543)			(9.16e-05)			(0.000481)	
$(GD_t /P_t – GD_t)^-$		-0.0246**			-0.00105**			-0.0292***	
		(0.0110)			(0.000487)			(0.0104)	
$(GD_t /P_t – GD_t)^+$		-0.0780**			0.00493**			-0.0831***	
		(0.0393)			(0.00243)			(0.0314)	
DD_t-DD_{t-1}			4.25e-05			-9.58e-07			-3.36e-05
			(3.88e-05)			(3.88e-06)			(2.59e-05)
$(DD_t /P_t – DD_t)^-$			-0.000561			3.29e-05			-0.000619
			(0.000914)			(2.61e-05)			(0.000662)
$(DD_t /P_t – DD_t)^+$			-0.00849*			-0.000492***			-0.00186
			(0.00481)			(0.000119)			(0.00197)
Constant	42.14*	31.94	61.81*	85.67***	-5.331	152.3***	35.63**	22.09	37.28**
	(23.68)	(25.26)	(32.07)	(24.93)	(29.07)	(29.96)	(16.06)	(14.99)	(15.56)
Number of observations	619	651	415	725	777	539	703	754	518
R^2	0.411	0.390	0.382	0.391	0.365	0.384	0.375	0.376	0.351
Number of Economies	15	13	11	17	15	13	17	15	13

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 Sample periods: Q1 2001 to Q3 2014 or Q4 2014, depending on the economy. Debt levels are estimated based on their ratios over GDP, GDP growth and setting GDP Q1 2001 equal to 100.

Sources: Own estimations with data from Eurostat, Gandelman and Hernández-Murillo (2014) and Haver Analytics.

Appendices: the (in)deflation risk-premium under a normal distribution

Sometimes it is useful to obtain an exact expression for the risk premium. This is possible if one is willing to make an assumption about the distribution of consumption growth and inflation. To see this, consider equation (3) in the main text, which we reproduce here for convenience:

$$\beta cov_t[\exp(-\gamma\Delta c_{t+1}), \exp(-\pi_{t+1})] + \exp(-r_t)\, E_t[\exp(-\pi_{t+1})] = \exp(-i_t)$$

Assuming that inflation and consumption growth have a joint conditional normal distribution with parameters (μ, Σ), and applying Stein's Lemma twice, we obtain the following expression for the left side of the equation:[43]

$$\beta\, E_t[\gamma \exp(-\gamma\Delta c_{t+1})] E_t[\exp(-\pi_{t+1})] cov_t[\gamma\Delta c_{t+1}, \pi_{t+1}] + \exp(-r_t)\, E_t[\exp(-\pi_{t+1})]$$

Hence, the exact expression for the risk premium is given by:

$$-\beta\gamma E_t[\gamma \exp(-\gamma\Delta c_{t+1})] E_t[\exp(-\pi_{t+1})] cov_t[\Delta c_{t+1}, \pi_{t+1}]$$

Thus, to the extent to which the normality assumption holds, the premium depends on the expected values of consumption growth and inflation, relative to their variances, and on the relative risk aversion and subjective discount coefficients.

Eurostat abbreviations

EU-15: Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, United Kingdom, Austria, Finland and Sweden.

EU-27: EU-15 and Bulgaria, Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia and Slovenia.

EU-28: EU-27 and Croatia.

[43] Charles Stein's Lemma states that if X and Y are two jointly normal distributed variables, then $cov(f(X), Y) = E\big(f'(X)\big) cov(X, Y)$, provided that f is such that all moments exist, i.e., they are finite.

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