Fitting the US term structure at the Federal Reserve Board

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1. Some notes on the US bond market

Treasury bills are discount securities with maturities of three and six months and one year. The three- and six-month bills are auctioned weekly and the year bill is auctioned monthly.

Treasury notes and *Treasury bonds* are coupon securities issued with various original terms to maturity in various auction cycles. Currently, the Treasury is issuing coupon securities with maturities of two, five, 10 and 30 years. The two-year note is auctioned monthly; the five-, 10- and 30-year notes are auctioned quarterly.

Reopenings. Not all auctions result in a new security. For bills, three-month bill auctions are always reopenings of outstanding six-month bills, and every fourth six-month bill auction is the reopening of an outstanding year bill. Coupon bonds are occasionally reopened. This occurs most often for the 30-year bond, but also for the 10-year note.

The bond price quotes are for *regular delivery*, for which the *settlement date* is the next business day ($t+1$). (Note that the yields on the FRBNY quote sheets are skip-day yields ($t+2$) computed from regular-delivery price quotes.)

Accrued interest. Notes and bonds pay coupons semiannually. Accrued interest is calculated as follows:

$$a = \left(\frac{c}{2}\right)\left(\frac{d}{b}\right) \tag{1}$$

where a is the accrued interest *per dollar* of face value,

b is the number of days in the coupon period (the basis),

c is the annual coupon rate, and

d is the number of days from issue or last coupon to settlement.

Tax treatment. Coupon payments and Treasury bill capital gains are subject to income tax as ordinary income. Capital gains on coupon securities are subject to income tax as capital gains, which have often been taxed at a lower rate than ordinary income.

Special features. In the past, the Treasury has issued notes and bonds with special features.

- Callable bonds are typically callable at par in the last five years of their life.
- Flower bonds are low-coupon bonds (typically) that can be redeemed at par for the payment of estate taxes.

Repo market. Government securities dealers finance their positions in the repurchase agreement market. They borrow funds (to finance their long positions) using the securities they own outright as collateral (so-called repos), and they take in securities (to deliver on their short positions) as collateral on loans they make (so-called reverses). When the aggregate short positions are large, dealers that are short may have to pay a premium to those who have possession of the collateral to acquire the specific collateral needed. The premium is paid by lending funds at less than the risk-free rate, in which case the security is said to be *on special* in the repo market. The (expected) difference between

[1] The views expressed herein are the author's and do not necessarily reflect those of the Board of Governors of the Federal Reserve System.

the risk-free rate and the special rate (over the life of the security) is capitalised into the security's price, pushing its yield below the yield curve.

The STRIPs (Separate Trading of Registered Interest and Principal) market provides direct observations on zero coupon securities at quarterly intervals out to 30 years. Ten-year notes and 30-year bonds are eligible for stripping (as well as some older 20-year bonds). Market participants strip (decompose) whole coupon bonds into a series of individual *coupon strips* and a final *principal strip*. While coupon strips with the same maturity date are fungible, principal strips are not - the coupon bond cannot be reconstituted without the principal strip. As a result, *principal* strips often trade at a premium relative to coupon strips.

When-issued trading is a forward market for a security that has yet to be issued. It starts when the Treasury announces the amount to be auctioned (about one week before the auction) and continues until the security is issued (about one week after the auction).

Futures market. There are a number of futures contracts for which various Treasury securities are deliverable. At times, the demand for an individual Treasury security may be affected by its deliverability.

2. Fitting the term structure

We have described in detail our technique for fitting the term structure in Fisher et al (1995). Here is a very brief summary:

- We fit a cubic spline to the forward rate curve.
- We minimise the weighted sum of two terms:
  - the sum of squared deviations of the fitted bond prices from the observed bond prices, and
  - a penalty for non-linearity - the integral of the squared second derivative of the forward rate curve.
- The weighting of the two terms is determined by minimising the generalised cross-validation (GCV) criterion:
  - the ratio of (i) a quasi-out-of-sample goodness-of-fit measure to (ii) the effective number of parameters.
- This can be thought of as a signal extraction technique.
  - The greater the forward rate signal in the bond prices, the more the forward rate curve may deviate from linearity.
  - The "amount" of signal extracted can be controlled by adjusting the trade-off between the two factors in the GCV criterion ratio.

Treasury bills vs notes and bonds. Currently, we fit separate yield curves for bills and coupon securities. We do not consider this to be an entirely satisfactory situation. The practice began because we were fitting the yield curve to measure deviations from the yield curve largely for coupon securities, and we noticed that coupon securities with less than one year to maturity were priced measurably differently from bills. This difference may be driven by liquidity, taxes, or other effects. We have estimated both curves daily starting in December 1987.

Coupon curve:

- We make no adjustments for tax effects.
- We exclude:
  (a) Treasury bills.
  (b) Notes and bonds with less than 30 days to maturity. We found that the prices of coupon securities very near maturity did not behave well.

(c) Callable bonds and flower bonds. The price of these securities reflects special features not captured by the discount function. As a result, we currently have no bonds in the estimation with maturities in the range of 10 to 18 years.

(d) The two most recently issued securities of each original term to maturity. We exclude these securities even though they are the most actively traded. The reason is that these securities are most likely to be on special in the repo market. Thus the price of these securities probably reflects special features not captured by the discount function.

Bill curve. There are three differences in the settings we use for the bill curve relative to the coupon curve.

(a) No bills are dropped. (There are about 32 bills outstanding each day.)

(b) We weight the observations by the inverse of the maturity.

(c) The GCV ratio has been adjusted to extract more signal.

Bills and coupons. We have a project under way to use the CRSP[2] daily bond file to estimate a curve daily starting in June 1961 that incorporates both bills and coupon securities. Given the limited number of long-term securities outstanding in the 1960s and 1970s, this curve will rely to some extent on callable bonds and flower bonds.

Linearised Nelson-Siegel. Prior to developing the smoothing spline approach to fitting the term structure using market prices for bonds, a simpler approach was developed, which could be applied directly to the *constant maturity yields*. This technique uses a linear approximation to the relationship between yields and prices.

STRIPs quotes. In principle, the STRIPs market is an excellent source of information. Currently, we have only limited historical data and no current data.

3. Modelling the term premia

It has been well documented that the expectations hypothesis does not hold: forward rates do not equal expected future spot rates. One reflection of the failure of the expectations hypothesis is that the slope of the yield curve is not an unbiased predictor of future changes in yields.[3] This failure results from the presence of time-varying term premia. Therefore, to extract the path of the expected short rate implicit in the term structure, we need to model these random term premia.

The approach we have undertaken is to fit multifactor models of the term structure that are based on absence-of-arbitrage conditions. (These models should not be confused with the so-called arbitrage-free models that exactly match a given term structure by fitting, for example, a deterministic time trend.) There is a class of numerically tractable models (the exponential-affine class, which includes the multifactor Cox, Ingersoll and Ross model, among many others) that has the potential to capture many of the stylised facts of the term structure, including the Campbell-Shiller regression results.

For this project, we take the zero coupon rates estimated with our smoothing spline techniques as "data". Thus, it is of prime importance that the cross-sectional and time series properties of the zero coupon rates not be obscured by the method used to obtain them. In particular, since intuition regarding the shape of the yield curve that relies on the expectations hypothesis may well be quite wrong, it is important not to impose an overly restrictive functional form that may obscure important relationships.

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[2] Center for Research in Security Prices.

[3] See, for example, the regression results in Campbell and Shiller (1991).

References

Campbell, J and R Shiller (1991): "Yield spreads and interest rate movements: a bird's eye view", *Review of Economic Studies*, 58, pp 495-514.

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