Yield curve estimation at the Bank of England

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The Bank of England estimates yield curves for the United Kingdom on a daily basis. Three basic curves are produced: a nominal and a real government liability curve and a nominal commercial bank liability curve. These curves are published daily on the Bank's website (www.bankofengland.co.uk). This note provides a brief description of the curves we produce and the methods used to derive them.

Government liability curve

The government liability curve is based on yields on UK government bonds (gilts) and yields in the general collateral repo market. The nominal yield curves are derived from UK gilt prices and General Collateral (GC) repo rates. The real yield curves are derived from UK index-linked bond prices. Using the Fisher relationship,[2] we are also able to estimate a term structure of inflation expectations for the United Kingdom.

Estimates for the nominal curve are available from 2 January 1979. Estimates for the real yield curve and implied inflation term structure are available from 2 January 1985. Depending on the range of available bonds, we aim to publish estimates of both the spot rate and the instantaneous forward rate out to a maturity of about 25 to 30 years.

GC repo rates are used to estimate the nominal curve down to a maturity of one week. By generating synthetic zero coupon bonds from the GC repo rates, we improve the estimation of the short end of the nominal curve, where gilts tend to be less liquid.

Commercial bank liability curve

The commercial bank liability curve[3] is based on sterling interbank rates (Libor) and on yields on instruments linked to Libor, specifically short sterling futures, forward rate agreements and Libor-based interest rate swaps. These commercial bank liability curves are nominal only.

Estimates of the commercial bank liability curve are available out to a maturity of 10 years from November 1990 to July 1997, out to a maturity of 15 years from July 1997, and out to a maturity of at least 25 years from January 1999.

Estimation

The technique currently used by the Bank to estimate its yield curves is the variable roughness penalty (VRP) method, which replaced the approach by Svensson.[4] The VRP methodology is based on the

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[2] The Fisher relationship means that an implied inflation term structure can be calculated as the difference between nominal and real yields.

[3] For further details, see Brooke et al (2000).

[4] Estimates of the nominal yield curve were first published by the Bank in the November 1999 *Inflation Report* and are discussed in an article by Anderson and Sleath in the November 1999 *Bank of England Quarterly Bulletin*.

spline-based technique proposed by Waggoner (1997). This method was considered superior to the Svensson method and other estimation techniques, based upon the criteria of smoothness, flexibility and stability.[5] An important innovation of this technique is that the degree of smoothing is a function of maturity, and in particular that the curve is more flexible at the short end (where the curve is likely to exhibit the greatest curvature) than at the long end (where expectations are likely to be more smooth).

The spline-based technique models forward rates as a piecewise cubic polynomial, with the segments joined at "knot points". The coefficients of the individual polynomials are restricted so that both the curve and its first derivatives are continuous at all maturities, and knot points are placed at the time of maturity of each bond. Although the cubic spline is more flexible than parametric forms, an unconstrained spline would be far too flexible to generate the type of yield curves necessary for the purpose of monetary policy. In order to control the trade-off between smoothness of the curve and goodness of fit, a roughness penalty is included to penalise excessive curvature of the forward curve. The size of this roughness penalty is determined by a time-invariant function, $\lambda(m)$, which varies with horizon m.

Formally, the VRP method minimises the objective function X_s:

$$X_s = X_p + \int_0^M \lambda_t(m)[f''(m)]^2 dm \qquad (1)$$

where*:*

$$X_p = \sum_{i=1}^{N} \left[\frac{P_i - \Pi_i(\beta)}{D_i} \right]^2$$

P_i is the observed price of the i-th bond, D_i is its modified duration, $\Pi_i(\beta)$ is the fitted price, m is the maturity, $f''(m)$ is the second derivative of the fitted forward curve, M is the maturity of the longest bond and β is the vector of spline parameters.

From the above objective function, it can be seen that the VRP technique minimises the sum of the squared bond price residuals (X_p)[6] subject to a penalty for curvature. In addition, the bond prices are weighted according to the inverse modified duration.

The optimisation procedure for the VRP technique has two steps. First, the parameters of the smoothing function, $\lambda(m)$, are optimised and then, holding these parameters constant, we estimate the spline parameters on a daily basis.

The parameters of the smoothing function were chosen to maximise out-of-sample goodness of fit[7] over the period 1 May 1996 to 31 December 1998. The function, $\lambda(m)$, is defined to be a continuous function of three parameters:

$$\log \lambda(m) = L - (L - S) \exp\left(-\frac{m}{\mu}\right) \qquad (2)$$

with L, S and μ the parameters maximised over the sample period.

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[5] A more in-depth comparison of the VRP method with other techniques can be found in Anderson and Sleath (2001).

[6] Minimising the squared bond price residuals weighted by duration is a first-order approximation to minimising yields. Proceeding in this way has the advantage of not needing to numerically calculate yields at each step of the optimisation.

[7] Following Waggoner, the main criterion for choosing the parameters was to maximise the out-of-sample goodness of fit averaged over the sample period. It was found that many combinations of these parameters gave similar measures of goodness of fit. We therefore opted for the set of parameters that led to the highest level of smoothing among these combinations.

References

Anderson, N and J Sleath (1999): “New estimates of the UK real and nominal yield curves”, Bank of *England Quarterly Bulletin*, November, pp 384-92.

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Brooke, M, N Cooper and C Scholtes (2000): “Inferring market interest rate expectations from money market rates”, *Bank of England Quarterly Bulletin*, November, pp 392-402.

Waggoner, D (1997): “Spline methods for extracting interest rate curves from coupon bond prices”, Federal Reserve Bank of Atlanta, *Working Paper series*, 97-10.

In addition to these references, a note entitled “Notes on the Bank of England yield curves” is available from our website (www.bankofengland.co.uk). This note accompanies the estimates we provide and is aimed at explaining the terminology and concepts behind our yield curves.