The estimation of forward interest rates and zero coupon yields at the Riksbank

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1. Overview

Since 2001 the forward interest rates reported in the inflation report have been calculated by means of the smoothing splines method. Earlier forward interest rates were calculated by the extended Nelson-Siegel method (or the Svensson method) and we continue to calculate Nelson Siegel parameters for the sake of continuity and to have a back-up method. Moreover, the extended Nelson Siegel model has also been used in macroeconomic studies. The quantitative difference between the extended Nelson Siegel and the smoothing splines are normally very small. Recently parameters of the original Nelson Siegel (1987) model have been estimated on monthly basis (from 1996 and onwards) in order to do analysis along the lines of Diebold and Li (2003) and Diebold, Rudebusch, and Auroba (2003).

2. Data

The data we use are benchmark government bonds from 2-10 years. In the short end, T-Bills with maturities (closest to) 3, 6, 9, and 12 months are used, in addition to the repo rate.

3. Description of the methods

3.1 Smoothing splines

When fitting the zero coupon bond (ZCB) curve, we do so in yield-space, that is we minimize the weighted sum of two terms: the squared deviations of the fitted yields from the quoted yields, and a penalty for roughness, which is the integral of the squared second derivatives of the ZCB curve. The weight on the penalty term is determined by the variable roughness penalty (VRP) method, described in Waggoner (1997). It follows the original method in Fisher, Nychka, and Zervos (1995) closely, but with different penalty weights for the shorter yield maturities in order to allow a more flexible specification of the short end of the curve. The penalty weights in the different segments are determined once and for all by a trial-and-error approach. Occasionally, we reinvestigate the choice of optimal penalty weights. With the ZCB curve at hand, we then calculate the forward curve.

3.2 Extended Nelson Siegel

The chosen objective in this estimation procedure is to minimise the sum of squared yield errors. The functional form for the forward rate curve at date t to be estimated is:

$$f(t;s) = \beta_0 + \left(\beta_1 + \beta_2 \frac{s}{\tau_1}\right) \exp\left(-\frac{s}{\tau_1}\right) + \beta_3 \frac{s}{\tau_2} \exp\left(-\frac{s}{\tau_2}\right) \quad (1)$$

where *s* is time to settlement.

In the estimations, we impose the restriction that $\beta_0 + \beta_1$ is equal to the official repo rate, in order to facilitate the interpretation of implied forward rates as expected future repo rates. The original Nelson Siegel model is obtained by the restriction $\beta_3 = 0$.

4. Reporting routines to BIS

The Riksbank reports the daily data on the six parameters in the extended Nelson Siegel model once a week (Mondays). The data set from which these parameters are estimated normally consists of 9 benchmark bonds and 4 treasury bills (see paragraph 2). The yields used in the computations are the average yields from bid and ask yields (close yields).

References

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