A technical note on the Merrill Lynch Exponential Spline model as applied to the Canadian term structure

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The purpose of this note is to describe the methodology used by the Bank of Canada to construct the Government of Canada yield curve. We generate zero coupon curves daily, for maturities from 0.25 to 30.00 years, by applying an estimation method based on the Merrill Lynch Exponential Spline (MLES) model to a selection of Government of Canada Treasury bill and bond prices.[2]

1. Data

The two fundamental types of Canadian dollar-denominated marketable securities issued by the government of Canada are Treasury bills and Canada bonds. Treasury bills, which do not pay periodic interest but rather are issued at a discount and mature at their par value, are currently issued at three-, six- and 12-month maturities. Government of Canada bonds pay a fixed semi-annual interest rate and have a fixed maturity date. Issuance involves maturities across the yield curve with original terms of maturity at issuance of two, five, 10 and 30 years.[3] Each issue is reopened several times to improve liquidity and achieve "benchmark status".[4] Canada bonds are currently issued on a quarterly "competitive yield" auction rotation with each maturity typically auctioned once per quarter.[5] In the interests of promoting liquidity, Canada has set targets for the total amount of issuance to achieve "benchmark status"; currently, these targets are CAD 7 billion to 10 billion for each maturity.

2. Data filtering

Our goal is to select only those bonds that are indicative of the current market yields. As a result, we use a system of filters to omit bonds which create distortions in the estimation of the yield curve.

- To avoid potential price distortions when large deviations from par exist, bonds that trade at a premium or a discount of more than 500 basis points from their coupon are excluded.[6]
- Bonds with less than CAD 500 million outstanding are excluded in order to include only those bonds with the requisite degree of liquidity. This amount was chosen in a fairly arbitrary manner to ensure a reasonable number of bonds in the sample.
- Canada benchmark bonds are the most actively traded Canada bonds in the marketplace and it is thus essential that the information contained in these bonds be incorporated into the yield curve.[7]

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[2] For a more detailed description of the approach, see Bolder and Gusba (2002), or Bolder, Johnson and Metzler (2004).

[3] Canada eliminated three-year bond issues in early 1997; the final three-year issue was 15 January.

[4] A "benchmark" bond is analogous to an "on-the-run" US Treasury security in that it is the most actively traded security for a given maturity.

[5] Government of Canada bond yields are quoted on an actual/actual day count basis net of accrued interest. The accrued interest, however, is by market convention calculated on an actual/365 day count basis.

[6] This value of 500 basis points is intended to reflect a threshold at which the tax effect of a discount or premium is not believed to have an economic impact.

- Additional "discretionary" filtering of bonds is possible. It should be noted, however, that the inclusion or exclusion of a bond is based on judgment and would occur after investigating the underlying reason for a problematic (or unusual) bond quote.

3. The model

The Bank of Canada uses the Merrill Lynch Exponential Spline (MLES) model, developed by Li et al.[8] The MLES model is a parametric model which specifies a functional form for the discount function, $d(t)$, as

$$d(t) = \sum_{k=1}^{9} z_k e^{-k\alpha t} \tag{1}$$

where z_k (k = 1,...,9) and α are the parameters to be estimated.

Once a functional form for the discount function has been specified, a zero coupon interest rate function is derived. The zero coupon curve, z(t), is given by

$$z(t) = -(\ln(d(t))/t \tag{2}$$

4. The estimation

The basic process of determining the optimal parameters for the discount function which best fits the bond data is outlined as follows:

- The sample of Government of Canada bond and Treasury bills is selected and the timing and magnitude of their cashflows are determined.
- The estimation involves 9 linear parameters (the z_k), and one non-linear parameter (α). The optimization normally takes less than one minute to complete. Maximum likelihood estimation is used. As a result of the fact that the discount function is a linear function of 9 of the 10 parameters, the majority of the maximum likelihood computations can be carried out as matrix multiplications, which are computationally efficient.
- Price residuals are calculated using theoretical Government of Canada security prices and the actual price data and inversely weighted by (modified) duration. The calculation of estimated prices is straightforward as the discount function permits us to discount any cashflow occurring throughout the term to maturity spectrum. The weighting on the i-th bond (w_i) is given as follows:

  $$w_i = 1/D_i$$

  where D_i is the (modified) duration of the i-th bond.[9]

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7 As previously discussed, the new issues may require two or more reopenings to attain "benchmark status". As a result, the decision as to whether or not a bond is a benchmark is occasionally a matter of judgment.

8 Li et al (2001)

9 This is consistent with the Bliss approach. For a complete explanation of the justification for weighting price errors, see Bliss (1996).

References

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