Zero-coupon yield curves estimated by central banks

Introduction

Following a meeting on the estimation of zero-coupon yield curves held at the BIS in June 1996, participating central banks have since been reporting their estimates to the Bank for International Settlements. The BIS Data Bank Services provide access to these data, which consist of either spot rates for selected terms to maturity or represent estimated parameters from which spot and forward rates can be derived. In the case estimated parameters are reported, the Data Bank Services provides, in addition to the parameters also the generated spot rates.

The purpose of this document is to facilitate the use of these data. It provides information on the reporting central banks' approaches to the estimation of the zero-coupon yield curves and the data transmitted to the BIS Data Bank. In most cases, the contributing central banks adopted the so-called Nelson and Siegel approach or the Svensson extension thereof. A brief overview of the relevant estimation techniques and the associated mathematics is provided below. General issues concerning the estimation of yield curves are discussed in Section 1. Sections 2 and 3 document the term structure of interest rate data available from the BIS. The final section provides examples of estimated parameter and selected spot and forward rates derived thereof. A list of contacts at central banks can be found after the references. The remainder of this document consists of brief notes provided by the reporting central banks on approaches they have taken to estimate the yield curves.

Since the last release of this manual in March 1999 there have been four major changes: Switzerland started to report their estimates of the yield curve to the BIS in August 2002. Furthermore, Sweden began to use a new estimation method in 2001, the United Kingdom since September 2002 and Canada since January 2005. These changes are included in Tables 1 and 2.

1. Zero-coupon yield curve estimation techniques

The estimation of a zero-coupon yield curve is based on an assumed functional relationship between either par yields, spot rates, forward rates or discount factors on the one hand and maturities on the other. Discount factors are the quantities used at a given point in time to obtain the present value of future cash flows. A discount function $d_{t,m}$ is the collection of discount factors at time t for all maturities m. Spot rates $s_{t,m}$, the yields earned on bonds which pay no coupon, are related to discount factors according to:

$$d_{t,m} = \exp(-s_{t,m}\ m) \text{ and } s_{t,m} = -\frac{1}{m}\log d_{t,m} \qquad (1)$$

Because spot interest rates depend on the time horizon, it is natural to define the forward rates $f_{t,m}$ as the instantaneous rates which, when compounded continuously up to the time to maturity, yield the spot rates (instantaneous forward rates are, thus, rates for which the difference between settlement time and maturity time approaches zero):

$$s_{t,m} = -\frac{1}{m}\int_0^m f(u)du \qquad (2)$$

or, equivalently:

$$d_{t,m} = \exp\left[-\int_0^m f(u)du\right] \qquad (3)$$

These relations can be inverted to express forward rates directly as a function of discount factors or spot rates:

$$f_{t,m} = s_{t,m} + m\dot{s}_{t,m} \text{ and } f_{t,m} = -\frac{\dot{d}_{t,m}}{d_{t,m}} \quad (4)$$

where dots stand for derivatives with respect to time to maturity.

However, the general absence of available pure discount bonds that can be used to compute zero-coupon interest rates presents a problem to practitioners. In other words, zero coupon rates are rarely directly observable in financial markets. Attempting to extract zero-coupon rates from the prices of those risk-free coupon-bearing instruments which are observable, namely government bonds, various models and numerical techniques have been developed. Such models can broadly be categorised into ***parametric*** and ***spline-based approaches***, where a different trade-off between the flexibility to represent shapes generally associated with the yield curve (*goodness-of-fit*) and the *smoothness* characterizes the different approaches. These main modelling approaches are now briefly discussed below.

Parametric Models

The underlying principle of parametric models, also referred to as function-based models, is the specification of a single-piece function that is defined over the entire maturity domain. Whilst the various approaches in this class of models advocate different choices of this function, they all share the general approach that the model parameters are determined through the minimisation of the squared deviations of theoretical prices from observed prices.

The Nelson and Siegel method

The method developed by Nelson and Siegel (1987) attempts to estimate these relationships by fitting for a point in time t a discount function to bond price data by assuming explicitly the following function form for the instantaneous forward rates:

$$f_{t,m} = \beta_{t,0} + \beta_{t,1}\exp\left(\frac{-m}{\tau_{t,1}}\right) + \beta_{t,2}\frac{m}{\tau_{t,1}}\exp\left(\frac{-m}{\tau_{t,1}}\right) \quad (5)$$

In this equation m denotes time to maturity, t the time index and $\beta_{t,0}$, $\beta_{t,1}$, $\beta_{t,2}$ and $\tau_{t,1}$ are parameters to be estimated.[1] The zero-coupon or spot interest rate curve s_m can be derived by integrating the forward rate curve:

$$s_m = \beta_0 + \beta_1\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} + \beta_2\left(\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right) \quad (6)$$

which is equivalent to:

$$s_m = \beta_0 + (\beta_1 + \beta_2)\frac{\tau_1}{m}\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \beta_2\exp\left(-\frac{m}{\tau_1}\right) \quad (7)$$

For long maturities, spot and forward rates approach asymptotically the value β_0 which must be positive. ($\beta_0 + \beta_1$) determines the starting value of the curve at maturity zero; β_1 thus represents the deviation from the asymptote β_0. In addition, ($\beta_0 + \beta_1$) must also be positive. The remaining two parameters β_2 and τ_1 are responsible for the "hump". The hump's magnitude is given by the absolute size of β_2 while its direction is given by the sign: a negative sign indicates a U-shape and a positive sign a hump. τ_1, which again must be positive, determines the position of the hump.

[1] To simplify the notation, the time index t is dropped below.

The Svensson method

To improve the flexibility of the curves and the fit, Svensson (1994) extended Nelson and Siegel's function by adding a further term that allows for a second "hump". The extra precision is achieved at the cost of adding two more parameters, β_3 and τ_2, which have to be estimated. The instantaneous forward rates curve thus becomes:

$$f_m = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} \exp\left(\frac{-m}{\tau_2}\right) \tag{8}$$

with β_3 and τ_2 having the same characteristics as β_2 and τ_1 discussed above. Again, to derive the spot rates curve the instantaneous forward rates curve is integrated:

$$\begin{aligned} s_m = \beta_0 &+ \beta_1 \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(-\frac{m}{\tau_1}\right)^{-1} + \beta_2 \left(\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right)\left(\frac{m}{\tau_1}\right)^{-1} - \exp\left(-\frac{m}{\tau_1}\right)\right) \\ &+ \beta_3 \left(\left(1 - \exp\left(-\frac{m}{\tau_2}\right)\right)\left(\frac{m}{\tau_2}\right)^{-1} - \exp\left(-\frac{m}{\tau_2}\right)\right) \end{aligned} \tag{9}$$

For zero-coupon bonds, spot rates can be derived directly from observed prices. For coupon-bearing bonds usually their "yield to maturity" or "par yield" only is quoted. The yield to maturity is its internal rate of return, that is the constant interest rate r_k that sets its present value equal to its price:

$$P_k = \sum_{i=1}^{n} \frac{CF_i}{(1 + r_k)^{t_i}} \tag{10}$$

where P_k is the price of bond k which generates n cash-flows CF at periods t_j (i = 1, 2, .. , n). These cash flows consist of the coupon payments and the final repayment of the principal or face value. Yields to maturity on coupon bonds of the same maturity with different coupon payments are not identical. In particular, the yield to maturity on a coupon-bearing bond differs from the yield to maturity - or spot rate - of a zero-coupon bond of the same maturity. Nevertheless, if the cash flow structure of a bond trading at the market ("at par") is known, it is possible to derive from estimated spot rates uniquely the coupon bond's theoretical yield to maturity, ie the rate the bond would require in order to trade at its face value ("at par"). Drawing on the spot rates $s_{t,m}$, the price equation can be expressed as:

$$P_k = \frac{C}{(1 + s_{t,1})} + \frac{C}{(1 + s_{t,2})^2} + \ldots + \frac{C}{(1 + s_{t,m})^m} + \frac{V}{(1 + s_{t,m})^m} \tag{11}$$

where C represents the coupon payments and V the repayment of the principal. The yield to maturity of a coupon-bearing bond is therefore an average of the spot rates which, in general, varies with the term to maturity.

To derive the term structure of interest rates, the discount function is estimated by applying a (constrained) non-linear optimisation procedure to data observed on a trade day. More important than the choice of a particular optimisation method (eg maximum likelihood, non-linear least squares, generalised method of moments) is the decision whether the (sum of squared) yield or price errors should be minimised. If one is primarily interested in interest rates, it suggests itself to minimise the deviation between estimated and observed yields. In this case the estimation proceeds in two stages: first, the discount function $d_{t,m}$ is used to compute estimated prices and, secondly, estimated yields to maturity are calculated by solving the following equation for each coupon-bearing bond k:

$$P_k = \sum_{i=1}^{m} C \exp(-r_k i) + V \exp(-r_k m) \tag{12}$$

At both stages, the starting point is from pre-selected values for the relevant parameters and to run through an iterative process until convergence is achieved. It is computationally easier to minimise price errors than yield errors, as this only requires finding a solution for the first stage. Unfortunately, minimising price errors can lead to large yield errors for financial instruments with relatively short remaining term to maturity. Considering how yield, price and term to maturity of a bond are related, it is not surprising to observe this heteroscedasticity problem: drawing on the concept of duration,[2] the elasticity of the price with respect to one plus the yield is equal to the duration of the bond. A given change in the yield corresponds to a small/large change in the price of a bond with a short/long term to maturity or duration. Fitting prices to each bond, given an equal weight irrespective of its duration, leads to over-fitting of the long-term bond prices at the expense of the short-term prices. One approach to correct for this problem is to weight the price error of each bond by a value derived from the inverse of its duration.

Other factors can also contribute to fairly large yield errors at the short end of the term structure. For instance, the trading volume of a bond can decrease considerably when it approaches its maturity date. The quoted price for such a bond may not accurately reflect the price at which trading would take place. For such reasons it may be appropriate to exclude price data of bonds close to expiration when fitting term structures.

Spline-based Models

Rather than specifying a single functional form over the entire maturity range, spline-based methods fit the yield curve by relying on a piecewise polynomial, the spline function[3], where the individual segments are joined smoothly at the so-called knot points. Over a closed interval, a given continuous function can be approximated by selecting an arbitrary polynomial, where the goodness-of-fit increases with the order of the polynomial. Higher-order polynomials, however, quite frequently display insufficient smoothing properties. This problem can be avoided by relying on a piecewise polynomial whereby the higher-order polynomial is approximated by a sequence of lower-order polynomials.

Consequently, spline functions are generally based on lower-order polynomials (mostly quadratic or cubic). A cubic spline, for instance, is a piecewise cubic polynomial restricted at the knot points such that their levels and first two derivatives are identical. One parameter corresponds to each knot in the spline.

The "smoothing splines" method

This method developed by Fisher, Nychka and Zervos (1995) represents an extension of the more traditional cubic spline techniques (eg Vasicek and Fong (1982)). In the case of "smoothing splines" the number of parameters to be estimated is not fixed in advance. Instead, one starts from a model which is initially over-parameterised. Allowing for a large number of knot points guarantees sufficient flexibility for curvature throughout the spline. The optimal number of knot points is then determined by minimizing the ratio of a goodness-of-fit measure to the number of parameters. This approach penalizes for the presence of parameters which do not contribute significantly to the fit. It is not convenient to draw on the (varying number of) parameters in disseminating yield curve information.

There is a broad range of spline-based models which use this "smoothing method" pioneered by Fisher et al. The main difference among the various approaches simply lies in the extent to and fashion by which the smoothing criteria are applied to obtain a better fix. The "variable penalty roughness" (VRP) approach recently implemented by the Bank of England allows the "roughness" parameter to vary with the maturity, permitting more curvature at the short end.[4]

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2 Recall that the duration of a zero-coupon bond is equal to its maturity. Assuming a flat yield curve, the sensitivity of a zero-coupon bond to a change in the term structure should be directly proportional to its maturity. A change in the interest rate divided by one minus the interest rate of 1% corresponds to a change in the price of 1% of a bond with a maturity of one year and of 10% of a 10-year bond.

3 Spline functions, such as basis or B-splines, are used in the context of yield curve estimation. At times there exists some confusion among practitioners between spline functions and spline-based interpolation. While the former technique uses polynomials in order to approximate (unknown) functions, the latter is simply a specific method to interpolate between two data points.

4 See James and Webber (2000) for a comprehensive overview and comparison of the various approaches and Anderson and Sleath (1999) for a detailed description of the VRP approach.

Generally, the estimation method largely depends on intended use of data: no-arbitrage pricing and valuation of fixed-income and derivative instruments vs information extraction for investment analytical and monetary policy purposes. One of the main advantages of spline-base techniques over parametric forms, such as the Svensson method, is that, rather than specifying a single functional form to describe spot rates, they fit a curve to the data that is composed of many segments, with the constraint that the overall curve is continuous and smooth.[5]

2. Provision of information on the term structure of interest rates

The term structure of interest rates, defined as the functional relationship between term to maturity and the spot interest rate of zero-coupon bonds, consists of an infinite number of points. In many respects forward interest rates are more interesting than spot rates, as implied by the spot rate curve or vice versa, as the former can pertain information about expected future time paths of spot rates. At any point along the maturity spectrum there exists an infinite number of forward rates which differ in terms of their investment horizon. The *instantaneous* forward rate represents just a special case, the one for which the investment horizon approaches zero.

Published information on term structure of interest rates usually consists of selected spot rates at discrete points along the maturity spectrum. Occasionally, these spot rates are complemented by a selection of specific forward interest rates.

Such limitations would be mitigated if information on the term structure of interest rates could be presented in terms of algebraic expressions from which spot and forward rates can be derived. This is straightforward for parsimonious approaches such as Nelson and Siegel and Svensson discerned above, for which spot and instantaneous forward rates can be calculated using the estimated parameters (β's and τ's).

Further information can be useful in interpreting the curves such as statistics on the quality of the fit, details on the debt instruments used in the estimations, and if and what kind of efforts were made to prevent that specific premia, eg tax premia, distort estimation results. Some of this information can be found below and in the notes provided by the central banks.

Comparability of central banks' term structures of interest rates

To estimate the term structure of interest rates, most central banks reporting data have adopted either the Nelson and Siegel or the extended version suggested by Svensson. Exceptions are Canada, Japan, (in part) Sweden[6], the United Kingdom, and the United States which all apply variants of the "smoothing splines" method.

Government bonds data are used in the estimations since they carry no default risk. Occasionally, central banks complement this information by drawing on money market interest rates or swap rates. Clearly, financial markets differ considerably in terms of the number of securities actively traded and their turnover, the variety of financial instruments and specific institutional features. Such differences can give rise to a variety of premia which should be taken into consideration in the estimation process but in practice this is difficult to do.

Premia induced by tax regulations are notoriously difficult to deal with. One could attempt to remove tax-premia from the observed prices/yields before they are used in estimations. In other instances it may be preferable to simply exclude instruments with distorted prices/yields from the data set. In cases where it is expected that tax distortions have only a minor impact on the estimation results the best approach may be to ignore this problem altogether. Occasionally, central banks prefer modifying the estimation approach instead of adjusting the data to deal with specific problems (see Table 1).

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[5] For example, at the long end of the yield curve, the Svensson model is constrained to converge to a constant level, directly implying that the unbiased expectation hypothesis holds.

[6] Although Sveriges Riksbank adopted the "smoothing splines" method in 2001, they still only report their Svensson method estimates to the BIS Data Bank.

Not all central banks estimate the term structure for the full maturity spectrum for which debt instruments are available. Although this concerns both ends of the curves, the short-end is usually more difficult to deal with than the long-end. In modelling the short-end of the term structure, the approaches taken by monetary authorities differ considerably. On the data side this concerns mostly the choice of the types of short-term instruments regarded to be the most suitable and the minimum remaining term to maturity allowed in the estimation. On the modelling side, it is this part of the term structure on which the decision for either Nelson and Siegel's "one-hump" or Svensson's "two-hump" model may have the greatest impact. The requirement of a minimum remaining term to maturity for a bond to be included in the estimations influence the fit of the very short-end of the curve. Considering the difficulties to consistently achieve a good fit for this part of the curve helps to explain why some central banks regard the short-end of their curves as less reliable than the rest. Across the board, the interval from one to 10 years is hardly controversial.

Table 1

The term structure of interest rates - estimation details

Central bank	Estimation method	Minimised error	Shortest maturity in estimation	Adjustments for tax distortions	Relevant maturity spectrum
Belgium	Svensson or Nelson-Siegel	Weighted prices	Treasury certificates: > few days Bonds: > one year	No	Couple of days to 16 years
Canada	Merrill Lynch Exponential Spline	Weighted prices	Bills: 1 to 12 months Bonds: > 12 months	Effectively by excluding bonds	3 months to 30 years
Finland	Nelson-Siegel	Weighted prices	≥ 1 day	No	1 to 12 years
France	Svensson or Nelson-Siegel	Weighted prices	Treasury bills: all Treasury Notes: : ≥ 1 month Bonds: : ≥ 1 year	No	Up to 10 years
Germany	Svensson	Yields	> 3 months	No	1 to 10 years
Italy	Nelson-Siegel	Weighted prices	Money market rates: O/N and Libor rates from 1 to 12 months Bonds: > 1 year	No	Up to 30 years Up to 10 years (before February 2002)
Japan	Smoothing splines	Prices	≥ 1 day	Effectively by price adjustments for bills	1 to 10 years
Norway	Svensson	Yields	Money market rates: > 30 days Bonds: > 2 years	No	Up to 10 years
Spain	Svensson Nelson-Siegel (before 1995)	Weighted prices Prices	≥ 1 day ≥ 1 day	Yes No	Up to 10 years Up to 10 years
Sweden	Smoothing splines and Svensson	Yields	≥ 1 day	No	Up to 10 years
Switzerland	Svensson	Yields	Money market rates: ≥ 1 day Bonds: ≥ 1 year	No	1 to 30 years

Table 1 cont

The term structure of interest rates - estimation details

Central bank	Estimation method	Minimised error	Shortest maturity in estimation	Adjustments for tax distortions	Relevant maturity spectrum
United Kingdom[1]	VRP (government nominal)	Yields	1 week (GC repo yield)	No	Up to around 30 years
	VRP (government real/implied inflation)	Yields	1.4 years	No	Up to around 30 years
	VRP (bank liability curve)	Yields	1 week	No	Up to around 30 years
United States	Smoothing splines (two curves)	Bills: weighted prices	–	No	Up to 1 year
		Bonds: prices	≥ 30 days	No	1 to 10 years

[1] The United Kingdom used the Svensson method between January 1982 and April 1998.

3. Zero-coupon yield curves available from the BIS

Table 2 provides an overview of the term structure information available from the BIS Data Bank. Most central banks estimate term structures at a daily frequency. With the exception of the United Kingdom, central banks which use Nelson and Siegel-related models report estimated parameters to the BIS Data Bank. Moreover, Germany and Switzerland provide both estimated parameters and spot rates from the estimated term structures. Canada, the United States and Japan, which use the smoothing splines approach, provide a selection of spot rates. With the exception of France, Italy and Spain, the central banks report their data in percentage notation. Specific information on the retrieval of term structure of interest rates data from the BIS Data Bank can be obtained from BIS Data Bank Services.

Table 2

The structure of interest rates available from the BIS Data Bank[1]

Central bank	Method[2]	Estimates available since	Frequency	Available yield series	Maturity interval	Parameters[3]	Parameter notation[4]
Belgium	SV-NS	1 Sep 1997	Daily	0 to 10 years	3 months	6	Per cent
Canada	SV	23 Jun 1998 to 15 Oct 2003	Daily	1 to 10 years	3 months	6	Per cent
	SS	1 Jan 1986	Daily	3 months to 30 years	3 months	na	na
Finland	NS	3 Nov 1997	Weekly; daily from 4 Jan 1999	1 to 10 years	3 months	4	Per cent
France[5]	SV-NS	3 Jan 1992 to 1 Jun 2004	Weekly	0 to 10 years	3 months	6	Decimal
Germany[6]	SV	7 Aug 1997	Daily	1 to 10 years	3 months	6	Per cent
	SV	Jan 1973	Monthly	na	—	6	Per cent
	SV	28 Aug 1997	Daily	1 to 10 years	1 year	6	Per cent
	SV	Jan 1973	Monthly	1 to 10 years	1 year	6	Per cent
Italy	NS	1 Jan 1996	Daily	0 to 10 years	3 months	4	Decimal
Japan[5]	SS	29 Jul 1998 to 19 Apr 2000	Weekly	1 to 10 years	1 year	na	na
Norway	SV	21 Jan 1998	Once a month	0 to 10 years	3 months	6	Per cent
Spain	NS	2 Jan 1991 to 30 Dec 1994	Daily	na	—	4	Decimal
	SV	2 Jan 1995	Daily	1 to 10 years	3 months	6	Decimal
Sweden	SV	9 Dec 1992 to 1 Mar 1999	Weekly	0 to 10 years	3 months	6	Per cent
	SV	2 Mar 1999	Daily	0 to 10 years	3 months	6	Per cent
Switzerland[7]	SV	4 Jan 1988	Daily	1 to 10 years	3 months	6	Per cent
	SV	4 Jan 1998	Daily	1 to 10, 15, 20, 30 years	1 year	6	Per cent
	SV	Jan 1988	Monthly	1 to 10, 15, 20, 30 years	1 year	6	Per cent
United Kingdom	SV	4 Jan 1982 to 30 Apr 1998	Daily	2 to 10 years	6 months	na	na
	SV	Jan 1982 to April 1998	Monthly	2 to 10 years	6 months	na	na
	VRP	4 Jan 1982	Daily	5, 10 years[8]	—	na	na
	VRP	Jan 1982	Monthly	5, 10 years[8]	—	na	na
	VRP	15 Jan 1985	Daily	20 years[8]	—	na	na
	VRP	Jan 1985	Monthly	20 years[8]	—	na	na
United States	SS	14 Jun 1961	Daily	0 to 10 years	6 months	na	na
	SV	01 Dec 1987	Daily	0 to 10 years	3 months	na	na

[1] As of August 2005. [2] NS = Nelson-Siegel, SV = extended Nelson-Siegel (Svensson), SS = smoothing splines, VRP = variable roughness penalty. [3] Where there is an indication of a parameter there is also a BIS generated yield available on the BIS Data Bank. Moreover, "na" means that the country is transmitting estimated yields and not parameters. [4] Estimated parameters define spot and forward interest rates expressed in either decimal notation or per cent. [5] The yield curve is currently not estimated. [6] BIS generated yields are available at three months interval in addition to the yearly yields reported by the Bundesbank. [7] BIS generated yields are available at three months interval in addition to the yearly yields reported by the Swiss National Bank. [8] The nominal and real yields as well as the implied inflation term structure are calculated for the corresponding maturities.

4. Spot interest rates and forward rates derived from estimation parameters

Spot interest rates and *instantaneous* forward rates can be derived directly from the equations for the Nelson-Siegel and Svensson approaches presented above: replace the parameters of the equations - β_0, β_1, β_2 and τ_1 in the Nelson and Siegel and β_0, β_1, β_2, τ_1, β_3 and τ_2 in the Svensson case - by their estimated values and evaluate the equations at terms to maturity *m* for which the spot or forward rates have to be derived (eg m = 1 for one year to maturity). Table 3 provides examples of estimated parameters and a selection of corresponding points on the term structures. For the calculation of spot and instantaneous forward rates, it is partly relevant if the term structure was estimated either in decimal or percentage notation; the only difference is that the β-parameters are rescaled by a factor of 100. Clearly, such rescaling has no impact on the location of the humps as determined by the τ-parameters. By setting β_3 = 0 and τ_2 to an arbitrary non-zero value (eg τ_2 = 1), the Svensson equations can be used to derive spot and forward rates of term structures estimated by the Nelson and Siegel approach. Thus it is sufficient to implement just the two Svensson equations to derive the spot and instantaneous forward rates for both approaches.

Table 3

Spot interest rates and instantaneous forward rates derived from estimation parameters

Estimation parameters	Svensson (in percentage notation)		Nelson and Siegel (in decimal notation)		Nelson and Siegel (in percentage notation)	
β_0	5.82		0.0769		7.69	
β_1	–2.55		–0.0413		–4.13	
β_2	–0.87		–0.0244		–2.44	
τ_1	3.90		0.0202		2.02	
β_3	0.45		–		–	
τ_2	0.44		–		–	
Term to maturity	**Spot rate (%)**	**Forward rate (%)**	**Spot rate**	**Forward rate**	**Spot rate (%)**	**Forward rate (%)**
0	3.27	3.27	0.0356	0.00356	3.56	3.56
1 year	3.61	3.78	0.0400	0.0444	4.00	4.44
15 months	3.65	3.84	0.0411	0.0465	4.11	4.65
18 months	3.69	3.91	0.0421	0.0486	4.21	4.86
21 months	3.72	3.98	0.0432	0.0506	4.32	5.06
2 years	3.76	4.05	0.0443	0.0526	4.43	5.26
5 years	4.17	4.80	0.0546	0.0683	5.46	6.83
10 years	4.68	5.45	0.0639	0.0758	6.39	7.58
∞	5.82	5.82	0.0769	0.0769	7.69	7.69

The calculation of forward rates with non-instantaneous term to maturity is slightly more complicated. It is important to notice that in those cases where the term structure parameters are readily available, forward rates can be derived for any desired term to maturity. To compute such *implied* forward rates, evaluate the spot rate equations at discreetly chosen term to maturity intervals, then calculate the implied forward rates recursively from the shortest to the longest term to maturity.

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