

Sectoral fragility: factors and dynamics

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1. Introduction

Business cycles can have a major impact on the credit portfolio of banks. Using re-sampling techniques on data from US banks, Carey (2002) suggests that mean losses during a period of distress such as the 1989-91 recession are 3.5 times larger than during an expansion and about the same as the expansion distributions' 0.5% tail. In terms of the capital that would be adequate for banks, Bangia et al (2002) find that, over a one-year horizon the banks' needs increase by 25-30% in a recession relative to expansions.

This interaction between the business cycle and the quality of banks' asset portfolios motivated substantial empirical research, primarily focused, at an aggregate level, on default rates.² Empirical evidence so far shows a strong negative relationship between realised defaults and the economic cycle, but it also suggests that the transmission channels are complex.³

In parallel to these results at the aggregate level, efforts to improve the modelling of risk in investment portfolios have produced another body of literature aiming at incorporating aggregate macroeconomic effects in value-at-risk (VaR) models.⁴ New developments in this approach also refine the modelling of the cyclical factors, including addressing international and cross-industry correlations, as in Pesaran et al (2003).

Two main motives fostering the development of new, or the improvement of existing, models are the need to provide realistic forecasting models and to test the resilience of a given bank's portfolio. Identifying in advance the impact on credit markets of macroeconomic developments calls for a forecasting infrastructure flexible and realistic enough to incorporate the main macroeconomic components. In the context of lending by the banking sector, advance information on credit risk developments is looked for with keen interest, as this would allow for a more flexible allocation of capital possibly better matching changes in requirements imposed by external developments. In a similar fashion, interest in testing the resilience of the banking sector has increasingly fuelled the incorporation of macroeconomic elements. Stress-testing models analyse the exposure of banks to lending and to credit risks associated with the lending, also including macroeconomic developments. Typically, initial efforts have looked at the behaviour of loan loss provisions or non-performing loans (for a given bank or

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² For example, Duffie and Singleton (2003, Section 3) provide an overview of the issues as well as references to related literature. In particular, taking as a starting point the seminal work of Fons (1991) (who considered the impact of GDP growth on the default rate of a number of rating pools of bonds), Jonsson and Fridson (1996), among others, extended the analysis by considering a larger number of macroeconomic variables and the ageing effect in the pools of bonds. Helwege and Kleiman (1996) refine the model further by introducing a new method of gauging macroeconomic effects on default behaviour. Also Wilson (1997a,b) models the impact of macroeconomic variables on the probability of defaults at the industry level.

³ Fridson et al (1997), to cite one example, find a relation between macroeconomic conditions and probability of default. In particular, they find that as real interest rates increase, asset values decrease, thereby increasing the estimate of the probability of default. Furthermore, interdependence may run through financial market shocks, as can be observed in periods of high instability.

⁴ Recent surveys on this literature include Saunders and Allen (2002), Crouhy et al (2000), Allen and Saunders (2002), Gordy (2000), and Koyluoglu and Hickman (1998).

groups of banks) in response to macroeconomic shocks.⁵ The main obstacle of this aggregated approach is the limited granularity and time availability of information on loan loss provisions.

Our objective is to contribute to developing a framework for stress-testing the resilience of the banking sector by exploiting information contained in available credit risk measures. In doing so, we construct an alternative forecasting engine that increases the granularity of past exercises by further breaking down exposure along sectoral lines. More specifically, we exploit the sensitivity of industry-wide expected default frequencies, or EDFs, from Moody's KMV to macroeconomic developments in order to model the evolution of bank portfolios' fragility. Imbalances in the performance of different industries suggest that greater granularity in the treatment of banks' corporate counterparties could increase the measurement accuracy of their fragility.⁶

When pursuing the analysis at a more granular level, it is also important to characterise the linkages interrelating risk across industries.⁷ This issue has recently been illustrated by the high degree of correlation and default transmission observed between industries. For example, default rates in the technology and telecommunications sectors often go hand in hand and have generally preceded problems in other industries. Also, albeit in a more subtle manner, other sectors such as insurance and banking can transmit problems to the rest of the economy owing to their important role in financial intermediation. Indeed, one can think of a number of micro- and macroeconomic relations tying the default performance across industries.

The capital allocation decision across sectors can also be seen from the perspective of an "aggregate investor". In the portfolio composed of sectoral assets, the latter are subject to both idiosyncratic (sector-specific) and common risks (such as macroeconomic or geopolitical factors or one-time events). In order to assess the risk for this portfolio, the aggregate investor would optimally account for the risk correlations between the different "assets". Such distinction is also characteristic of VaR modelling of bank portfolios, where very granular information on loan characteristics, including the corporate client sector, is used in specifying the risk correlation within a given portfolio.

Developing a minimum understanding of sectoral risk relationships and their dynamics appears high on the research agenda, but progress is marred by the absence of data and a workable multivariate framework for assessing the nature and extent of the interaction. In principle, the firm-level exercise could be mimicked at the aggregate level, by replicating cross-portfolio channels and thus incorporating risk dynamics between different (aggregate) portfolio elements. However, the lack of sufficiently detailed aggregate and consolidated information on exposures and default rates precludes progress in this direction. Alternatively, information on sectoral lending volumes and asset quality could be analysed in the light of industry-specific developments and wider economic and idiosyncratic shocks, possibly exploiting information on cross-industry channels. Alas, this would also require a substantial amount of information at the sectoral level, currently available only to some bank supervisors, on the sectoral exposures and non-performing assets of financial institutions.

The use of market-based information for assessing sectoral risk is a feasible way forward out of the data bottleneck. Sectoral EDFs provide a measure of sectoral risk similar to that of default rates. In contrast to default rates or information on non-performing assets, EDFs contain information on the ex ante expectation of default as embodied in the asset valuation of a firm, thus making a distinction between latent and realised risk. In addition to being a micro-founded indicator of fragility, EDFs have been observed to have good early warning properties.⁸ Being widely used in the assessment of asset

⁵ Many national banking supervisors have carried out this exercise in one form or another. For instance, Morin (2003) and Trucharte and Saurina (2002) follow this procedure, the latter also incorporating some of the elements considered in this paper.

⁶ Moody's, for example, reports insolvencies in the telecommunications and technology sectors during 2001 and 2002 representing about 70% of the total dollar-weighted defaults in Europe. In contrast, the financial sectors (excluding the insurance sector) have experienced a very small number and volume of defaults. In addition, one could expect that non-cyclical sectors are slower to respond to changes in the macroeconomic environment, thus making them less sensitive to aggregate fluctuations.

⁷ The theory underlying one such possible interlinkage of default processes is presented in Jarrow and Yu (2001). They model the default intensity as depending on macroeconomic factors and an interdependence term linking firms across industries and sectors.

⁸ See Delianedis and Geske (1998) and Kurbat and Korablev (2002) and references in the latter for recent studies of the relationship between EDFs and actual default rates, or Gropp et al (2002) for a comparison applied to the European banking sector.

quality and available for a large number of firms, they can be easily incorporated into a simple econometric model to obtain a sensible measure of the nature, direction and magnitude of risk cross-dynamics, and assist in modelling the future evolution of exposures.⁹

This paper explores both the interaction between risk factors across different economic sectors through time and their joint and idiosyncratic sensitivity to macroeconomic and systemic developments, and therefore represents the first step in setting up a macro VaR model (derivation of probabilities of default and cross-correlations). In particular, the modelling exercise aims to better depict sectoral risk interactions, the duration of shocks, and the relationship between industry-specific risk and macroeconomic activities.

Section 2 presents the cointegrated autoregressive (CVAR) framework used to model the risk interactions as well as the data used. In Section 3 the procedures are explained and the results are derived. The model's forecasting abilities are discussed in Section C. Section 4 concludes.

2. Elements of the model

2.1 A reduced-form dynamic model

The characteristics of the industry risk indicators described above point to the need to account for the strong cross-sectoral risk correlations. One relatively simple model capturing such strong interaction is the so-called *cointegrated vector autoregression* (CVAR) model. This linear representation of a system of interrelated variables is widely used for modelling problems with similar characteristics.¹⁰ Indeed, this model accounts for properties characterising sectoral risk, thus offering a richer representation of the system-wide counterparty risk facing the banking sector.

We consider a set of n industries $i \in I = \{1, \dots, n\}$ to which the EU banking sector has an exposure $x_i, i \in I$. The risk in industry i is denoted by the random variable r_i . Accordingly, the vector of industry risks is denoted by \mathbf{r} and has dimension n . The various elements of \mathbf{r} interact contemporaneously (the risk level in some industries may serve as a factor to that of others) and through time (difficulties in an industry may affect the originating and other industries with a delay). In addition, sectoral risk clearly depends on the overall macroeconomic environment. The macroeconomic and/or systemic factors are represented in our framework by a vector \mathbf{y} of exogenous processes. Finally, extraordinary events affecting sectoral risk profiles, such as the events of 11 September 2001, can be described by a vector of shock dummies \mathbf{d} .¹¹

In order to illustrate the elements of the model, consider two industries with a cross-impact taking place over one period only, and whose risk is only affected by one current (not past) macroeconomic variable y_t and one deterministic shock d_t . The risk processes for these two industries can be represented by the following system of equations:

$$r_{1,t} = b_{10} - b_{12}r_{2,t} + \gamma_{11}r_{1,t-1} + \gamma_{12}r_{2,t-1} + c_1y_t + \psi_1d_t + \varepsilon_{1,t}$$

$$r_{2,t} = b_{20} - b_{21}r_{1,t} + \gamma_{21}r_{1,t-1} + \gamma_{22}r_{2,t-1} + c_2y_t + \psi_2d_t + \varepsilon_{2,t},$$

which, in a more compact matrix form, can be written as

⁹ See also Appendix A for a summary of the construction of EDFs. Combined with sectoral exposure information (for example from countries with credit registries), these EDF-based measures can provide a first approximation of the nature and magnitude of lending exposure (to default). Depending on the availability of recovery rates by sector, especially if these are modelled dynamically, the notion of VaR would be exact.

¹⁰ Johansen (2000, p 361), for example, suggests that

[if] we want to find relations between simultaneous values of the variables in order to understand interactions of the economy one would get a lot more information by relating the value of the variable to the value of other variables at the same time point rather than relating it to its own past. One can say that if we want to discuss relations between variables, then one should take combinations of simultaneous values and if we want to discuss dynamic development of the variables we should investigate the dependence on the past.

¹¹ This approach has much in common with autoregressive, distributed-lag (ARDL) models, a survey of which can be found in Pesaran and Smith (1998).

$$B\mathbf{r}_t = \Gamma_0 + \Gamma_1\mathbf{r}_{t-1} + C\mathbf{y}_t + \Psi\mathbf{d}_t + \varepsilon_t, \quad (1)$$

where

$$B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}; \quad \Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}; \quad \Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}; \quad C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}; \quad \Psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}; \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}.$$

Equation (1) is the primitive form of the vector autoregression (VAR) process. The standard form can be obtained by premultiplication by the inverse of matrix B , resulting in:

$$\mathbf{r}_t = \pi + \Pi_1\mathbf{r}_{t-1} + Z\mathbf{y}_t + \varphi\mathbf{d}_t + \mathbf{e}_t, \quad (2)$$

where

$$\pi = B^{-1}\Gamma_0; \quad \Pi_1 = B^{-1}\Gamma_1; \quad Z = B^{-1}C; \quad \varphi = B^{-1}\Psi; \quad \mathbf{e}_t = B^{-1}\varepsilon_t.$$

In general, the interaction across risk processes and with the exogenous macroeconomic factors may take place over p (monthly) periods. In the presence of a vector of exogenous processes \mathbf{y}_t (with an impact over k periods) and a dummy vector of exogenous shocks \mathbf{d}_t (effect over l periods), equation (2) can be written as:

$$\begin{aligned} \mathbf{r}_t = & \pi + \Pi_1\mathbf{r}_{t-1} + \Pi_2\mathbf{r}_{t-2} + \dots + \Pi_p\mathbf{r}_{t-p} \\ & + Z_0\mathbf{y}_t + Z_1\mathbf{y}_{t-1} + \dots + Z_k\mathbf{y}_{t-k} \\ & + \varphi_0\mathbf{d}_t + \dots + \varphi_l\mathbf{d}_{t-l} + \mathbf{e}_t. \end{aligned} \quad (3)$$

If the equilibrium is to be meaningful, the risk series need to be stationary, ie should not be characterised by a unit root. If any of the series \mathbf{r}_t is integrated, denoted by $I(1)$, the regression results are not valid. In particular, whereas the estimated coefficients are still unbiased, their t-values are overrepresented.¹²

Integrated series could be brought back to stationarity by (the linear transformation of) differencing, $\mathbf{r}_t - \mathbf{r}_{t-1} = \Delta\mathbf{r}_t$. However, there may exist a non-zero linear combination of the integrated risk series, $\beta\mathbf{r}_t$, that is stationary. If this is the case, differencing the integrated series would ignore valuable information about long-term relationships that may exist between the series, such as both real and financial microeconomic linkages tying the different sectors. If such linear combinations exist, then the system is said to be cointegrated (CVAR) and the dynamic restrictions they impose on the system are testable. The linear relations between the sectoral risk measures are often called long-run equilibria.¹³

Cointegration is more easily visualised through an error correction representation of equation (3) above. For example, with restrictions of only one exogenous process and shock respectively, and $p = 2$, $k = 0$ and $l = 0$, equation (3) simplifies to:

$$\Delta\mathbf{r}_t = \Phi_1\Delta\mathbf{r}_{t-1} - \Pi\mathbf{r}_{t-1} + \pi + Z\mathbf{y}_t + \varphi\mathbf{d}_t + \mathbf{e}_t, \quad (4)$$

where $\Pi = I_p - \Pi_1 - \Pi_2$ and $\Phi_1 = -\Pi_2$. The term Π embodies the long-term effects in levels (adjustment to previous disequilibria in the risk profile across sectors), whereas Φ_1 represents the short-term or transitory shocks (adjustment to previous changes in risk).¹⁴

The hypothesis of cointegration is formulated as a reduced rank of the Π matrix:

$$H_1(r) : \Pi = \alpha\beta', \quad (5)$$

where α and β are $p \times r$ matrices of full rank. The cointegration hypothesis implies that the process \mathbf{r}_t is non-stationary, but that $\Delta\mathbf{r}_t$ and $\beta'\mathbf{r}_t$ are stationary.

¹² For details on the nature of the problem, see for instance Hendry and Juselius (2000a).

¹³ See Hendry and Juselius (2000b) and Doornik et al (1998) for a comprehensive and clear exposition of cointegration analysis of VARs.

¹⁴ A number of alternative and equivalent error correction representations are possible, each emphasising a different aspect of the dynamic relationship. While they have all equivalent explanatory power and can be estimated by ordinary least squares, inference on some parameters will not be standard when the risk processes are integrated. See Hendry and Juselius (2000b, Section 4) for details on the different representations.

In the context of risk management, the cointegrating vectors in β can be thought of as *optimal portfolios*, as the risk profile of $\beta' r_t$ is, for each vector in β , constant in expectation. That is, holding assets of the different sectors in proportions given by the inverse of the coefficients in the columns of β creates portfolios with stationary risk. The presence of several cointegrating relationships does not necessarily mean that there is more than one long-run equilibrium position. More likely, it may hint at the existence of a long-run equilibrium with embedded sectoral equilibria or cointegrated subsets of variables.

2.2 Data elements

As described above, our model consists of two main components, one endogenous and the other exogenous, and some additional elements that allow modelling one-time events. The first element is a *set of sectoral risk indicators* constituting a closed system, ie possibly interdependent, which we construct on the basis of firm-level EDFs. The second element is a *set of exogenous variables* that are orthogonal to the space of sector-specific shocks. These macro variables can also be thought of as a toolbox to be used in forecasting, scenario building and stress testing the system. We first specify the properties of the closed system before incorporating the exogenous elements and running the scenarios and stress tests.

2.2.1 Sectoral measures of risk

The chosen sectoral aggregation relating firm-specific EDF information to industry-specific risk measures r_t needs to weigh the positive information content of a possibly large set of indicators and their cost in terms of modelling requirements (allowing distinct characteristics across sectors). We define seven broad industries. EDFs for firms are available from KMV on a common methodology from January 1992 until May 2003 (137 monthly observations). Using as a basis the EU classification of economic activities (NACE Rev. 1), the over 1,500 SIC codes were mapped to a simpler classification of *seven broad industries* characterising the largest distinct economic sectors of interest (see Table 1).

Table 1
Economic industries

BaC	Basic goods and construction
EnU	Energy and utilities
Cap	Capital goods
CCy	Consumer cyclicals
CNC	Consumer non-cyclicals
Fin	Financial
TMT	Technology, media and telecommunications

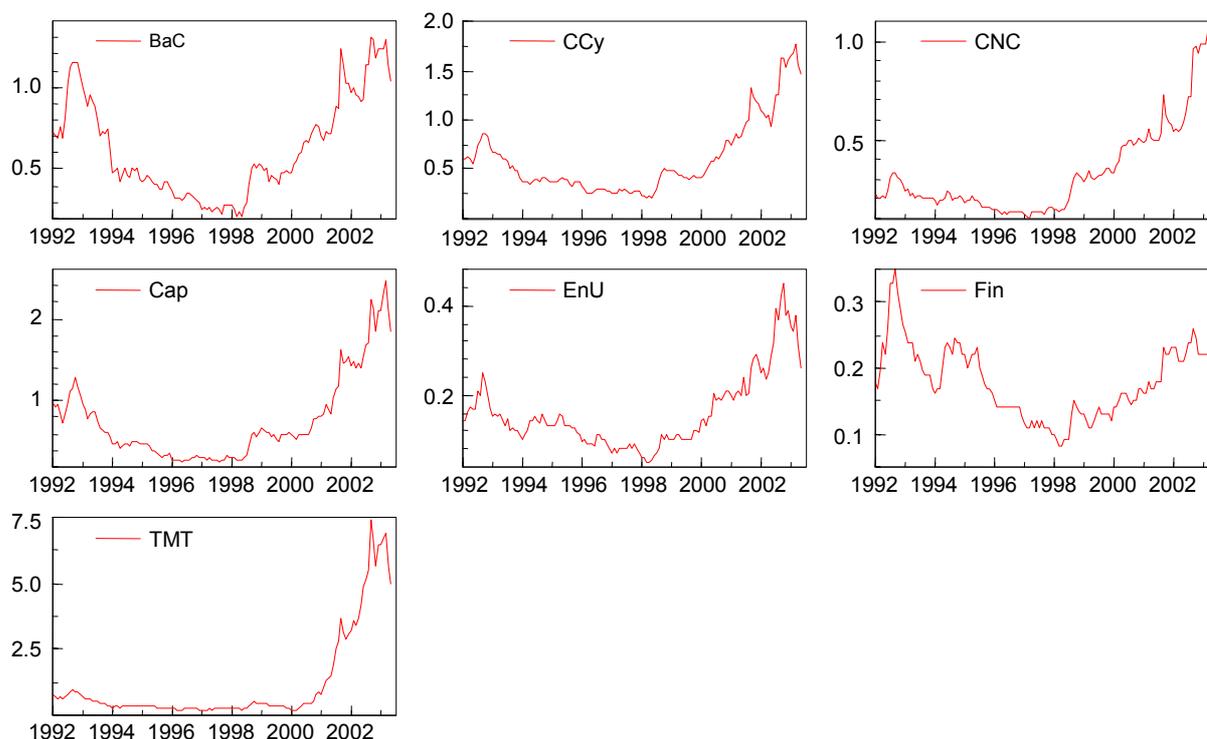
Once the industries (also referred to as sectors in what follows) have been defined, there are a number of ways of aggregating the firm-level EDF information into measures of sectoral default probability.¹⁵ Of these, the simplest to implement is the sector's sample median (Graph 1).¹⁶

¹⁵ See Appendix A for a discussion of the construction of the sectoral fragility indices.

¹⁶ Although the sample median is not a sufficient statistic for the population mean, it converges (eg in mean) to the population mean when the population's distribution is symmetric, as described by Rose and Smith (2002). The median is robust to outliers in the sample, but has two weaknesses: it does not account for the potential risky tail of the distribution (does not weigh in information on the very risky firms) and ignores the size of the exposure to individual names. These issues remain to be addressed in future work.

Graph 1

Industry median EDF measures



Sectoral (median) EDF series are highly correlated across industries, denoting the close interaction of their measures of risk, and their possible sensitivity to common systemic or macroeconomic effects. The financial (Fin) and consumer non-cyclical (CNC) industries show the lowest correlation coefficients (Table 2), whereas those with the stronger contemporaneous correlations are the basic goods and construction (BaC), capital goods (Cap), consumer cyclical (CCy) and energy and utilities (EnU) industries.

Table 2

Industry EDF correlations

BaC						
0.9068	CCy					
0.7039	0.8875	CNC				
0.9223	0.9739	0.8196	Cap			
0.8596	0.9448	0.8483	0.9040	EnU		
0.7576	0.5726	0.2850	0.6085	0.6685	Fin	
0.6721	0.8800	0.7853	0.8738	0.8246	0.3789	TMT

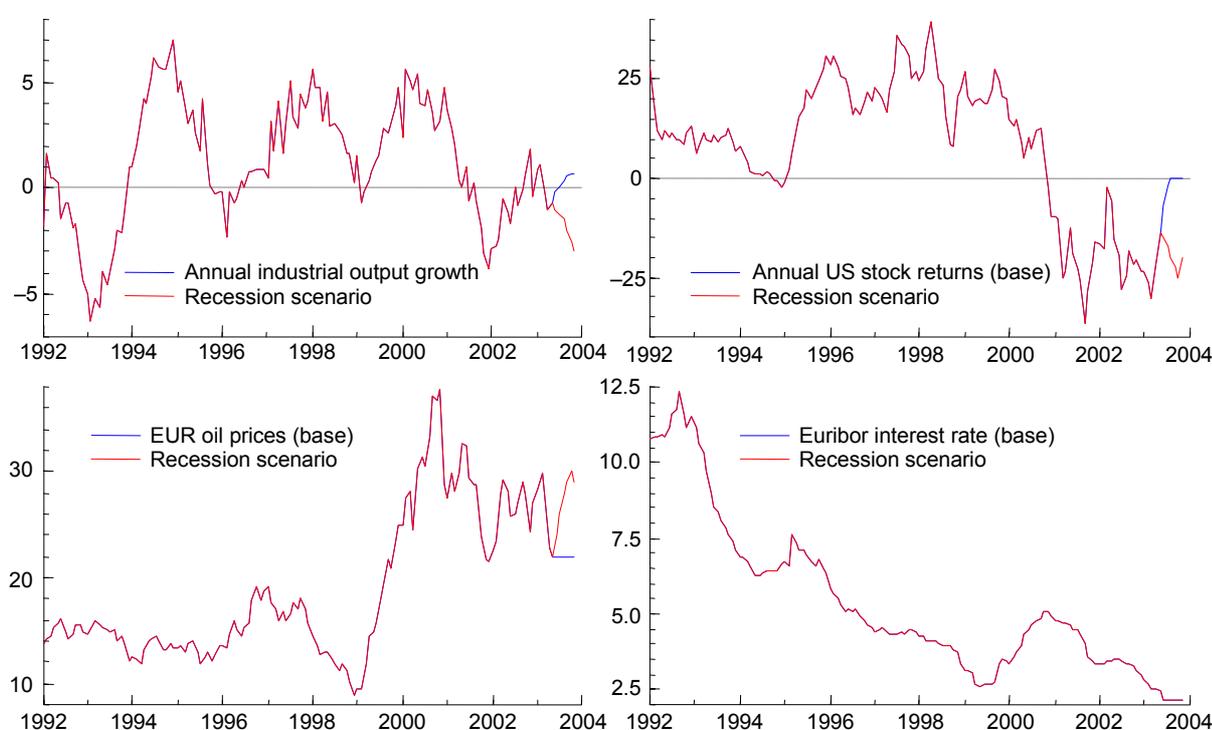
2.2.2 Exogenous variables

Three broad macroeconomic measures and a common stock market element are considered. The impact of *aggregate demand* changes is proxied by industrial output innovations.¹⁷ Shocks emanating from *aggregate supply* are captured by innovations in oil prices, denominated in euros. Finally, *monetary shocks* are described by changes in a benchmark interest rate, which we have chosen to be the three-month Euribor rate. By construction, firm-level EDF information is negatively correlated to the firm's stock valuation. Movements in the stock markets that are widespread are therefore likely to have an impact on the sectoral measure of risk, suggesting the need to identify distinctly this type of *common equity shock*. In order to avoid endogeneity problems, we rely on recent results in the contagion literature suggesting that large shocks to equity markets are transmitted internationally, therefore hinting at the use of innovations in a foreign benchmark to proxy for economy-wide stock effects. We selected innovations in the DATASTREAM benchmark US stock index for this purpose.

The four exogenous variables are displayed, together with some assumed scenarios (see Appendix C below), in Graph 2.

Graph 2

Exogenous macroeconomic variables and scenarios



2.2.3 Additional elements

In addition to the endogenous and systemic exogenous indicators, we specify purely exogenous idiosyncratic shocks, such as the 11 September or ERM crisis shocks. Such events clearly affect the risk profile of firms and industries in quite a fundamental way and need to be accounted for in the exercise. We model these shocks by resorting to dummies, making careful note of the significant events underlying periods of “unusual” activity.¹⁸

¹⁷ The choice of industrial production instead of GDP enables the analysis to be carried out on a monthly basis. We consider the 12-month growth rate, so as to avoid seasonal effects.

¹⁸ These elements are not only desirable from a model-specification point of view, but they could also potentially serve to replicate similar shocks in the context of a historical stress test.

3. Econometric estimation

The risk series r_t do not show a trend but exhibit a high degree of persistence, with unit root Dickey-Fuller tests failing to reject the null hypothesis of an integrated process. The integrated nature of the risk processes requires differencing the risk series or incorporating cointegration analysis. On the basis of the high degree of covariance in the series (Graph 1), we look for cointegrating relationships across sectors, estimate the full cointegrated model, and analyse the cointegrating relationships.

It is evident from the preliminary model selection exercise in Appendix B that errors are significantly larger in some episodes of weak economic activity, active monetary policy, supply shocks and systemic stock market instability. For example, both periods of economic underperformance in the early 1990s and 2002 are marked by clearly larger levels of risk across sectors. Conditioning on external macroeconomic and financial market factors, as in equation (3), provides a set of “instruments” for carrying out stress-testing exercises on sectoral risk.¹⁹

3.1 Basic setup with exogenous factors y_t

We consider the impact of the four exogenous variables described in Section 2.2.2, namely the 12-month change in the log of industrial output (proxying demand shocks), the Euribor three-month interest rate (proxying monetary policy changes), the price of oil in euros (identifying supply shocks), and the 12-month change in the log of the DATASTREAM benchmark US stock index (capturing large system-wide stock-related factors that are orthogonal to the industry-specific shocks). In equation (2), the lag specification of y_t is that of the endogenous variables, ie $p = k = 2$.

The diagnostic statistics on the individual equations improve relative to the closed model of Appendix B following the incorporation of systemic and macroeconomic variables (Table 3). The VAR(1) specification appears overall congruent with the data. Some problems remain with the EnU and, especially, the TMT sector.

Table 3

Model diagnostics for individual equations in the full model

Test	BaC	CCy	CNC	Cap	EnU	Fin	TMT
AR 1-12	0.82283	1.2097	1.6449	1.0137	0.70774	1.3571	4.5086
F(12,109)	[0.6267]	[0.2857]	[0.0897]	[0.4416]	[0.7410]	[0.1978]	[0.0000]**
ARCH 1-12	1.3075	0.94573	1.1426	1.6383	2.9640	0.75391	5.3021
F(12,97)	[0.2267]	[0.5056]	[0.3358]	[0.0935]	[0.0015]**	[0.6955]	[0.0000]**
Normality	3.9161	3.0566	8.0917	7.2450	14.709	2.3625	71.637
$\chi^2(2)$	[0.1411]	[0.2169]	[0.0175]*	[0.0267]*	[0.0006]**	[0.3069]	[0.0000]**
Hetero	1.9279	2.7686	3.4031	2.9249	3.3417	2.3137	5.1990
F(22,98)	[0.0154]*	[0.0003]**	[0.0000]**	[0.0002]**	[0.0000]**	[0.0027]**	[0.0000]**
Hetero X	1.5412	1.6428	1.3300	1.9305	4.2282	1.5192	2.7440
F(77,43)	[0.0621]	[0.0392]*	[0.1554]	[0.0103]*	[0.0000]**	[0.0686]	[0.0003]**

¹⁹ A framework with emphasis on the interdependence between sectoral risk dynamics and macroeconomic variables would consider these factors as additional endogenous variables in the VAR, possibly testing their (weak) exogeneity. We assume full exogeneity from the outset, because we are interested in the impact of systemic events on the sectoral risk profile of a chosen set of scenarios. An enhanced general equilibrium specification could be the focus of future work.

3.2 Characteristics of the system

Overall, the model is quite satisfactory in terms of its econometric properties. But how does it square with the intuition in terms of how macroeconomic events affect the risk profile of the different sectors? The significance of the retained regressors and the estimated coefficients is reported in Table 4.

First, the risk processes show strong and significant persistence (underpinned by the significant and large coefficients on their own lags), indicating the non-stationary nature of sectoral risk. In addition, all sectors show some degree of interconnection, even after conditioning on the macroeconomic processes. The significant and persistent impact of the CCy and TMT sectors on the remaining sectors also stands out. Whereas the former is significant at the 1% level in all equations in both lags, the first lag of the latter is significant in four other equations and mostly at the 5% level. Second in importance are the first lags of the CNC and EnU sectors, which show a significant impact on the Cap and TMT (at the 1% level) and Fin and TMT sectors (at the 5% level) respectively. Somewhat weaker is the effect of risk profile changes in the BaC and Cap sectors, which only appear significant at the 5% level in the CNC and TMT equations respectively. The risk profile of the Fin sector, while sensitive to the CCy and EnU sectors, does not appear to affect any of the other sectors.

The model is also congruent with some stylised facts regarding the systemic variables. First, the sign of the coefficients of the exogenous variables is, where significant, as expected: higher money market interest rates increase the risk of the given industry, whereas higher output and stock exchange growth rates decrease it. Positive shocks in the US stock market decrease the contemporaneous risk in any one industry (positive stock exchange impact), but appear to have some ripple effect (the lag of the opposite sign which is then reverted again in the second lag). The somewhat surprising result is that risk does not seem to be affected by oil prices (output shocks), except in the TMT sector, and then with an unexpected sign (higher oil prices lower the risk in the TMT sector). The second notable result is that deterministic shocks affect all the different industries, as denoted by the significance of the coefficients of the deterministic dummies in each equation of Table 4.

The results of Table 4 also suggest, however, that the estimation can be improved, in particular regarding the inference about the degree of interaction between sectors and the overall congruence of the model. This is illustrated in particular by the strongly significant one-lag coefficients in all equations. In order to address this potential shortcoming and so as to extend the cointegration analysis of the previous section, we explore the cointegrated relationships of the expanded open model.

3.3 Cointegrating relationships

Testing the cointegration rank r of the seven industry risk systems suggests the presence of a number of cointegration relationships ($r \neq 0$). The estimated eigenvalues, the log likelihood and the trace and maximum likelihood tests are tabulated in Table 5 below.

The trace test suggests that all λ_i are different from zero, indicating that all risk series are stationary. The variance in the value of the λ points out that the adjustment to the cointegration relationships varies substantially across cointegrating vectors. Since the λ_i can be interpreted as a squared canonical correlation coefficient, it provides a measure of the correlation with the stationary part. Accordingly, the drop in the magnitude of λ_i relative to λ_{i-1} would suggest a stronger correlation in the first $i-1$ relationships. However, the estimated λ_i for i sufficiently large are still showing substantial correlation with the stationary components. Notwithstanding this, some notable drops can be observed between the third and the fourth and between the fifth and the sixth λ . The maximum likelihood tests also pick up these drops, but still fail to reject the hypothesis of zero λ_i for i sufficiently large. We remain, therefore, suspicious about the significance of the fourth and higher cointegrating relationships.

Table 4
Full preliminary VAR(1) model

		Equation													
		BaC		CCy		CNC		Cap		EnU		Fin		TMT	
Regressor	t	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val
BaC	-1	0.817	0.000	0.070	0.503	0.002	0.976	0.081	0.685	-0.039	0.474	-0.042	0.279	-0.159	0.620
	-2	0.049	0.740	0.042	0.672	0.153	0.038	0.176	0.347	0.098	0.059	0.030	0.407	0.369	0.221
CCy	-1	0.621	0.002	1.024	0.000	0.232	0.020	1.143	0.000	0.256	0.000	0.072	0.150	1.924	0.000
	-2	-0.700	0.001	-0.406	0.003	-0.460	0.000	-0.686	0.008	-0.284	0.000	-0.147	0.004	-1.426	0.001
CNC	-1	-0.312	0.155	-0.081	0.579	0.682	0.000	-0.742	0.008	-0.100	0.188	-0.053	0.330	-1.779	0.000
	-2	0.438	0.071	0.263	0.104	0.212	0.077	0.690	0.025	0.097	0.247	0.047	0.431	0.581	0.238
Cap	-1	0.064	0.543	0.054	0.444	0.016	0.765	0.574	0.000	-0.047	0.202	0.039	0.132	-0.505	0.020
	-2	-0.019	0.849	-0.052	0.446	0.032	0.532	0.001	0.996	0.040	0.263	0.015	0.562	0.381	0.069
EnU	-1	0.126	0.652	0.265	0.158	0.267	0.056	-0.076	0.829	0.604	0.000	0.138	0.049	1.132	0.049
	-2	-0.065	0.839	-0.083	0.699	0.064	0.689	-0.580	0.154	0.069	0.536	0.060	0.450	0.753	0.251
Fin	-1	0.246	0.537	-0.004	0.987	0.024	0.904	0.348	0.489	0.112	0.419	0.812	0.000	-0.487	0.549
	-2	-0.367	0.332	-0.259	0.306	-0.322	0.087	-0.376	0.432	-0.095	0.468	-0.121	0.197	-0.518	0.502
TMT	-1	0.007	0.815	0.049	0.018	0.030	0.046	0.083	0.032	0.028	0.010	-0.007	0.351	1.188	0.000
	-2	-0.004	0.891	-0.017	0.380	-0.014	0.355	-0.042	0.260	-0.017	0.094	0.003	0.693	-0.227	0.000

Table 4 (cont)
Full preliminary VAR(1) model

Regressor	t	Equation													
		BaC		CCy		CNC		Cap		EnU		Fin		TMT	
		coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val
i		0.034	0.108	0.013	0.359	0.014	0.191	0.018	0.506	0.011	0.130	0.012	0.023	0.076	0.077
	-1	-0.021	0.450	-0.003	0.861	-0.022	0.119	-0.002	0.964	-0.007	0.474	-0.010	0.168	-0.106	0.067
	-2	-0.004	0.843	-0.004	0.756	0.005	0.630	-0.018	0.490	-0.004	0.566	0.001	0.798	0.002	0.958
y		-0.008	0.046	-0.008	0.003	-0.003	0.174	-0.002	0.693	-0.002	0.133	-0.001	0.306	-0.009	0.266
	-1	0.000	0.968	0.001	0.665	0.001	0.778	0.008	0.119	0.002	0.127	0.001	0.545	0.014	0.090
	-2	0.003	0.377	0.004	0.082	0.004	0.027	0.000	0.918	0.001	0.685	0.000	0.916	-0.001	0.897
us		-0.002	0.045	-0.003	0.000	-0.001	0.021	-0.003	0.004	-0.001	0.030	-0.001	0.000	-0.009	0.000
	-1	0.003	0.011	0.003	0.001	0.002	0.019	0.006	0.000	0.001	0.042	0.001	0.002	0.010	0.000
	-2	-0.002	0.043	-0.002	0.013	-0.001	0.274	-0.002	0.088	-0.001	0.109	-0.001	0.001	-0.003	0.127
oil		0.002	0.534	0.000	0.813	0.000	0.847	0.004	0.219	0.002	0.068	0.000	0.650	0.010	0.062
	-1	0.000	0.963	0.001	0.788	0.000	0.915	-0.001	0.761	-0.001	0.686	0.000	0.690	0.001	0.840
	-2	-0.002	0.519	-0.001	0.745	0.000	0.756	-0.005	0.178	-0.001	0.417	-0.001	0.372	-0.012	0.031
c		0.026	0.534	0.058	0.041	0.046	0.028	-0.001	0.986	0.018	0.220	0.039	0.000	0.161	0.064
FinT		0.190	0.000	0.242	0.000	0.145	0.000	0.426	0.000	0.052	0.000	0.032	0.000	1.557	0.000
S11		0.301	0.000	0.247	0.000	0.187	0.000	0.332	0.000	0.008	0.535	0.026	0.006	0.784	0.000
ERM		0.084	0.005	0.071	0.000	0.042	0.005	0.076	0.041	0.032	0.002	0.042	0.000	0.099	0.097

Table 5
Cointegration tests of the open model

rank (<i>i</i>)	λ_i	loglik	$H_0: r \leq i$			
			Trace test		Max test	
0		2084.185	324.93	[0.000]**	101.05	[0.000]**
1	0.56619	2140.558	223.88	[0.000]**	61.94	[0.000]**
2	0.40066	2175.113	161.93	[0.000]**	54.38	[0.000]**
3	0.36199	2205.448	107.55	[0.000]**	36.31	[0.002]**
4	0.25927	2225.706	71.24	[0.000]**	35.02	[0.000]**
5	0.25132	2245.243	36.22	[0.000]**	22.33	[0.002]**
6	0.16853	2257.701	13.89	[0.000]**	13.89	[0.000]**
7	0.10842	2265.447				

In addition to the trace and maximum likelihood tests of Table 5, other criteria for specifying the cointegration rank include (Hendry and Juselius (2000b, p 22)):

1. the t-values of the α coefficients;
2. the graph of the cointegrating relation;
3. the recursive graph of the trace statistic for $\tilde{r} = 1, 2, \dots, p$;
4. the characteristic roots of the model;
5. economic interpretability of the results.

The rank estimation of the unrestricted cointegrated model yields the cointegration matrices reported in Table 6. The t-values for the α coefficients would support ignoring the last two cointegrating vectors (no coefficients above the benchmark value of 3).

The graphs of the cointegrating vectors $\beta_j r_t$ (Graph 3) indicate that the last three vectors appear to be non-stationary. This is also the case, but to a lesser degree, with the fourth cointegrating vector towards the end of the period, and, sporadically, also for the third vector. As the cointegrating relations are supposed to be stationary, this suggests that the cointegration rank is unlikely to exceed 4.

Since the variable $T_j \ln(1 - \lambda_j)$ for $j = T_1, \dots, T$ grows linearly over time when $\lambda_j \neq 0$, the recursively calculated components of the trace statistic should increase linearly for the first r components, but stay constant for the remainder. The recursive components of the unrestricted model's trace statistics shown in Graph 4 would support the correct specification to be $r = 1$, as the trace statistics for $r \geq 2$ all seem to be constant.

The information derived from the characteristic roots (Table 7) does not provide any conclusive information about the correct rank specification. In principle, if the r th + 1 cointegration vector is non-stationary and is wrongly included in the model, then the largest characteristic root will be close to the unit circle. The largest characteristic root when $r = 1$ has the smallest value among the largest common roots for different restrictions on the cointegration rank, supporting the possibility of a single cointegration relationship. Other troughs are found at $\tilde{r} = 4$ and 6.

Table 6
Cointegration matrices of the open model

	β							α						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
BaC	1.00	0.54	1.12	1.99	-0.31	0.05	-48.50	0.019	-0.044	0.016	-0.028	0.103	0.038	0.001
CCy	-3.83	1.00	-2.50	0.27	-0.15	-0.15	86.41	0.021	0.058	0.144	-0.024	0.129	0.008	0.000
CNC	-5.39	-0.45	1.00	-6.22	0.23	-0.26	3.51	0.017	0.015	0.089	0.008	-0.110	0.087	0.000
Cap	2.82	-0.76	0.08	1.00	0.04	0.10	-19.98	0.011	0.515	0.061	-0.020	-0.031	-0.216	0.001
EnU	18.47	-1.26	0.79	15.14	1.00	-0.47	-16.18	0.004	0.014	0.012	-0.012	-0.166	0.095	0.000
Fin	-15.10	0.59	-0.91	-3.57	0.33	1.00	31.02	0.014	-0.044	0.002	-0.006	-0.055	-0.081	0.000
TMT	-0.26	0.05	0.21	-0.26	-0.02	0.02	1.00	0.131	0.711	-0.171	0.007	0.503	0.271	0.000
i	0.65	-0.01	-0.05	-0.31	-0.02	0.02	11.95							
-1	-0.83	0.01	0.05	-0.23	0.02	-0.03	-10.72							
-2	0.15	-0.03	0.01	0.31	0.00	0.00	0.16							
y	-0.08	0.00	-0.02	0.13	-0.01	-0.01	-1.22							
-1	0.05	0.01	-0.01	-0.05	-0.01	0.00	-0.67							
-2	0.03	0.00	0.03	0.02	0.00	0.01	1.88							
us	-0.06	0.00	0.00	0.03	0.00	0.00	-0.06							
-1	0.06	0.01	0.00	-0.04	0.00	0.00	1.11							
-2	-0.03	0.00	0.00	0.04	0.00	0.00	-0.46							
oil	0.03	0.01	-0.01	-0.05	0.00	0.00	0.28							
-1	0.00	0.00	0.00	0.00	0.00	0.00	-0.26							
-2	-0.05	-0.01	0.01	0.02	0.00	0.00	-0.55							

The greyness of the alpha value is determined by its t-value: if in excess of five, it is dark grey; if in excess of four, it is grey; and if in excess of three, it is light grey.

Graph 3

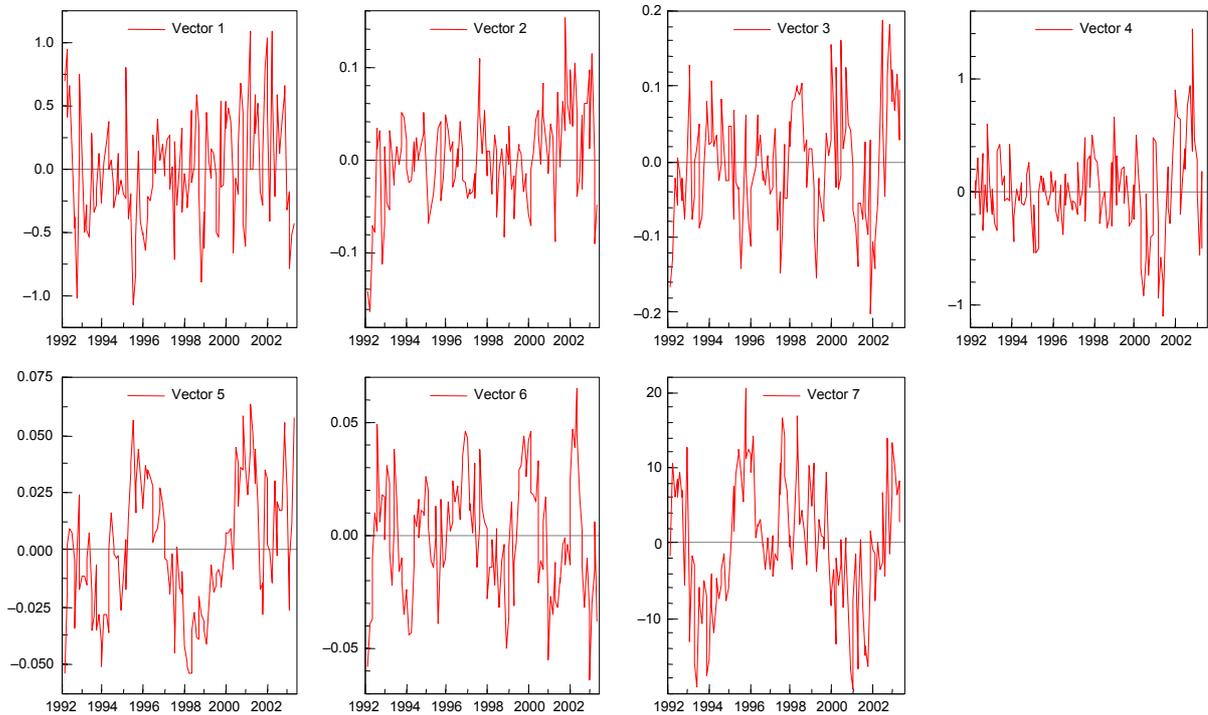
Unrestricted cointegration relations $\beta_i r_t$ 

Table 7

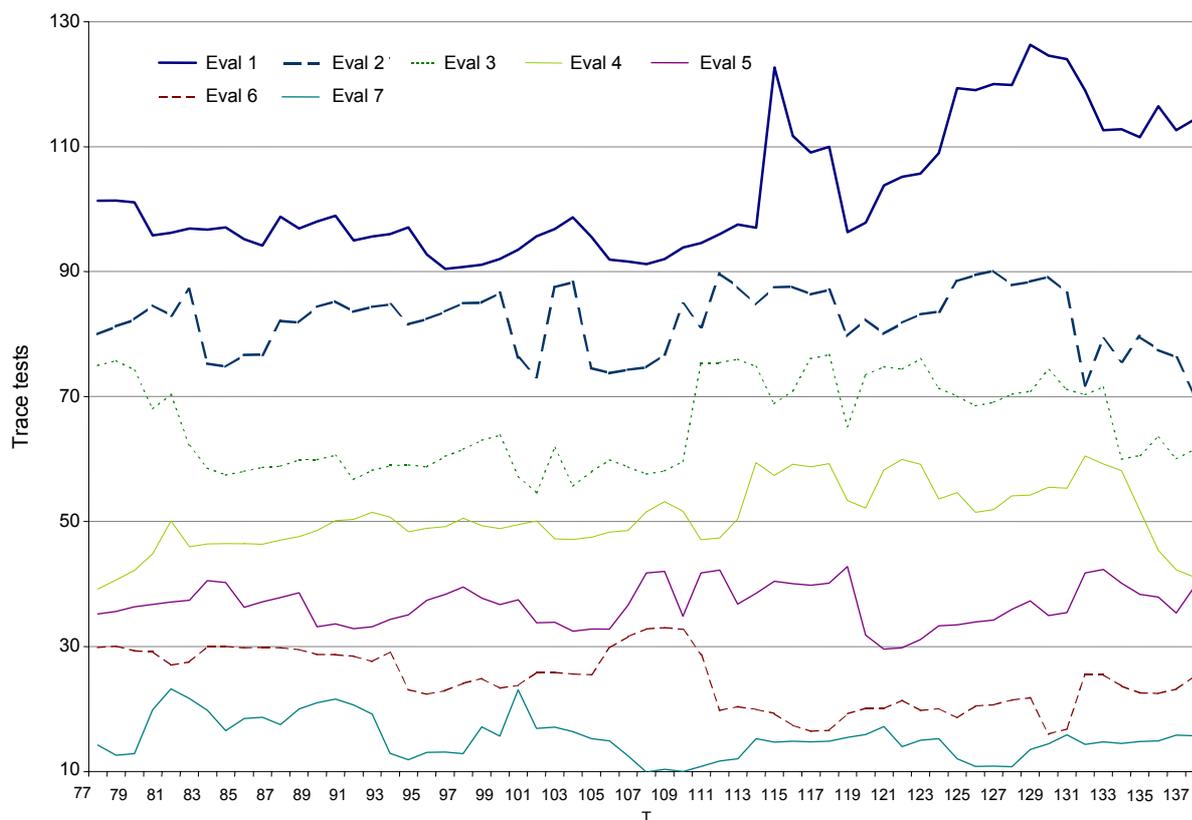
Largest characteristic roots of model

$H_0: r =$						
1	2	3	4	5	6	7
1.00	1.00	1.00	1.00	1.00	1.00	0.96
1.00	1.00	1.00	1.00	1.00	0.86	0.96
1.00	1.00	1.00	1.00	0.88	0.86	0.79
1.00	1.00	1.00	0.86	0.77	0.77	0.75
1.00	1.00	0.91	0.86	0.77	0.77	0.75
1.00	0.92	0.75	0.61	0.65	0.64	0.65
0.76	0.43	0.44	0.61	0.65	0.64	0.65

In order to assess the correct rank, we check the economic interpretability of the higher-order ranks. In this task, the significance of the adjustment parameter α (Table 6) is useful, as it identifies the sector whose risk process is correlated with the stationary part. Only the EnU sector adjusts to the fifth cointegration vector. Indeed, looking at the coefficients of β_5 suggests that it is essentially a unit vector describing risk in the EnU sector. Looking at the evolution of this vector in Graph 3 and the evolution of r_{EnU} in Graph 1 supports this view, especially until 2001 (where the series have not been filtered for deterministic shocks). Similar reasoning would also apply to β_4 (tying r_{CCY} and r_{TMT} , possibly through some consumption goods channel), β_3 (tying r_{Cap} and r_{TMT} possibly through an investment channel), and β_2 (a unit vector for r_{TMT} , with a strong impact on the Cap sector).

Graph 4

Recursive trace statistic $T \ln(1 - \lambda_i)$



With the prerogative of treating near unit roots as unit roots (thereby increasing the accuracy of statistical inference),²⁰ we adopt a bias to reduce the cointegration rank wherever the criteria do not strongly support the advantage of increasing the rank. The strict implementation of these criteria would indicate $r = 1$, with a somewhat more lenient alternative of $r = 4$. So as to facilitate discussion, we take the strict route and consider a unitary cointegration rank.

3.3.1 Identification, hypothesis testing and weak exogeneity

A simple rotation of the cointegration space (leaving the estimated long-run matrix Π in equation (5)) is sufficient to uniquely determine the cointegrating space (in this case, vector).²¹ We set $\beta_1 = 1$, thus obtaining the cointegrating vector²²

$$\hat{\beta}' = \begin{bmatrix} \text{BaC} & \text{CCy} & \text{CNC} & \text{Cap} & \text{EnU} & \text{Fin} & \text{TMT} \\ 1 & -3.83 & -5.39 & 2.82 & 18.50 & -15.10 & -0.26 \\ (0.00) & (1.76) & (1.31) & (0.76) & (3.29) & (2.30) & (0.11) \\ \\ \dot{i}_t & \dot{i}_{t-1} & \dot{i}_{t-2} & y_t & y_{t-1} & y_{t-2} & us_t & us_{t-1} & us_{t-2} & oil_t & oil_{t-1} & oil_{t-2} \\ 0.65 & -0.83 & 0.15 & -0.08 & 0.05 & 0.03 & -0.06 & 0.06 & 0.03 & 0.03 & 0.00 & 0.05 \\ (0.20) & (0.28) & (0.20) & (0.04) & (0.04) & (0.04) & (0.01) & (0.01) & (0.01) & (0.03) & (0.03) & (0.03) \end{bmatrix}.$$

²⁰ See Hendry and Juselius (2000b, pp 22-24) for an elaboration on the preferability of treating near unit roots as unit roots.

²¹ In general, just identification can be achieved by imposing one appropriate normalisation (ensuring that this coefficient is non-zero) and $r - 1$ restrictions on each β_i .

²² Standard errors in parenthesis.

Table 8
Model in its non-integrated form

		Equation													
		ΔBaC		ΔCCy		ΔCNC		ΔCap		ΔEnU		ΔFin		ΔTMT	
Regressor	t	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val	coeff	p-val
ΔBaC	-1	-0.055	0.682	-0.065	0.511	-0.144	0.044	-0.167	0.343	-0.057	0.238	-0.050	0.181	-0.334	0.295
ΔCCy	-1	0.573	0.001	0.325	0.010	0.337	0.000	0.937	0.000	0.213	0.001	0.108	0.023	1.909	0.000
ΔCNC	-1	-0.442	0.025	-0.061	0.672	-0.068	0.511	-0.735	0.005	-0.090	0.204	-0.033	0.548	-1.417	0.003
ΔCap	-1	0.021	0.807	0.046	0.473	0.017	0.709	-0.084	0.463	-0.042	0.182	-0.002	0.931	-0.422	0.045
ΔEnU	-1	0.080	0.768	0.224	0.265	0.080	0.579	0.574	0.110	-0.177	0.072	-0.043	0.572	-0.693	0.287
ΔFin	-1	0.308	0.376	-0.021	0.936	0.063	0.733	-0.013	0.978	0.092	0.461	0.078	0.424	-0.039	0.962
ΔTMT	-1	0.020	0.437	0.031	0.107	0.014	0.301	0.071	0.041	0.030	0.002	-0.003	0.667	0.312	0.000
CI	-1	0.002	0.688	0.000	0.900	-0.003	0.210	0.016	0.015	0.002	0.240	-0.003	0.014	-0.043	0.000
c		-0.007	0.703	0.004	0.733	0.014	0.127	-0.052	0.023	-0.008	0.225	0.012	0.017	0.170	0.000
FinT		0.184	0.000	0.257	0.000	0.159	0.000	0.479	0.000	0.056	0.000	0.031	0.000	1.689	0.000
S11		0.329	0.000	0.281	0.000	0.193	0.000	0.388	0.000	0.019	0.137	0.035	0.001	0.895	0.000
ERM		0.114	0.000	0.092	0.000	0.038	0.005	0.098	0.004	0.039	0.000	0.050	0.000	0.082	0.180

Also a factor important to the fragility of a number of sectors, changes in the risk profile of the CNC sector show a *negative* impact on the risk profile of the sectors that are statistically significant to them, notably BaC, Cap and TMT. The negative and generally large coefficient in these relations may capture some sort of impact of investment strategies seeking more cyclical targets once the non-cyclical sectors show signs of fragility. This stylised view of seeking “faster-rebounding” investments is reinforced by the large coefficient in the TMT sector, usually a strong performer in economic rebounds. The other sector whose fragility shows correlation with that of other industries, the TMT industry, has positive coefficients, denoting the standard positive sign of “contagion”, albeit the scale is smaller, except for the autoregressive term, which shows how strong the persistence is in the risk process of this sector.

The adjustment coefficients do not point to any peculiarity other than long-term adjustment not significantly affecting more than three sectors, of which the negative impact on the TMT sector stands out. The deterministic shocks show a significant impact on risk across sectors. Only the EnU and Fin industries show a small response to the deterministic components.

Overall, the model performs reasonably well, as portrayed by the goodness of fit measures: the R^2 based on the likelihood ratio is 0.83, whereas that based on the Lagrange multiplier reports a lower 0.19. Model diagnostics also suggest that some problems are still present in the TMT equation, where the errors show some degree of kurtosis, and serial correlation, thereby suggesting that the results from this equation be viewed with caution. Notwithstanding this, the remaining equations perform well.

Two main points stand out in the analysis so far. The first underlines the fact that the very simple structure we imposed on the system already reveals great complexity in the cross-industry risk linkages. The interaction between the median EDF measures across sectors depicted by the VAR model has two interesting components: across industries the correlations may be negative, suggesting some “complementarity” property probably driven by large capital movements across sectors, and the time dimension suggests that some sectors, notably consumer cyclicals, serve as shock transmission channels, also potentially signalling early warning properties.

The second property supported by the model is the perception that systemic variables, in particular those related to the macroeconomy, do not appear to influence the behaviour of the series at the monthly frequency. This result is to be taken with caution, as there are a number of simplifications assumed in constructing the systemic variables. Nonetheless, the results are at least suggestive of other factors driving the behaviour of a significant proportion of risk in the sectors. Also supporting this perception is the important role of deterministic components (dummies), which capture the sensitivity of risk to events outside the economic framework.

4. Concluding remarks

This exercise is an example of the use of market-based information in the assessment of fairly aggregated sectoral fragility. While the preliminary results are encouraging in terms of both statistical fit and modelling usefulness, the model could benefit from having longer time series covering a full macroeconomic cycle, currently not fully available.

The results from the model underline three factors defining risk across economic sectors. The first one refers to the important observation that risk modelling ought to consider the important cross-dynamics transmitting risk across industries and time. It appears that some progress can be made in modelling the structure through which risk is propagated across sectors and time, and that imposing further restrictions on the reduced form of the VAR may well provide further insight about the structure of risk transmission. The second element is the notion that risk exhibits evidence of evolving to a long-run equilibrium. Systemic and macroeconomic factors affect the steady state levels and thus represent important determinants of the steady state risk profile of most of the sectors. Ignoring this interaction weakens the strength of the forecasts, which benefit from incorporating this significant adjustment factor. The final element is a word of caution. The model fails to detect a large significant impact stemming from macroeconomic and systemic elements, and a substantial share of the variation in risk across industries remains unaccounted for. This outcome may be uncovering some uncomfortable results, namely that much of the change in risk profiles is driven by elements that are independent of the economic performance, possibly owing to some herding factor.

A number of potential applications for the framework are possible. For example, and in combination with aggregate sectoral exposure data, one could assess the exposure at risk of the banking sector's lending portfolio, as well as its variation in selected scenarios for the systemic variables (scenario or stress testing). In addition, one may want to use the model to assess future levels of risk by sector, or in the aggregate, benefiting from information on the interaction of risk across industries and the forces driving these to the long-run equilibrium. Furthermore, the sensitivity of sectoral risk to other factors, including risk in other sectors, can be further refined, thus allowing for a more refined assessment of optimal investment strategies across sectors.

The exercise lends itself to a number of improvements and variations. A key characteristic of the model is the substantial amount of volatility experienced in periods of economic distress. We have accounted for this by carefully constructing deterministic variables characterising the main aspects of these periods. Some preliminary analysis with Markov-switching VARs suggests that this may indeed lead to a potential improvement in the model by pinning down the factors affecting the transition probabilities between states of high and low volatility. This strategy would require careful consideration of the type of switching mechanism that would best suit the properties of the median EDF. A second constraint on this exercise is the high number of parameters needing estimation. A possible way forward is to implement Bayesian estimation techniques reducing the number of parameters by imposing prior restrictions based on the experience gained so far. It is difficult to assess how much mileage to extract from this, however, given that the persistence of the estimated VAR appears to be quite low (one lag sufficed to remove the serial correlation of the errors, except perhaps in the TMT sector) and the substantial differences in the behaviour of the probability of default across sectors. Finally, and in the light of the weak explanatory power of the systemic indicators (exogenous variables), further investigation would be appropriate in determining the nature of the common factors driving the movement in the median EDF measures. Two aspects affecting risk developments are (i) performance announcements and (ii) estimated future outlooks. These elements suggest that one looks at either contemporaneous *expected* values, or future values of the exogenous variables (assuming that forecasts are correct). It is difficult to argue, however, that relatively high-frequency information will be driven by lower-frequency announcements, so it may be desirable to look at the behaviour of trends (smoothed data).

Each of these issues may deserve some attention in the future.

Appendix A: Financial fragility

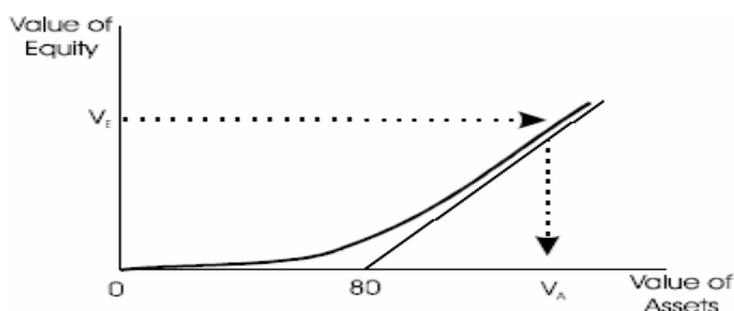
Among the measures that have been proposed to gauge corporate fragility, option-based indicators, such as KMV's expected default frequency (EDF), have shown to have desirable leading-indicator properties. Theoretically founded on the well known Black-Scholes option-pricing equation, they combine three elementary components (assets' value, their risk and the firm's leverage) into a unique measure of default risk.²³ Their ability both to predict overall levels of defaults and to discriminate between defaults and non-defaults is known and valued among practitioners.²⁴ EDFs can be directly used in calculating exposures at risk whereas alternative forward-looking indicators (subordinated debt spreads or equity price implied volatility) have to first be converted into a meaningful measure of probability of default. Together with proper recovery rates, they can also serve to assess loss-given-default. In this section we first derive the firm's EDF from high-frequency market and financial information on the firm. For further details, see Crosbie and Bohn (2002).

A.1 Corporate fragility

The practical approach implemented by KMV rests on three basic steps: the estimation of asset value and volatility, the calculation of the distance-to-default measure, and the derivation of the EDF. The first step in this derivation is based on the observation that *equity is essentially the same as a call option on the firm's assets with a strike price equal to the book value of the firm's liabilities* (at liquidation). The option nature of equity serves to derive the underlying market value and volatility of the firm's assets, the volatility of equity, and the book value of liabilities.²⁵ This process is similar in spirit to the procedure used by option traders in the determination of the implied volatility of an option from the observed option price and exploits the close relationship between the value of debt and that of equity as they are both really derivative securities on the underlying assets of the firm. The option nature of equity can be thus exploited to relate the market value of equity and the book value of debt to determine the implied market value of the underlying assets. Graph 5 illustrates the derivation of the market value of assets V_A from the value of equity V_E and an option-pricing relationship (thick line) for a simple leveraged mutual fund.

Graph 5

Derivation of the value of assets V_A from the value of equity V_E



²³ See Crosbie and Bohn (2002) for further details on the construction of the distances to default on which EDFs are based.

²⁴ KMV has produced a number of technical documents on the subject. See, for instance, Kurbat and Korablev (2002) for further references on the subject.

²⁵ The model was developed at KMV by Oldrich Vasicek and Stephen Kealhofer as an extension of the Black-Scholes-Merton framework and is known as the Vasicek-Kealhofer (VK) model. This model assumes that the firm's equity is a perpetual option with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point, the firm is assumed to default. Multiple classes of liabilities are modelled: short-term liabilities, long-term liabilities, convertible debt, preferred equity, and common equity. When the firm's asset value becomes very large, the convertible securities are assumed to convert and dilute the existing equity. In addition, cash payouts such as dividends are explicitly used in the VK model. See Crosbie and Bohn (2002) for further details.

In fact, accounting for more complicated examples, the value of assets V_A and its volatility σ_A are derived from the following simultaneous relationships:

$$V_E = \text{Option Function} \left(V_A, \sigma_A, \begin{bmatrix} \text{Capital} \\ \text{Structure} \end{bmatrix}, \begin{bmatrix} \text{Interest} \\ \text{rate} \end{bmatrix} \right)$$

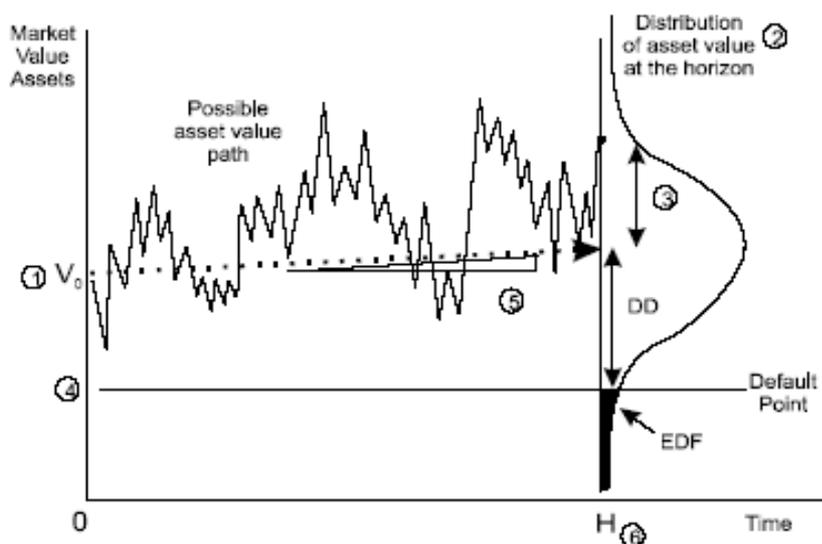
$$\sigma_E = \text{Option Function} \left(V_A, \sigma_A, \begin{bmatrix} \text{Capital} \\ \text{Structure} \end{bmatrix}, \begin{bmatrix} \text{Interest} \\ \text{rate} \end{bmatrix} \right)$$

from where it is clear that only the market value of assets V_A and its volatility σ_A are unknown. Both are derived by solving the relationships from the other known variables.

The second step involves the calculation of the distances to default and requires six measures. Considering the horizon from now until H , the variables required are: (1) the current asset value, (2) the distribution of the asset value at time H , (3) the volatility of the future assets at time H , (4) the level of the default point (book value of liabilities), (5) the expected rate of growth in the asset value over the horizon, and (6) the length of the horizon H . These elements are illustrated in Graph 6.

Graph 6

Calculation of the distance to default



The first four (asset value, future asset distribution, asset volatility and the level of the default point) are the main variables, as the expected growth in the asset value has little default discriminating power and the analyst defines the length of the horizon. If the future distribution of the distance to default were known, the default probability (EDF value) would simply be the likelihood that the final asset value was below the default point (the shaded area in Graph 6). In practice, however, the distribution of the distance to default is difficult to access, as the usual normal or log-normal distributional assumptions cannot be used. The likelihood of large adverse changes in the relationship of asset value to the firm's default point is critical to the accurate determination of the default probability. These changes may come about from changes in asset value or changes in the firm's leverage. In fact, changes in asset value and changes in firm leverage may be highly correlated. Consequently, the distance to default is first measured as the number of standard deviations the asset value is away from default:

$$DD = \frac{V_A - \begin{bmatrix} \text{Default} \\ \text{Point} \end{bmatrix}}{V_A \sigma_A}$$

Empirical data are then used in the third and final stage to determine the corresponding default probability. KMV obtains the relationship between distance to historical default and bankruptcy frequencies from a database including over 250,000 company years of data and over 4,700 incidents of default or bankruptcy. From these data, a look up or frequency table can be generated which relates the likelihood of default to various levels of distance to default.²⁶

A.2 Sectoral fragility

Forward-looking indicators assist in predicting the trend of the expected bank losses in the near future. As we have seen, EDFs provide an approximation of the expected probability of default for individual firms. Each firm is associated with an industry and thus *industry risk* measures can be constructed in a number of ways. For example, we could resort to the industry's median EDF, a weighted average (by market asset value or liabilities, for instance) EDF, or other kernel measures aggregating firms' EDFs. The weighted average effectively incorporates information on the *large players* affecting the sector's riskiness, but is subject to spurious variation due to classification changes, especially of large players. Because the problem with weighted averages may be significant in our data sample, we instead obtain sectoral measures of risk by grouping firms into sectors and taking the median of those.²⁷ Likewise, an aggregate measure of risk can be derived by considering the whole population in the sample from where the median is drawn. Denoting a group of firms in our data set by J and a firm j 's EDF by p_j , our measure of risk for group J , r_J , is therefore the median p_j of the group.

²⁶ The relationship between distance to default and default frequency for industry, size, time and other effects was tested by KMV and was found constant. This is not to say that there are no differences in default rates across industry, time and size but only that it appears that these differences are captured by the distance-to-default measure.

²⁷ Other measures have been suggested that are less subject to the spurious fluctuation due to classification changes while at the same time providing a greater significance to the large players. One measure that could be implemented is the median of the n largest (by liabilities) corporations. Some sensitivity analysis would be required for establishing the optimal n and this option could be considered in future work.

Appendix B: Preliminary model specification

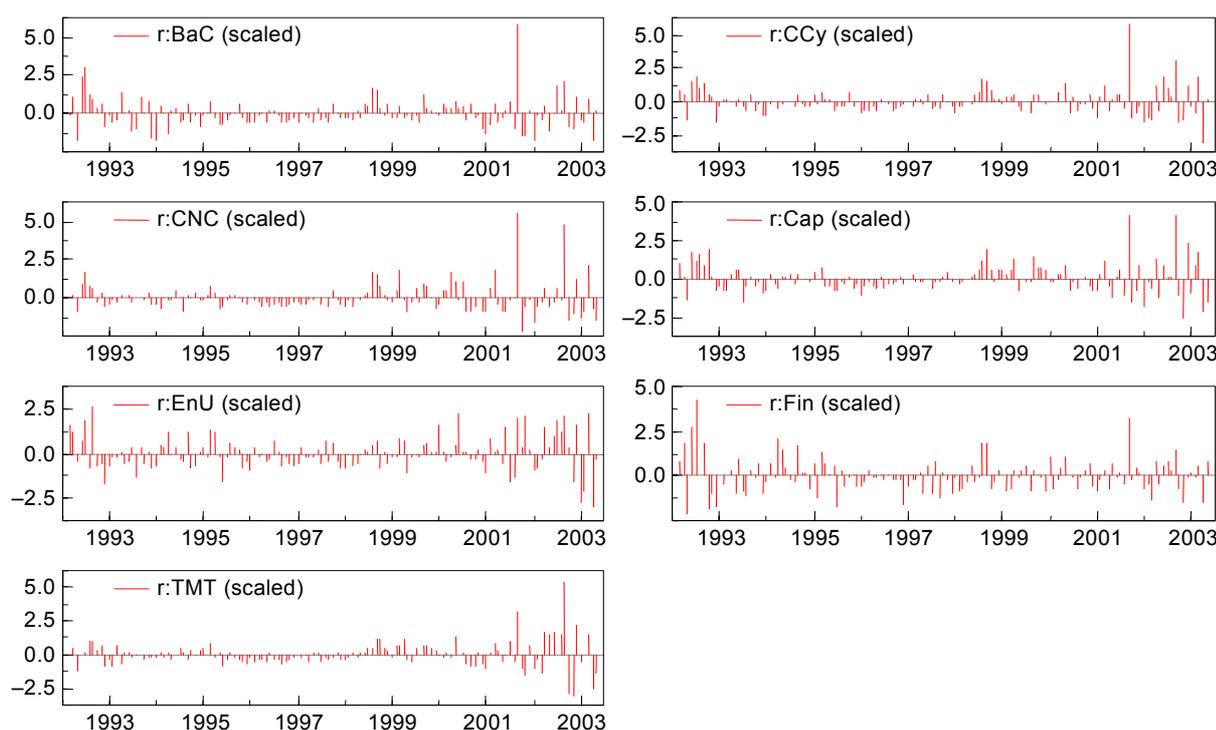
In the spirit of Hendry and Juselius (2000b), we begin by first tentatively estimating a VAR system under the presumption that the risk processes are not integrated, as in equation (3), excluding the dependency of systemic and macroeconomic effects.²⁸

In this appendix, we verify the model's specification congruency with the data on the basis of the three core criteria for statistical inference identified by Hendry and Juselius (2000b): parameter constancy, serially uncorrelated residuals, and residual skewness.²⁹

Two monthly lags suffice to account for the 12-month autocorrelation of four of the sectoral equations.³⁰ The remaining large errors (Graph 7) are concentrated in summer 1992 (ERM crisis), early autumn 2001 (events of 11 September), and autumn 2002 (financial market turbulence associated with uncertainty over the impact of the bursting of the equity market bubble). In addition, the volatility of the residuals appears higher towards the end of the sample, suggesting some form of non-linearity in the system.

Graph 7

Residuals in the closed VAR with no deterministic or exogenous variables



²⁸ We thereby first focus on selecting the most parsimonious specification that does not show residual serial correlation. Selecting a parsimonious lag order is also recommended for testing the cointegration rank, as elaborated by Ho and Sorensen (1996). One could alternatively resort to standard tests for the optimal lag. See Doornik and Hendry (2001), Hansen and Juselius (2002) or Enders (1996) for further material on this subject and references to original theoretical work.

²⁹ The software package used for estimation is GiveWin version 2.2.

³⁰ The tests for 12-lag serial correlation do not reject, at the 1% level of significance, the residuals for the consumer cyclical (CCy) and non-cyclical (CNC) goods and technology and telecommunications (TMT) sectors, which are still subject to very large shocks affecting the tests for serial correlation. Because of these shocks, incorporating additional lags does not correct for the pattern captured by the serial correlation tests.

We therefore correct for the presence of deterministic exogenous shocks by adding shock dummies for the periods of exogenous instability (summer 1992, autumn 2001, and the turbulent period in financial markets in autumn 2002).³¹ The dummies used in the exercise are “neutral” over time or mean-zero, as for example the Fall01 dummy, which equals 1 in September and –1 in October 2001 (and zero otherwise). In addition, and in agreement with the discontinuities observed in Graph 1, the FinT dummy has scaled values in the period from June 2002 to May 2003 which add up to zero, similar to the ERM dummy in the period June–December 1992.³²

Accounting for deterministic shocks noticeably corrects the congruence of the model, with the three shocks being significant at the 1% level. Serial correlation (12 lags) remains insignificant at the 1% level of significance in all industries except for the technology and telecommunications (TMT) sector, where large shocks in April and May 2002 continue to account for errors larger than 3.5 standard deviations, suggesting that some sector-specific factors have still not been accounted for.³³ The persistent presence of large shocks also affects multivariate tests on serial correlation. As pointed out by Hendry and Juselius (2000b, p 6), however, in economic applications the multivariate normality and serial correlation are seldom satisfied, and accurate inference must rely on the careful interpretation of remaining problems.

Most one-step residuals of a recursive estimation are within two standard deviations, indicating parameter constancy.³⁴ Owing to the more volatile environment surrounding the bursting of the equity bubble, errors also appear heteroskedastic, except for the energy and utilities (EnU) and TMT sectors, where the assumption of homoskedastic errors cannot be rejected at the 5% level of significance. The assumption of normality can be rejected at the 5% level, with the exception again of the EnU and TMT sectors. A closer inspection of Graph 8, displaying the errors’ residual density, suggests that some unusual spikes (long tails in the distributions) are at the source in the case of the TMT industry. For the EnU sector, and less markedly for the consumer non-cyclical goods (CNC) sector, the distribution of the errors is clearly skewed to the left, suggesting that a number of “negative” shocks have not been accounted for yet.

It is worth noting that, overall, error distributions are not skewed. Lack of residual skewness, in contrast to lack of kurtosis, is identified by Hendry and Juselius (2000b, p 7) as an important requirement for correct model specification. Statistical inference is moderately more robust to the validity of the latter.

The eigenvalues of the companion matrix suggest that the system is stable (see, for example, Hendry and Juselius (2000b, Section 3.4)), as all of the 14 ($2 * p = 14$) moduli of the eigenvalues of the companion matrix are inside the unit circle (two moduli are close to 0.98, and four above 0.92). The fact that a number of eigenvalues are close to the unit circle also indicates the possibility of a stochastic trend, and suggests that the processes may be cointegrated.

³¹ The econometric implications of using indicators (dummy) variables are discussed in Doornik et al (1998, Section 2.2).

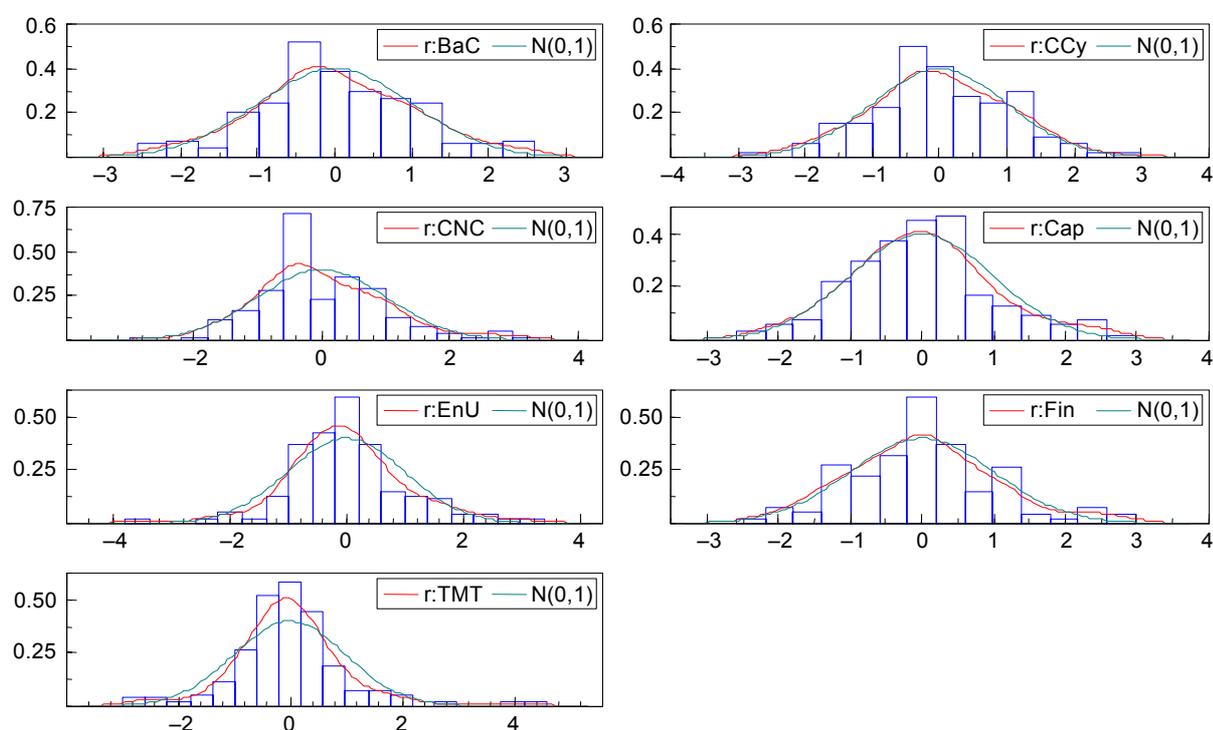
³² For example, within FinT June has weight 0.25, September 2002 1, October 2002 –0.74, November 2002 –0.5, December 2002 0.25, March 2003 0.25, and April 2003 –0.75.

³³ The very peculiar behaviour of the TMT sector requires some special attention. Clearly, developments in this sector have influenced to a significant degree risk in financial markets after March 2000.

³⁴ The recursive estimation was carried out over 50 periods. Only the TMT sector has a number of spikes outside the two-standard deviation benchmark for the one-step residuals. These are only towards the end of the sample, denoting the profound change that has taken place in this sector since late 2000.

Graph 8

Residual error density in the closed model with deterministic components



By way of a prelude to the cointegration analysis of the full model, we briefly look at the integrated nature of the risk series. Testing the cointegration rank r of the closed system suggests the presence of two cointegration relationships between the risk processes ($r = 2$).³⁵ The results are tabulated in Table 9 below.

Table 9

Cointegration analysis of the closed model

rank (i)	λ_i	loglik	$H_0: r \leq i$			
			Trace test		Max test	
0		2019.149	173.01	[0.000]**	61.70	[0.000]**
1	0.38017	2051.675	111.30	[0.002]**	48.00	[0.003]**
2	0.31070	2076.976	63.31	[0.147]	33.41	[0.053]
3	0.22815	2094.585	29.90	[0.725]	14.27	[0.800]
4	0.10475	2102.109	15.63	[0.743]	7.39	[0.928]
5	0.055647	2106.003	8.24	[0.447]	4.40	[0.811]
6	0.033546	2108.323	3.84	[0.050]	3.84	[0.050]
7	0.029321	2110.347				

³⁵ The sequence of trace tests used in the determination of the cointegration rank represents a consistent procedure, as elaborated in Doornik and Hendry (2001). We report in Table 9 the T-nm tests.

Both the rank trace and maximum likelihood tests detect at least one cointegrating relationship and possibly two (the third one is only detected by one test). We consider two cointegrating relationships and will turn to a more refined procedure for selecting the cointegrating rank for the full model. On the basis of the rank estimation, the cointegrated model (rank 2) yields cointegration matrices α and β given in Table 10.

Table 10
Cointegration matrices of the closed model

	β		α	
BaC	1.00	-0.97	-0.064 (0.029)	-0.013 (0.050)
CCy	-3.71	1.00	-0.029 (0.022)	-0.124 (0.039)
CNC	-0.12	-1.14	0.006 (0.014)	-0.053 (0.025)
Cap	1.12	0.62	-0.185 (0.034)	-0.167 (0.060)
EnU	5.45	1.76	-0.025 (0.011)	-0.036 (0.018)
Fin	-2.56	-0.75	-0.001 (0.008)	0.024 (0.014)
TMT	0.11	-0.20	-0.388 (0.072)	0.107 (0.124)

Two groups of industries are identified by the two cointegrating relationships β in the closed system: the BaC, CNC, Cap and TMT sectors on the one hand, and the CCy, EnU and Fin sectors on the other. Within the first group, BaC, CNC and TMT are “substitutes” for each other (their β coefficients share a common sign in each vector β) and “complements” to Cap across cointegrating relationships. In the second group, CCy and Fin are “substitutes” for each other but “complements” to EnU across relationships. The complementarity/substitutability relationship is constant within groups across cointegrating vectors, but reverts between groups across cointegrating relationships. This suggests that the cointegrating relationships capture distinct effects of different types of shocks on the correlation across groups (even though they affect members within any one group equally). The degree of this complementarity/substitutability is slightly different in each cointegrating relationship, also pointing to the distinct magnitude of the shocks’ impact.

The adjustment to the different cointegrating relationships is also revealing. Only the Cap, Fin and TMT sectors adjust to the first error correction relationship (as denoted by the significance of the α coefficients in each equation). The CCy, CNC and (again) Cap and TMT adjust to the other relationship. The BaC and EnU sectors do not appear to adjust to either of the error correction factors, and are therefore weakly exogenous, ie risk levels in the latter sectors do not respond to deviations from long-term risk “equilibria”. Both sectors being at the first stage of the production chain suggests that they enjoy greater independence from the economic relationships tying the remaining sectors’ long-term equilibria. A closer look at the sectors that adjust to deviations from long-term equilibria could provide an indication of the nature of the cointegrating factor. In this light, the latter group (CCY, CNC, Cap and TMT) appears to capture a cointegration resulting from consumption, whereas the second group (Cap, Fin and TMT) one from investment cycles.

Appendix C: A forecasting framework

The error correction specification (cointegration relation) significantly restricts the model and its forecasts, as the variables in the cointegrating relationship will adjust to their long-run equilibrium. This is an important factor driving some of the short-run dynamics. Naturally, the forecasts are affected by this characteristic, tending to convergence to the level specified by the long-run components. Importantly from the results of the previous section, this long-run equilibrium will also embody the path that we exogenously assumed for the systemic variables, as they will drive the stochastic trend.

In order to make a forward assessment of the evolution of risk in the different industries, we require assumptions about the behaviour of the exogenous variables in our model. Indeed, Graph 2 above presents a *baseline scenario* for the six months following May 2003 (the last date for which data are available on EDFs), together with a *scenario of a deepening recession*. Future values on industrial output, oil prices and the US stock exchange consistent with these two scenarios are fed into the model to obtain out-of-sample forecasts.³⁶

C.1 Forecasts with the integrated series

We first look at the forecasts from the restricted integrated model. The implied baseline scenario risk measures from June to November 2003 are displayed in Graph 9, from where it is clear that the model foresees the gradual reduction of the expected default frequency. This trend extends the very strong correction in April and May 2003, whose impact appears to persistently drive risk down across industries.

Standard error bars³⁷ are displayed around the forecast values to illustrate their uncertainty. Whereas the model suggests that risk will decrease to different degrees in all sectors, significant uncertainty surrounds the forecast values. With the exception of the Fin sector, the model still considers a deterioration in risk possible within a standard deviation. It must be noted, however, that much of the improvement forecast in the risk series following the substantial easing in risk that materialised in April and May 2003 is also reinforced by the high persistence in the industrial risk measures.

Indeed, much of the same persistence drives the forecasts under the more adverse recession scenario, where much of the same pattern applies to forecast industry risk. The high persistence in the risk series dominates the negative pull of the assumed systemic variables. Whereas risk in all industries is higher than under the baseline scenario, the change is small in relation to the levels in the risk measures. These forecasts are shown in Graph 10.

As already emphasised in Section 3.4 above, the limited effect of the systemic factors is also present in the forecasts: substantially distinct paths for the exogenous variables do not cause a reversal in the trend of the risk measures. However, their impact is not negligible, as noted by the higher forecast risk levels across industries under the recession scenarios. In particular, the possibility of a trend reversal in the financial sector (Fin) is well within a standard deviation under the more adverse conditions.

C.2 Forecasts with non-integrated series (in the $I(0)$ space)

The forecasting in the $I(0)$ space should be more accurate, as the cointegrating vectors become endogenous in a simultaneous equation model, and indeed the pattern for the risk forecasts depicted has some interesting differences.

The baseline forecast in the $I(0)$ space is given in Graph 11, showing no remarkable differences with the regression in the integrated space shown in Graph 9.

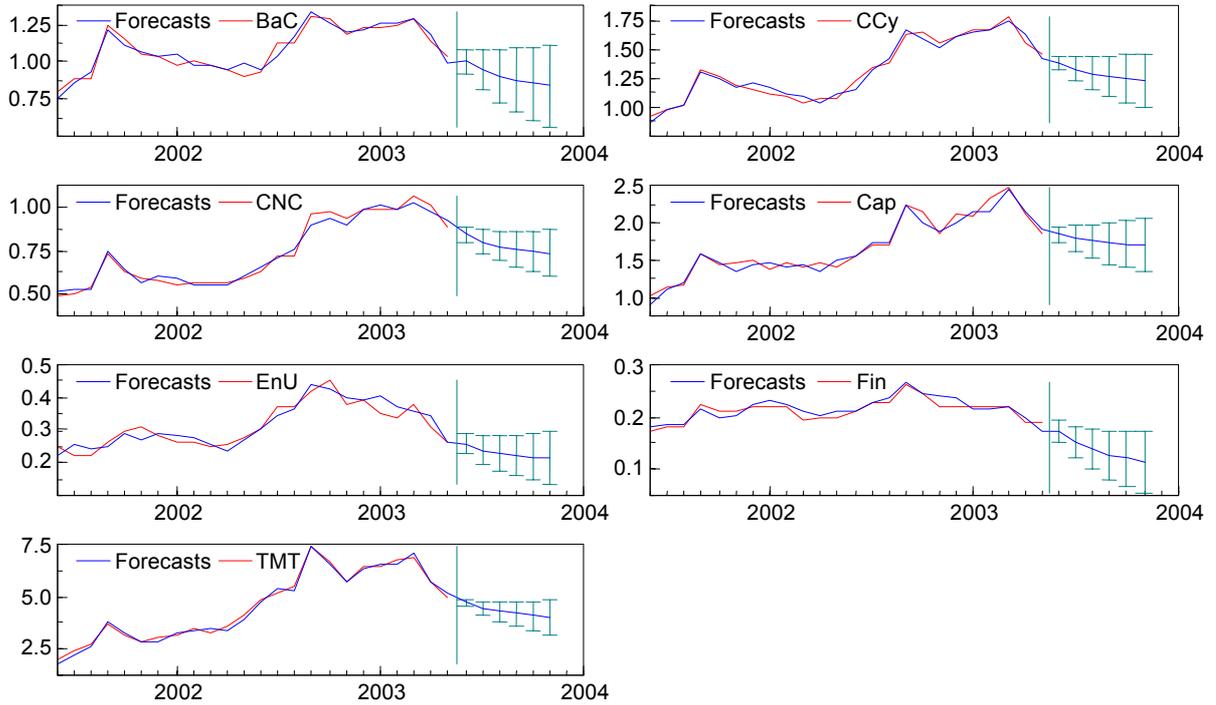
Similar comments apply to the forecast under the recessionary scenario.

³⁶ The baseline scenario is based on the April 2003 ECB forecast exercise.

³⁷ On the basis of error variance only, ie does not include parameter uncertainty.

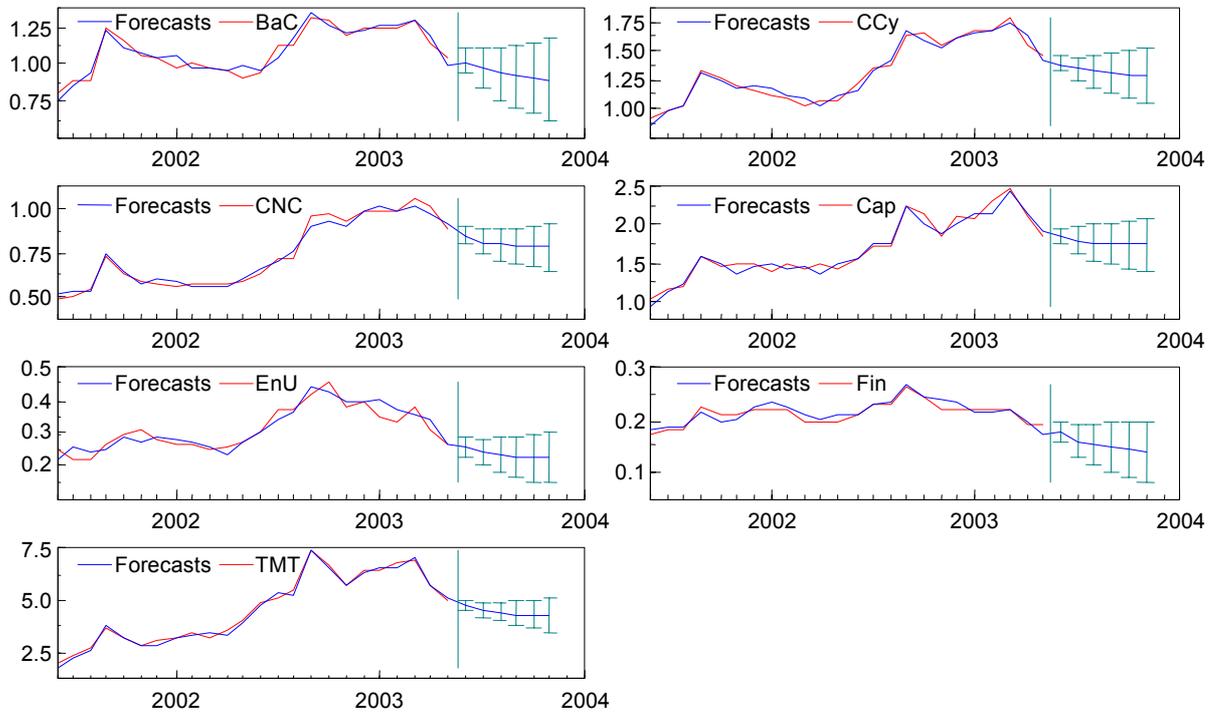
Graph 9

Baseline out-of-sample forecasts for industrial risk measures



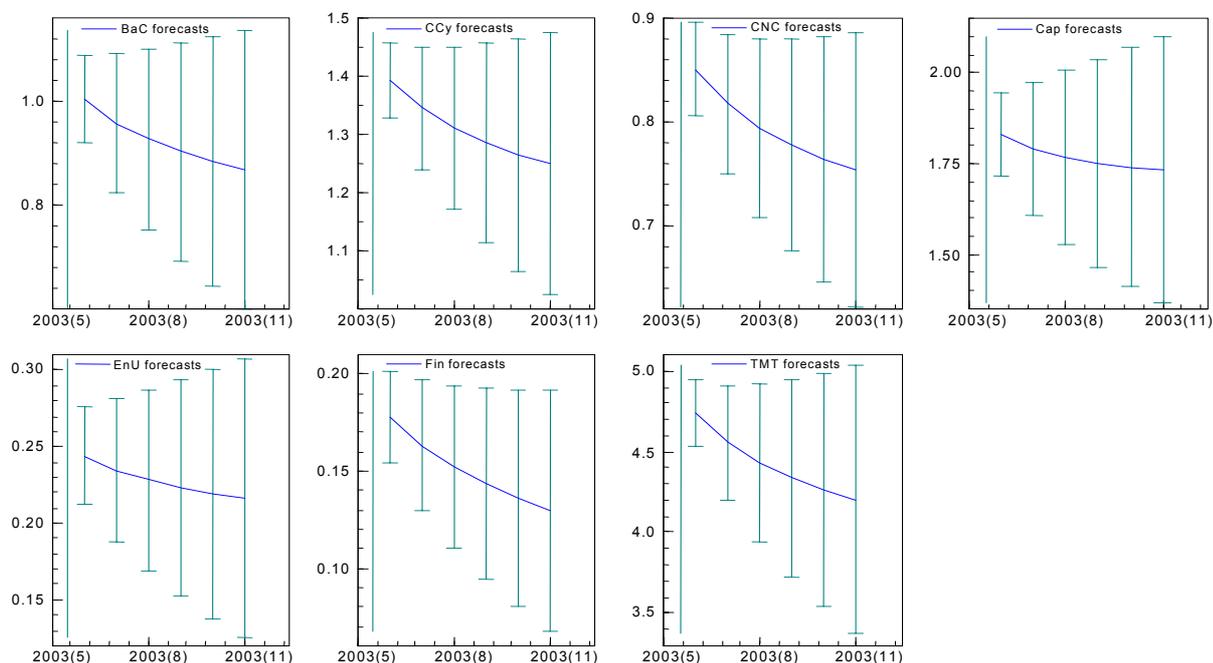
Graph 10

Recessionary out-of-sample forecasts for industrial risk measures



Graph 11

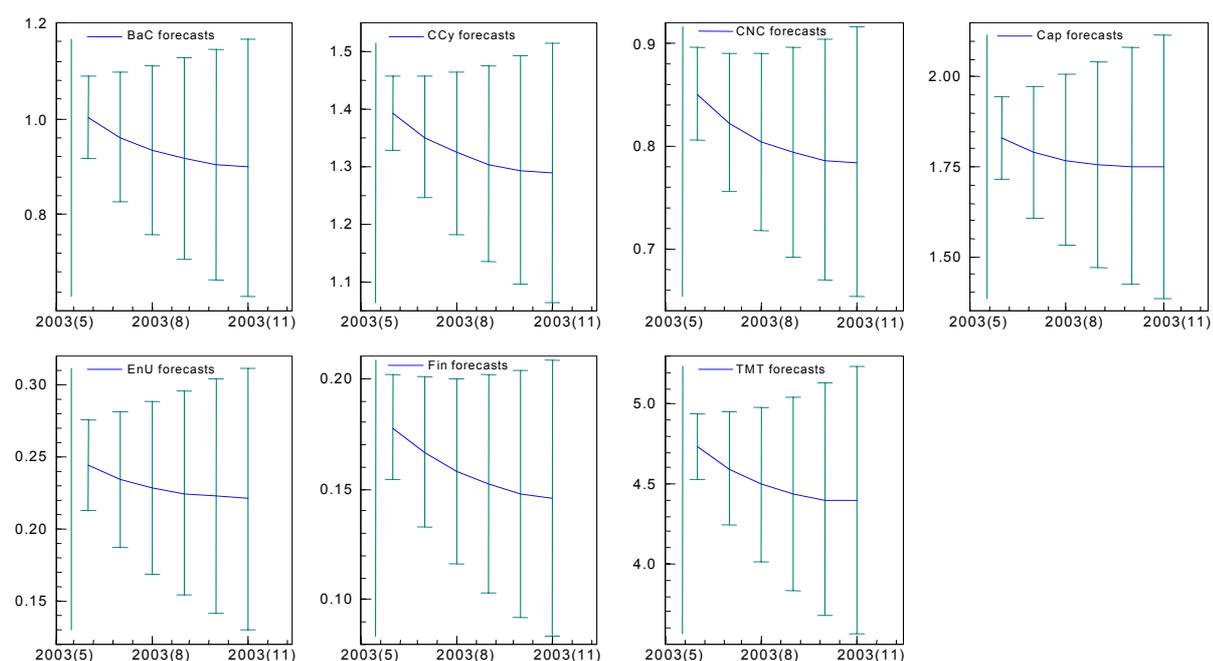
Baseline out-of-sample forecasts for industrial risk measures with error correction



Overall, the combined effect of strong common factors driving the dynamics of the system and the significant feedback mechanisms linking risk across sectors appear as overwhelmingly more predominant in determining the short outlook of sectoral risk.

Graph 12

Recessionary out-of-sample forecasts for industrial risk measures with error correction



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