Business cycle, credit risk and economic capital determination by commercial banks

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1. Introduction

Regular assessments of the default risk of bank clients and estimations of credit risk at the portfolio level are becoming a necessity for banks in their daily operations. The design of optimal lending contracts and the need to conform to new regulatory trends constitute at least two reasons why banks have to pay closer attention to quantitative methods for assessing the credit risk of their clients. While primarily designed for use in commercial banks, credit risk models have recently started to attract the attention of other groups of economic professionals. It is the supervisory function of central banks that is mostly triggering the interest in examining credit risk models in this environment. In addition, an overall assessment of the creditworthiness of domestic firms has implications for the conduct of monetary policy. These and other reasons have prompted several central banks in Europe to develop and implement their own models for monitoring the financial situation of domestic firms and the lending performance of domestic banks.[2]

The objective of this paper is to develop an assessment technique for analysing the impact of different risk-based capital requirement rules on the potential needs for capital in the Czech banking sector. For this purpose, we apply these methods to an artificially constructed risky loan portfolio. The latter reflects a number of prominent features of Czech non-financial borrowers.

When defining the creditworthiness characteristics of the loan portfolio, we apply the Moody's KMV method for rating private firms. To determine capital requirements for this portfolio, we use the New Basel Capital Accord (NBCA) and the CreditMetrics and CreditRisk+ models. In the context of CreditMetrics, we are able to conduct stress testing to gauge the impact of interest rate uncertainty (eg caused by changes in monetary policy and different reactions of the yield curve to these changes) on economic capital calculations. In addition, we describe an independent debt valuation model similar to that of KMV and outline the techniques for its numerical implementation. The proposed model has a substantial advantage over the previously mentioned ones in that it addresses three key problems of credit risk modelling. Namely, this model, although remaining in the KMV line of analysis:

- incorporates macroeconomic systemic factors, such as position in the business cycle, interest rate and exchange rate volatility and the monetary policy stance, when deriving a valuation of bank lending risks;
- combines the features of structural and reduced-form models of debt valuation;
- offers a framework for assessing the influence of market risk factors on credit risk in a bank loan portfolio.

In the latter respect, our model advances towards an *integrated financial risk assessment methodology*, which has recently been called for in the risk analysis literature (see, for instance, Barnhill and Maxwell (2002), or Hou (2002)).

The principal feature of the paper is a comparative analysis of the predictions of these models when applied to an artificially created loan portfolio constructed using Czech data. Another important

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[2] Rating systems and creditworthiness assessment models for firms have been developed, among others, by the central banks of Austria, France, Germany, Italy and the United Kingdom.

contribution is a demonstration, even if in an incipient manner, of the way in which scarce and usually unavailable variables can be estimated or proxied to obtain the inputs required by the credit risk models. One oft-mentioned drawback of credit risk modelling is the difficulty with which the credit risk analyst can access the required input data. This problem was also present in our case. Although overcome, the data problem has had negative implications for the robustness of our results. Thus, from this perspective we have to look at the paper's findings with caution. However, the insight into credit risk modelling that is offered here can be extended at a later stage when more data is available. We also hope that our findings may be of use to banking supervisors when these issues become a matter of regulatory practice.

Although credit risk models often prove useful for other purposes, their main merit rests in estimating the capital level that banks have to maintain over the given risk horizon. The outcome is called regulatory capital in regulatory terms and economic capital in terms of credit risk modelling. Both regulatory and economic capital are supposed to cover unexpected losses resulting from banks' lending operations to clients with different levels of default risk. Whereas holding regulatory capital is compulsory as a part of adherence to prudential regulations, holding economic capital beyond the minimum required level is the banks' own choice. Worth mentioning, however, is the regulatory tendency to come closer to credit risk modelling and to allow banks to develop their own models for determining the amount of regulatory capital to hold. These models will most probably adopt and synthesise many features of the credit risk models already in use. This is one reason why comparing regulatory and economic capital today is becoming an insightful exercise for the regulatory decisions of the future.

1.1 Literature review

In June 1999, the Basel Committee on Banking Supervision released a proposal to replace the 1988 Basel Capital Accord with a more risk-sensitive framework. A concrete proposal in the form of a consultative document, the New Basel Capital Accord (NBCA), was presented in January 2001. This document proposed new regulatory rules for banks' capital adequacy evaluations. The main innovations related to credit and operational risk. In terms of credit risk, the NBCA revised the 1988 Accord by proposing a more risk-sensitive methodology for assessing the default risk of banks' clients. The risk inputs entering the final capital adequacy computations were closely related to the risk characteristics of individual bank clients. In this sense, the proposed methodology opted for the adoption of ratings (developed by external agencies or by banks themselves) in quantifying and signalling to the bank the default risk of individual borrowers. In a simpler version of the methodology (the standardised approach), ratings are directly associated with risk weights (for example, an A-rated asset would be assigned a risk weight of 50%, a BBB-rated asset would be assigned a risk weight of 100%, and so on). In the more advanced internal ratings-based (IRB) approach, ratings represent the basis for computing the probability of an obligor's default. Default probabilities and other risk characteristics (loss-given-default, exposure at default) enter more complicated formulas for determining the risk weights of individual assets in regulatory capital estimations.

In the banking industry, credit risk modelling has also been explored and extended since the release of the four major credit risk models at the end of the last decade.[3] In this paper we consider only two such models, CreditMetrics and CreditRisk+, which utilise, respectively, the structural and the reduced-form approach to modelling default risk (see Duffie and Singleton (1998)). In the structural approach, it is assumed that default is triggered when an unobserved variable (obligor's firm asset value) falls below a certain threshold level (firm's outstanding debt). CreditMetrics extends this reasoning to rating downgrades by defining rating class-specific threshold levels that mark the switch from one rating class to another in the event that the firm's standardised asset returns cross these threshold values. In the reduced-form literature, default is modelled as an autonomous stochastic process that is not driven by any variable linked to the obligor firm's capital structure or asset value. Particular formulations for the default process were considered in Jarrow and Turnbull (1995; exponential distribution), Jarrow et al (1997; a continuous Markov chain) and Duffie and Singleton (1998; a stochastic hazard rate process). CreditRisk+ represents the reduced-form approach by

[3] We refer to JPMorgan's CreditMetrics/Credit Manager model, Credit Suisse Financial Products' CreditRisk+, KMV Corporation's KMV model, and McKinsey's CreditPortfolioView.

assuming that the average number of defaults in each homogeneous class of obligors follows a Poisson distribution. The unifying element of the CreditMetrics and CreditRisk+ models is the value-at-risk (VaR) methodology used in quantifying and provisioning for credit risk at the portfolio level. Even though CreditMetrics derives the portfolio value distribution and CreditRisk+ the portfolio loss distribution at the end of the risk horizon, both models estimate economic capital such that unexpected losses are covered by the estimated economic capital within an acceptable confidence level.

The KMV model represents another step towards market-based derivation of economic capital. Similarly to CreditMetrics, it uses the obligor's equity price statistics to derive the value distribution of a given loan. Correlations are obtained automatically from the risk factors that determine the obligor firm value (equity). However, this method requires the assumption of complete markets, the validity of risk neutral asset valuation and tradability of both the obligors' equities and their debt in the bank portfolio. The KMV team offers unspecified remedies in cases where one of these preconditions is not satisfied, but open sources of credit risk literature offer no general solution of these problems.

This paper proposes a way around the said difficulties in the KMV approach by resorting to the so-called pricing kernel methods of asset pricing (comprehensive expositions can be found in, for instance, Campbell et al (1997) and Cochrane (2001)). Asset tradability and market completeness are no longer necessary, and there are numerous possibilities for modelling default events that depend on systemic and idiosyncratic risk factors. Numerical approaches to calculating pricing kernel-based asset values have also been developed in recent years (see, for instance, Ait-Sahalia and Lo (2000), or Rosenberg and Engle (2002)).

1.2 Methodology

Three pillars make up the main structure of our analysis. First, a tested bank loan portfolio is constructed in such a way as to reflect with some degree of realism the rating distribution of a pool of Czech bank clients. Second, we take into account the random nature of interest rates and other economic fundamentals that enter the loan valuation. Among other things, this means that market risk factors (interest rates and exchange rates) were an integral part of the capital calculations as far as each of the tested approaches allowed. Third, when conducting model-based economic capital calculations, we follow the market loan pricing point of view wherever possible (ie when the corresponding model allows it either explicitly or implicitly). This is done because we want to identify those elements of capital requirements which may be seen differently from the credit risk modelling and regulatory perspectives.

Our analysis utilises a hypothetical portfolio containing 30 loans. This simplified portfolio mirrors the rating structure of a real loan portfolio obtained on the basis of a pool of corporate customers of six Czech banks.[4] Since ratings are the key input in many credit risk approaches, a simplified version of Moody's rating methodology for private firms has been applied to obtain ratings in our real sample of bank clients. Estimates of other inputs required by credit risk modelling which were not available in the real bank data set were obtained using aggregate data from Czech National Bank (CNB) databases.

In an earlier paper (Derviz et al (2003)), we examined and compared the predictions of the NBCA with those delivered by the CreditMetrics and CreditRisk+ models. Following the January 2001 consultative version of the NBCA guidelines we found that in our particular example the standardised approach of the NBCA predicted approximately the same level of capital as the credit risk models at the 95% confidence level. At the 99% confidence level, the internal credit risk models predicted a higher level of economic capital than the NBCA standardised approach, but these estimates were still lower than the estimates of the NBCA IRB approach. We obtained different results when applying the NBCA guidelines as formulated by the third quantitative impact survey (QIS 3, October 2002). Here, the results of both NBCA approaches (standardised and IRB) were more similar to each other, with the IRB requirement being slightly lower than that of the standardised approach. The results of both regulatory approaches were even lower than the level of capital required by the various credit risk models. For ease of reference, we reproduce the regulatory capital results along with the modelled economic capital ones at the end of the present paper (Table 8).

[4] We would like to thank Alena Buchtíková for making this data set available for our research purposes.

In the context of the CreditMetrics model, we have extended the analysis of economic capital by allowing both the bank lending rates and the forward zero coupon rates used as discount factors in asset valuations to become random variables (a form of stress testing; see the details in Appendix A). Floating lending rates and random changes in the forward curves were all implemented using Monte Carlo simulations. As expected, more uncertainty associated with the evolution of these variables required more economic capital to be held by banks. However, the proposed changes in forward zero curves did not impose significantly different levels of credit risk-related economic capital as compared with the case of stable forward yield curves (but maintaining floating interest rates in both cases). Downward movements in the forward zero curves required higher levels of capital at all confidence levels. This is an inconvenient consequence of the existing credit models (CreditMetrics and KMV in particular) which we strive to overcome by proposing a model of our own.

Our approach to modelling financial and real uncertainties is similar to Ang and Piazzesi (2003), although we do not orient our state-space estimation on fitting the observed yield curve. Instead, we estimate the pricing kernel parameters that fit the returns of a number of basic infinite maturity assets. The reason for this is that we need a direct connection between macroeconomic risk factors, asset prices and bank loan values. Looking for this connection through the prism of yield curve dynamics would be too circumspect for our purposes, since extracting business cycle information from the yield curve is a misspecification error-laden process in itself. In contrast, by allowing the model to reflect a one-to-one correspondence between a vector of basic assets and another vector of unobserved factors, we are likely to capture a latent principal component responsible for the economic activity. We believe that this estimated model, which we later use for simulations, contains less noise in the identified business cycle position of the economy than most multifactor yield curve models in the literature.

The paper is structured as follows. In Section 2 we briefly describe the main characteristics of the real bank and test portfolios and their estimation. We also mention the reasons why the models could not be implemented entirely on the basis of real Czech bank data. In Section 3 we outline the methodologies proposed by two popular credit risk models (CreditMetrics and CreditRisk+) and present their economic capital estimations. Section 4 outlines our own model of risky debt, its valuation and the resulting economic capital requirements, going along the structural lines of the original KMV model. Section 5 discusses estimation procedures and outcomes. Section 6 concludes.

2. The test portfolio

Our bank data set contains the balance sheets and profit and loss accounts of non-financial firms that were granted bank loans between 1994 and 2000. The CZ-NACE classification,[5] legal form and CNB loan classification[6] (from 1997) were also available for each bank customer. Six Czech banks provided the data to the CNB from 1994 until 1999, of which two banks terminated cooperation in 2000. The banks reported only a fraction of their corporate portfolios. The exact selection procedure used by banks to choose particular firms is not known. Also unknown is the proportion of reported versus unreported clients satisfying certain criteria. In this sense, we observed a certain bias of the data providers towards non-reporting of loans in the last two categories (4 and 5) but were not able to assess the direction and magnitude of this sampling bias in our results.

Since our main goal was to assign ratings to banks' corporate clients, we primarily focused on their default behaviour. Default was defined as a credit event in which the loan classification of a certain company migrated from the first or second category to any of the third, fourth or fifth categories over the considered risk horizon. Due to the short time length of our data set we had to focus on annual default rates. The largest number of defaults occurring over a one-year period was recorded between

5 CZ-NACE (Czech abbreviation: OKEČ) represents the industry classification of economic activity in the Czech Republic.

6 The CNB's loan classification ranges from 1 to 5, with category 1 meaning standard, 2 watch, 3 non-standard, 4 doubtful and 5 loss loans.

1997 and 1998, representing 8% of all firms in the sample.[7] The sample-based annual default rates were 0.07% between 1998 and 1999 and 0% between 1999 and 2000. These low default rates may be partially explained by the Czech economic recovery and by more prudent bank lending behaviour during 1999-2000. Nevertheless, we think that the main reason is insufficient default reporting by banks. Therefore, we preferred to restrict the reference data set only to the accounting information collected in 1997 and the default events observed in 1998, assuming that default reporting by banks in that period was closer to the reality. While analysing a longer time period would have been highly valuable, we considered that the sample-based information over 1998-2000 painted a biased picture about corporate default and, consequently, it was not used in modelling the rating structure of the test portfolio.

The annual default rate of 8% in the reference data set was significantly higher than the average value of 1.5% usually used by Moody's in the context of western European economies. However, volume-based information about the loan defaults of individuals and corporates in the entire banking sector revealed an annual default rate of approximately 20%. We considered that neither 1.5% nor 20% would be the appropriate annual default rate for our artificial portfolio[8] and, in general, for a typical Czech bank portfolio of corporate loans. Without any other more reliable source of information, we used the 8% default rate as indicated by our sample bank data to calibrate the probit model.

To assign ratings to each firm we used the calculated default rates and the tables containing cumulative default rates published by different rating agencies. Even though Moody's rating methodology was used, we preferred to calibrate our results to Standard & Poor's (S&P) ratings. This was done since (a) the NBCA assigned risk weights based on S&P ratings and (b) our inputs into the credit risk models were, to a great extent, based on S&P data. For calibration purposes we used the 1996 S&P cumulative one-year default rate matrix shown in Table 1.

Table 1

S&P's rating class-specific one-year default rates

Rating	One-year default rate (%)	Cutoff values for defining ratings in the bank portfolio (%)
AAA	0	–
AA	0	–
A	0.06	0.03
BBB	0.18	0.12
BB	1.06	0.62
B	5.2	3.13
CCC	19.79	12.495

Source: CreditMetrics - Technical Document.

The probabilities given in the second column are rating class-specific default probabilities published by S&P. The third column contains the probabilities that mark the transition from one rating class to another in our model. They are the midpoints in the intervals determined by the one-year default probabilities given in the second column. For example, if the estimated default probability of a certain firm belonged to the interval [0, 0.03), an AA rating grade was assigned to that firm. If the estimated

7 The reference sample included only bank clients that were present both in 1997 and 1998 (663 firms). To examine only one-year default behaviour, those enterprises which were already in default in 1997 were also eliminated. In the end we obtained a data set containing 606 firms.

8 These figures reflected assumptions that were not applicable in our case. For example, the 1.5% level was based on the western experience, while the 20% level was volume-based and represented both firms' and individuals' default behaviour.

default probability fell into the interval [0.03, 0.12), then an A grade was assigned and so on. Based on this mapping procedure, each firm present in the 2000 data set was marked with a certain rating grade. The resulting rating structure and the loan classification of the pooled bank portfolio for 2000 are presented in Table 2.

Table 2

The pooled bank portfolio structure in 2000 according to loan classification and ratings (number of firms/percentage)

	AA	A	BBB	BB	B	CCC	Total
1	15/1.45	5/0.48	26/2.51	87/8.41	580/56.04	89/8.60	802/77.49
2	1/0.10	0	1/0.10	19/1.84	150/14.49	23/2.22	194/18.74
3	0	0	0	0	12/1.16	11/1.06	23/2.22
4	0	0	0	0	3/0.29	3/0.29	6/0.58
5	0	0	0	0	1/0.10	90.87	10/0.97
Total	16/1.55	5/0.48	27/2.61	06/10.24	746/72.08	135/13.04	**1,035/100**

Next, we constructed an artificial portfolio incorporating as much real information as possible. We could not perform our risk capital estimations on the pooled bank portfolio because (a) this would have been extremely time-consuming and (b) many inputs required by the credit risk models were not available for the real bank customers. For instance, outside the ratings, the bank data set did not contain information regarding loan volumes or maturities, charged interest rates and borrower asset returns or recovery rates. Since these parameters represent required inputs into many regulatory and internal credit risk models, we constructed proxy variables based on data available at the macro level or obtained them as random drawings from known distributions. While in a reduced portfolio (like our testing one) the construction of the proxy variables is easily done, this construction would have been more difficult to produce based on a portfolio of 1,035 bank clients. In what follows we describe the manner in which these inputs were generated. The main information source was the CNB supervisory database, which contains yearly data on residual maturity of Czech bank loans, their category and the borrower CZ-NACE code. It also categorises loans according to the charged interest rate.

Ratings and exposures

The rating structure displayed in Table 2 was adjusted to reflect the following changes:

- All bank clients in loan category 5 (loss loans) were eliminated. Such loans are usually covered by provisions created in the current period. Moreover, the fifth category is an absorbing state: a loan falling into this category has a negligible probability of recovery. These loans pose a vacuous problem from the risk management perspective, since their future status is not associated with any uncertainty.
- The 8.6% of firms with a CCC rating were removed from category 1 and added to categories 3 and 4. We assumed that the 8.6% outcome reflected the imperfections of our model. Czech banks monitor the creditworthiness of their clients, thus loans falling in the first category are unlikely to be granted a CCC grade.
- The rating structure was adjusted to resemble the loan volume configuration at the end of December 2000 as closely as possible (as shown in Table 3).

Now the rating structure of our test portfolio takes the form shown in Table 4. To have a fair representation of all ratings in each loan category, we needed a minimum of 30 assets.

The exposure of an asset in a certain loan category represents the ratio of the total loan volume to the number of assets in that category. For example, all assets belonging to category 1 (21 in the test portfolio) would have an exposure of CZK 607.235 billion/21 = CZK 28.915 billion.

Table 3

Loan volumes by category granted by Czech banks, as of end-2000

Category	Volume (CZK bn)	Proportion (%)
1	607.235	68.58
2	85.811	9.69
3	54.577	6.16
4	26.982	3.05
5	110.834	12.52

Source: Czech National Bank.

Table 4

Rating structure of the artificial portfolio: number of assets in each loan category and rating class

Loan category	AA	A	BBB	BB	B	CCC	Total
1	1	1	2	3	14	0	21
2	0	0	0	1	2	1	4
3	0	0	0	0	1	2	3
4	0	0	0	0	0	2	2
Total	1	1	2	4	17	5	30

Maturities, lending and recovery rates

Loans with a maturity exceeding five years are sparsely represented in the Czech bank portfolios. For this reason, we considered maturities that ranged from one to five years only. Maturity was assigned to individual assets by drawing random numbers from the interval [1, 5] according to the uniform distribution and then rounding these numbers to the nearest integer.

We computed the mean and standard deviation of the lending rates for each loan category (using the SUD data set). To assign lending rates to assets in our portfolio, we randomly drew numbers from normal distributions described by the estimated means. Standard deviations were in general reduced to prevent interest rates from deviating too much from these mean values.

We also generated collateral and recovery rates (see Derviz et al (2003) for details). The main characteristics of the artificial test portfolio are shown in Table 5.

Asset return correlations

Firms in the bank data set were grouped according to ratings and loan classification. Then we found the CZ-NACE category that was the most frequently represented in each group. For example, in the group of firms with rating AA and loan classification 1 the largest number of firms belonged to CZ-NACE 51. If in a certain group no dominating CZ-NACE could be found, we randomly selected the representative figure from those that were present in that group. The resulting CZ-NACE structure was mapped to the test portfolio. Having assigned a CZ-NACE label to each asset in the test portfolio, we used the price index characteristic of the corresponding branch as a proxy variable for that asset's returns.[9] Asset return correlations were determined by computing correlations among price indices.

Table 5

Portfolio composition and individual loan characteristics

Loan	CZ-NACE	Regulatory loan class	Rating	Loan volume[1]	Maturity	Lending rate (%)	Type of collateral	Recovery rate (%)
1	51	1	AA	28.916	3	6.5	6	0
2	36	1	A	28.916	1	7.3	6	0
3	74	1	BBB	28.916	3	7.5	6	0
4	31	1	BBB	28.916	5	7.6	6	0
5	74	1	BB	28.916	5	8.2	4	71.43
6	20	1	BB	28.916	5	8.5	1	94.34
7	51	1	BB	28.916	1	8.7	6	0
8	28	1	B	28.916	3	8.8	2	100
9	15	1	B	28.916	4	9.5	6	0
10	51	1	B	28.916	2	10.5	1	94.34
11	52	1	B	28.916	2	11.0	5	50
12	29	1	B	28.916	1	11.1	6	0
13	70	1	B	28.916	1	11.3	5	50
14	74	1	B	28.916	2	11.6	4	71.43
15	50	1	B	28.916	2	11.7	1	94.34
16	24	1	B	28.916	1	11.9	5	50.00
17	45	1	B	28.916	2	12.1	1	94.34
18	60	1	B	28.916	2	12.2	1	94.34
19	40	1	B	28.916	3	12.5	3	89.29
20	25	1	B	28.916	2	12.6	1	94.34
21	65	1	B	28.916	4	13.0	4	71.43
22	51	2	BB	21.452	2	8.4	1	94.34
23	25	2	B	21.452	5	10.4	1	94.34
24	29	2	B	21.452	3	11.5	6	0
25	55	2	CCC	21.452	3	13.5	5	50
26	45	3	B	18.192	2	11.9	1	94.34
27	28	3	CCC	18.192	5	12.4	1	94.34
28	21	3	CCC	18.192	4	12.2	1	94.34
29	21	4	CCC	13.491	5	14.3	4	71.43
30	37	4	CCC	13.491	2	15.4	1	94.34

[1] In billions of Czech korunas.

[9] At the outset, all price indices were deflated by the PPI in order to eliminate the systemic inflationary influence in their evolution.

3. Portfolio value and economic capital according to commercial risk measurement models

3.1 CreditMetrics

In the CreditMetrics model, risk is associated with changes in the portfolio value caused by changes in the credit quality of individual obligors (downgrades or default) over the considered risk horizon (usually one year). We have followed the two standard pillars of the CreditMetrics approach. The analytical pillar requires a derivation of primary risk measures such as means, variances and standard deviations at the asset and portfolio level. The estimation pillar implies generating a simulated portfolio value distribution at the risk horizon. Based on this distribution, estimates of economic capital can be obtained at different confidence levels. Application of the Monte Carlo simulation method to our portfolio in accordance with the CreditMetrics approach is described next.

Monte Carlo simulation

Individual random draws from a multinomial normal distribution ("scenarios" in CreditMetrics terminology) of asset returns contain the same number of components (real numbers) as the number of assets in the portfolio. Each component of each scenario is compared with predefined, rating class-specific threshold values marking the switch from one rating class to another. In this way, within each scenario, a new rating is assigned to each asset (obligor) in the portfolio.

In case of non-default, each asset i is revalued according to the formula:

$$V_i^{g'} = \sum_{t=1}^{T_i - 1} \frac{r_i}{\left(1+f_t^g\right)^t} + \frac{r_i + F_i}{\left(1+f_{T_i}^g\right)^{T_i}}, \tag{1}$$

where r_i and F_i are the loan interest payments and the face value of the loan respectively, and f_t^g are the annualised forward zero rates for the years one to T_i, applicable to the rating class g' (here T_i is the maturity of the loan). In this specification it is assumed that the present rating changes from g to g' over the one-year period. In the event of default the present value of the loan is computed as the product of the face value of the loan and a recovery rate.

Note that the way of defining the future loan value as a random variable implies that the distribution of this variable would be taken with respect to the *risk neutral probability* (RNP). CreditMetrics works with the assumption that such a probability is well defined for the studied economy (we take a different view in our own model; see subsections 4.1 and 4.2). Under the risk neutral probability, in contrast to the "physical" one, zero forward rates are unbiased estimates of the future spot interest rates. That is, the capital requirements under CreditMetrics are also derived from the RNP. This might lead to certain discrepancies between the CreditMetrics interpretation by the market (which is based on the RNP) and its interpretation by the regulator (based on the physical probability).

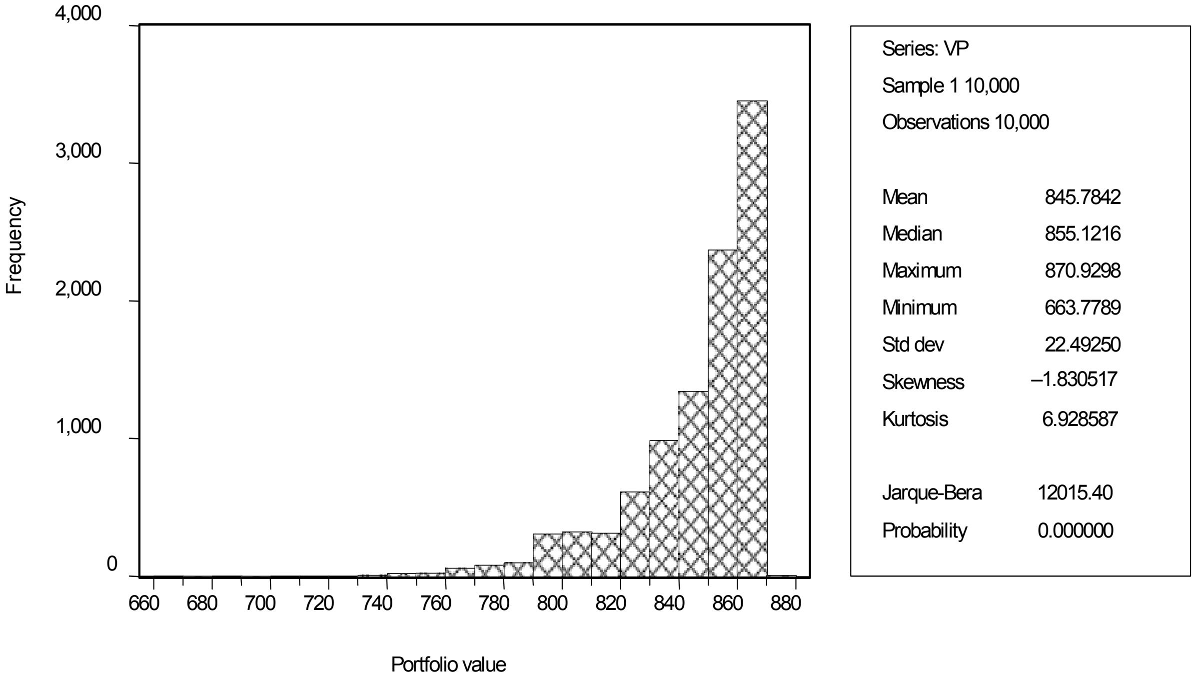
To obtain the portfolio value distribution and derive the economic capital requirement in accordance with the CreditMetrics model, we conducted a Monte Carlo simulation with 10,000 random draws. Each scenario contained 30 correlated random draws from the standard normal distribution. Each element of each scenario represented a standardised return corresponding to one of the 30 assets belonging to the portfolio. Comparing the elements of the scenario with the threshold values characteristic of each rating category, new ratings were assigned to each of the 30 assets. Summing up the values of the 30 assets thus obtained, a new portfolio value resulted for each scenario. Since eight potential new ratings were possible for each of the 30 assets at the end of the year, the total number of potential portfolio values was 8^{30}. In practice, this number was far lower, as some rating class migrations had a zero probability of realisation. Figure 1 shows the distribution of our portfolio value expected at the end of 2000 for the year 2001. On the horizontal axis are the non-overlapping intervals within which the portfolio value falls, while on the vertical axis are the frequencies with which these realisations occurred within each interval in our simulation.

The economic capital is obtained as the difference between the mean of the portfolio value and a p-percentile (p is usually assumed to be 1%, 2% or 5%):

Economic capital = Mean of the portfolio distribution – p-percentile

Figure 1

Monte Carlo simulation for the loan portfolio value



In the case of a discrete portfolio distribution, the p-percentile is obtained by looking at the lowest portfolio value whose cumulative frequency exceeds p%. An interpretation of the p-percentile is that in p% of cases we can expect the portfolio value to take values lower than the p-percentile over the one-year period. For example, in our case we can expect that only 100 times in 10,000 cases could the portfolio value reach a value lower than CZK 767.89 billion (in other words we know with 99% probability that the portfolio value will be higher than CZK 767.89 billion at the year-end). To cover this high loss, however unlikely it is, the bank must keep economic capital of CZK 77.88 billion.

The 1%- and 5%-percentiles take the values CZK 767.89 billion and CZK 796.62 billion respectively. Based on these estimations, the bank’s need for economic capital at different confidence levels is:

1%-percentile	**5%-percentile**	**Mean**	**99% economic capital**	**95% economic capital**
767.89	796.62	845.78	77.88	49.16

Macroeconomic fundamentals and the CreditMetrics-based economic capital

The nature of CreditMetrics makes it rather difficult to analyse the impact of shocks to real economic activity on the economic capital allocation. The most natural type of systemic factor analysis that can be conducted in the context of this model concerns the interest rate uncertainty. Subsequently, different business cycle developments can be accommodated in a CreditMetrics environment by assigning them an appropriate change in the yield curve. In Appendix A, we report the results of the corresponding Monte Carlo simulations for our artificial portfolio.

It turns out that the economic capital values are very sensitive to the yield curve movements, and often exhibit a counter-intuitive reaction on certain interest rate developments. For instance, an average downward (clockwise) rotation of the yield curve, which corresponds to the expected loosening of monetary policy, leads to an economic capital increase. This is an inevitable consequence of the fact that CreditMetrics does not work with the duration characteristics of the loans. The model to be described in Section 4 strives to overcome this deficiency.

3.2 CreditRisk+

CreditRisk+ is suitable for assessing credit risk in portfolios containing a large number of obligors with small default probabilities. The model groups bank customers according to their common exposure. The common exposure of an obligor represents the ratio between his bank exposure and a selected unit of exposure (CZK 1 billion in our case). Bank clients can be grouped in homogeneous “bands” that contain obligors with the same common exposure.

For obligor i, we introduce the following notations, taken from the CreditRisk+ technical document: L_i - exposure, P_i - default probability, $\nu_i' = \frac{L_i}{L}$ - common exposure, ν_i - rounded common exposure, $\varepsilon_i = \nu_i' \times P_i$ - expected loss. In addition, at the given Band j-level, ν_j is the common exposure in units of L, $\varepsilon_j = \sum_{i \in Band\ j} \varepsilon_i$ - expected loss in Band j in units of L, $\mu_j = \frac{\varepsilon_j}{\nu_j}$ - expected number of defaults in Band j.

The risk assessment at the asset level consists in estimating the expected loss (ε_i). At the band level, the model estimates the average number of defaults (m_j) as the ratio of the total expected loss in the band (ε_j) to the common exposure characteristic to the obligors from that band (ν_j).

By assumption, the distribution of the number of defaults in each band is of the Poisson type:

$$P_j = P(number\ of\ defaults\ in\ Band\ j = k) = \frac{m_j^k e^{-m_j}}{k!} \quad k = 0,\ 1,\ldots$$

Further, using the properties of probability-generating functions, the model estimates recursively the probabilities that the portfolio loss reaches values expressed as multiples of the unit of exposure.

For our test portfolio, the analytical risk assessments at the asset and band level are contained in Tables 6 and 7. Specifically, Table 7 illustrates the partition of the portfolio into bands and the risk characteristics of each band. Four different bands have thus been obtained, with rounded common exposures of 14, 19, 22 and 29.

For each band a probability-generating function is given by:

$$G_j(z) = \sum_{n=0}^{\infty} \Pr\{L_j = n\} z^n = \sum_{k=0}^{\infty} P(k\ defaults) z^{k\nu_j} = \sum_{k=0}^{\infty} \frac{m_j^k e^{-m_j}}{k!} z^{k\nu_j} = e^{-m_j + m_j z^{\nu_j}}.$$

The probability-generating function for the entire portfolio is the product of the individual probability-generating functions displayed in the last column of Table 7. In this particular example we get:

$$G(z) = e^{-\sum_{j=1}^{m} m_j + \sum_{j=1}^{m} m_j z^{\nu_j}} = e^{-1.877 + 0.381 z^{14} + 0.429 z^{19} + 0.305 z^{22} + 0.762 z^{29}}.$$

To derive the probabilities that loss equals multiples of the unit of exposure, CreditRisk+ constructs a recurrence relationship:

$$P_n = \sum_{\substack{j \\ s.t.\nu_j \le n}} \frac{\nu_j \times m_j}{n} P_{n-\nu_j}$$

that starts with the probability of no loss:

$$P_0 = P(\text{No loss}) = e^{-m} = e^{-\sum_{j=1}^{m} m_j} = e^{-1.877} = 0.153$$

The resulting loss distribution in the case of our portfolio is shown in Figure 2.

The economic capital is given by the difference between the p-percentile and the expected mean of the loss distribution:

Economic capital = p-percentile – Expected loss.

Table 6

Risk assessment at the asset level according to CreditRisk+

Asset i	**Exposure (CZK bn)**	**Common exposure (ν_i')**	**Common exposure rounded to multiples of CZK 1 bn (ν_i)**	**Probability of default (P_i)**	**Expected loss ($\varepsilon_i = \nu_i' \times P_i$)**
1	28.92	28.92	29	0	0.00
2	28.92	28.92	29	0.0006	0.02
3	28.92	28.92	29	0.0018	0.05
4	28.92	28.92	29	0.0018	0.05
5	28.92	28.92	29	0.0106	0.31
6	28.92	28.92	29	0.0106	0.31
7	28.92	28.92	29	0.0106	0.31
8	28.92	28.92	29	0.052	1.50
9	28.92	28.92	29	0.052	1.50
10	28.92	28.92	29	0.052	1.50
11	28.92	28.92	29	0.052	1.50
12	28.92	28.92	29	0.052	1.50
13	28.92	28.92	29	0.052	1.50
14	28.92	28.92	29	0.052	1.50
15	28.92	28.92	29	0.052	1.50
16	28.92	28.92	29	0.052	1.50
17	28.92	28.92	29	0.052	1.50
18	28.92	28.92	29	0.052	1.50
19	28.92	28.92	29	0.052	1.50
20	28.92	28.92	29	0.052	1.50
21	28.92	28.92	29	0.052	1.50
22	21.45	21.45	22	0.0106	0.23
23	21.45	21.45	22	0.052	1.12
24	20.38	20.38	22	0.052	1.12
25	21.45	21.45	22	0.1979	4.25
26	18.19	18.19	19	0.052	0.95
27	18.19	18.19	19	0.1979	3.60
28	18.19	18.19	19	0.1979	3.60
29	13.49	13.49	14	0.1979	2.67
30	13.49	13.49	14	0.1979	2.67

Source: Own computation.

Table 7

Band partition and risk assessment at the band level according to CreditRisk+

Band j	Rounded common exposure in Band j (ν_j)	Number of obligors in Band j	Expected loss in Band j (ε_j)	Expected number of defaults in Band j ($m_j = \varepsilon_j / \nu_j$)	Probability-generating function for Band j
1	14	2	5.340	0.381	$\exp(-0.381 + 0.381z^{14})$
2	19	3	8.147	0.429	$\exp(-0.429 + 0.429z^{19})$
3	22	4	6.704	0.305	$\exp(-0.305 + 0.305z^{22})$
4	29	21	22.092	0.762	$\exp(-0.762 + 0.762z^{29})$

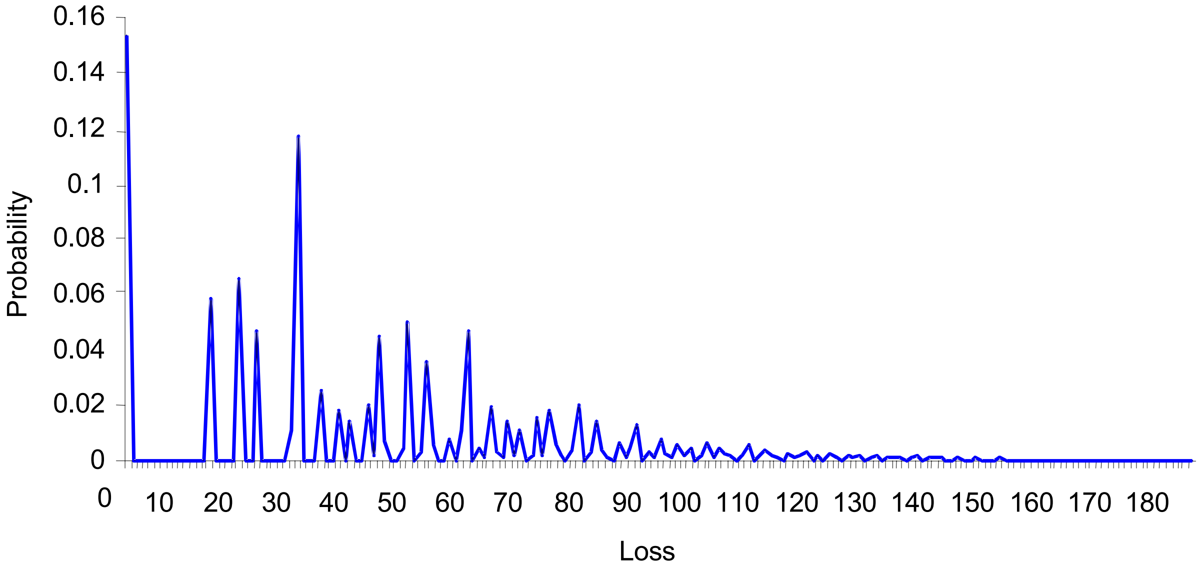
Source: Own computation.

Applying the CreditRisk+ approach to our portfolio, we got estimates of risk capital at different confidence levels as shown below.

1%-percentile	5%-percentile	Expected loss	99% capital	95% capital
133	**101**	**42.18**	**90.82**	**58.82**

Figure 2

Loss distribution based on the CreditRisk+ model



4. A structural model of risky debt with a random default arrival

We next give an outline of a model of risky debt, its valuation and the resulting economic capital requirements, going along the "structural" lines of the original KMV model and its ramifications. The term "structural" means that we make the default explicitly dependent on loan and obligor characteristics. However, we borrow an additional element from the so-called “reduced-form” models of default (for a survey of both types of model, see Bohn (1999)), by working with a default process in Poissonian form. This technique was introduced by Jarrow and Turnbull (1995). We follow the variant utilised by Madan and Unal (1998, 1999), in that the default event arrival rate becomes a function of the same obligor fundamentals as the ones that drive the asset prices. However, this principle is developed in a way to establish a link between the default process of the abstract reduced-form models and the empirics inspired by the Expected Default Frequency notion of KMV. The proposed model allows one to deal with a loan portfolio with correlated defaults in a natural way.

4.1 Definitions

Consider a portfolio of n loans issued by n different companies (obligors). Loan i pays a coupon c_t^i at time t and the coupon plus the face value F^j at maturity T^i ($t = 1,\ldots, T^i$). The value of the loan to firm i at time t is denoted by V_t^i. Then, $B_t = \sum_{i=1}^{n} V_t^i$ is the value of the loan portfolio to be found.

There are N assets traded in the market, with prices P_t^j at time t ($j = 1,\ldots, N$). These assets represent all sources of aggregate uncertainty in the economy independent of the actions of obligors defined above. In this sense, the financial markets outside the considered borrower set are complete. These uncertainty factors will be represented by random variables x^j, $j = 1,\ldots, N$. By x, we denote the vector of the unobserved state variables of the model.

The loans are risky. If firm i generates period t-cash flow $Y_t^i = f^i(x_t)$ net of all the other debt service obligations, then the probability of default π_t^i on the loan is an inverse function of the difference $Y_t^i - C_t^i : \pi^i(x) = \pi(Y^i - C^i)$. Here, $C_t^i = c_t^i$ if $t < T^i$ and $C_t^i = c_t^i + F^i$ if $t = T^i$. Hence, the variable driving the default rate in our model is an analogue of the *distance-to-default* measure used in KMV. One shall think of π as approaching unity when the distance to default falls to minus infinity, and approaching zero when it increases to plus infinity.

The space of random events in our model is formed by pairs $\omega = (\chi, b)$, where χ is a realisation of x and $b = [b^1,\ldots, b^n]$, $b^i = S$ if there is no default (survival) and $b^i = D$ if firm i defaults on the loan. The arrival *fact* of the default event itself (which we represent by the Bernoulli process B^i) is assumed independent of x, ie only *the probability value* of the default is x-dependent through the cash flow variable Y^i.

For each loan, there is collateral that is tradable and depends on the same sources of uncertainty as the basic assets. That is, the collateral price for loan j is equal to $Z^i = \zeta^i(\omega)$. If the loan defaults, the bank seizes the collateral, ie receives the value of Z^i.

There are two important cases to be distinguished with regard to the collateral prices. One possibility is to allow ζ^i to depend on both x and b. That is, this collateral is worth different amounts depending on whether the debt has defaulted or not. This would be the case if there were a separate structural factor behind the realisation of b, correlated with the market risk factors x. The same factor should be responsible for the value of the collateral. This situation would allow loan i to be priced in accordance with the risk neutral valuation principles (see an example of such a valuation in Derviz and Kadlčáková (2002, Section 3). It may occur when the collateral is very obligor-specific.

However, an equally legitimate case is that of the collateral being totally unrelated to the operation of the firm (eg securities in its investment portfolio). Then $Z^i = \zeta^i(x)$ and a unique risk neutral valuation of the loan is impossible. In that case, one must resort to pricing techniques based on explicit individual portfolio optimisation. This is done next.

4.2 The individual loan and the portfolio value

Let us consider an optimising investor in discrete time who decides upon allocating his/her wealth between the existing marketable assets, ie the N traded securities, the n collateral assets and the n company loans (all defined above). This is a standard optimisation problem under uncertainty in discrete time.

Let g^j be the stream of coupons/dividends paid out by the basic security j, and h^i the same thing for the collateral security i. These values are unknown at the beginning of each period, when the investor makes the portfolio allocation decisions. Define R^j as the current yield on the basic security and z^i as the current yield on the collateral. In addition, it is convenient to use the notation y^j for the continuously compounded current yield on basic asset j:

$$e^{y^j_{t+1}} = 1 + R^j_{t+1} = \frac{g^j_{t+1} + P^j_{t+1}}{P^j_t},\ 1 + z^i_{t+1} = \frac{h^i_{t+1} + Z^i_{t+1}}{Z^i_t}.$$

r_{t+1} will denote the risk-free short rate between periods t and $t+1$. The period utility function of the investor is a function of the dividend rate withdrawn after the investment strategy gains are realised: $u = u(\tilde{\rho})$. The ρ-dependence is of the standard Inada form. If the time preference rate of the investor is $\beta \in (0, 1)$, the pricing kernel (stochastic discount factor, see Campbell et al (1997), or Cochrane (2001)) is given by:

$$M_t^{t+1} = \frac{\beta u'(\rho_{t+1})}{u'(\rho_t)},\ M_t^\tau = \prod_{k=t}^{\tau} M_k^{k+1},\ \tau > t,\ M_t^t = 1. \tag{2}$$

The information available to the investor at time t consists of the trajectories of g, h, P and Z as well as the default event realisations b, all up to time t. The no-default up to time t subset of the event space for loan i will be denoted by N_t^i. Let the x-dependent statistics of the survival, $S^i_{t,\tau}$ between times t and $\tau \geq t+1$, be defined as:

$$S^j_{t,\tau} = \prod_{k=t+1}^{\tau}\left(1 - \pi^i_k\right). \tag{3}$$

Now we apply the standard asset pricing theory results. The optimal investor behaviour implies the following asset pricing formulae (special cases of the discrete time consumption-based CAPM):

$$E_t\left[M_t^{t+1}(1 + R^j_{t+1})\right] = 1,\ E_t\left[M_t^{t+1}(1 + z^i_{t+1})\right] = 1,\ E_t\left[M_t^{t+1}\right](1 + r_{t+1}) = 1, \tag{4}$$

$$Z_t^i = E_t\left[\sum_{\tau=t}^{\infty} M_t^\tau h^i_\tau\right], \tag{5}$$

$$V_t^i = Z_t^i + E_t\left[\sum_{\tau=t}^{T^i} M_t^\tau B^i_\tau\left(C^i_\tau - h^i_\tau\right)\right]. \tag{6}$$

In view of our assumptions about default arrival independence on x, equation (6) can be rewritten in the form:

$$V_t^i = Z_t^i + E_t{}^*\left[\sum_{\tau=t}^{T^i} S^j_{t,\tau+1} M_t^\tau\left(c^i_\tau - h^i_\tau\right)\right] - E_t{}^*\left[S^i_{t,T^i+1} M_t^{T^i+1} Z^i_{T^i+1}\right], \tag{7}$$

where, now, the conditional expectation $E_t{}^*$ is taken only with respect to the market-wide risk factors. Thus, we have eliminated the firm-specific default event process B^i from the debt pricing formula (6). Also note that the asset pricing equations (4) could be written with expectation $E_t{}^*$ instead of E_t from the outset, since they are default event-independent.

Next, utilising the previously made assumption about market (in)completeness, we note that the value of individual loans and the loan portfolio as a whole can be calculated as soon as one reconstructs a formula for the pricing kernel M_t^τ from the pricing equations (4). Formally, we will be estimating and using *the empirical pricing kernel* in (7) instead of the theoretical pricing kernel (2). The empirical pricing kernel is a projection of the theoretical one on the space of modelled market uncertainty factors x. Due to the law of iterated expectations, the left-hand side of (7) is fully determined by this projection (since the external expectation is taken with respect to x-generated random events). One goes about calculating asset prices in the pricing kernel context by either setting up a parametric model for it or applying orthogonal basic decomposition methods known from numerical mathematics, under a non-parametric approach. Examples in the literature include Ait Sahalia and Lo (2000), Jackwerth (2000), or Rosenberg and Engle (2002).

We proceed by constructing a Gaussian state-space model for the pricing kernel and estimating it on the Czech asset data.

4.3 A hidden factor asset pricing model

The observation process y of our model consists of the above-mentioned traded asset yields $y^1,\ldots, y^N$. The observation equations are:

$$y_{t+1}^j = a_0^j + a_1^j x_t + A^j x_{t+1},\ j=1,\ldots, N \tag{8}$$

Here, $a_0 = \left[a_0^1,\ldots,a_0^N \right]^T$ is an $N \times 1$-vector of intercepts, a_1 and A are $N \times n$-matrices of coefficients with rows $a_1^j = \left[a_{11}^j,\ldots,a_{1n}^j \right]$ and $A^j = \left[A_1^j,\ldots,A_n^j \right]$ respectively. The n-dimensional vector x of unobserved state residuals follows the VAR(1)-process:

$$x_{t+1} = bx_t + B\varepsilon_{t+1}. \tag{9}$$

Coefficient matrices b and B in (9) are of size $n \times n$. Process ε is an n-dimensional vector of mutually independent standard normal errors. In general, $n \neq N$ and, if there is a reason to assume eg cyclical components in the observations, one will need to take $n > N$.

Another unobserved state variable is the log of the one-period pricing kernel $m_{t+1} = \log M_t^{t+1}$:

$$m_{t+1} = \lambda_0 + \lambda_1 x_t + \Lambda x_{t+1}. \tag{10}$$

Here, λ_0 is a scalar constant, whereas λ_1 and Λ are row vectors of dimension n. The observation equation system (8) together with the state equation system (9), (10) constitutes the state-space representation of the present model. However, this is not the definitive representation to be estimated, since one must incorporate the coefficient restrictions following from the no-arbitrage pricing equations (4) for returns y.

Proposition 1 *The no-arbitrage pricing conditions for the asset return model defined by (8)-(10) above are equivalent to the following constraints on the model coefficients:*

$$a_0^j = -\lambda_0 - \frac{\left|(\Lambda + A^j)B\right|^2}{2},\ a_1^j = -\lambda_1 - (\Lambda + A^j)\mathrm{b},\ j=1,\ldots, N. \tag{11}$$

The proof is given in Appendix B. Note that, whereas the first equality in (11) is scalar, the second one is for N-dimensional row vectors.

We are able to reduce the number of estimated parameters by simplifying the covariance structure of the state equation through a change of variables. Specifically, assume that B is non-singular and put $x_t = Bu_t$ for all t. Then:

$$u_{t+1} = \Phi u_t + \varepsilon_{t+1},\ \Phi = B^{-1}bB. \tag{12}$$

The log-pricing kernel equation is now given by:

$$m_{t+1} = c_0 + c_1 u_t + Cu_{t+1} \tag{13}$$

instead of (10), with $c_0 = \lambda_0$, $c_1 = \lambda_1 B$, $C = \Lambda B$. Put $\gamma^j = (\Lambda + A^j)B$. It is easily checked that the no-arbitrage pricing conditions (11) of Proposition 1 imply the following equations for the observed yields:

$$\begin{aligned} y_{t+1}^j &= -\lambda_0 - \frac{|\gamma^j|^2}{2} - \left(c_1 + \gamma^j \Phi\right)u_t + \left(\gamma^j - C\right)u_{t+1} \\ &= -m_{t+1} - \frac{|\gamma^j|^2}{2} + \gamma^j \varepsilon_{t+1},\ j=1,\ldots, \mathrm{N}. \end{aligned} \tag{14}$$

Process u will be the state process of the definitive formulation of our model. The state equations for its components are in (12). The observation equations are in (14). The model written in state-space form in (12)-(14) contains restrictions on the fixed coefficients. After having estimated it by means of the Kalman filter method and obtained the (hyper)coefficients Φ, λ_0, c_1, C, γ, we can reconstruct the observation equation coefficients a_0, a_1 by means of (10) and the coefficient matrix A by using the definition of γ:

$$A^j = \gamma^j B^{-1} - \Lambda = (\gamma^j - C)B^{-1}, j = 1,\ldots, N. \tag{15}$$

This will complete the estimation procedure for the pricing kernel and allow us to price the non-traded debt by means of (5).

One-period interest rate as a basic security

As was already mentioned, the present model is not constructed by directly fitting the yield curve. However, if one works with monthly data, it is convenient to take the one-month risk-free interest rate as one of the basic securities. Let us assign it superscript 1. Then y^1_{t+1} is the continuously compounded one-period rate between t and $t+1$. We incorporate it into the model by imposing the requirement $A^1 = 0$ in (8) for $j = 1$. Equivalently, one must have $\gamma^1 = \Lambda B = C$, then the first observation equation degenerates to:

$$y^1_{t+1} = -\lambda_0 - \frac{|C|^2}{2} - (c_1 + C\Phi)u_t,$$

and the coefficient recovery formulae (15) are applicable for $j = 2,\ldots, N$.

All other points in the yield curve (with longer maturities) generate uncertain one-period yields, so that there is no need for further specialisation in the event that one would want to include any of them in the list of basic securities.

5. Asset pricing data and economic capital estimation

The sample period 1999-mid-2003 is characterised by a negative trend in the Czech interest rate and bond yield data (the disinflation process and the monetary policy rate convergence to the EU level). The model described in the previous section would require a number of messy technical adjustments to accommodate these non-stationary yields along with the stock return data. Since our objective is the modelling of real shock effects on credit risk valuation, we need a pricing kernel projected on the space of most relevant security returns. That is, for our purposes it is sufficient to select the stationary assets with a clear relation to the economic cycle and avoid the problems with fitting the yield curve evolution. Therefore, we have selected a four-factor model based on the following asset returns:

- PX50 stock index return;
- Česká pojišťovna (a major Czech insurance company) stock return;
- DAX stock index return;
- Altana (the pharmaceutical company) stock return.

This choice is motivated by the effort to capture both internal and external risk factors for the small open Czech capital market with a high degree of dependence of the corresponding markets in Germany and the European Union. The stock index returns reflect the direct link to the Czech and euro area business cycles. The additional stocks (Česká pojišťovna on the Czech side and Altana on the EU side) were found to be less than perfectly correlated with the major indices. Therefore, they serve in the model as proxies for the countercyclical risk factors priced in the corresponding markets.

Estimation of the empirical pricing kernel

The unobserved state-space model covered in Section 4, implemented for the four named assets, contains four state variables $u1$, $u2$, $u3$ and $u4$, and their four one-period lags. According to the estimation outcome, the pricing kernel log, m, has the following dependence on these variables:

$$\begin{aligned} m = {} & 0.023 - 0.00376\, u_1(-1) + 0.0002\, u_2(-1) + 0.00274\, u_3(-1) + 0.00681\, u_4(-1) \\ & + 0.00993\, u_1 + 0.00065\, u_2 + 0.0148\, u_3 + 0.0203\, u_4. \end{aligned}$$

The estimated autoregression matrix Φ for the states (cf (12) in Section 4) is equal to:

$$\Phi = \begin{bmatrix} 1.786 & -0.320 & 0.192 & -0.135 \\ 2.370 & 0.038 & 0.541 & -0.432 \\ -0.0570 & 0.031 & 0.936 & -0.032 \\ -0.023 & -0.010 & 0.004 & 0.997 \end{bmatrix}.$$

Using the estimated state variable series, one can now establish the dependence of the obligor company cash flow (cf (7)) on the hidden risk factors by regressing these flows on the four state series (with first-order lag terms). Considering the way in which the portfolio was constructed, individual asset cash flows are proxied by the price indices corresponding to the relevant industries. We implement a seemingly unrelated regression (SUR) with the price indices acting as the dependent variables and the state vectors as the explanatory variables:

$$Y^i = \beta^i + \sum_{k=1}^{4} \gamma_k^i \cdot u_k + \sum_{k=1}^{4} \delta_k^i \cdot u_k(-1) + \varepsilon_i, \qquad i = 1,\ldots, 30.$$

The coefficients $\beta^i, \gamma_k^i, \delta_k^i$ facilitate the computation of the annual default probabilities for each asset. By assumption, year t default probability of the *i-th* asset is linked to the cash flow Y_t^i according to the formula:

$$\pi_t^i = \frac{e^{-\alpha^i - Y_t^i}}{e^{-\alpha^i - Y_t^i} + e^{\alpha^i + Y_t^i}}, \quad i = 1,\ldots, 30, \; t = 1,\ldots, 5. \tag{16}$$

Intuitively, (16) reflects the fact that high cash flow realisations would yield probabilities of default (PDs) close to zero while highly negative cash flows would yield PDs close to unity. The PD formulae are calibrated to become compatible with the S&P rating structure of the portfolio. This means that the parameters α^i are estimated in such a way as to produce the S&P asset-specific ratings when the cash flows in (16) are zero.

The recovery rates are also modelled as state-dependent variables. The last four of the six types of collateral considered (securities, commercial real estate, other and no collateral) are proxied by the following variables:

- yields on five-year Czech government bonds;
- the real estate price index;
- a linear combination of the PPI and industrial production index (reflecting uncertainty regarding other types of collateral); and
- an insurance sector price index.

As in the case of the assets' cash flows, these variables are regressed on the state variables. By assumption, the recovery rates are exponential functions of the hidden risk factors u:

$$RR_t^i = e^{\left(a^i + b^i + \sum_{k=1}^{4} c_k^i u_k + \sum_{k=1}^{4} d_k^i u_k(-1)\right)}, \quad i = 1,\ldots, 30, \quad t = 1,\ldots, 5 \tag{17}$$

where the coefficients b^i, c_k^i and d_k^i are obtained from the regressions and a^i are parameters that calibrate the recovery rate formulae to the real recovery rates as considered in Table 5.

The annual values of individual loans up to maturity can be calculated as soon as these intermediate valuation elements are available:

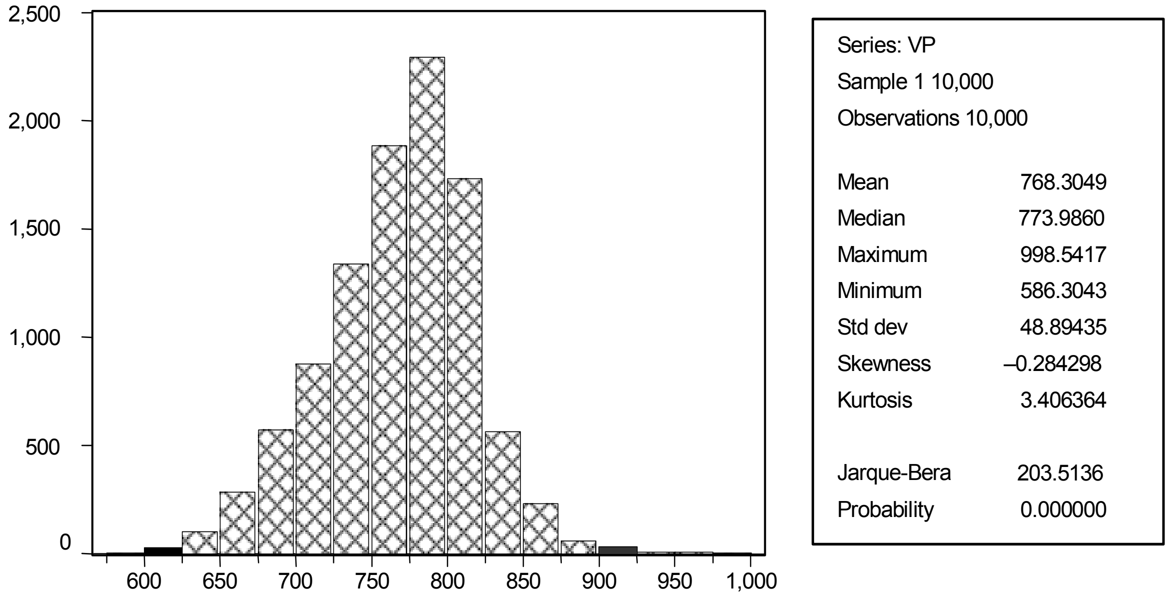
$$V_t^i = U^i \cdot \sum_{\tau=1}^{t} M_\tau \left[c_\tau^i \left(1 - \pi_1^i\right) \ldots \left(1 - \pi_\tau^i\right) + RR_\tau^i \cdot \left(1 - \pi_1^i\right) \ldots \left(1 - \pi_{\tau-1}^i\right) \pi_\tau^i \right], \quad t = 1,\ldots, T^i. \tag{18}$$

Here U^i is the contractual loan volume and c_τ^i is the loan annual payment (the lending rate in the years prior to maturity and one plus the loan lending rate at maturity).

We conduct a Monte Carlo simulation with 10,000 scenarios to generate the distribution of the portfolio value at the risk horizon. Each replica starts with the four-component state vector at the end of 2000. Then, the annual u vectors for the subsequent five-year period are determined using autoregressive state variable formulae as in (12). Additionally, each component of the vector u is shocked each year with an error term randomly drawn from the standard normal distribution. Given the *u*-dependence of the pricing kernel given by (15), default probabilities (see (16)) and recovery rates (see (17)), each loan's valuation is fully determined by (18). The portfolio value is obtained by summing up the individual loan values. This procedure is repeated 10,000 times. The mean of the portfolio distribution thus obtained (see Figure 3) is then used to estimate the economic capital. In our understanding the economic capital represents the difference between the riskless value of the portfolio and the mean value mentioned above. The riskless value of the portfolio is estimated assuming no default events taking place over the risk horizon, thus making the π_t^i equal to zero in (18) for each i and t.

Figure 3

Portfolio distribution according to the pricing kernel model, baseline case



Business cycle events and economic capital in the pricing kernel model

We are now ready to investigate the consequences of the domestic and foreign economic cycle. The latter case will be modelled by constructing the real shocks for Germany since there is no generally accepted industrial production index for the European Union or the euro area as a whole.

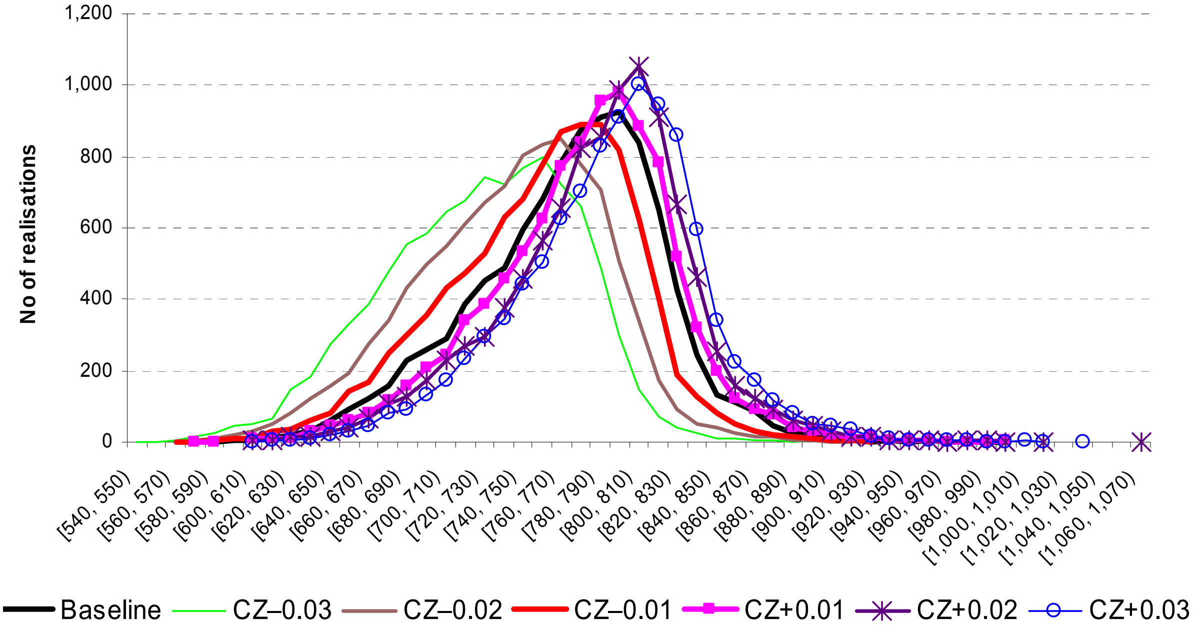
The loan portfolio distributions under different macroeconomic developments in the Czech Republic and Germany are shown in Figure 4. In addition, Figure 5 shows the pricing kernel baseline and the most extreme positive/negative economic activity shock cases in comparison to the CreditMetrics distribution.

When we model the different business cycle developments in the pricing kernel model, we rely on a shock to a corresponding state variable (the first one for the Czech business cycle and the third one for the German). These shocks were selected since the underlying risk factors roughly correspond to the normalised Czech and German industrial production indices. They also produce an almost isolated response in the first (PX50 exchange index) and the third (Dax) of the modelled assets. It turns out that the shocks we model (denoted by CZ $\pm$ 0.01, CZ $\pm$ 0.02, CZ $\pm$ 0.03 and DE $\pm$ 0.01, DE $\pm$ 0.02, DE $\pm$ 0.03 in Figures 4 and 5) correspond to the 1, 2 and 3% rise/decline of the Czech and German Industrial Production Index (IPI), respectively.

Figure 4

Pricing kernel model: loan portfolio distribution under domestic and foreign real economic activity shifts

(a) Shocks to the Czech business cycle



(b) Shocks to the German business cycle

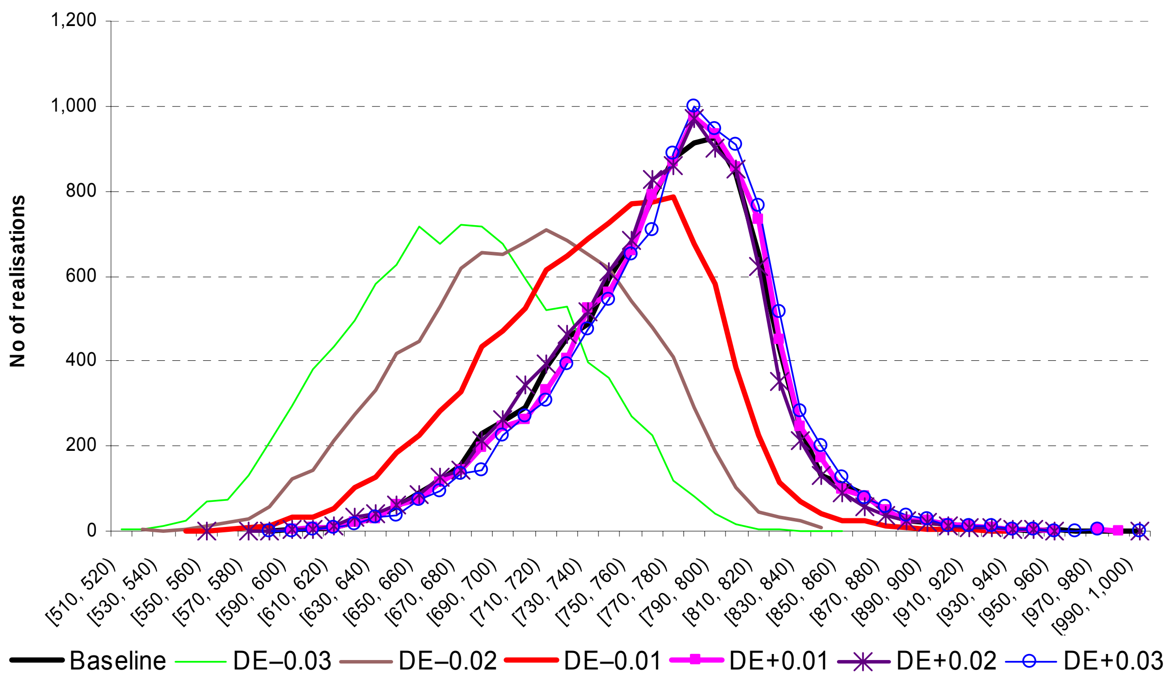


Figure 5

Portfolio value distributions according to the CreditMetrics and pricing kernel models, different growth scenarios

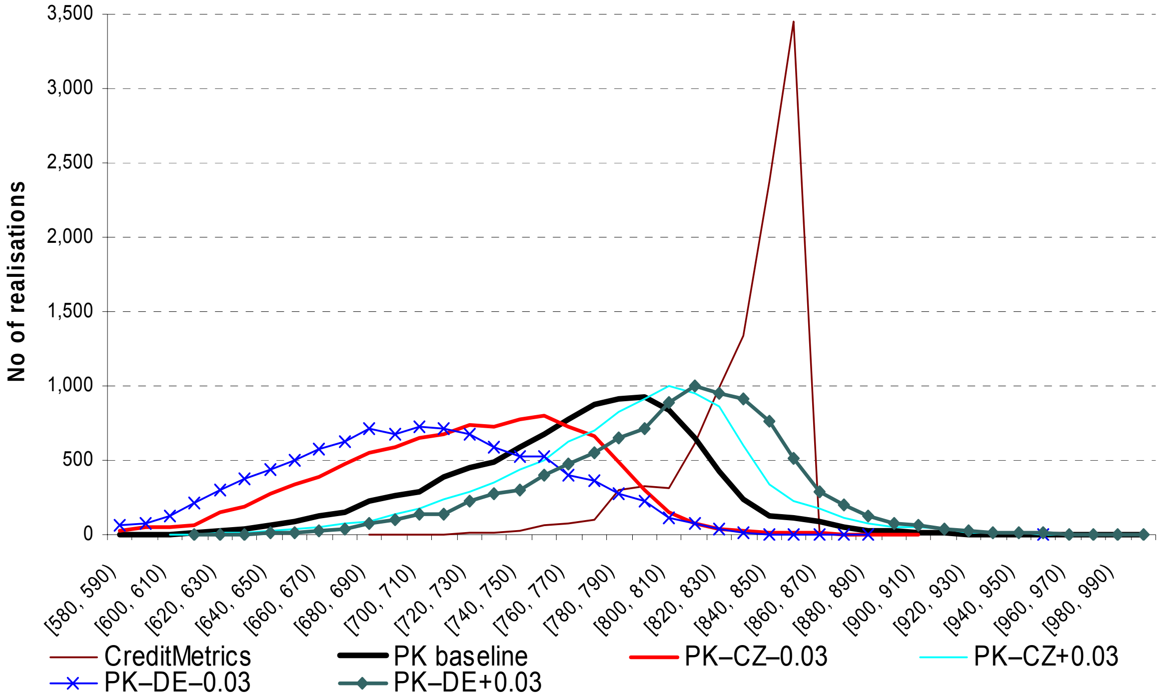


Table 8 sums up the credit risk-related estimations of economic capital in all the cases considered. For comparison, we have also included the regulatory capital measures for the same artificial portfolio (both standardised and IRB approaches) according to the original NBCA guidelines of January 2001 and the 3rd Quantitative Impact Study of October 2002. These were obtained in our earlier paper (Derviz et al (2003)), where a detailed account of the calculation procedure can be found.

6. Conclusion

This paper has an applied objective of analysing the impact of business cycle and monetary policy on credit risk valuation. It does not aspire to create an empirically waterproof econometric model of business cycle effects on asset prices as such. That is why we are not dealing with processes for real economic activity, inflation and monetary policy as (either observable or hidden) explanatory factors for the observed security yields. Instead, we take a shortcut by assuming that at least a subset of the chosen basic asset yields provides a sufficient statistic of either present or future (expected) economic growth and of the monetary policy stance. We are interested in modelling a domestic economic up-/downturn with foreign development being stable, and an expansion/recession abroad with the domestic environment being stable. This has been identified with the corresponding behaviour of the chosen asset returns in both economies considered. Namely, current high/low growth is reflected in the high/low current values of the leading asset return.

The model outlined in the paper is likely to be free of certain deficiencies typical to the two standard ones, whose application to economic capital calculations was described in detail in Section 3. For instance, a counter-intuitive negative dependence of the capital requirement on the market interest rate is an unfortunate feature of the way CreditMetrics works with the relation between the debt value and the economic capital. That model does not have any link from the interest rates to the firm's ability to repay the loan. On the other hand, our model is able to create this link because the obligor's net cash flow will usually be negatively related to the market rates of interest. Therefore, the reduction of the latter (such as the down-translation of the forward zero curve) increases the cash flow and, therewith, reduces the default event rate.

Table 8

Summary of economic capital estimations

	Regulatory capital	**1%-percentile**	**5%-percentile**	**Mean**	**99% economic capital**	**95% economic capital**	**Non-VaR economic capital**
NBCA - Standardised approach (Jan 2001)	51.84						
NBCA - IRB approach (Jan 2001)	165.46						
NBCA - Standardised approach (Oct 2002)	46.90						
NBCA - IRB approach (Oct 2002)	44.79						
CreditMetrics		767.90	796.62	845.78	77.89	49.16	
CreditRisk+ (Loss)		133	101	42.18	90.82	58.82	
Pricing kernel model baseline				**768.30**			**64.56**
CZ–0.03				723.00			109.86
CZ–0.02				737.94			94.93
CZ–0.01				754.27			78.59
CZ+0.01				775.68			57.19
CZ+0.02				783.32			49.54
CZ+0.03				790.24			42.63
DE–0.03				673.73			163.14
DE–0.02				704.80			132.06
DE–0.01				738.20			99.66
DE+0.01				770.62			68.24
DE+0.02				766.15			72.71
DE+0.03				774.14			65.72

Another important advantage of the model is its ability to handle correlated defaults in a natural way. This is because default correlation in the model is not an exogenously given property or an ad hoc assumption, but instead follows by construction from the dependence of default rates on common risk factors.

A certain difficulty lies in the necessity of calculating the pricing kernel recursively for multi-period loan contracts. However, the calculations themselves are routine and are based on well developed numerical techniques, allowing one to apply relatively standard software.

Once the distribution of the loan portfolio market price has been calculated, it can be used to derive the bank-internal measure of economic capital. The latter shall be subsequently compared with the prudential capital values derived directly from regulatory principles. The difference would tell us the degree of discrepancy between internal risk measurement and regulatory mechanisms of risk-based capital allocation by the bank.

The main conclusions of this study can be summarised as follows.

- Bank capital on the basis of our model would react procyclically. However, this reaction differs substantially from loan to loan, so that certain "countercyclical" loans may even be assigned lower capital values under a downturn. Also, stochastic properties of the collateral (their non-zero covariances with the underlying systemic uncertainties) mitigate the procyclical economic capital allocation in our model.

- When floating interest rates and changes in yield curves were modelled, the estimate of economic capital was generally higher. This result is intuitive, as increased uncertainty should generally impose higher required levels of capital on banks.
- The particular changes in the forward zero curves analysed in this paper in the CreditMetrics context did not impose significantly different levels of economic capital than did the case with stable forward zero curves. The scenario where forward zero rates fell (both translation and clockwise rotation) required slightly more capital, and this situation persisted at all considered confidence levels.
- Risky debt valuation by the traditional asset pricing methods currently in use by the banking industry tends to generate higher loan values and reduce economic capital requirements, compared to other possible regulatory and model-based risk measurement methods. Therefore, the regulator may see an effort on the part of the banks to treat different parts of loans on their balances differently in terms of economic capital. The difference will go in the direction of reducing capital allocation (and specialising collateral requirements) in those segments of the loan portfolio that exhibit strong correlation with traded risks.
- Asset pricing methods of risk measurement may lead to a better recognition of the role of the business cycle and other systemic macroeconomic factors in economic capital determination. Therefore, the feedback from the business cycle-related events in the security markets to economic capital, as captured in our model but also present in many existing risk management procedures, may make natural (and desirable) countercyclical economic capital adjustments possible.
- Those methods of credit risk measurement which explicitly deal with market incompleteness (ie the lack of market valuation of both the loan itself and the assets of the obligor) lead to a better recognition of the role of the business cycle and other systemic macroeconomic factors in economic capital determination. Therefore, the regulator should encourage the use of methods that allow for countercyclical adjustments by banks of procyclically biased risk management procedures.

Appendix A:
Yield curve fluctuations, CreditMetrics and economic capital

A.1 CreditMetrics - allocation of economic capital under floating lending rates and fixed forward zero curves

We assume that the mechanism of future changes in lending interest rates is as follows:

$$1 + \frac{r_t^{g'}}{100} = \left(1 + \frac{r_t}{100}\right) * e^{\sigma_t}$$

Here σ is a normally distributed random variable, so the exponential follows a log-normal distribution.

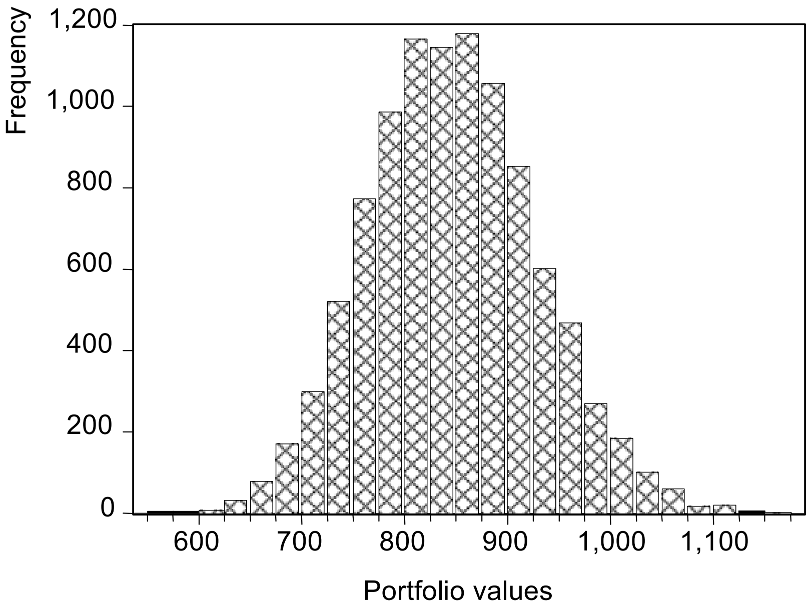
In this formulation, interest rates on loans preserve the markups that capture obligor-specific risk or liquidity premium (already incorporated in the old interest rates), but also contain a random component reflecting uncertainty related to the future course of the money market rate (Pribor). The random component does not vary across obligors. In our simulations it was obtained as a random draw from a log-normal distribution. The parameters (mean and standard deviation) describing the log-normal distribution were estimated on actual Czech data (1Y Pribor over the year 2000, a period of relative rate stability and no monetary policy changes).

A.2 Monte Carlo simulation with floating interest rates

A total number of 10,000 random draws from the standard log-normal distribution were performed to determine the random component of Pribor and thus the loans' interest rates. Due to the floating nature of the interest rates, the valuations of each asset and of the portfolio varied in line with the particular values of the random draws. A total number of 10,000 portfolio values at the end of the risk horizon were thus obtained. Figure A1 shows the relative frequencies of these values at the year-end.

Figure A1

The empirical distribution of the portfolio value with floating interest rates



Series: VP	
Sample 1 10,000	
Observations 10,000	
Mean	848.3348
Median	845.5493
Maximum	1167.615
Minimum	554.5349
Std dev	81.97142
Skewness	0.195979
Kurtosis	3.005247
Jarque-Bera	64.02450
Probability	0.000000

Note: The horizontal axis shows non-overlapping intervals that cover the entire range of the estimated portfolio values, and the vertical axis shows the frequencies with which the portfolio values fell into those intervals.

The estimation of economic capital in this case is given in Table A1.

Table A1

Capital requirements assuming different changes in interest rates and forward zero curves (FZCs)

(10,000 random draws)

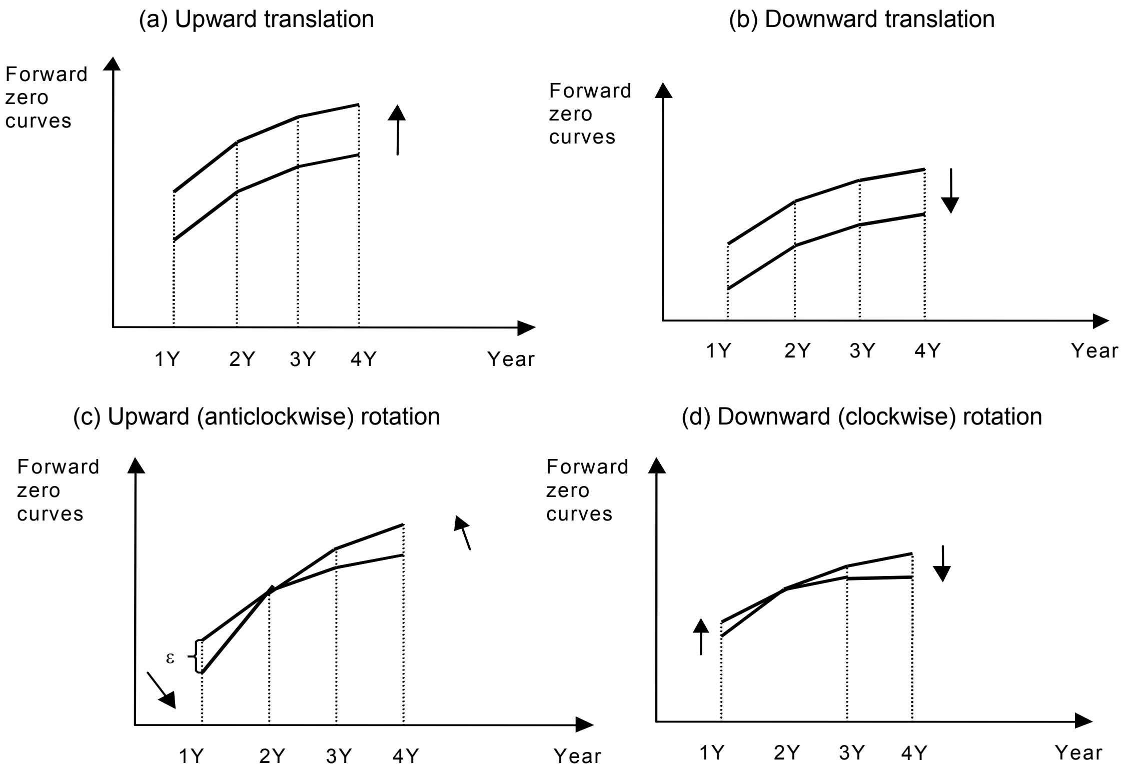
	1%-percentile	5%-percentile	Mean	99% ec capital	95% ec capital
Fixed interest and fixed FZC	767.90	796.62	845.78	77.89	49.16
Floating interest and fixed FZC	669.65	718.48	848.33	178.69	129.85
Upward translation of FZC	655.15	705.70	834.25	179.11	128.56
Downward translation of FZC	672.44	724.32	856.94	184.50	132.62
Anticlockwise rotation of FZC	668.83	722.92	839.14	170.31	116.22
Clockwise rotation of FZC	668.83	722.92	853.01	184.18	130.09

A.3 Monte Carlo simulation with floating interest rates and stochastic forward zero curves

The next four cases retain the assumption that floating interest rates were charged by the bank on its loans. Additional uncertainty is added with regard to changes in forward zero curves (the discount factors entering the loan valuations) over the one-year period. We analysed the impact on the bank's need for economic capital under the following changes in the forward zero curves: upward translation, downward translation, clockwise rotation and anticlockwise rotation. Rotations of the forward zero curves are around the point determined by the two-year forward rate. These changes are illustrated in Figure A2.

Figure A2

Simulated changes in the forward zero curves



We assumed that these changes reflected subjective expectations concerning the evolution of the Czech forward zero curves over a one-year period. They became rating class-specific by adding the US forward spread (the difference between the US rating class-specific and the US forward zero curves). We incorporated these changes into the forward zero curves in the model according to the formula:

$$1+\frac{f_t^{g'}}{100}=\left(1+\frac{f_t}{100}\right)*\left(1+\frac{s_t^g}{100}\right)*e^{\varphi_t} \quad t=1,\,2,\,3,\,4 \tag{A1}$$

Here f_t is the original Czech forward zero rate at year t, s_t^g is the spread in forward zero rates characteristic of the g-th rating class at time t, and φ_t is a random draw from the normal distribution (thus $e^{\varphi}{}_t$ is log-normally distributed).

The proposed changes in the forward zero curves were captured in the model by considering particular random variables ϕ_t in (A1):

$\phi_t = \mu + \varepsilon$, $t=1, 2, 3, 4$, for an upward translation;

$\phi = -\mu + \varepsilon$, $t=1, 2, 3, 4$, for a downward translation;

$\phi_1 = -\mu + \varepsilon$, $\phi_3 = \mu + \varepsilon$, $\phi_4 = 2\mu + \varepsilon$, for an upward rotation;

$\phi_1 = \mu + \varepsilon$, $\phi_3 = -\mu + \varepsilon$, $\phi_4 = -2\mu + \varepsilon$, for a downward rotation.

The overall effect of an upward shift in the forward zero curve is a decrease in the present value of all loans in the portfolio. If the bank heavily discounts the future, the opportunity cost of granting loans increases, since alternative assets may provide higher returns in the future. Accordingly, the present value of the cash flows accrued from the loans is lower compared with the case where the forward zero curves remain unchanged. The effect of a downward shift of the forward zero curve is the opposite of the one mentioned above. Rotations of the forward zero curve affect assets' valuations depending on maturity. For example, the anticlockwise (upward) rotation discounts assets with a short maturity less and assets with a long maturity more. Therefore, the valuation of the portfolio is very sensitive to the portfolio composition. If more assets fall into the long-maturity category the present value of the portfolio tends to decrease, while if they fall into the short-maturity category the present value of the portfolio tends to increase.

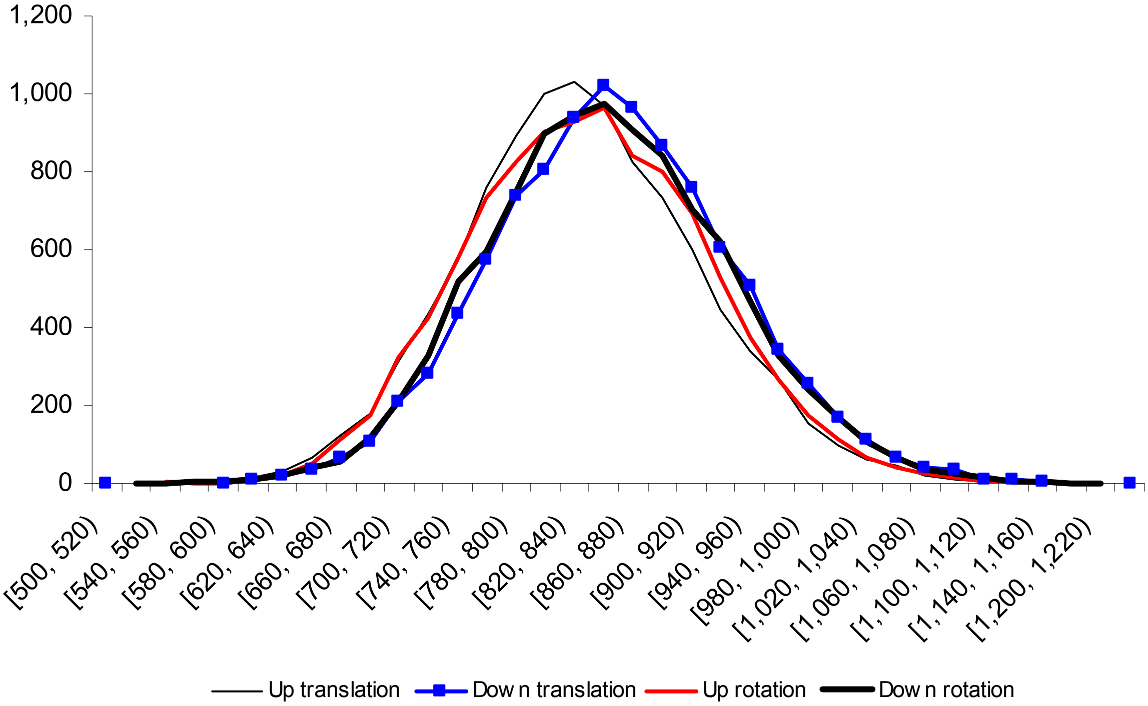
In all previous formulations of the φ_t distribution, the parameter μ determined the magnitude of the deterministic change in forward zero curves and ε added random deviations. We wanted changes in the deterministic part of the discount factors not exceeding 1% (thus, if for a given maturity the forward zero rate was 3.4%, we wanted it to deviate upward to 4.4% only). This assumption implied a value for μ of 0.01. In each case, ε was assumed to follow a normal distribution with mean $-\frac{\sigma^2}{2}$ and standard deviation σ, so that the mean of the log-normally distributed factor e^{ε} is equal to 1. Under these conditions φ became normally distributed with mean $\mu+m$. The standard deviation σ of ε was estimated by computing the standard deviation of the log(1+1Y Pribor/100) variable using daily observations over the year 2000 after removing the trend. The same estimated values of the parameters μ and σ were used in all cases of random changes in forward zero curves.

We performed Monte Carlo simulations containing 10,000 scenarios that simultaneously accounted for random changes in interest rates and forward zero curves. Shown next are the portfolio value distributions and the estimates of economic capital based on simulations that incorporated the proposed changes into the forward zero curves.

Figure A3 displays the portfolio value distributions when the four forward zero curve change cases discussed above are compared.

Figure A3

Portfolio distributions under different forward zero curve movements



Economic capital estimations at different confidence levels are displayed in Table A1. The downward translation and the clockwise rotation of the forward zero curves impose the highest requirements of economic capital in this particular example. However, economic capital seems to converge towards the fixed forward zero curves (with floating lending rates) case when the confidence level is reduced.

Appendix B:
Proof of Proposition 1

The pricing relation (4) for asset j can be written as $E_t\left[e^{m_{t+1}+y^j_{t+1}}\right]=1$. Since both m and y^j are normally distributed with known conditional expectations and variances for each date t, the last equation can be rewritten as:

$$E_t\left[m_{t+1}+y^j_{t+1}\right]+\frac{1}{2}Var_t\left[m_{t+1}+y^j_{t+1}\right]=0.$$

In accordance with (8)-(10), this is equivalent to:

$$\lambda_0+\lambda_1 x_t+\Lambda b x_t+a_0^j+a_1^j x_t+A^j b x_t+\frac{1}{2}\left|(\Lambda+A^j)B\right|^2=0 \qquad \text{(B1)}$$

for all t for each j. Equations (B1) can be considered as identities involving a non-trivial vector autoregressive state process x. They are satisfied if and only if all coefficients on the left-hand side of (B1) are identically zero. This means:

$$a_0^j+\lambda_0+\frac{\left|(\Lambda+A^j)B\right|^2}{2}=0,\ a_1^j+\lambda_1+(\Lambda+A^j)b=0$$

for all j, which is equivalent to (11).

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