# Linking real activity and financial markets: the first steps towards a small estimated model for Canada

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## 1. Introduction

The crudest feature of many models' treatment of financial markets is that they aggregate all financial markets into only two: the market for money and the market for everything else. This aggregation allows us to summarise asset market equilibrium in a single LM curve but hides the structure needed to achieve a good understanding of how financial markets and the real economy are interrelated.

Another weakness of most models that purport to describe the transmission mechanism is their failure to pass the simple test of generating a different steady state rate of inflation in response to a series of monetary policy actions.[2] Such models with an unique steady state rate of inflation are very difficult to reconcile with the unit root test results found in the empirical literature.[3] One goal of this paper is to identify permanent shocks causing inflation to reach a new steady state rate of growth. A second goal is to model equilibrium values of financial variables through their long-run relationships with real variables in a tractable macroeconomic model.

Over the past two decades, there has been a growing interest in developing tractable macroeconomic models with transparent theoretical foundations. As written in Garratt et al (2001): "There are two main theoretical approaches to the derivation of long-run, steady state relations of a core macroeconomic model. One possibility is to start with the inter-temporal optimisation problems faced by 'representative' households and firms and solve for the long-run relations. […] An alternative approach […] is to work directly with the arbitrage conditions which provide inter-temporal links between prices and asset returns in the economy as a whole. […] The strength of the inter-temporal optimisation approach lies in the explicit identification of macroeconomic disturbances as innovations (shocks) to processes generating tastes and technology. However, this is achieved at the expense of often strong assumptions concerning the form of the underlying utility and production functions." In contrast, the approach that Garratt et al (2001) and the present paper adopt, focuses on long-run theory restrictions and leaves the short-run dynamics largely unrestricted (in the context of a VECM model), thus providing a much more flexible modelling strategy.

Our aim is to combine Garratt et al's (2001) approach with King et al's (1991) methodology allowing the identification of permanent shocks in a cointegrated system. Crowder et al (1999), Dhar et al (2000), Jacobson et al (2001) and Cassola and Morana (2002) all follow this *route* and show the degree of "structure" that may be assigned to a simple vector autoregression (VAR) framework characterised by cointegration if one embraces sufficient identifying restrictions.

The building blocks of the model consist of three cointegrating relations: (1) a money market equilibrium relation, (2) an arbitrage relation between short- and long-term bonds, and (3) a long-run relation between the stock market and real output. This last relation allows the identification of a supply shock as the only shock permanently affecting the stock market and a demand shock leading to significant transitory stock market overvaluation. We also identify a nominal shock defined as the only shock having a permanent impact on the level of inflation. In future work, we will study the

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[2] More details on this point are made in Selody (2001).

[3] This is also a very difficult issue as inflation is expected to become stationary in a successful inflation targeting environment.

behaviour of a monetary policy reaction function consisting in reversing any identified nominal shock causing inflation to permanently deviate from the target.

Our paper is organised as follows. The theoretical foundations of the model are presented in Section 2. The cointegration analysis and specification test results are given in Section 3. Section 4 presents the econometric formulation of the core model. Section 5 analyses the impulse response functions. A conclusion follows.

## 2. The theoretical foundations of the model

In this section, we describe the long-run relations used as the building blocks of our model. We base our core model on Blanchard (1981), who develops a simple model of the determination of output, the stock market and the term structure of interest rates. The model is an extension of the IS-LM model. However, whereas the IS-LM model emphasises the interaction between “the interest rate” and output, Blanchard’s model emphasises the interactions between output and four marketable asset values. These are shares which are titles to the physical capital, private short- and long-term bonds issued and held by individuals, and money.

### Linking the real economy and the stock market

We assume that there are two main determinants of spending.[4] The first is the value of shares in the stock market. It may affect spending directly through the wealth effect on consumers, or indirectly through its impact on the borrowing capacity of consumers and investors (the credit channel effect); determining the value of capital in place relative to its replacement costs, it affects investment. The second is current income, which may affect spending independently of wealth if consumers are liquidity-constrained. Total spending is expressed as:

$$d_t = \alpha sm_t + \beta y_t;\ \ \alpha > 0;\ \ \beta > 0 \qquad (1)$$

All variables are real, $d$ denotes spending, $sm$ is the stock market value, and $y$ is income.[5] We can see equation (1) as a forward-looking aggregate spending curve with $sm$ being a function of the present value of expected future profits, the latter being a function of expected future output. Hence, aggregate spending is implicitly a negative function of actual and expected interest rates and a positive function of actual and future expected output. Output adjusts to spending over time:

$$\dot{y}_t = \sigma(d_t - y_t) = \sigma(\alpha sm_t - by_t);\ \ \sigma > 0;\ \ b \equiv 1 - \beta \qquad (2)$$

where a dot denotes a time derivative. Since output growth is a stationary variable and the level of output and the stock market price are both $I(1)$ variables, equation (2) can be seen as an error correction equation positively linking the short-run dynamics of output to deviations of the stock market from the real economy. Such a long-run relation between output and the stock market implies that transitory changes in output cannot permanently affect the level of the stock market.

### Money market equilibrium

Portfolio balance is characterised by a long-run relation between money, output, interest rate and inflation:

$$M_t - p_t = cy_t - hi_t - \beta\pi_t;\ \ c > 0;\ \ h > 0;\ \ \beta > 0 \qquad (3)$$

where $i$ denotes the short-term nominal rate, $y$ is real income, $M$ and $p$ denote the logarithms of nominal money and the price level, and $\pi$ is the level of inflation. The parameter $c$ is positive because an increase in output shifts the money demand for transaction purposes upwards; an increase in the

[4] Blanchard also includes a balanced budget change in public spending as a third determinant of total spending.

[5] No stochastic error terms are included in this section to simplify the presentation.

interest rate and an increase in inflation both increase the opportunity cost of holding money, which decreases real balance. Given that all the variables in equation (3) are better characterised as $I(1)$ variables, if deviations of real money from its determinants are transitory, then this equation represents a cointegrating relationship.

**Arbitrage between short- and long-term bonds**

The expectations hypothesis is perhaps the best known and most intuitive theory of the term structure of interest rates. If $lr_t$ is the nominal yield to maturity of a discount bond and $i_t$ is the period-t one-period rate, the expectations hypothesis in the absence of uncertainty implies that

$$(1+lr_t)^n = \prod_{i=0}^{n-1}(1+i_{t+i}) \tag{4}$$

This is an arbitrage condition ensuring that the holding-period yield on the n-period bond is equal to the yield from holding a sequence of one-period bonds. Taking logs of both sides and recalling that $\ln(1+x) \sim x$ for small $x$, yields a common approximation:

$$lr_t = \frac{1}{n}\sum_{i=0}^{n-1} i_{t+i} \tag{5}$$

The long-term yield is equal to the average of one-period yields. Hence, a permanent shock to the short-term yield will, in the long run, be reflected one for one in the long-term yield, once the shock is correctly perceived as permanent by the financial markets. This defines a third cointegration relationship.

## 3. Cointegration analysis

We estimate a monthly VECM over the 1975-2002 period with six endogenous variables and one exogenous variable and two lags.[6] The endogenous variables are the following Canadian variables: real gross domestic product (GDP) at basic prices,[7] the over 10-year marketable bond rate, the overnight rate,[8] a broad money aggregate (CPI deflated M2++),[9] the real stock market price (CPI deflated Toronto Stock Exchange (TSE)), and the CPI year-over-year inflation rate. Given the strong economic links between Canada and the United States, we incorporate as an exogenous variable the real US industrial production index, one available monthly proxy for US activity. This will allow simulation of different US scenarios. Unit root tests indicate that all variables can be treated as $I(1)$ variables. We add a dummy equalling one from 1993 onwards to capture the change in the trend of inflation after the adoption of an inflation target in 1991.

Based on the theoretical foundations of the core model described in the above section, we expect to find three cointegrating relations in the estimated VECM (as described by equations (2), (3) and (5)). The cointegration tests corrected for the presence of one exogenous variable, as proposed by Pesaran et al (2000), are presented in Table 1. The L-max test indicates the presence of three cointegration vectors, supporting our a priori expectations based on Blanchard's model, while the Trace test suggest only two vectors.

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6 Two lags minimise the Hannan-Quinn and Schwartz information criteria and are sufficient to remove the correlation in residuals. We use monthly data because the Bank of Canada has adopted a fixed action date schedule eight times a year. A series of specification tests have been done and will be included in the next version of this paper.

7 This series has been merged with real GDP at factor cost for the period 1975-80.

8 As noted in Selody (2001), a good monetary policy instrument must be under the direct or close control of the central bank.

9 M2++ includes mutual funds, whose importance increased continuously in consumer portfolios over the 1990s, and which are relatively liquid. Using a broad aggregate like M2++ in the model avoids interpreting a precautionary portfolio adjustment from mutual funds to money as inflationary. Moreover, Longworth (2003) finds that, since 1992, both core inflation and M2++ have been remarkably stable.

Table 1

**Cointegration tests**[1]

| L-max | Trace | $H_0$: r = | L-max (0.10) | Trace (0.10) |
|---|---|---|---|---|
| 63.12 | 151.48 | 0 | 40.2 | 104.4 |
| 46.36 | 88.36 | 1 | 34.1 | 76.9 |
| 35.84 | 42.00 | 2 | 28.3 | 54.8 |
| 10.39 | 16.17 | 3 | 22.2 | 35.9 |
| 2.97 | 5.78 | 4 | 15.9 | 20.8 |
| 2.81 | 2.81 | 5 | 9.5 | 9.5 |

[1] The critical values corrected for the presence of one exogenous variable are taken from Table T.3 in Pesaran et al (2000).

To discriminate between our cointegration tests, we looked at the t-values of the $\alpha$ coefficients for the third vector, as suggested in Hendry and Juselius (2001); when these are small, say less than 3.0, then one would not lose greatly by excluding that vector as a cointegration relation in the model. Given that many of these t-values are greater than 3.0 for all three vectors and that our theoretical model also suggests three vectors, we proceed under the assumption that there are three cointegrating vectors in our model.

The Johansen (1992) procedure allows us to identify the number of cointegrating vectors. However, in the case of existence of multiple cointegrating vectors, an interesting problem arises: $\alpha$ and $\beta$ are only determined up to the space spanned by them. Thus for any non-singular matrix $\zeta$ conformable by-product:

$$\Pi = \alpha\beta' = \alpha\zeta\zeta^{-1}\beta'$$

In other words, $\beta$ and $\beta'\zeta$ are two observationally equivalent bases of the cointegrating space. The obvious implication is that before solving such an identification problem, no meaningful economic interpretation of coefficients in cointegrating space can be proposed. The solution is imposing a sufficient number of restrictions on parameters such that the matrix satisfying such restrictions in the cointegration space is unique. Such a criterion is derived in Johansen (1992) and discussed in Hamilton (1994). Our restrictions are based on Blanchard's model and suggest more than a sufficient number of constraints on the cointegration space. The overidentification restrictions can therefore be tested. The results are in Table 2.

Table 2

**Testing restrictions on the cointegrating vectors**[1]

**The LR test, $\chi^2(10) = 7.02$, p-value = 0.72**

| *inf* | *y* | *onr* | *m* | *sm* | *lr* | $y^{US}$ |
|---|---|---|---|---|---|---|
| 2.41 (0.27) | –1.18 (0.08) | 2.41 (0.27) | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | –1 | 0 |
| 0 | –1 | 0 | 0 | 1 | 0 | 0 |

[1] Standard errors are given within parentheses.

The restricted core model is strongly accepted with a p-value of 0.72. These results are consistent with the theoretical foundations presented in Section 2. The first cointegrating relation corresponds to the money market equilibrium, and the second to an approximation of the pure expectations hypothesis based on an arbitrage relation between short- and long-term bonds, while the third relation links real activity with the real stock market. The coefficients of the cointegrating relation cannot usually be interpreted as elasticities even if the variables are in logs, since a shock to one variable implies a shock to all variables in the long run. Hence the coefficients do not in general allow for a ceteris paribus interpretation (see Lutkepohl (1994)). Interpreting the coefficients in the first cointegrating relation is thus meaningless. However, given that the last two cointegrating relations involve only two variables, we do not need the ceteris paribus interpretation. The second long-run relation specifies that a permanent 1% increase in the short-term interest rate is associated with the equivalent increase in the long-run interest rate. The third cointegrating relation suggests that a 1% permanent increase in output (or a 1% increase in potential output) is associated with a permanent 1% increase in the stock market. Interestingly, this last relation also implies that transitory changes in real output can only lead to transitory changes in the level of the stock market.

The economy is in a long-run equilibrium when those three cointegrating relationships are respected, that is, when there is no gap between money, output, inflation and the overnight rate (or no money gap), the overnight rate is equal to the long rate (no interest rate gap), and the stock market level is consistent with potential output (no stock market gap).

Graph 1 illustrates the money gap[10] over the sample period. The two surges in inflation, in 1981 and 1991, were preceded by an increasing money gap around two years before. It is also interesting to note that since the Bank of Canada adopted an explicit inflation target in 1991, the money gap has been much more stable, deviating only slightly from equilibrium and for short periods of time in 1995 and 2000. This is in line with the results in Longworth (2003), who reports that, since 1992, both core inflation and M2++ have been remarkably stable.

The interest rate gap is defined as the yield spread (the long minus the overnight rate), well known as a good monetary policy stance measure. With this definition of the interest rate gap, the short rate is at its neutral level, or at its long-run equilibrium value, when it is equal to the long rate. According to this definition, the Bank of Canada was restrictive at the end of the 1980s to achieve the following disinflation and was accommodative for most of the 1990s except for a short period in 1999-2001. The overnight rate was back below equilibrium at the end of 2002 by almost 2%.

The stock market gap in Graph 2 illustrates periods of "mis-valuation" of the stock market.[11] Our results show that the stock market led the 1981 and 1991 recessions and became strongly undervalued (close to 40%) after the 1981 recession. It became relatively less depressed after the 1991 recession (around 20%), but took longer to recover; the market got back to its fair value only in 1994. Graph 2 also shows that the stock market was about 20% overvalued before the 1987 crash and undershot by about 10% afterwards. The market was overvalued for most of the 1996-2000 period, except for the strong correction following the Asian crisis in 1998. By far the most significant departure from equilibrium happened at the beginning of 2000 when the stock market appeared to have been close to 60% higher than what was justified by "fundamentals". Finally, the bubble burst and the market overreacted again. Graph 2 suggests it was about 10% undervalued at the end of 2002. These results are in line with Dupuis and Tessier (2003), who estimate a three-variables VECM linking the US stock market to dividends and the long-term interest rate.

## 4. Econometric formulation of the core model

The three long-run equilibrium relationships can be written in the following form:

$$m_t = c_{11} + c_{12}y_t + c_{13}onr_t + c_{13}inf_t + \xi_{1t+1} \quad (6)$$

$$lr_t = c_{21} - onr_t + \xi_{2t+1} \quad (7)$$

[10] The gaps in this section are simply defined as the error correction term from the cointegrating relations. Gaps based on permanent components of the variables will be presented in Section 5.

[11] Note that the permanent components of the variables have yet to be identified before we can tell if a positive error correction term is due to the stock market being too high or output too low (or both). This is done below.

$$sm_t = c_{31} - y_t + \xi_{3t+1} \quad (8)$$

The three long-run relations of the core model, equations (6), (7) and (8), can be written as

$$\xi_t = \beta' z_{t-1} - c_0 \quad (9)$$

where $z_t = (inf_t, y_t, onr_t, m_t, sm_t, lr_t, y_t^{US})'$, $c_0 = (c_{11}, c_{21}, c_{31})'$, $\xi_t = (\xi_{1t}, \xi_{2t}, \xi_{3t})'$, and

$$\beta' = \begin{bmatrix} -c_{13} & -c_{12} & -c_{13} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (10)$$

Let $x_t = (inf_t, y_t, onr_t, m_t, sm_t, lr_t)'$. We base our analysis on the following conditional error correction model:

$$\Delta x_t = a - \alpha\xi_t + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + b\Delta y_t^{US} + u_{xt} \quad (11)$$

where $a$ is a $6 \times 1$ vector of fixed intercepts, $\alpha$ is a $6 \times 3$ matrix of error correction coefficients, $b$ is a $6 \times 1$ vector representing the impact effects of changes in US output on $\Delta x_t$, and $u_{xt}$ is a $6 \times 1$ vector of disturbances assumed to be $IID(0, \Sigma_x)$, with $\Sigma_x$ being a positive definite matrix.

From equations (9), (10) and (11), we have

$$\Delta x_t = a_y + \alpha_y c_0 - \alpha_y \beta' z_{t-1} + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + b_y \Delta y_t^{US} + u_{yt} \quad (12)$$

where $\beta' z_{t-1}$ is a $3 \times 1$ vector of error correction terms. This specification implies the economic theory's long-run predictions by construction. The estimations of the parameters in equation (12) are obtained by using the estimation procedure of vector error correction models with exogenous $I(1)$ variables (Pesaran et al (2000)).

## 5. Shock analysis

### The impact of a change in US industrial production

The response functions to a permanent increase of 1% in US industrial production are shown in Graph 4. Small inflation pressures are generated as output is boosted by almost 0.2% on impact. Interest rates are increased by around 25 basis points to keep demand in line with short-run supply. The Canadian stock market is negatively affected by the higher interest rate. It nevertheless increases by 0.12% in the long run, in line with the permanent increase in output.[12] Broad aggregate money is negatively affected in the short run by the slight increases in inflation and real interest rates. Only output is significantly affected in the long run.

### Identification of the permanent shocks

Given the presence of three cointegrating vectors and six endogenous variables, there are three stochastic trends or permanent shocks to be identified.[13] The first permanent shock, $\varepsilon_{\pi t}$, labelled an inflation shock, is the only shock having a permanent impact on inflation. According to the "monetarist" view, the long-run money growth and inflation rate are ultimately set exogenously by monetary authorities. So the inflation shock relates to central bank monetary policy. A positive inflation shock

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12 US industrial production represents about 15% of US total GDP. Under the assumption that a permanent increase of 1% in US industrial production translates into an increase of 0.15% in US total GDP, our results suggest that a 0.15% increase in US GDP is associated with an increase of about 0.12% in Canadian GDP.

13 Details on identification in the presence of exogenous variables will be published in a future version of this paper.

reflects the central bank's decision to permanently increase the inflation rate. Hence, the structural inflation shock is identified by assuming that the long-run system has the following recursive structure:

$$\lim_{s\to\infty}\begin{bmatrix} inf_{t+s} \\ y_{t+s} \\ onr_{t+s} \\ m_{t+s} \\ sm_{t+s} \\ lr_{t+s} \end{bmatrix} = \begin{bmatrix} \tau_{11} & 0 & 0 \\ \tau_{21} & \tau_{22} & 0 \\ \tau_{31} & \tau_{32} & \tau_{33} \\ \tau_{41} & \tau_{42} & \tau_{43} \\ \tau_{51} & \tau_{52} & \tau_{53} \\ \tau_{61} & \tau_{62} & \tau_{63} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{yt} \\ \varepsilon_{dt} \end{bmatrix}$$

Note that $\tau_{ij}$ is the long-run response of the *i*th endogenous variable to the *j* element in the vector of structural disturbances $\varepsilon_t$. The restrictions $\tau_{12} = 0$ and $\tau_{13} = 0$ mean that only an inflation shock, $\varepsilon_{\pi t}$, affects the long-run level of inflation. The mainstream view would predict that the decision to change inflation permanently has no permanent impact on real variables and thus that $[\tau_{21}\ \tau_{41}\ \tau_{51}] = 0$. However, economic theory provides no clear-cut predictions on that question. In several theoretical models, the superneutrality result due to Sidrausky (1967) breaks down as inflation can have either positive or negative effects on real variables such as consumption and investment, depending on the exact assumptions concerning preferences. Additionally, in these models the real interest rate may or may not be independent of inflation in the long run (see Orphanides and Solow (1990) for a survey). Some recent empirical results (see, for example, Rapach (2003) and Gauthier et al (2003)) find support for the Mundell-Tobin effect, suggesting that an unexpected increase in inflation has a permanent negative impact on the real interest rate. We let the data talk on this point by leaving unconstrained the parameters in $[\tau_{21}\ \tau_{31}\ \tau_{41}\ \tau_{51}\ \tau_{61}]$.

Most theoretical models define supply shocks as being governed by technology innovations determining the technical capacity of the economy. We thus identify a supply shock as a shock allowed to have a permanent effect on output but not on inflation. The long-run effects on all the other real variables are left unconstrained. Note that all shocks are allowed to impact all the variables in the short run. In particular, a supply shock is expected to decrease inflation in the short run.

The third structural shock is a shock having no permanent impact either on output or on inflation. This shock is labelled a demand shock.

### The inflation shock

A positive inflation shock reflects the central bank's decision to permanently increase the inflation rate.[14] Given the instrument used by the central bank, this can only be achieved by decreasing the overnight rate. Graph 5 shows that our results are consistent with this view. To achieve a typical unexpected inflation increase of around 0.3% in the long run, the central bank has to decrease the overnight rate by about 25 basis points. Given the expectations hypothesis of the term structure in our core model, the long rate is persistently depressed as well. The bank's intervention leads to a small output stimulus in the short run. The shock also significantly hurts the stock market and decreases *real* broad aggregate money in the short run.

The permanent significant negative effect of inflation on interest rates may be explained through the Mundell effect: an *unexpected* increase in inflation decreases real wealth, which increases savings. Real interest rates must then fall to restore goods market equilibrium. Our results are also consistent with the focus on stabilising output in the 1970s and the beginning of the 1980s even at the cost of higher inflation. Furthermore, they are in line with the need to persistently increase the interest rate in disinflation periods and in the first years of inflation targeting in order to gain credibility. Rapach (2003)

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[14] Of course, such a shock can always be reversed by a negative inflation shock of the same size, if the central bank decides to do so.

also finds that an unexpected permanent increase in inflation is associated with permanently lower long-run real interest rates in every industrialised country in a sample of 14, including Canada.[15]

When inflation is forecast to deviate permanently from the actual target of 2%, the historical estimated reaction function (the equation for the overnight rate) may be adjusted using the estimated impact over time of the typical permanent inflation shock in such a way as to eliminate the expected long-run deviation from target.

### The supply shock

The typical supply shock increases the productive capacity of the economy by around 0.9% in the long run. Inflation is pushed downwards in the short run as production costs are decreased (Graph 6) but goes back to its initial level in the long run. The central bank has, over the sample, accommodated the shock by decreasing interest rates to eliminate the excess supply in the goods market and bring inflation back to target.[16] The stock market leads output and overshoots somewhat. Broad money is higher in the short run because of the accommodative stance of monetary policy and remains higher in the long run because of both higher money demand for transaction purposes and higher real value of the stock market.

### A demand shock[17]

The demand shock increases inflation, output and the stock market in the short run. Short and long interest rates increase in the short run as expected. This can be seen as the result of a standard textbook open market operation with a disinflationary objective. When inflation and output turn out to be higher than expected, an inflation targeting central bank has to increase interest rates. It is interesting to note that since a demand shock has no permanent impact on output, the significant stock market surge in the first months following the shock slowly dissipates as investors realise that higher profits cannot be sustained without a permanent increase in productivity.

The permanent positive impact on the overnight rate implies that the so-called demand shock induces, on average, a higher equilibrium interest rate. This, again, is consistent with the need to persistently increase the interest rate in disinflation periods and in the first years of inflation targeting in order to gain credibility. Furthermore, as predicted by the long-run theory of growth models, any shock that persistently lowers the share of product going into investment is associated with higher real interest rates in the long run.[18] For example, fiscal shocks crowding out investment persistently will be associated with persistently higher interest rates.

### Output gap

An output gap is easily obtained from our model as the difference between actual output and the historical contribution of permanent shocks to output (determining potential output). Potential output and the output gap are plotted in Graphs 8 and 9 respectively. According to these results, the Canadian economy was in excess demand before both the 1982 and the 1991 recessions and was in excess supply for most of the 1990s. The gap was closed at the end of 1999 and the economy turned

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15 Note that a permanent inflation shock represents an *unexpected* persistent deviation of inflation from its deterministic trend. This source of increase in inflation is associated in the long run with a decrease in interest rates. That, of course, does not mean that *expected* changes in inflation have the same effect on interest rates.

16 In some SDGE models with adjustment costs on capital (see Neiss and Nelson (2001, p 23) for an example), productivity shocks would decrease the *neutral* rate in the short run. This provides further incentives to decrease the actual interest rate after a productivity shock.

17 Other demand shocks having only transitory effects may also be identified.

18 King et al (1991) estimate a significant cointegration relationship negatively linking the ratio of investment to output and the real interest rate in the United States and identify what they call a “real interest rate shock” with long-run properties very similar to our “demand” shock. They also identify what they call a “balanced-growth” shock, which is very similar to our supply shock increasing output permanently while leaving the ratios of investment and consumption to output as well as the real interest rate and inflation unchanged in the long run.

to excess demand for the following two years. The economy was back in excess supply (though close to zero) at the end of 2002. What may be more surprising is the period over which supply shocks contributed to increasing output permanently. Graph 8 suggests that it started around 1985 and lasted until 1996, the year Chairman Greenspan first talked of irrational exuberance. From 1996 until the end of 2000 and the strong stock market correction, the economy was demand-driven and potential would have been growing at a rate lower then the deterministic rate.[19] This result, in line with Dueker and Nelson (2002) and the latest economic developments, casts some doubts on the purported “new economy” in the second half of the 1990s.

## 6. Conclusion

We have estimated a small monthly VECM to study the interactions between the real and financial sectors of the Canadian economy. To take into account the high degree of economic integration between Canada and the United States, the US industrial production index has been included as an exogenous variable. Identification of permanent shocks in a VECM with exogenous variables represents a technical contribution to the literature.

Our principal results are: (1) the identification of a long-run relation between the stock market and real output which allows the identification of a supply shock as the only shock permanently affecting the stock market and a demand shock leading to significant transitory stock market overvaluation; (2) the money gap defined as the error correction term from the first cointegrating relation has been much more stable since the adoption of inflation targets in Canada.

The next step in this project is to study the behaviour of a reaction function that would reverse any identified nominal shock causing inflation to persistently deviate from the target. The model could also be used to build a financial condition index for Canada using the stock market and money gaps from the core model together with the deviation of the actual real interest rate from the neutral interest rate recommended by the proposed reaction function. This index could also possibly be completed with the deviation of the Canadian exchange rate from equilibrium provided in Gauthier and Tessier (2002) and tested against those proposed in Gauthier et al (2003). This is left for future research.

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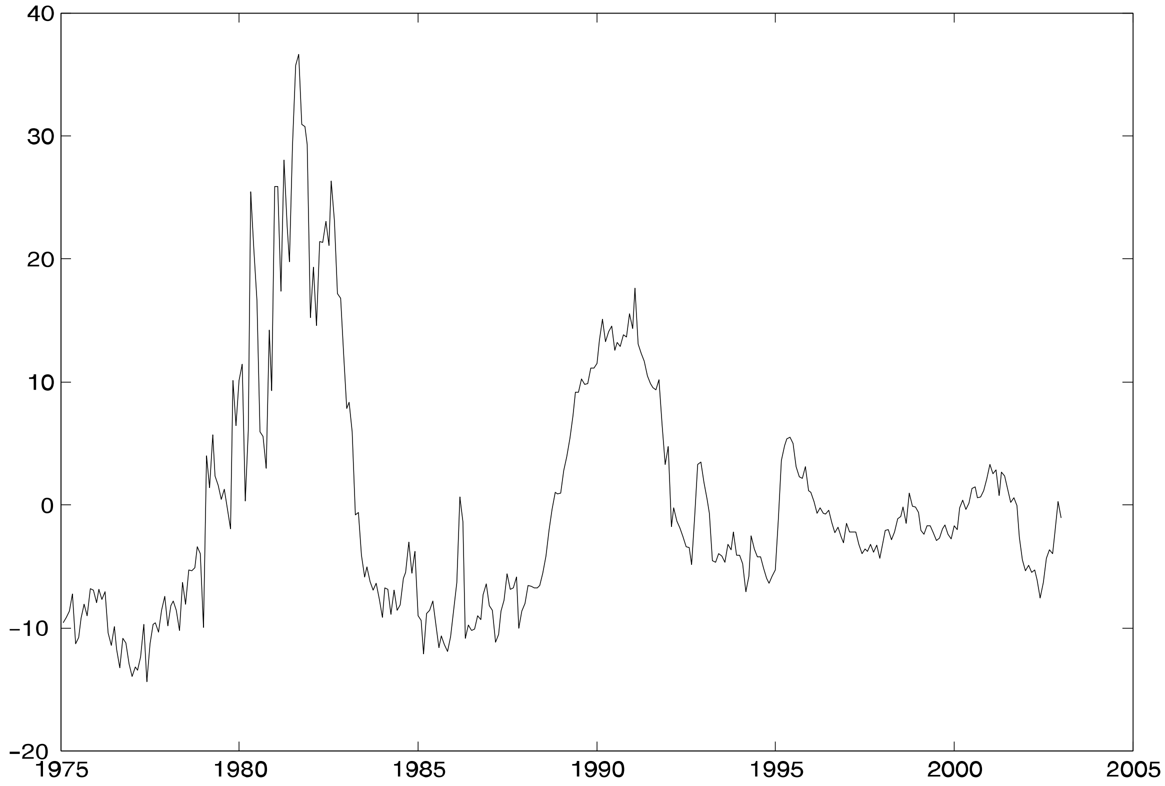
[19] It should be noted, however, that a shift in the deterministic trend in output is estimated in 1993. Hence, the growth of potential in the second half of the 1990s is lower compared with a relatively higher growth in trend. Depending on our judgment on the source of this shift, the story can be completely different. If the higher deterministic output growth is attributed to supply shocks, then potential output would have increased continuously in the 1990s and the Canadian economy would currently be in considerable excess supply. Nevertheless, given the deterministic nature of this shift and the recent economic developments, we proceed under the assumption that this change in trend should be considered as demand-driven, implying that potential output and the output gap are well approximated by Graphs 8 and 9. The fact that potential has been below the higher growth trend for the last seven years is also an indication that the higher trend should be seen as transitory.

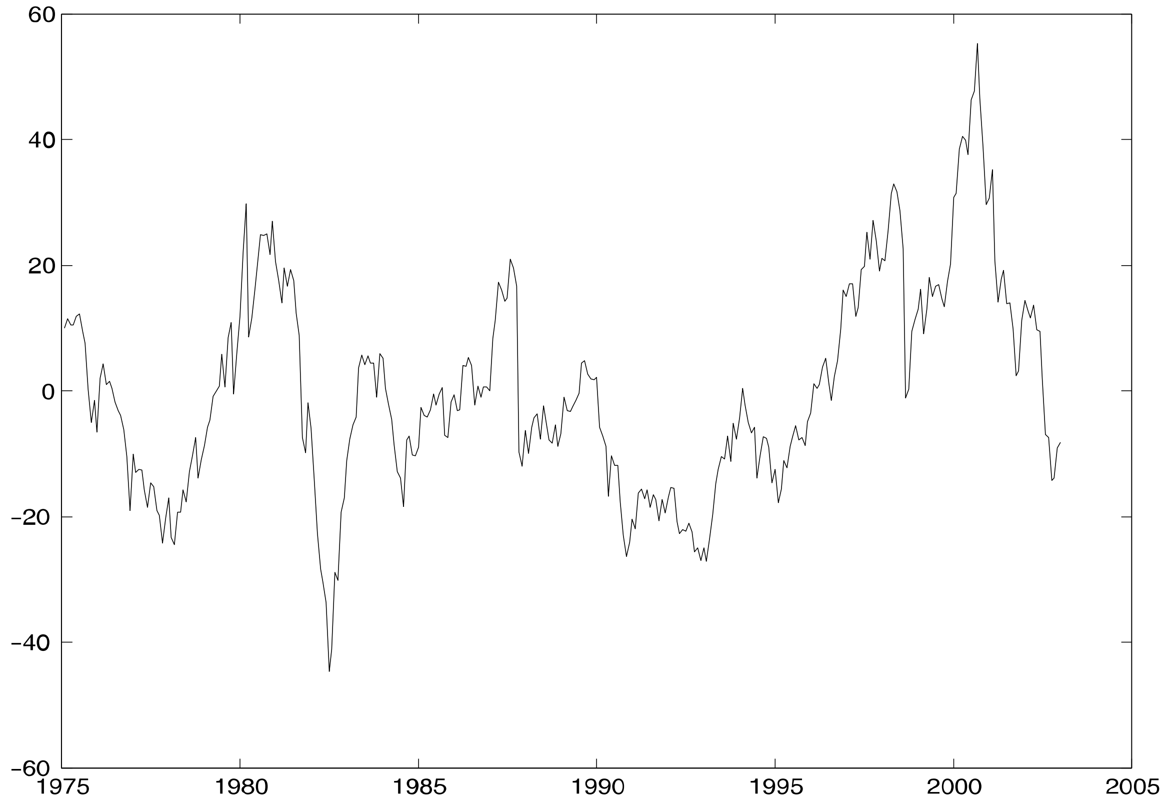
Graph 1

**Money gap**



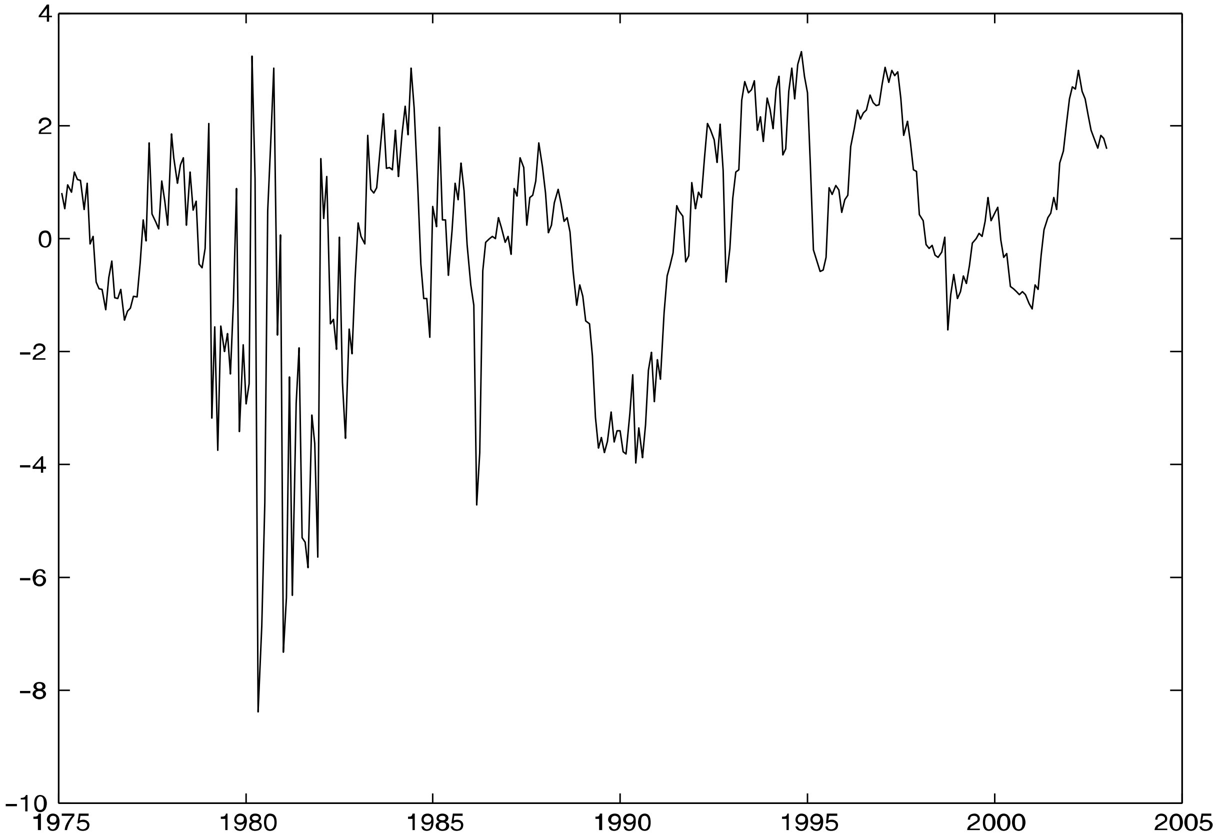

Graph 2

**Stock market gap**



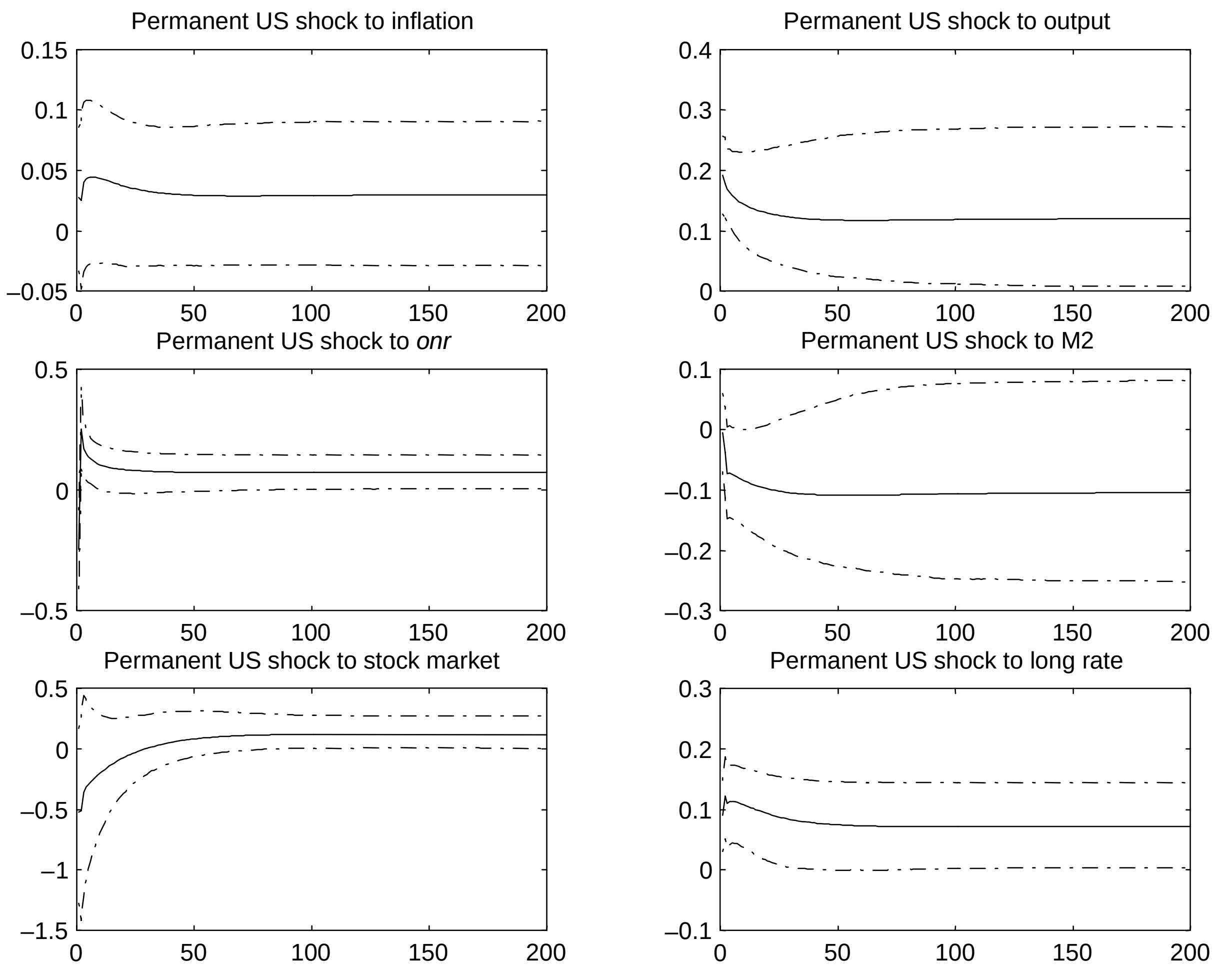
Graph 3

**Interest rate gap**



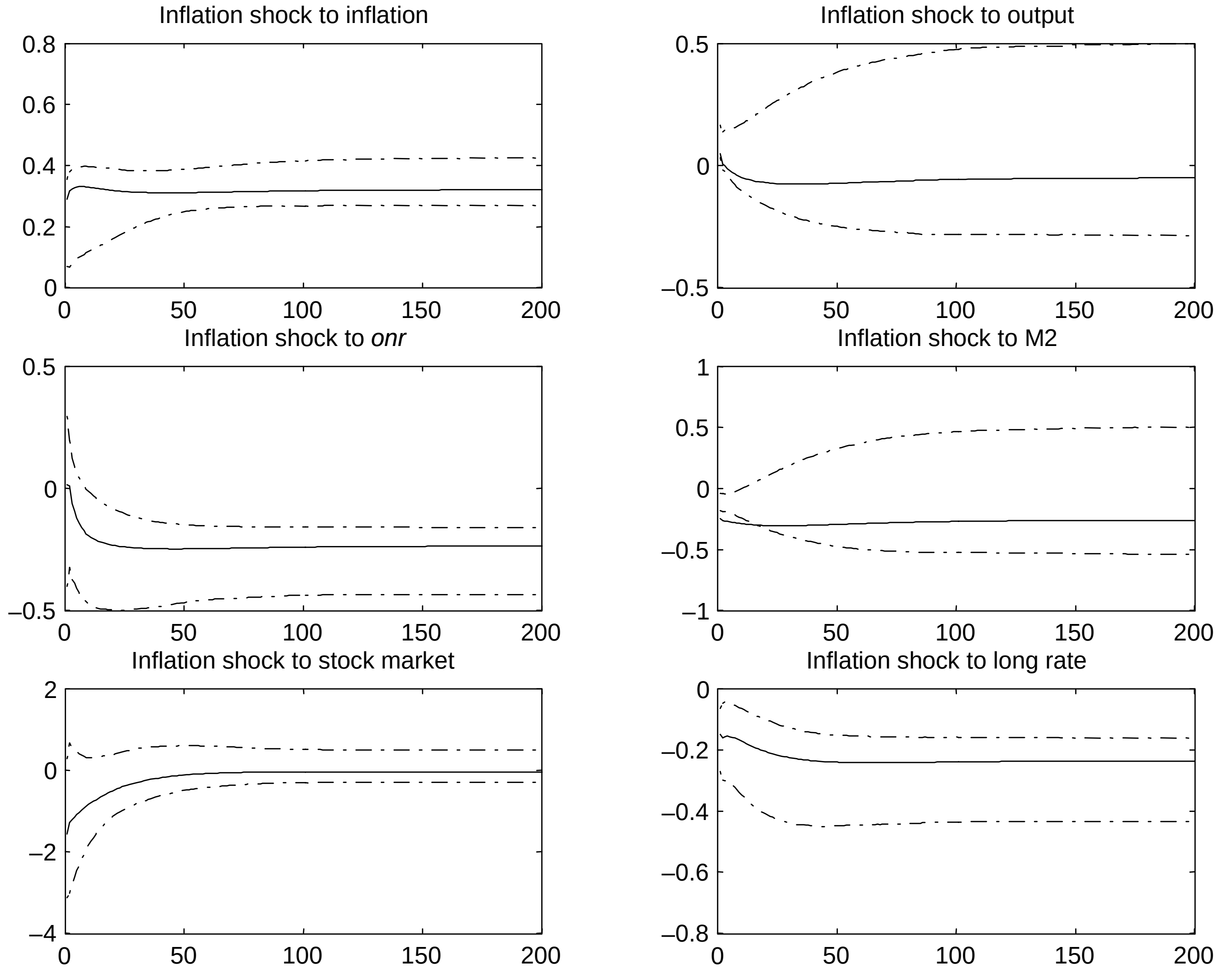
Graph 4

**Responses to a permanent increase in US industrial production**



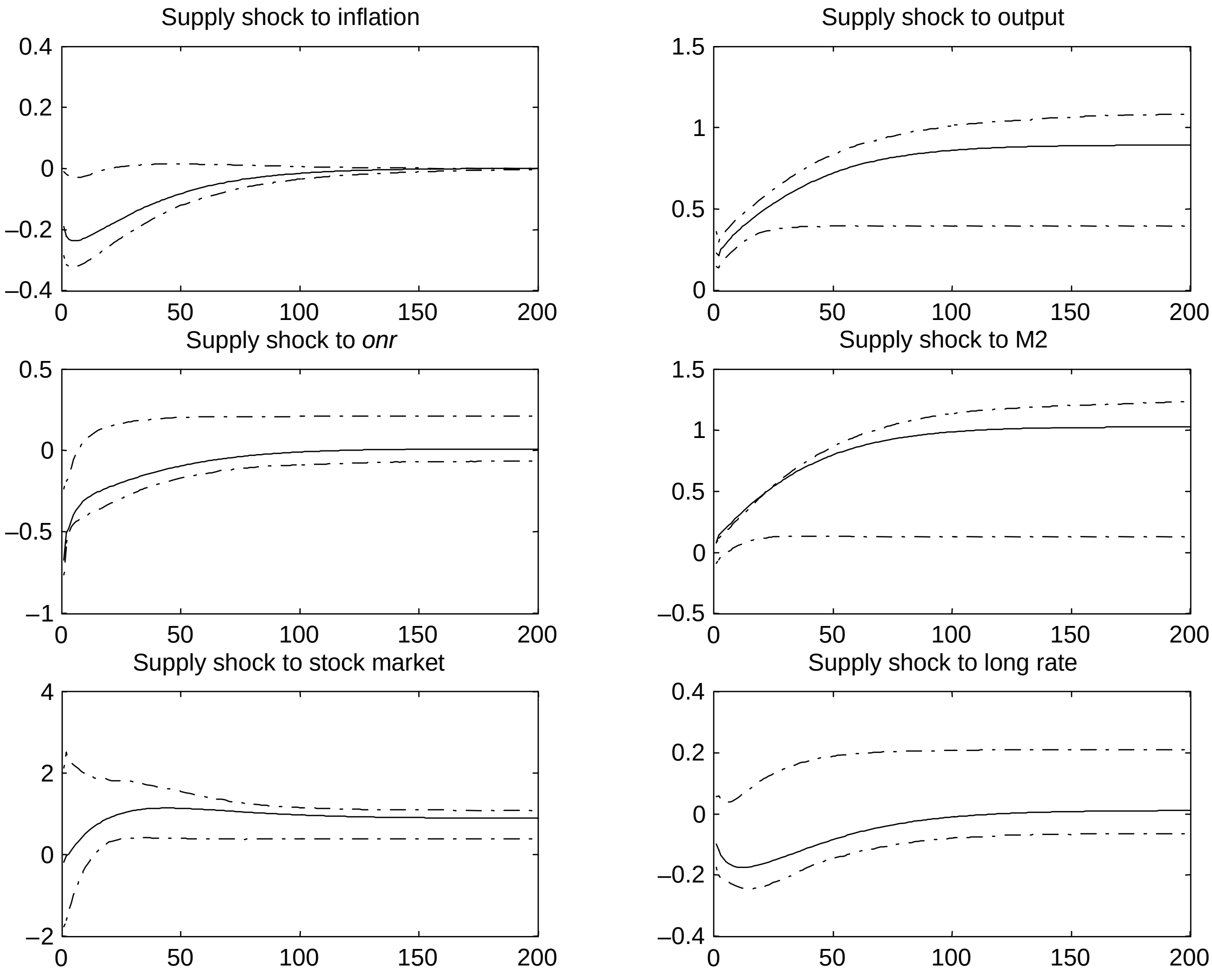
Graph 5

**Impulse responses to an inflation shock**



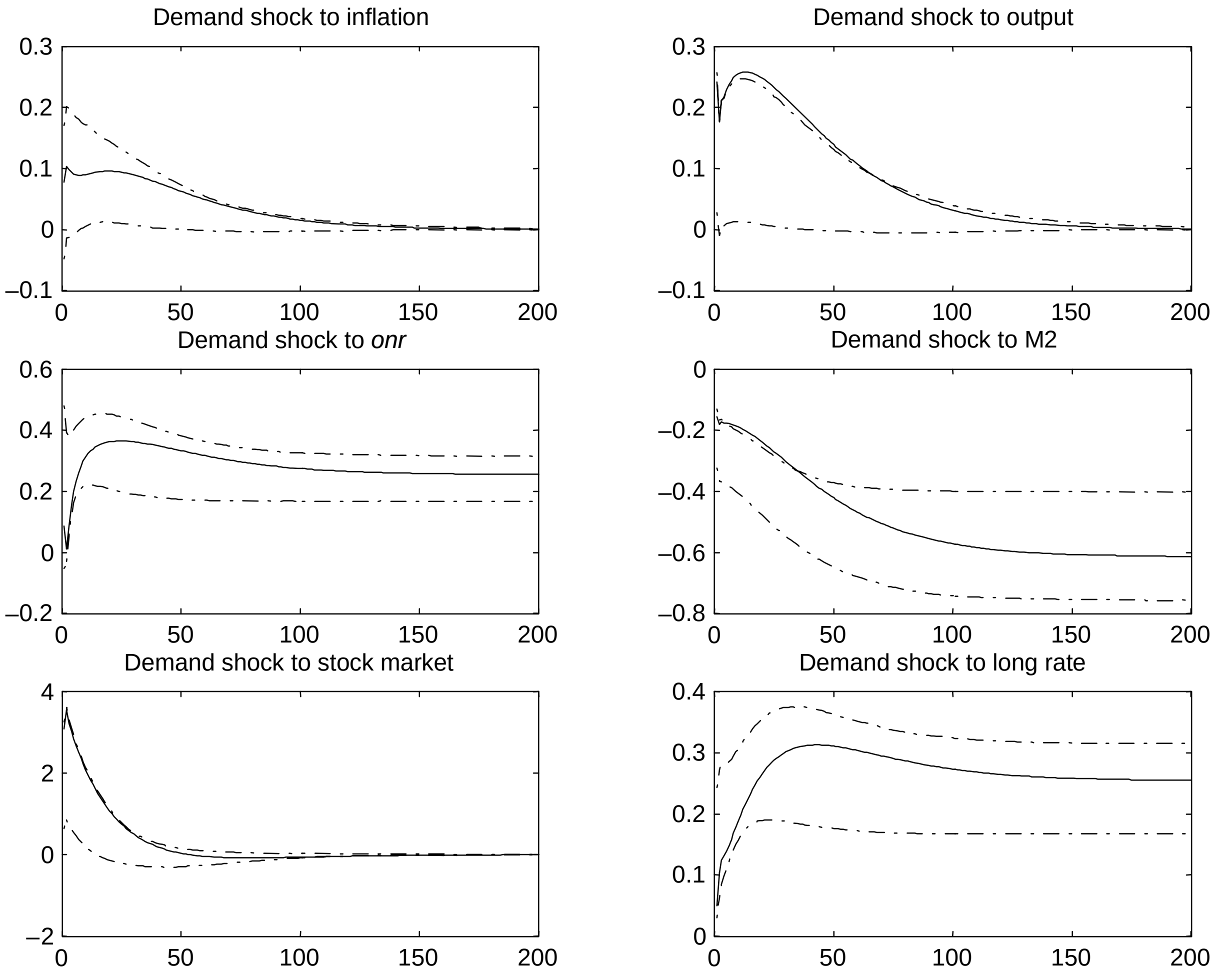
Graph 6

**Impulse responses to a supply shock**



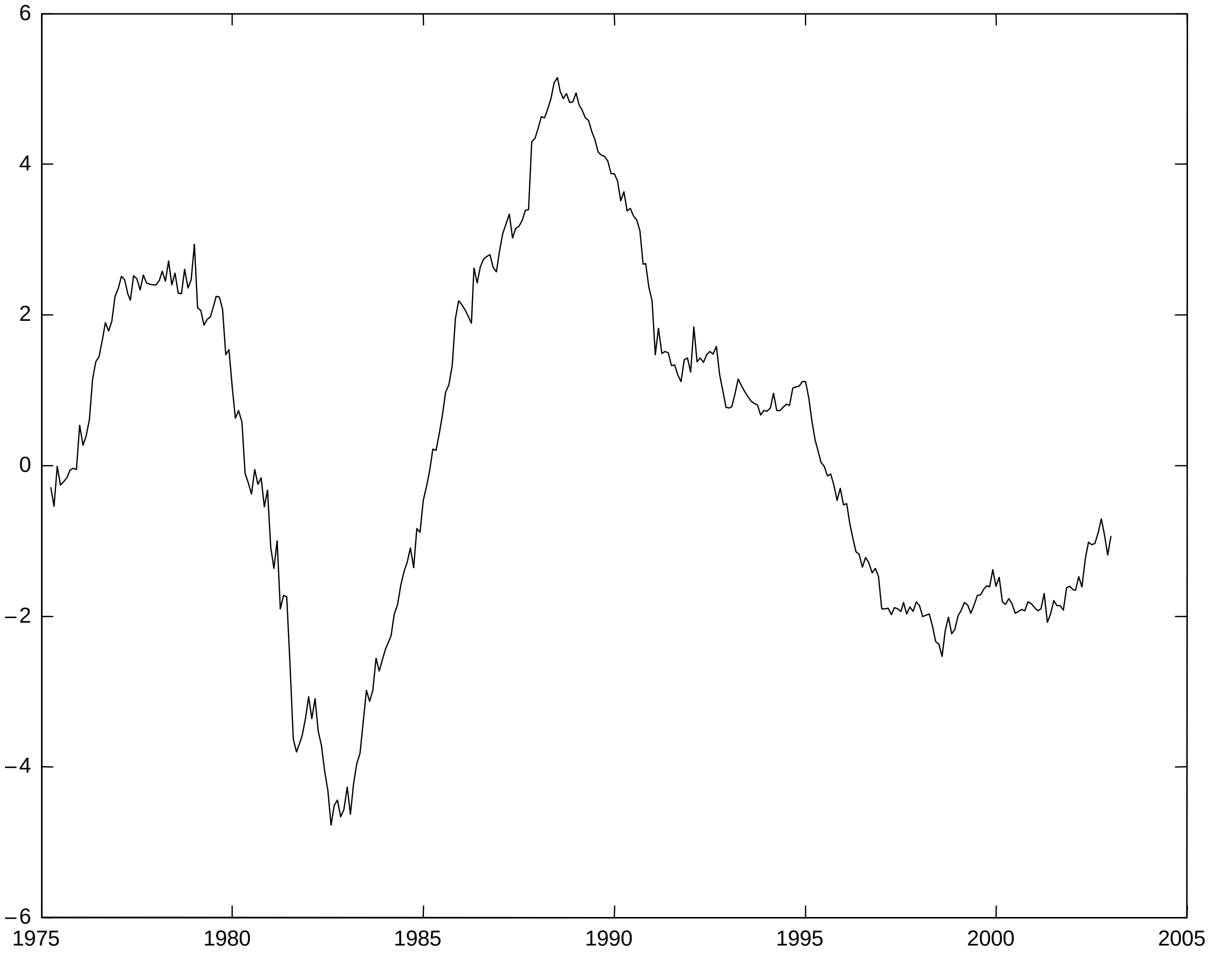
Graph 7

**Impulse response to a demand shock**



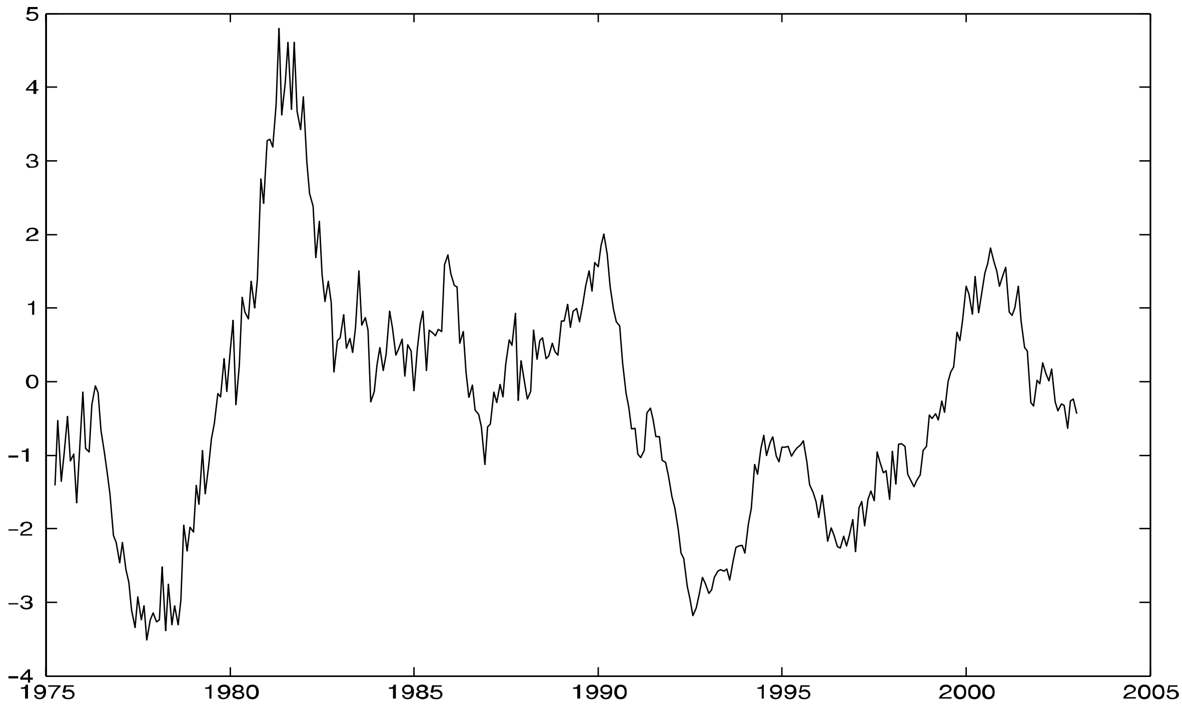
Graph 8

**Potential output**



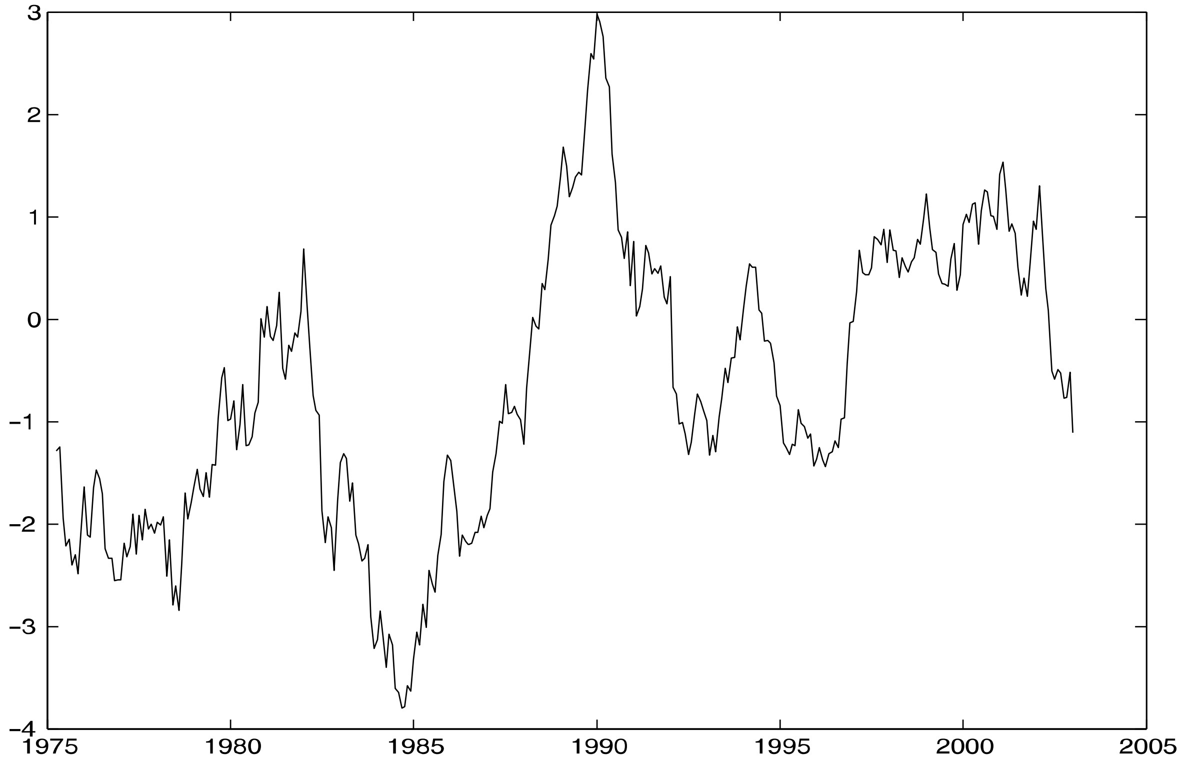
Graph 9

**Output gap**



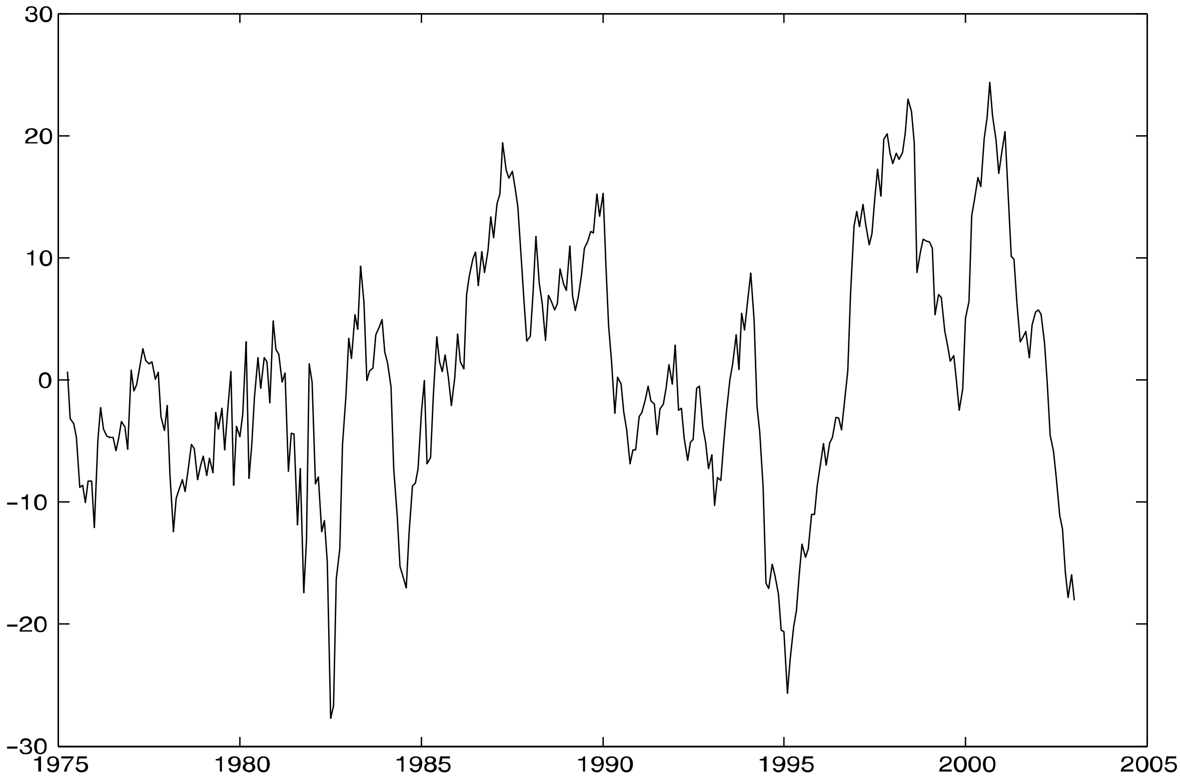

Graph 10

**Money gap (based on permanent components)**



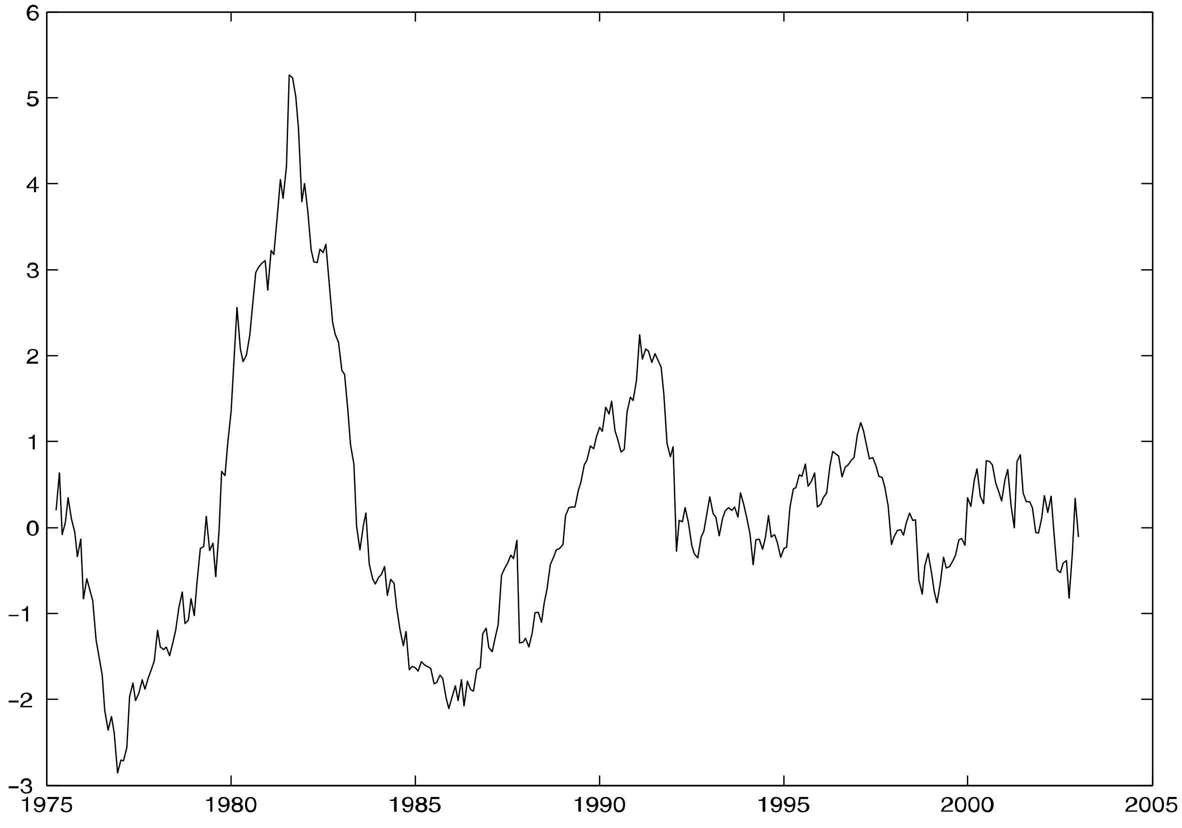
Graph 11

**Stock market gap (based on permanent components)**




Graph 12

**Transitory component of inflation**



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