Monetary policy with unobserved potential output

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Abstract

This paper studies some effects of real-time information on the implementation of monetary policy. We consider an economy in which several sources of uncertainty, such as measurement errors and imperfectly observable states, do not allow the policymaker to identify the true state of the economy. Optimal policy thus requires the policymaker to jointly solve a filtering and an optimisation problem. We focus in particular on the case of a non-observable measure of potential output and analyse the consequences of this assumption for the macroeconomy (policy, output and inflation). The paper provides a benchmark model to assess the claim that conditioning policy on a potential output measure using real-time data may be at the root of a biased policy stance, as recently suggested by Orphanides. More generally, it offers a rigorous framework to analyse the effects of imperfect information and to assess the role of macroeconomic indicators in alleviating information problems.

1. Introduction and main findings

This note discusses recent results concerning monetary policy with real-time information.[2] The theme of this paper is that the implementation of monetary policy is often faced with the difficult task of taking decisions in the presence of high uncertainty. Policy decisions require knowledge of a structural economic model and of the state of the economy (the realisation of the different shocks impinging on it). Such information is rarely available to the policymaker. Taking decisions in real time, when the latest data on some target variables (eg inflation, employment and output) may not be available, or may be subject to substantial measurement errors, requires an efficient filtering of the available information to ensure the best possible inference on the state of the economy is formed.

An example illustrates the nature of this basic problem faced by central banks. A stabilising role for monetary policy crucially hinges on some notion of "potential output", a non-observable economic variable representing the desirable (or target) level at which actual output should be. The conduct of monetary policy requires, therefore, that the central bank estimates, and continually updates, its potential output forecast. Orphanides (2000a,b, 2001) provides persuasive support for the view that a significant overestimation of potential output during the oil shocks of the 1970s aggravated inflation at that time by leading to a monetary policy stance which turned out to be, with the benefit of hindsight, excessively loose ex post. Somewhat symmetrically, the strong productivity gains recorded in the United States during the second half of the 1990s raised the possibility, again with the benefit of hindsight, that the subsequently greater than expected increases in potential output could have allowed for a less restrictive monetary policy stance than that initially suggested by real-time estimates of inflation and the output gap.

The work of Orphanides sheds interesting new light on monetary policy during the 1970s and raises an important question about whether such retrospective policy mistakes can be avoided in the future. If they were due to poor but correctable forecasting procedures or to an inefficient specification of the "policy rule", a likely answer to this question is yes. Assessing the extent to which such mistakes were

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[2] These results draw on the findings of Cukierman and Lippi (2002) and Gerali and Lippi (2002), where several of the technical details are discussed.

due to “bad policies” rather than to “bad luck” requires a model which identifies optimal monetary policy under imperfect information. Once this benchmark is defined, and its properties are established, one can proceed to evaluate the extent to which (retrospective) policy errors were avoidable.

This paper contributes to the debate on the effects of imperfect information by proposing such a benchmark model and analysing its properties. It is shown that, given the structure of information, some policy decisions which are judged ex post to be mistakes may be unavoidable even if the central bank utilises the most efficient forecasting procedures. Moreover, such retrospective mistakes are small during periods in which changes in potential output are small, and large during periods characterised by substantial changes in the long-run trend of output. During the latter episodes, policy mistakes in a given direction are likely to persist for some time.

The evidence in Orphanides (2001) supports the view that monetary policy during the 1970s was excessively loose since a permanent reduction in potential output was interpreted for some time as a negative output gap. The analytical framework of this paper provides an “optimising” analytical foundation for this mechanism and identifies the conditions under which it operates.[3] Interestingly, a large permanent decrease in potential output does not lead to an excessively loose policy stance under all circumstances. Whether it does or not depends on the relative persistence of demand and of cost shocks, and on other parameters like the degree of conservativeness of the central bank.

While the theoretical analysis suggests that imperfect information may lie at the root of a “biased” policy stance (judged with the benefit of hindsight), a preliminary quantitative assessment of the effects of imperfect information indicates that the effects of such biases on the main macroeconomic variables are not very large. While preliminary, this finding seems to suggest that it is difficult to “explain” the high inflation of the 1970s as a consequence of imperfect information alone.

These results are first presented by means of a simple model by Cukierman and Lippi (2002), which captures the conception of many central banks about the transmission process of monetary policy. The advantage of this simple formulation lies in the tractability of the analytical framework. That model identifies conditions under which the presence of imperfect information leads monetary policy to be *systematically* tighter than under perfect information in periods of permanent increases in potential output and to be too loose relative to this benchmark in periods of permanent reductions in potential output. The reason is that, even when they filter available information in an optimal manner, policymakers as well as the public at large detect permanent changes in potential output only *gradually*. When, as was the case in the 1970s, there is a permanent decrease in potential output, policymakers interpret part of this reduction as a negative output gap and loosen monetary policy too much in comparison to the no permanent-temporary confusion (PTC) benchmark. Thus, in periods of large permanent decreases in productivity, inflation accelerates because of the relatively expansionary monetary policy stance. Conversely, when - as might have been the case in the United States during the 1990s - a “new economy” permanently raises the potential level of output, inflation goes down since, as policymakers interpret part of the permanent increase in potential output as a positive output gap, policy is tighter than under perfect information. A main novel result of the paper is that, even when the information available to policymakers in real time is used efficiently and monetary policy chosen optimally, errors of forecast in real-time estimates of potential output and of the output gap are serially correlated retrospectively. In general, this serial correlation is induced by shocks to potential output, as well as to the cyclical components of output.

We subsequently show how similar results can be produced by a more up-to-date forward-looking model of the “new synthesis” variety developed by Woodford (1999) and Clarida et al (1999).

2. The background analytical framework

The problems analysed in the following can be framed within the setup and notation used by Svensson and Woodford (2000) to model a linear-quadratic economy with two agents, a government

3 Related work on the effects of imperfect information for monetary policy appears in Ehrmann and Smets (2001), who develop a quantitative assessment of the effects of imperfect information using a numerical analysis based on a calibrated model for the euro area.

and an aggregate private sector, which are assumed to have the same imperfect information on the state of the economy. We use the algorithms developed by Gerali and Lippi (2002) to solve this problems numerically using MATLAB.

The economy is described by:

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + Bi_t + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix} \tag{2.1}$$

where X_{t+1} is a vector of n_X predetermined variables in period t (natural state variables), x_t is a vector of n_x forward-looking variables, i is a vector of n_i policy instruments, u_t is a vector of n_X iid shocks with mean zero and covariance Σ_u^2, and A^1, A^2 and B are matrices of appropriate dimension. For any variable z_t, the notation $z_{t|\tau}$ denotes the expectation, $E[z_t|I_\tau]$, ie the rational expectation of z_t with respect to the information I_τ available in period τ.

Let Y_t represent the vector of target variables that enter the government criterion function,

$$Y_t = C^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + C_i i_t \tag{2.2}$$

where C^1, C^2 and C_i are matrices of appropriate dimension. Let the quadratic form describing the period loss function be given by:

$$L_t \equiv Y_t' W Y_t \tag{2.3}$$

where W is a positive semi-definite matrix of weights. The government actions are aimed at minimising the intertemporal loss function

$$\Lambda_t = E\left[\sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} \middle| I_t\right] \tag{2.4}$$

where $\delta \in (0, 1)$ is the intertemporal discount factor.

Finally, let the vector of observable variables Z_t be given by:

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t \tag{2.5}$$

where the "noise" vector v_t is assumed to be iid with covariance matrix Σ_v^2. Information I_t in period t is:

$$I_t \equiv \left\{ Z_\tau, \tau \le t; A^1, A^2, B, C^1, C^2, C_i, D^1, D^2, W, \delta, \Sigma_u^2, \Sigma_v^2 \right\}$$

3. Application 1 (from Cukierman and Lippi (2002))

This section presents a simplified version of the backward-looking sticky price model presented in Svensson (1997). Although the model is not rooted in explicit microfoundations, it is likely to reflect the views of several central banks about the transmission process of monetary policy. Its main advantage is that it allows the basic consequences of imperfect information to be illustrated analytically in a relatively simple manner. We therefore maintain the assumption that this reduced form model captures the actual behaviour of the economy. A richer economic structure, incorporating transmission lags or forward-looking variables, does not eliminate the effects described in the paper (eg Ehrmann and Smets (2001)) but may introduce new ones. Although such models may be preferable for theoretical and empirical reasons, they would prevent us from illustrating our main points analytically.

3.1 The economy

In this framework (the logarithm of) output (y_t) and inflation (π_t) are determined, respectively, as follows:

$$y_t = z_t - \varphi r_t + g_t \tag{3.1}$$

$$\pi_t = \lambda(y_t - z_t) + u_t \tag{3.2}$$

Here z_t denotes (the log of) potential output at period t, r_t is a *real* short-term interest rate, g_t is a demand shock and u_t a cost push shock. This framework postulates that potential output z is a fundamental long-run determinant of actual output. But, in addition, actual output is also affected by a demand shock and by the real rate of interest, which for given inflationary expectations is determined in turn by the (nominal) interest rate policy of the central bank.

In line with conventional macroeconomic wisdom, we postulate that the demand and cost shocks are less persistent than changes in potential output, which are affected by long-run factors like technology and the accumulation of physical and human capital. The permanence of shocks to potential output is modelled by assuming that z_t is a random walk.[4] More specifically, we postulate the following stochastic processes for the shocks:

$$g_t = \mu g_{t-1} + \hat{g}_t \quad 0 < \mu < 1; \quad \hat{g}_t \sim N(0, \sigma_g^2) \tag{3.3}$$

$$u_t = \rho u_{t-1} + \hat{u}_t \quad 0 < \rho < 1; \quad \hat{u}_t \sim N(0, \sigma_u^2) \tag{3.4}$$

$$z_t = z_{t-1} + \hat{z}_t \quad \hat{z}_t \sim N(0, \sigma_z^2) \tag{3.5}$$

To reiterate, the main purpose of this simple model is to characterise the macroeconomic consequences of optimally chosen monetary policy (ie a sequence for r_t) when policymakers cannot identify with certainty (not even retrospectively) the sources of output changes.

3.2 Monetary policy

The policy instrument is the nominal interest rate. But since prices are temporarily sticky, the policymaker can bring about the real rate he desires by setting the nominal rate. For convenience and without loss of generality, we can therefore consider the policymaker as setting the real interest rate r_t. This policy instrument is set at the *beginning* of period t before output, inflation (y_t and π_t) and period t shocks are realised. The policy objective is to minimise the objective function:

$$L_t \equiv \tfrac{1}{2} E\left\{ \sum_{j=0}^{\infty} \beta^j \left[\alpha (x_{t+j})^2 + (\pi_{t+j})^2 \right] \middle| J_{t-1} \right\} \quad \alpha > 0 \tag{3.6}$$

where $x_t \equiv y_t - z_t$ denotes the output gap (defined as the difference between (the logarithms of) actual and potential output) and J_{t-1} is the information set available at the beginning of period t, when r_t is chosen. The first-order condition for the discretionary (time-consistent) monetary policy $(\min_{r_t} L_t)$ implies:

$$x_{t|t-1} = -\frac{\lambda}{\alpha} \pi_{t|t-1} \tag{3.7}$$

Here $\pi_{t|t-1}$ and $x_{t|t-1}$ are the expected values of inflation and of the output gap conditional on the information available at the beginning of period t, J_{t-1}. At this stage we note that J_{t-1} contains, inter alia, observations on actual inflation and output up to and including period t – 1. A full specification of J_{t-1} appears below. Since the values of inflation and of the output gap at period t are not known with

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[4] Nothing in our results would change if we added a (more realistic) deterministic trend growth to the potential output process.

certainty at the beginning of period t, those variables (which are indirectly controlled by policy) appear in equation (3.7) in expected terms.

The equilibrium outcomes for the interest rate, output and inflation obey:

$$r_t = \frac{1}{\varphi}\left[g_{t|t-1} + \frac{\lambda}{\alpha+\lambda^2}u_{t|t-1}\right] \tag{3.8}$$

$$y_t = z_t + (g_t - g_{t|t-1}) - \frac{\lambda}{\alpha+\lambda^2}u_{t|t-1} \tag{3.9}$$

$$\pi_t = \frac{\alpha}{\alpha+\lambda^2}u_t + \lambda(g_t - g_{t|t-1}) + \frac{\lambda^2}{\alpha+\lambda^2}(u_t - u_{t|t-1}) \tag{3.10}$$

3.3 The structure of information and optimal policy

The interest rate rule in equation (3.8) implies that the optimal real interest rate policy for period $t+1$, r_{t+1}, requires the policymaker to form expectations about the values of the demand shock and the cost push shocks, g_{t+1} and u_{t+1}. Although he does not observe those shocks directly, the policymaker possesses information about economic variables from which noisy, but optimal, forecasts of the shocks can be derived. In particular, we assume that policymakers know the true structure of the economy: $\Omega \equiv \left\{\varphi, \lambda, \rho, \mu, \sigma_u^2, \sigma_g^2, \sigma_z^2\right\}$ but do not know the precise stochastic sources of fluctuations in output and inflation.

Thus, when the interest rate r_{t+1} is chosen, at the beginning of period $t+1$, the policymaker forms expectations about g_{t+1} and u_{t+1} using historical data. The latter consists of observations on output and inflation up to and including period t. The information available at the beginning of period $t+1$ is summarised by the information set:

$$J_t = \{\Omega, y_{t-i}, \pi_{t-i} \,|\, i = 0, 1, 2, \ldots\} \tag{3.11}$$

which is used to form the conditional expectations: $g_{t+1|t}$ and $u_{t+1|t}$. Past observations on output and inflation are equivalent to past observations on the two signals, $s_{1,t}$ and $s_{2,t}$ (obtained by rearranging equations (3.9) and (3.10)):

$$s_{1,t} \equiv y_t + g_{t|t-1} + \frac{\lambda}{\alpha+\lambda^2}u_{t|t-1} = z_t + g_t \tag{3.12}$$

$$s_{2,t} \equiv \pi_t + \lambda g_{t|t-1} + \frac{\lambda^2}{\alpha+\lambda^2}u_{t|t-1} = \lambda g_t + u_t \tag{3.13}$$

where variables to the left of the equality sign are observed separately while those to the right are not. Clearly, $s_{1,t}$ and $s_{2,t}$ contain (noisy) information on g_t and u_t which can be used to make inferences on g_{t+1} and u_{t+1}, using the fact that $g_{t+1|t} = \mu g_{t|t}$ and $u_{t+1|t} = \rho u_{t|t}$.

Notice how the optimal estimates of g_t and u_t conditional on J_t, $g_{t|t}$ and $u_{t|t}$ respectively, follow immediately from the two signal equations (3.12) and (3.13), once the optimal estimate of potential output, $z_{t|t}$, is known. Therefore, the signal extraction (or filtering) problem solved by the policymaker reduces to an inference problem concerning the level of potential output.

3.4 Mismeasurement of potential output and policymakers' views about the state of the economy

Let policymakers' forecast errors concerning the variables z_t, g_t and u_t conditional on the information set J_t be:

$$\tilde{u}_{t|t} \equiv u_t - u_{t|t} \tag{3.14}$$

$$\tilde{g}_{t|t} \equiv g_t - g_{t|t} \tag{3.15}$$

$$\tilde{z}_{t|t} \equiv z_t - z_{t|t} \tag{3.16}$$

Using equations (3.12) and (3.13) the following useful relationship between these errors can be derived:

$$\lambda\tilde{z}_{t|t} = -\lambda\tilde{g}_{t|t} = \tilde{u}_{t|t} \tag{3.17}$$

The last equation shows that overestimation of potential output ($\tilde{z}_{t|t} < 0$) simultaneously *implies* an overestimation of the cost push shock and an underestimation of the demand shock. This is summarised in the following remark.

Remark 1. *Potential output overestimation ($\tilde{z}_{t|t} \equiv z_t - z_{t|t} < 0$) implies:*

(i) *demand shock underestimation ($\tilde{g}_{t|t} \equiv g_t - g_{t|t} > 0$)*

(ii) *cost push shock overestimation ($\tilde{u}_{t|t} \equiv u_t - u_{t|t} < 0$)*

Inequalities with opposite signs hold when $\tilde{z}_{t|t} > 0$.

The intuition underlying this result can be understood by reference to equations (3.12) and (3.13). The first equation implies that an increase in $s_{1,t}$ is always and **optimally** interpreted as being due partly to an increase in z_t and partly to an increase in g_t. Similarly, an increase in $s_{2,t}$ is interpreted as partly due to an increase in g_t and partly to an increase in u_t. Thus, when only z_t increases, part of this increase is interpreted as an increase in potential output, but the remainder is interpreted as an increase in g_t. As a consequence the error in forecasting z_t is positive and the error in forecasting g_t is negative, producing a **negative** correlation between the forecast errors in those two variables. Since $s_{2,t}$ does not change the (erroneously) perceived increase in g_t is interpreted as a decrease in u_t, producing a positive forecast error for this variable and, therefore, a **positive** correlation between the forecast errors in u_t and in z_t.

3.5 Consequences of forecast errors in potential output for monetary policy, inflation and the output gap

Remark 1 shows how mismeasurement of potential output distorts policymakers' perceptions about cyclical conditions (cost push and demand shocks). The purpose of this subsection is to answer the following question: how do such noisy perceptions about the phase of the cycle affect monetary policy, inflation and the output gap? We do this by comparing the values of those variables in the presence of the PTC with their values in the benchmark case in which there is no such confusion. In the benchmark case, policymakers possess in each period *direct information* about the realisations of the shocks up to and including the previous period. Formally, in the absence of the PTC policymakers possess, at the beginning of period $t+1$, the information set J_t^* that is defined by:

$$J_t^* = \{J_t, g_{t-i}, u_{t-i} \,|\, i = 0, 1, 2, \ldots\} \tag{3.18}$$

3.5.1 Consequences for monetary policy

We begin by studying the determinants of the difference between the settings of monetary policy in the presence and in the absence of the PTC. Using equations (3.8), (3.14), (3.15) and (3.17), the *deviation* of the optimal interest rate in the presence of the PTC from its optimal value in the absence of this confusion $\left(\text{ie } r_{t+1}^* = \frac{1}{\varphi}\left[\mu g_t + \frac{\lambda}{\alpha+\lambda^2}\rho u_t\right]\right)$ can be written as:

$$\Delta r_{t+1} \equiv r_{t+1} - r_{t+1}^* = -\frac{1}{\varphi}\left[\mu\tilde{g}_{t|t} + \frac{\lambda\rho}{\alpha+\lambda^2}\tilde{u}_{t|t}\right] \tag{3.19}$$

$$= \frac{\left(\mu - \frac{\rho\lambda^2}{\alpha+\lambda^2}\right)}{\varphi}\tilde{z}_{t|t} \tag{3.20}$$

It follows immediately from equation (3.19) that if demand shocks are sufficiently persistent in comparison to cost shocks (ie $\mu > \frac{\rho\lambda^2}{\alpha+\lambda^2}$), the deviation of the real interest rate from its full information counterpart moves in the same direction as the forecast error in potential output ($\tilde{z}_{t|t}$). Although one cannot rule out the possibility that, when the persistence in cost shocks is sufficiently larger than that of demand shocks, the opposite occurs, it appears that the first case seems more likely a priori. The reason is that the persistence parameter of the cost shocks is multiplied by a fraction implying that Δr_{t+1} and $\tilde{z}_{t|t}$ are positively related even if ρ is larger than μ, but not by too much. Note that the smaller the (Rogoff (1985) type) conservativeness of the central bank (the higher α), the more likely it is that Δr_{t+1} and $\tilde{z}_{t|t}$ are positively related even when ρ is larger than μ. Hence, for central banks which are (using Svensson's (1997) terminology) relatively flexible inflation targeters, the case in which Δr_{t+1} and $\tilde{z}_{t|t}$ are positively related is definitely the more likely one for most or all values of ρ and μ in the range between zero and one. The various possible effects of imperfect information are summarised in the following proposition:

Proposition 1. *(i) When the persistence of demand shocks is sufficiently high* $\left(\mu > \frac{\rho\lambda^2}{\alpha+\lambda^2}\right)$ *monetary policy is driven mainly by "demand shocks" considerations. This implies that potential output over/underestimation (causing the demand shock to be under/overestimated) leads to real rates which are lower/higher than the rate which is optimal in the absence of the PTC*.

(ii) When the persistence of demand shocks is sufficiently low $\left(\mu < \frac{\rho\lambda^2}{\alpha+\lambda^2}\right)$ *monetary policy is driven mainly by "cost push shocks" considerations. This implies that potential output over/underestimation (causing the cost push shock to be over/underestimated) leads to a real rate which is higher/lower than the rate that is optimal in the absence of the PTC*.

To understand the intuition underlying the proposition, it is useful to consider the case in which there is, in period *t*, a negative shock to potential output and no changes in the cyclical shocks, *g* and *u*. This leads, as of the beginning of period $t+1$, to overestimation of potential output in period t ($\tilde{z}_{t|t} < 0$). Remark 1 implies that this overestimation is associated with an overestimation of the cost shock and an underestimation of the demand shock of period *t*.

The policy chosen at the beginning of period $t+1$ aims to offset the (presumed) deflationary impact of the demand shock on the output gap and the (presumed) inflationary impact of the cost shock on inflation. In comparison to the no PTC benchmark, the first objective pushes policy towards expansionism while the second pushes it towards restrictiveness. If demand shocks are relatively persistent, the first effect dominates since policymakers believe that most of what they perceive to be a negative demand shock in period *t* is going to persist into period $t+1$ while what they perceive to be a positive cost shock in period *t* is not going to persist into period $t+1$. Hence, in this case monetary policy is more expansionary than in the no PTC benchmark and Δr_{t+1} and $\tilde{z}_{t|t}$ are positively related (case (i) in the proposition). But if the reverse is true (cost shocks are relatively more persistent), beliefs about the cost shock in period $t+1$ dominate policy, pushing it towards tightening. As a consequence, monetary policy is more restrictive than in the no PTC benchmark and Δr_{t+1} and $\tilde{z}_{t|t}$ are negatively related and case (ii) of the proposition obtains.

3.5.2 Consequences for the output gap and inflation

We turn next to the consequences of mismeasurement of potential output for the output gap and inflation. The objective is, as in the previous subsection, to analyse the deviations of the outcomes obtained in the presence of the PTC from those that arise in its absence. Using equations (3.9) and (3.10), the logical next step is to relate these deviations to the interest rate deviations studied above. This yields:

$$\Delta x_{t+1} \equiv x_{t+1} - x^*_{t+1} = -\varphi \Delta r_{t+1} \tag{3.21}$$

$$\Delta\pi_{t+1} \equiv \pi_{t+1} - \pi^*_{t+1} = -\varphi\lambda\Delta r_{t+1} \tag{3.22}$$

where x^*_{t+1} and π^*_{t+1} are the values of the output gap and inflation under optimal monetary policy in the absence of the PTC. These equations show that when the interest rate is below (above) its value in the absence of the PTC both inflation and the output gap are above (below) their no PTC values.

The case of overexpansionary monetary policy (case (i) of proposition 1) is consistent with the empirical results in Orphanides (2000a,b, 2001), according to which, during the 1970s, US monetary policy was overly expansionary due to an overestimation of potential output and an associated underestimation of the output gap. Obviously, this underestimation could have been due to inefficient forecasting procedures on the part of the Fed. A main message of this paper is that this effect is present even if monetary policy is ex ante optimal and forecasting procedures are as efficient as possible. In normal times, during which the change in potential output is not too far from its mean, this effect is likely to be small and short-lived. But when large permanent shocks to potential output occur, this effect is likely to be large and more persistent. This point is discussed in detail in the next section.

3.6 Optimal potential output forecasts

This section describes the solution to the signal extraction problem faced by policymakers. To convey the intuition of the basic mechanisms at work, we focus in the text on the particular (but simpler) case in which demand and cost push shocks are equally persistent ($\mu = \rho$), which yields a tractable closed form solution without affecting the key properties of the predictor.[5]

The conditional expectation of z_t based on J_t, $z_{t|t}$, is given by (see Cukierman and Lippi (2002)):

$$z_{t|t} = aS_t + (1-a)(1-\kappa)\sum_{i=0}^{\infty}\kappa^i S_{t-1-i} \tag{3.23}$$

where:

$$\kappa \equiv \frac{2}{\phi + \sqrt{\phi^2 - 4}} \in (0,1) \quad \phi \equiv \frac{2 + T(1+\mu^2)}{1+\mu T} \geq 2\,; \quad T \equiv \left(\frac{\sigma_z^2}{\sigma_g^2} + \frac{\lambda^2\sigma_z^2}{\sigma_u^2}\right)$$

$$a \equiv \frac{[(1-\mu)+(1-\kappa)+T(1-\mu\kappa)]T}{[T(1-\mu-\mu\kappa)+(1-\mu-\kappa)](1+T)+(T+\mu)(1+\mu T)} \in (0,1) \tag{3.24}$$

$$S_{t-i} \equiv s_{1,t-i} - \frac{\lambda\sigma_g^2}{\sigma_u^2 + \lambda^2\sigma_g^2}s_{2,t-i} = z_{t-i} + \frac{\sigma_u^2 g_{t-i} - \lambda\sigma_g^2 u_{t-i}}{\sigma_u^2 + \lambda^2\sigma_g^2} \tag{3.25}$$

S_{t-i} is a combined signal that summarises all the relevant information from a period's $t - i$ data. Note that it is positively related to that period's potential output and demand shock, and negatively related to that period's cost shock. As a consequence, the optimal predictor generally responds positively to current as well as to all past shocks to demand and potential output, and responds negatively to current as well as to all past, cost shocks.

The conditional forecast equation (3.23) possesses several key properties. First, since a and κ are both bounded between zero and one, the current optimal predictor is positively related to the current as well as to all past signals. Second, the weight given to a past signal is smaller the further in the past that signal is. Third, since $a < 1$, when a positive (negative) innovation to current potential output (z_t) occurs, the potential output *estimate* increases (decreases) *by less* than actual potential output. Fourth, the sum of the coefficients in the optimal predictor in equation (3.23) is equal to one. Finally, note that although the true value of potential output is contained only in the signals $s_{1,t-i}$, the optimal predictor assigns positive weights **also** to the signals $s_{2,t-i}$. The intuitive reason is that, by allowing a

[5] The solution when the degrees of persistence differ ($\rho \neq \mu$), based on the Kalman filter, appears in Cukierman and Lippi (2002).

more precise evaluation of the demand shock, g_t, the utilisation of $s_{2,t-i}$ facilitates the separation of g_t from z_t in the signals $s_{1,t-i}$.

3.6.1 Serial correlation in forecast errors of potential output

The form of the optimal predictor in equation (3.23), in conjunction with the fact that all coefficients are positive and sum to one, implies that when a single shock to potential output occurs (say) in period *t* and persists forever without any further shocks to potential output, policymakers do not recognise its full impact immediately. Although their forecasting is optimal, policymakers learn about the permanent change in potential output gradually. Initially (in period $t + 1$) they adjust their perception of potential output by the fraction *a*. In period $t + 2$ they internalise the larger fraction $a + (1 - a)(1 - \kappa)$, in period $t + 3$ they internalise the even larger fraction $a + (1 - a)(1 - \kappa) + (1 - a)(1 - \kappa)\kappa$, and so on. After a large number of periods this fraction tends to 1, implying that after a sufficiently large number of periods the full size of the shock is ultimately learned. Thus, equation (3.23) implies that there is gradual learning about potential output and that forecast errors are, therefore, on the same side of zero during this process.

Conversely, when a single relatively large shock to one of the cyclical components of demand occurs it is partially interpreted for some time as a change in potential output. This too creates ex post serial correlation in errors of forecast in the output gap and in potential output. In general, two kinds of errors can be made. A change in potential output may be partly misinterpreted as a cyclical change, or a cyclical change may be partly misinterpreted as a change in potential output. Both types of errors tend to create ex post serial correlation in errors of forecast. But this serial correlation cannot be utilised in real time to improve policy because, contrary to errors of forecasts of variables which become known with certainty one period after their realisation, the potential output of period *t* is not known with certainty even after that period. As a consequence the forecast error committed in period *t* cannot be used to “correct” future forecasts of potential output in the same manner that errors of forecast of a variable that is revealed one period after the formation of that forecast are normally used to update future forecasts.[6]

As a matter of fact, it can be shown that forecast errors of potential output and of the output gap are generally serially correlated even in the population. The remainder of this subsection establishes this fact more precisely and identifies conditions under which this serial correlation is dominated by the variability of innovations to potential output. Note first, from equation (3.17), that the error in forecasting the output gap is equal to minus the error of forecast in potential output. Hence, if errors of forecast of potential output are serially correlated, so are errors of forecast of the output gap. It is shown in Appendix C in Cukierman and Lippi (2002) that the covariance between two adjacent forecast errors is:

$$E[\tilde{z}_{t|t}\tilde{z}_{t-1|t-1}] > 0$$

This leads to the following:

Proposition 2. *Errors in forecasting potential output and the output gap generally display a positive serial correlation*.

Interestingly, this proposition is consistent with recent empirical findings in Orphanides (2000a). Orphanides utilises real-time data on the perceptions of policymakers about potential output during the 1970s and compares those perceptions with current estimates of the historical data. Taking the “current” rendition of estimates of potential output as a proxy for the true values of potential output during the 1970s, he finds highly persistent deviations between the current and the real-time estimates of the output gap (see his Graph 3 in particular).

6 When the true value of the variable that is being forecasted is revealed with certainty with a lag of one period, as is often assumed, the general principle that forecast errors are serially uncorrelated in the population applies. This feature has been used extensively to test for the efficiency of financial markets. However, when, as is the case here, the true value of the variable that is being forecasted is not revealed afterwards, forecast errors are in general serially correlated.

3.7 A quantitative illustration

As a practical illustration of the effects described above, we present an impulse response analysis of the effects of a potential output shock under imperfect information. The numerical implementation of this exercise relies on the algorithms discussed in Gerali and Lippi (2002). We parametrise our model economy using the settings reported in Table 1 (corresponding to the long-run elasticities reported in the model of Rudebusch and Svensson (1999)).

Table 1

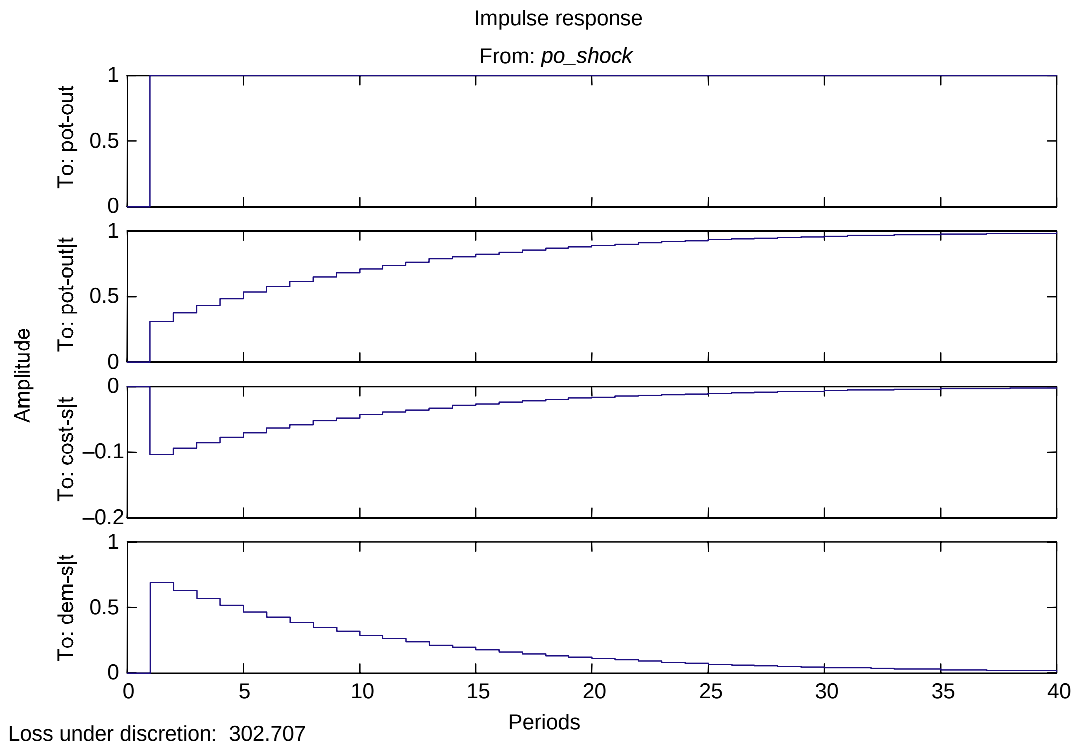
Baseline parameter values for CL model

Parameters						**Innovations (std)**		
β	α	λ	ρ	μ	φ	σ_z	σ_u	σ_g
0.99	1	0.14	0.7	0.7	1	0.5	1.5	1.5

The example below illustrates the impulse response of the main variables in the system to a unit shock in potential output. Graph 1 illustrates how, with imperfect information, the signal extraction problem faced by the policymaker creates confusion about the sources of the business cycle fluctuations.

Graph 1

Perceived state of the economy in response to a potential output shock



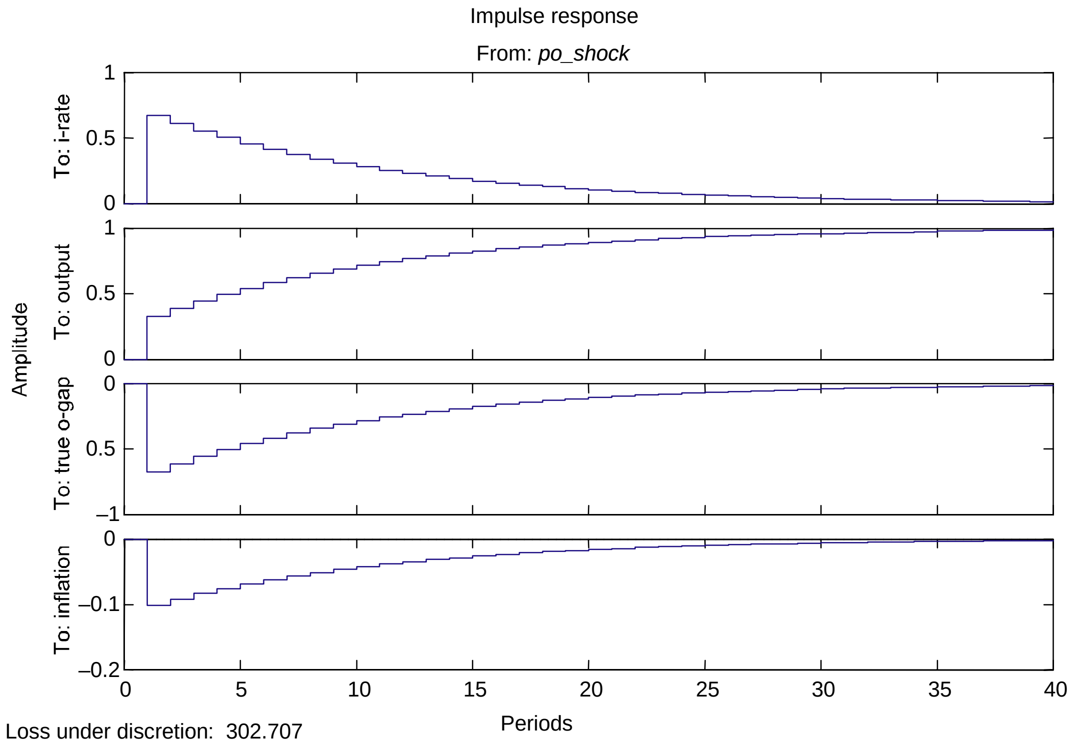
The upper panel displays the true pattern followed by the (unit root) potential output shock. The estimated pattern for this shock (computed with a Kalman filter) is traced out in the second panel. As the theory showed, the learning process is gradual and the forecast errors display a positive serial correlation. The two remaining panels illustrate how misperceptions about potential output cause misperceptions about the cost push and demand shocks, the true value of which is identically zero in this experiment (these relationships obey equation (3.17)). It is evident that an underestimated potential output level implies an overestimated demand shock (to “explain” the currently high output level observed) and an underestimated cost shock (consistent with the relatively low realised inflation).

Quantitatively, of the true 1% increase in potential output only 0.3 is estimated initially, while about 0.7 percentage points of the output rise are attributed to the demand shock.

The policy consequences of these misperceptions are depicted in Graph 2. The parameters chosen are such that the inequality $\left(\mu > \frac{\rho\lambda^2}{\alpha + \lambda^2}\right)$ is satisfied, implying that monetary policy is driven mainly by "demand shocks" considerations (see Proposition 1). Recall that, under the complete information benchmark, there should be no policy response following this shock, ie the optimal interest rate path should be identically zero. The graph shows how, under imperfect information, a positive innovation in potential output causes interest rates to rise above their optimal level in the absence of PTC. This causes the true output gap to fall (even though the policymaker perceives a positive output gap!) and inflation to be lower than under the no PTC benchmark. This is how the model rationalises a situation like the 1990s, when high output growth is associated with low inflation.

Graph 2

Macro effects of a potential output shock with imperfect information

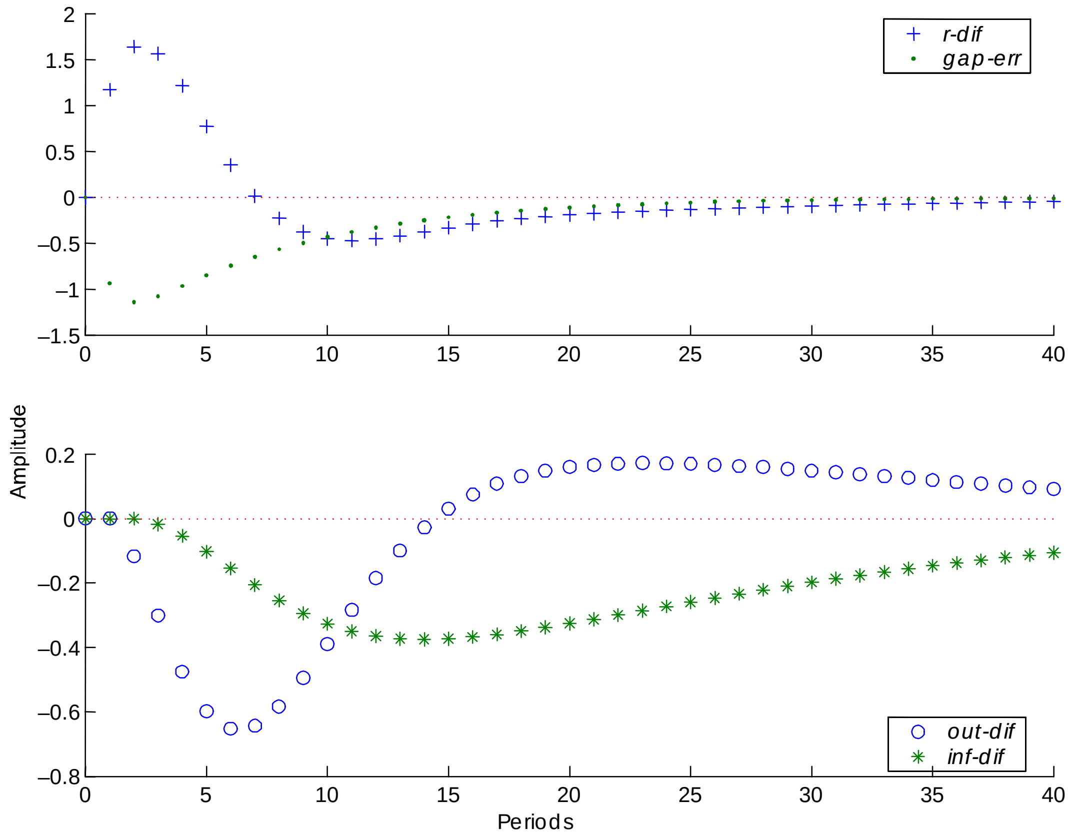


For a more realistic assessment of the quantitative effect of imperfect information, we repeated the exercise developed above using the model of the US economy of Rudebusch and Svensson (1999). Graph 3 reports the values of a given variable under imperfect information in deviation from the values recorded under the full information benchmark (ie potential output is known) after a 1% increase of the output gap. The upper panel shows that, following the shock in potential output, the policymaker's forecast error for the output gap is very large (almost none of the shock is predicted initially) and highly persistent (it takes about two years to learn half of the shock). The interest rate is higher than under full information, as almost all of the output expansion is interpreted as a cyclical shock. As a consequence, both output and inflation are below their full information counterpart (lower panel).

A back-of-the-envelope calculation can be used to compare the magnitudes predicted by our model with the events of the 1970s. If we take Orphanides' measures of the forecast errors in the output gap for the 1970s, hovering about 5 percentage points, we have to scale all the effects in Graph 3 by a factor of –5 (so that the measured forecast error in the output gap is matched in size and sign). This implies that the interest rate under incomplete information is more than 5 percentage points below the full information counterpart during the year following the shock. Moreover, the exercise indicates that inflation and the output gap record a maximum deviation from the full information benchmark of about 2 and 3.5 percentage points, respectively. While those numbers are not too small, indicating that imperfect information might contribute to explain the higher than average inflation recorded in the mid-1970s, they admittedly only go part of the way, leaving a significant part of that inflationary burst unexplained.

Graph 3

Effects of imperfect information (potential output shock)



4. Application 2: a "new synthesis" model (from Gerali and Lippi (2002))

In this section, we use the toolkit derived in Gerali and Lippi (2002) to analyse the real-time information problem of monetary policy within a version of the sticky price framework developed by, among others, Woodford (1999) and Clarida et al (1999). In that framework, output (y_t) and inflation (π_t) are determined, respectively, by a dynamic IS curve and a Phillips curve, according to:[7]

$$y_t = y_{t+1|t} - \sigma[i_t - \pi_{t+1|t}] + g_t \tag{4.1}$$

$$\pi_t = \delta\pi_{t+1|t} + \kappa(y_t - \bar{y}_t) + u_t \tag{4.2}$$

[7] These conditions are derived from the optimising behaviour of consumers (ie an intertemporal Euler equation) and price-setting monopoly firms facing a randomly staggered price adjustment mechanism as in Calvo (1983).

where $\bar{y}_t$ denotes potential output as of period t (ie the output level that would obtain under flexible prices), i_t the nominal interest rate, g_t a demand shock and u_t a cost push shock. The output gap is defined as the difference between actual and potential output, $y_t - \bar{y}_t$.

Following Clarida et al (1999, henceforth referred to as the CGG model), we assume the economy is subject to three types of shocks: demand (g_t), cost push (u_t) and potential output ($\hat{y}_t$). They obey the following processes:

$$\bar{y}_t = \gamma \bar{y}_{t-1} + \hat{y}_t \quad 0 < \gamma < 1; \quad \hat{y}_t \sim N(0, \sigma^2_{\hat{y}}) \tag{4.3a}$$

$$g_t = \mu g_{t-1} + \hat{g}_t \quad 0 < \mu < 1; \quad \hat{g}_t \sim N(0, \sigma^2_{g}) \tag{4.3b}$$

$$u_t = \rho u_{t-1} + \hat{u}_t \quad 0 < \rho < 1; \quad \hat{u}_t \sim N(0, \sigma^2_{u}) \tag{4.3c}$$

where the innovations $\hat{y}_{t+1}$, $\hat{u}_{t+1}$ and $\hat{g}_{t+1}$ are iid. Let us assume the measurable variables are given by:

$$\bar{y}^o_t = \bar{y}_t + \theta_{\bar{y}t} \tag{4.4a}$$

$$y^o_t = y_t + \theta_{yt} \tag{4.4b}$$

$$\pi^o_t = \pi_t + \theta_{\pi t} \tag{4.4c}$$

where the measurement errors θ_{jt} are iid. Finally, let the central bank period loss function be:

$$L_t \equiv \tfrac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t - x^*)^2\right] \tag{4.5}$$

which allows us to encompass some special cases of interest, as done theoretically by Clarida et al (1999).[8]

We introduce imperfect information by adding noise to the measurement block (equation (4.4)). This amounts to assuming that potential output, actual output and inflation are subject to the measurement errors reported in Table 2. With imperfect information, the policymaker uses the available information to form an estimate about the true state of the economy (ie $X_{t|t}$).

Table 2

Baseline parameter values for CGG model (from Gerali and Lippi (2002))

Parameters

δ	γ	ρ	μ	κ	σ	λ_y	x^*	π^*
0.99	0.7	0.4	0.3	0.05	2.0	0.25	0.0	0.0

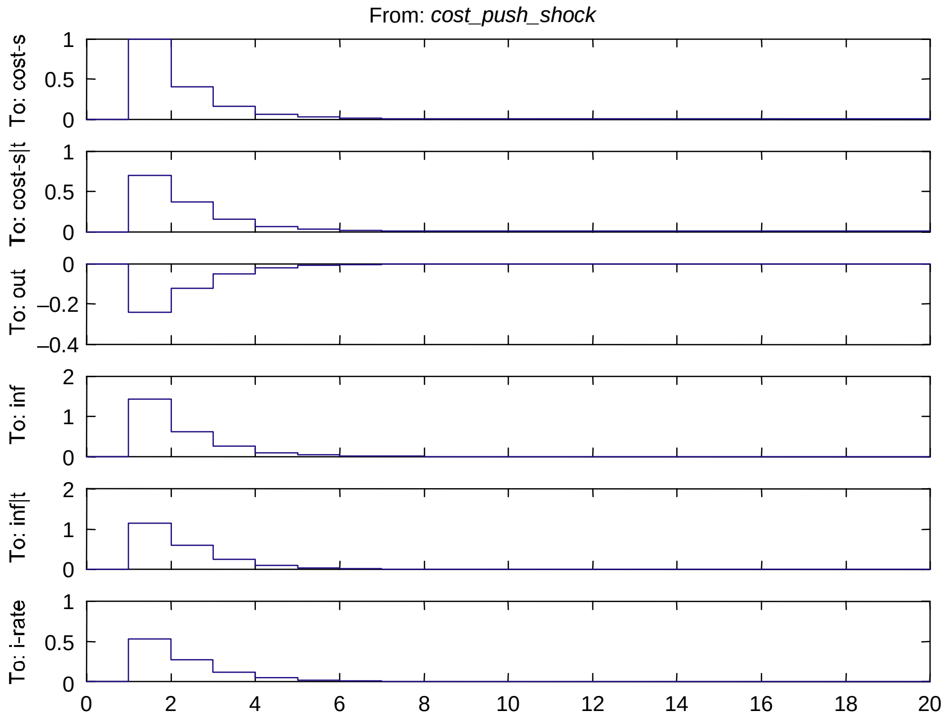
Innovations (std)			**Measurement errors (std)**		
$\sigma_{\bar{y}}$	σ_u	σ_g	$\sigma_{\theta\bar{y}}$	$\sigma_{\theta y}$	$\sigma_{\theta\pi}$
0.001	0.004	0.004	0.004	10^{-8}	10^{-8}

Graph 4 illustrates the effect of a cost push shock under discretion. The first obvious difference with respect to the complete information case is that the true pattern of the shock now differs from the one estimated by the policymaker, as shown in the two upper panels.

8 Among these is the presence of a systematic inflation bias, $x^* > 0$.

Graph 4

Cost push shock with discretion and imperfect information



The signal extraction problem solved with the Kalman filter leads the policymaker to learn only gradually about the realisation of the cost push shock: in the current setup, after a unitary cost push shock ($u_t = 1$) occurs, the contemporaneous estimate of the shock by the policymaker is $u_{t|t} = 0.70$. Naturally, the magnitude of the forecast errors induced by imperfect information depends on the assumptions about the properties of the fundamental processes (eg the persistence of the various structural shocks *g*, *u* and *y* and the signal to noise ratios encoded in Σ_u^2 and Σ_v^2). For instance, if we double the amount of noise in the inflation equation (ie raise $\sigma_{\theta\pi}$), the estimated value of the shock is much smaller ($u_{t|t} = 0.38$), as one would expect in the presence of more noise in the cost push shock indicator, π_t^o.[9]

Through its effect on the expectations about the state of the economy (eg $X_{t|t}$), imperfect information affects the dynamics of the forward-looking variables. First, the policy response of i_t is less strong than in the full information case, as the perceived size of the cost push shock is smaller.[10] The response of output and of inflation is also muted in comparison to the complete information case: output falls by 0.24 (versus 0.32) while inflation increases by 1.4 (versus 1.6). This is due both to the policy response and to the fact that the future expected values of the cost push shock are smaller under incomplete information than under complete information, thus inducing the private economy to expect a different pattern of future shocks and policy.

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9 Several key objects produced by the filtering problem are computed by our MATLAB code, such as the matrices *P* and *Po* corresponding, respectively, to the one-step-ahead and contemporaneous forecast errors in X_t.

10 Due to the certainty equivalence feature of our problem, policy differences stemming from imperfect information arise entirely from the estmates of the states as the coefficient *F* in the optimal control function ($I_t = FX_{t|t}$) does *not* depend on the uncertainty.

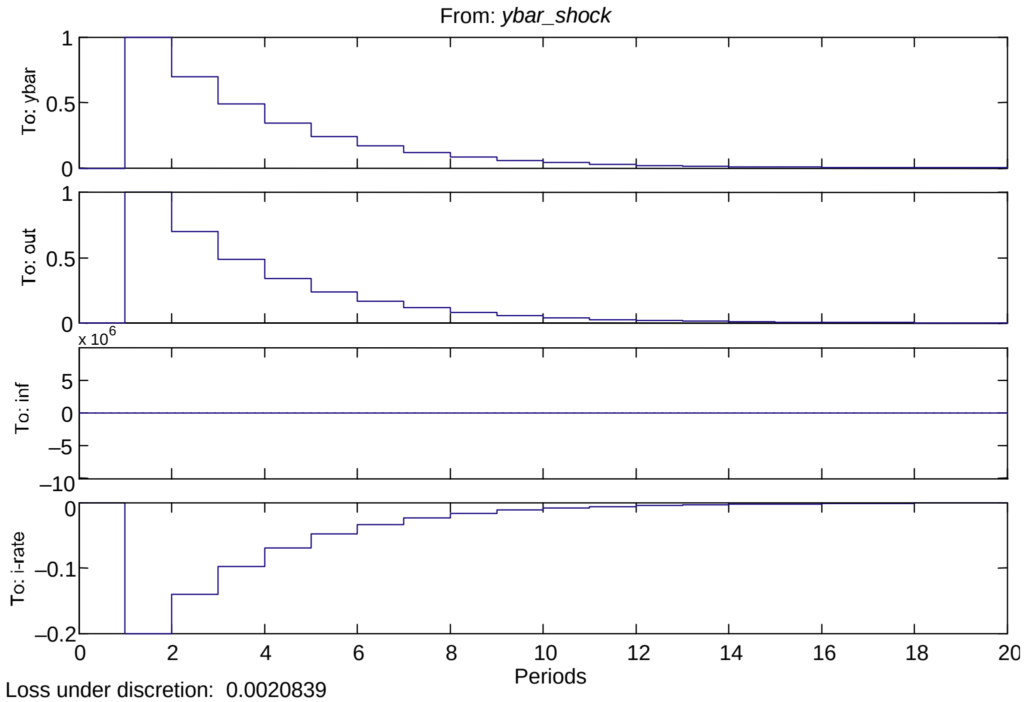
4.1 The macroeconomic consequences of unobservable potential output in the CGG model

We next explore the effects of imperfect information about potential output in the CGG model (under discretionary policy). Several contributions of Orphanides show that potential output estimates are very imprecise in real time. It is argued that basing policy on the estimates of such an unobservable (and noisy) variable may be at the root of important differences between policy based on real-time information and the optimal policy under complete information. To formalise this argument within the CGG model, we compute the effects of a potential output shock in the presence of, respectively, full and incomplete information. The difference in the dynamics of the endogenous variables between these two settings measures the effect of imperfect information.

Graph 5 shows the effect of a potential output shock with full information. The interest rate adjusts in such a way that the dynamics of actual output optimally replicate those of potential output (compare the upper two panels in the graph), eg the "output gap" is nil. This policy poses no trade-off between the objectives of the policymaker, and therefore inflation remains constant at its steady state level.

Graph 5

Potential output shock with full information

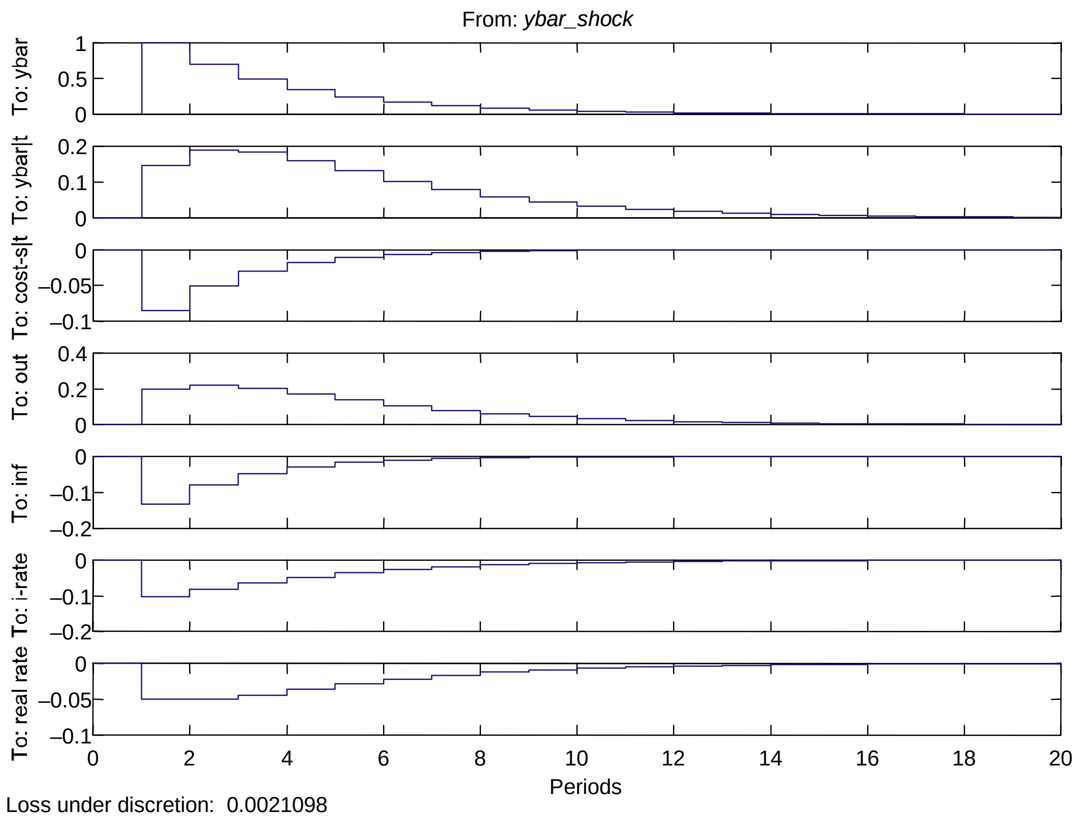


The same potential output shock leads to different consequences under imperfect information, as shown in Graph 6. The upper two panels reveal that the true shock is only partially identified by the policymaker in real time, and that a cost push shock (third panel) is perceived by the policymaker while no such a shock has occurred in reality.

Graph 7, which compares the dynamic response of interest rates, output and inflation under incomplete versus complete information, shows that the interest rate (both nominal and real) is relatively loose (ie is reduced by a smaller amount) under incomplete information. This occurs because, as potential output is underestimated (with incomplete information), the policymaker's perception of how much the interest rate needs to be lowered is smaller than under complete information (recall that the interest rate is proportional to the expected output growth - see equation (4.1)). Therefore, the interest rate under complete information is loose in comparison to the full information benchmark.[11] As a consequence of different policy and expectations, the dynamics of inflation and the output gap under imperfect information differ from their complete information benchmark. The lower panel shows that, following a positive potential output shock, both inflation and the output gap are lower than their full information counterpart.

Graph 6

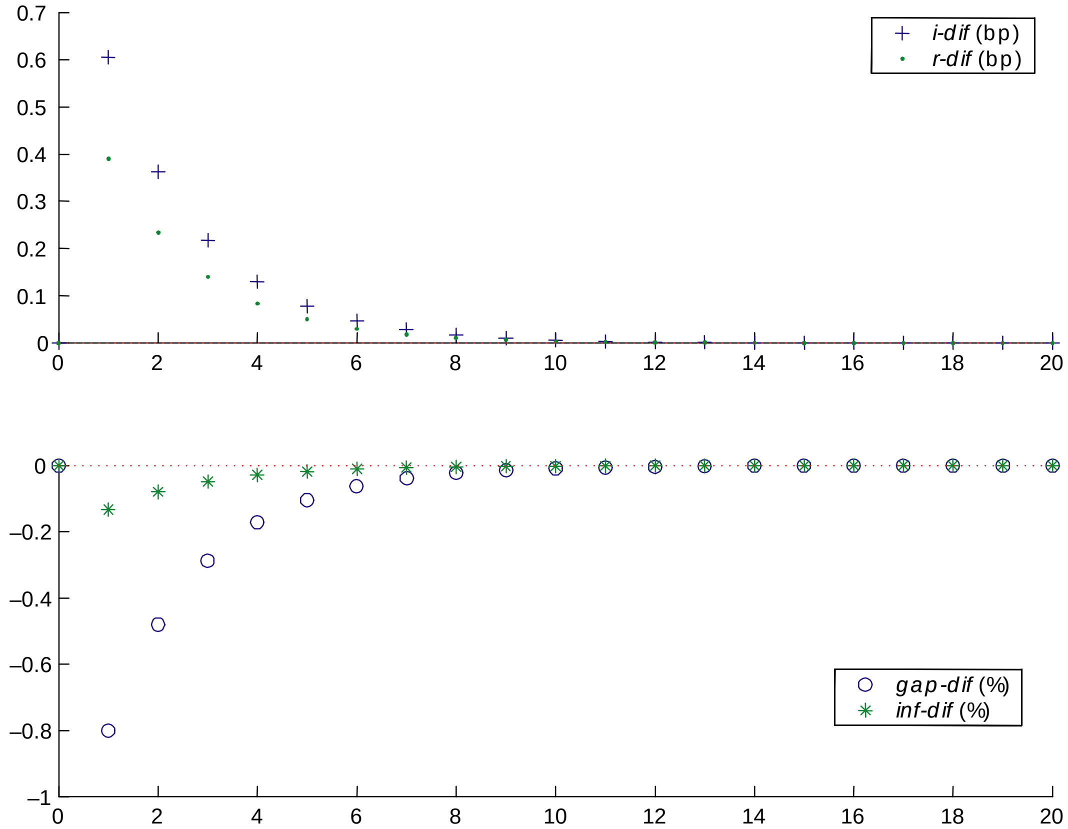
Potential output shock with imperfect information



[11] There is a second effect which goes in the opposite direction but is dominated under most plausible parameter values. It arises because the perceived negative cost push (under incomplete information) leads the policymaker to lower the interest rate (no effect under full information since there is no cost push shock).

Graph 7

Effects of imperfect information



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