The effects of bank consolidation on risk capital allocation and market liquidity[1]

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Abstract

This paper investigates the effects of financial market consolidation on risk capital allocation in a financial institution and the implications for market liquidity in dealership markets. We show that an increase in financial market consolidation can have ambiguous effects on liquidity in foreign exchange and government securities markets. The framework employed assumes that financial institutions use risk management tools (for example value-at-risk) in the allocation of risk capital. Capital is determined at the firm level and allocated among separate business lines, or divisions. Market-makers' ability to supply liquidity is influenced by their risk-bearing capacity, which is directly related to the amount of risk capital allocated to this activity. A model of inter-dealer trading is developed similar to the framework of Volger (1997). However, we allow for heterogeneity among dealers with respect to their risk-bearing capacity.

The allocation of risk capital within financial institutions has implications for what types of mergers among financial institutions can be beneficial for market quality. This effect depends on the correlation among cash flows from business activities that the newly merged financial institution will engage in. A negative correlation between market-making and the new activities of a merged firm suggests the possibility of increased market liquidity. Our results suggest that, when faced with a proposed merger between financial institutions, policymakers and regulators would want to examine the correlations among division cash flows.

1. Introduction

Change in financial markets is ubiquitous. Historically, regulatory restrictions have often inhibited the ability of financial institutions operating in one area of the financial services industry to expand their product set into other areas, but deregulation has allowed financial institutions to offer a broader range of banking, insurance, securities and other financial services.[4] Innovations in financial engineering and evolving market structures have altered the way financial markets and institutions operate. At the same time, deregulation in the industry has increased competition, prompting financial institutions to look for new profitable lines of business. Some financial institutions have found it advantageous to merge in order to generate higher returns through economies of scope or scale. The impact of consolidation on market liquidity, in particular liquidity in government securities and foreign exchange markets, is of increasing importance to policymakers. Ensuring liquidity in these two markets is important to governments and central banks interested in maintaining or enhancing the functioning of these markets so that they can effectively implement fiscal and monetary policies. In Canada,

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4 For example, the Glass-Steagall Act of 1933 sought to impose a rigid separation between commercial banking (deposit-taking and loan-making) and investment banking (underwriting, securities issuance). The Act limited the ability of banks and securities firms to engage directly or indirectly in each other's activities.

policymakers are concerned with the declining number of dealers in both Government of Canada fixed income markets and foreign exchange markets, and worry that increased consolidation among financial institutions will cause liquidity in these markets to fall.

This paper analyses the impact of financial consolidation on market liquidity by studying the effects of consolidation on the risk-bearing capacity of market-makers, or dealers, in dealership markets. To carry out our analysis, two previously separate areas of research are bridged. The first, market microstructure theory, focuses on how market participants and the trading mechanism affect price discovery and market liquidity.[5] The second, risk management, influences the way in which firms look at both the returns and risks of individual business operations. Our analysis traces the impact of a merger on the capital allocation decisions of the new merged financial institution and the resulting change in the behaviour of dealers.

Using a model in which a financial institution, henceforth referred to as a bank in this paper, allocates risk capital across its business activities in order to satisfy a firm-wide capital requirement, we show that the optimal capital allocation conditions the risk aversion of division managers and traders. This key result relates risk management by the bank to the behaviour of its market-makers in asset markets. The risk-bearing capacity of a dealership market depends on the number of market-makers present as well as the risk aversion of each market-maker. Since market liquidity in dealership markets is determined by the inherent riskiness of the market and the risk-bearing capacity of the market, capital allocation affects market liquidity by influencing the risk aversion of market-makers.

We apply this framework to examine the effects of financial consolidation on market liquidity. We find that consolidation has an ambiguous effect on market liquidity. In particular, market liquidity can increase upon consolidation. Whether this happens depends on the correlation among the cash flows from the merged bank's division. This is in contrast to other results in the literature, which argue that market liquidity will necessarily deteriorate with consolidation. These other studies only consider the effects of a reduction in market-makers on risk-sharing, while our paper shows that the effect on liquidity of a bank merger will also depend critically on the risk-bearing capacities of the old and new banks. Therefore, policymakers and regulators faced with a proposed merger between banks would want to examine the correlations among division cash flows. A negative correlation between market-making and the new activities of a merged firm suggest the possibility of increased market liquidity.

Capital allocation decisions are more complicated than a simple application of the capital asset pricing model (CAPM), since frictions exist in capital markets. Imperfect capital markets impose deadweight costs that must be covered by the cash flows of a business line if the business line is to be profitable. Froot and Stein (1998) and Perold (2001) model the capital structure decision by positing frictions in capital markets and/or in the internal management of firms that lead to deadweight costs. In Froot and Stein (1998) firms engage in risk management to avoid ex post penalties resulting from a cash flow shortfall. Perold (2001), on the other hand, derives ex ante deadweight costs associated with actions undertaken by the firm to provide performance guarantees on its customer contracts through the purchase of insurance and a cash cushion. Both papers demonstrate that there is a trade-off between managing risk via ex ante capital structure policies and via capital budgeting and hedging policies. Hence, the capital structure, hedging and capital budgeting policies of a firm are interrelated and jointly determined. In a multi-divisional firm, risk management tools are also used for performance evaluation. Specifically, risk capital allocation is an important component of the process of determining the risk-adjusted rate of return and ultimately the economic value added of each business unit. Such calculations can then form the basis for incentive compensation. Stoughton and Zechner (1999) examine performance evaluation and managerial compensation issues but we will abstract from those issues in this paper. In addition to the internal risk management that financial institutions engage in, regulators impose capital requirements on banks. Externalities from bank failures, risk-shifting in the presence of fixed premium deposit insurance, and the protection of uninformed investors who hold most of a bank's debt are the main justification for regulating bank capital. For all these reasons, financial institutions often maintain capital levels over and above the amounts they need to finance their operations.

[5] See O'Hara (1995) for a survey of the theoretical literature on market microstructure and Madhavan (2000) for a survey of the empirical literature.

Market liquidity is influenced by the way the market is structured. For example, most foreign exchange and government bond markets are characterised by price competition (quote-driven) among multiple dealers and inter-dealer trading rather than by Cournot competition (order-driven), and the actions of the dealers in the public and inter-dealer markets provide much of market liquidity. Such markets are referred to as *dealership markets*. This paper develops a dealership market model similar to the framework of Volger (1997). However, we allow for heterogeneity among dealers with respect to their risk-bearing capacity.

The paper is organised as follows: Section 2 presents the capital allocation model for a bank with multiple divisions; Section 3 provides an analysis of a dealership market model in which dealers can be heterogeneous with respect to their risk aversion; Section 4 looks at the effects of financial consolidation on capital allocation and liquidity in the dealership market; and Section 5 suggests implications for regulators and policymakers, and for further research.

2. Capital allocation in a multi-divisional firm

Effective risk management promotes both financial institution and industry stability by protecting a financial institution against market, credit, liquidity, operational and legal risk. The primary means of protection is the financial institution's risk (or economic) capital. One goal of risk management is to determine the firm's optimal capital structure. This process involves estimating how much risk each business unit, or division, contributes to the total risk of the firm and thus to overall capital requirements. Since investment decisions and risk exposures are determined at the division level, correlations between portfolios held by different divisions are externalities among units that create a need for centralised risk management. Hence, risk management in a multi-divisional firm also involves determining the capital charge to each division whose activities contribute to firm risk in order to induce the appropriate risk-taking behaviour by division managers.

The framework that we use in this section is adapted from Stoughton and Zechner (1999). Consider an economy with N + 1 banks, indexed i = 0,. . , N.6 Each bank is engaged in a number of financial activities that generate income or cash flows. These activities are indexed by j. We denote the set of all possible financial activities by J. One activity that all banks participate in is market-making in a dealership market for a particular security.[7] In this section, we analyse the problem of a bank that engages in a subset K J of activities, each of which generates cash flows to equity holders.[8] Each business line j is undertaken by a division, so we will denote the division by the same index j.

Each division j has an expected cash flow μ_j determined by the function

$$\mu_j = \mu_j\left(\sigma_j^2\right), j \in K \tag{1}$$

where σ_j^2 is the variance of cash flow from division j. It is assumed that more risk-taking by a division yields a higher expected return,

$$\frac{\partial \mu_j}{\partial \sigma_j^2} > 0, \ j \in K.$$

In addition, we assume that the function μ_j is strictly concave in σ_j^2 and the Inada conditions hold, or

$$\frac{\partial^2 \mu_j}{\partial \sigma_j^2} < 0, \left.\frac{\partial \mu_j}{\partial \sigma_j^2}\right|_{\sigma_j^2=0} \to \infty, \lim_{\sigma_j^2 \to \infty} \frac{\partial \mu_j}{\partial \sigma_j^2} = 0, \ j \in K.$$

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6 Later, we analyse the case where a merger occurs and N banks remain in the economy.

7 This may be a foreign exchange, fixed income or equity market.

8 We will drop for now the index i corresponding to the N + 1 banks. In later sections of the paper, the index is brought back into our notation.

To simplify the analysis without loss of generality, investment activities are assumed to require zero cash outlay, or cash flows are defined after the appropriate interest costs. Furthermore, for any j, $h \in J$, the correlation between project j's and project h's cash flows is given by ρ_{hj}.

The opportunity cost of equity capital, $r > 0$, is assumed to be constant and identical across banks. Financial institutions must allocate the scarce equity capital without violating regulatory constraints. A bank's equity capital requirement is determined as a fixed proportion of the risk of its portfolio, as measured by the variance of its total cash flows,

$$C \geq \alpha\sigma_p^2, \alpha > 0 \tag{2}$$

The bank's overall risk, σ_p^2, can be expressed as

$$\sigma_p^2 = \sum_{j \in K} \sigma_j^2 + \sum_{h \in K} \sum_{j \in K,\, j \neq h,} \rho_{hj}\sigma_h\sigma_j \tag{3}$$

The bank's objective function is to maximise the net present value of cash flows, taking into account the opportunity cost of capital. This is equivalent to maximising the economic value added (EVA) or the contribution to shareholder value, where

$$\text{EVA} = \sum_{j \in K} \mu_j - rC \tag{4}$$

2.1 First-best: centralised investment decisions

Before we examine the problem of allocating capital across divisions in a delegated environment, we first derive the solution to the bank's centralised problem. Continuing with our analysis of a bank with multiple divisions, indexed by $j \in K$, the centralised problem is

$$\begin{aligned} &\max_{\{\sigma_j\}_{j \in K}} \sum\nolimits_{j \in K} \mu_j\left(\sigma_j^2\right) - rC \\ &\quad s.t. \qquad C \geq \alpha\sigma_p^2 \end{aligned} \tag{5}$$

where σ_p^2 is defined in (3). Since capital is costly, the constraint in the maximisation problem is always satisfied with equality.

The first-order conditions (for an interior optimum) are

$$\frac{\partial\mu_j\left(\hat{\sigma}_j^2\right)}{\partial\sigma_j^2} = r\alpha\left[1 + \sum_{h \neq j, h \in K} \rho_{hj}\left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2}\right)^{1/2}\right], j \in K \tag{6}$$

where $\hat{\sigma}_j, j \in K$ is the optimal risk level for division j.

The right-hand side of the above equation is just the marginal contribution to the overall risk of the bank by division j's activities multiplied by the cost of capital. At a given risk level, σ_j^2, division j's marginal contribution to the overall risk of the bank is

$$\frac{\partial\alpha\sigma_p^2}{\partial\sigma_j^2} = \alpha\left[1 + \sum_{h \neq j, h \in K} \rho_{hj}\left(\frac{\sigma_h}{\sigma_j}\right)\right], j \in K \tag{7}$$

Intuitively, investment (in terms of risk undertaken) will occur up to the point where the marginal increase in expected returns from activities by division j is balanced by the marginal cost of risk undertaken by that division.

2.2 Delegated investment decisions

In an environment where investment decisions are delegated to each division, the bank's problem is one of allocating the appropriate amount of risk capital across divisions to maximise the bank's economic value added. It is straightforward to determine the capital allocation function that implements the first-best solution to the delegated problem. Suppose that the bank establishes a capital allocation rule for each division, T_j, $j \in K$. The formal delegation problem can be written

$$\max_{\{T_j\}_{j\in K}} \sum_{j\in K}\left[\mu_j\left(\sigma_j^2\right) - rC - U_j\right] \tag{8}$$

$$s.t.\ \sigma_j^2 \in \arg\max_{\sigma^2} \mu_j\left(\sigma^2\right) - rT_j\left(\sigma_j^2\right), j \in K \tag{9}$$

$$C \geq \alpha\sigma_p^2 \tag{10}$$

where U_j denotes the compensation that the firm transfers to division managers. This compensation function is designed so that each division makes optimal investment decisions and is assumed to consist of a fixed (salary) component, S_j, and a performance component in the form of a share of the EVA generated by the division. The division's EVA is, in turn, defined as the mean return from the division's project adjusted for the appropriate capital charge, rT_j, that is,

$$U_j = \tau\,[\mu_j - rT_j] + S_j,\ j \in K \tag{11}$$

This compensation scheme induces each division to solve the problem

$$\max_{\sigma_j^2} \mu_j\left(\sigma_j^2\right) - rT_{j,}\ j \in K$$

which yields constraint (9) in the bank's delegation problem. We present the solution to the delegation problem and its implication in the following proposition.

Proposition 1. *The optimal capital allocation function to each division j, j $\in$ K, is a linear function of the risk undertaken by the division:*

$$T_j\left(\sigma_j^2\right) = \theta_j\sigma_j^2,\ j \in K \tag{12}$$

where

$$\theta_j = \alpha\left[1 + \sum_{h\neq j, h\in K} \rho_{hj}\left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2}\right)^{1/2}\right] \tag{13}$$

This capital allocation function conditions the division manager's risk aversion so that the manager, by maximising his utility, behaves like a risk-averse agent with exponential utility and risk-aversion parameter given by

$$\gamma_j = r\theta_j,\ j \in K \tag{14}$$

in the presence of normally distributed cash flows.

Proof: See Appendix.

The optimal capital allocation to a division is thus proportional to the risk undertaken by the division, where risk is measured by the variance of the cash flow generated by the division. One can think of this as a charge to the division for the risk imposed on the bank by the division's activities. More importantly, this proposition relates the risk management of a bank to the behaviour of its dealers in the bank's trading activities. We elaborate on this point in the next section, where a dealership market model is presented.

3. Model of the dealership market

Liquidity is an important dimension of all financial markets.[9] For example, government securities markets perform several important functions that hinge on the fact that they are very liquid. They are the markets in which governments raise funds and are thus of particular interest to central banks with fiscal agency responsibilities. Furthermore, because of their virtually riskless nature, government securities serve as the pricing benchmark and hedging vehicle for other fixed income securities. While market liquidity is a concept that is difficult to measure or define because of its multidimensional nature, most market participants would agree on the following characterisation. A *liquid* market is *one in which large transactions can be completed quickly with little impact on prices*.[10] The various dimensions of liquidity also tend to interact.[11] In this paper, we focus on bid-ask spreads as a measure of liquidity.

In this section, we develop a model of the dealership market in which banks provide market-making services. Each bank's market-making activity is carried out by a dealer who is constrained in his risk-taking behaviour by the bank's capital allocation. This allows us to study how the capital allocation decisions by individual banks impact market liquidity. We will then apply this framework to a merger between two banks by first examining how capital allocation is affected by the merger and the consequences of that for market liquidity.

Consider a security that trades in a dealership market with $N + 1$ market-makers (or dealers). The security is traded at price p between dealers and outside investors in the public market, and at price p_d among dealers in a separate inter-dealer market. The exogenously given liquidation value of the security is denoted by v, a random variable which is normally distributed with zero mean and standard deviation σ_v. We assume that no market participant has private information about the future liquidation value of the traded security. There is one investor in the market who trades a quantity w, the realisation of a random variable which is independent of the asset's liquidation value, v, and distributed with zero mean and standard deviation σ_v. By convention, $w > 0$ denotes an investor's buy order and $w < 0$ a sell order. We consider one trading period where trade takes place in two stages.

In stage 1, all dealers simultaneously quote a price schedule over customer orders, w, in the public market. The investor observes the quotes of all the competing dealers and submits the whole order w to the dealer quoting the best price. It is a defining characteristic of dealership markets that market-makers compete for the whole order. We assume that each dealer starts with a zero inventory position which is observable to all dealers.[12] Bargaining between investors and market-makers is resolved by assuming that market-makers compete à la Betrand. That is, all bargaining power is on the investor's side.

Trading between market-makers to reallocate inventories takes place in stage 2. Once an investor gives the whole order to one of the $N + 1$ competing dealers, that particular dealer's inventory changes by $-w$.This dealer now has an incentive to trade in the inter-dealer market to reduce his risk exposure. Hence, inter-dealer trading allows dealers to risk-share. Dealers are assumed to behave as strategic competitors by submitting their demand functions in the inter-dealer market. That is, they take into account the effect their quantities are expected to have on the market-clearing price. The equilibrium concept we employ is that of a non-competitive rational expectations Nash equilibrium in demands (Kyle (1989)). The security will be liquidated at the end of the inter-dealer trading in stage 2.

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9 The discussion of liquidity is taken from Gravelle (1999, 2001).

10 In the literature, market liquidity is typically defined over four dimensions: immediacy, depth, width (bid-ask spread) and resiliency. Immediacy refers to the speed with which a trade of a given size at a given width is completed. Depth refers to the maximum size of trade that can be carried out for any given bid-ask spread. Width refers to the cost of providing liquidity, with narrower spreads implying greater liquidity. Resiliency refers to how quickly imbalances in transaction flows dissipate. An imbalance in transaction flows means that there is a one-way market, or prices are gapping. If imbalances tend to persist, or when imbalances tend not to generate a counterbalancing order flow (once prices have moved enough to attract this counterbalancing order flow), the market is not resilient.

11 For example, width will generally increase with the size of a given trade, or, for a given bid-ask spread, all transactions under a given size can be executed immediately with no movement in the price or spread.

12 Allowing for different initial inventories only complicates the analysis without qualitatively affecting our merger analysis results in the next section.

The $N + 1$ dealers are indexed by $i = 0,...,N$. Each dealer behaves like a risk-averse agent with a coefficient of absolute risk aversion given by γ_i.[13] Specifically, dealer i with risk aversion parameter γ_i maximises the exponential utility function

$$U(\pi_i) = -e^{-\gamma_i \pi_i}$$

where $\pi_i = v(x_i - w) + pw - p_d x_i$ is the profit of dealer i if he gets an order from the investor, while $\pi_i = vx_i - p_d x_i$ is his profit if he gets no customer order, x_i is the demand of dealer i in the inter-dealer market, p is the price at which the customer order is transacted, and p_d is the price that prevails in the inter-dealer market.

We will solve the model under two different scenarios. In the first, we assume that dealers have identical risk aversion, γ. In the second, we assume that dealers are one of two types. Type 1 dealers have risk aversion $\gamma_1 = \delta\gamma$, $\delta > 0$ and type 2 dealers have risk aversion $\gamma_2 = \gamma$. There are N_1 type 1 dealers and N_2 type 2 dealers. Naturally, $N_1 + N_2 = N + 1$.

Later, when we analyse the effects of a merger between two banks (and consequently two dealers), the starting point is a market with identical dealers. This is the case when potentially differentiated banks allocate the same amount of capital to market-making. When two of those banks merge, we allow for the case that the merged entity engages in a different set of activities, thus allocating risk capital differently across business lines. To this end, we will need an analysis of a dealership market with heterogeneous dealers.

3.1 Dealership market with identical market-makers ($\delta = 1$)

In this section, dealers have an identical coefficient of absolute risk aversion, given by γ. We solve the model by backward induction. That is, we first solve stage 2 of the model for a symmetric equilibrium in the inter-dealer market, taking the equilibrium price in the public market as given. Then, we solve stage 1 of the model for the equilibrium reserve prices (the price that leaves the dealer indifferent between getting the customer order and not getting it) in the public market. For reservation prices that differ, the equilibrium price in the public market is given by the second-best reservation price which is quoted by the dealer with the best reservation price. When all dealers have the same reservation price, they each quote their reservation prices and receive the customer order with equal probability.

3.1.1 Equilibrium in the inter-dealer market (stage 2)

We simplify the analysis by assuming that the inter-dealer market is a call market. All market-makers submit their orders simultaneously to an inter-dealer broker who executes the set of multilateral transactions at one market-clearing price.

A symmetric linear equilibrium in the inter-dealer market is obtained if the demand schedules of the each dealer i can be written

$$x_i = \alpha - \beta p_d + \eta w_i, \qquad i = 0,...,N \tag{15}$$

Proposition 2. *There exists a linear equilibrium in the inter-dealer market in which market-makers' demand is given by (15). The parameters are given by*

$$\alpha = 0, \qquad \beta = \frac{N-1}{N\Psi}, \qquad \eta = \frac{N-1}{N} \tag{16}$$

where $\Psi = \gamma\sigma_v^2$. *The equilibrium price in the inter-dealer market is*

$$p_d = \frac{\Psi}{N+1} w. \tag{17}$$

Proof: See Appendix.

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[13] In our framework, dealers' risk aversion is determined by the amount of risk capital allocated to market-making in that security by the financial institutions that own the dealers.

The equilibrium price in the inter-dealer market depends on the size of the investor's order. η is the proportion of the investor's order that a dealer passes on in the inter-dealer round of trading and hence a measure of risk-sharing. It is increasing in N. Therefore, risk-sharing improves as the total number of dealers in the market increases and the inter-dealer market becomes more competitive.

Using the fact that $w_i = w$ if dealer i receives the customer order and $w_i = 0$ otherwise, demands and inventories after inter-dealer trade are

$$x_i = \begin{cases} \dfrac{N-1}{(N+1)}w & \text{if } w_i = w \\ -\dfrac{N-1}{N(N+1)}w & \text{if } w_i = 0 \end{cases} \tag{18}$$

$$I_i = \begin{cases} \dfrac{N-1}{(N+1)}w - w = -\dfrac{2}{N+1}w & \text{if } w_i = w \\ -\dfrac{N-1}{N(N+1)}w & \text{if } w_i = 0 \end{cases} \tag{19}$$

Notice that risk-sharing is not perfect in this model. Since dealers are ex ante identical, perfect risk-sharing implies that all dealers will end up with identical inventory levels after inter-dealer trade, or $I^* = -\dfrac{w}{N+1}$. However, $|I_i| > |I^*|$ if $w_i = w$ and $|I_i| < |I^*|$ if $w_i = 0$.

Perfect risk-sharing does not obtain because of imperfect competition in the inter-dealer market. That is, each dealer has an incentive to restrict the quantity he trades in the inter-dealer market relative to what he would trade if the inter-dealer market was competitive. Notice that perfect risk-sharing will obtain if $N \to \infty$.

3.1.2 Equilibrium in the public market (stage 1)

To solve for the equilibrium in the public order market, we first determine each dealer's reservation quotes, in anticipation of inter-dealer trading in the next stage. The dealer with the best reservation price receives the public order by quoting the second-best reservation price. Recall that if dealer i gets the public order, he has a final inventory $-\dfrac{2}{N+1}w$ while if he does not get the public order, he has a final inventory $-\dfrac{N-1}{N(N+1)}w$.

Denote dealer i's reservation price by p_i^r, $i = 0,..,N$. If dealer i gets the public order at his reservation price, his expected utility will be given by

$$EU_i^w = p_i^r w - p_d x_i - \frac{\Psi}{2}\left(\frac{2}{N+1}w\right)^2 \tag{20}$$

$$= p_i^r w - \frac{(N-1)\Psi}{(N+1)^2}w^2 - \frac{\Psi}{2}\left(\frac{2}{N+1}w\right)^2 \tag{21}$$

while if he does not, his expected utility is

$$EU_i^0 = -p_d x_i - \frac{\Psi}{2}\left(\frac{N-1}{N(N+1)}w\right)^2 \tag{22}$$

$$= \frac{(N-1)\Psi}{N(N+1)^2}w^2 - \frac{\Psi}{2}\left(\frac{N-1}{N(N+1)}w\right)^2. \tag{23}$$

At his reservation price, dealer *i* is indifferent between getting the public order and not getting it. Equating $EU_i^w = EU_i^0$ and simplifying, we get dealer *i*'s reservation price, given in the next proposition.

Proposition 3. *For* $N > 0$*, the equilibrium price in the public market is*

$$p = \frac{(2N-1)\Psi}{2N^2}w \tag{24}$$

and the market bid-ask spread for a customer order of size $|w|$ *is*

$$s = \frac{(2N-1)\Psi}{N^2}|w|. \tag{25}$$

Since all dealers are identical, they quote the same price (equal to their reservation price) and have equal chances of receiving the public order. The market bid-ask spread for an order of size $|w|$ is just s = 2 $|p|$.

If dealers are risk-neutral $(\Psi = \gamma\sigma_v^2 = 0)$, the equilibrium price is equal to the expected value of the security, which is normalised to zero. For risk-averse dealers and multiple dealers, $N > 0$, the equilibrium price is increasing in the size of the customer order and decreasing in *N*. The larger the size of the customer order, the higher the risk premium required by dealers to absorb this quantity. An increase in the number of competing dealers leads to better risk-sharing and hence a lower risk premium is required.

3.2 Dealership market with heterogeneous market-makers

In this section, there are two types of dealers: type 1 dealers have risk aversion $\gamma_1 = \delta_\gamma$, and $\delta > 0$ type 2 dealers have risk aversion $\gamma_2 = \gamma$. There are N_1 type 1 dealers and N_2 type 2 dealers, where $N_1 + N_2 = N + 1$. We begin this section by characterising the equilibrium for the case where N_1 and N_2 can take any values but we solve explicitly for the equilibrium with only one type 1 dealer and *N* type 2 dealers. This minimal amount of heterogeneity is all we need in order to perform the merger analysis that comprises the next part of this paper.

3.2.1 Equilibrium in the inter-dealer market (stage 2)

Let the set of type 1 dealers be denoted by χ_1 and the set of type 2 dealers be denoted by χ_2. From here on, we will denote an arbitrary type 1 dealer by *i* and an arbitrary type 2 dealer by *j*. Since each of the two types of dealers is identical with respect to the other dealers in his group, a linear equilibrium in the inter-dealer market is obtained if the demand schedules of each dealer *i* can be written

$$x_i = \alpha_1 - \beta_1 p_d + \eta_1 w_i, \qquad \forall\ i \in \chi_1 \tag{26}$$

$$x_j = \alpha_2 - \beta_2 p_d + \eta_2 w_j, \qquad \forall\ j \in \chi_2 \tag{27}$$

Proposition 4 (General case). *There exists a linear equilibrium in the inter-dealer market in which the market-makers' demand is given by (26) and (27). The parameters are given implicitly by*

$\alpha_1 = \alpha_2 = 0, \qquad \eta_1 = \delta\Psi\beta_1, \qquad \eta_2 = \Psi\beta_2,$

$$\beta_1 = \frac{(N_1-1)\beta_1 + N_2\beta_2}{1+\delta\Psi\left[(N_1-1)\beta_1 + N_2\beta_2\right]} = \frac{\overline{\beta} - \beta_1}{1+\delta\Psi\left(\overline{\beta} - \beta_1\right)} \tag{28}$$

$$\beta_2 = \frac{N_1\beta_1 + (N_2 - 1)\beta_2}{1 + \Psi(N_1\beta_1 + (N_2 - 1)\beta_2)} = \frac{\overline{\beta} - \beta_2}{1 + \Psi(\overline{\beta} - \beta_2)} \tag{29}$$

where $\overline{\beta} = N_1\beta_1 + N_2\beta_2$ and $\Psi = \gamma\sigma_\upsilon^2$. *Denoting the type of the dealer who received the customer order in stage 1 by y, the equilibrium price in the inter-dealer market is:*

$$p_d = \frac{\beta_1\delta\Psi w}{\overline{\beta}} \qquad \text{if } y \in \chi_1 \tag{30}$$

$$= \frac{\beta_2\Psi w}{\overline{\beta}} \qquad \text{if } y \in \chi_2. \tag{31}$$

The proof of proposition 4 follows the same steps as for proposition 1. Note that the equilibrium inter-dealer price is lower if the dealer with the public order is the one with lower risk aversion. For $\delta < 1$ ($\delta > 1$), a type 1 dealer has a lower (higher) risk aversion than a type 2 dealer. As well, the equilibrium inter-dealer price is increasing in the size of the customer order, $|w|$.

Although the solutions to the two equations for $\beta 1$ and $\beta 2$ are difficult to derive explicitly, we can characterise the solutions.

1. β_1, β_2 and $\overline{\beta}$ are increasing in δ.
2. $\beta_1 - \beta_2 > 0$ and $\delta\Psi\beta_1 - \Psi\beta_2 < 0$ for $N > 2$ and $\delta < 1$.

   $\beta_1 - \beta_2 < 0$ and $\delta\Psi\beta_1 - \Psi\beta_2 < 0$ for $N > 2$ and $\delta > 1$.
3. β_1, β_2 and $\overline{\beta}$ are decreasing in Ψ.
4. Holding N constant, β_1, β_2 and $\overline{\beta}$ are decreasing in N_1 if $\delta < 1$ and increasing in N_1 if $\delta > 1$.

For the special case of $N_1 = 1$, the explicit solutions for β_1 and β_2 are given in the next proposition. This special case is relevant when we analyse a merger between two banks, and hence two dealers.

Proposition 5 (Special case). *For* $N_1 = 1$ and $N > 3$, *there exists a linear equilibrium in the inter-dealer market in which the market-makers' demand is given by (26) and (27). The parameters are given implicitly by*

$\alpha_1 = \alpha_2 = 0, \qquad \eta_1 = \delta\Psi\beta_1, \qquad \eta_2 = \Psi\beta_2,$

$$\beta_1 = \frac{N_2\beta_2}{1 + \delta\Psi N_2\beta_2} \tag{32}$$

$$\beta_2 = \frac{\delta N_2^2 + 1 - 2N_2(1+\delta) + \sqrt{\left[\delta N_2^2 + 1 - 2N_2(1+\delta)\right]^2 + 4\delta N_2(N_2 - 1)(2N_1 - 2)}}{2\delta N_2(N_2 - 1)} \tag{33}$$

where $\Psi = \gamma\sigma_v^2$.

3.2.2 *Equilibrium in the public market (stage 1)*

In this section, we carry through the assumption that $N_1 = 1$ and $N_2 = N$. As before, we first determine each dealer's reservation quotes in the public market, in anticipation of inter-dealer trading in the next stage. Since there are two types of dealers, there will be two different reservation prices. The dealer with the best reservation price receives the public order by quoting the second-best reservation price.

Let p_1^r be the reservation price of the type 1 dealer and p_2^r be the reservation price of a type 2 dealer.

Proposition 6. *Let* $\Psi_1 = \delta\gamma\sigma_v^2$ *and* $\Psi_2 = \gamma\sigma_v^2$. *The type 1 dealer has a reservation price given by*

$$p_1^r = \left\{\left(1 - \frac{\beta_1}{\overline{\beta}}\right)\frac{\beta_1^2\Psi_1^2}{\overline{\beta}} + \frac{\beta_1}{2\overline{\beta}^2}\beta_2^2\Psi_2^2(2 - \beta_1\Psi_1) - \frac{\Psi_1}{2}\left[1 - \left(1 - \frac{\beta_1}{\overline{\beta}}\right)\beta_1\Psi_1\right]^2\right\}w \tag{34}$$

while a type 2 dealer has a reservation price given by

$$p_2^r = \left\{ \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \frac{\beta_2^2 \Psi_2^2}{\bar{\beta}} + \frac{\beta_2}{2\bar{\beta}^2} \beta_1^2 \Psi_1^2 \left(2 - \beta_2 \Psi_2\right) - \frac{\Psi_2}{2} \left[1 - \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \beta_2 \Psi_2 \right]^2 \right\} w \tag{35}$$

if the winning dealer is a type 1 dealer, and

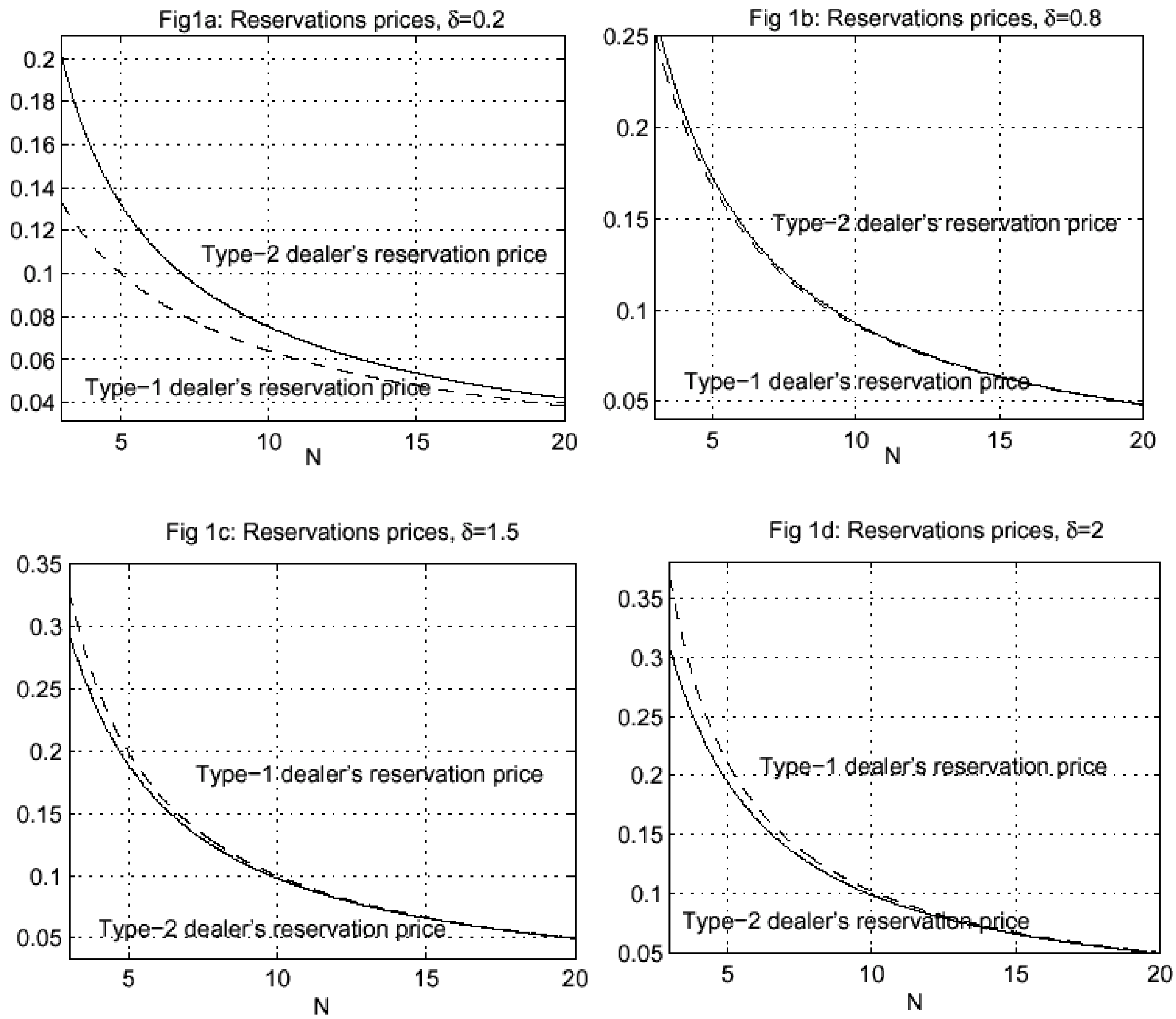
$$p_2^r = \left\{ \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \frac{\beta_2^2 \Psi_2^2}{\bar{\beta}} + \frac{\beta_2}{2\bar{\beta}^2} \beta_2^2 \Psi_2^2 \left(2 - \beta_2 \Psi_2\right) - \frac{\Psi_2}{2} \left[1 - \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \beta_2 \Psi_2 \right]^2 \right\} w \tag{36}$$

if the winning dealer is another type 2 dealer.

We turn to numerical examples (Figure 1) to illustrate the following proposition, which lays out the equilibrium price in the public market.

Figure 1

Reservation prices for type 1 and type 2 dealers



Proposition 7. *Assuming that* $N \geq 3$*, then the following is true.*

(1) If $\delta < 1$*, the type 1 dealer has the better reservation price,* $p_1^r < p_2^2$*. Hence, the equilibrium price is given by (35) and the type 1 dealer receives the customer order.*

(2) If $\delta > 1$*, type 2 dealers have the better reservation price,* $p_1^r > p_2^2$*. Hence, the equilibrium price is given by (34) and a type 2 dealer receives the customer order.*

In addition, the gap between type 1 and type 2 dealers' reservation prices is increasing in the difference in the two types' risk parameter. That is, the gap in reservation prices is decreasing in δ for δ > 1 and increasing in δ for δ > 1. Note that the two types are identical when δ = 1.

4. Merger analysis

In order to analyse the effects of a merger between two banks on capital allocation and market making, we impose restrictions on the model in order to derive closed-form solutions with which we can perform numerical simulations.

It is assumed that before a merger, a bank engages in two of three available financial activities. As before, we assume that all banks are engaged in market-making. In addition, each bank chooses one of two activities, project X or project Y. Hence, banks can be differentiated according to whether they are engaged in project X (type X banks) or in project Y (type Y banks). However, we will impose symmetry between project X and Y so that both types of banks will end up allocating the same amount of risk capital towards market-making.

We will conduct the analysis of a merger assuming that prior to the merger, all *N* + 1 banks have the same capital allocation functions and hence have the same amount of risk capital allocated to market-making. This is the case of *N* + 1 dealers with identical risk preferences. A merger is then considered between two banks which results in (1) a reduction in the number of firms and hence dealers in the dealership market considered, and, potentially, (2) the creation of a new type of dealer with a different risk preference from all the other dealers, in which case we have a dealership market with heterogeneous dealers.

4.1 The stylised model

Prior to a merger, all banks consist of two divisions, or business lines. Type X banks are engaged in market-making and project X while type Y banks are engaged in market-making and project Y. We denote the variance and expected return from project X by σ_X^2 and $\mu_X(\sigma_X^2)$, from project Y by σ_Y^2 and $\mu_Y(\sigma_Y^2)$ and from market-making by σ_M^2 and $\mu_M(\sigma_M^2)$. We assume that the relationship between the expect return and variance of any particular project's cash flow, $\mu_j(\sigma_j^2), j \in \{X,Y,M\}$, is increasing and satisfies the Inada conditions. We further restrict the relationship between expected return and variance to be the same across projects X and Y.

$\mu_X(\sigma^2) = \mu_Y(\sigma^2) = \mu(\sigma^2)$

Finally, the correlation between cash flows from projects X and Y is denoted by $\rho_{XY} \in [-1,1]$, the correlation between cash flows from project X and market-making by $\rho \in [-1,1]$ and the correlation between cash flows from project Y and market-making by $\rho \in [-1,1]$.

The assumption of symmetry between activities X and Y implies that type X and type Y banks are identical from a risk-return perspective. Hence, each bank allocates the same amount of risk capital to market-making and this results in *N* + 1 identical dealers in the dealership market with risk aversion coefficient denoted by γ.

Recall from equation (6) that the first-order conditions for optimality for a bank engaging in set *K* of financial activities are

$$\frac{\partial \mu_j(\hat{\sigma}_j^2)}{\partial \sigma_j^2} = r\alpha \left[1 + \sum_{h \neq j, h \in K} \rho_{hj} \left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2} \right)^{1/2} \right], j \in K.$$

Specialising this condition for our two-division banks yields

$$\frac{\partial \mu\left(\hat{\sigma}_X^2\right)}{\partial \sigma_X^2} = r\alpha \left(1 + \rho \frac{\hat{\sigma}_M}{\hat{\sigma}_X} \right) \tag{37}$$

and

$$\frac{\partial \mu_M\left(\hat{\sigma}_M^2\right)}{\partial \sigma_M^2} = r\alpha \left(1 + \rho \frac{\hat{\sigma}_X}{\hat{\sigma}_M} \right) \tag{38}$$

for a type X bank. The conditions are identical for a type Y bank since $\hat{\sigma}_Y = \hat{\sigma}_X$ due to the symmetry assumptions.

4.2 Effect of a merger on capital allocation

In this section, we show that *a merger between two banks can have ambiguous effects on the risk-taking it undertakes in its various divisions*. The results depend on the correlation in cash flow among different divisions of the newly merged bank, and hence on the types of banks that participate in the merger.

4.2.1 Merger between two banks of the same type

Consider a merger between two type X (or type Y) banks. By assumption, there are no economies of scale to any of the banking activities considered. We proceed by assuming that the risk-return characteristics of all banking activities, represented by the functions $\mu(\cdot)$and $\mu_M(\cdot)$, are unchanged by the merger.[14] Since the merged bank remains engaged in the same two activities with the same risk-return characteristics as before the merger and there are no economies of scale present, the merged bank is identical to each of the banks prior to the merger. That is, the merged bank's optimisation and capital allocation problem is the same as before. The only change is that there are now N banks instead of N + 1. This will have an unambiguously negative impact on market quality in the dealership market considered since the decrease in the number of dealers results in less efficient risk-sharing than before and hence higher risk premiums charged by dealers, or higher spreads.

Recall that the pre-merger market spread is given by

$$s = \frac{2N-1}{N^2}\Psi|w|.$$

This is decreasing in N since

$$\frac{\partial s}{\partial N} = -\frac{2(2N-1)}{N^3}\Psi|w|.$$

4.2.2 Merger between two banks of different types

Consider a merger between a type X and a type Y bank. Assuming that the merged bank retains all three business activities, M, X and Y, this merged bank is now different from all the other banks in the economy. The merged bank's market-making activities are now carried out by a dealer who is potentially different from the rest of the dealers in the market. This dealer is denoted as a type 1 dealer who has a risk aversion coefficient denoted by $\delta\gamma$.

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[14] These are rather restrictive assumptions but they are made so as to focus attention on the economies of scope effects from a bank merger.

The risk associated with the merged bank's total cash flow is

$$\sigma_p^2 = \sigma_X^2 + \sigma_Y^2 + \sigma_M^2 + 2\rho_{XY}\sigma_X\sigma_Y + 2_{\rho}(\sigma_X + \sigma_Y)\sigma_M. \tag{39}$$

The first-order conditions (for an interior optimum) facing the merged bank are simply

$$\frac{\partial\mu_X(\tilde{\sigma}_X^2)}{\partial\sigma_X^2} - r\alpha\left(1 + \rho_{XY}\frac{\tilde{\sigma}_Y}{\tilde{\sigma}_X} + \rho\frac{\tilde{\sigma}_M}{\tilde{\sigma}_X}\right) = 0 \tag{40}$$

$$\frac{\partial\mu_Y(\tilde{\sigma}_Y^2)}{\partial\sigma_Y^2} - r\alpha\left(1 + \rho_{XY}\frac{\tilde{\sigma}_X}{\tilde{\sigma}_Y} + \rho\frac{\tilde{\sigma}_M}{\tilde{\sigma}_Y}\right) = 0 \tag{41}$$

$$\frac{\partial\mu_M(\tilde{\sigma}_M^2)}{\partial\sigma_M^2} - r\alpha\left(1 + \rho\ \frac{\tilde{\sigma}_X}{\tilde{\sigma}_M} + \rho\frac{\tilde{\sigma}_Y}{\tilde{\sigma}_M}\right) = 0. \tag{42}$$

The symmetry between activities X and Y implies that, at the optimum, $\tilde{\sigma}_X^2 = \tilde{\sigma}_Y^2$, where the ~ denotes optimal risk levels for the merged bank. The next proposition outlines two factors driving the change in risk-taking by the merged bank in each if its business lines.

Proposition 8. *Suppose that a unique solution exists to the capital allocation problem. Then, the merged bank tends to undertake more risk in projects X and Y relative to its pre-merger level* ($\tilde{\sigma}_X^2 > \hat{\sigma}_X^2$)

(1) The more negatively correlated are the cashflows of division X and the new division Y, and

(2) The more positively correlated are the cash flows from market-making and division X or Y if more risk is undertaken in market-making, or the more negatively correlated are the cash flows from market-making and division X or Y if less risk is undertaken in market-making after the merger.

The merged bank tends to undertake more risk in market-making ($\tilde{\sigma}_M^2 > \hat{\sigma}_M^2$)

(1) the more negatively correlated are the cash flows from market-making nd the new division, Y, and

(2) the more positively correlated are the cash flows from market-making and division X if more risk is undertaken in division X after the merger, or the more negatively correlated are the cash flows from market-making and division X if less risk is undertaken in project X after the merger.

Proof: See Appendix.

A lower (higher) level of risk-taking in any division corresponds to less (more) risk capital being allocated to that division by the merged bank. Finally, since the merged bank's market-making activities are carried out by a type 1 dealer with a coefficient of risk aversion given by $\delta\gamma$, the merged bank undertakes more (less) risk in market-making if and only if $\delta < 1$ ($\delta > 1$).

Numerical examples for a given function form for the expected return to each division illustrate the proposition. Suppose that the expected return to risk-taking in each division can be expressed as

$$\mu_X\left(\sigma_X^2\right) = a_X\left(\sigma_X^2\right)^{b_X},$$

$$\mu_M\left(\sigma_M^2\right) = a_M\left(\sigma_M^2\right)^{b_M}.$$

We find that the following is true.

(i) The merged firm undertakes more risk in market-making if and only if the cash flow from market-making is (strictly) negatively correlated with cash flows from the other divisions, X and Y. If that correlation is zero, there is no change in the level of risk-taking in market-making.

(ii) The merged bank undertakes more risk in division X if the cash flows from division X and Y are not too correlated.

Graphs from the numerical examples are presented in the Appendix.

4.3 Effect of a merger on market liquidity

Here, we show that the impact on market liquidity of a merger between two banks depends crucially on whether more or less risk capital is allocated to market-making by the newly merged bank. *If the newly merged bank allocates more capital to market-making, market liquidity can improve.* On the other hand, market liquidity will deteriorate if the newly merged bank allocates less capital to market-making.

Prior to the merger, the equilibrium price in the public market is given by (24), or

$$p^0 = \frac{(2N-1)\Psi}{2N^2} w.$$

After the merger, the equilibrium price that prevails depends on whether the type 1 dealer has a higher ($\delta > 1$) or a lower ($\delta < 1$) risk aversion. This depends on whether the merged firm allocates more or less risk capital to market-making. If more risk capital is allocated, the type 1 dealer has a higher risk-bearing capacity and hence $\delta < 1$. The reverse is true if the merged firm allocates less risk capital to market-making. From proposition 6, we know that if $\delta < 1$, the new equilibrium price in the public market is given by

$$p^m(\delta < 1) = \left\{ \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \frac{\beta_2^2 \Psi_2^2}{\bar{\beta}} + \frac{\beta_2}{2\bar{\beta}^2} \beta_1^2 \Psi_1^2 (2 - \beta_2 \Psi_2) - \frac{\Psi_2}{2} \left[1 - \left(1 - \frac{\beta_2}{\bar{\beta}}\right) \beta_2 \Psi_2 \right]^2 \right\} w \tag{43}$$

However, if $\delta > 1$, the new equilibrium price in the public market is given by

$$p^m(\delta > 1) = \left\{ \left(1 - \frac{\beta_1}{\bar{\beta}}\right) \frac{\beta_1^2 \Psi_1^2}{\bar{\beta}} + \frac{\beta_1}{2\bar{\beta}^2} \beta_2^2 \Psi_2^2 (2 - \beta_1 \Psi_1) - \frac{\Psi_1}{2} \left[1 - \left(1 - \frac{\beta_1}{\bar{\beta}}\right) \beta_1 \Psi_1 \right]^2 \right\} w. \tag{44}$$

Proposition 9. *Market liquidity improves, in the sense that market spreads are smaller, when δ is small enough, and the number of dealers in the market is large enough. That is, for values of $\delta < 1$ small enough, a merger improves liquidity for any number of dealers. For intermediate values of $\delta < 1$ a merger improves liquidity only if the number of dealers in the market is large enough. For $\delta > 1$ a merger always results in a deterioration of liquidity.*

Figures 2 to 4 illustrate the proposition. What the figures show is the following. For any $\delta < 1$ there is a critical N^* for which liquidity improves if $N > N^*$ while liquidity deteriorates if $N < N^*$. $N^*(\delta)$ is implicitly defined as the N that solves

$$p^0 - p^m(\delta) = 0 \tag{45}$$

where p^0 and $p^m(\delta)$ are defined by equations (24), (43) and (44). For small values of δ, this critical N^* is negative. For intermediate values of $\delta < 1$, this critical N^* becomes positive (and finite) and is increasing in δ. For $\delta > 1$, however, $N^* \to \infty$.

As stated at the beginning of this section, the merger that we consider between two banks has the following consequences: *(1) a reduction in the total number of dealers in the dealership market*, and *(2) the creation of a new type of dealer with a different risk preference from all the other dealers* (a type 1 dealer and N type 2 dealers). The first effect reduces the efficiency of risk-sharing among dealers in the market. As we have already argued, risk-sharing is inefficient in this market because of imperfect competition among dealers. That is, the efficiency of risk-sharing is increasing in the number of dealers in the market and tends towards first-best as the number of dealers tends towards infinity. Hence, a reduction in the number of dealers will have a negative impact on market prices and spreads.

The second effect induces a change in the risk-bearing capacity of the market since there is a change in the risk preference of one dealer, the newly created type 1 dealer. If the type 1 dealer has a larger capacity for bearing risk (that is, a lower risk aversion parameter or $\delta < 1$), the second consequence of a merger has a positive impact on market price and spreads. In this case, the net impact of a merger on market prices and spreads is ambiguous. If the type 1 dealer has a smaller capacity for bearing risk ($\delta > 1$), the second consequence of the merger has a negative impact on market prices and spreads. In this case, the net impact of a merger on prices and spreads is negative.

When the type 1 dealer has a larger risk-bearing capacity, $\delta < 1$, the two consequences of the merger as outlined have offsetting effects. The greater the increase in the type 1 dealer's ability to bear risk (or, the smaller δ), the more important is the impact of the increased risk-bearing capacity in the market. Moreover, the larger the number of dealers in the market to start with (N), the less important will be the reduction in the efficiency of risk-sharing from a merger. Hence for any $\delta < 1$, the larger N, the more likely is the merger to improve market liquidity. As well, for any N, the smaller $\delta < 1$, the more likely is the merger to improve market liquidity.

Figure 2

Equilibrium prices before and after a merger, $\delta < 1$

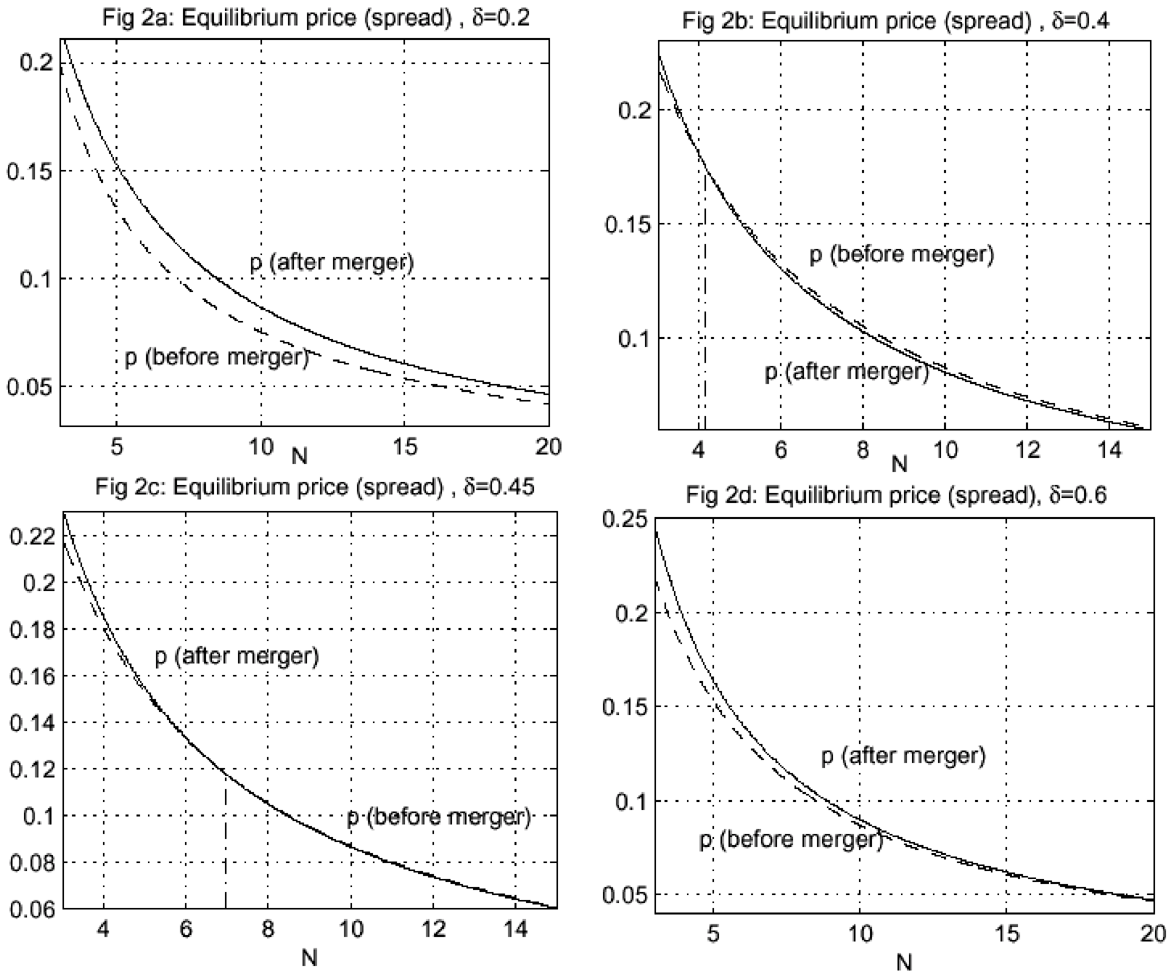


Figure 3

Equilibrium prices before and after a merger, $\delta > 1$

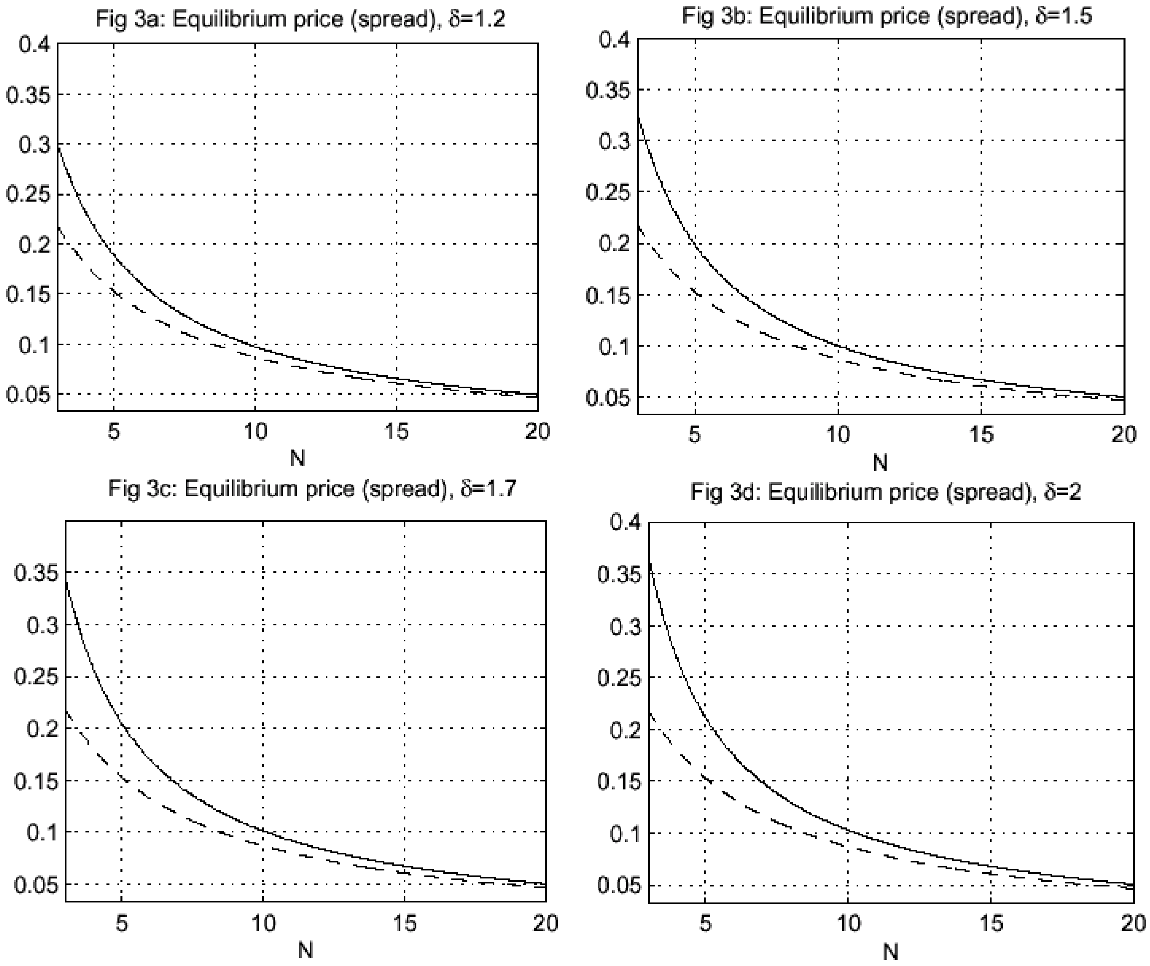
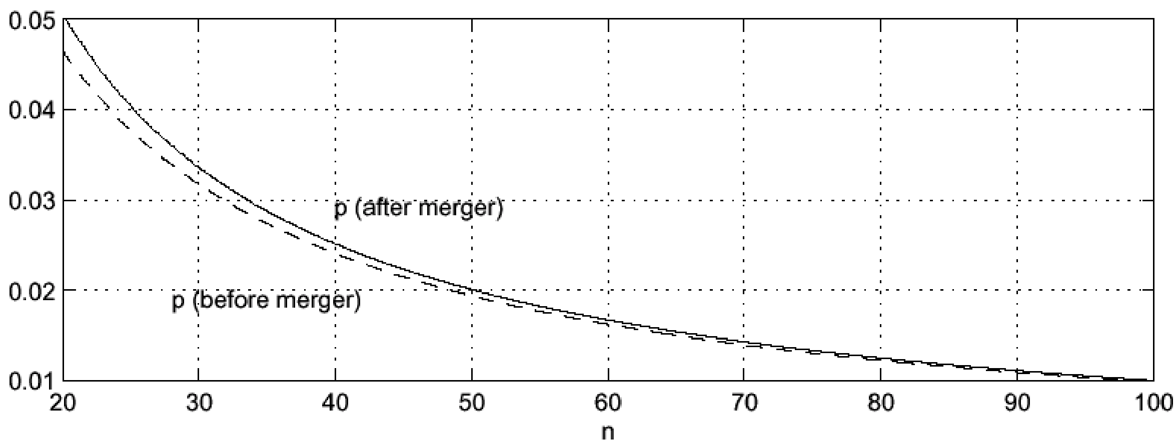


Figure 4

Equilibrium price (spread), $\delta = 2$



5. Conclusion

The paper bridges two topics in financial economics that until now have evolved as separate areas of research: market microstructure models and capital allocation decision-making within financial institutions. This is a first step towards consolidating advances made in the individual fields of study and will be a useful framework for understand how financial institutions and markets are interrelated through the interaction of risk management of institutions and the risk-bearing capacity of markets.

Although there are many possible applications for the framework we introduce, we focus in this paper on the impact of financial market consolidation on liquidity in dealership markets. Liquidity is characterised by bid-ask spreads in a model of inter-dealer trading that has been extended to allow for heterogeneity among dealers. The impact on market-making behaviour from a change in the allocation of capital across bank divisions is explicitly modelled so that we are able to characterise the potential effects of financial market consolidation on dealership markets such as foreign exchange and government securities markets.

We find that a merger of two banks can lead to increased market liquidity in dealership markets, even in highly concentrated markets, if the merger results in a sufficient increase in the risk-bearing capacity of the market. The risk-bearing capacity of the dealership market in turn depends on how the capital allocation decision in a financial institution is affected by the merger. This depends on the correlation among cash flows from business activities that the newly merged financial institution will engage in. A negative correlation between market-making and the new activities of a merged firm suggests the possibility of increased market liquidity. Our results suggest that, when faced with a proposed merger between financial institutions, policymakers and regulators would want to examine the correlations among division cash flows.

Appendix

A.1 Proof of proposition 1

The first-order condition (for an interior optimum) to division *j*'s problem is

$$\frac{\partial \mu_j\left(\sigma_j^2\right)}{\partial \sigma_j^2} = r\,\frac{\partial T_j\left(\sigma_j^2\right)}{\partial \sigma_j^2}, j \in K. \tag{46}$$

In order to induce the first-best choice of σ_j^2 at the division level, the bank's risk manager has to choose a capital allocation function T_j so that, at the optimal risk level for the division,

$$\frac{\partial T_j\left(\hat{\sigma}_j^2\right)}{\partial \sigma_j^2} = \alpha\left[1 + \sum_{h \neq j, h \in K} \rho_{hj}\left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2}\right)^{1/2}\right]$$

Integrating over σ_j^2 yields the optimal capital allocation functions

$$T_j\left(\sigma_j^2\right) = \alpha\left[1 + \sum_{h \neq j, h \in K} \rho_{hj}\left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2}\right)^{1/2}\right]\sigma_j^2, j \in K. \tag{47}$$

The salary component of the division manager's compensation, S_j, is chosen so that the manager obtains at least his reservation utility, $\underline{U}_j$,

$$S_j = \underline{U}_j - \tau\left[\mu\left(\hat{\sigma}_j^2\right) - rT_j\left(\hat{\sigma}_j^2\right)\right].$$

If we define γ_j such that

$$\gamma_j = r\alpha\left[1 + \sum_{h \neq j, h \in K} \rho_{hj}\left(\frac{\hat{\sigma}_h^2}{\hat{\sigma}_j^2}\right)^{1/2}\right], j \in K,$$

we can rewrite the objective function of the manager of division *j* as

$$\max_{\sigma_j^2} \mu_i\left(\sigma_j\right) - \frac{\gamma_j}{2}\sigma_j^2. \tag{48}$$

Hence, the optimal capital allocation function induces the (otherwise risk-neutral) manager of division *j* to behave like a risk-averse agent with exponential utility function and risk-aversion parameter γ_j with net payoff $\mu_j\left(\sigma_j^2\right)$.[15]

A.2 Proof of proposition 2

Each dealer takes into account the effect his trade has on the equilibrium inter-dealer price when determining his trading strategy. A dealer's strategy is an excess demand function. These functions are communicated to the inter-dealer broker who chooses a market-clearing price. By changing the excess demand function he sends to the inter-dealer broker, a dealer changes the equilibrium price. Therefore, each dealer has an incentive to restrict the quantity he trades in comparison with the competitive level, since he is trading against an upward-sloping residual supply curve, much like a monopolist.

Consider the problem of a dealer *i*. Market clearing implies that $x_i + \Sigma_{j \neq i}\alpha - \beta p_d + \mu w_j = 0$, or

$$P_d = \frac{x_i + N\alpha + \mu\sum_{j\neq i} w_j}{N\beta}. \tag{49}$$

The assumption that the market-clearing price and the investor's order do not convey any information and are normally distributed random variables along with an exponential utility implies that maximising expected utility of profits is equivalent to maximising the certainty equivalent, given by $E[\pi_i] - \frac{\Psi}{2}\mathrm{var}(\pi_i)$ or,

$$(x_i - w_i)E\,[v] + pw_i - p_d x_i - \frac{\Psi}{2}(x_i - w_i)^2. \tag{50}$$

The first-order condition with respect to X_i yields

$$-p_d\frac{\partial p_d}{\partial x_i}x_i - \Psi(x_i - w_i) = -p_d - \frac{x_i}{N\beta} - \Psi(x_i - w_i) = 0$$

or

$$x_i = \frac{N\beta(-p_d + \Psi w_i)}{1 + \Psi N\beta} \tag{51}$$

Equating coefficients yields the desired results.

A.3 Proof of proposition 8

The pre-merger problem for a bank is the following.

$$\max_{\sigma_M^2,\,\sigma_X^2} \mu_M + \left(\sigma_M^2\right) + \mu_X\left(\sigma_X^2\right) - r\alpha\left[\sigma_M^2 + \sigma_X^2 + 2\rho\sigma_M\sigma_X\right]. \tag{52}$$

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[15] For a risk-averse manager with risk-aversion parameter λ who maximises $U_j - \frac{\lambda}{2}var(U_j) \equiv \mu_j - rT_j - \frac{\tau\lambda}{2}\sigma_j^2$. The capital allocation function is $T_j(\sigma_j^2) = \left\{\alpha\left[1 + \sum_{h\neq j,\ h\in K}\rho_{hj}\left(\frac{\sigma_h^2}{\sigma_j^2}\right)^{1/2}\right] + \frac{\tau\lambda}{2}\right\}\sigma_h^2$.

The first-order conditions are

$$\mu_M'\left(\hat{\sigma}_M^2\right)=r\alpha\left[1+\rho\frac{\hat{\sigma}_X}{\hat{\sigma}_M}\right] \tag{53}$$

$$\mu_X'\left(\hat{\sigma}_X^2\right)=r\alpha\left[1+\rho\frac{\hat{\sigma}_M}{\hat{\sigma}_X}\right]. \tag{54}$$

Note that $\mu_M\left(\sigma_M^2\right)$ is a function of σ_M^2 only and $\mu_X\left(\sigma_X^2\right)$ is a function of σ_X^2 only.

Now, we turn to the merged bank's problem.

$$\max_{\sigma_M^2,\sigma_X^2,\sigma_Y^2} \mu_M\left(\sigma_M^2\right)+\mu_X\left(\sigma_X^2\right)+\mu_Y\left(\sigma_Y^2\right)-r\alpha\left[\sigma_M^2+\sigma_X^2+\sigma_Y^2+2\rho\sigma_M\left(\sigma_X+\sigma_Y\right)+2\rho_{XY}\sigma_X\sigma_Y\right]. \tag{55}$$

Taking derivatives with respect to σ_M^2, σ_X^2, and σ_Y^2 yields

$$\mu_M'\left(\sigma_M^2\right)-r\alpha\left[1+\frac{\rho\sigma_X+\rho\sigma_Y}{\sigma_M}\right]\equiv D_M\left(\sigma_M^2\middle|\sigma_X^2,\sigma_Y^2\right) \tag{56}$$

$$\mu_X'\left(\sigma_X^2\right)-r\alpha\left[1+\frac{\rho_{XY}\sigma_Y+\rho\sigma_M}{\sigma_X}\right]\equiv D_X\left(\sigma_X^2\middle|\sigma_M^2,\sigma_Y^2\right) \tag{57}$$

$$\mu_Y'\left(\sigma_Y^2\right)-r\alpha\left[1+\frac{\rho_{XY}\sigma_X+\rho\sigma_M}{\sigma_Y}\right]\equiv D_Y\left(\sigma_Y^2\middle|\sigma_M^2,\sigma_X^2\right) \tag{58}$$

Post-merger optimum $\tilde{\sigma}_M^2,\tilde{\sigma}_X^2=\tilde{\sigma}_Y^2$ is implicitly defined by

$$D_M\left(\tilde{\sigma}_M^2\middle|\tilde{\sigma}_X^2\right)=D_X\left(\tilde{\sigma}_X^2\middle|\tilde{\sigma}_M^2\right)=0. \tag{59}$$

Now, consider the function

$$D_M\left(z\middle|\tilde{\sigma}_X^2\right)=\mu_M'(z)-r\alpha\left[1+\frac{\rho\tilde{\sigma}x+\rho\tilde{\sigma}y}{z^{1/2}}\right]. \tag{60}$$

Notice we have taken $\tilde{\sigma}_X=\tilde{\sigma}_Y$ as (fixed) parameters. This defines a function in z that we know is decreasing in z (from the existence and uniqueness of the solution to the merged bank's problem). We also know that evaluated at $z=\tilde{\sigma}_M^2$ the function is equal to zero.

Therefore, for a value $z = z'$, $D_M\left(z'\middle|\tilde{\sigma}_X^2\right)<0$ implies that $z'>\tilde{\sigma}_M^2$ and $D_M\left(z'\middle|\tilde{\sigma}_X^2\right)>0$ implies that $z'<\tilde{\sigma}_M^2$. Since we are interested in comparing $\hat{\sigma}_M^2$ with $\hat{\sigma}_M^2$, we want to know whether the function $D_M\left(z\middle|\tilde{\sigma}_X^2\right)$ evaluated at $z=\hat{\sigma}_M^2$ is positive or negative.

$$D_M\left(\hat{\sigma}_M^2\middle|\tilde{\sigma}_X^2\right)=\mu_M'\left(\hat{\sigma}_M^2\right)-r\alpha\left[1+\frac{\rho\tilde{\sigma}_X+\rho\tilde{\sigma}_Y}{\hat{\sigma}_M}\right]. \tag{61}$$

From equation (2), we can substitute $r\alpha\left[1+\rho\frac{\hat{\sigma}_X}{\hat{\sigma}_M}\right]$ for $\mu_M'\left(\hat{\sigma}_M^2\right)$. (Remember that $\mu_M'\left(\hat{\sigma}_M^2\right)$ does not depend on σ_X^2 or σ_Y^2.) So the above works out to

$$D_M\left(\hat{\sigma}_M^2\middle|\tilde{\sigma}_X^2\right)=-r\alpha\left[\frac{\rho\left(\tilde{\sigma}_X-\hat{\sigma}_X\right)+\rho\tilde{\sigma}_Y}{\hat{\sigma}_M}\right]. \tag{62}$$

The same can be done for project X to obtain

$$D_X\left(\hat{\sigma}_X^2 \middle| \tilde{\sigma}_M^2\right) = -r\alpha\left[\frac{\rho_{XY}\tilde{\sigma}_Y + \rho\left(\tilde{\sigma}_M - \tilde{\sigma}_M\right)}{\hat{\sigma}_X}\right]. \tag{63}$$

Expression (63) is more likely to be positive if ρ_{XY} is negative and large in absolute value. That is, *post-merger risk-taking in division X is influenced by the correlation of its cash flow with division Y's cash flow.* In addition, it is also more likely to be positive when ρ is negative if $\tilde{\sigma}_M - \hat{\sigma}_M > 0$ and vice versa. That is, *post-merger risk-taking in division X is also influenced by the correlation of its cash flow with market-making and by what happens to the risk-taking level in market-making as a result of the merger*. The net effect on σ_X^2 of the merger, of course, depends on the combination of the two effects. The same analysis can be made for market-making: expression (62) is positive if $\rho\left(2\tilde{\sigma}_X - \hat{\sigma}_X\right) < 0$ and negative if the reverse is true.

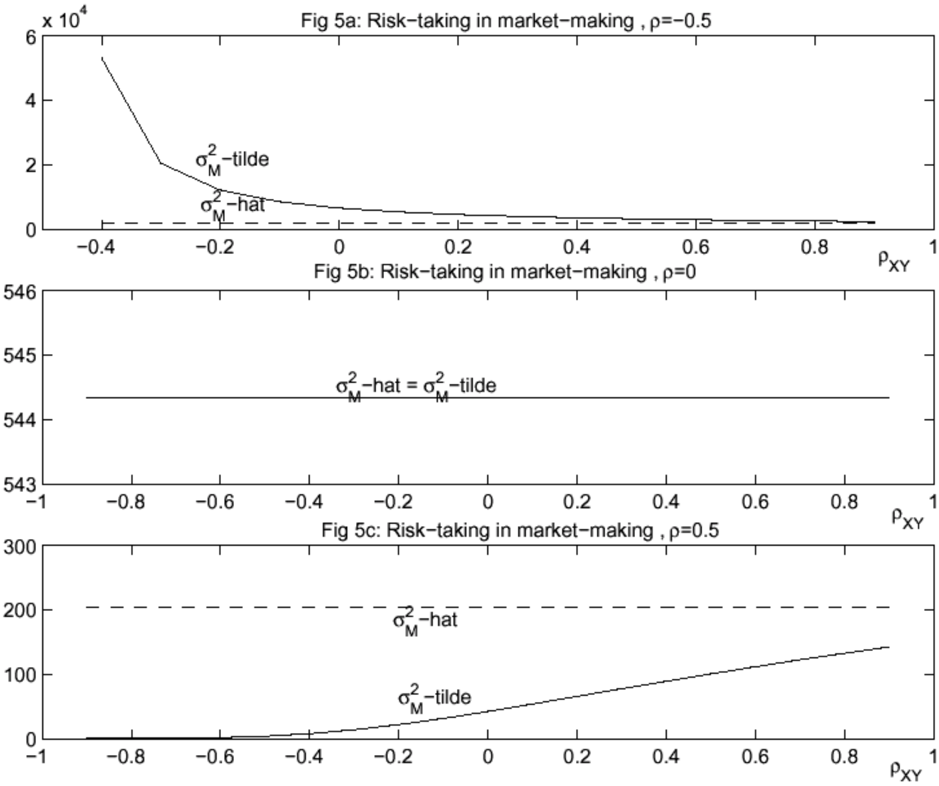
A.4 Numerical examples

For return functions characterised by

$$\mu_X\left(\sigma_x^2\right) = a_X\left(\sigma_X^2\right)^{b_X},$$

$$\mu_M\left(\sigma_M^2\right) = a_M\left(\sigma_M^2\right)^{b_M},$$

we obtain the following results with the parameter values: a_X = 1.5, a_M =1, b_X = b_M = 1/3, r = 0.1, and a = 0.05. Changing the parameters does not affect the results.



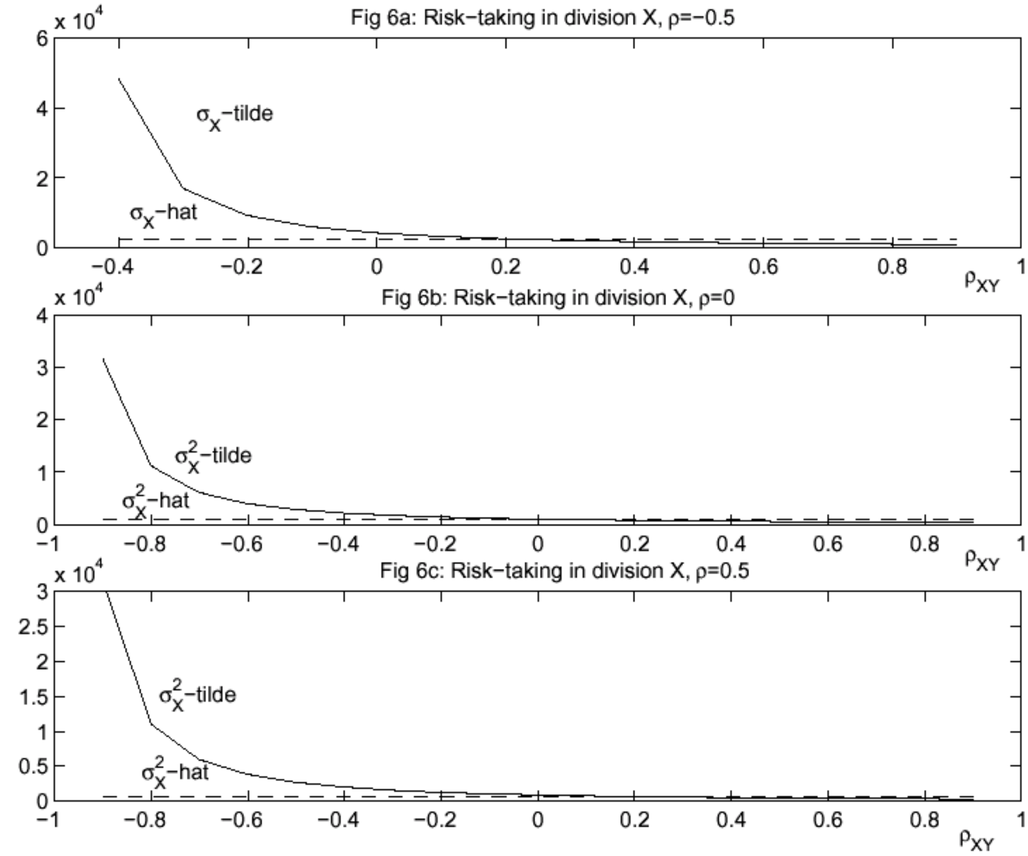


Fig 6a: Risk−taking in division X, ρ=−0.5
x 10^4
σX−tilde
σX−hat
ρXY
Fig 6b: Risk−taking in division X, ρ=0
σ2X−tilde
σ2X−hat
Fig 6c: Risk−taking in division X, ρ=0.5

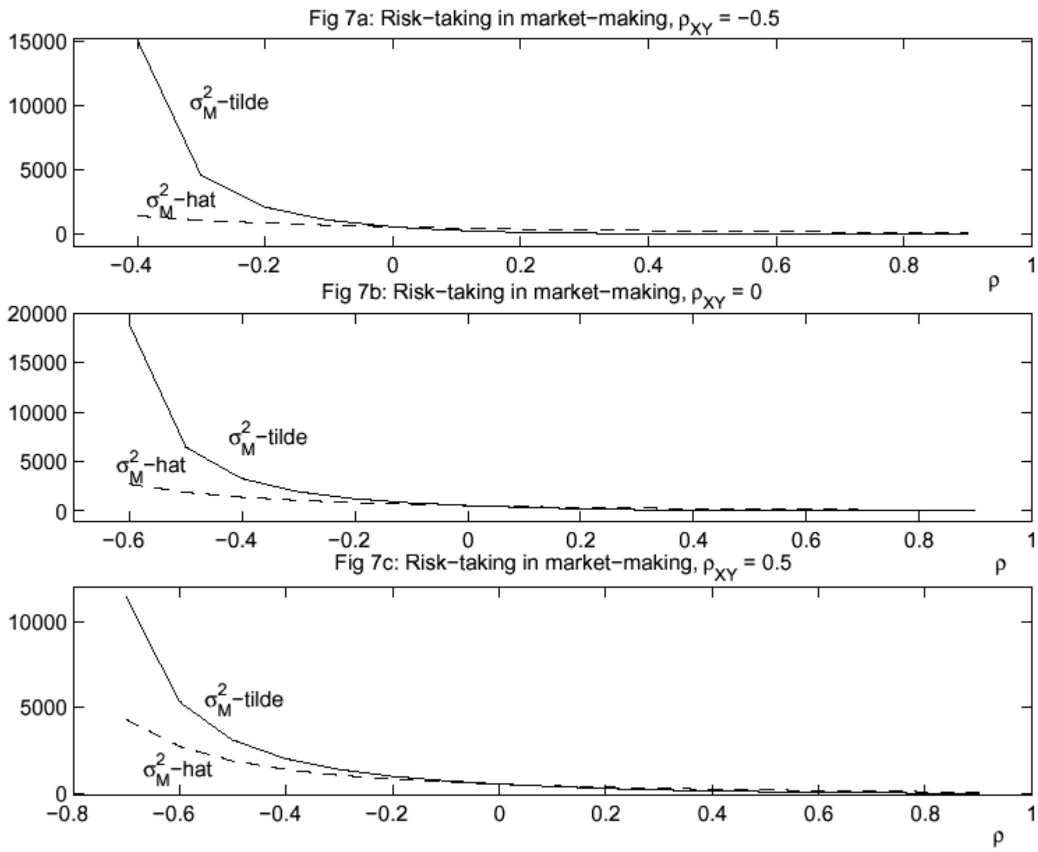
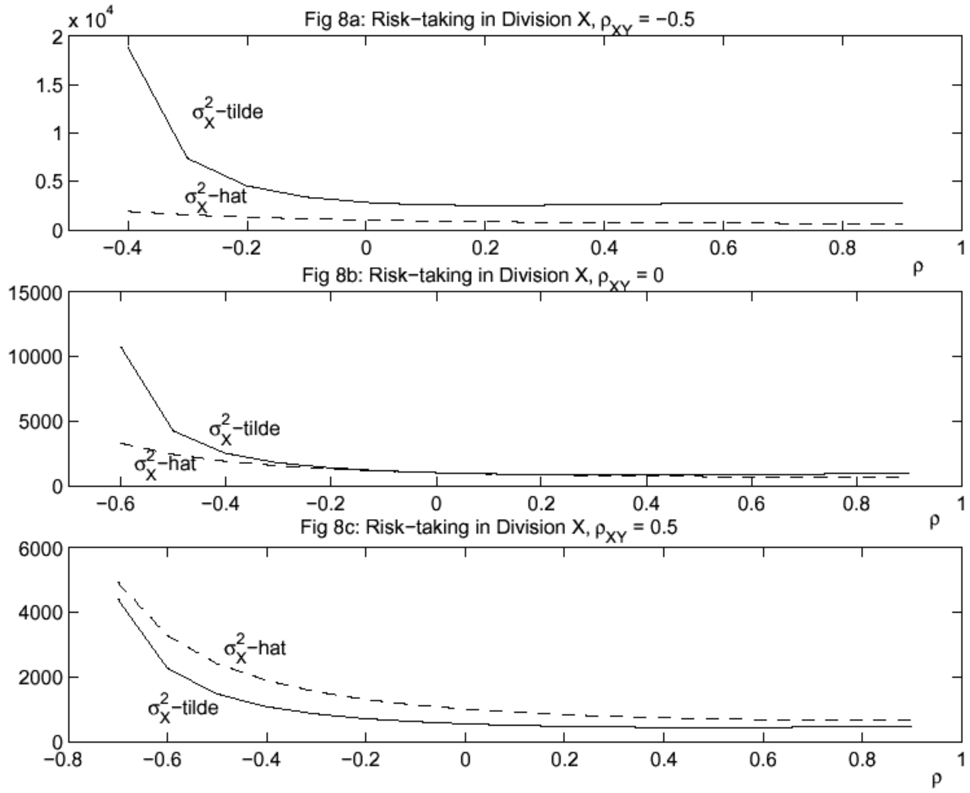


Fig 7a: Risk−taking in market−making, ρXY = −0.5
σ2M−tilde
σ2M−hat
ρ
Fig 7b: Risk−taking in market−making, ρXY = 0
Fig 7c: Risk−taking in market−making, ρXY = 0.5



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