Estimating liquidity premia in the Spanish Government securities market[1]

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Abstract

This paper investigates the presence of liquidity premia in the relative pricing of assets traded on the Spanish government securities market. Firstly we propose a classification of bonds consisting on four categories that identify groups of bonds with a different degree of liquidity. Secondly we estimate liquidity premia, including liquidity parameters in the estimation of the zero-coupon yield curve. The results suggest the existence of a significantly higher yield (adjusted for differences in cash flows) for non-strippable bonds that mostly reflects their lower degree of liquidity. Secondly, post-benchmark bonds display a positive liquidity premium over benchmark issues. Thirdly, the lack of liquidity of pre-benchmark bonds does not seem to be priced. We also show that these pricing discrepancies are robust to the impact of taxes on bonds. The size of relative liquidity premia seems relatively small.

1. Introduction

The liquidity of an asset is generally understood as the ease of its conversion into money.[2] In practice, the conversion of an asset into money involves certain costs: searching costs, delays, broker's commissions, etc. The higher these costs the lower the degree of liquidity of the asset. In most financial markets there is a class of agents known as market makers whose function is to provide liquidity. These agents are ready to buy and sell securities up to a maximum amount and make their profit on the difference between the bid and ask prices. These latter are the prices at which the other market participants can execute surely and immediately sell and buy transactions, respectively. As a consequence, the bid-ask spread reflects the cost incurred by a typical investor to unwind an asset position, which is part of the cost of converting an asset into money. This is why the bid-ask spread is one of the most widely used measures of liquidity.

According to market microstructure models, the bid-ask spread may reflect three different costs faced by market-makers: asymmetric information costs, inventory costs, and order processing costs. However, as Gravelle (BIS, 1999) points out, asymmetric information costs should not be very significant in the case of government securities (GS). This implies that liquidity in the GS markets should be closely linked to the market-makers' inventory risk and order processing costs which ultimately depend on the level of risk of the asset (duration) and the frequency with which a transaction will be executed (turnover). Sometimes, liquidity is measured by some indicators of market activity (turnover, turnover ratio, benchmark status, age).

Because investors value asset liquidity we can expect liquidity and differences in liquidity to be priced. In other words, investors may require a *liquidity premium* for holding illiquid assets in order to compensate them for bearing higher transaction costs. In the literature, there are some papers that test for the existence of liquidity premia in securities markets. The bulk of this literature focuses on equity markets,[3] whereas there are only a few papers that focus on debt markets, most of them using US market data.

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[2] See *Market liquidity: Research Findings and Selected Policy Implications* in BIS (1999) for the various dimensions of liquidity.

[3] See, for example Amihud and Mendelson (1986,1989).

Papers that test for the existence of a liquidity premium in the GS markets have followed a number of approaches. For example, in Amihud and Mendelson (1991), Kamara (1994) and Garbade (1996) the liquidity premium is only estimated for short-term US Treasuries. They compare the yield of notes with that of bills with the same term to maturity. Since both instruments are identical, except for the fact that notes are more actively traded, yield differences are attributed to differences in liquidity. The liquidity premia in these papers are found to be high and significant. Elton and Green (1998) use volume as a proxy for liquidity and introduce it as an explanatory variable when fitting a zero-coupon curve to the US market. They find that the coefficient of this variable is significant for most of the days considered, although the implied liquidity premium is very small. Warga (1992) proxies liquidity by indicating whether or not an issue is on-the-run (the most recently issued security of a particular maturity). He compares the ex-post monthly excess return over the 30-day Treasury bill rate for two series of constant duration portfolios made up of securities traded on the US market, one containing on-the-run bonds and the other containing the other available bonds. Warga (1992) finds that the portfolios having the on-the-run bonds exhibit a lower return and interprets this as evidence of a liquidity premium. In other papers, like Shen and Starr (1998), liquidity is proxied by the bid-ask spread. In that paper the term premium, proxied by the excess ex-post returns of the six-month US T-bill over the 3-month bill, is estimated and it is found that the bid-ask spread accounts for a substantial portion of this premium.

The estimation of liquidity premia in GS markets is important because among other reasons, it might improve the information content of prices. For example, the existence of liquidity premia may distort the information extracted from the estimated term structure or the estimates of implied inflation expectations obtained by comparing fixed-coupon and inflation linked bonds.

Against this background, the main goal of this paper is to investigate whether there is a liquidity premium in the relative pricing of assets traded on the Spanish GS market. To do this, we first characterise the relative liquidity of bonds. The strippability of the asset and its benchmark status appear to be two relevant determinants of securities' liquidity within each maturity zone. Given this property, we consider four categories of bonds that take into account these elements. The categories are non-strippable bonds and pre-benchmark (bonds that will have the benchmark status in the future), benchmark and post-benchmark (bonds that had the benchmark status in the past) strippable bonds. We find that there are important liquidity differences among these categories of bonds according to different measures of liquidity based on activity measures and bid-ask spread. Benchmark bonds are the most liquid, followed by strippable non-benchmark bonds. And finally non-strippable bonds appear to be very illiquid.

In the second part of the paper we estimate relative liquidity premia. A traditional approach consists on calculating yield spread between non-benchmark and benchmark bonds. However, this estimation of the liquidity premia do not control for risk and tax factors derived from different cash flow structures of bonds. Our methodology follows that of Elton and Green (1998) which is based on the estimation of the term structure of interest rate. This approach allows us a better control of effects related to the cash flow structure of bonds. Concretely, we incorporate liquidity effects in the estimation of zero-coupon yield curve introducing dummy variables for the different categories of bonds. The results suggest the existence of a significantly higher yield, adjusted for differences in cash flows, for non-strippable bonds that seems to mostly reflect their lower degree of liquidity. Second, post-benchmark bonds display a positive liquidity premium over benchmark issues. Third, the lack of liquidity of pre-benchmark bonds does not seem to be priced. Even, in some periods, we find evidence of a negative premium over benchmark issues for pre-benchmark bonds. We also show that these pricing discrepancies are robust to the impact of taxes on bonds. Thus, these results point to the existence of a liquidity premium in the relative pricing of bonds traded on the Spanish GS market, although its size seems relatively small.

The remainder of the paper is organised as follows. Section 2 describes the structure of the Spanish GS market. Section 3 describes the data. Section 4 proposes a classification of bonds which tries to identify bonds with a different degree of liquidity. Section 5 estimates liquidity premia. And, finally Section 6 summarises the main conclusions.

2. Structure of the Spanish government securities market

Two types of instruments are issued by the Spanish Treasury: *Letras del Tesoro* (Treasury bills), which are short-term securities issued at a discount, and *Bonos y Obligaciones del Estado* (State bonds) which are medium and long-term securities with annual coupon payments. Both types of instruments are represented by book entries and issued via regular competitive auctions. Bills are issued at 6-, 12- and 18-month maturities, whereas bonds are issued at 3-, 5-, 10-, 15- and 30-year maturities. Auctions take place on a monthly basis, except for 30-year issues, which are auctioned every two months, and 12- and 18-month issues, which are auctioned fortnightly. In the case of medium and long-term securities issues are reopened over several consecutive auctions until the outstanding amount reaches a minimum level .The securities allocated at such auctions have identical nominal coupon and interest payment and redemption dates.

Since July 1997, the Spanish Treasury has been issuing strippable bonds. They enjoy a more favourable tax treatment for payers of the corporate income tax because the latter are not subject to withholding tax on coupon payments. These securities can be stripped during their life into n+1 zero -coupon assets, where n stands for the number of remaining coupon payments, arising from the cash flow generated by the bond's coupons and principal. The stripping process is conducted by a type of market participant, known as market makers, who assume a number of commitments subject to annual review. They are also allowed to conduct the reverse process (reconstitution). All outstanding bonds issued before July 1997 are non-strippable and do not enjoy the favourable tax treatment of strippable bonds.

Secondary market trades are conducted through three systems, the first two being reserved for market members, while the third is for transactions between market members and their clients. In the first system (known as a blind market) trading is electronically conducted without knowledge of the counterparty's identity. Only those market members who comply with certain requirements and assume a number of commitments can participate in this market segment. For instance, they are obliged to quote during at least 60% of the time of each session the five references of bonds with benchmark status subject to a maximum bid-ask spread. Blind market trades can only be outright transactions, whether spot or forward.

The second trading system (known as second tier) channels all the remaining transactions between market members. Trading is conducted directly between traders or through brokers. Some brokers post indicative bid and ask prices on electronic market information systems such as Reuters and Bloomberg. In this market segment, participants can trade outright (in spot or forward transactions) or enter into repos. Two types of repo transactions are allowed: ordinary repos and blocked repos, the difference being the fact that under the second type the buyer cannot transact freely with the securities purchased, regardless of the buy-back date set.

Clearing and settlement of GS transactions is carried by the *Central de Anotaciones en Cuenta* (Book-Entry System). Each market member holds an account in this system. Individuals or institutions who are not members have to channel transactions through a Managing Institution. The procedure used is one of cash on delivery and transactions are settled three business days later.

In addition to the secondary market, there is a futures and options market (MEFF RF) where the underlying assets are Spanish GS. At present traded contracts include 5- and 10-year futures and options and 30-year futures.

Table 1 provides turnover figures for 1999 for the different market segments and for the different types of transactions on the Spanish GS market. Repos are the type of transaction with the highest activity, which is mainly concentrated in the very short-term (mainly overnight), followed by outright spot purchases. By contrast, trading activity through forward, futures, and options transactions, is more limited.

Table 2 gives a breakdown of the holders of Spanish GS at end-1999. Financial institutions own a very large proportion of the outstanding amount (61.2% for bonds and 91.3% for bills). Non-residents also have a significant share of the market in the case of bonds (32%). By contrast, the share of non-financial companies and households is very low.

3. Data

In this paper we use daily data of prices (quoted and traded), outstanding amounts, and trading activity (trading volume and number of trades) for the Spanish GS market from January 1999 to

April 2000. This information is collected for 34 bonds,[4] 24 of them being outstanding through out the sample period (see Table 3 for a description of the issues). For the short-term, the data refer to Treasury bills and repo transactions.

The price data include the following:

(i) Daily mean prices traded in the blind and second tier markets. This information was provided by the Central de Anotaciones del Banco de España.

(ii) Daily quoted bid and ask prices collected from two sources:

   (a) Reuters-BDE: since August 1998 the Banco de España Research Department has been building a database from prices posted by Reuters at 2 p.m. It includes best quoted bid and ask prices for long and short-term issues; and bid and ask rates for repo transactions. Bid and ask daily prices are calculated using the quotations of four brokers: three[5] of them are pre-determined and the fourth is the one who traded most recently. From these quotations, the highest bid and the lowest ask prices are selected for each issue. If the bid price is higher than the ask,[6] then prices are sequentially removed until a non-negative spread is obtained.

   (b) Bloomberg: this agency provides daily quoted prices calculated as the average bid and ask quotations at the close (determined by Bloomberg to be about 5 p.m.). In this case, by construction, the bid-ask spread is always positive. Bloomberg also provides bid and ask rates for a wide range of repo transactions.

It is worth noting that these databases of quoted prices use different methodologies to calculate daily bid-ask spreads for each instrument. The Reuters-BDE spread is the best proxy for market spread, but it employs non-contemporaneous quotations that may introduce noise and may bias downwards the size of the spread. On the other hand, the Bloomberg spread cannot be interpreted as the market spread, but its main advantage is that it reduces non-contemporaneity problems due to the fact that quotes are averaged. Both the Reuters-BDE and Bloomberg databases suffer from the shortcoming that quotations are indicative, not firm, and therefore may not reflect actual prices. This problem arises from the specific microstructure of this market and means that it is desirable to use traded prices as complementary information.

Trading activity (number of trades and trading volume) is provided by Central de Anotaciones del Banco de España and refers only to transactions between market members, in both, the blind and the second tier market.

4. Classifying bonds by their relative degree of liquidity

4.1. A classification of bonds according to their life cycle and tax treatment

In this section we propose a classification of bonds that tries to identify a number of bonds' groups with a different degree of liquidity. It is important to stress that the approach we follow in this paper of classifying bonds by their relative liquidity is not intended to compare liquidity across different maturity zones but within them. This decision is justified by the methodology we use in the next section to estimate liquidity premia, which consists of fitting a zero-coupon yield curve. With this methodology it would be very difficult to separate term and liquidity effects if we used a classification of liquidity across different maturities.

Our classification takes into account two elements of an issue: its stage of the life cycle and its strippability (whether or not the issue is strippable). We consider the stage of the life because, in general, trading activity and, consequently, liquidity is related to this. It is well-known that in GS markets trading activity tends to be concentrated in a small group of securities known as *benchmark*

[4] Strips are not included. The percentage of stripped bonds is very low in the Spanish market.

[5] Four in the case of short-term instruments.

[6] In the information provided by Reuters it is not uncommon to find negative bid-ask spreads, possibly due to the lack of contemporaneity in quotations.

issues. The prices of these instruments are used for extracting information for macroeconomic analysis and pricing purposes. In the Spanish GS market there are five benchmark issues corresponding to 3-, 5-, 10-, 15- and 30-year maturities. Since all issued bonds become benchmark issues, the life cycle of bonds consists of three stages. In the first stage, the bond is initially issued and its outstanding volume is relatively small, and henceforth, its trading activity is very low and does not have benchmark status. Henceforth bonds in this stage will be referred to as *pre-benchmark* issues. New fungible auctions increase the outstanding amount, and trading activity also increases until benchmark status is achieved and the issue becomes the most liquid one for a particular maturity. Finally, in the third stage, the bond is replaced by a new benchmark and its activity begins to decrease. Bonds in this stage will be referred to as *post-benchmark* issues.

More specifically, our classification consists on four categories, namely: pre-benchmark, benchmark, post-benchmark and non-strippable issues. The first three categories are only made up of strippable bonds, whereas the last category includes all non-strippable bonds. We have decided to group all non-strippable bonds in one specific category for the following two reasons. Firstly, in our sample, all non-strippable bonds are at a very advanced third stage of their life cycle, therefore they are very illiquid. And, secondly they are subject to a less favourable tax treatment that could be reflected in a tax premium.

Over our sample period there was no official classification of benchmark issues in Spain. As a consequence, the identification of the benchmark issue may be difficult during transition periods. The criterion we use in this paper considers that a new bond becomes the benchmark issue when it has been traded more than the old benchmark for at least three consecutive days. Table 4 gives the dates on which a benchmark bond changes according to this criterion, and compares them with those reported by Reuters and Bloomberg. It can be observed that dates do not coincide for the different criteria, but differences between them rarely exceed one month.

Figure 1 shows the life cycle for all the strippable 10-year bonds of our sample. Over this period two replacements take place, one in February 1999 when the 5.15% bond becomes the new benchmark. The other takes place in October 1999, where the most recently issued bond (4% with maturity at 31/1/10) acquires the benchmark status. This figure displays the evolution of trading activity, outstanding volume and bid-ask spread. It can be seen that the life cycle in trading activity appears also to be incorporated in the evolution of bid-ask spreads, suggesting a very different degree of liquidity of bonds depending on the stage of life they are at.

Table 5 gives a descriptive analysis of previously defined bond categories over our sample period. It can be seen that benchmark issues account, on average, for almost 53% of the daily trading volume (see panel c). Regarding non-strippable bonds, despite the high number of issues (20 issues at the beginning of the period and 16 at the end), they account for just 7% of daily trading volume (see panel c). Finally, the remaining 40% of the trading volume corresponds to strippable non-benchmark bonds - of which pre-benchmark issues account for a 15% and post-benchmark issues for a 25%. There are 4 such bonds at the beginning of the period increasing to 9 by April 2000.

Looking at trading activity by maturity, a high concentration around the 10-year zone is observed, with approximately 33% of the total market activity. The 3- and 5-year maturities account for approximately 25% and 23% respectively. And finally, the zones of 15- and 30-year residual maturity have the lowest activity, with approximately 5% and 3% of total market activity respectively.

Regarding outstanding volume, panel b of Table 5 shows a very different distribution. In particular, a high concentration (more than 50%) is observed in the 0-6 years zone.

4.2. Is this classification useful to identify groups of bonds with a different degree of liquidity?

We now show to what extent the four categories we propose are useful to identify groups of bonds with a different degree of liquidity. To do this we compute some liquidity measures for the different categories of bonds.

Table 6 reports various measures of liquidity, all based on trading activity, with a breakdown by bond category and maturity. These measures are the number of trades, trading volume and turnover ratio. All these indicators are computed per bond and, therefore, can be interpreted as liquidity measures. A feature shared by all the liquidity measures of Table 6 is the remarkable differences in liquidity between the various categories considered.

When comparing liquidity within each maturity zone benchmark issues are the bonds with the greatest liquidity, followed by strippable non-benchmark bonds (for both pre-benchmark[7] and post-benchmark). Finally, non-strippable bonds are notable for their very scarce trading activity.

When comparing liquidity among maturity zones the most liquid bonds are those with a residual maturity between 3 and 10 years. In this regard, it is worth noting that strippable non-benchmark bonds are sometimes even more liquid than 15 and 30-year benchmark issues.

Figure 2 displays the bid-ask spread[8] as a function of the duration for the different categories of bonds. This indicator is used as an additional measure of liquidity. A similar pattern to that reported for trading activity also appears here when comparing the liquidity of bonds within each maturity zone (proxied by duration), although now pre-benchmark issues appear to be more similar, in terms of liquidity, to non-strippable bonds than to post-benchmark strippable issues.

However, note that trading activity and the bid-ask spread would provide a very different ranking of bonds by liquidity if we did not control for time to maturity or duration. As Figure 2 illustrates, the bid-ask spread is positively correlated with time to maturity. This effect clearly dominates the previously reported pattern (benchmark status of the bond). This relationship between spread and time to maturity, which has also been identified in other GS markets,[9] can theoretically be explained by the market-makers' inventory risk and order processing costs which ultimately depend on the level of risk of the asset proxied by time to maturity. Accordingly, a ranking of bonds using the bid-ask spread would classify short-term securities as the most liquid and long-term issues as relatively illiquid. Conversely, a ranking of bonds by the liquidity measures of Table 6 would classify the 10-year bonds as the most liquid issues.

It is worth noting that our classification of bonds can be justified either by the bid-ask spread or by the trading activity measures because, as was mentioned before, we do not intend to compare liquidity across different maturity zones but within them.

Three conclusions may be drawn from the previous analysis. Firstly, the benchmark status and strippability of bonds are two relevant determinants of the liquidity differences within a maturity zone for the bonds in our sample. Secondly, the four categories we consider seem useful to identify groups of bonds with a different degree of liquidity. And, finally there seems to be strong differences in liquidity between bonds, so that it makes sense to test for the existence of a liquidity premium. This is the aim of the next section of the paper.

5. Estimating liquidity premia

5.1 Preliminary analysis

Our aim in this second part of the paper is to find out if differences in liquidity are priced in the Spanish GS market. One simple approach is to approximate the liquidity premium by the yield spread between non-benchmark and benchmark bonds. Applying this measure to the 3,5, 10 and 15-year bonds of our sample always gives a negative figure suggesting the existence of negative liquidity premia. However, it has to be taken into account that some factors may bias this measure. Differences in coupons between bonds imply different risk and tax burdens that may make the investor to demand different yields to maturity for other reasons not related to liquidity effects. The main difficulty when identifying the liquidity premium is the separation of the factors explaining differences in yields to maturity. With this in mind, our proposal is based on the estimation of the term structure of interest rate, because this allows us to control any effect associated with differences in maturity and cash flows. This approach is similar to that followed by Elton and Green (1998).

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7 Although in the case of the turnover ratio no significant differences are observed between pre-benchmark and benchmark issues.

8 For this analysis we use Reuters-BDE database because it provides a spread measure more closely linked to the concept of market spread (see Section 3 for a description of this database and a comparison with the Bloomberg data).

9 BIS (1999) compares bid-ask spreads for "on-the-run" and "off-the-run" issues for Canada, Italy, Japan, UK and US. In all cases they find the same results as ours, except for Japan, where the liquidity of on-the-run 10-year bonds makes their spread narrower than the one for "on-the-run" 5-year bonds. (See Table 3 of the document *Market Liquidity: Research Findings and Selected Policy Implications*)

According to the analysis developed in Section 4, we distinguish four types of bonds in relation to their life cycle and tax-treatment: non-strippable bonds, pre-benchmark, post-benchmark, and benchmark strippable bonds.

As a preliminary analysis of the existence of liquidity premia we estimate a zero-coupon yield curve using the Svensson model[10] and analyse yield errors for bonds. We find a very interesting pattern. Figure 3 shows the estimation for a representative day of the sample, (29/3/99) and Table 7 reports average yield errors for each bond in our sample distinguishing its classification over the sample. The first thing we observe is that most non-strippable bonds appear to have on average positive yield errors, i.e. they are located above the estimated yield curve. The second point to be stressed is the tendency of pre-benchmark bonds to exhibit a lower yield error than benchmark bonds. Finally, the yield error of post-benchmark bonds lies between yield errors of benchmark and non-strippable bonds. These results suggest a preliminary evidence of a positive premium for post-benchmark and non-strippable bonds over benchmark issues. Next section tries to estimate them using a more formal approach.

5.2 Estimating the yield curve incorporating liquidity effects

Previous results indicate that the estimation of the yield curve for the Spanish GS market may be improved if we introduce liquidity and withholding tax affects. Our proposal consists of estimating the term structure of interest rates using the Svensson model, and introducing additional parameters to capture these effects. Elton and Green (1998) follow a similar procedure[11] introducing volume as a measure of liquidity. Given that the relationship between traded volume and liquidity may not be linear, we prefer using dummy variables to classify securities according to their degree of liquidity and tax treatment.

The instantaneous forward rate of term *m* is modelled in the following way:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_0 PREBENCH + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

where *PREBENCH*, *POSTBENCH* and *NONSTRIP* are dummy variables which take value 1 respectively for pre-benchmark, post-benchmark and non-strippable bonds. Parameters for dummy variables must be interpreted in relation to the remaining instruments (3, 5, 10, 15 and 30-year benchmark bonds and short-run securities).

Parameters γ_0, γ_1 and γ_2 capture the excess yield demanded on pre-benchmark, post-benchmark and non-strippable bonds respectively. Parameters γ_0 and γ_1 can be directly interpreted as liquidity premia since the tax treatment is the same for all strippable issues. Conversely, parameter γ_2 may incorporate a tax premium in addition to a liquidity premium due to the relatively less favourable tax treatment of non-strippable bonds. The interpretation of remaining parameters is the standard in the Svensson model.

Once we have modelled the instantaneous forward rate including withholding tax and liquidity effects, we estimate the parameter vector $\beta = (\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2, \gamma_0, \gamma_1, \gamma_2)$ for each day *t* as

$$\hat{\beta} = \operatorname{argmin} \sum_{i=1}^{N} \left[\frac{p_{ti} - p_{ti}(\beta)}{p_{ti}} \right]^2$$

where p_{ti} is the actual price of bond *i* at time *t* and N is the number of securities considered in the estimation. And $p_{ti}(\beta)$ is the theoretical price obtained from previous specification of the

10 See Annex 1 for a description of this model. We use the mid-point between the bid and ask price provided by Bloomberg database. Bonds with a remaining life of under a year are excluded from the estimation. For short-term we use repo and bill rates.

11 They use non-linear least squares to fit a cubic spline to the after-tax cash flows of bonds.

instantaneous forward rate. We minimise price errors instead of yield errors to obtain a better fit for the maturities we are most interested in, i.e. medium and long-term.

We distinguish four nested models. The basic model, called Model 1, does not consider any liquidity or tax effects, i.e. γ_0, γ_1 and γ_2 are restricted to zero. In Model 2 all these parameters are freely estimated. Model 3 eliminates the dummy variable for pre-benchmark bonds. And finally, Model 4 estimates freely the parameter of the dummy variable for non-strippable bonds, and restricts to zero the parameters of other dummy variables.

We estimate these four models for the whole sample, from January 1999 to April 2000. Over the period 4/1/99 to 30/9/99 the Svensson model does not improve on the Nelson-Siegel model[12] and consequently, we estimate the latter model for this period. Table 8[13] reports a descriptive analysis of the estimated parameters for dummy variables, the root mean square error (RMSE), and the reduction in the RMSE with respect to the basic model.

When all the three dummy parameters are freely estimated (Model 2) the average RMSE is 63% less than that of the basic model. The estimated parameter for the non-strippable bonds dummy is positive and significant for almost the whole sample, indicating that these bonds incorporate a premium over the benchmark issues that ranges from 2 to 10 basis points in our sample. On average, the excess yield demanded on these bonds is 7 basis points.

Regarding post-benchmark bonds, they also appear to incorporate a premium. It takes the value of 5 basis points on average, but in this case there are some periods where the parameter of this variable is not positive and statistically significant. Figure 4 shows the evolution of estimated parameters, $\gamma_0, \gamma_1, \gamma_2$. Shaded areas indicate that γ_2 is not statistically significant. These areas coincide with transition periods in benchmark bonds. During January-February 1999 it occurs a change in 3, 5 and 10-year benchmark bonds. During October-December 1999 there is a change of all benchmark bonds except for 30-year term. Replacements take place gradually in the markets and during these transition periods frontiers between strippable categories are blurred given the small number of bonds in each one. The approach we follow changes the category of a bond in a specific date (see table 4) generating sharp movements in the estimations of premium parameters. The sensitivity of our estimations to benchmark changes does not allow interpreting the evolution of the parameters during transition periods. In any case, results when the categories of bonds are enough differentiated show that post-benchmark bonds include a liquidity premium over benchmark issues.

Regarding the parameter of pre-benchmark bonds, it turns out to be negative on average, suggesting the existence of a negative premium for these bonds over benchmark issues. However, some care must be taken in the interpretation of this parameter. It is negatively significant just for 54% of the sample, and for some periods there is just one bond included in this category. The conclusion drawn from the analysis of this parameter is that pre-benchmark bonds do not seem to include any positive liquidity premium in spite of its low liquidity in terms of trading activity and bid-ask spreads. This result may possibly be explained by the forward-looking behaviour of investors. It is costly to trade with these securities, but investors do not demand a higher yield because, in the future, these costs will be much lower when the bond acquires benchmark status. In addition, the limited supply of these issues may contribute to sustain its price.

Given that pre-benchmark bonds do not appear to have a positive liquidity premium over benchmark issues, and given that there are not many pre-benchmark bonds in our sample we proceed to group pre-benchmark and benchmark bonds into the same category. Model 3 includes only two dummy variables to capture liquidity differences, one for post-benchmark and the other for non-strippable bonds. The reduction of RMSE with respect to the basic model continues to be very high (51%). Non-strippable bonds appear to have a premium of 8 basis points on average, ranging from 5 to 12 basis points. This parameter is statistically significant for the whole sample. For post-benchmark bonds, parameter γ_1, is positive and significant for 76% of the sample, showing on average a liquidity

[12] This model is a particular case of the Svensson model, where the instantaneous forward rate is modelled with just one hump, i.e. β_3 and τ_2 are set to zero and one respectively.

[13] We concentrate on the interpretation of parameters γ_0, γ_1, γ_2, which will be reported in different tables. The remaining parameters of the Svensson and Nelson-Siegel model are presented in Table 12 for all models estimated throughout the paper.

premium of 5 basis points. Therefore, inclusion of pre-benchmark bonds with benchmark bonds increases the size of the premia.

Finally, Model 4 includes just one dummy variable for non-strippable bonds. It allows an average reduction of 37% in the RMSE with respect to the basic model. In this model the average premium for non-strippable bonds over the remaining bonds is 6 basis points. The size of this parameter is lower than in Model 3 because in this case the group of bonds excluded from dummy variables includes post-benchmark issues.

Summing up, the results of this section suggest the existence of a positive liquidity premium for post-benchmark issues over benchmark bonds, although its size is very small - similar to that found by Elton and Green (1998) in the US market. Conversely, the lack of liquidity of pre-benchmark issues does not seem to be priced and, even a negative premium over benchmark issues is found in some periods. Finally, there is clear evidence of a positive premium for non-strippable bonds in the whole sample. However, as noted before, this premium could not be interpreted directly as a liquidity premium since the excess yield demanded on non-strippable bonds may also include a withholding tax premium. In the next section we try to separate these two components.

5.3 Separation of withholding tax and liquidity effects for non-strippable bonds

Previous results show strong evidence of a premium for non-strippable bonds. This premium may be due to liquidity or withholding tax effects. In order to separate these two elements we re-estimate Model 3 taking into account the withholding tax (Model 5). To do this we modify the cash flows of non-strippable bonds in the following way. We reduce the amount of coupon payments by 18% (withholding tax rate during the sample period) and include a new cash flow stream corresponding to the compensation for the withholding tax. The dates we assign to these cash flows are selected to coincide with the quarterly tax returns that Spanish corporations are obliged to make. It has to be noted that this procedure considers the maximum cost faced by non-strippable bond holders since they can reduce it through *coupon washing transactions*.[14] Consequently, the estimated premium for non-strippable bonds can be interpreted as a lower bound for the true liquidity premium.

Table 9 reports the estimated parameters, which are comparable with those for Model 3. This model reduces the premium for non-strippable bonds by only 1.2 basis points, in comparison with Model 3, whereas the premium for post-benchmark bonds does not change significantly. These results suggest that most of the estimated premia for non-strippable bonds in Models 2 to 4 correspond to liquidity premia. According to the results of Table 9, the average liquidity premium for non-strippable bonds is at least 6.8 basis points. This minimum liquidity premium for non-strippable bonds is still higher, on average terms, than the one found for post-benchmark bonds. This fact is logical given their strong differences of liquidity.

5.4 Robustness to tax effects

The above results may be sensitive to including general tax effects in the estimation of the zero-coupon yield curve. That is, they may be affected by the estimation of a pre-tax instead of a post-tax yield curve. It is very difficult to estimate a post-tax yield curve, because tax treatment is very different across investors. In this section we estimate a post-tax yield curve considering the tax treatment for Spanish corporations, which hold two thirds of the outstanding volume of bonds (see Table 2).

The theoretical price is expressed as the present value of after-tax cash flows imposing a tax rate of 35% for both interest payment and capital gains. Capital gains or losses at redemption date are distributed over the life of the bond using a constant yield method, and assuming that the bond is held until maturity.

Table 10 shows the main results of the re-estimation of Model 3 using after-tax flows (Model 6). It can be seen that in this new model the sign and significance of estimated premia remain unchanged, but their absolute size is reduced by around 35%. The parameters that capture the asymptotic value of the

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[14] These transactions consist of selling non-strippable bonds to non-resident investors, who are not subject to withholding tax, before the date of the coupon and buying them again after that date.

instantaneous forward rate ($\beta_0 = \varphi_\infty$) and the instantaneous forward rate for an infinitely small term to maturity ($\beta_0 + \beta_1 = \varphi_0$) are also reduced by around 35%. Consequently, the liquidity premia expressed as a percentage of these interest rates remain almost unchanged (see Table 11). These results indicate that previously estimated relative liquidity premia are robust to tax effects.

Finally, it is interesting to see that the model with taxes further reduces the RMSE. In average the reduction over the basic model is 77%, which compares with the 51% reduction of the model with two dummies without taxes (model 3). This result suggests the relevance of introducing tax effects when fitting zero-coupon yield curves.

5.5 Impact of liquidity premia on the estimated yield curve

In this subsection we study the impact of the introduction of liquidity effects on the estimated zero-coupon yield curve. To do this we compare the estimated parameters for the different models we have considered in subsections 5.2 to 5.4.

Table 12 reports the mean and the standard deviation of the basic parameters of the term structure.[15] The mean of the parameters' estimates for the models that introduce liquidity effects without taxes (Models 2 to 5) are very close to those of the basic model (Model 1). This is not a surprising result given the relatively small liquidity premia we have found. Conversely, when taxes are introduced (Model 6) some parameters change significantly. β_0 and β_1 are now around 35% (the tax rate) below the estimated level for Model 3, whereas the changes in the other parameters are less dramatic. These changes imply that after-tax zero-coupon rates are around 35% below before-tax rates.

Figure 5 depicts the zero-coupon yield curve at 29/3/99 for two of the models we have estimated, using the Nelson-Siegel approach. The models we consider in this figure are: the basic model (Model 1), and the model that introduces dummy variables for both post-benchmark and non-strippable bonds (Model 3). The yield curve of the model that accounts for liquidity (Model 3) stands slightly above (with a maximum difference of 9 basis points) that of the basic model for short-term maturities and slightly below (with a maximum difference of 12 basis points) for medium-term horizons. For long-term maturities both models display very similar rates.

Figure 6 shows the estimated 1-year forward curves implied by the yield curves depicted in Figure 5. Estimated forward rates for horizons shorter than 6 years are slightly lower for the model that accounts for liquidity effects, whereas the opposite is the case for longer horizons.

Figures 7 and 8 are the same as Figures 5 and 6, except for the fact that they are estimated at 31/3/00 using the Svensson model. In this case, the zero-coupon rates are lower for the model that introduces liquidity effects in comparison with the basic model, for all horizons. It is worth noting that for the medium and long-term horizons the difference between the two curves is smaller than that found with Figure 5. This is not surprising given the reduction in the estimated liquidity premia in the second half of the sample.

The introduction of liquidity effects has allowed us to identify liquidity premia and to reduce the error in the estimation of the zero-coupon yield curve. However, the implications for information content of the estimated yield curve are not very important. This is an expected result given the small size of the estimated liquidity premium.

5.6 Biases in quoted prices

The results of the previous analysis have been derived from the quoted prices posted by brokers in Bloomberg. We know that these prices are not firm, but merely informative and, consequently, we do not know how accurate they are. The existence of a bias in quoted prices may have an impact on the liquidity premia we have estimated. In this subsection we study the existence of biases in quoted prices in comparison with traded prices and derive conclusions on their impact on the estimated liquidity premia. To do this, for each day and security we compute the difference in yield between the mean traded price and the mid-quoted price. Table 13 shows the average of these yield differences for

[15] From 4/01/99 to 30/9/99 we estimate Nelson-Siegel model, and from 1/10/99 to 14/4/00 Svensson model.

each security, distinguishing its classification along the sample in the four categories we have considered along this paper (pre-benchmark, benchmark, post-benchmark and non-strippable).

Three important features emerge from Table 13. First, average yield differences are very small for all securities (between 0 and 2.3 basis points), most of them being not significantly different from zero. Second, yield differences tend to be positive for most bonds (those figures statistically different from zero are all positive). Third, yield differences tend to be higher for pre-benchmark and non-strippable bonds.

These results suggest the existence of a small negative bias in quoted prices in comparison with traded prices, which seems to be more important for pre-benchmark and non-strippable issues. This would mean that there is a negative bias in the estimated liquidity premia for non-strippable and pre-benchmark bonds. That is, the actual liquidity premium for non-strippable bonds is higher than the estimated, whereas the absolute value of the liquidity premium for pre-benchmark issues is lower. However, these biases in the liquidity premia should be relatively small given the very small size of the bias in quoted prices. Consequently, the reported results of sections 5.1 to 5.5 seem to be robust to the data set we have used.

6. Conclusions

The analysis developed in the first part of the paper has showed that there are important liquidity differences among securities traded in the Spanish GS market. The strippability of the asset –strippable bonds are the most liquid- and the benchmark status –benchmark bonds are more liquid- appear to be two relevant determinants of the securities' liquidity within each maturity zone.

The second part of the paper analyses the presence of liquidity premia in the relative pricing of assets traded in the Spanish GS market. The estimation is carried out introducing liquidity parameters in the Svensson model of the zero-coupon yield curve. These liquidity parameters allow us to estimate the excess yield of pre-benchmark, post-benchmark and non-strippable bonds over benchmark issues. This methodology improves the traditional approach of the estimation of liquidity premium – which consists on computing the yield spread between non-benchmark and benchmark bonds - because it allows an appropriate control of effects associated with differences in maturity and cash flows.

The main conclusions that can be drawn from the estimations made are the following. Firstly, results suggest the existence of a positive and significant premium for post-benchmark bonds (both strippable and non-strippable). However, these premia are very small and similar to those found by Elton and Green (1998) in the US market. Secondly, the lack of liquidity of pre-benchmark bonds does not seem to be priced in the market. Even, in some periods, a negative premium over benchmark issues arises for these bonds. This is a somewhat surprising result that can possibly be explained by the forward-looking behaviour of market participants. Thirdly, estimated liquidity premia seem robust to tax effects and to the data set we have used (quoted prices instead of actual traded prices). Finally, regarding the implications for the estimated zero-coupon yield curve, the introduction of liquidity effects do not have significant effects due to small size of the estimated liquidity premia.

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Annex

The Svensson model

Svensson (1994) specifies a smooth parametric function for the yield curve. The functional form for the instantaneous forward rate of term *m*, φ_m , is the following:

$$\varphi_m(\beta) = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} \qquad \text{(A1)}$$

where

$\beta = (\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2)$ is the vector of parameters.

m is the term to maturity.

$\beta_0 = \varphi_\infty$ is the asymptotic value of the instantaneous forward rate.

$\beta_0 + \beta_1 = \varphi_0$ is the instantaneous forward rate for an infinitely small term to maturity.

τ_1 indicates the position of the first internal maximum or minimum.

β_2 determines the magnitude and direction of the first hump. If $\beta_2 > 0$, there is a maximum at τ_1, and a minimum if $\beta_2 < 0$.

τ_2 indicates the position of the second internal maximum or minimum.

β_3 determines the magnitude and direction of the second hump. If $\beta_3 > 0$, there is a maximum at τ_2, and a minimum if $\beta_3 < 0$.

The zero-coupon rate is the mean of integration of the instantaneous forward rate between period 0 and the term to maturity, $r_m = \frac{1}{m}\int_0^m \varphi_\theta \, d\theta$:

$$r_m(\beta) = \beta_0 + (\beta_1 + \beta_2)\frac{\tau_1}{m}(1 - e^{-\frac{m}{\tau_1}}) - \beta_2 e^{-\frac{m}{\tau_1}} + \beta_3 \frac{\tau_2}{m}(1 - e^{-\frac{m}{\tau_2}}) - \beta_3 e^{-\frac{m}{\tau_2}} \qquad \text{(A2)}$$

The continuously compounded discount factor is $d(m) = e^{-mr_m}$:

$$d(m,\beta) = e^{-\beta_0 m - (\beta_1+\beta_2)\tau_1(1-e^{-\frac{m}{\tau_1}}) + \beta_2 m e^{-\frac{m}{\tau_1}} - \beta_3\tau_2(1-e^{-\frac{m}{\tau_2}}) + \beta_3 m e^{-\frac{m}{\tau_2}}} \qquad \text{(A3)}$$

The theoretical price of bond *i* at time *t* is the present value of its cash flows:

$$p_{it}(\beta) = \sum_{j=t}^{M} d(j-t,\beta) f_{ij} \qquad \text{(A4)}$$

where f_{ij} represents the coupon and principal payments of bond *i* at date *j*, and *M* is the redemption date.

The parameter vector is estimated by the minimisation of some distance between the theoretical and actual prices of the bonds. Sometimes the minimisation problem is specified in terms of yield errors instead of price errors, or some combination of both. Another possibility is estimation by maximum likelihood.

Table 1

Turnover in the Spanish government securities market

1999, EUR millions

	Total	Between market members	Through registered dealers
Outright spot purchases	1,817,465	498,958	1,318,507
Bills	79,839	31,609	48,230
Bonds	1,737,626	467,349	1,270,277
Repo transactions	11,254,309	2,785,567	8,468,742
Forward transactions	37,901	–	–
Futures and options transactions	356,146	–	–

Source: Banco de España

Table 2

Holders of Spanish government securities

End-1999, percentages

	Bills	Bonds
Banking sector	49.5	31.1
Other financial institutions	41.8	30.1
Mutual funds	35.8	16.8
Pension funds	0.5	2.9
Insurance companies	0.3	10.1
Other	5.2	0.3
Non-financial companies	1.7	3.8
Households	4.6	1.2
Non-residents	2.3	32.0
Other	0.1	1.8

Source: Banco de España.

Table 3
Bonds included in the sample

Coupon (%)	Issue Date	Maturity Date	Remaining life at 4/1/99 (years)
10.65	30/07/86	30/07/01	2.57
12.25	25/03/90	25/03/00	1.22
11.3	15/11/91	15/01/02	3.03
10.3	15/04/92	15/06/02	3.45
10.9	15/02/93	30/08/03	4.65
10.5	17/05/93	30/10/03	4.82
8.2	15/12/93	28/02/09	10.16
8.0	17/01/94	30/05/04	5.41
7.4	15/02/94	30/07/99	0.57
10.0	15/11/94	28/02/05	6.16
10.1	15/09/95	28/02/01	2.15
10.15	15/09/95	31/01/06	7.08
9.4	15/12/95	30/04/99	0.32
8.4	15/03/96	30/04/01	2.32
8.8	15/03/96	30/04/06	7.32
7.8	17/06/96	31/10/99	0.82
8.7	15/07/96	28/02/12	13.16
7.9	15/10/96	28/02/02	3.15
6.75	15/11/96	15/04/00	1.28
7.35	16/12/96	31/03/07	8.24
5.0	15/07/97	31/01/01	2.08
5.25	15/07/97	31/01/03	4.08
6.0	15/07/97	31/01/08	9.08
6.15	15/07/97	31/01/13	14.08
6.0	15/01/98	31/01/29	30.10
5.15	10/07/98	30/07/09	10.58
4.25	7/08/98	30/07/02	3.57
4.5	10/08/98	30/07/04	5.57
4.75	7/12/98	30/07/14	15.58
4.0	11/05/99	31/01/10	11.08
3.0	13/07/99	31/01/03	4.08
3.25	12/07/99	31/01/05	6.08
4.6	15/02/00	30/07/03	4.57
4.95	14/02/00	30/07/05	6.57

Table 4

Dates of change in benchmark bonds according to different criteria

Term	Benchmark		Criteria		
	Coupon (%)	Maturity date	Ours	Bloomberg	Reuters
3	5.00	31/1/01			
	4.25	30/7/02	28/1/99	12/1/99	2/2/99
	3.00	31/1/03	2/12/99	4/1/00	19/11/99
5	5.25	31/1/03			
	4.50	30/7/04	13/1/99	12/1/99	2/2/99
	3.25	31/1/05	2/12/99	4/1/00	19/11/99
10	6.00	31/1/08			
	5.15	30/7/09	26/2/99	12/1/99	2/2/99
	4.00	31/1/10	6/10/99	4/1/00	19/11/99
15[1]	6.15	31/1/13			
	4.75	30/7/14	2/12/99		
30[1]	6.00	31/1/29			

[1] Bloomberg and Reuters do not provide benchmark information on these terms.

Table 5

Main figures for Spanish GS bonds by categories and term to maturity

a) Number of issues

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark	total
0-2		0.9	5.9		6.9
2-4	1.0	1.5	4.8	1	8.3
4-6	0.5	0.3	3.1	1	4.9
6-9		0.9	3.1		4.0
9-11	0.4	0.4	0.8	1	2.6
11-16	0.7	0.3	1.0	1	3.0
16-30				1	1.0
Total	**2.6**	**4.4**	**18.7**	**5**	**30.7**

b) Total outstanding amount

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark	total
0-2		4.7%	13.4%		18.1%
2-4	1.2%	6.5%	8.6%	4.0%	20.4%
4-6	1.3%	4.6%	7.2%	4.1%	17.2%
6-9		7.9%	7.1%		15.0%
9-11	1.6%	5.6%	2.7%	4.7%	14.5%
11-16	1.5%	5.0%	0.8%	4.2%	11.6%
16-30				3.3%	3.3%
Total	**5.6%**	**34.3%**	**39.7%**	**20.3%**	**100.0%**

c) Aggregate trading volume

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark	total
0-2		4.5%	2.1%		6.6%
2-4	2.9%	6.8%	2.2%	12.7%	24.7%
4-6	3.5%	4.2%	0.7%	14.6%	23.0%
6-9		3.5%	1.7%		5.3%
9-11	7.6%	4.4%	0.1%	20.4%	32.6%
11-16	1.0%	1.4%	0.3%	2.6%	5.4%
16-30				2.5%	2.5%
Total	**15.0%**	**24.9%**	**7.3%**	**52.9%**	**100.0%**

d) Total number of trades

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark	total
0-2		3.4%	1.4%		4.8%
2-4	1.8%	6.5%	1.9%	12.5%	22.6%
4-6	3.5%	5.1%	0.8%	15.4%	24.8%
6-9		3.7%	1.7%		5.4%
9-11	6.2%	4.8%	0.1%	21.9%	33.0%
11-16	1.3%	1.4%	0.3%	3.2%	6.1%
16-30				3.2%	3.2%
Total	**12.8%**	**24.9%**	**6.2%**	**56.2%**	**100.0%**

For each day in the sample bonds are classified according to their residual life and category. Panel (a) reports the average number of bonds in each class. In the remaining panels, activity measures are added for the bonds included in each class. Values are expressed in percentage terms with respect to the whole market. Figures in this table are averages of the daily data from January 1999 to April 2000.

Table 6

Liquidity measures for the Spanish Government Bonds

a) Number of transactions per bond

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark
0-2		11.9	0.8	
2-4	4.5	17.4	1.4	43.9
4-6	6.3	9.5	0.9	54.3
6-9		13.2	1.9	
9-11	28.0	17.0	0.5	77.2
11-16	4.6	4.9	0.9	11.3
16-30				11.3
Total	**16.5**	**16.9**	**0.3**	**39.6**

b) Trading volume per bond (EUR millions)

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark
0-2		90.5	7.0	
2-4	50.0	106.4	9.5	258.2
4-6	49.5	55.0	5.3	295.9
6-9		72.0	11.3	
9-11	199.0	89.8	2.5	414.3
11-16	20.2	28.5	6.8	53.6
16-30				51.1
Total	**45.5**	**63.2**	**6.1**	**153.3**

c) Turnover ratio

residual life (years)	pre-benchmark	post-benchmark	non-strippable	benchmark
0-2		0.9%	0.1%	
2-4	2.1%	1.0%	0.2%	2.9%
4-6	2.5%	0.8%	0.1%	3.2%
6-9		0.4%	0.2%	
9-11	4.3%	0.7%	0.0%	4.0%
11-16	0.6%	0.3%	0.4%	0.6%
16-30				0.7%
Total	**2.4%**	**0.7%**	**0.2%**	**2.3%**

For each day in the sample bonds are classified according to their residual life and category. For each class we compute the average of the activity measures for the bonds included. Figures in this table are averages of the daily data from January 1999 to April 2000.

Table 7

Errors in the estimated zero-coupon yield curve

Issue			Average estimated yield errors (basis points)			
Coupon (%)	**Maturity Date**	**Average duration**	**Pre-bench**	**Benchmark**	**Post-bench**	**Non-strip**
12.25	25/3/00	1.02				22.9
6.75	15/4/00	1.09				20.5
10.1	28/2/01	1.47				3.1
5.0	31/1/01	1.49		0.6	– 1.0	
8.4	30/4/01	1.58				– 2.5
11.3	15/1/02	2.16				2.7
7.9	28/2/02	2.31				1.3
10.3	15/6/02	2.49				1.6
4.25	30/7/02	2.81	– 3.7	– 7.1		– 3.7
3.0	31/1/03	3.09	– 5.6	0.0		
5.25	31/1/03	3.17		– 7.8	– 3.8	
4.6	30/7/03	3.26	– 1.4			
10.9	30/8/03	3.39				0.9
10.5	30/10/03	3.53				0.8
8.0	30/5/04	4.05				– 2.2
10.0	28/2/05	4.46				2.5
4.5	30/7/04	4.52	–14.0	– 9.5	– 4.6	
3.25	31/1/05	4.86	– 4.9	– 3.4		
4.95	30/7/05	4.94	– 4.6			
10.15	31/1/06	5.05				5.2
8.8	30/4/06	5.28				2.7
7.35	31/3/07	6.01				3.2
6.0	31/1/08	6.77		3.0	3.9	
8.2	28/2/09	7.04				1.3
5.15	30/7/09	8.02	– 7.1	– 4.2		– 4.1
4.0	31/1/10	8.43	– 7.1	– 6.5		
8.7	28/2/12	8.44				5.4
6.15	31/1/13	9.48		2.1	2.2	
4.75	30/7/14	10.87	– 2.0	– 1.3		
6.0	31/1/29	14.57		0.0		
Average			**– 5.6**	**– 2.8**	**– 1.6**	**4.3**

Figures are average of daily errors. Zero coupon-yield curves are estimated with the Svensson model for the whole sample.

Table 8

Liquidity premia estimates

(basis points)

	Model 1 Basic model	Model 2			Model 3		Model 4
		γ_0	**γ_1**	**γ_2**	**γ_1**	**γ_2**	**γ_2**
Mean		– 3.1	4.6	6.9	5.3	8.0	6.3
Max		4.5	10.7	10.4	12.4	11.5	9.2
Min		–11.1	–6.0	2.5	– 3.7	5.1	3.4
Std Dev		4.0	3.5	1.7	3.9	1.6	1.5
% Signif (-)		54%	0%	0%	0%	0%	0%
2% Signif (+)		0%	72%	99%	76%	100%	100%
Mean RMSE	*1.61*	*0.59*			*0.79*		*1.02*
Mean error reduction		*63.35%*			*50.90%*		*36.60%*

Zero-coupon yield curves are estimated using Nelson-Siegel model until September 1999 and Svensson model for the remaining sample. Dummy variables are introduced to capture liquidity differences. The instantaneous forward rate of term *m* is modelled in the following way for Svensson model:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_0 PREBENCH + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

Model 1 (basic model) does not consider any liquidity effect i.e. γ_0, γ_1 and γ_2 are restricted to zero. Model 2 estimates freely all parameters. Model 3 eliminates the dummy variable for pre-benchmark bonds, γ_0. Model 4 estimates the parameter for non-strippable bonds, and restricts to zero the parameters of the other dummy variables. Premium figures are expressed in basis points. Mean error reduction refers to the reduction of RMSE with respect the basic model.

Table 9

Impact of withholding tax on liquidity premia estimates

(basis points)

	Model 5	
	γ_1	γ_2
Mean	5.4	6.8
Max	12.6	10.5
Min	– 3.4	3.8
Std Dev	4.0	1.7
% Signif (-)	0%	0%
% Signif (+)	78%	100%
Mean RMSE	0.82	
Mean error reduction	49.06%	

Zero-coupon yield curves are estimated using Nelson-Siegel model until September 1999 and Svensson model for the remaining sample. Dummy variables are introduced to capture liquidity differences. The instantaneous forward rate of term *m* is modelled in the following way for Svensson model:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

Model 5 is the same as Model 3 but taking into account withholding tax effects for non-strippable bonds. Liquidity premium figures are expressed in basis points. Mean error reduction refers to the reduction of RMSE with respect the basic model (Model 1).

Table 10

Impact of taxes on liquidity premia estimates

(basis points)

	Model 6	
	γ_1	γ_2
Mean	3.3	5.0
Max	8.1	7.4
Min	– 2.3	3.3
Std Dev	2.6	1.0
% Signif (-)	0%	0%
% Signif (+)	76%	100%
Mean RMSE	0.37	
Mean error reduction	77.01%	

Zero-coupon yield curves are estimated using Nelson-Siegel model until September 1999 and Svensson model for the remaining sample. Dummy variables are introduced to capture liquidity differences. The instantaneous forward rate of term *m* is modelled in the following way for Svensson model:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

Model 6 is the same as Model 3 but using after-tax cash flows. Liquidity premium figures are expressed in basis points. Mean error reduction refers to the reduction of RMSE with respect the basic model (Model 1).

Table 11

Relative liquidity premia with and without taxes

(percentage points)

	γ_1/β_0	γ_2/β_0	$\gamma_1/(\beta_0+\beta_1)$	$\gamma_2/(\beta_0+\beta_1)$
Model 3, before-tax yield curve	0.85	1.30	2.07	3.05
Model 6, after-tax yield curve	0.83	1.26	1.98	2.93

Table 12

Estimated parameters in different term structure models

		β_0		β_0		β_0		τ_0		β_0		τ_0	
		mean	**sd**	**mean**	**sd**	**mean**	**sd**	**mean**	**sd**	**mean**	**sd**	**mean**	**sd**
Model 1	4/1/99-30/9/99	6.23	0.18	–3.74	0.15	–0.66	1.16	4.89	2.29				
	1/10/99 - 14/4/00	6.42	0.23	–3.28	0.59	40.64	0.37	2.96	1.22	–40.42	0.36	3.03	1.27
Model 2	4/1/99-30/9/99	6.18	0.27	–3.60	0.33	–1.89	1.57	3.34	0.72				
	1/10/99 - 14/4/00	6.43	0.24	–3.42	0.64	40.72	1.15	2.33	0.43	–40.89	1.30	2.39	0.43
Model 3	4/1/99-30/9/99	6.29	0.20	–3.80	0.18	–1.22	1.46	4.42	1.97				
	1/10/99 - 14/4/00	6.42	0.24	–3.42	0.64	40.89	1.50	2.31	0.45	–41.06	1.64	2.36	44.32
Model 4	4/1/99-30/9/99	6.27	0.20	–3.79	0.18	–1.01	1.39	4.57	2.02				
	1/10/99 - 14/4/00	6.40	0.24	–3.36	0.65	40.62	1.02	2.31	0.74	–40.82	1.07	2.36	0.75
Model 5	4/1/99-30/9/99	6.29	0.20	–3.82	0.18	–1.18	1.47	4.47	1.99				
	1/10/99 - 14/4/00	6.42	0.24	–3.44	0.63	40.42	0.27	2.23	0.43	–40.63	0.27	2.28	0.43
Model 6	4/1/99-30/9/99	3.92	0.15	–2.28	0.14	–0.70	1.19	4.01	1.52				
	1/10/99 - 14/4/00	4.11	0.14	–2.16	0.40	40.46	0.16	2.06	0.40	–40.58	0.16	2.09	0.40

Zero-coupon yield curves are estimated using Nelson-Siegel model until September 1999 and Svensson model for the remaining sample. Dummy variables are introduced to capture liquidity differences. The instantaneous forward rate of term *m* is modelled in the following way for Svensson model:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_0 PREBENCH + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

Model 1 (basic model) does not consider any liquidity effect i.e. γ_0, γ_1 and γ_2 are restricted to zero. Model 2 estimates freely all parameters. Model 3 eliminates the dummy variable for pre-benchmark bonds, γ_0. Model 4 estimates the parameter for non-strippable bonds, and restricts to zero the parameters of the other dummy variables. Model 5 is the same as Model 3 but taking into account withholding tax effects for non-strippable bonds. Model 6 is the same as Model 3 for after-tax cash flows. Interest rates are expressed in percentage points.

Table 13

Biases in quoted prices

Issue			Average differences in yield (basis points)			
Coupon (%)	**Maturity Date**	**Average duration**	**Pre-bench**	**Benchmark**	**Post-bench**	**Non-strip**
12.25	25/3/00	1.02				1.0
6.75	15/4/00	1.09				1.2[1]
10.1	28/2/01	1.47				1.0[1]
5.0	31/1/01	1.49		0.2	0.1	
8.4	30/4/01	1.58				1.2[1]
11.3	15/1/02	2.16				0.2
7.9	28/2/02	2.31				0.2
10.3	15/6/02	2.49				0.4
4.25	30/7/02	2.81	1.0[1]	0.0	0.1	
3.0	31/1/03	3.09	1.9[1]	0.8[1]		
5.25	31/1/03	3.17		– 1.1	0.2	
4.6	30/7/03	3.26	0.0			
10.9	30/8/03	3.39				0.6[1]
10.5	30/10/03	3.53				0.4
8.0	30/5/04	4.05				0.2
10.0	28/2/05	4.46				0.1
4.5	30/7/04	4.52	– 0.1	0.0	0.3	
3.25	31/1/05	4.86	2.3[1]	0.5		
4.95	30/7/05	4.94	1.2[1]			
10.15	31/1/06	5.05				– 0.2
8.8	30/4/06	5.28				0.0
7.35	31/3/07	6.01				0.2
6.0	31/1/08	6.77		– 0.4	0.1	
8.2	28/2/09	7.04				1.1[1]
5.15	30/7/09	8.02	– 0.3	– 0.2	0.4	
4.0	31/1/10	8.43	0.3	0.6[1]		
8.7	28/2/12	8.44				0.9[2]
6.15	31/1/13	9.48		0.0	0.4	
4.75	30/7/14	10.87	– 0.2	0.0		
6.0	31/1/29	14.57		0.0		
Average			0.3	– 0.1	0.2	0.3

[1] Significant at 5%. [2] Significant at 10%

For each day and security we compute the difference in yield between the mean traded price and the mid quoted price. Figures are daily averages for the whole sample (January 1999 to April 2000).

Figure 1

Life cycle for strippable 10-year bonds included in the sample

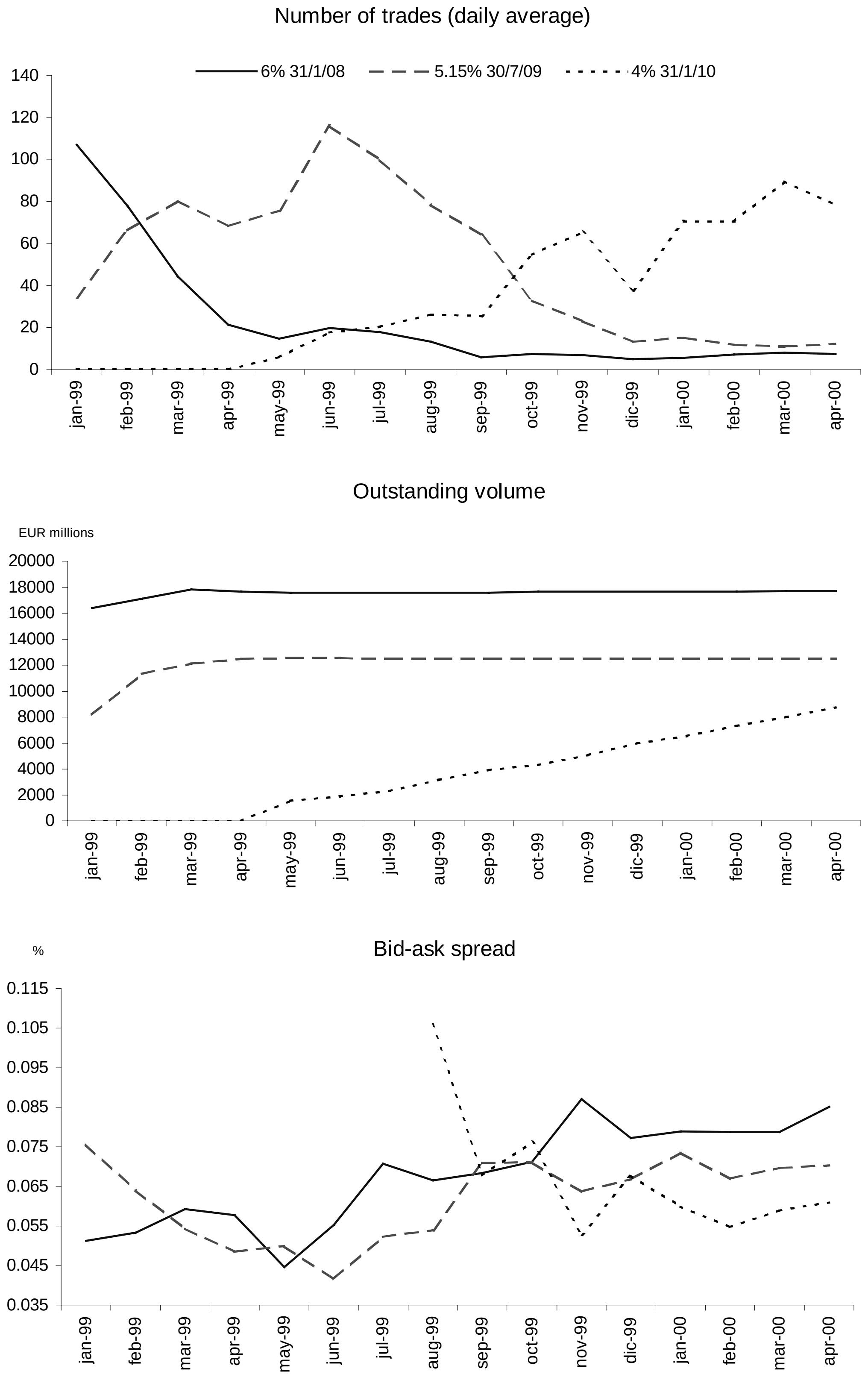


Figure 2

Average of daily bid-ask spread

(percentage points)

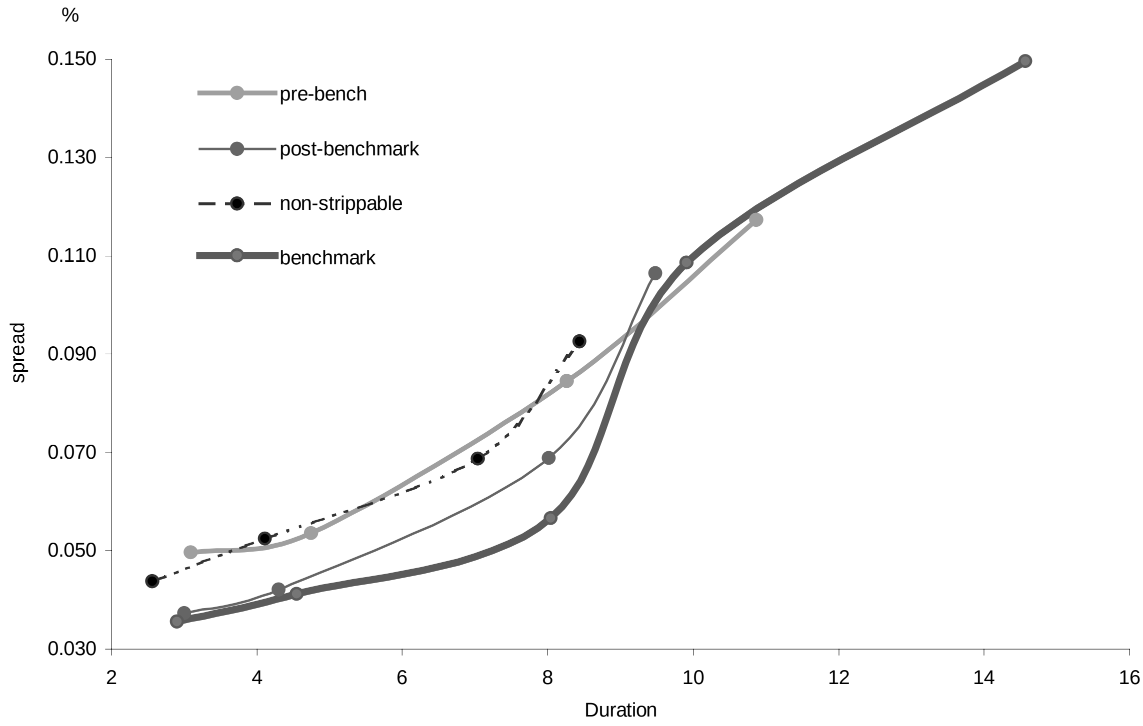


Figure3

Zero-coupon yield curve

(Nelson-Siegel model, 29 March 1999)

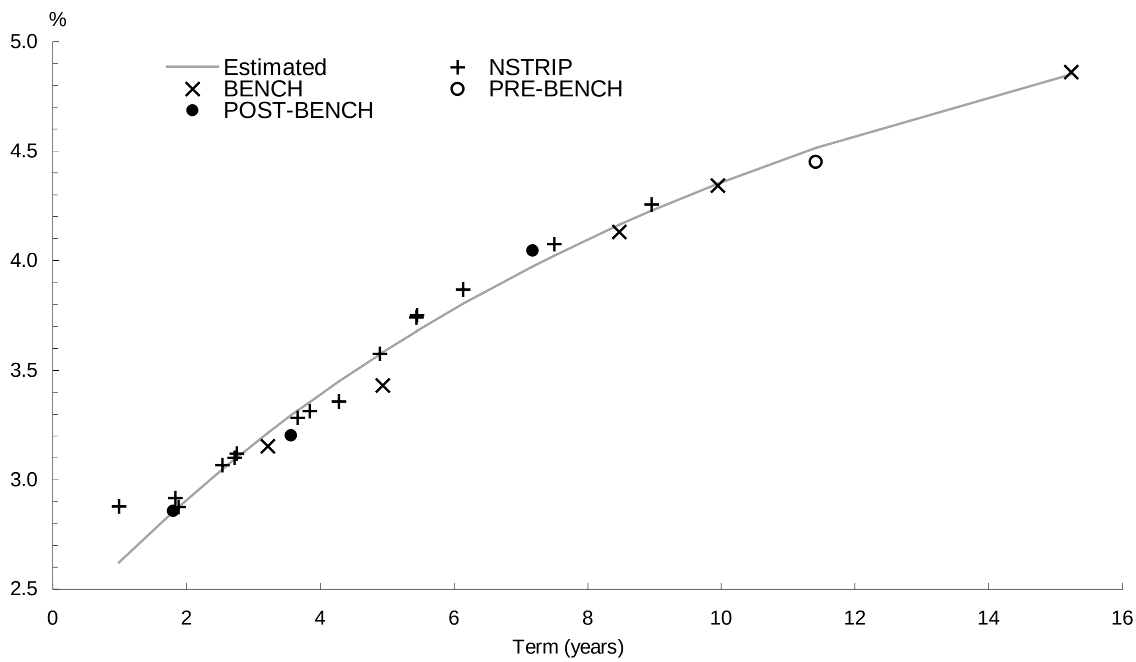
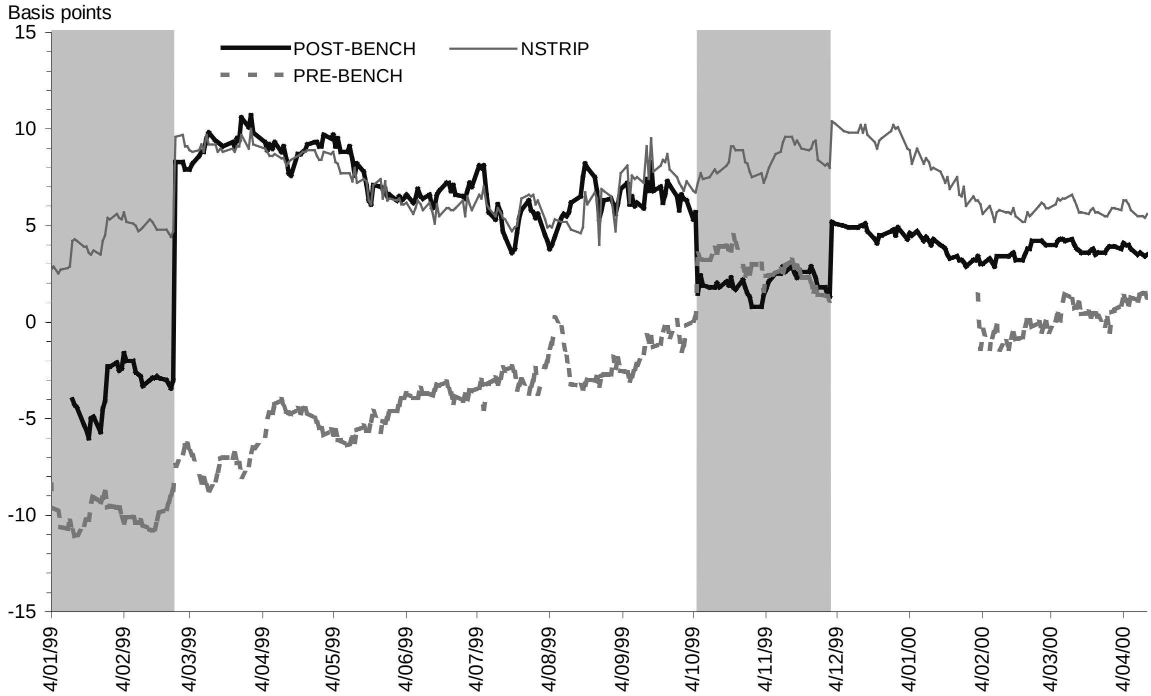


Figure 4

Evolution of estimated parameters



Zero-coupon yield curves are estimated using Nelson-Siegel model until September 1999 and Svensson model for the remaining sample. Dummy variables are introduced to capture liquidity differences. The instantaneous forward rate of term *m* is modelled in the following way for Svensson model:

$$\varphi_m = \beta_0 + \beta_1 e^{-\frac{m}{\tau_1}} + \beta_2 \frac{m}{\tau_1} e^{-\frac{m}{\tau_1}} + \beta_3 \frac{m}{\tau_2} e^{-\frac{m}{\tau_2}} + \gamma_0 PREBENCH + \gamma_1 POSTBENCH + \gamma_2 NONSTRIP$$

This estimation corresponds with Model 2. Shaded areas indicate that γ_2 is not statistically significant.

Figure 5

Zero-coupon yield curve

(Nelson-Siegel model, 29 March 1999)

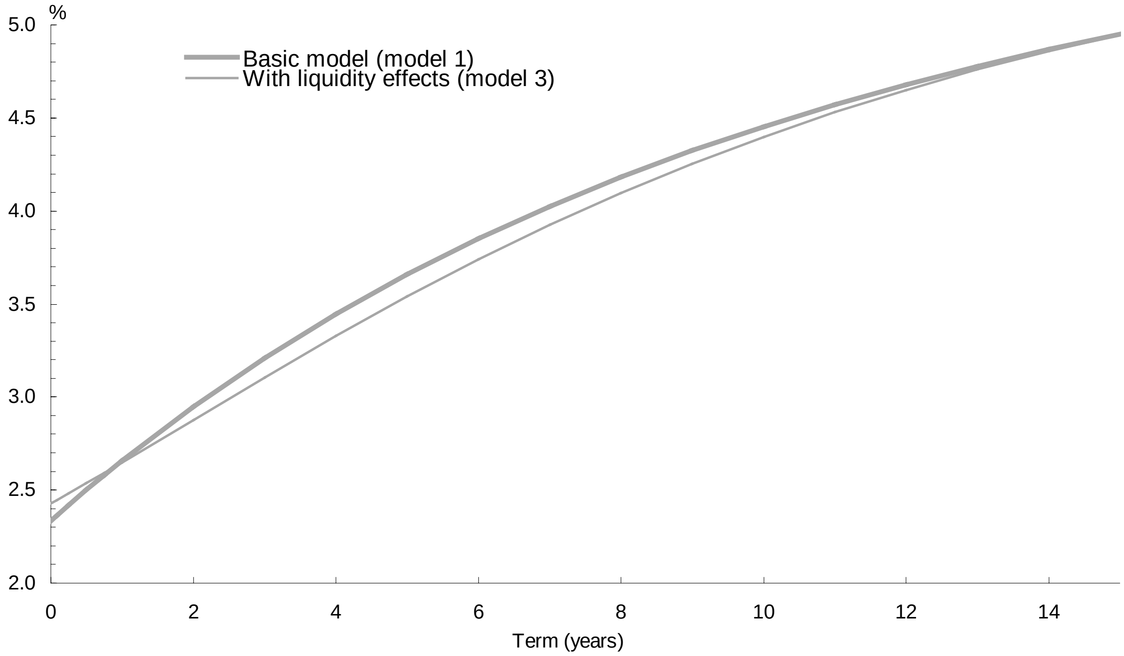


Figure 6

One-year forward curve

(Nelson-Siegel model, 29 March 1999)

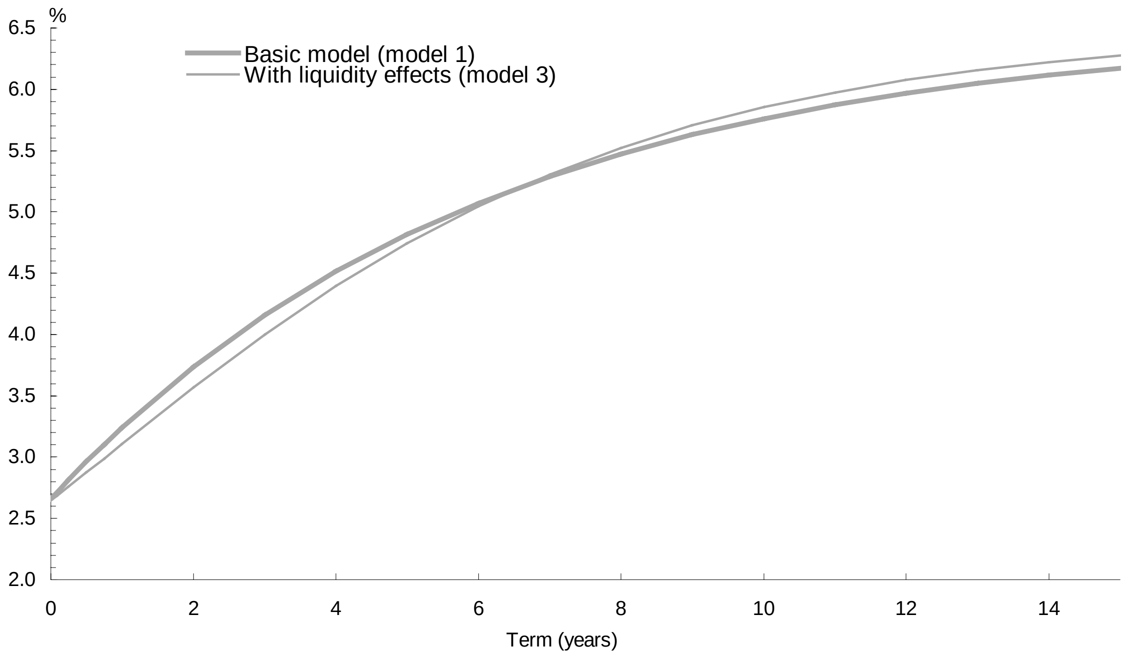


Figure 7

Zero-coupon yield curve

(Svensson model, 31 March 1999)

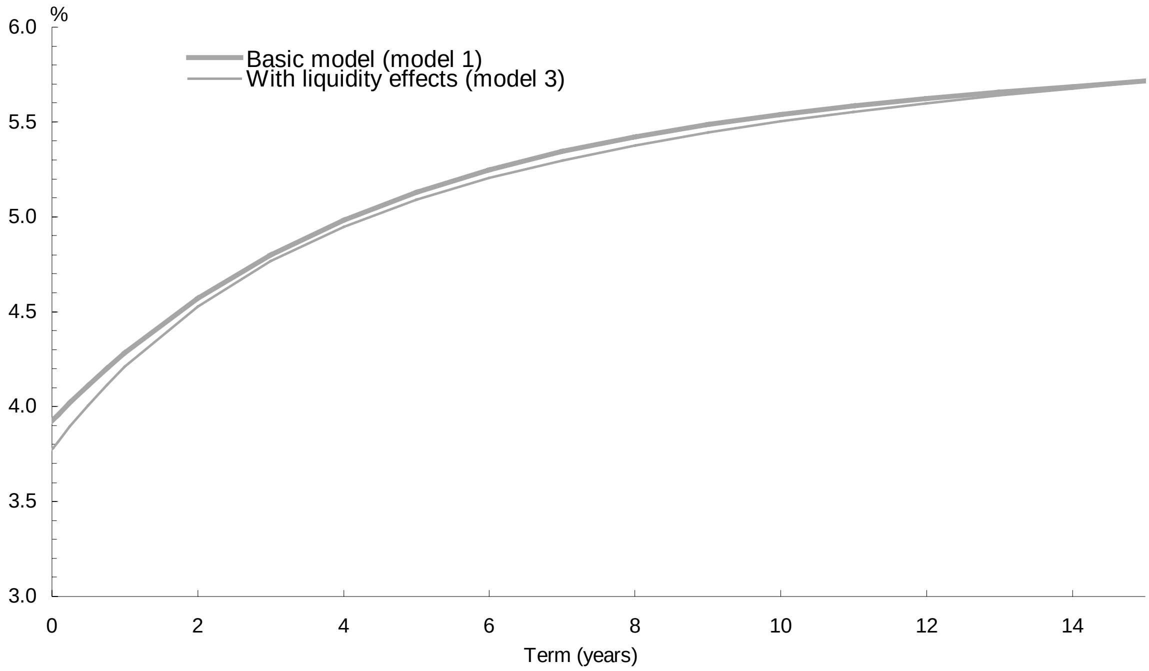
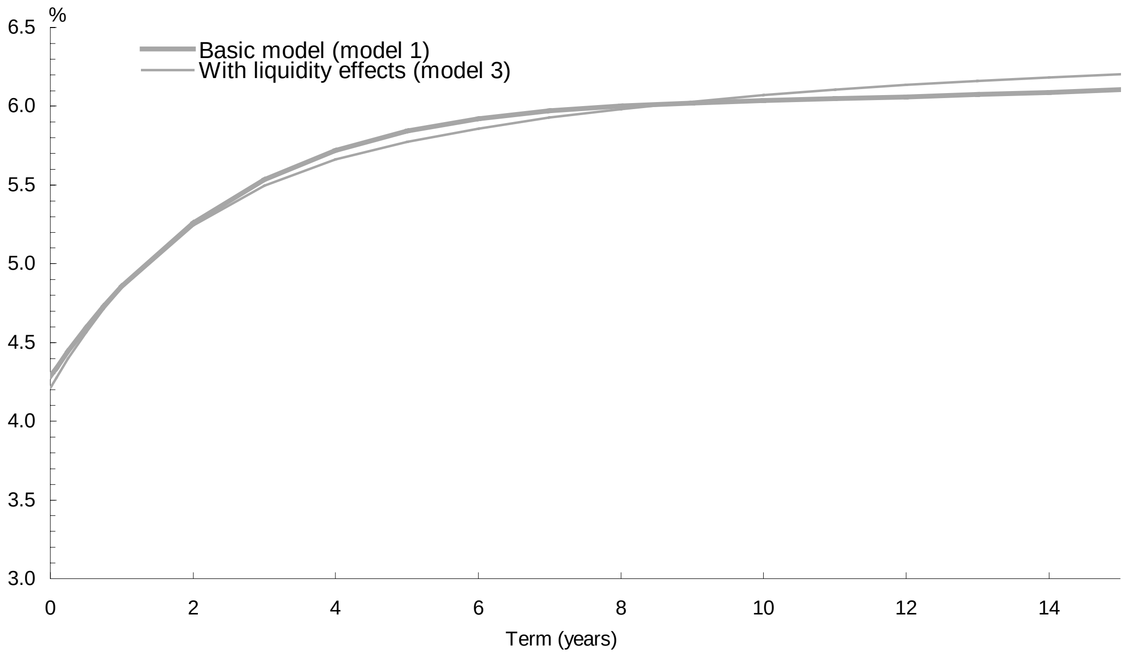


Figure 8

One-year forward curve

(Svensson model, 29 March 1999)



Comments on "Estimating liquidity premia in the Spanish Government securities market" by F Alonso, R Blanco, A del Río and A Sanchís

Christian Upper, Deutsche Bundesbank

Liquidity premia for fixed income securities have commanded increasing attention since their sharp increase during the summer of 1998. They normally obtained by comparing the yields of two bonds with similar maturity and default risk but different liquidity. Blanco et al follow a more sophisticated course. Instead of pairwise comparisons of two securities, they estimate a zero-coupon yield curve for Spanish government securities and include dummy variables for three categories of bonds which they deem to be less liquid. The categories are non-strippable post-benchmark bonds, strippable post benchmark bonds and strippable pre-benchmark securities. The coefficients of the dummies give an estimate for the premium commanded by the corresponding category over a hypothetical benchmark security of the same maturity.

Yield spreads may arise due to a variety of factors, and liquidity differences is only one of them. The authors follow a two-step strategy in order to make sure that what they estimate does indeed reflect liquidity premia and not something else. They begin by showing that market liquidity does vary across the four categories of bonds. For my taste, they rely too much on activity indicators such as trading volume or turnover ratios. Such variables show that the market was liquid enough in order to handle the amount of activity observed, but not how easily securities could be traded. However, given the scarcity of readily available measures for market liquidity, the use of activity indicators may be unavoidable. Nonetheless, perhaps the fact that bid-ask spreads increase with maturity does indicate that shorter term paper is more liquid than longer term paper, although they will not be able to identify this with their methodology. As a second step, the authors rule out the main alternative factor – differences in taxation - by repeating the estimation with post tax returns. They find that the estimates for the coefficients barely change. Hence the interpretation of the yield spread as a liquidity premium seems acceptable.

The authors find that non-strippable bonds (all bonds issued before July 1997) and post-benchmark bonds command a yield premium in the range of 5 to 10 basis points over the benchmark, while pre-benchmark issues (on-the-run issues which eventually will become benchmarks) show a negative yield spread over benchmark paper, although this is significant only about one half of the time.

Let me first make some remarks on the methodology before discussing the results. The procedure used requires considerably more effort than simple pairwise comparisons. So what is the value-added and is it worth the effort?

Firstly, by using a yield curve as a baseline, the authors reduce the importance of pricing errors for individual bonds, e.g. due to non-synchronised trading. Such ‚noise' should be present mainly in the prices of the less liquid bonds, so that the gain is not so much by using a better benchmark but by aggregating over several bonds in the individual categories. Secondly, using a yield curve reduces the bias that arises because of differing maturities that are sometimes unavoidable in pairwise comparisons. This should improve the accuracy of the estimates in particular for shorter maturities, where the yield curve tends to be steeper. Thirdly, their approach allows them to test whether the estimates for the yield curve are sensitive to the presence of liquidity premia. Thankfully, they find that they are not. This should be of great relieve to all those users of yield curves who do not adjust for liquidity differences.

A final advantage is that their approach allows them to distinguish between the relative importance of different attributes of a bond in determining the liquidity premium. This would enable us to break down the liquidity premium into different components. Unfortunately, the authors do not make much out of this possibility since the categories into which they group the bonds are mutually exclusive. For example, they do not included the non-strippable bonds in the post-benchmark category. As a consequence, we don't know whether the liquidity premium of non-strippable bonds over strippable post-benchmark securities is statistically significant. Using non-exclusive categories would permit them to identify the marginal effects of each attribute.

As in any empirical paper, the question arises whether the variables (in their case the dummies) included are the correct ones. Now I don't know much about the market for Spanish government securities beyond what I learned in the paper, so I'm not qualified to make judgements on this issue. However, I wonder why they omitted the volume outstanding of the securities. In Germany, the on-the-run/off-the-run spread is quite sensitive to the size of the issues. Is this not the case in Spain? Another possibility would be to include measures of liquidity in a more direct manner. The authors are right to point out that the relationship between liquidity premia and liquidity itself is likely to be non-linear, but this could be dealt with by using dummies e.g. for low/average/high liquidity. Of course, one always has to bear in mind that with only about 30 bonds at any point in time, the number of variables that one may include is rather limited.

Let me now briefly comment on the results. Figure 4 of the paper shows the evolution of the estimated liquidity premia over time. Two rather distinct periods can be identified, separated by two months in which the coefficient of POST-BENCH turned out to be insignificant. Between March and September 1999, the coefficients of NSTRIP and POST-BENCH are virtually identical. As I mentioned before, using non-exclusive categories could have told us whether this was statistically significant. In the second period, from December 1999 to April 2000, the two coefficient differ by a much greater extend. That of POST-BENCH declines from 5 to 3 basis points, while that of NSTRIP halves from 10 to 5 basis points. Unfortunately, the paper does not discuss the evolution of liquidity premia over time, although this may be due to its more methodological focus. All in all, the movement over time of the estimated premia is rather different from the pattern followed by a (admittedly very simple) measure of liquidity spreads in the German market, which has seen a sharp rise in liquidity premia during the summer of 1999. It would be interesting to know why.

Let me conclude by saying that there is a lot of value added in their approach. I found it a very inspiring paper and I'm looking forward to follow ups. There are two extensions in which I would be particularly interested. The first would be to go beyond the Spanish market and consider yield differences between bonds of different nationalities. At the beginning of August, Spanish 10 year benchmark bonds yielded 18 basis points and Portuguese and Italian bonds 26 basis points more than German paper. Why is this the case? Which role does the presence of a liquid futures market for the German Bundesanleihe play? These are questions which wait to be answered, although admittedly the task of putting together a EMU-wide dataset seems aweinspiring. The second extension would be to include credit spreads into the analysis. So far, it has been difficult to disentangle the effect of creditworthiness and liquidity. For example, at the Bundesbank, we have argued that the yield spread of German bank bonds over Bunds mainly reflect liquidity rather than credit premia, but we haven't been able to test this in a rigorous manner. With your methodology, this could be done.

Comments on “Estimating liquidity premia in the Spanish government securities market” by F Alonso, R Blanco, A del Río and A Sanchís

Oreste Tristani, European Central Bank[1]

This is an interesting paper that presents quantitative estimates of the premium implicit in the price of Spanish government securities to compensate investors for liquidity risk and tax factors. The main results are obtained fitting a yield curve model through bond prices, measured daily, over the maturity spectrum. The estimation is repeated for the number of available days, from January 1999 to April 2000. Liquidity premia are identified through a number of dummy variables attached to less liquid bonds, i.e. issues that are not on-the-run.

The results show that the latter category of bonds appears to command a relatively small premium, of the order of a few basis points, broadly comparable to the estimates obtained by Elton and Green (1998) for US government securities. This is reassuring, since it confirms that in developed economies, excluding times of market stress, liquidity premia on government securities can be considered as negligible in terms of their macroeconomic implications. Nevertheless, their precise measurement can be useful in the management of government debt, for example allowing a more informed assessment of the relative desirability of alternative issuing practices.

The authors (henceforth ABRS) also mention that knowing the size of liquidity premia on government bonds is important for central banks, when they try to extract macroeconomic information (mainly on inflation expectations) from the yield curve. This activity crucially involves filtering out from the curve any kind of risk-premia, amongst which liquidity premia, that are unrelated to macroeconomic factors. The more precise the technique adopted to implement the filtering, the more precise the estimation of macroeconomic information reflected in the yield curve.

To filter out liquidity premia, central banks typically concentrate on benchmark issues. However, this methodology is relatively crude, since liquidity premia (compensation for the lack of immediacy, depth, tightness and resiliency of the market) are likely to characterise all bonds, not just the *relatively* more illiquid ones. The assumption that benchmark bonds are not just *more* liquid, but completely (liquidity) risk-free is not necessarily warranted. To improve the precision of the measurement of inflationary expectations implicit in the yield curve, one would really need an estimate of the liquidity premium implicit in the return on benchmark issues.

Unfortunately, ABRS’ methodology does not provide indications in this respect, but their exercise remains interesting for the other reasons emphasised above.

Is the Svensson model precise enough?

As already mentioned, the magnitude of the premia measured by ABRS is “moderately” small, of the order of a few basis points. In this respect, ABRS’ paper is a commendable effort to look for refinement and precision in the measurement and estimation of yield curves. A fundamental choice in the analysis is therefore the selection of the yield curve model. As is often the case in economics, we have many available models to fit one phenomenon, and no model appears to unequivocally dominate the others under all circumstances.

ABRS dismiss altogether dynamic models, presumably because, although theoretically more appealing, their simpler versions do not provide an accurate description of empirical yield curves. The choice falls instead on static “estimated” models, such as simple polynomial splines (e.g. the cubic one suggested by McCulloch, 1971, 1975), which have the advantages of being both manageable and to only require estimation of a small number of parameters to fit the data. Among the available static models, ABRS favour Svensson’s (1994) modification of the Nelson and Siegel (1987) model. The main advantage of the Nelson and Siegel model with respect to simple polynomial splines is that the latter, as noted by Shea (1984), tend to produce unstable forward rates, especially at longer

[1] The opinions expressed are personal and should not be attributed to the European Central Bank.

maturities. Nelson and Siegel's formulation, on the contrary, imposes an asymptote on the forward rates curve, so that the latter always stabilises as the maturity increases. Svensson's modification of this model essentially represents a way to increase the degrees of freedom available in the estimation, so that a larger number of shapes for the yield curve can be accommodated.

Though more stable at long maturities Nelson and Siegel's methodology is not, however, "perfect". Like spline models, it essentially amounts to fitting a certain functional form through bond prices. As in any estimation, there will typically be a discrepancy between actual and fitted values. As a result of Svensson's modification, the functional form used by ARBS is very flexible and it can accommodate a large number of shapes for the yield curve. Nevertheless, it will not, excluding exceptional circumstances, be able to replicate *exactly* observed prices. In fact, it is not uncommon for it to produce fitted yields to maturity that are a few basis points different from observed yields.

Whether this imperfect fit is acceptable will ultimately depend on the scope of the analysis. When zero coupon rates and long-term forward rates are used to gauge average future inflationary expectations, a discrepancy between actual and fitted values of up to 10 basis points can probably be considered negligible. Indeed, Svensson takes the view that "for financial analysis, the estimation of forward rates is done with a number of different methods, some rather complex to achieve sufficient precision. For monetary policy analysis, the demand for precision is arguably less, which can be traded for increased robustness and simplicity of the estimation method".

Is the extended Nelson and Siegel model sufficiently precise for the estimation of liquidity premia? I believe that the answer is not obvious, since the magnitude of pricing errors generated by the Svensson model is comparable to that of the liquidity premia resulting from ABRS' estimation. Indeed, liquidity premia are defined as the discrepancy between the observed price of an illiquid bond and the estimated price of a perfectly liquid theoretical bond with the same maturity and coupons. An insufficiently precise estimate of the latter would obviously bias the calculation of the premia.

The authors might therefore want to spend some time convincing the reader that it is indeed liquidity premia that they are measuring, and not (at least in part) a mispricing error due to an insufficiently precise model. Since they repeat their yield curve estimation for a large number of days, thus a large variety of shapes for the yield curve, it should be possible to argue that any potential mispricing error is likely to have zero mean over the days contained in the sample. Hence, even if daily premia could occasionally be affected by a pricing error, the average of these premia over the sample period is likely to genuinely represent the compensation for liquidity risk requested by investors.

An alternative model

There are, however, alternative methodologies, such as that presented in Brousseau and Sahel (1999), that allow for a more precise replication of the yield curve.

The fundamental difference between the Nelson and Siegel methodology and that adopted by Brousseau and Sahel is that the latter provides an exact representation of the yield curve, and not a statistical approximation of it. Rather than postulating a certain functional form for the curve, and then fitting it to the data, Brousseau and Sahel use a non-parametric procedure that allows the shape of the curve to be (almost) completely data-determined.

This *bootstrapped* yield curve is defined as one in which the observed prices of all the available securities are replicated without any pricing error. Since only a finite number of bonds is available for the construction of a continuous curve, the maturities for which no traded bonds are available must, like in the other methodologies, be interpolated. In this case, however, the interpolation is such as to leave the estimates of prices of existing securities equal to their observed values.

A bootstrapped curve obviously bears a more complex mathematical representation than one obtained through, for example, the Svensson methodology. Specifically, it cannot be represented through a small number of parameters and it can typically only be obtained numerically. Brousseau and Sahel construct the curve in an iterative fashion, starting from the available security with the shortest maturity and then moving on by increasing maturity. For illustrative purposes, they concentrate on the US and German securities markets and report the results of some bootstrapped curves obtained assuming different sorts of interpolation methods (see Brousseau and Sahel, for details). They then compare these results with those estimated through a simple spline model.

Interestingly, the difference between the yield curves obtained through the two methodologies turns out to be, on average, of the order of a few decimals of a percentage point, i.e. potentially large enough to produce significant differences in the estimation of liquidity premia. This seems to confirm that some caution is necessary in the interpretation of ABRS's results. It must be taken into account, however, that the comparison presented in Brousseau and Sahel (1999) is with respect to a simple spline model. Compared to the latter, Svensson's model is likely to improve significantly the fit of the observed price data, so that it remains unclear whether its pricing errors can be considered negligible for the purpose of estimating liquidity premia.

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