

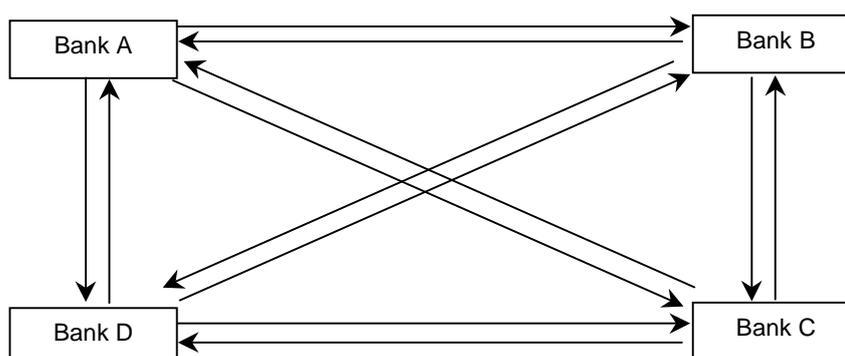
Estimating bilateral exposures in the German interbank market: is there a danger of contagion?

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1. Introduction

Credit risk associated with interbank lending may lead to domino effects, where the failure of a bank results in the failure of other banks even if they are not directly affected by the initial shock. Recent work in economic theory shows that this risk of contagion depends on the precise pattern of interbank linkages. In Allen and Gale (2000), banks hold deposits with banks of other regions in order to insure against liquidity shocks in their own region. Here a “region” should not necessarily be interpreted in geographical terms but could in principle refer to any grouping of banks. If a bank is hit by a shock, it tries to meet the liquidity need by drawing on its deposits at other banks before liquidating long-term assets. This pecking order follows from the assumption that the premature liquidation of the long-dated asset is costly, for example because real investment projects have to be abandoned or because long-term lending relationships are interrupted.

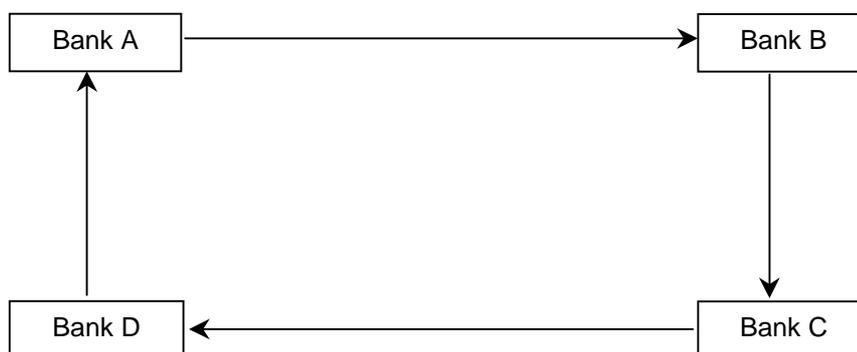
Figure 1
“Complete market structure” according to Allen and Gale (2000)



In aggregate, the interbank market can only redistribute liquidity and does not create liquidity of its own. While this is not a problem if the aggregate liquidity need is lower than the aggregate holdings of liquid assets, it may give rise to contagion if the opposite is true. Instead of liquidating their long term assets, banks withdraw their deposits at other banks, thus spreading their illiquidity throughout the financial system. The possibility of contagion depends strongly on the precise structure of interbank claims. Contagion is less likely to occur in what Allen and Gale term a “complete” structure of claims, in which every bank has symmetric linkages with all other banks in the economy (see Figure 1). “Incomplete” structures, where banks have links to only some neighbouring institutions (see Figure 2 for an extreme example), are shown to be much more fragile.

¹ We are grateful to Hans Bauer, Ralf Körner, Karl-Heinz Tödter and Benno Wink for their invaluable comments and help. The opinions expressed in this article are the authors’ own and do not reflect those of the Deutsche Bundesbank.

Figure 2
“Incomplete market structure” according to Allen and Gale (2000)



In a related paper by Freixas et al (2000), interbank lending arises from consumers’ uncertainty not about *when* to consume (as in Allen and Gale) but about *where* to consume. Their model can be interpreted as a payment system, where interbank credit lines are used to reduce the overall amount of liquid (but costly) reserves. In their setting, contagion can occur even if all banks are solvent. If the depositors in a sufficiently large number of banks believe that they will not obtain payment, it is optimal for them to withdraw their deposits. Since this forces their banks to liquidate their investments, it triggers a run in which all other depositors withdraw their deposits and the banking system reaches gridlock.² Besides contagion driven by non-banks’ behaviour, the authors also consider the impact that an insolvent bank has on the banking system. They find that interbank connections generally enhance the resiliency (ie the ability to withstand shocks) of the financial system. Interbank credit lines provide an implicit subsidy to the insolvent bank, which is able to spread part of its losses to other banks. Interbank lending thus contributes to loosening market discipline.³ As in Allen and Gale, a “complete” structure of claims reduces the risk of contagion, while incomplete structures, or “credit chains” (like the one in Figure 2), increase the fragility of the system.⁴

Unfortunately, very little is known about the actual structure of *bilateral* exposures in the interbank market as banks do not report their counterparties. As a consequence, empirical studies concentrate mainly on the payment system, for which such data is available.⁵ An exception is Furfine (1999), who uses settlement data to compute bilateral exposures in the interbank federal funds market. He finds that even in his worst case scenario (failure of the most significant bank and a 40% loss rate⁶) only two to six other banks fail, accounting for less than 0.8% of total bank assets. No contagion occurs at all if the loss rate is 5% - such as the one estimated by Kaufman (1994) for Continental Illinois. The results have to be interpreted with care, however, since the fed funds market accounts for only 10 to 20% of total interbank exposures in the United States, although its share is likely to be higher if only uncollateralised positions are considered.

An alternative approach followed by the present paper is to estimate the matrix of bilateral credit relationships from bank balance sheet data. In contrast to studies based on settlement data, this

² This can be prevented by central bank guarantees for interbank credit lines. These lines are never used in equilibrium, since by assumption all banks are solvent.

³ This provides a second rationale for central bank involvement in financial supervision, namely to organise the orderly closure of insolvent banks.

⁴ Rochet and Tirole (1996) and Aghion et al (1999) consider the incentives to bail out failing banks by providing interbank loans. The absence of a bailout could be due to a weak financial position of the banking system and thus serves as a signal triggering a bank run. They do not say much about how the precise direction of interbank linkages affects the possibility of contagion.

⁵ For example, Humphrey (1986) and Angelini et al (1996).

⁶ The 40% loss rate corresponds to the typical loss on assets of a failing bank estimated by James (1991).

permits us to cover interbank lending completely.⁷ This comes at a cost, however. Due to the fact that we do not have complete information on the individual counterparties in the interbank business, we need to make assumptions concerning the distribution of bank *i*'s interbank loans and deposits over the other banks. In order to bias our estimates against the hypothesis of contagion, we assume that interbank lending is as dispersed as possible, given the observed distribution of loans and deposits.

Our paper is closely related to Sheldon and Maurer (1998), who estimate a matrix of interbank loans for Switzerland.⁸ They come to the conclusion that the interbank loan structure that existed among Swiss banks posed little threat to the stability of the Swiss banking system in the period under consideration. Our work differs from Sheldon and Maurer's in several respects. For computational reasons, they aggregate the individual banks into 12 categories. Our approach is much more disaggregated, because we want to make use of virtually all information on the interbank market that is available to us. Firstly, we consider all German banks individually. Secondly, our data permits us to estimate separate matrices for (i) loans of savings banks to their regional giro institutions, (ii) loans of giro institutions to affiliated savings banks, (iii) loans of cooperative banks to cooperative central banks, (iv) loans of cooperative central banks to cooperative retail banks, and (v) loans and deposits of the remaining "normal" interbank loans. The interbank assets and liabilities in each of these five cases are divided into five maturity categories, giving a total of 25 separate matrices for a given month, which we add up to a system-wide matrix of interbank relationships. Since we fully use this supplementary information - consisting mainly of zero restrictions in the single maturity and loan classes - our estimate should be much more accurate than a matrix estimated with figures for total lending and deposits alone, while maintaining the assumption of a maximum amount of diversification consistent with the data.

The system-wide matrix differs considerably from a matrix estimated from aggregate exposures (across all maturities and banking groups) alone. Interbank lending is much more concentrated, thus making the banking system potentially more vulnerable to contagion according to the results of the theoretical literature. We assess the danger of contagion by letting every bank go bankrupt one at a time and compute the effect of this failure on the other banks. We find that credit exposures in the interbank market can lead to domino effects. At worst, the failure of a single bank triggers a chain that ends with the bankruptcy of almost 15% of the banking system in terms of assets. If we ignore the existing safety mechanisms such as the institutional guarantees in the savings and cooperative bank sector, the results are even more devastating. Our analysis takes the initial shocks leading to contagion as given. We are therefore not able to attach an explicit probability to our scenarios.

It is important to stress that our analysis concentrates exclusively on contagion due to credit exposures in the interbank market. We rule out other channels of contagion like bank runs, which have commanded the attention of much of the theoretical literature and have dominated the discussion on banking regulation.⁹ We believe that this omission is justified not only for methodological reasons, in that it permits us to isolate one specific channel of contagion. Contagion due to bank runs by non-bank depositors in the wake of the breakdown of a single institution are highly unlikely in Germany, where virtually all deposits by non-financial institutions are insured. The type of contagion we analyse is dependent less on the *behavioural* interdependence of the respective parties and more on the interdependence due to interbank linkages. In this sense, it can be described as being "mechanical".

The paper is structured as follows. After a description of the data set, we present the estimation methodology. This is followed by a section on the structure of the German banking system in general and the interbank market in particular. In Section 5, we then estimate the danger of contagion in the German interbank market, leaving aside for a moment banking supervision and the existence of "safety nets". We find that in this case, contagion may lead to the breakdown of a large part of the German banking system. We also confirm the proposition given by the theoretical literature that symmetric (in the extreme: "complete") structures are less vulnerable to contagion than asymmetric ("incomplete") structures. Having done this, we measure the importance of the relevant "safety nets"

⁷ We focus only on direct lending relationships and do not capture exposures arising in the payment or security settlement systems or exposures due to the cross-holding of securities.

⁸ They also estimate the probability of failure, and thus the likelihood of contagion, which we ignore.

⁹ The seminal theoretical paper is perhaps Diamond and Dybvig (1983).

existing in Germany in order to prevent such scenarios. Our results suggest that these institutions and regulations dramatically reduce, but not eliminate, the danger of contagion. A final section concludes.

2. Description of the data

The analysis is based on balance sheets, which all German banks have to submit to the Bundesbank every month. At this stage, we only consider the balance sheets from end-December 1998, but we plan to make use of the time series dimension of the data in future work. In their submissions, banks have to state whether their counterparty in the interbank market is a domestic or a foreign bank, a building society (*Bausparkasse*) or the Bundesbank. Savings banks and cooperative banks also have to identify lending to giro institutions (*Landesbanken*) and cooperative central banks, respectively (and vice versa). In addition, all banks have to break down their interbank business into five maturity categories (listed in Table 1)¹⁰. Since our data only cover domestic banks and the German branches of foreign banks, we exclude loans to and deposits from building societies, the Bundesbank and foreign banks. This leaves us with a closed system for each maturity category, where all interbank loans and deposits add up to zero in principle.

In practice, discrepancies between assets and liabilities do arise (see row “All banks” in Table 1). They are particularly acute for overnight loans and deposits, where the latter consistently exceed the former by around 10 to 15%. One possible reason could be the existence of floating transactions. Since the German payment system is mainly transfer-based, the interbank liabilities of the payer’s bank tend to increase before the corresponding asset position of the payee’s bank. For this reason, the individual asset positions were scaled such that their sum matches that of the liability positions within the same maturity category. Another possible source of the discrepancies between assets and liabilities are errors in the data. The database has been checked for consistency (eg all positions on the balance sheet have to satisfy an adding-up constraint), but entries in the wrong category remain a possibility.

Table 1 shows the maturity structure of the interbank assets and liabilities of all German banks, both vis-à-vis all other banks, including foreign banks, building societies and the Bundesbank, and vis-à-vis the banks contained in our sample only. It is apparent that over all banks, more than half of the interbank assets and liabilities have a maturity of at least four years.

However, this share varies widely across bank types: whereas long-run interbank liabilities are very important for savings banks (91.5%) and for cooperative banks (91.7%), they are less important for commercial banks (36.2%), Landesbanken (45.5%) and cooperative central banks (27.8%). The picture differs considerably when it comes to interbank assets: here, only 8.8% (savings banks) and 11.5% (cooperative banks) of interbank loans have a maturity of at least four years. For the Landesbanken and the cooperative central banks, the corresponding figures are 60.5% and 67.7%, respectively. On the whole, Table 1 shows that the interbank market consists of far more than just the exchange of liquidity on a day-to-day basis. In order to assess the danger of contagion, it is therefore necessary to consider not just these short-run relationships but also, and especially, the longer-term assets and liabilities.

¹⁰ Deposits from banks are actually broken down into six categories, which we consolidate into five in order to be able to compare them to the lending side.

Table 1
Interbank assets and liabilities by maturity and broad bank category
(end-December 1998)

	Notes	maturity				
		daily	> 1 day & < 3 mths	≥ 3 mths & ≤ 1 yr	> 1 yr & < 4 yrs	≥ 4 yrs
All banks						
Liabilities	1	11.7	13.2	14.5	5.6	55.0
	2	12.6	24.3	17.4	4.5	41.1
Assets	1	10.6	14.1	17.2	5.6	52.5
	2	11.2	18.8	18.0	6.0	46.0
Commercial banks						
Liabilities	1	21.1	18.0	21.3	3.4	36.2
	2	18.6	32.6	23.7	3.2	21.9
Assets	1	22.3	30.3	27.1	5.3	14.9
	2	20.4	35.5	26.7	5.8	11.6
Savings bank sector						
<i>Landesbanken</i>						
Liabilities	1	9.6	19.0	17.3	8.6	45.5
	2	9.9	26.7	18.8	6.7	37.9
Assets	1	4.6	7.7	15.7	4.3	67.7
	2	4.8	12.2	18.0	5.2	59.8
<i>Savings banks</i>						
Liabilities	1	2.7	2.2	2.0	1.5	91.5
	2	2.5	15.3	2.9	1.4	77.9
Assets	1	20.7	38.5	28.7	3.2	8.8
	2	20.5	39.2	27.3	3.7	9.3
Cooperative sector						
<i>Central banks</i>						
Liabilities	1	26.5	20.5	14.3	10.8	27.8
	2	26.3	25.9	17.1	8.2	22.5
Assets	1	9.7	5.7	18.5	5.6	60.5
	2	9.8	10.5	22.7	5.3	51.7
<i>Cooperative banks</i>						
Liabilities	1	7.0	9.3	18.9	5.9	58.9
	2	6.5	12.8	18.0	5.2	57.5
Assets	1	2.9	4.5	5.1	7.4	80.0
	2	2.9	4.0	5.4	6.6	81.1
Other banks						
Liabilities	1	7.0	9.3	18.9	5.9	58.9
	2	6.5	12.8	18.0	5.2	57.5
Assets	1	2.9	4.5	5.1	7.4	80.0
	2	2.9	4.0	5.4	6.6	81.1

1: Excluding interbank assets and liabilities vis-à-vis foreign banks, building societies and the Bundesbank.

2: All interbank assets and liabilities

3. Estimation methodology

The lending relationships in the interbank market can be represented by the following $N \times N$ matrix:

$$\mathbf{X} = \begin{array}{c} \left[\begin{array}{cccc} X_{11} & \dots & X_{1j} & \dots & X_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{i1} & \dots & X_{ij} & \dots & X_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{N1} & \dots & X_{Nj} & \dots & X_{NN} \end{array} \right] \begin{array}{c} \Sigma_j \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_N \end{array} \\ \hline \begin{array}{cccc} \Sigma_i & I_1 & \dots & I_j & \dots & I_N \end{array} \end{array} \quad (1)$$

where x_{ij} is the credit exposure of bank j vis-à-vis bank i and N is the number of banks. We cannot observe the bilateral exposures x_{ij} , but we do know the sum of each bank's interbank loans and deposits,

$$a_i = \sum_j x_{ij} \quad \text{and} \quad l_j = \sum_i x_{ij}, \quad (2)$$

respectively. This information does not suffice to identify the elements of X , so we are left with $N^2 - 2N$ unknowns. If we want to estimate the bilateral exposures, we have to make assumptions on how banks spread their interbank lending.

With the appropriate standardisation, we can interpret the a 's and l 's as realisations of the marginal distributions $f(a)$ and $f(l)$, and the x 's as their joint distribution, $f(a, l)$. If $f(a)$ and $f(l)$ are independent, then $x_{ij} = a_i l_j$. The assumption of independence implies that interbank loans and deposits are as equally spread over banks as is consistent with the observed marginal distributions. It can thus be interpreted as an equivalent to Allen and Gale's (2000) "complete" structure of claims, where banks symmetrically hold claims on all other banks in the economy, conditioned on the size structure of the banks (see Figure 1).

There are many reasons to believe that independence is a rather poor description of reality, for example because assessing the creditworthiness of a borrower is costly. Nevertheless, it is a convenient way to bias our test against the hypothesis of contagion, and is therefore used whenever we do not have additional information on bilateral exposures.

The independence matrix X in (1) has the unattractive feature that the elements on the main diagonal are non-zero if a bank is both lender and borrower. In this case, using X to compute bilateral exposures would amount to assuming that banks lend to themselves. This unattractive feature does not necessarily disappear as the number of banks increases, if interbank lending or borrowing is relatively concentrated. We therefore need to modify the independence assumption by setting $x_{ij} = 0$ for $i = j$ (see (1*)).¹¹

¹¹ Setting the elements on the diagonal equal to zero also reduces the number of coefficients to be estimated to $N^2 - 3N$ by imposing more structure on X .

$$\mathbf{X}^* = \begin{array}{c} \begin{matrix} \mathbf{0} & \dots & X_{1j}^* & \dots & X_{1N}^* \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{i1}^* & \dots & \mathbf{0} & \dots & X_{iN}^* \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{M1}^* & \dots & X_{Nj}^* & \dots & \mathbf{0} \end{matrix} & \begin{matrix} \Sigma_j \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_N \end{matrix} \\ \hline \Sigma_i & I_1 & \dots & I_j & \dots & I_N \end{array} \quad (1^*)$$

Unfortunately, it is not obvious a priori how to redistribute the mass of the elements on the diagonal of X to the other elements of the matrix. In principle, we could estimate the elements of X^* by minimising the sum of the squared deviations from the independence matrix X subject to a non-negativity constraint on the elements of X^* and the adding-up constraints given in (2). In matrix notation,

$$\min_x \mathbf{x}' \mathbf{I} \mathbf{x} - 2 \mathbf{a} \mathbf{x} \quad (3)$$

$$\text{s.t. } \mathbf{x} \geq 0 \quad \text{and} \quad \mathbf{A} \mathbf{x} = [\mathbf{a}', \mathbf{l}']',$$

where \mathbf{x} is a $(N^2 - N) \times 1$ vector containing the off-diagonal elements of X^* , I is the identity matrix of dimension $(N^2 - N)$, \mathbf{a} and \mathbf{l} are the marginals, and \mathbf{A} is a matrix containing the adding-up restrictions (2). Since the objective function is strictly concave, programme (3) yields a unique solution for the structure of interbank lending X^* that is as close to the independence matrix X as possible.

The cost of solving (3) for a banking system of more than 3,000 banks is prohibitive. One possibility would be to follow Sheldon and Maurer (1998) and aggregate the individual banks into broad categories. Since this would entail a considerable loss of information, we follow an alternative strategy which involves approximating the solution to (3) by means of an iterative algorithm with much lower computational requirements. Our aim is to obtain a matrix that is as close as possible to the independence matrix X and that satisfies the following two conditions:

- (A) The elements on the diagonal are equal to zero, and
- (B) (B) the elements of row i sum to a_i and the elements of column j to l_j (corresponding to the conditions in (2)).

Our algorithm works as follows:¹²

We begin by constructing two weighting matrices A and L , with elements

$$\alpha_{ij} = \begin{cases} 0 & \text{for } i = j \\ \frac{a_i}{\sum_{k \neq j} a_k} & \text{otherwise} \end{cases} \quad \text{and} \quad \lambda_{ij} = \begin{cases} 0 & \text{for } i = j \\ \frac{l_j}{\sum_{k \neq i} l_k} & \text{otherwise} \end{cases}$$

respectively. Each column of A contains the relative weights of the a_i 's given that $a_j = 0$, while the columns of L give the relative weights of l_j 's given that $l_i = 0$. These weights add up to 1, that is, $\sum_j \alpha_{ij} = 1$ and $\sum_i \lambda_{ij} = 1$.

We compute a starting matrix $X^{(0)}$ by distributing half of the interbank assets according to the relative weights of the liabilities, and half of the liabilities using the relative weights of the asset positions. Consequently, the off-diagonal elements of $X^{(0)}$ are

¹² The algorithm was developed by Karl-Heinz Tödter.

$$x_{ij}^{(0)} = \frac{1}{2} (a_i \lambda_{ij} + l_j \alpha_{ij}) \quad (4)$$

and the elements on the diagonal are zero by construction. $X^{(0)}$ has a unit mass, but its rows and columns will not generally add up to a_i and l_j , respectively.¹³

For each row, we calculate the adding-up “error”

$$d_i^a = \sum_j x_{ij}^{(0)} - a_i \quad (5)$$

and redistribute it using L . We obtain $x_{ij}^{(1)} = x_{ij}^{(0)} - d_i^a \lambda_{ij}$.

Likewise, we redistribute the adding-up error for each column, yielding $x_{ij}^{(2)} = x_{ij}^{(1)} - d_j^l \alpha_{ij}$. We repeat this procedure until the adding-up errors fall below a specified convergence criterium.¹⁴

4. The structure of the German interbank market

As mentioned above, our data contain information on interbank lending and borrowing in each of five maturity categories for:

- (1) deposits and loans from savings banks to regional giro institutions (Landesbanken);
- (2) deposits and loans from regional giro institutions (Landesbanken) to savings banks;
- (3) deposits and loans from cooperative banks to cooperative central banks;
- (4) deposits and loans from cooperative central banks to cooperative banks;
- (5) all interbank loans and deposits that do not belong to (1) to (4).

This permits us to compute a total of 25 matrices of bilateral exposures, which add up to give the total amount of interbank exposures. We use this added-up, “full information”, matrix to test for the possibility of contagion. For (1) to (4), we can compute interbank exposures using the independence matrix X (equation (1)) because no bank appears both as lender and depositor. Where banks do appear on both sides, in (5), we use the algorithm presented in the previous section. In addition, we also compute a matrix of bilateral exposures using interbank borrowing and lending aggregated over cases (1) to (5) and over all maturities. This latter matrix (“baseline matrix”) serves as a benchmark, against which we measure deviations from the “complete” claims structure.

Since most banks borrow and lend only at specific maturities (which are usually different from each other), most of the elements of a and l for the different maturity categories are zero. These zero restrictions considerably reduce the number of possible counterparts for each bank, and consequently yield much more precise estimates of the ‘true’ structure of interbank lending than could be obtained from using aggregate exposures alone. In addition, the full information matrix differs from the baseline matrix in that it uses the available information on the intra-lending patterns of the two giro systems. Tables 2a and 2b show that the structure of interbank borrowing and lending in Germany differs considerably from the benchmark of a “complete” structure of claims (the numbers in each row sum up to 100). The figures refer to the exposures of the average bank within a given category (they are therefore not comparable to the figures from the consolidated balance sheet of each group presented in Deutsche Bundesbank (2000a)).

¹³ The choice of the starting values does not matter provided requirement 1 is met and the elements of $X^{(0)}$ add up to one.

¹⁴ We do not show analytically that the algorithm always converges, but we have not encountered any convergence problems in the course of our computations.

Table 2a
Exposures in the German interbank market: deposits
 (average % share of banks by category, end-December 1998)

	Inter-bank matrix	Commercial banks	Savings bank sector		Cooperative sector		Other banks
			Landesbanken	Savings banks	Central banks	Cooperatives	
All banks	full info	5.1	20.3	0.9	58.2	0.5	15.0
	baseline	15.9	30.3	7.5	8.0	6.9	31.4
Commercial banks	full info	31.7	24.6	5.3	6.2	2.2	30.0
	baseline	15.9	30.3	7.5	8.0	6.9	31.4
Savings bank sector							
Landesbanken	full info	18.6	21.5	19.2	4.1	1.6	35.0
	baseline	16.3	28.6	7.7	8.1	7.1	32.1
Savings banks	full info	2.8	75.2	0.5	0.5	0.3	20.7
	baseline	15.9	30.3	7.5	8.0	6.9	31.4
Cooperative sector							
Central banks	full info	10.0	10.7	1.8	1.4	59.0	17.0
	baseline	16.2	30.8	7.7	6.5	7.0	31.9
Cooperative banks	full info	1.9	5.1	0.3	81.7	0.2	10.8
	baseline	15.9	30.3	7.5	8.0	6.9	31.4
Other banks	full info	18.6	27.6	4.0	4.1	1.6	44.2
	baseline	16.0	30.5	7.6	8.0	6.9	30.9

Note: Linkages *within* a giro system are shaded in grey.

In the baseline, all banks hold virtually the same portfolio of interbank loans and deposits. The only difference comes from the restriction that banks do not lend to themselves. As a consequence, the share of banks within the same category is somewhat lower than average.

By contrast, the full information matrix implies that interbank exposures vary considerably between banks and bank categories. We find that both commercial banks and other banks transact much more with institutions of the same category than would be predicted by the baseline matrix. What is particularly striking, however, is the large share of the head institutions of the two giro systems in the interbank loans and deposits of the institutions at the base level. Seventy-five per cent of the interbank deposits at savings banks are held by Landesbanken, who also receive 81% of their loans. Cooperative central banks account for 82% of the deposits at and 94% of the loans of cooperative banks.

It is also striking to see that there are almost no deposits held between banks on the base level of the same giro system (savings banks, 0.5%; cooperative banks, 0.2%) and across the two giro systems (savings banks at cooperatives, 0.3%; cooperatives at savings banks, 0.3%). The full information estimate therefore shows that the interbank deposit market is organised in two tiers: The first tier consists of most savings banks and virtually all cooperative banks, who transact mainly with the Landesbanken and cooperative central banks. The second tier consists of the head institutions of the two giro systems, the commercial banks and the "other banks".

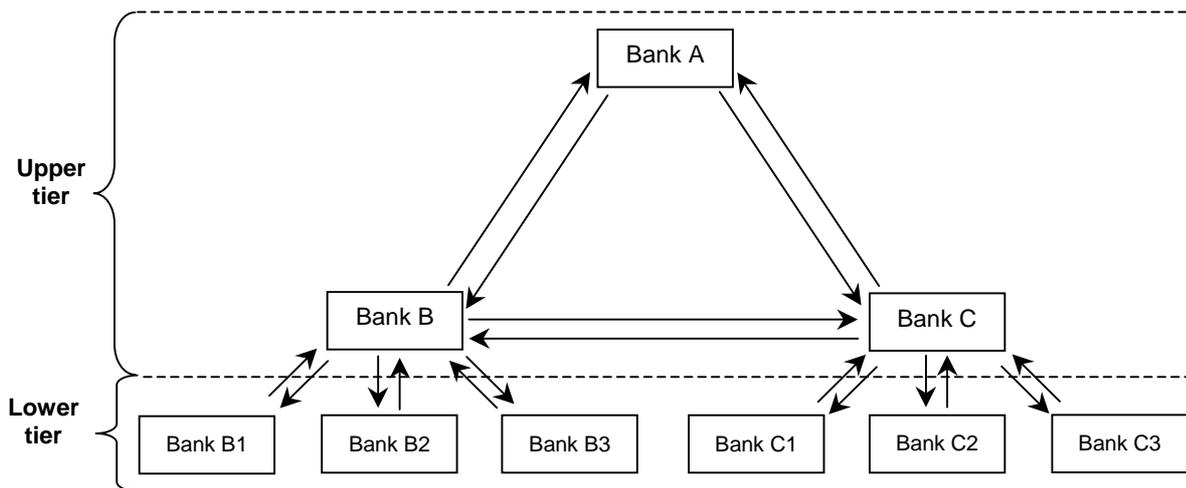
Tables 2a and 2b point to the existence of two relatively closed systems, with very few direct linkages to banks of other categories except those with the respective head institutions and deposits from "other" banks, which comprise mainly mortgage and development banks. We estimate that about half of these deposits represent credit lines that serve to refinance development loans. The upper tier consists of the commercial banks, a small number of savings banks (around 10), the Landesbanken and cooperative central banks plus a variety of other banks. Instead of focused relationships with a small number of head institutions, banks belonging to the upper tier entertain lending relationships with a variety of other banks belonging to the same tier, including those of other categories. As a consequence, in the upper tier the pattern of interbank exposures is much closer to a "complete" structure of claims than that in the lower tier.

Table 2b
Exposures in the German interbank market: loans
 (average % share of banks by category, end-December 1998)

	Inter-bank matrix	Commercial banks	Savings bank sector		Cooperative sector		Other banks
			Landesbanken	Savings banks	Central banks	Cooperatives	
All banks	full info baseline	7.6 19.5	19.0 26.8	1.3 16.6	67.1 11.4	0.4 6.2	4.7 19.5
Commercial banks	full info baseline	43.1 19.4	23.0 26.8	2.9 16.6	10.3 11.4	1.3 6.2	19.5 19.5
Savings bank sector							
Landesbanken	full info baseline	18.4 19.9	19.7 25.3	35.6 16.9	5.0 11.6	1.7 6.4	19.7 19.9
Savings banks	full info baseline	8.3 19.5	81.0 26.8	1.8 16.5	1.9 11.4	0.4 6.2	6.5 19.5
Cooperative sector							
Central banks	full info baseline	17.5 19.9	15.6 27.5	1.8 17.0	3.2 9.3	49.8 6.4	12.0 20.0
Cooperative banks	full info baseline	1.8 19.5	1.8 26.8	0.5 16.6	94.2 11.4	0.1 6.2	1.5 19.5
Other banks	full info baseline	25.9 19.6	27.6 26.9	10.4 16.6	8.5 11.4	2.7 6.3	24.9 19.2

Note: Linkages *within* a giro system are shaded in grey.

Figure 3
“Two-tier” structure of the German interbank market



The link between the two tiers is provided by the Landesbanken (for the savings banks) and the cooperative central banks (for the cooperative sector). On the one hand, they provide long-term loans to and take short-term deposits from their affiliated institutions. This part accounts for 36% (Landesbanken) and 50% (cooperative central banks) of their interbank loans, and 19% and 59% of their interbank deposits, respectively. On the other hand, they operate in the upper-tier interbank market as any commercial bank does. We can interpret the giro systems as some sort of “internal interbank market” whereas the “outside interbank market” consists only of the commercial banks, the head institutions of the giro systems and the “other banks”.

The “two-tier” structure of the interbank market in Germany depicted in Figure 3 differs considerably from the “complete” (see Figure 1) and “incomplete” structures considered in the theoretical literature. It is not clear a priori whether such a structure leads to a significantly higher danger of contagion compared to the “complete” structure as indicated by the baseline matrix. The subsequent estimations will deal with this question.

5. Estimating the danger of contagion in the absence of any “safety net”

We estimate the possibility of contagion by letting banks go bankrupt one at a time and measuring the number of banks that fail due to their exposure to the failing bank. More precisely, the failure of bank j triggers the failure of bank i if $\theta x_{ij} > c_i$, where θ is the loss rate and c_i is bank i 's book capital.

Contagion is not confined to such first-round effects, however. Instead, the failure of a single bank can potentially trigger a whole chain of subsequent failures (the domino effect) even if the initial impact is relatively weak. For example, suppose that bank i fails due to its exposure to bank j . This will cause the breakdown of bank k if its exposure to banks i and j multiplied by the loss rate exceeds its capital, that is, if $\theta(x_{ki} + x_{kj}) > c_k$. This line of argument can also be applied to higher orders. Clearly, the danger of such a chain depends crucially on the structure of the existing interbank relationships, as well as on the loss rate θ .

A necessary condition for contagion to occur is that the volume of a bank's interbank loans exceeds its capital. As can be seen from Table 3, this is generally the case. Of the 3,246 banks that existed in Germany at the end of 1998, 2,758 (85%) had interbank loans in excess of their capital. The average ratio of interbank loans to capital was just below 3, although this is driven by the large number of small cooperative banks in our sample, which tend to hold relatively few interbank assets. The corresponding figures for the Landesbanken, cooperative central banks and other banks are well above 10. This suggests that there may be scope for domino effects.

Table 3
Interbank lending and capital, by broad banking category
(end-December 1998)

	All banks	Commercial banks	Savings bank sector		Cooperative sector		Other banks
			Landesbanken	Savings banks	Central banks	Coop. banks	
No of banks	3,246	331	13	594	4	2,256	48
Of which with loans > cap	2,758	228	13	432	4	2,043	38
Interbank loans/capital	2.96	4.64	13.73	1.95	14.39	2.69	12.94

The choice of the loss rate θ is crucial to the danger of domino effects. Ultimately, the losses incurred in bank failures vary, so the correct choice of θ is by no means obvious. The average loss realised in bank failures in the United States in the mid-1980s was 30% of the book value of the bank's assets. In addition, creditors had to bear administrative and legal costs amounting to a further 10% (James (1991)).¹⁵ Other estimates are much lower. For example, Kaufman (1994) estimates that the creditor banks of Continental Illinois - a bank with large interbank operations that failed due to its exposure to Latin America - would have suffered a loss of only 5% had it not been bailed out by the Fed. When BCCI failed in the early 1990s, creditors at first expected losses of up to 90%, but ended up recovering

¹⁵ This cost represents the discount of the market value of the failed bank's assets relative to their book value.

more than half of their deposits - albeit many years after the failure.¹⁶ Creditor banks of Herstatt have so far received 72% on their assets, with the liquidation of the bank continuing to drag on even a quarter of a century after the closure in 1974.¹⁷ These examples show that it may not be the actual losses borne by the creditor banks that matter, but the expected losses at the moment of failure which determine to which extent the exposure to the failing bank has to be written down, and hence whether the creditor bank becomes technically insolvent or not.¹⁸

The loss rate also depends on the availability of collateral, for example in repos. Unfortunately, our data do not provide information on the share of collateralised positions. Only in 1999, that is, after the end of our sample period and after the major structural change of EMU, was the monthly balance sheet amended to include interbank repos. In summer 2000, such collateralised positions accounted for 6.2% of all interbank lending, although anecdotal evidence suggests that these are mostly with foreign banks.

Table 4
Incidence of contagion
(based on end-December 1998 matrix)

		loss ratio $\theta =$					
		0.75	0.50	0.40	0.25	0.10	0.05
Contagion occurs in of 3,245 cases	full info <i>baseline</i>	3,245 3,245	3,245 3,245	3,245 3,245	3,245 3,245	3,245 3,245	3,245 3,245
Average number of banks affected	full info <i>baseline</i>	30.3 22.8	22.2 17.3	20.0 17.2	18.0 17.0	17.0 17.0	17.0 17.0
% of total assets of banking system affected on average	full info <i>baseline</i>	0.85 0.45	0.66 0.27	0.58 0.27	0.30 0.26	0.26 0.25	0.25 0.25
Maximum number of banks affected	full info <i>baseline</i>	2,444 2,159	1,740 1,047	115 520	31 19	19 17	18 17
Maximum share of total assets affected	full info <i>baseline</i>	76.3 70.9	61.6 55.8	5.0 48.9	0.75 0.57	0.57 0.25	0.30 0.25

Given the difficulties in determining the appropriate loss rate, we follow Furfine (1999) and test for the possibility of contagion using a variety of values for θ , which we assume to be constant across banks. We perform this exercise both for the baseline matrix that uses only aggregate information and for the full information matrix. The difference in the results is due to the additional information that is included in the full information matrix compared to the baseline matrix. This additional information refers to the relationships within the two giro systems and the breakdown into maturity categories. At this stage, we do not explicitly take into account any institutional safeguards present in the German system, which may affect the possibility of contagion, although they may in part be reflected in the bilateral lending matrix X. The results are reproduced in Table 4.

The results of this exercise are rather surprising at first sight. Contagion always occurs *no matter which loss rate we choose*, although the choice of θ determines how many banks are affected on average. There are 17 commercial banks which fail, irrespective of which bank is the first to break down. However, we should not overvalue this result. Firstly, the banks in question are relatively small,

¹⁶ Financial Times, 8 July 1998.

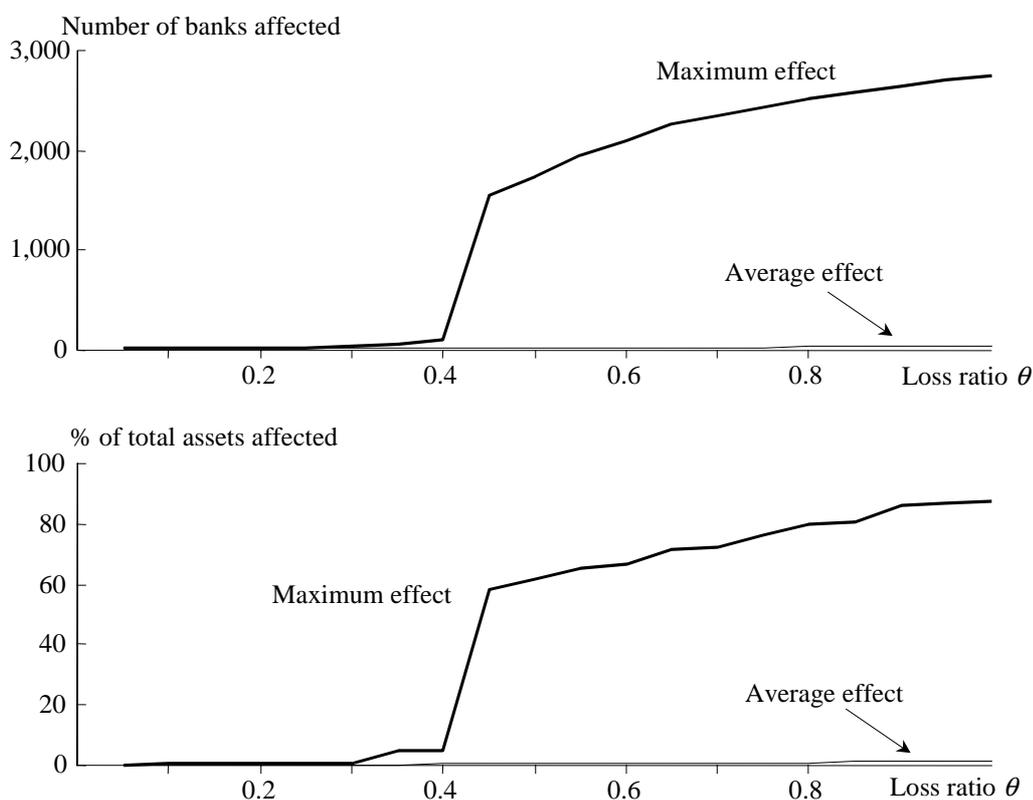
¹⁷ Frankfurter Allgemeine Zeitung, 26 June 1999.

¹⁸ There are several cases where regulators have been lenient in forcing banks to write down assets even though hardly anybody seriously expected their value to recover. Examples are US banks in the wake of the Mexican debt moratorium in 1992 and Japanese banks in the 1990s.

accounting for only a quarter of a percentage point of the total assets of the banking system. Secondly, the fact that 10 of them always break down in the first round, that is, due to the immediate impact, suggests that this is a reflection of our assumption that interbank exposures are as equally spread as is consistent with the data, rather than a description of reality. We would expect that small banks are much more likely to concentrate their lending on a small number of counterparties, which would preclude that the same bank always breaks down in the first round.

While the finding that contagion always occurs may be an artifact of our methodology and should not be taken too seriously, the other results are more interesting. Assuming an (admittedly high) loss ratio of 75%, the maximum number of bank failures caused by domino effects is 2,444. This means that 88% of the institutions where contagion is a possibility because interbank loans exceed capital are affected. As is to be expected, the number of breakdowns decreases with the loss rate. This decline is not linear, however. Figure 4 shows that there is a jump in the severity of contagion if the loss ratio exceeds 40%. This points to the existence of some “critical” θ : if θ is smaller than 0.40, even in the worst case the effects on contagion are rather small. With a loss ratio larger than 0.45, the increase in the damage caused by contagion again seems to be rather moderate. But, if θ lies between 0.40 and 0.45, the loss of assets due to contagion is very sensitive to changes in θ .

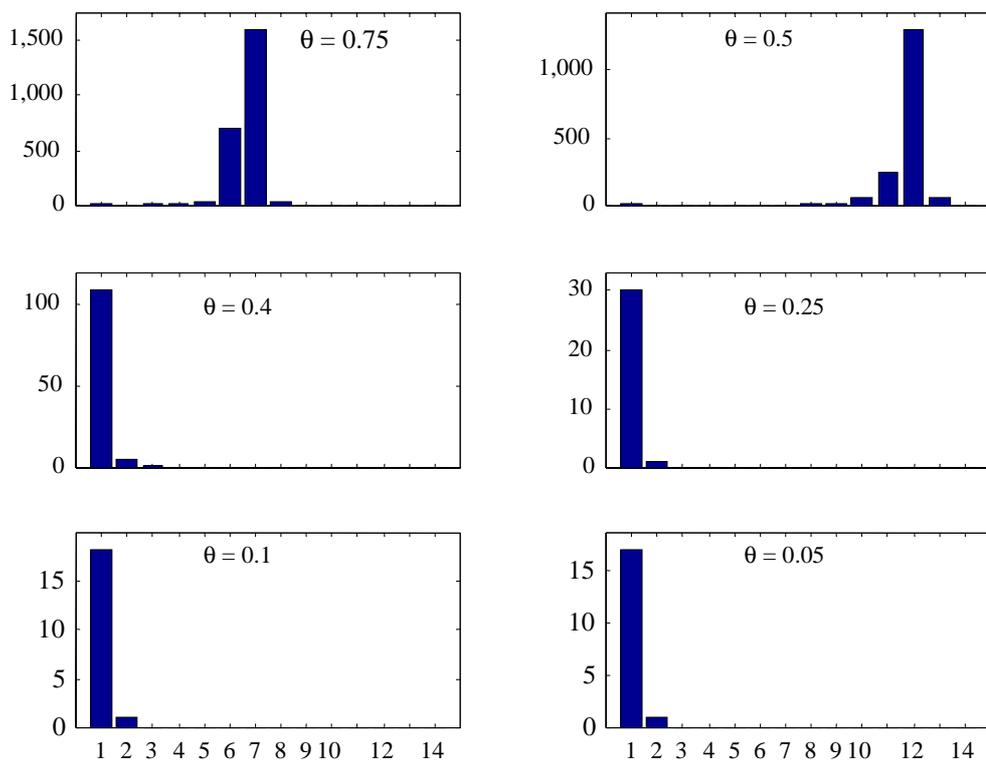
Figure 4
Loss ratio and the severity of contagion
 (full information matrix)



The kink in the relationship between θ and the maximum number of failures is more to the right if one considers the baseline compared to the outcome with the full information matrix. Otherwise, our findings largely confirm the theoretical results of Allen and Gale (2000) that contagion is less likely in a banking system characterised by a “complete” structure of claims relative to a more concentrated system.

Figure 5 plots the spreading of contagion over time.¹⁹ The number of banks that fail in the worst case are plotted on the vertical axis (note the different scales), and the round of failure on the horizontal axis. For small θ s, that is, when the severity of contagion is limited even in the worst possible case, the bulk of banks fail in the first round, with some minor second- and, possibly, third-round effects. For $\theta > 40\%$, the path of contagion is very different. While the first-round effects are only slightly larger than before, they do not peter out but at some point reach a critical mass that leads to the collapse of much of the banking system. However, such widespread contagion is rare even for high θ s. In most cases, the initial shock leads to the breakdown of a number of banks, which in turn causes a smaller number of further failures. Then the process stops. This, together with the high values of θ necessary to cause prolonged chains of failure may explain why Furfine (1999) does not obtain higher than second-round effects. The reason why we find such a striking difference between the contagion patterns of “low” and “high” loss ratios - with the critical value of θ somewhere around 0.40 - can intuitively be explained by looking at what the failure of a single bank means for the continuation of the contagion process. Such an event generally has two implications: on the one handside, it reduces the pressure that drives the contagion process; on the other, the failure of the bank at hand also contributes to the continuation of this process, because it can affect other banks as well. These two opposing effects jointly determine the probability with which the failure of a single bank adds to the dynamics of the contagion process or dampens them. Their relative importance is determined mainly (1) by the interbank asset structure of the bank (which indicates how much the bank is affected by the preceding failures of other banks), (2) by the interbank liability structure of the bank (which is central to how the shock is transferred to other banks) and (3) by the loss ratio θ (which determines the strength of the transmission of the shock).

Figure 5
Number of banks affected in round r =
(Worst case, full information matrix)



¹⁹ Here we equate “time” and “rounds”.

This “switching” in the time pattern of contagion has important implications for the regulation of banks and the design of institutional safeguards. It shows that it may be possible to stop the most severe scenarios with relatively low costs at an early stage, that is, before the dramatic wave of bank failures sets in. It must nevertheless be borne in mind that the “rounds” on the horizontal axis of Figure 5 are not necessarily comparable to discrete “time periods” in the usual sense. Even very “late” rounds can actually already occur a very short (calendar) time period after the initial shock, so that there may be virtually no possibility for a regulator to react to a process once it has started.

Table 5 shows the contagion effects by bank category of the initial shock for the loss rates of 0.50 and 0.10, respectively. As one would expect, the failure of savings banks or cooperative banks has very little impact on other banks. Even in the case of a loss rate of 0.50, the damage would remain below 1% of total assets. The largest contagion effects occur if a head institution of one of the giro systems fails. With $\theta = 50\%$, the failure of a Landesbank could trigger the failure of up to 1,740 banks and more than 60% of total assets. On average, 286 banks are affected, corresponding to 10% of total assets. The effects of the failure of a cooperative central bank are similar. Again, 1,740 banks - or more than 60% of the banking system in terms of assets - would fail in the worst case, and 370 banks (13% of total assets) on average. The severity of contagion is much smaller if a loss rate of only 0.10 is assumed: in neither case would the damage be more than 0.6% of total assets. What is striking is the limited effect of the failure of a commercial bank (the category that includes the large banks). Even in the worst case, it would only cause a loss of 40 banks or 1.2% of total assets. This may in part be due to the fact that a large proportion of the interbank claims of large commercial banks are on foreign banks which are not included in our data set.

Table 5
Incidence of contagion, by bank category
(based on end-December 1998 full information matrix)

	Loss rate $\theta =$	Max no of failures (“worst” case) in brackets: % of total assets		Mean no of failures (“normal” case) in brackets: % of total assets		Average% share in total interbank loans
		in $r = 1$	sum $r > 1$	in $r = 1$	sum $r > 1$	
Commercial banks	0.50	22 (0.61)	18 (0.58)	13.9 (0.22)	6.2 (0.36)	16.0
	0.10	17 (0.25)	15 (0.25)	13.9 (0.22)	3.1 (0.04)	
Savings bank sector	0.50	20 (0.75)	1,720 (60.17)	17.9 (0.37)	268.4 (9.51)	30.0
	0.10	17 (0.25)	0 (0.00)	17.0 (0.25)	0.0 (0.00)	
Savings banks	0.50	17 (0.25)	18 (0.58)	11.8 (0.21)	8.2 (0.37)	7.7
	0.10	17 (0.25)	15 (0.25)	11.8 (0.21)	5.2 (0.05)	
Cooperative sector	0.50	195 (1.56)	1,545 (60.06)	57.6 (0.54)	312.2 (12.28)	7.8
	0.10	18 (0.30)	1 (0.27)	17.2 (0.26)	0.2 (0.05)	
Cooperative banks	0.50	17 (0.25)	18 (0.58)	9.3 (0.17)	10.7 (0.41)	7.0
	0.10	17 (0.25)	15 (0.25)	9.3 (0.17)	7.7 (0.08)	
Other banks	0.50	25 (3.6)	1,715 (56.95)	15.8 (0.32)	40.6 (1.52)	31.5
	0.10	17 (0.25)	15 (0.25)	15.5 (0.24)	1.5 (0.02)	

6. Banking supervision, regulation and “safety nets”

The preceding analysis ignored the prudential regulation of banks and the existence of “safety nets”, both of which may limit the probability of our scenarios. In particular, as will become clear below, the relevant safety mechanisms that are in place in Germany are designed to prevent the failure of Landesbanken and cooperative central banks. The worst case scenarios identified in the previous section are therefore virtually impossible. In this section, we address this omission and extend our analysis to incorporate the safeguards present in the German system.

Prudential supervision limits the danger of contagion in a number of ways.²⁰ Firstly, banking supervision aims to reduce the incidence of failures by forcing (or encouraging) banks to behave more prudently. In our model, this corresponds to minimising the probability that a “shock” (ie the initial failure of a bank) will occur in the first place. Secondly, if banks do fail, swift action by the regulator could ensure that banks are liquidated before the losses become too large. This would be reflected by a low loss ratio θ . Finally, banking regulation may limit the exposure of banks to any single debtor or group of debtors, which in turn reduces the scope for contagion. For example, in Germany bank loans to a single debtor should in principle not exceed 25% of the capital of the creditor, although interbank loans are partly exempted. For the purpose of our paper, such regulations are reflected in the pattern of interbank exposures.

While prudential supervision does play an important role in reducing the risk of contagion *ex ante*, it cannot stop this process once it is under way. In this case, two additional mechanisms could step in. The first is the Liko-Bank, a bank owned by the banking system with participation by the Bundesbank, which exists solely to provide liquidity to illiquid but solvent banks. The second mechanism is the insurance of interbank deposits.

Deposit insurance can halt contagion through credit exposures in the interbank market - as opposed to contagion through bank runs - only if it covers deposits by banks as well as deposits by non-banks. While the statutory deposit insurance applies to non-bank deposits only (Deutsche Bundesbank [2000b]), for savings and cooperative banks, including their respective head institutions, this is supplemented by so-called “institutional guarantees”. Both the savings banks’ and cooperative banks’ associations operate funds backed by mutual guarantees which serve to recapitalise member institutions in the event that they become insolvent.²¹ In addition to the guarantee fund, savings banks are also explicitly guaranteed by the corresponding local or regional government. There are also a (small) number of public banks guaranteed by the federal government.²²

We incorporate these safeguards into our analysis by assuming that:

- savings banks never fail;
- public banks guaranteed by the federal government never fail; and
- cooperative banks never fail in the first round and hence never trigger contagion. In further rounds, they fail only if the exposure of the aggregate cooperative sector to the banks which failed in previous rounds exceeds aggregate capital. This is equivalent to assuming unlimited cross-guarantees and may thus overstate the effectiveness of the “safety net”. As a consequence, either no cooperative bank fails or the complete cooperative sector, including the cooperative central banks, breaks down.²³

Since in the previous section we found that the largest effects were caused by the breakdown of a head institution of the savings or the cooperative sector, we should expect much lower contagion once

²⁰ We use the terms “supervision” and “regulation” interchangeably.

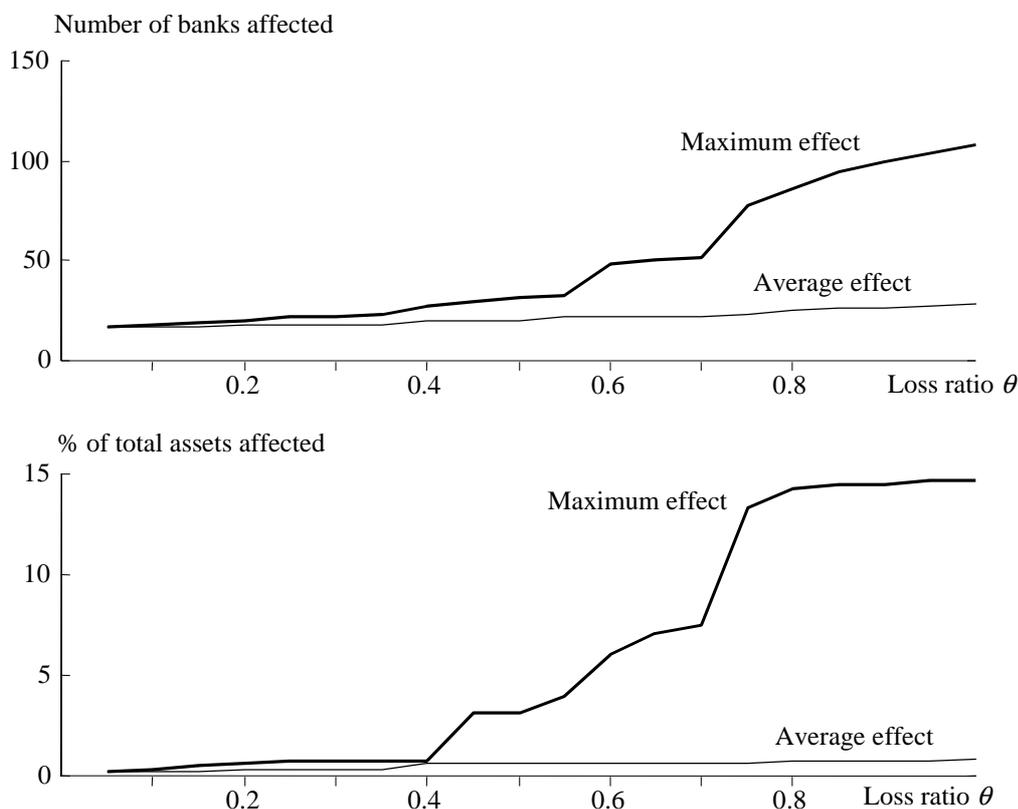
²¹ In order to alleviate moral hazard problems, failed cooperative banks usually lose their independence and are merged with stronger banks, which obtain support from the guarantee fund. A detailed description of the guarantee schemes is contained in Deutsche Bundesbank (1992) and (2000b).

²² For example, the Kreditanstalt für Wiederaufbau (KfW).

²³ The charter of the safety fund for cooperative banks stipulates that each bank has to provide guarantees amounting to 60% of its statutory credit provisions. This is in addition to the contributions to the deposit guarantee fund, which covers non-bank deposits.

we incorporate the existing safeguards. The fact that during the past half-century the stability of the German banking system has never been called into question despite a number of bank failures can be seen as an indication that the existing banking supervision and safety mechanisms have worked well in the past. This effectiveness is clearly confirmed by our results, presented in Figure 6.

Figure 6
Loss ratio and the severity of contagion in the case of “perfect safety nets”
 (full information matrix)



We find that contagion is much more limited in scope but remains a possibility even if we incorporate safety mechanisms into our analysis. As before, we find that the maximum percentage of assets affected remains relatively flat for loss ratios below 40% but increases for higher values. For θ s in excess of 75%, about 100 banks may be affected in the worst case of contagion. This corresponds to 15% of the banking system in terms of assets, which is considerably below the corresponding values of up to 80% if one ignores the safety mechanisms. In particular, we find that the cooperative system never fails, even for the highest θ s.

When interpreting our results, it has to be borne in mind that we do not say anything about the *efficiency* of “safety nets”, since we do not incorporate their direct and indirect costs.

7. Conclusions

Credit risk associated with interbank lending may lead to domino effects, where the failure of one bank results in the failure of other banks not directly affected by the initial shock. Recent work in economic theory shows that this risk of contagion depends on the precise pattern of interbank linkages. We use balance sheet information to estimate matrices of bilateral credit relationships for the German banking system. In contrast to commercial data sets, our data covers the entire banking system, so all domestic interbank loans and deposits add up to zero in principle. In their submissions to the Bundesbank, which provide the basis of our analysis, banks have to give a detailed breakdown of their

interbank assets and liabilities, showing maturity categories and whether or not the counterparty is a head institution of the giro system the respective bank belongs to. This permits us to estimate a matrix of bilateral credit exposures for each maturity and banking group, thus imposing much more structure on the problem than would be possible with aggregate interbank loans and deposits alone. The estimated system-wide matrix differs considerably from a matrix estimated from aggregate exposures (across all maturities and banking groups) alone.

We find that interbank lending in Germany is characterised by a two-tier structure. The first tier consists of most savings banks and virtually all cooperative banks, which transact mainly with the Landesbanken and cooperative central banks. The second tier consists of the head institutions of the two giro systems, the commercial banks and the “other banks”.

Our results suggest that domino effects through interbank credit exposures are possible.²⁴ While the danger of contagion is normally confined to a limited number of relatively small banks, bank failures that affect a sizeable part of the banking system remain a possibility even if we explicitly take into account safety mechanisms like the institutional guarantees for savings banks and cooperative banks. In the absence of such mechanisms, the effects of the breakdown of a single bank could potentially be very strong indeed. This cannot be taken as a statement on the desirability of these mechanisms or a system with publicly owned banks, though, as we do not consider the incentive effects they are associated with.

Not surprisingly, the danger of contagion crucially depends on the losses experienced by the creditor bank in the case of insolvency of the debtor bank. We find that large-scale contagion can in any case only occur if the loss rate on interbank loans exceeds a value of approximately 40%.

Our findings have important implications for banking regulation. The regulator can minimise the danger of contagion in a number of ways. Firstly, it can reduce the probability of the initial shock that could trigger contagion by encouraging banks to behave more prudently. Secondly, if a bank does fail, the regulator should ensure a quick and orderly liquidation before the ratio of losses to assets becomes too large. And finally, banking regulation can limit the exposure of banks to individual debtors.

²⁴ We do not consider other channels for contagion, like runs by non-banks.

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