Interest Rate Spreads Implicit in Options:

Spain and Italy against Germany*

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Abstract

The options premiums are frequently used to obtain probability density functions (pdfs) for the prices of the underlying assets. When these assets are bank deposits or notional Government bonds it is possible to compute probability measures of future interest rates. Recently, in the literature there have been many papers presenting methods of how to estimate pdfs from options premiums. Nevertheless, the estimation of probabilities of forward interest rate functions is an issue that has never been analysed before. In this paper, we propose such a method, that can be used to study the evolution of the expectations about interest rate convergence. We look at the cases of Spain and Italy against Germany, before the adoption of a single currency, and conclude that the expectations on the short-term interest rates convergence of Spain and Italy vis-à-vis Germany have had a somewhat different trajectory, with higher expectations of convergence for Spain.

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I. Introduction

Derivative prices supply important information about market expectations. They can be used to obtain probability measures about future values of many relevant economic variables, such as interest rates, currency exchange rates and stock and commodity prices (see, for instance, Bahra (1996) and Söderlind and Svensson (1997)).

However, many times market practitioners and central bankers want to know the probability measure of a combination of economic variables, which is not directly associated with a traded financial instrument. This paper presents and illustrates a simple method of obtaining a density function of a combination of economic variables, $f(x,y)$, when there is an option for the variable x and another for the variable y but there is not an option for the variable $f(x,y)$.

To get the implied probability density function on the combination $f(x,y)$, when no options are traded on $f(x,y)$ itself, requires knowledge of the implied correlation between x and y. If variables x and y are exchange rates the implied correlation may be obtained from currency option implied volatilities. Campa and Chang (1997) show how correlations can be calculated via the triangular relationship that exists between options on different currency pairs. However, there is no known procedure of computing implied correlations when variables x and y are not exchange rates. In such cases, proxies (for instance, historical correlations or cross-section correlation between the futures prices for different maturities) are the only alternative. Given that the proxies may not coincide with the market implied correlations, it is necessary to carry out a sensitivity check of the results to the correlation assumption.

We restrict our attention to the case $f(x,y) = x-y$, being x a 3-month forward interest rate of the Spanish peseta or the Italian lira and y a 3-month forward interest rate of the German mark. Such method is particularly relevant for assessing convergence probabilities of monetary policies. This was a relevant issue in the assessment of the transition towards EMU, having been an important subject of empirical work by central banks and investment banks over the last years.

In this context, "EMU convergence calculators" were developed to compute the probability of a given country joining the EMU.[1] These convergence calculators have as domain the set of forward interest rate differentials and as counterdomain the interval [0,1]. However, this procedure of computing convergence indicators is controversial, to say the least, since the link between forward interest rate differentials and convergence probabilities is not clear. In fact, those differentials change due to other factors than the referred probabilities, namely those related with the business cycle and the economic convergence.

Moreover, the EMU calculators are based on the estimation of the interest rate spread that would happen if the country did not join the EMU. This spread has been estimated using the estimators of a regression between the observed spreads and term structure variables related to international risk-aversion and liquidity during a recent period in which the EMU probability was near zero. The identification of such sample period is difficult. Besides, the estimators' values are subject to the Lucas critique or other structural breaks. Therefore, the regression results are evidently conditioned by the sample period considered, as well as by the explanatory variables chosen.

The method presented in this paper goes further than previous methods that compute probabilities of future events, as it is based on risk-neutral probabilities that can be estimated directly from the prices of traded financial instruments. However, it has the disadvantage of the information about options premiums traded in exchanges being limited to one year horizons, while, for instance, indicators built from the current term structure of interest rates enable the computation of forward interest rates for longer horizons.

We focus on the probability of short-term interest rate convergence instead of on the probability of EMU participation for two reasons. The first has to do with the fact that a zero interest rate differential does not imply a unitary probability of monetary union. The second has to do with the fact that the monetary integration in January 1999 did not preclude positive interest rate differentials in earlier settlement dates.[2]

In fact, the European monetary unification, on 1/1/1999, implied a zero interest rate differential between Germany and the remaining participating countries only at that time. The announcement of the participating countries took place in the Brussels summit of 1-3 May 1998. The bilateral parities were defined as the ERM-EMS central exchange rates, which were different from the spot exchange rates. Thus, the associated spot interest rate differentials would likely be different from zero and would converge to zero, as EMU starting date would approach. Therefore, a small probability of a country short-term interest rates convergence for dates before 1/1/1999 did not necessarily mean that the market expected that country would stay out of the European Monetary Union.

The evidence presented in the paper suggests that the options market participants did not consider likely that there would be complete convergence of interest rates between Germany, Spain and Italy before June 1998. There is also evidence that the Italian short-term interest rate convergence was expected to be behind the Spanish one. Moreover, there is evidence that the option markets expected both spreads to be smaller in June of 1998 than in March 1998.

This paper has four more sections. The second section describes the model; the third section contains the estimation technique; the data and the empirical results obtained are presented in the fourth section and the fifth section concludes.

II. Model

Let S_1 and S_2 be two futures contracts. We assume that $(\ln S_{1t}, \ln S_{2t})$ are stochastic variables distributed as a mixture of two bivariate normal distribution. Thus, the bivariate probability density function (pdf) of (S_{1t}, S_{2t}) is a mixture of two bivariate lognormal, given by:

(1)

$$f(S_{1t}, S_{2t}) = \theta_t \left((2\pi)^2 |\mathbf{\Sigma}_{1t}|\right)^{-1/2} \exp\left(-\frac{1}{2}\left([\ln S_{1t} \ \ln S_{2t}] - \boldsymbol{\mu}'_{1t}\right)\mathbf{\Sigma}_{1t}^{-1}\left([\ln S_{1t} \ \ln S_{2t}]' - \boldsymbol{\mu}_{1t}\right)\right) \times$$
$$\times \frac{1}{S_{1t}S_{2t}} + (1-\theta_t)\left((2\pi)^2 |\mathbf{\Sigma}_{2t}|\right)^{-1/2} \times$$
$$\times \exp\left(-\frac{1}{2}\left([\ln S_{1t} \ \ln S_{2t}] - \boldsymbol{\mu}'_{2t}\right)\mathbf{\Sigma}_{2t}^{-1}\left([\ln S_{1t} \ \ln S_{2t}]' - \boldsymbol{\mu}_{2t}\right)\right)\frac{1}{S_{1t}S_{2t}}$$

where $\boldsymbol{\mu}'_{kt} = [\mu_{k1t} \ \mu_{k2t}]$, $\mathbf{\Sigma}_{kt} \equiv \begin{bmatrix} \sigma^2_{k1t} & \rho_{kt}\sigma_{k1t}\sigma_{k2t} \\ \rho_{kt}\sigma_{k1t}\sigma_{k2t} & \sigma^2_{k2t} \end{bmatrix}$ and $\theta \in [0,1]$, being k = 1,2. The parameters $\boldsymbol{\mu}'_{kt}$ are the expected value vectors of the corresponding bivariate normal distributions, $\mathbf{\Sigma}_{kt}$ are the covariance matrices of the corresponding bivariate normal distributions and θ_t and $(1-\theta_t)$ are the weights of the distributions.

The marginal pdf for S_{it}, $f'(S_{it})$, is:

(2)
$$f'(S_{it}) = \theta_t \left((2\pi)^2 \sigma_{1it}^2\right)^{-1/2} \exp\left(-\frac{1}{2\sigma_{1it}^2}(\ln S_{it} - \mu_{1it})^2\right)\frac{1}{S_{it}} + \\ + (1-\theta_t)\left((2\pi)^2 \sigma_{2it}^2\right)^{-1/2} \exp\left(-\frac{1}{2\sigma_{2it}^2}(\ln S_{it} - \mu_{2it})^2\right)\frac{1}{S_{it}}$$

To get an unconditional pdf for S_{1t} - S_{2t} we transform the variables.[3] Define $Y_{1t}=S_{1t}-S_{2t}$ and $Y_{2t} = S_{2t}$. The bivariate pdf of Y_{1t} and Y_{2t} is:

(3)
$$f^*(Y_{1t}, Y_{2t}) = \theta_t \left((2\pi)^2 |\mathbf{\Sigma}_{1t}|\right)^{-1/2} \times \\ \times \exp\left(-\frac{1}{2}\left([\ln(Y_{1t}+Y_{2t})\ \ln Y_{2t}] - \boldsymbol{\mu}'_{1t}\right)\mathbf{\Sigma}_{1t}^{-1}\left([\ln(Y_{1t}+Y_{2t})\ \ln Y_{2t}]' - \boldsymbol{\mu}_{1t}\right)\right) \times \\ \times \frac{1}{(Y_{1t}+Y_{2t})Y_{2t}} + (1-\theta_t)\left((2\pi)^2 |\mathbf{\Sigma}_{2t}|\right)^{-1/2} \times \\ \times \exp\left(-\frac{1}{2}\left([\ln(Y_{1t}+Y_{2t})\ \ln Y_{2t}] - \boldsymbol{\mu}'_{2t}\right)\mathbf{\Sigma}_{2t}^{-1}\left([\ln(Y_{1t}+Y_{2t})\ \ln Y_{2t}]' - \boldsymbol{\mu}_{2t}\right)\right) \times \\ \times \frac{1}{(Y_{1t}+Y_{2t})Y_{2t}}$$

Thus, the marginal pdf for Y_{1t} corresponds to the pdf for the interest rate differential, being determined as:

(4)
$$f^{**}(Y_{1t}) = \int_{-\infty}^{+\infty} f^*(Y_{1t}, Y_{2t})\, dY_{2t}$$

Since the algebraic integration of $f^*(Y_{1t}, Y_{2t})$ with respect to Y_{2t} revealed itself complex, we decided to get the marginal pdf of Y_{1t} through a computer code, which integrates numerically $f^*(Y_{1t}, Y_{2t})$ with respect to Y_{2t}.[4]

III. Estimation

To compute the conditional distribution of interest rates, as well as the marginal distribution of interest rate differentials, it is necessary to know the vectors of expected values $\boldsymbol{\mu}'_{kt}$ and the matrices of covariances $\boldsymbol{\Sigma}_{kt}$. The expected values and the variances in the matrices $\boldsymbol{\Sigma}_{kt}$ can be estimated from the options premiums on futures contracts. As it is well known, when investors are risk-neutral, an European call option on a futures contract at time t with strike price X and term to maturity τ, $C(X,\tau)$, obeys the expression:

$$\text{(5)} \qquad C(X,\tau) = E_t e^{-r_{t,\tau}\tau} \max[S_{iT} - X, 0]$$

$$= e^{-r_{t,\tau}\tau} \int_X^{\infty} (S_{iT} - X) q(S_{iT}) dS_{iT}$$

where E_t is the conditional (on information known at date t) expected value operator, S_{iT} is the price of the underlying asset at maturity date T, r a riskless interest rate for maturity τ (being $\tau = T\text{-}t$), and $q(S_{iT}) \sim \theta LN(\mu_{1i},\sigma_{1i}) + (1-\theta) LN(\mu_{2i},\sigma_{2i})$ (i.e., a combination of two lognormal pdfs, being i = 1, 2). The solution to the problem below gives an estimate for the nine parameters of $q(S_{iT})$:

$$\text{(6)} \qquad \underset{\substack{\theta,\mu_{11t},\mu_{12t},\mu_{21t},\mu_{22t},\\ \sigma_{11t},\sigma_{12t},\sigma_{21t},\sigma_{22t}}}{Min} \sum_{j=1}^{M} \left[\hat{C}_{1,j}(X_{1,j},\tau) - C_{1,j}^{0}\right]^2 + \sum_{j=1}^{N} \left[\hat{C}_{2,j}(X_{2,j},\tau) - C_{2,j}^{0}\right]^2,$$

where M and N represent the number of strike prices available for each option, $X_{1,j}$ and $X_{2,j}$ the strike prices observed for each option, $\hat{C}_{1,j}$ and $\hat{C}_{2,j}$ are the estimated option premiums for each option and strike price for the maturity τ and $C_{1,j}^{0}$ and $C_{2,j}^{0}$ are the premiums observed for each option and strike price.[5]

There is a relationship between the elements off the diagonal of the matrices $\mathbf{\Sigma}_{1t}$ and $\mathbf{\Sigma}_{2t}$ and the correlation between $\ln S_{1t}$ and $\ln S_{2t}$. It can be shown, using the moment generating function technique, that the resulting correlation between $\ln S_{1t}$ and $\ln S_{2t}$ is:

$$(7)\ \rho = \frac{\theta(\mu_{11t}\mu_{12t} + \rho_{1t}\sigma_{11t}\sigma_{12t}) + (1-\theta)(\mu_{21t}\mu_{22t} + \rho_{2t}\sigma_{21t}\sigma_{22t}) - (\theta\mu_{11t} + (1-\theta)\mu_{21t})(\theta\mu_{12t} + (1-\theta)\mu_{22t})}{\prod_{j=1}^{2}\left[\theta(\mu_{1jt}^2 + \sigma_{1jt}^2) + (1-\theta)(\mu_{2jt}^2 + \sigma_{2jt}^2) - (\theta\mu_{1jt} + (1-\theta)\mu_{2jt})^2\right]^{1/2}}$$

The assumptions about the elements off the diagonal matrices, $\mathbf{\Sigma}_{kt}$'s, will be discussed in the next section.

IV. Data and empirical results

We applied the model described to the conditional and unconditional differentials between 3-month German mark and Italian lira interest rates, on one hand, and German mark and Spanish peseta, on the other. For that purpose we used daily quotes between 18/3/1997 and 7/7/1997 of LIFFE's futures options on 3-month interest rates (for the mark and the lira) and MEFF Renta Fija's futures options on 3-month interest rates (for the peseta), with maturity on March 1998 and June 1998.[6]

Given that a fast nominal convergence process marked this period, we suspected that neither the historical correlation nor the futures prices correlation might estimate correctly the true correlation between the interest rates.[7] Thus, we opted for assuming several correlation figures and assessing the sensitivity of the results to those figures. We verified that the higher the values for •$_1$ and •$_2$ we consider, the more probability would be concentrated in higher differentials. That has to do with the fact that during the sample period the 3-month interest rate differentials were still substantial (see figure 1).

Consequently, the upper and the lower bounds of the interest rate convergence probability are obtained when the $\bullet_K$'s are near –1 and 1, respectively. Therefore, we initially considered $\bullet_1$, $\bullet_2$ = 0.99,[8] in order to characterise the less favourable scenario to lower interest rate differentials. The unconditional interest rate differential pdfs, estimated according to (6), evidence lower expected differentials for the Spanish peseta and for June 1998 (figure 2).

According to the shape of the estimated pdfs, the statistical measures of the distributions evidence a significant positive skewness in both differential pdfs for March 1998, as the mode is consistently lower than the median and the latter is consistently lower than the mean (figure 3). This difference is reduced only in the last sample days. The shape of the estimated pdfs is different for June 1998, as they consistently exhibit a negative skewness.

The results suggest that major improvements about the prospects on interest rate convergence of the Spanish peseta in March 1998 were achieved between March 1997 and July 1997, as the mean decreased from around 2.1 to below 1.5. Further convergence was expected to be done between March 1998 and June 1998, given that the mean of the distribution for June 1998 was between 0.75 and 1 in the sample considered. In spite of the large Italian lira differential, the evolution was not so remarkable, but the results obtained also show an expectation of lower spreads in June 1998 than in March 1998.

The distribution functions for the interest rate differentials can be used to get an indicator about the expectations of the lira and the peseta short-term interest rate convergence. This indicator could be the cumulative probability in zero, or any low enough differential that may be considered as corresponding to interest rate convergence.

The distribution functions for the interest rate differentials (peseta-mark and lira-mark) revealed small lower bounds to the probabilities for non-positive interest rate differentials (figure 4). However, the figures for the differentials in June 1998 (which were for Spain between 10% and 14% and for Italy up to 4%) were clearly higher than for March 1998. This may reflect that the markets were not completely sure of the peseta and lira integration in the European Monetary Union, or that they expected the continuation of convergence between June 1998 and the starting of the European Monetary Union in 1/1/1999.

As it was referred at the beginning of this section, until now we have assumed a correlation coefficient for both distributions of 0.99. Consequently, it is important to assess the sensitivity of the results to the correlation coefficient figures.

That analysis was performed for the Spanish differentials in March 1998. The results obtained confirm the conjecture that the higher the correlation coefficient, the smaller the probability of small interest rate differentials. Furthermore, our results also show that higher correlation coefficients imply lower dispersion and less smooth curves (figure 5).[9]

We also performed a sensitivity test to the estimation method. We compare the results obtained with a mixture of two lognormal distributions specification, previously presented, with those obtained with a lognormal distribution specification. In general, the pdfs obtained with the two specifications are rather different. The two-lognormal distribution is more asymmetric, sometimes has more than one mode and has fatter tails. As a result, the indicator we used to assess interest rate convergence, the probability of non-positive interest rate differential, assumes higher values in the two-lognormal specification.

In figure 6 we have the Spanish differential pdfs for March 1998 in 20/3/97 and 7/7/1997 (assuming 0.99 correlation coefficients). The day 7/7/1997 represents better the more frequent differences associated with the two estimation methods. In that sense, that day is a more standard day than the day 20/3/97.

V. Conclusion

The option premiums have been recently used to extract information about the expected future behaviour of many economic variables, in particular interest rates. Nevertheless, this literature does not have anything to say about the expected future behaviour of interest rate differentials. It is this paper objective to perform that task, with the estimation of the density function of short-term interest rate differentials.

The estimation of unconditional pdfs for the peseta-mark and lira-mark differentials shows that the conjecture that Spain was ahead of Italy in the convergence process was correct, at least with respect to short-term interest rates.[10] Moreover, the exercise shows that financial markets expected further convergence in the short-term interest rates after June 1998.

The interest rate differentials pdfs can also be used, for instance, to identify the convergence of long-term interest rates, supplying, in this case, useful information about the expectations of sovereign and liquidity risk and/or accomplishment of the long-term interest rates convergence criteria.[11]

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[1] See, for instance, De Grauwe (1996), Dillén and Edlund (1997), Favero *et al.* (1997) and JPMorgan (1997).

[2] At the time the exercise was done March and June 1998 were the maturity dates available nearest to January 1999.

[3] The conditional pdfs are obtained from the bivariate and marginal pdfs. The conditional pdf for S_{1t} given S_{2t} is the ratio $\frac{f(S_{1t}, S_{2t})}{f(S_{2t})}$. If we had chosen θ = 1, after substitution and algebraic manipulation, the following expression for the conditional pdf of S_{1t}, could have been obtained:

$$\exp\left\{-\frac{1}{2\sigma_{11t}^2\left(1-\rho_{1t}{}^2\right)}\left[\ln S_{1t} - \mu_{11t} - \frac{\rho_{1t}\sigma_{11t}}{\sigma_{12t}}\left(\ln S_{2t} - \mu_{12t}\right)\right]^2\right\} \times \left[2\pi\sigma_{11t}^2\left(1-\rho_{1t}^2\right)\right]^{-1/2}\frac{1}{S_{1t}}.$$

[4] The grid for Y_{2t} was chosen between with an interval of 10 basis point.

[5] About different alternative estimation techniques of the distribution for S_T see, e.g., Bahra (1996).

[6] Even though the options traded in LIFFE are American, since they are pure options, they can be treated as European options.

[7] Simple historical correlation coefficients, as well as exponentially weighted moving average of the correlation coefficients (between the futures contracts prices with term to maturity between September 1996 and June 1998, in the period between March 14, 1995 and the day for which the pdf is estimated), are in general positive and high. On the other hand, daily cross-section correlation coefficients between the interest rates implicit in futures prices for different settlement dates are in general non-positive.

[8] For the period under consideration this corresponds to a correlation coefficient between $\ln S_{1t}$ and $\ln S_{2t}$ in the interval [0.3,0.7]. These values are almost always below the corresponding historical correlation coefficients.

[9] Similar conclusions are also obtained for conditional density functions.

[10] As we are estimating risk-neutral density functions, any difference in risk-aversion patterns is not taken into account, which means that some of the probability differences between the Italian and the Spanish spread, as well as their time changes, may have been motivated by risk-aversion differences or variations.

[11] It is also possible to extract information about long-term interest rate differentials expectations from the prices of *DIFF* future contracts prices traded at MEFF Renta Fija. It would be interesting to evaluate the consistency of the information obtained by this method, confronting the expected values estimated with the differentials implicit in the quotes of the *DIFF* contracts.

Figure 1
3-month interest rates

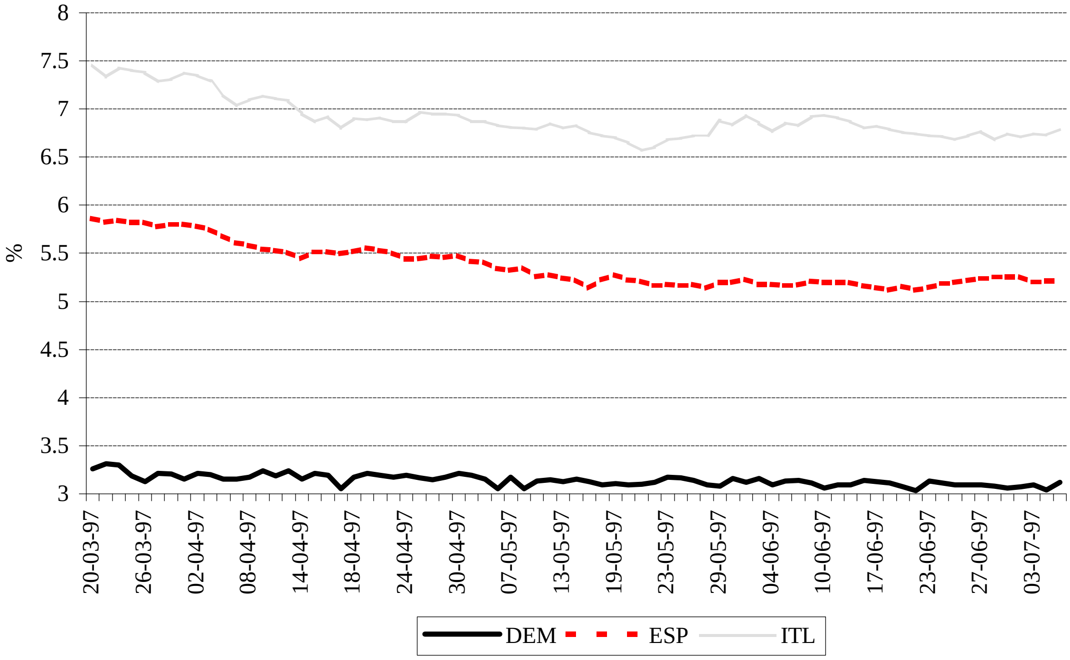


Figure 2
Unconditional density functions of 3-month interest rate differentials vis-à-vis German mark (ρ1, ρ2=0.99)

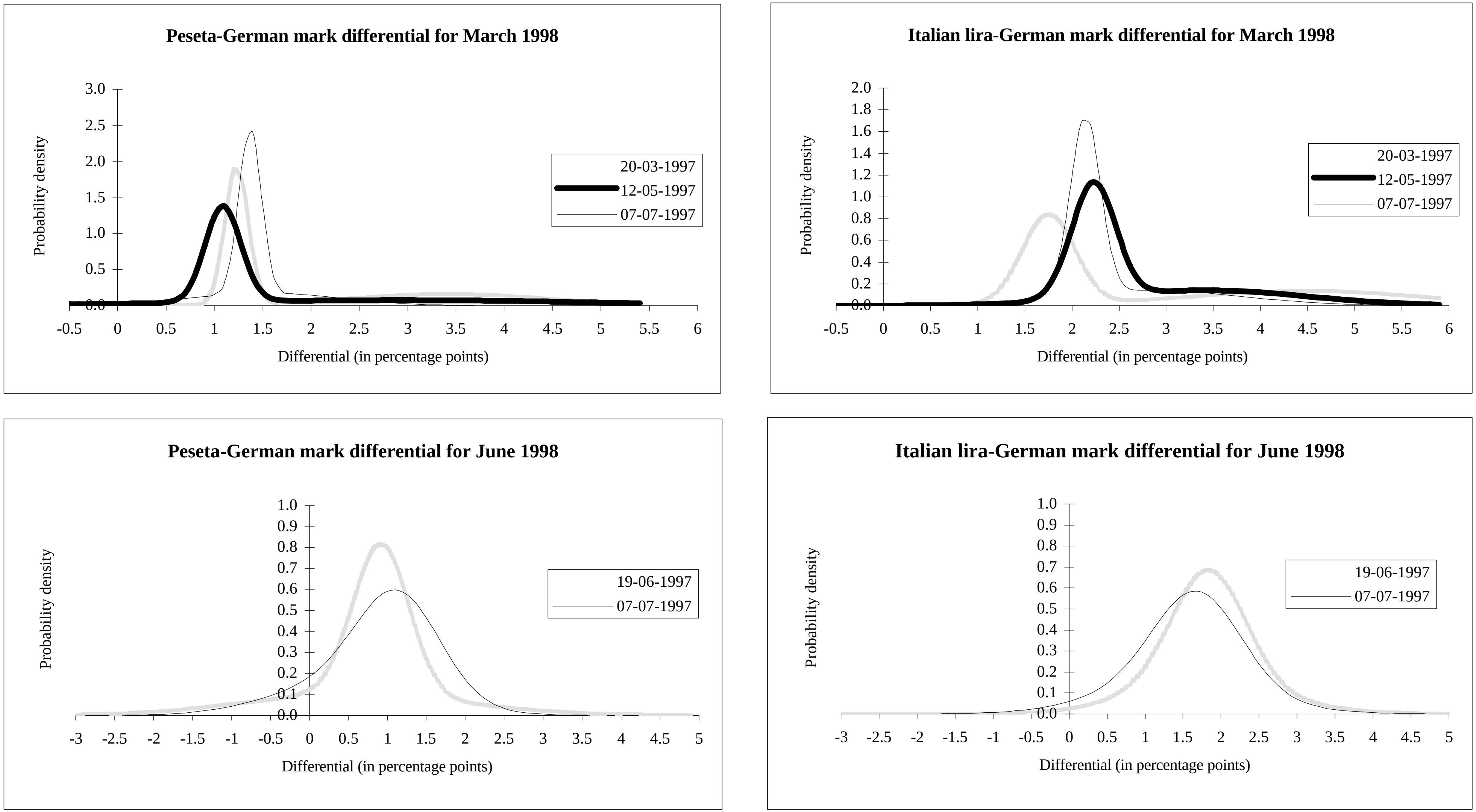
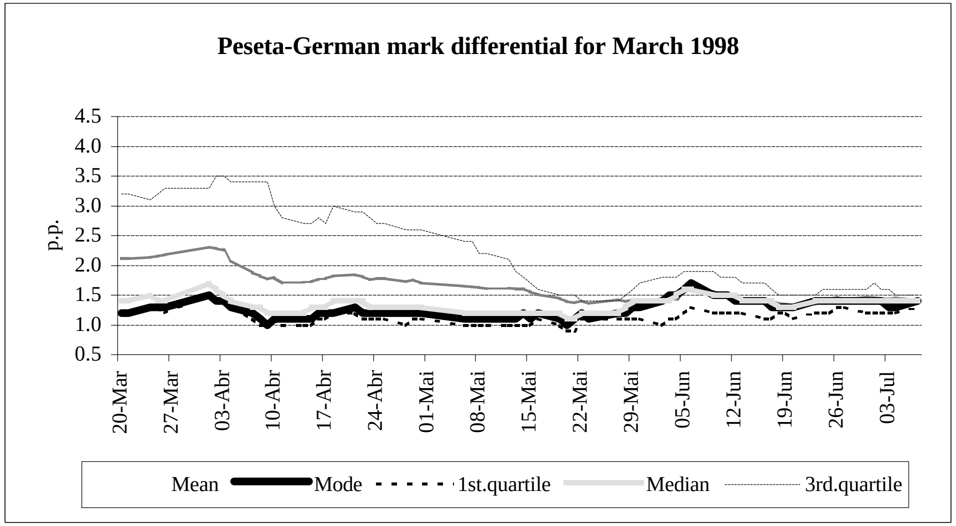
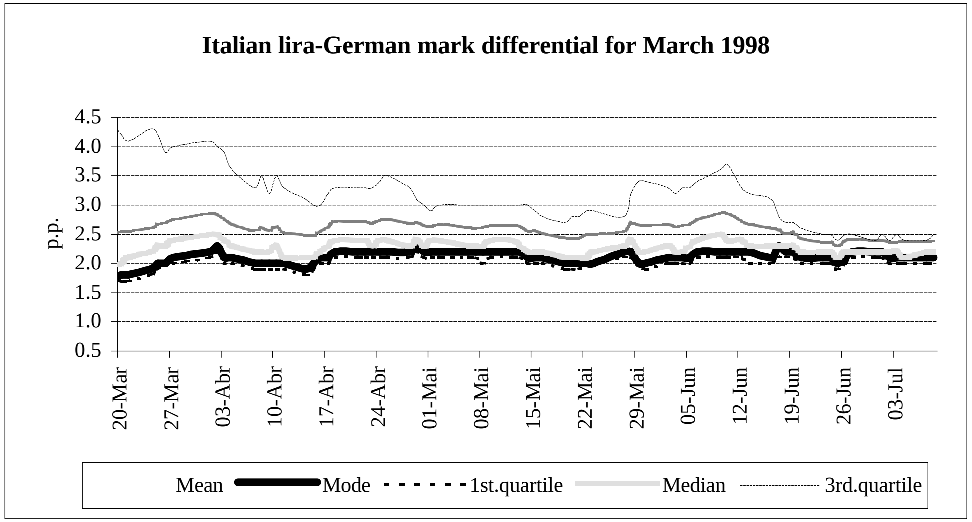
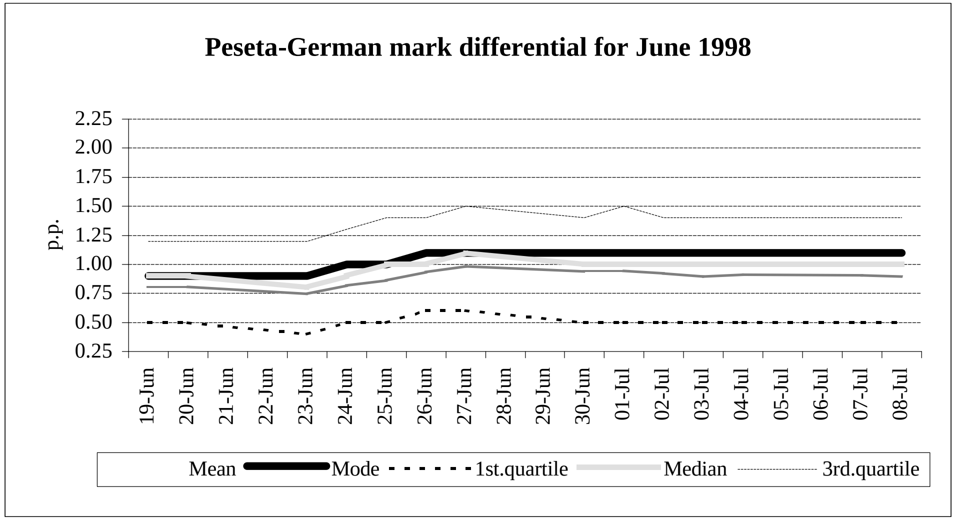


Figure 3

Statistical measures of 3-month interest rate differentials vis-à-vis German mark (ρ1, ρ2=0.99)







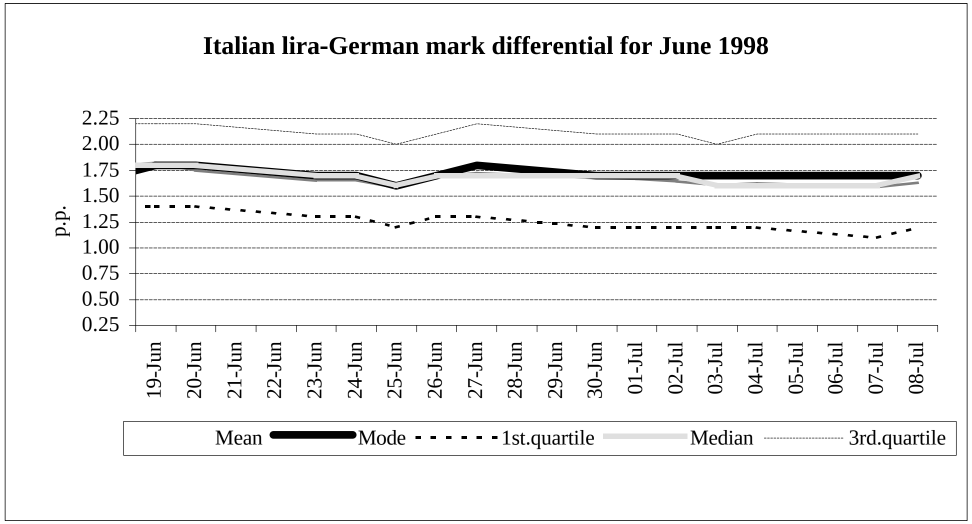
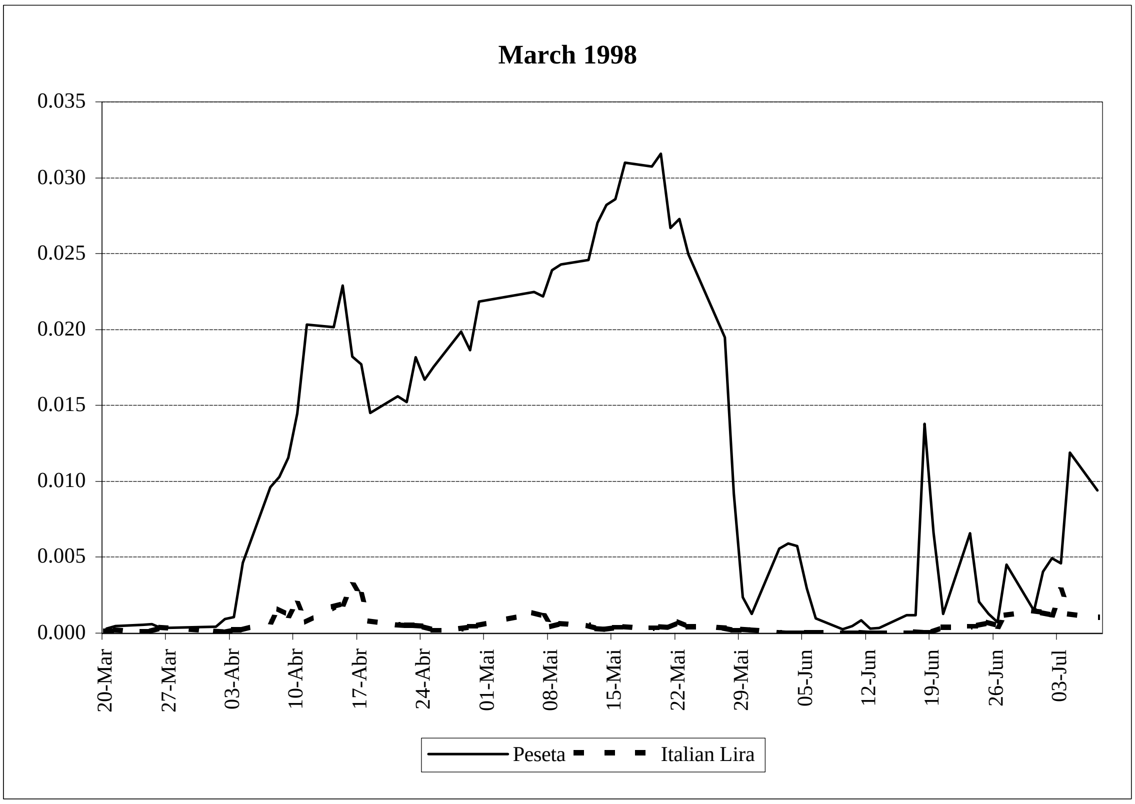


Figure 4
Probability of a non-positive interest rate spread vis-à-vis Germany (ρ1, ρ2=0.99)



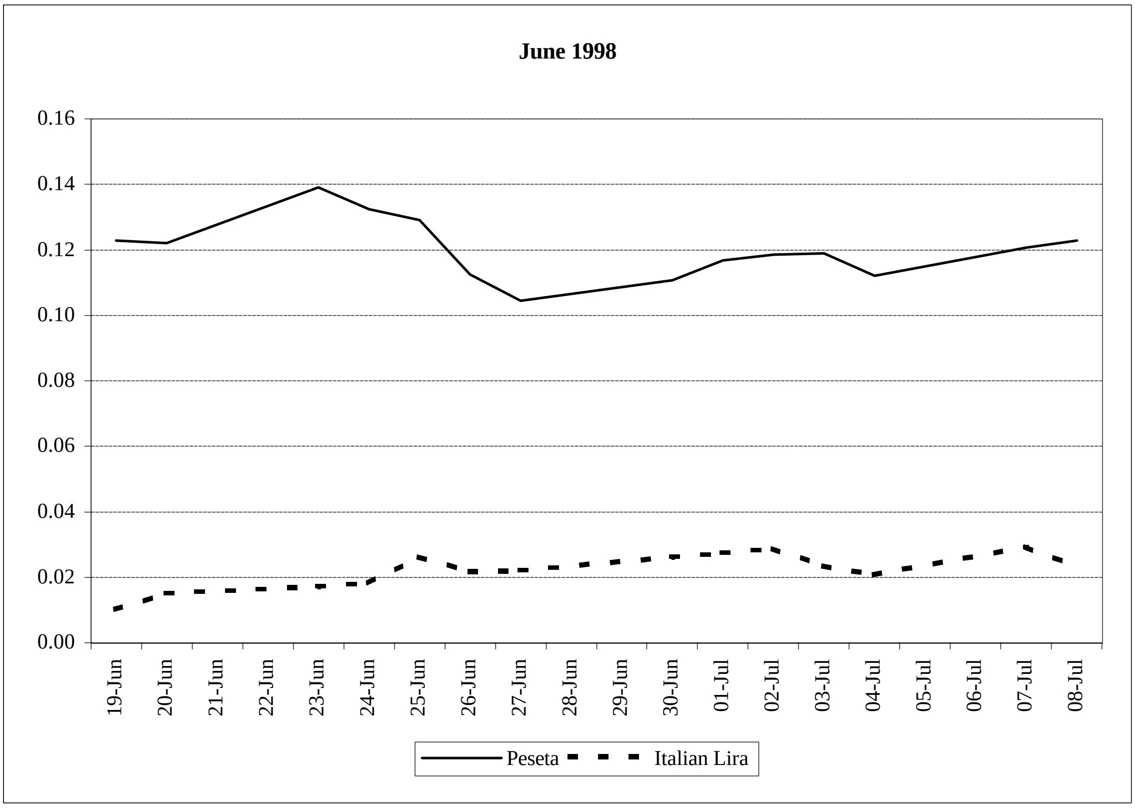
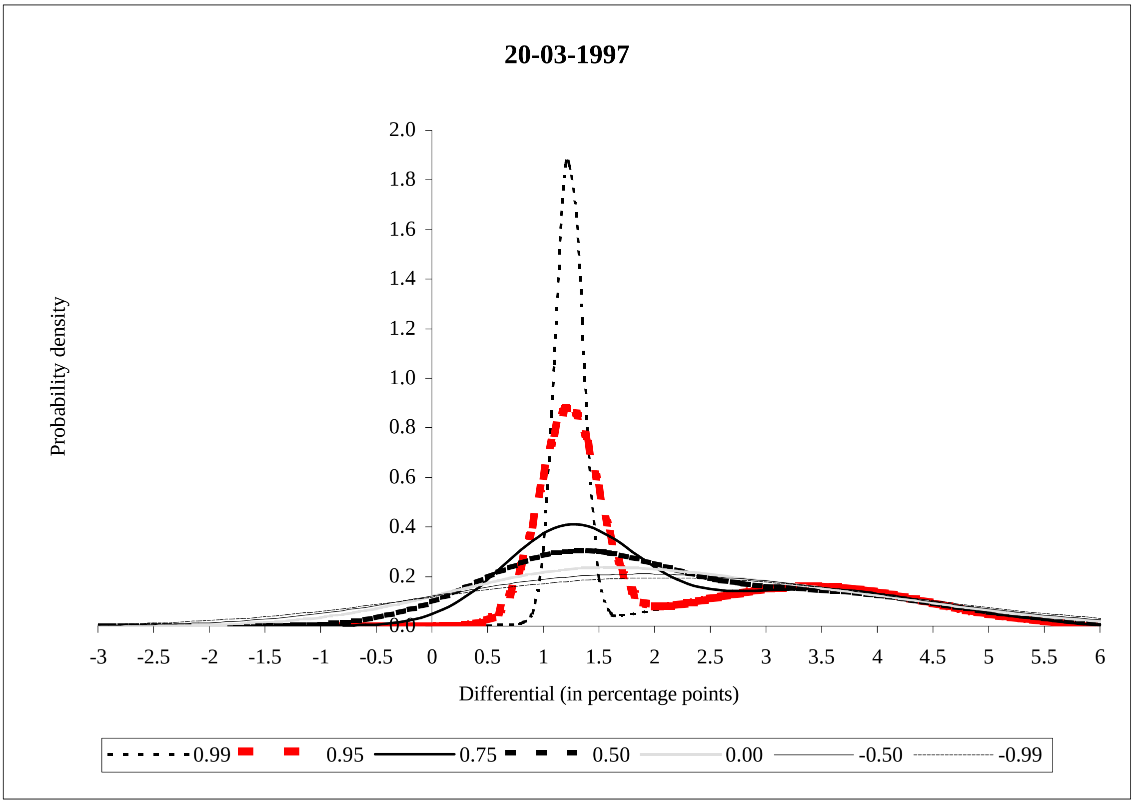


Figure 5

Sensitivity to the correlation coefficient of the unconditional density functions of 3-month interest rate differentials between the Spanish peseta and the German mark for March 1998



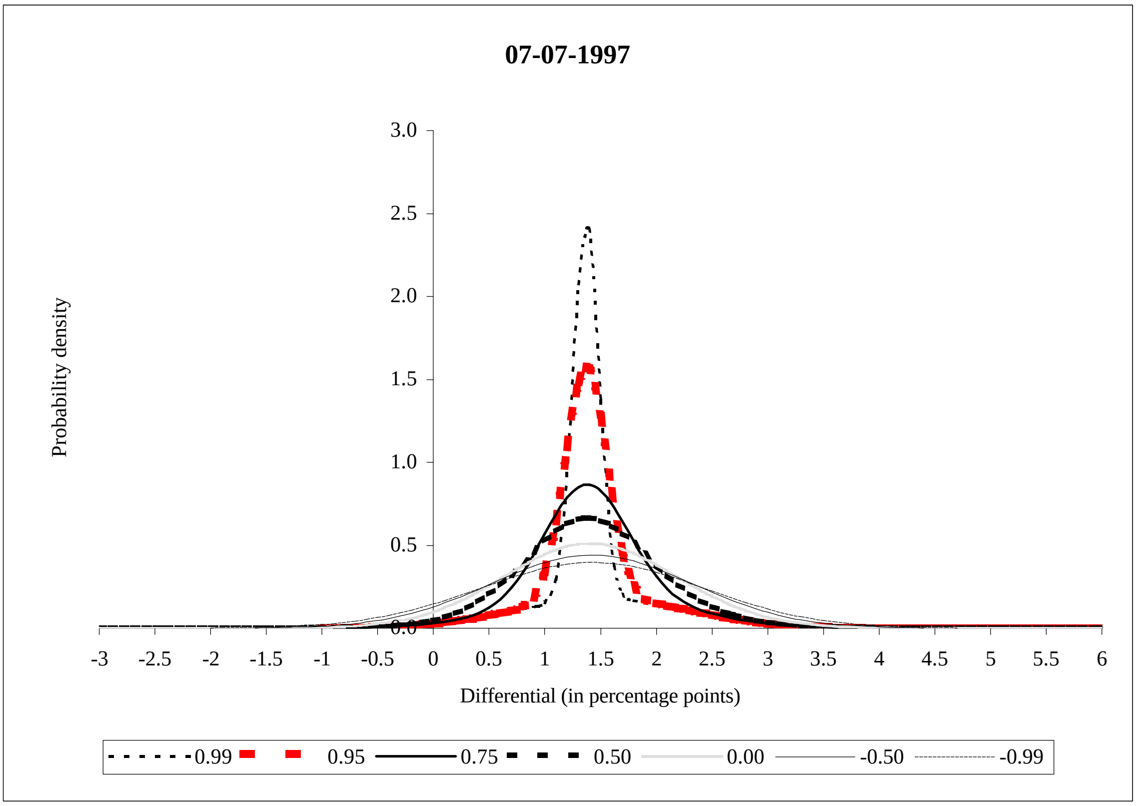
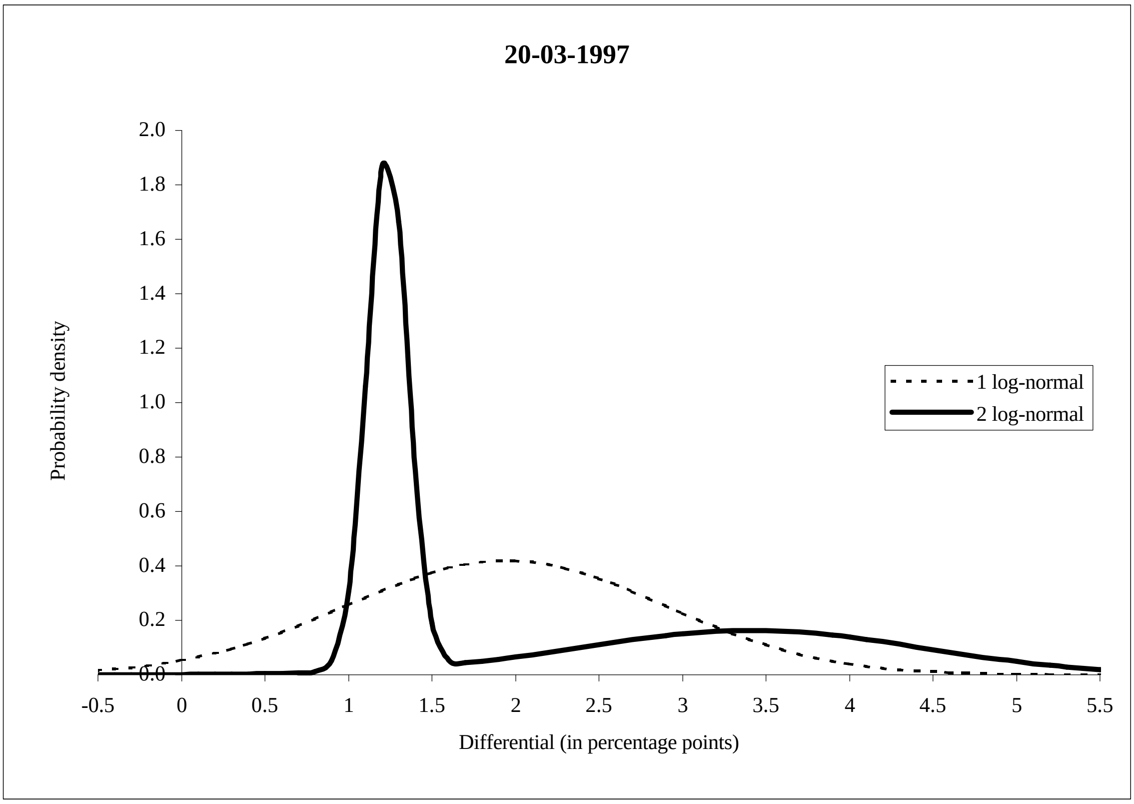
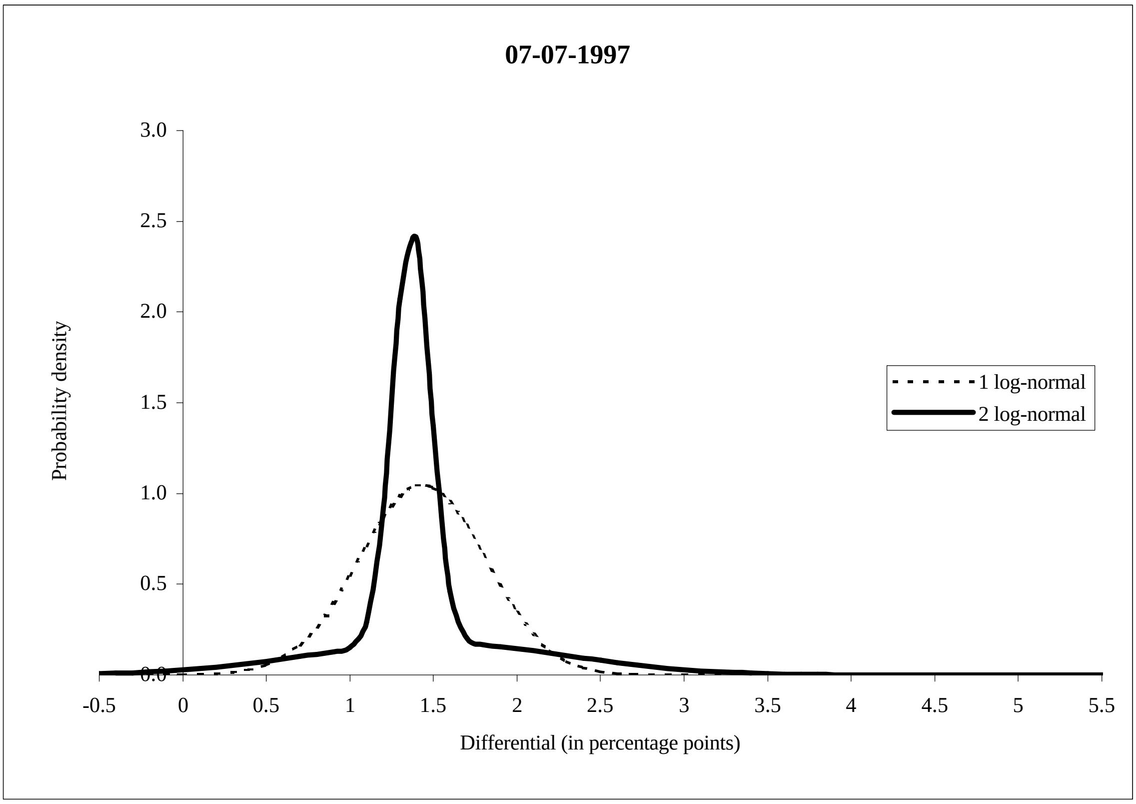


Figure 6

Sensitivity to the estimation method of the unconditional density functions of 3-month interest rate differentials between the Spanish peseta and the German mark for March 1998





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Comments on the Adão and Barros paper

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This paper combines market data on a pair of interest rates with an estimate of the correlation of those rates to arrive at estimates of the probability distribution of the interest rate spread. It applies this method to interest rate spreads between German mark money markets on the one hand and Italian lira and Spanish peseta rates on the other, with a view to assessing expectations regarding convergence of interest rates in the approach to European Monetary Union (EMU) at the beginning of 1999. Since perfect interest rate convergence was presumed to be a consequence of credibly fixing exchange rates with no fluctuation limits, market views on interest rate convergence serve also as indicators of expectations regarding the credibility, timing and membership of EMU.

The estimation exercise requires an estimate of the expected correlation of interest rates. The authors base most of the results they report on a “benchmark" estimate in which the correlation is assumed to be close to 0.99. Given the estimated parameters, this assigns the lowest possible probabilities to narrow interest rate spread outcomes. It thus represents the most cautious assessment from the point of view of a policy maker aiming at convergence.

The authors estimate the probability density function by fitting a mixture of bivariate joint lognormals to prices of options on short-term interest rate futures. This is a natural extension of the by-now standard procedure of applying mixture distributions in PDF estimation. The correlation between the two interest rates is then determined by both the estimated parameters and the postulated values of the correlations of the lognormal distribution entering into the mixture. The highest possible correlation between the rates themselves is arrived at by postulating the correlations among the mixture components to be +0.99.

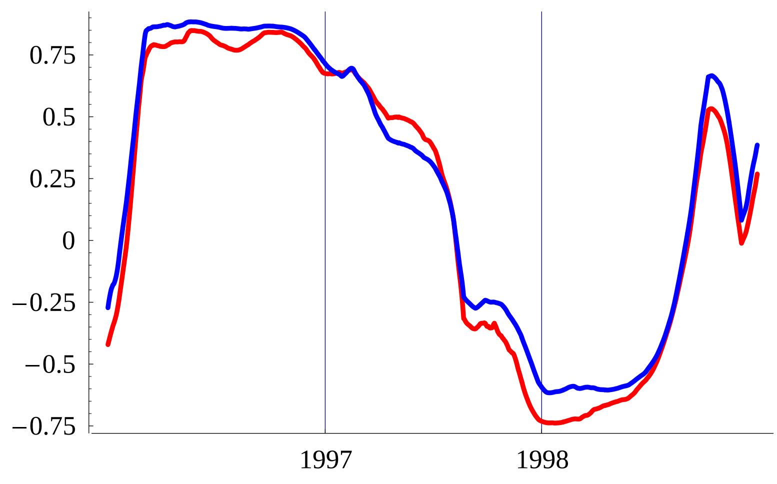
The paper draws several broad conclusions from the estimates:

- The market expected substantial interest rate convergence to occur between March and June 1998.
- Convergence was not, however, expected to be complete by June 1998.
- Interest-rate convergence was lower for the lira than for the peseta

One consequence of this procedure is that the outcome of a negative future interest-rate spread vis-à-vis the mark is assigned a positive, even substantial, probability. The probability of negative spreads is highest for the authors' benchmark case in which the mixture components have correlations of +0.99. However, a negative spread is difficult to interpret: the flip side of the “cautious" benchmark is this rather implausible scenario.

What were the correlations? They varied widely over 1996-1998, as seen in the accompanying chart. The red line represents lira and the blue line peseta. Correlations are based on one year of daily data.

Figure 1: Correlations of 3-month cash rates



As can be seen, correlations were moving fast towards -1 during the period studied. Later, in 1998, with much convergence already attained and rates across Europe dropping, the correlations moved back towards +1. The chart suggests that market participants may have anticipated much smaller correlations that +0.99.

It would have been interesting to know whether the PDFs of the individual interest rates were also skewed, or if only the German or only the non-German PDFs were skewed.

The paper pushes the envelope on the uses of risk-neutral PDFs. It would be useful, however, to display additional information pertinent to convergence, since it is difficult to interpret PDFs in isolation from other markets and other dimensions of the money markets. In particular, it would be interesting to see

- The evolution of the term structure: How rapidly had longer term rates converged? How had the slopes of the yield curves changed over time? How do forward rate agreements behave?
- Exchange rates and deviations from central parity.
- Historical implied volatility for interest rates and currencies.

It would also be useful to discuss the liquidity of the lira and peseta contracts.

Comments on Adão & Barros Luís

by Christian Upper[1]

Outline of the paper

The paper by Adão and Barros Luís extends the literature of implied probability density functions (PDFs) to variables for which no options. More precisely, they derive an implied PDF for spread between Spanish and German, and Italian and German 3-month interest rates. Since there do not exist any options on the spread, this information has to be extracted from the prices of options on the underlying interest rates.

For this purpose, they set up a bivariate probability density function for the underlying interest rates, which is then transformed into the bivariate density of the spread and of one of the interest rates. Integrating over the latter yields the implied PDF for the spread. They then estimate the parameters of the implied *marginal* PDFs for each contract. This does not, however, yield any estimate for the *correlation* between the two contracts. Instead, they try out different values and check to which extend this affects their results. They find that high values for the correlation coefficient lead to more mass being concentrated in higher differentials.

Their results indicate that option market participants did not expect complete convergence of interest rates by either March or June 1998, although the mean differential implied in the March contracts declines over the sample period, more so for the peseta than for the lire. There did not find any equivalent decline for the June 1998 contract.

[1] The opinions expressed here are the author's own and should not be attributed to the Deutsche Bundesbank.

Comments

Let me first comment on the methodology. The main difficulty is the choice of ρ. Historical correlations cannot be used due to non-stationarity under the null hypothesis of convergence. Implied correlations a la Campa & Chang (1998) could in principle be used if exchange rate at maturity were known, but this was not the case here. Hence the approach of choosing a ρ and then undertaking a sensitivity analysis seems reasonable. The problem is that densities become very flat for low values of ρ, which makes it very difficult to say anything about convergence. A possible alternative would have been to wait a bit until the January 1999 contract became traded, assume that the currencies would enter monetary union at their EMS central rates, and use Chang & Campa's approach to compute implicit correlations.

Let me turn to data issues. The different results they get for the two contract maturities may be due to the low liquidity of the June contract. In fact, even the March contract had a large residual maturity and may thus not be very liquid. Unfortunately, I cannot offer any solution to this problem. Perhaps we should take seriously only implied PDFs that are computed for short residual maturities.

A more serious question is why we should have expected any convergence in short term interest rates by March or June 1998. For both Italy and Spain it seemed clear, although possibly for different reasons, that they would attempt to keep interest rates higher than those in Germany as long as possible. There was certainly no point in aligning monetary policy as early as half a year before EMU.