Underlying inflation measures in Spain (*)

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(*) We would like to thank Eilev S. Jansen (Norges Bank) as well as the rest of participants at the BIS Meeting of Central Bank Model Builders and Econometricians held in February 1999. This paper draws on Álvarez and Matea (1997).
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A. Signal extraction with reduced-form models

B. Econometric methodology of the economic measures of underlying inflation with multivariate models
Applying the concept of underlying inflation can be thought of as an attempt to capture the general trend in inflation more accurately than with readily available data on headline inflation. In this paper a number of approaches to the analysis of underlying inflation are examined from a unifying standpoint, stressing their complementary nature, and empirical results are presented for the Spanish economy. Different measures differ from each other in the information set which is considered to be relevant for estimating the underlying rate of inflation. We first examine the simplest of the procedures that amounts to ignoring price developments in the most volatile sub-components of the CPI and then consider limited-influence estimators that take advantage of the information contained in the cross-sectional distribution of individual prices. Statistical methods of extracting the trend component of inflation are also discussed. Finally, measures that allow for the interplay of are other economic variables considered.
1. Introduction

It is widely acknowledged that the adequate assessment of inflationary trends is a complex undertaking and no single variable covers it fully. The Banco de España therefore uses a relatively complex analytical approach based on the examination of various economic, monetary and financial indicators, with the consumer price index (CPI) serving as a key element.

It is also well established that various shocks may, at least temporarily, produce noise in inflation statistics. Furthermore, there is general consensus that in view of the lags in the transmission mechanism, monetary policy should have a medium term orientation and thus transitory inflationary developments should not unduly affect policy decisions. The existence of short-term volatility in prices which cannot be controlled by monetary policy points to the need of developing measures of underlying inflation aimed at minimising this type of problem. This need has recently become even more important as central banks focus their attention on inflation as the primary goal of monetary policy.

While the terms "underlying inflation" and "core inflation" enjoy widespread use, they appear to have no widely accepted definition. Therefore, we think that it would be useful to present the main approaches that have been proposed in the literature. In our view, however, no single approach is able to summarise all relevant information; therefore, the different available measures should be jointly examined, taking into account their complementary nature. Moreover, since the various methods present different advantages and limitations we feel that users of these underlying inflation measures should be fully aware of them.

Solutions to the problem of high-frequency noise in price data include excluding certain prices in the calculation of the index based on the assumption that these are the ones with a high-variance noise component. This is the "ex. unprocessed food and energy" strategy which is discussed in section 2. Alternatively, it has been suggested to employ limited-influence estimators motivated by the observation that sizeable individual price changes tend to reflect transitory supply shocks and that these shocks may originate in any sector of the economy. Underlying inflation measures based on this type of estimators are discussed in section 3. Another approach, which is presented in section 4, involves calculating a low-frequency trend over which the noise is reduced. The fifth section describes two approaches based on a multivariate model and which are consistent with the existence of a vertical long-run Phillips curve and a monetary view of inflation. Finally, the sixth section presents the main conclusions drawn on the paper. A number of Tables summarising the use of underlying inflation measures by other authors and institutions are also included.

2. Underlying inflation by exclusion

From a monetary policy standpoint, a drawback to the direct use of the CPI is that this index is obscured by transitory price movements which hamper the description of lasting and more permanent price trends. To avoid, or at least to reduce, this problem it was initially proposed in the literature to exclude highly volatile prices from the CPI.

A possibility would be to adjust headline inflation for the estimated impact on prices of specific disturbances when they occur. However, it might be argued that transparency would be enhanced if reported inflation were adjusted for specific price disturbances according to a pre-specified rule. Depending on the structure of the economy, institutional arrangements and the methodology employed in the calculation of the CPI, European Union central banks (see Table 1) exclude different sub-components of the CPI to obtain measures of underlying inflation. Here, we will focus our attention in the case of Spain. In particular, following the Banco de España traditional breakdown\(^1\) of the CPI into five major sub-components (unprocessed food, processed food, energy, non-energy industrial goods and services) it seems reasonable to exclude the most

\(^1\) This is also the breakdown that the European Central Bank has adopted to employ.
volatile sub-components\textsuperscript{2}: unprocessed food and energy. In this section we put forward the arguments that are typically employed when trying to justify the exclusion of these sub-components.

\textbf{TABLE 1. - Underlying Inflation (UI) Measures of EU Central Banks Underlying Inflation by Exclusion}

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Underlying inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i.e. headline inflation adjusted for:)</td>
</tr>
<tr>
<td>Bank of England</td>
<td>- RPIX (mortgage interest payments (mips))</td>
</tr>
<tr>
<td></td>
<td>- RPIY (mips, indirect and local taxes)</td>
</tr>
<tr>
<td></td>
<td>- RPIXFE (mips, food, fuel, light)</td>
</tr>
<tr>
<td></td>
<td>- TPI (direct taxes)</td>
</tr>
<tr>
<td></td>
<td>- THARP (indirect and local taxes)</td>
</tr>
<tr>
<td>Sveriges Riksbank</td>
<td>- UND1 (interest costs for owner-occupied housing, indirect taxes, subsidies, depreciation after float)</td>
</tr>
<tr>
<td></td>
<td>- UND2 (ditto, plus heating oil and propellants)</td>
</tr>
<tr>
<td>Suomen Pankki</td>
<td>- IUI (capital costs in owner-occupied housing, indirect taxes, subsidies)</td>
</tr>
<tr>
<td>Banco de España</td>
<td>- IPSEBENE (energy, unprocessed food)</td>
</tr>
<tr>
<td></td>
<td>- Case-by-case (direct taxes, exogenous prices)</td>
</tr>
<tr>
<td>Deutsche Bundesbank</td>
<td>- CPI net (most indirect taxes)</td>
</tr>
<tr>
<td></td>
<td>- Case-by-case (food and/or energy)</td>
</tr>
<tr>
<td>Oesterreichische Nationalbank</td>
<td>- Case-by-case (indirect taxes, seasonal food)</td>
</tr>
<tr>
<td>De Nederlandsche Bank</td>
<td>- ULI (vegetables, fruit and energy)</td>
</tr>
<tr>
<td></td>
<td>- CPI market (public services, natural gas, rents, indirect and consumption-linked taxes)</td>
</tr>
<tr>
<td>Banque Nationale de Belgique</td>
<td>- CPI net (main indirect taxes)</td>
</tr>
<tr>
<td></td>
<td>- ULI1 (food and energy)</td>
</tr>
<tr>
<td></td>
<td>- ULI2 (energy)</td>
</tr>
<tr>
<td></td>
<td>- ULI3 (energy, main indirect taxes)</td>
</tr>
<tr>
<td>Institut Monétaire Luxembourgeois</td>
<td>- ULI (oil)</td>
</tr>
<tr>
<td></td>
<td>- Case-by-case (indirect taxes)</td>
</tr>
<tr>
<td>Banque de France</td>
<td>- ULI (food, energy, tobacco and taxation effects)</td>
</tr>
<tr>
<td>Danmarks Nationalbank</td>
<td>- CPI net (indirect taxes, subsidies)</td>
</tr>
<tr>
<td></td>
<td>- ULI1 (indirect taxes, subsidies, food, energy, rents, public services, effect of imports)</td>
</tr>
<tr>
<td></td>
<td>- ULI2 (indirect taxes, subsidies, food, energy, rents, public services)</td>
</tr>
<tr>
<td>Central Bank of Ireland</td>
<td>- ULI1 (mortgage interest payments)</td>
</tr>
<tr>
<td></td>
<td>- ULI2 (mips, food and energy)</td>
</tr>
<tr>
<td>Banco de Portugal</td>
<td>- ULI (unprocessed food and energy)</td>
</tr>
<tr>
<td>Banca d'ITALIA</td>
<td>- CPI net (indirect taxes)</td>
</tr>
<tr>
<td>Bank of Greece</td>
<td>- ULI (food and energy)</td>
</tr>
<tr>
<td></td>
<td>- Case-by-case (oil, public utilities, regulated prices, indirect taxes, subsidies, etc.)</td>
</tr>
</tbody>
</table>

Source: Ravnkilde Erichsen and van Riet (1995)

\textsuperscript{2} See, for example, Espasa et al. (1987) and Matea (1993).
As can be seen in Figure 1, which depicts the year-on-year rates for the CPI excluding unprocessed food and energy, the CPI and its major sub-components, the two sub-components whose year-on-year rates fluctuate most are those corresponding to energy and unprocessed food prices. This graphical evidence is also supported by the quantitative results in Table 2. Furthermore, among non-energy components, for which ARIMA models are available [see Table 3], the unprocessed food index is also the one with the largest residual standard deviation.
<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>MEASURE OF VOLATILITY (*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unprocessed food</td>
<td>0.96</td>
</tr>
<tr>
<td>Processed food</td>
<td>0.18</td>
</tr>
<tr>
<td>Non-energy industrial goods</td>
<td>0.13</td>
</tr>
<tr>
<td>Services</td>
<td>0.14</td>
</tr>
</tbody>
</table>

(*) Residual standard deviation (multiplied by 100) of ARIMA models with intervention analysis, built on the logarithmic transformation.

TABLE 2. VOLATILITY IN MAJOR CPI SUB-INDICES (*)

<table>
<thead>
<tr>
<th>Component</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>1.42</td>
</tr>
<tr>
<td>Goods</td>
<td>1.38</td>
</tr>
<tr>
<td>Food</td>
<td>2.03</td>
</tr>
<tr>
<td>Unprocessed food</td>
<td>2.73</td>
</tr>
<tr>
<td>Processed food</td>
<td>2.07</td>
</tr>
<tr>
<td>Industrial goods</td>
<td>1.41</td>
</tr>
<tr>
<td>Non-energy industrial goods</td>
<td>1.37</td>
</tr>
<tr>
<td>Energy</td>
<td>4.55</td>
</tr>
<tr>
<td>Services</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Memo item:
CPI excluding unprocessed food and energy 1.46

(*) Standard deviation of year-on-year rates
The volatility of the unprocessed food index is generally seen as the result of two factors. On the one hand, changes in weather conditions determine changes in the supply of unprocessed food. On the other hand, a relatively low demand elasticity make supply shifts cause relatively large changes in prices. These two reasons justify the exclusion of unprocessed food from the all items CPI to obtain a clearer picture of the inflationary process.  

A number of authors have recently made a case for the exclusion of all food prices. As regards the Spanish CPI, the intense impact of the 1995 drought on various processed foods (e.g., olive oil and wine) has indeed caused some to wonder whether the entire food component should not be excluded. However, it would probably be going too far to exclude all processed food prices since demand conditions and other input prices, besides those of agricultural products, generally play a non-negligible role in their determination.

The volatility of energy prices is determined by several factors. First, energy prices on international markets fluctuate considerably. Second, imports of energy products are, to a large extent, priced in dollars and the exchange rate of the peseta vis-à-vis the dollar is far from constant. Third, indirect taxes are a major component of energy prices and changes in excise duties generally result in sizeable price changes; and fourth, the energy index has mainly included regulated prices, which are only changed from time to time, but by quite a large extent. This last factor has recently lost some relevance since, following the entry into force of the Hydrocarbons Act, only electricity prices are fully regulated.

As a result of these factors it is not surprising that the energy index remains highly volatile. It may therefore be well to use the CPI ex. unprocessed food and energy as a measure of underlying inflation.

To end the brief discussion of this underlying inflation measure it may be well to present its main advantages and shortcomings. Adjustment by exclusion has the advantage that it increases the transparency and verifiability of the underlying inflation measure by completely pre-specifying its construction. On the contrary, the main criticisms levelled at this type of measure are that temporary disturbances are not necessarily limited to some sub-components, that prior exclusion of specific prices requires the use of non-controversial elements of judgement and, also, that there is a potential risk that significant information will not be taken into account. In any case, it should be stressed that a careful analysis of the inflation process may not be obtained exclusively from this underlying inflation measure.

3. Underlying inflation measures with limited-influence estimators

It has been argued that measured CPI is affected by monetary factors but also by changes in relative prices when there is some nominal rigidity. When these changes in relative prices are sizeable and result mainly from transitory supply shocks that are unrelated to the general trend in inflation, it may be advisable to follow Bryan and Cecchetti (1994) and use limited-influence estimators of a measure of the central tendency of the cross-sectional distribution of individual product price changes. Specifically, these authors argue that the weighted median and the trimmed mean should be used, rather than the weighted mean, for computing a measure of underlying inflation. By reducing the weight of extreme values and the distorting influence of shocks, these two statistics may provide a clearer signal of price level changes. The use of these measures reflects the intuition that the types of shocks that may cause problems with price measurement are infrequent but are not concentrated in some sectors of the economy. Compared to the underlying inflation measures obtained by exclusion of sub-components, these limited-influence estimators present the strength of not requiring prior determination of the origin of shocks that have a distorting influence in the measurement of trend inflation.

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3 It should also be borne in mind that some components of the unprocessed food index present methodological differences with the rest of the sub-indices of the CPI. Specifically, in its fresh fruit and vegetables sub-components weighted averages of twelve terms are used.
3.1. Theoretical model

In general terms, Ball and Mankiw’s single period model (1995) focuses on the problem of price setting for firms that incur costly price adjustment. Typically, firms do not instantly adjust prices to every change in circumstances; instead, they adjust only if their desired price change is large enough to justify the costs of adjustment (“menu costs”). Therefore, firms have a range of inaction in response to shocks. In this model, shocks that affect relative prices may have an impact on the aggregate price level; this will depend on the distribution of the shocks (see Figure 2). Specifically, if the distribution is symmetric the average effect will be zero, as price increases of some firms will be offset by price cuts made by others. By contrast, if the distribution of shocks is skewed, the aggregate price level will temporarily increase or decrease depending on the importance of firms raising prices relative to those lowering them. In this case, costly price adjustment may result in transitory movements of headline inflation from its long-run trend.

Fig. 2
To be more specific we present the version of Bryan and Cecchetti (1994) of Ball and Mankiw’s model of price setting. This model takes place in a single period and there is a large number of firms which face the same "menu costs" when adjusting their prices. Besides, money growth ($\dot{m}$) is constant and exogenously determined, velocity of circulation is constant, and trend output growth is normalised to zero. Under these assumptions, at the outset of the period, each firm will decide to increase its price by $\dot{m}$. As a result, aggregate inflation in this model will equal monetary inflation. Therefore, in this case, core or underlying inflation ($\pi^c$) may be defined as:

$$\pi^c = \dot{m}$$

Following this initial price-setting exercise, each firm (i) experiences a shock to $\epsilon_i$ either its product demand or its production costs. In general, however, the distribution of shocks across firms may have any shape. After the shock is realised only firms for which $\epsilon_i$ in absolute value exceeds the "menu cost" will adjust their price. For these firms, the growth rate of prices $\pi_i$ will be:

$$\pi_i = \dot{m} + \epsilon_i$$

If it is also assumed that the level $\bar{\epsilon}$ at which firms decide to adjust their prices is the same, then the observed inflation rate will be:

$$\pi^o = \dot{m} + \sum_{i=1}^{n} \frac{\epsilon_i}{|\epsilon_i|}$$

where $n$ is the number of firms in the economy. If the distribution of shocks is symmetrical, the second term of the right hand side of the above equation cancels, but if it is skewed, actual inflation does not match monetary inflation. As the difference between $\pi^o$ and $\dot{m}$ arises from the tails of the distribution of $\epsilon_i$, one way to reduce the impact of shocks on measured inflation is to use limited-influence estimators.

With regard to the theoretical model that is used to motivate the limited-influence estimators Zeldes (1994) points out that changes in relative prices do not have to be necessarily transmitted to aggregate inflation. If this were not the case, then there would be no compelling reason to exclude extreme values. This author also notes that there may be permanent shocks to inflation associated with the existence of skewness in the distribution of relative price changes. However, if skewness were caused by permanent shocks it would clearly be misleading to exclude extreme values to obtain a measure of underlying inflation.

From an statistical point of view, it is well known that a small change in the tails of a distribution may entail a sizeable change in the arithmetic mean, while trimmed means and weighted medians are celebrated estimates of location in situations where the occurrence of outliers is suspected. Robustness arguments favour medians over trimmed means. However, under certain assumptions, heavily trimmed means have smaller asymptotic variance and hence are superior to medians. Therefore, from a statistical point of view, neither estimator is clearly preferable. This consideration suggests that both measures should be examined.

3.2. Estimates

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4 In classical economic theory, the price level is determined by the money supply and changes in supply and demand for various products affect not the price level, but relative prices.

5 See Oosterhoff (1994)
Disaggregated consumer price data for Spain were used to construct these types of underlying inflation measures. Specifically, the sub-indices of the CPI were considered for calculating the weighted median and the trimmed mean. To minimise the effect of seasonality on the cross-sectional distribution, we follow Matea (1994a) and use year-on-year rates. As each of the sub-indices includes the prices of various goods and services, we assume that the weight of each sub-index in the CPI basket represents the percentage of the distribution of all prices that experiences that price change. To calculate the weighted median at a given time, the year-on-year rates of the individual sub-indices are multiplied by their weights and the resulting figures are then ordered from small to large; then the central point in the cross-sectional histogram is chosen. The trimmed mean is obtained by excluding a chosen proportion of unusually large and small price changes before the average is computed.

To determine whether or not the cross-sectional distribution of the CPI sub-indices’ year-on-year rates is symmetrical, the skewness coefficient was computed. As can be seen in Figure 3, skewness has changed considerably over time, and in some periods it is quite important. This suggests the usefulness of considering limited-influence estimators.

To select the size of the trimmed mean, 5%, 10% and 15% trimmed means were considered. Finally, a 5% trimmed mean was chosen (see Figure 4) as the resulting time-series showed the smallest variance. This result differs from that of Bryan and Cecchetti (1994) for the CPI of the United States, as they obtain a series with minimum variance with a 15% trimmed mean (see also Table 4 for further evidence). In any case, with the three alternatives considered for the Spanish CPI, very similar time-series were obtained. Also, as one would expect, the volatility of this underlying inflation measure is lower than that of headline CPI.

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Bryan and Cecchetti (1994) used 36 components of the U.S. CPI. Here these 156 indices resulted from crossing the two types of breakdown by sub-indices used by the INE [National Statistics Institute]. Thus, in each case, the classification that produces most disaggregation was used.

Bryan and Cecchetti (1994) employ seasonally adjusted series. However, this has the disadvantage that including fresh data involves recomputing limited influence estimators for the whole sample period.

Note, however, that if fixed weights are used, then, in general, the 0% trimmed mean of the cross-sectional year-on-year rates is not equal to the year-on-year rate of headline inflation. This is so because the (fixed-weight) weighted average of year-on-year rates does not equal the year-on-year rate of the (fixed-weight) weighted average. Since it seems advisable that the 0% trimmed mean and the year-on-year rate be equal, we have used variable weights. These weights correspond to the share of each sub-index in the CPI level twelve months ago. These shares will only equal the fixed weights if all prices grow by exactly the same amount every month.

A distribution is symmetrical when this coefficient is zero, whereas if it is positive (negative), the area on the right-hand (left-hand) side of the distribution is greater than that on the left-hand (right-hand) side.
In analysing the 5% trimmed mean, no sub-index fails to be covered in the whole sample period under consideration. On the basis of this, it could be argued that there should be no prior exclusion of any sub-index, to say nothing of any of the 5 major components of the CPI. Even so, an examination of sub-indices grouped under the major sub-components shows that all those comprising the unprocessed food index were at some time in the tails of the distribution (see Table 5). By contrast, in the period under consideration, 28% of the sub-indices of the energy index, 31% of the processed food index, 36% of the services index, and 58% of the non-energy industrial goods index have always been considered in the trimmed mean. This result tallies with the exclusion

<table>
<thead>
<tr>
<th>Central Bank or Paper</th>
<th>Underlying inflation measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of England</td>
<td>- RPY 15% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- RPIY weighted median</td>
</tr>
<tr>
<td>Sveriges Riksbank</td>
<td>- CPI 15% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- CPI weighted median</td>
</tr>
<tr>
<td>Banco de España</td>
<td>- CPI 5% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- CPI weighted median</td>
</tr>
<tr>
<td>Banca d'Italia</td>
<td>- CPI 20% trimmed mean</td>
</tr>
<tr>
<td>Bryan and Cecchetti (1994)</td>
<td>- CPI 15% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- CPI weighted median</td>
</tr>
<tr>
<td>Cecchetti (1996)</td>
<td>- CPI 10% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- CPI 25% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- CPI weighted median</td>
</tr>
<tr>
<td>Mayes and Chapple (1995)</td>
<td>- CPI weighted median</td>
</tr>
<tr>
<td>Roger (1995)</td>
<td>- PXIG 10% trimmed mean</td>
</tr>
<tr>
<td></td>
<td>- PXIG weighted median</td>
</tr>
<tr>
<td>Shiratsuka (1997)</td>
<td>- CPI 15% trimmed mean</td>
</tr>
</tbody>
</table>

Sources: For Central Banks: Spain (Banco de España), other EU countries (Ravnkilde Erichsen and van Riet [1995])
of the unprocessed food index and, to a lesser extent, the energy index, i.e. the use of the CPI excluding unprocessed food and energy as a measure of underlying inflation.

The five major sub-components of the CPI are captured in the weighted median. If, a comparison is made with the CPI, then we generally find that the weighted median shows markedly lower rates than the CPI. This clearly shows how substantial sector-specific price increases have affected the CPI. On the other hand, the weighted median shows substantial volatility; a feature that without additional treatment, could complicate an accurate analysis of the inflationary process.

To conclude, limited-influence estimators present a drawback as a result of the presence of goods and services whose prices do not change often and not always in the same month of the year. This causes the rate of change of these prices to be zero in some months and to be quite high in others. It is therefore not surprising that they are found in the tails of the cross-sectional distribution and, in practice, are commonly not taken into account in these measures of underlying inflation. For example, at the beginning of the sample period a sizeable portion of energy items, which had a regulated price, are usually excluded by a trimmed mean.

### 4. Underlying inflation by smoothing

Statistical signal extraction techniques have been used by economists to break down a time-series into its trend\(^{10}\), seasonal and irregular components. Nonetheless, for monetary policy, it is particularly important to know whether price changes are transitory or, more importantly, whether they have a permanent nature. Consequently, since seasonal effects are cancelled out within a year and irregular movements disappear even sooner, it is the trend component which is crucial in the analysis of inflation\(^{11}\). Furthermore, as can be seen in Figure 5, the trend component fluctuates considerably less than the seasonally adjusted series.

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10 As a trend is an unobservable component, it has no single definition. Indeed, the concept used in this section is univariate whereas the ones employed in section 5 are multivariate.

11 Since a seasonally adjusted series may be seen as a trend contaminated by noise, it is conceptually hard to find a convincing argument to base a descriptive analysis of inflation on a seasonally adjusted series.
In the literature, several signal extraction techniques have been proposed, probably the most popular ones being X11-ARIMA, ARIMA model based procedures and those based on structural time series models. Matea and Regil (1994) applied these different techniques to the Spanish CPI and found that they resulted in highly similar trend components. However, since seasonal factors seemed to be slightly better estimated with an ARIMA model based procedure, we will use here the program SEATS [see Gómez and Maravall (1998a)].

Our preferred underlying inflation measure by smoothing, in the case in which inflation is appropriately characterised by a purely stochastic process, will be defined as the centred year-on-year growth rate of the trend of a price index. Centring a rate is necessary if it is deemed desirable to synchronise it with month-on-month growth. It is often the case, however, that deterministic and stochastic elements are thought to be present in the series under study. Although several techniques exist to decompose a series into its deterministic and stochastic components, we will focus our discussion on ARIMA models with intervention analysis and their associated model-based signal extraction technique. Those models may be identified and estimated for all the major subindices of the CPI and enable a breakdown into stochastic components (associated with the ARIMA models) and deterministic components (associated with intervention analysis). Correspondingly, unobservable components (e.g. trends), which may be estimated by using an ARIMA model-based signal extraction procedure,

\[ \text{Note: Month-on-month inflation for January 1992 is adjusted for the impact of VAT changes} \]

12 Note that this technique does not require to specify beforehand a particular functional form for the trend. Readers interested in this method may see Appendix A and Gómez and Maravall (1998b).

13 A rate of change calculates growth between two periods. Centering consists in assigning said growth to the intermediate point in the period of time under consideration. As a consequence, when computing a centered rate for the most recent data either some information is lost or forecasts are required. The interest of policymakers in the most recent information makes the use of forecasts desirable.

14 In other words, so that the maxima and minima of the year-on-year rate match those of month-on-month growth.

15 The energy index may be an exception. This is due to the fact that these prices have been regulated during most of the sample period.
may have stochastic and deterministic elements. Specifically, the trend must not only capture the stochastic trend, but also those deterministic elements (interventions) with a permanent nature. As has been mentioned above, a growth rate on a stochastic trend should be centred. However, use of a centred growth rate on a series that has a deterministic component associated with the trend (e.g. the effect of VAT changes on the price level) would imply dating the exceptional event before it actually occurs. Therefore, the growth rate on permanent deterministic components should not be centred. Specifically, the following equation\(^\text{16}\) may be used to obtain a measure of underlying inflation:

\[
UIS_t = T^1_{12} \left( ST_t \right)^C + \left[ \frac{1}{100} T^3_{12} \left( SAI_t \right)^C + 1 \right] T^1_{12} \left( PI_t \right)^{NC}
\]

\[
T^3_{12} \left( X_t \right)^C = \left[ \frac{X_{t-6} - X_{t-6}}{X_{t-6}} \right] 100 \quad T^1_{12} \left( X_t \right)^{NC} = \left[ \frac{X_t - X_{t-12}}{X_{t-12}} \right] 100
\]

Where UIS, is the underlying inflation measure by smoothing, \(ST_t\) is the stochastic trend, \(PI_t\) is the effect of permanent interventions, \(T^1_{12}\) denotes year-on-year growth and superscripts \(C\) and \(NC\) indicate, respectively, whether the rate is centred or not.

A possible approximation to the year-on-year rate of the stochastic trend may be obtained by using a rate of growth on the series adjusted for intervention analysis. In this case a considerable simplification in the calculation of the measure is obtained. The rationale behind this approximation is based on the fact that the optimal estimator of the trend component involves the use of a centred weighted moving average (a two-sided filter). In practice, a growth rate on the original series adjusted for intervention analysis averaging a large enough number of observations may be a satisfactory approximation.

In particular, as can be seen in Álvarez and Matea (1997) for all the major CPI sub-indices, with the exception of unprocessed food\(^\text{17}\), the centred \(T^3_{12}\) provides a very good approximation to the year-on-year rate of the stochastic trend\(^\text{18}\). However, rather than adopt a different rate for each CPI sub-component it is simpler to use a single growth rate. Therefore, we use the centred \(T^3_{12}\) for all major sub-components\(^\text{19}\).

As a result, a measure of underlying inflation by smoothing may be approximated on the basis of the following equation:

\[
UIS_t \equiv T^3_{12} \left( SAI_t \right)^C + \left[ \frac{1}{100} T^3_{12} \left( SAI_t \right)^C + 1 \right] T^1_{12} \left( PI_t \right)^{NC}
\]

\[
T^3_{12} \left( X_t \right)^C = \left[ \frac{X_{t+5} + X_{t+6} + X_{t+5}}{X_{t+6} + X_{t+7}} \right] \left( X_{t-5} + X_{t-6} + X_{t-7} \right) \left( X_{t-5} + X_{t-6} + X_{t+7} \right) 100
\]

\(^\text{16}\) See Espasa and Cancelo (1993).

\(^\text{17}\) For this component, due to its larger variability, a longer moving average is required. Note, however, that the validity of this approximation may be too country-specific. For other countries, there may not be a satisfactory approximation or the one valid may differ from the one used in Spain.

\(^\text{18}\) The uncentered \(T^3_{12}\) represents the growth of the average of three consecutive months vis-à-vis the average of the same three months in the previous year. The centered \(T^3_{12}\) rate assigns said growth to the intermediate point in the period of time employed to compute the rate.

\(^\text{19}\) This rate has the advantage of requiring at most 7 forecasts. Other rates involving longer moving averages require more forecasts, so that revisions in the measure will be more important at the end of the sample.
where SAI is the original series adjusted for all interventions.

Figure 6 depicts the measure of underlying inflation for the headline CPI, the CPI excluding food and energy and the major CPI components obtained using the approximation described above.

![Graph of underlying inflation by smoothing](image)

Although the procedure outlined above is suitable for series with an important stochastic component, indices with regulated prices present a complication stemming from the fact that such prices rather than evolving smoothly change suddenly at specific times. In view of this peculiarity, which was especially notable in the energy index, a case has been made for estimating its underlying rate using the year-on-year rate of the original series.

With this underlying inflation measure, forecasts are required for the last observations. Therefore, as fresh data are released the measure is accordingly revised.

To conclude, it should be noted that the underlying inflation by smoothing measure involves greater complexity than the simple exclusion of some components. Moreover, being a model-based approach,
different authors may obtain different underlying inflation estimates of this type by not considering the same deterministic elements in the CPI, by using different statistical techniques to estimate the trend or by considering a different sample period. However, it should be borne in mind that complexity and possible differences among researchers simply reflect the flexibility of the approach and the possible lack of agreement among econometricians. On the other hand, this approach eliminates transitory elements and yields a satisfactory way of analysing inflation trends.

5. Underlying inflation measures with multivariate models

As outlined above, we have been examining various measures of underlying inflation, either by excluding specific index components, as in the case of measures of underlying inflation by exclusion (see section 2) and limited-influence estimators (see section 3) or by smoothing (see section 4). The common denominator of all these approaches is their univariate nature; that is, they are constructed using only the information contained in price series.

Recently, however, some authors (see Table 6) have proposed using supplementary measures obtained from structural vector autoregressive (SVAR) models. These procedures are characterised by the use of restrictions based on propositions set forth by economic theory with regard to the long-run behaviour of several variables and, also, by their multivariate nature. This means that in determining the measures of underlying inflation they take into account information that supplements price series data (e.g. that contained in real activity or in a given monetary aggregate).

Specifically, two procedures are examined in this section which, even though they are not without drawbacks, supplement the methods discussed above. These approaches are consistent with a monetary view of inflation in the long run and meet the generally accepted condition that the long-run Phillips curve is vertical, i.e. that there is no long-run trade-off between output and inflation, so that changes in nominal magnitudes do not have real effects in the long run. However, these approaches also permit an economy to be hit by shocks in the short run which, depending on their origin and duration, may affect both the cyclical component and the trend of inflation and output. Thus, two alternative measures of underlying inflation are obtained based on a structural dynamic model of inflation and output: permanent inflation and core inflation.

Permanent inflation captures the impact of disturbances which in the long run determine inflation. Assuming rationality, these shocks are incorporated in the expectations of economic agents and are therefore the driving force that determines the growth rate of nominal variables.

Core inflation\(^{20}\) captures the impact on inflation of shocks which do not have a long-run effect on output. Although, no long-run inflation rate\(^{21}\) can be obtained directly using this technique, a highly relevant by-product that is obtained is an estimate of the economy’s trend output and, as a residual, an estimate of the output gap\(^^{22}\).

\(^{20}\) Quah and Vahey (1995) first proposed this measure.

\(^{21}\) From a theoretical standpoint, core inflation cannot strictly be interpreted as long-run inflation, as not all transitory shocks on real output are necessarily transmitted to inflation. Strictly speaking, this measure considers not only permanent demand shocks, but also shocks associated with the business cycle. However, as is shown further on, permanent and core inflation closely resemble each other in the Spanish economy. As a result, it would seem proper in practice to interpret core inflation as long-run inflation.

\(^{22}\) This measure is discussed in Álvarez and Sebastián (1998).
### TABLE 6. Literature on multivariate measures of underlying inflation

<table>
<thead>
<tr>
<th>PAPER</th>
<th>COUNTRY COVERED</th>
<th>VARIABLES USED</th>
<th>MEASURE OF UNDERLYING INFLATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. Inflation with all disturbances having a permanent effect on output eliminated (core inflation). [Quah and Vahey (1995) core inflation]</td>
</tr>
<tr>
<td>Gartner and Wehinger (1998)</td>
<td>Austria, Belgium, Germany, Finland, France, Italy, the Netherlands, Sweden, United Kingdom</td>
<td>Consumer prices, Gross Domestic Product, Short term interest rate</td>
<td>Quah and Vahey (1995) Core inflation</td>
</tr>
<tr>
<td>Quah and Vahey (1995)</td>
<td>United Kingdom</td>
<td>Consumer prices, Industrial output</td>
<td>Inflation with all disturbances having a permanent effect on output eliminated (core inflation).</td>
</tr>
<tr>
<td>Roberts (1993)</td>
<td>United States</td>
<td>GDP deflator, Unemployment rate, Velocity of circulation</td>
<td>&quot;Monetary&quot; inflation</td>
</tr>
</tbody>
</table>
It should be pointed out that the structural interpretation of the shocks that permit identification of these underlying inflation measures is not straightforward. Specifically, it is not possible to distinguish directly between supply shocks and demand shocks. The permanent inflation procedure distinguishes between disturbances according to their long-run effect on inflation. However, disturbances that affect inflation in the long run may arise from both aggregate demand (e.g. changes in the growth rate of the money supply) and the supply side (e.g. changes in the trend growth of the economy). By contrast, the core inflation procedure distinguishes among shocks on the basis of their long-run impact on output. However, shocks which do not have a long run impact on output may arise from both the demand side (e.g. monetary disturbances) and the supply side (e.g. transitory technological shocks). A comparison of the two measures of permanent core and inflation with observed inflation nevertheless facilitates an interpretation of the type of shocks predominant in the economy.

In any case, these measures, like any others seeking to approximate a phenomenon as complex as the inflation process, must be assessed and interpreted with due prudence and caution. The approaches discussed in this section are also limited by their initial assumption that there are only two types of disturbances that affect inflation and output. Actually, it seems likely that there are many sources of shocks and that some of them have differential effects on the economy. Therefore interpretations must be made in terms of the effect of groups of shocks. However, on the basis of the estimated transmission mechanisms hypotheses may be advanced as to the nature of the shocks. Moreover, these measures are constructed on the basis of changes in inflation, so that an additional hypothesis is needed to recover their level.

5.1. Permanent inflation

The unrestricted VAR model, common to both permanent and core inflation estimates, uses a sample period that is long enough. Since our identification schemes are based on long-run restrictions, we require enough data to plausibly claim that we can estimate long-run phenomena. Specifically, we begin in the first quarter of 1970 and end in the third quarter of 1998. Four lags for each of the variables are used. As deterministic variables, in addition to a constant term, it must be borne in mind that the GDP growth rate series shows different means across subsamples. Thus, breaks are included in the mean during the first quarter of 1976 and the last quarters of 1984 and 1991.

To obtain the structural shocks and transmission mechanisms (impulse response functions) that provide the basis for these measures of underlying inflation, the identification procedure first proposed by Blanchard and Quah (1989) is used. These authors decompose output movements into permanent and transitory components. One of our structural models also breaks down output movements into permanent and transitory components, although our main interest is the effect of these shocks on inflation. The other model performs a similar decomposition for inflation. The method involves the use of long-run identification restrictions in a VAR model which captures the main interactions between inflation and output.

---

23 In Álvarez and Sebastián (1995) the sample period ends in the fourth quarter of 1993. Results are almost identical to the ones discussed here.

24 Four lags adequately cover the dynamics of the process. Using five lags does not practically change the results.

25 This method is outlined in Appendix B.
It should be pointed out that the long-run identification used in studies of this kind involves no specific assumption with regard to the short-run transmission mechanism. Therefore, in order to give an economic interpretation of structural shocks, and as an informal test, not only must transmission mechanisms associated with each shock be examined; it must also be checked whether the signs and time patterns of the responses are in line with the interpretation being made.

Two types of shocks are identified for permanent inflation (see Table 7) and defined on the basis of their long-run effect on inflation. These disturbances and their transmission mechanisms may be obtained using the procedure outlined in Appendix B. Once these are known, the inflation rate \( \pi_t \) may be broken down into the sum of two terms: permanent inflation \( \pi_t^p \) and transitory inflation \( \pi_t^t \).

\[
\pi_t = \pi_t^p + \pi_t^t
\]

An analysis of the transmission mechanism of shocks associated with permanent changes in inflation (which determine \( \pi_t^p \) ) shows that they have a positive but relatively mild impact on real activity. Such effect is significant in the short run, but not in the long run, so that long-run superneutrality would hold. This would be consistent with nominal disturbances having a short-run expansionary effect on activity but unable to modify potential output. On the other hand, the effect on real output of disturbances having no long-run effect on inflation (which determine \( \pi_t^t \) ) is also positive but much larger. These disturbances may be associated with technological shocks by their positive and permanent effect on output.

<table>
<thead>
<tr>
<th>TABLE 7. - Identification schemes used to obtain the measures of permanent inflation and latent inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCHEME 1</td>
</tr>
<tr>
<td>Disturbances NOT having a long run effect on inflation (Identification restriction)</td>
</tr>
<tr>
<td>Disturbances HAVING a long run effect on inflation</td>
</tr>
<tr>
<td>SCHEME 2</td>
</tr>
<tr>
<td>Disturbances NOT having a long run effect on output (Identification restriction)</td>
</tr>
<tr>
<td>Disturbances HAVING a long run effect on output</td>
</tr>
</tbody>
</table>

By contrast, when inflation is examined, shocks that permanently affect it are, logically, more important than those having a transitory effect. Moreover, what we identified as technological shocks have a transitory downward impact on inflation. Thus, they do not affect the potential growth rate.26

Using the methodology outlined in Appendix B, it is also possible to obtain an estimate of permanent inflation. Two separate considerations must be borne in mind when analysing this series. First, the difference between actual inflation and permanent inflation, and second, the time path of permanent inflation. The first

26 In fact, considering output as an I(1) variable rules out the possibility of shocks with a long-run effect on the growth rate of output.
factor may be controversial as the level of permanent inflation is not identified and requires an additional hypothesis\textsuperscript{27}. However, the second factor is independent of such an assumption. Therefore, in the economic assessment of this measure, prime consideration should be given to whether permanent inflation is actually speeding up or slowing down, and not whether it is above or below actual inflation\textsuperscript{28}.

As may be seen in Figure 7, except for very specific periods, the time path of permanent inflation is generally similar to that of actual inflation. This result squares with the fact that, in relative terms, transitory shocks have a less important effect on inflation, so that inflation is dominated by its trend component.

In turn, the time path of the estimated permanent inflation series shows the effect of permanent disturbances in both demand (e.g. monetary disturbances) and supply, which are reflected in changes in the long-run inflation rate. It also bears noting that transitory inflation, even when not very great, is procyclical and lagged, which may be interpreted as reflecting the presence of demand shocks having transitory effects on inflation and output.

5.2. Core inflation

For core inflation, the two types of structural shocks are defined according to their long-run effect on real activity. The first type does not have a long-run effect on output, although it affects actual inflation. The

\textsuperscript{27} The number of possible hypotheses is, theoretically, unlimited. In this paper, we use the hypothesis that the sum of deviations between both rates of inflation is zero. The rationale of using this assumption is that, by definition, deviations of the actual inflation rate from permanent inflation can only be temporary.

\textsuperscript{28} This line of reasoning is also valid for core inflation.
second type affects the long-run trend of output, but not core inflation. Using again the method outlined in Appendix B, measured inflation may be, alternatively, broken down into the sum of core inflation \( \pi^c \) and non-core inflation \( \pi^n \). Core inflation is defined as the contribution to inflation of shocks which have no long-run effect on the level of output and is the time path of inflation that would have obtained in the absence of permanent shocks on real activity.

\[
\pi = \pi^c + \pi^n
\]

An analysis of the transmission mechanisms shows that disturbances which do not affect real activity in the long-run (which determine \( \pi^c \)) do have a significant short-run impact on output, although it is quantitatively small. The transitory nature of the effect of these shocks on output and its explanatory power on real activity make it possible to associate these shocks with the business cycle. However, the impact of disturbances having a long run effect on output (which determine \( \pi^n \)) is considerably larger. The permanent effect of these shocks is due to the fact that output is a non-stationary series and, by the identification restriction, the other shocks have a temporary effect on output. The considerable explanatory power of these disturbances on output is such that they may be associated with technological changes that permanently affect factor productivity or with increases in the use of productive factors.

Temporary shocks on output (which determine \( \pi^c \)) have a powerful effect on inflation. Ninety-two percent of the variance of the one-year-and-a-half forecast error is due to these shocks, which suggests that they are ultimately responsible for changes in measured inflation. This result is consistent with its characterisation as a measure of underlying inflation and also with the results discussed in section 5.1. On the other hand, shocks with permanent effects on real activity also have permanent effects on inflation. Nevertheless, their explanatory power is considerably lower.

A further application of the methodology set out in Appendix B yields the core inflation series depicted in Figure 8. Just as before in the case of permanent inflation, core inflation represents the major portion of the reported inflation rate during this period. The similarity of changes in core inflation to those of actual inflation indicates that inflation dynamics in Spain has shown an inertial behaviour minimally determined by disturbances having a permanent effect on the level of output.

---

The assumption used in determining the level is more controversial inasmuch as, a priori, the fact that core inflation deviations from the actual inflation rate must just be transitory is not explained.
As discussed above, core inflation reflects the impact of shocks without a long-run effect on the level of output. In other words, this is the component of inflation which is determined by permanent demand shocks and the business cycle\textsuperscript{30}. Non-core inflation is determined by shocks which have a permanent effect on the level of output. These may be technological or be determined by public or private investment decisions which affect the level of output through the accumulation of capital.

5.3. Comparison of results: the determinants of inflation

Starting from the above results, the determinants of inflation may be interpreted on the basis of two elements: first, a comparison of permanent and core inflation and second, a comparison of the time path of these measures of underlying inflation with that of actual inflation.

As mentioned above, permanent inflation is caused by permanent changes in the growth rate of monetary aggregates or technological factors which change the growth potential. Furthermore, core inflation develops on the basis of shocks to the growth rate of monetary aggregates, and business cycle shocks, which do not have a long-term effect on the level of output.

Therefore, if changes in the rate of core inflation resemble those of actual inflation, inflation is mainly determined by shocks that have no long-run effect on real activity. In turn, if changes in permanent inflation resemble those of actual inflation, transitory factors play a relatively minor role.

Moreover, if both underlying inflation measures are similar, it seems reasonable to believe that, on average, inflation in the chosen sample period was dominated by shocks that have a permanent effect on inflation and do not affect long-run output (e.g. permanent changes in the growth rate of monetary aggregates). On the other hand, the difference between the two measures provides information on the shocks specific to each of the concepts: i.e. as regards core inflation, temporary technological shocks, and, as regards permanent inflation, shocks with a permanent effect on the potential growth rate.

As can be seen in Figures 7 and 8 the time paths of permanent inflation and core inflation are quite similar; nor do they differ excessively from actual inflation, except at specific times. The similarity of the two measures of underlying inflation therefore indicates that permanent nominal shocks have played a key role in determining the path of inflation in the Spanish economy.

6. Conclusions

Direct use of the actual inflation rate in the analysis of the inflation process may be problematic, owing to the fact that inflation is contaminated by transitory factors, which obscure its true state. With a view to avoiding, or at least reducing, this shortcoming, the literature has developed various measures for capturing the most permanent signals of the inflation process. In this paper, we have examined various procedures in their application to the Spanish economy.

First of all, we discuss the standard measure of underlying inflation by exclusion which is obtained by excluding from the CPI its two most variable components: the unprocessed food and energy indices. Alternative measures have recently been proposed that attempt to overcome some of the inadequacies of the standard underlying inflation measure. Thus, with limited-influence estimators (i.e. trimmed means and weighted medians), rather than always excluding the prices of the same articles, sub-indices are excluded if they exhibit outlying price changes. Another possibility would be to obtain an underlying inflation measure by smoothing.

\textsuperscript{30} It is therefore not correct to interpret this measure as a cyclically adjusted measure of inflation.
Specifically, a rate of change is applied to the trend component of a price index. However, it may be better to calculate it on the basis of a sub-component of the CPI, rather than on the CPI itself. Specifically, its calculation on the basis of the CPI excluding its most volatile components may be informative. However, if large price changes in some periods originate in sectors whose prices are generally relatively stable, it may be better to use limited-influence estimators. Finally, we have also presented, using a multivariate perspective, a permanent inflation measure, which shows the explanatory power of shocks having a long-run effect on inflation, and a core inflation measure, which is determined by the effect on inflation of shocks that do not have a long-run effect on output. Besides providing underlying inflation measures, the joint examination of these multivariate approaches permits a reading of the economic determinants of inflation.

In any case, as all these measures have advantages and disadvantages (see Table 8) and none of them takes priority over the rest, it is well to examine them all in order to obtain a more reliable description of the state of inflation. While time-specific circumstances may make it advisable to focus on one of them in particular, it is nevertheless true that diagnosis of the inflation process gains in solidity insofar as they all convey the same message.

| TABLE 8. - Main advantages and limitations of the various underlying inflation measures |
|-------------------------------------------------|---------------------------------|---------------------------------|
| **MEASURE OF UNDERLYING INFLATION** | **ADVANTAGES** | **LIMITATIONS** |
| Underlying inflation by exclusion | . Readily understandable | . A prior decision must be made as to articles whose prices should be excluded |
| | . Easy to compute | |
| | . No need for long time series | |
| Trimmed mean | . No need for a prior decision as to articles whose prices should be excluded | . Choice of where to trim the tails of the cross-sectional distribution |
| | . Easy to compute | |
| | . No need for long time series | |
| Weighted median | . No need for a prior decision as to articles whose prices should be excluded | . Fluctuates excessively in practice |
| | . Easy to compute | |
| | . No need for long time series | |
| Underlying inflation by smoothing | . Gives a clear signal of the trend of inflation | . Potential differences in the assessment of outliers and in the estimation of the trend |
| Permanent inflation | . Consistent with a widely accepted economic theory (vertical long run Phillips curve) | . An additional hypothesis required to determine its level |
| | . Multivariate nature | |
| Core inflation | . Consistent with a widely accepted economic theory (vertical long run Phillips curve) | . An additional hypothesis required to determine its level |
| | . Multivariate nature | |
REFERENCES


APPENDIX A

Signal extraction with reduced-form models

In signal extraction by reduced-form models, the unobserved components of a series are constructed from the roots of the ARIMA model which best fits that series, assuming that these components, in turn, follow ARIMA processes. However, not all ARIMA models can be decomposed in this way. For example, an airline process where \( \theta_2 \) is not positive or very close to zero\(^{31}\) is not permissible. Moreover, to be able to go from the ARIMA model to models of its components, certain restrictions must be imposed, which are set out below.

Suppose that series \( X_t \) follows a process of the type:

\[ \phi(L) X_t = \theta(L) a_t \quad (A.1) \]

where \( \phi(L) \) may have unit roots and \( a_t \) is a white noise process, and it is wished to decompose it into its trend, seasonal and irregular components, i.e.:

\[ X_t = T_t + S_t + I_t \quad (A.2) \]

The roots of the polynomials \( \phi(L) \) and \( \theta(L) \) shall be assigned to each of these three components, taking into account the cycle of each root and the component to which it theoretically corresponds\(^{32}\). For this purpose, it is initially hypothesised that the three components, in turn, follow ARIMA processes of the form:

\[ \phi_T(L) T_t = \theta_T(L) a_t \]
\[ \phi_S(L) S_t = \theta_S(L) a_2 \]
\[ \phi_I(L) I_t = \theta_I(L) a_3 \quad (A.3) \]

where \( a_1, a_2, \) and \( a_3, \) are white noise processes independent of each other, and the polynomials of the trend and seasonal components may have unit roots.

In addition, it must hold that:

\[ \phi(L) = \phi_T(L) \phi_S(L) \phi_I(L) \quad (A.4) \]

without the autoregressive polynomials appearing to the right of the equals sign sharing roots in common.

When each of the roots of the polynomial \( \phi(L) \) has been assigned to the three unobserved components, the restrictions are imposed that the maximum orders of \( \theta_T(L) \) and \( \theta_S(L) \) shall not exceed the maximum orders

\(^{31}\) This result, as demonstrated by Hillmer and Tiao (1982), is common to all ARIMA \( (0,1,1) \times (0,1,1)_s \) models, and is, in turn, extendable to ARIMA \( (0,0,1) \times (0,1,1)_s \) models. These authors also establish from which values of \( \theta_s \), for different \( s \), the ARIMA \( (0,1,1)_s \), models are consistent with a decomposition by reduced form models, the sufficient condition being that \( \theta_s > -0.1010 \).

\(^{32}\) However, in some situations it may not be clear to which component a particular root corresponds.
of $\phi_t(L)$ and $\phi_s(L)$, respectively. Finally, as the system is not identified by these order restrictions only, it is usually required that the variance of the innovation of the irregular component $\sigma_{aI}^2$ be maximised. This latter condition is called the canonical property, and implies that most of the variability is concentrated in the irregular component, while the other two components are as stable as possible.

When the ARIMA models, including their parameters, have been obtained for the components, a time series needs to be generated for each of them. To do this, the theoretical estimators of the components with minimum average quadratic error are obtained by applying symmetric filters to the original series. The filter for the trend component is:

$$
\frac{\sigma_{aI}^2 \theta_t(L) \theta_t(F) \phi_s(L) \phi_s(F) \phi_t(L) \phi_t(F)}{\sigma_o^2 \theta(L) \theta(F)}
$$

(A.5)

where $F$ is the forward operator, i.e. $F=L^{-1}$.

In practice, it is necessary to apply the above filters (which are characterised by being symmetric and infinite, although convergent) to a finite sample, to obtain the empirical estimators of the components. For this purpose, they are approximated by finite filters, and forecasts are inserted at the ends of the series where values are not known.
APPENDIX B

Econometric methodology of the economic measures of underlying inflation with multivariate models

To obtain the various types of shocks on which the economic measures of underlying inflation will be based a bivariate time-series model is estimated, including logarithmic changes of output in real terms and absolute changes in the rate of inflation, using for this the logarithmic year-on-year rate\(^{33}\). We use the notation \(X_t = (\Delta \pi_t, \Delta y_t)'\) where \(\Delta\) is the first difference operator, \(\pi_t\) the inflation rate and \(y_t\) output; we assume that \(X_t\) has a structural interpretation\(^{34}\).

\[X_t = A(0) e_t + A(1) e_{t-1} + \ldots = \sum_{j=0}^{\infty} A(j) e_{t-j}\]

\[\text{Var}(e) = I\]

where \(e_t\) is the vector of structural disturbances in the system \((e_t, e_{t-1})'\). This vector shows no serial correlation and is normalised to the identity matrix\(^{35}\). Equation (B.1) shows the transmission mechanism through which structural disturbances affect the economy.

Nevertheless, these structural disturbances \(e_t\) are not observed directly, but must be recovered on the basis of the moving average representation of the estimated VAR model:

\[X_t = v_t + C(1) v_{t-1} + \ldots = \sum_{j=0}^{\infty} C(j) v_{t-j}\]

with the first matrix of the polynomial \(C(j)\) being the identity matrix and \(\Omega\) the covariance matrix of \(v_t\), the vector of reduced-form innovations.

Comparison of (B.1) and (B.2) shows that reduced-form shocks are linear combinations of the structural shocks

\[v_t = A(0) e_t\]

and, moreover, the transmission mechanisms are related through \(A(j) = C(j) \cdot A(0)\) for any \(j\). As \(v_t\) is computed on the basis of residuals of the VAR model, knowing \(A(0)\) allows us to recover structural shocks. The matrices \(A(j)\) that define the transmission mechanism may also be recovered. Once the structural shocks and their transmission mechanisms have been recovered, actual inflation may be broken down into two terms. Depending

\(^{33}\) These transformations are used, in line with Augmented Dickey-Fuller and Phillips-Perron unit root tests to ensure that we are dealing with a stationary process. It should be pointed out that year-on-year inflation in Spain seems to be nonstationary, so that there have been permanent shocks to the inflation rate. Moreover, use of the year-on-year rate reflects a nonstationary stochastic seasonality of the CPI, as suggested by the Franses seasonal unit root tests run by Matea (1994). On the other hand, it is assumed, on the basis of the hypothesis of a vertical long-run Phillips curve and the results of the Johansen and Dickey Fuller cointegration tests, that there is no long-run relationship between inflation and output.

\(^{34}\) To simplify notation, the determinist elements of the model are not included.

\(^{35}\) Note that we are assuming that structural components are uncorrelated.
on which identification hypothesis is used, the measures of permanent and core inflation may be obtained. To do so, it is therefore necessary to identify the 4 elements of the matrix $A(0)$.

From (B.3), we have

$$
\Omega = A(0) \cdot A(0)'
$$

(B.4)

which yields three restrictions. The fourth restriction required is obtained from the long-run identification restriction.

Thus, with regard to permanent inflation, the two types of disturbances are defined according to their long-run effect on the inflation rate: the first group has a transitory effect, while the impact of the second is permanent. To identify the first group of disturbances we restrict the long-run multiplier for $e_t^i$ on $\pi$ to be identically equal to zero, because this shock is not allowed to have a permanent effect on inflation. Restricting the sum of parameters in $a_{11}(L)$ to be zero achieves this condition.

$$
\sum_{j=0}^{\infty} a_{11}(j) = 0
$$

(B.5)

where $a_{11}(j)$ is the (1,1) element of $A(j)$. To understand this restriction, it should be noted that $a_{11}(j)$ shows how $\pi_t$ is affected after $j$ periods following a unit innovation of $e_t^i$. Therefore, $\sum_{j=0}^{\infty} a_{11}(j)$ is the effect on inflation after $k$ periods, so that in order for $e_t^i$ not to have a long-run impact on inflation, it must be that $\sum_{j=0}^{\infty} a_{11}(j) = 0$.

Once the structural disturbances and their transmission mechanisms have been obtained, we may compute the desired breakdown of the change in the inflation rate into two components:

$$
\Delta \pi_t = \sum_{j=0}^{\infty} a_{11}(j)e_{t-j}^i + \sum_{j=0}^{\infty} a_{12}(j)e_{t-j}^p
$$

(B.6)

$$
\Delta \pi_t = \Delta \pi_t^i + \Delta \pi_t^p
$$

The first term of the right hand side shows the effect on the change in the temporary component of inflation. The second term of the right hand side shows the effect on the change in the permanent component of inflation$^{36}$.

With regard to core inflation, the two types of structural shocks are defined on the basis of their long-run effect on real output. The first type does not have a long-run effect on output, although it affects actual inflation. The second type has a long-run effect on output, but does not affect core inflation. Core inflation is defined as the contribution of the first type of shocks on actual inflation.

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$^{36}$ As the model is estimated in first differences, it is not permanent inflation which is identified, but the change in permanent inflation. To obtain its level it is necessary to make a further assumption. The same applies to core inflation.
In formal terms, to obtain core inflation, the long-run restriction \[ \sum a_{22}(j) = 0 \] must be replaced by \[ \sum \hat{a}_{22}(j) = 0 \], so that the disturbance we now denote as \( \hat{\varepsilon}_t^c \), does not have a long-run effect on output.

Similar to equation (B.6), the inflation rate breaks down\(^{38}\) as:

\[
\Delta \pi_t = \sum_{j=0}^{\infty} \hat{a}_{12}(j) \hat{\varepsilon}_t^c + \sum_{j=0}^{\infty} \hat{a}_{12}(j) \hat{\varepsilon}_t^r
\]

\[ \Delta \pi_t = \Delta \pi_t^c + \Delta \pi_t^r \] \hspace{1cm} (B.7)

The first term of the right hand side shows the effect on the change in core inflation and the second term, the difference between the changes in actual and core inflation.

\(^{37}\) It should be noted that the coefficients and structural shocks change as the identification scheme changes. In this second scheme, we denote structural coefficients with a circumflex.

\(^{38}\) In an analogous way, the output equation may be broken down into one term associated with the business cycle and another associated with trend or potential output. See Álvarez and Sebastián (1998).