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Online appendix: model description

The SIR-macro model we use to simulate the effect of alternative containment strategies is a modified version of Jones et al (2020).

The economy is made up of *N* identical households. Each household starts with a continuum of mass 1 of family members and maximises its expected discounted stream of future utility, given by:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, i_t, d_t) = \sum_{t=0}^{\infty} \beta^t \left[(1-d_t)\log(c_t) - (1-d_t-\kappa i_t)\frac{l_t^{1+\varphi}}{1+\varphi} - u_k i_t - u_d d_t\right] \quad (1)$$

where c_t is the per capita consumption of the household, l_t is the per capita labour supply of those who are alive and not sick, d_t is the number of household members who are deceased, i_t is the number of household members who are infected with the virus, β is the household's discount rate, κ is a parameter that controls the proportion of infected household members who are too sick to work, and u_k and u_d are the utility lost from being sick and from dying.

The production side of the economy consists of a representative firm that uses the production function:

$$Y_t = L_t \quad (2)$$

where Y_t is aggregate production and L_t is aggregate labour supply. Given this production function, and noting that the economy consists of a single competitive firm, the price of the final good equals the wage per unit of effective labour, which we normalise to one.

Market clearing requires that all production be consumed:

$$Y_t = C_t \quad (3)$$

Noting that each household is identical, and normalising *N* = *1*, we can rewrite these constraints in per capita terms as:

$$(1-d_t)c_t = (1-d_t-\kappa i_t)l_t \quad (4)$$

At period $t = 0$, a new virus emerges, to which all household members are initially susceptible. The spread of the virus is governed by the process:

$$s_{t+1} = s_t - \gamma e_t I_t s_t \quad (5)$$

$$i_{t+1} = (1-\rho-\delta_t)i_t + \gamma e_t I_t s_t \quad (6)$$

$$d_{t+1} = d_t + \delta_t i_t \quad (7)$$

$$r_{t+1} = r_t + \rho i_t \quad (8)$$

where s_t is the number of household members who are susceptible to the virus, r_t is the number who have recovered from the virus, γ is the infection rate per unit of exposure, ρ is the recovery rate and δ_t is the mortality rate. I_t is the infection rate of the population as a whole, which households take as given in forming their own decisions.

A household's exposure rate is given by:

$$e_t = \bar{e} + (1-d_t)e^c c_t C_t \quad (9)$$

The first term, $\bar{e}$, is an unavoidable level of exposure, independent of market activities. The second term, $(1-d_t)e^c c_t C_t$, is exposure through market activities, where C_t is aggregate consumption, which the household takes as given in making its decisions.

Finally, the mortality rate is an increasing function of the level of aggregate infections:

$$\delta_t = \bar{\delta} + \exp(\chi I_t) - 1 \tag{10}$$

We solve the model under three assumptions about the extent to which households take account of the virus in their economic decision-making.

Case 1: Myopic

In this case, households take no account of the virus, so that a household's problem is:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg[& (1-d_t)\log(c_t) - (1-d_t-\kappa i_t)\frac{l_t^{1+\varphi}}{1+\varphi} - u_k i_t - u_d d_t \\ & + \lambda_t((1-d_t-\kappa i_t)l_t - (1-d_t)c_t)\Bigg]\end{aligned}$$

The first-order conditions for this problem are:

$c_t\colon \frac{1}{c_t} = \lambda_t$

$l_t\colon l_t^{\varphi} = \lambda_t$

Case 2: Precautionary

In this case, households take account of the direct effect of the virus on their own welfare, but do not take account of the effects of their decisions on aggregate variables. The household's problem is then:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg[& (1-d_t)\log(c_t) - (1-d_t-\kappa i_t)\frac{l_t^{1+\varphi}}{1+\varphi} - u_k i_t - u_d d_t \\ & + \lambda_t((1-d_t-\kappa i_t)l_t - (1-d_t)c_t) \\ & + \lambda_t^e(e_t - \bar{e} - (1-d_t)e^c c_t C_t) \\ & + \lambda_t^s(s_{t+1} - s_t + \gamma e_t I_t s_t) \\ & + \lambda_t^i(i_{t+1} - (1-\rho-\delta_t)i_t - \gamma e_t I_t s_t) \\ & + \lambda_t^d(d_{t+1} - d_t - \delta_t i_t)\Bigg]\end{aligned}$$

The first-order conditions for this problem are:

$c_t\colon \frac{1}{c_t} = \lambda_t + \lambda_t^e e^c C_t$

$l_t\colon l_t^{\varphi} = \lambda_t$

$e_t\colon \lambda_t^e = \gamma(\lambda_t^i - \lambda_t^s) I_t s_t$

$s_{t+1}\colon \lambda_t^s = \beta[\lambda_{t+1}^s + \gamma(\lambda_{t+1}^i - \lambda_{t+1}^s)e_t I_t]$

$i_{t+1}\colon \lambda_t^i = \beta\left[-\kappa\frac{l_{t+1}^{1+\varphi}}{1+\varphi} + u_k + \lambda_{t+1}\kappa + (1-\rho-\delta_{t+1})\lambda_{t+1}^i + \delta_t \lambda_{t+1}^{\mathrm{d}}\right]$

$d_{t+1}\colon \lambda_t^d = \beta\left[\log(c_{t+1}) - \frac{l_{t+1}^{1+\varphi}}{1+\varphi} + u_d + \lambda_t(l_{t+1} - c_{t+1}) - \lambda_{t+1}^e e^c c_{t+1} C_{t+1} + \lambda_{d,t+1}\right]$

Case 3: Benevolent

In this case, households take account of the full effects of their decisions on aggregate outcomes. The household's problem is:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg[& (1-d_t)\log(c_t) - (1-d_t-\kappa i_t)\frac{l_t^{1+\varphi}}{1+\varphi} - u_k i_t - u_d d_t \\ & + \lambda_t((1-d_t-\kappa i_t)l_t - (1-d_t)c_t) \\ & + \lambda_t^e(e_t - \bar{e} - (1-d_t)e^c c_t^2) \\ & + \lambda_t^s(s_{t+1} - s_t + \gamma e_t i_t s_t) \\ & + \lambda_t^i(i_{t+1} - (1-\rho-\delta_t)i_t - \gamma e_t i_t s_t) \\ & + \lambda_t^d(d_{t+1} - d_t - \delta_t i_t)\Bigg]\end{aligned}$$

The first-order conditions for this problem are:

$$c_t: \frac{1}{c_t} = \lambda_t + 2\lambda_t^e e^c c_t$$

$$l_t: l_t^{\varphi} = \lambda_t$$

$$e_t: \lambda_t^e = \gamma(\lambda_t^i - \lambda_t^s) I_t s_t$$

$$s_{t+1}: \lambda_t^s = \beta[\lambda_{t+1}^s + \gamma(\lambda_{t+1}^i - \lambda_{t+1}^s) e_t I_t]$$

$$i_{t+1}: \lambda_t^i = \beta \left[-\kappa \frac{l_{t+1}^{1+\varphi}}{1+\varphi} + u_k + \lambda_{t+1}\kappa + (1 - \rho - \delta_{t+1} - \chi \exp(\chi i_{t+1}) i_{t+1}) \lambda_{t+1}^i \right.$$
$$\left. + \gamma e_{t+1} s_{t+1} (\lambda_{t+1}^i - \lambda_{t+1}^s) + (\delta_t + \chi \exp(\chi i_{t+1}) i_{t+1}) \lambda_{t+1}^{\mathrm{d}} \right]$$

$$d_{t+1}: \lambda_t^d = \beta \left[\log(c_{t+1}) - \frac{l_{t+1}^{1+\varphi}}{1+\varphi} + u_d + \lambda_t (l_{t+1} - c_{t+1}) - \lambda_{t+1}^e e^c c_{t+1}^2 + \lambda_{d,t+1}\right]$$

Parameterisation

Table A1 shows the parameter values that we use for the exercises in Graph 2 of the Bulletin.

Model parameter values — Table A1

Parameter	Description	Value
β	Household discount rate	0.996
κ	Proportion of infected individuals who are unable to work	0.15
φ	Disutility of labour	2
u_k	Disutility of illness	0.4
u_d	Disutility of death	1.6
$\bar{e}$	Exogenous level of exposure	0.33
e^c	Exposure through market activities	0.67
γ	Infection rate per unit of exposure	0.7
ρ	Recovery rate	0.35
δ	Base mortality rate	0.02
χ	Curvature of mortality function	0.13