Presenting a new Market VaR measurement methodology providing separate Tail risk assessment

Conseil Optimum Heuristique Inc. / Logiciel ZbBRA Software - V0.2.1

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1. Introduction and context:

Conseil Optimum Heuristique Inc. has been working in the past years on the definition of new methodologies for the accurate and unbiased assessment of different forms of business risk for financial institutions.

The Value at Risk is of major importance in the monitoring of the financial system’s resilience. Accurate assessment of this risk not only ensures economic stability but may also free capital mistakenly retained due to deficient evaluation methods.

« ZbBRA » (Zero-bias Business Risk Assessment) now presents ZbBRA-VaR, a set of new methodologies to achieve proper measurement of Market Risk, including the assessment of the extreme Tail risk, and a properly founded technique for long-horizon evaluations.

These unprecedented innovations in Market risk assessment are supported by a proof of concept software application.

ZbBRA-VaR is a unique piece of software incorporating numerous proprietary algorithms and finally resolving the issue of identifying and measuring stock market entities risk components. This application is an extension of the historical approach, not just eliminating the methodology flaws, but also expanding it by introducing a new modelling of Stock Market symbols variations, through a unique true dichotomy of the risk, allowing separate and accurate measurements of the standard and extreme (aka Tail Event) risks.

ZbBRA-VaR also provides a new way of extending the VaR measurement to multi-days horizons which is not based on convenience but actually results from a realistic simulation of the stock Market fluctuations and modern controls.

This document describes the essential differences between ZbBRA-VaR (also called simply Z-2) and other existing methodologies.

2. The Market Risk:

None of the existing methodologies, from the simplest to the most complex, appears to be resting on a properly founded economic modelling of Stock market variations and the related risks. Whether using the «Risk-Metrics » or the Monte-Carlo method or a “Historical” approach, all ultimately rely Normal and Gauss laws measurement tables. These techniques can lead to under-evaluation of the tail (extreme) events risk, and a poor estimation of the VaR.
ZbBRA presents major improvements to the historical approach by identifying a complete modeling of the Stock Market symbols variations distribution, and a methodology for the accurate measurement of its risk elements:

- Separating and calculating individually the primary standard variation risk and the extreme (tail) event risk, and
- Introducing a new way to extend the historical approach for long term evaluations.

Both methodologies introduce truly innovative approaches based on emulation of the Stock market components and participants’ behaviour, achieved through actual understanding of the distribution components, risk triggers, and confirmed by the application of true scientific methods.

3. Modeling the Market variation and the VaR:

Modeling of the VaR is actually – so far – very limited. As explained above, current methodologies for VaR measurement fail to pass mathematical scrutiny. The practiced methods appear to lack an actual understanding of the nature of the tail risk, nor provide any kind of modeling of the actual distributions. This last plural is not a typo.

The general consensus is (obviously) that stock market variations do not obey the Normal Law, and only vaguely resemble the more generalized form of the Gauss Law. The major discrepancy with these distributions comes from the « Fat rat tails » exhibited by the historical data distribution of these variations. To illustrate the issue and the resulting measurement difficulty, we can look at an actual example. The vast majority of stock symbols exhibit distribution curves very similar to this example:

1  Graphics are not produced directly by the ZbBRA software, but instead done with a worksheet application based on a worksheet interface report generated by the ZbBRA-VaR application.
It is obvious when looking at such graphics that using the Normal law to assess the Stock Market risks cannot be considered accurate.

The generalized Gauss Law, implemented using properly calculated median and asymmetric semi-deviations, at the limits of its adaptability, does slightly better, but remains clearly insufficiently accurate to be relied on for large financial risk measurements\(^2\):

One should note also that in both cases, because the curve fails to “incorporate” properly the “tail” fluctuations, the resulting threshold line (in the above example the 95\% threshold or a 5\% tolerance level), is located much further to the right than it would be with a purely historical analysis.

The preliminary research phases leading to the development of ZbBRA-VaR, aimed at:

1. Identifying a distribution model and Law which better approximates actual reality;
2. The model must have *logical, mathematical and financial justification*;
3. The distribution must fit the quasi-totality of free floating stock symbols;
4. An accompanying methodology must enable accurate VaR assessment of both “regular” and “extreme” risks; and finally
5. Be applicable for single day VaR and for longer horizons.

### 4. The ZbBRA-VaR model:

In order to resolve these issues and meet the modeling requirements we chose to base our approach on standard mathematical function analysis techniques and established statistical methods, similar to those often used in Magnetism and Quantum Mechanics. This means relying mostly on differentiation to decompose natural and economic data behaviour and achieve unbiased measurements.

\(^2\) *Graphics are not produced directly by the ZbBRA software, but instead done with a worksheet application based on a worksheet interface report generated by the ZbBRA-VaR application.*
Using these techniques, we were able to extrapolate directly distribution curves for any stock symbol and any time period (as shown in the examples in this document) and were able to test their level of mean squares correlation to various mathematical probabilities laws.

Our analysis further confirmed that the location of inflexion points is not compatible with existing probability laws distributions. The successive differentials show clearly the presence of peaks in the variation curves slopes leading us to conclude the necessary presence of a second probability law, relating to tail events, originally hidden behind the large fat tail variations of the historical curves.

From there, based on mathematical findings as well a pure economics reasoning (with abundant amounts of trial and error) we successfully identified the proper mathematical model to match the tail events distribution. As both mathematical and economic/financial logic strongly support the correspondence and applicability of this law to extreme tail events, it was then only a matter of finding a technique to separate the two laws while optimizing the overall correlation with historical distributions.

This led us to a proper modeling of Stock variations which:

- Is parameterized separately for each stock symbol, and
- Separates and measures individually both types of applicable risks:
  - The standard Gaussian risk of daily moderate stock variations, and
  - The extreme tail event risk causing the distribution fat tails shape.

This does not just provide better measurement of the risks. It also allows for:

- Adjustment, normally lowering it, of the Value at Risk (VaR);
- True dichotomy of the two risks allowing to better value these based on their nature;
- And of course increasing the availability of capital and improving your profitability.
Using this model we found that the near totality of stock symbols tested showed a very high correlation level with our Hybrid law. As an example, the fitted ZbBRA-VaR Hybrid Gauss+Terp distribution curve for the MIND symbol shows a 0.9766 (97.7%) correlation level with historical data, while the Normal, Gauss and Cauchy laws respectively only reach 0.898 and 0.931.

5. Tail event risk measurement:
The dichotomy of the probability laws allows our process and software to calculate the VaR at any threshold and exactly apportion the regular and extreme probabilities. In this «MIND» example, as sample (partial) outputs from our ZbBRA-VaR software show, at the 95% tolerance level, the total Value at Risk is mainly Tail (extreme) event risk (5.338%), which becomes 8.43% at the 99% tolerance level, while “regular” or “Gaussian” VaR at these levels is less than one tenth of one percent:

![Tail Risk graphic for the above example](image-url)
6. VaR and longer horizons

The basic mathematical quandary is that stocks can vary any day, and these variations are triggered by a large number of extraneous as well as economical factors, and stock variation assessment goes way beyond simply evaluating the financial viability of a firm, product or even commodity. To make things even more complex, the sometimes less than ethical actions of some people or firms can further compromise any attempt at directly modeling the Stock market environment. Hence the recourse to mathematical probability laws as the only method for risk assessment.

6.1 Long term stocks behaviour

We showed how a combination of two well known and established probability laws lead to an unbiased and accurate method for measuring the 1-day Value at Risk. But, as the horizon expands, other factors become dominant, making stock fluctuations more akin to a vibration than a somewhat predictable curve. Professionals and software algorithms will likely divest themselves from losing stocks, further accentuating their fall, and switch to better options, causing these to rise.

These somewhat predictable reactions will secondary effects. The result is clearly that past the first few (2 to 4) days, a look back at stock variations now shows patterns very similar to waves observed in acoustics and quantum physics. This brings back into the alternatives for modeling long term variations a third probability law: The Cauchy-Lorentz Law, specifically applied by Physicists for the measurement of such behaviour.

6.2 Historical correlation

To take advantage of this natural correspondence between the Stock Market variations and the applicability of the Cauchy-Lorentz law, the ZbBRA methodology uses a simple historical simulation of the reality and actions of professionals and computerized algorithms.

ZbBRA-VaR rebuilds a new historical table for each horizon, looking at the proper matching time interval to define the frequency of each stock movement (again in percentage, by 0.1% increments), and matches this histogram to the best correlated probability law, Hybrid, Gauss or Cauchy-Lorentz.

Repeated tests show that the Gauss-TeRp hybrid remains the best method to model stock variation for short (1-3) days horizons, but that the Cauchy-Lorentz often provides an adequate model for longer horizons (6 and up).

The intermediate zone (3-6) result varies from symbol to symbol. A sample output is shown in section 7.3.2, displaying the table of the VaR results at thresholds 95% and 90%, for 1, 2, 5, 8 and 12 days horizons for symbol HIBB (2014/3/14)
7. **The ZbBRA_VaR_V0.2.1 software**

Following our conclusions, largely based on numerous tests of our probability laws using various software, we designed and developed an application base. This application – ZbBRA-ii “VaR” - is fully functional and was designed to be adaptable to any institutions specific needs. So although it executes and processes any stock symbol or list of symbols (one or several portfolios) standalone on any Windows platform, it is built to be usable either as a Windows (from XP and up) application or to be wrapped into another application.

With a wide array of options and parameters, is can easily be adapted to user preferences. The application is fully bilingual (French and English), and executes extremely quickly on any platform (Windows, Linux, Unix or Aix) processing any stock code in around ten to fifteen (10-15) seconds.

### 7.1 Basic ZbBRA-ii “VaR” features

- Processes single or multiple (portfolio) meta-stock format data of any type,
- Automatic determination of historical data relevance (selects the time period to use),
- Determination of the main data curve characteristics, growth factors and ratios,
- Extreme events history analysis;
- Detection of impending « doom » extreme events by autocorrelation;
- Conversion to variations distribution for multiple horizons;
- Comparison of the distribution to all applicable probability laws, including hybrid law;
- Automatic optimization of each one to maximize the History-to-Law correlation factor;
- Automatic VaR (price) calculation and reporting at two threshold levels,
- Multi-horizon VaR projection (Day 1 + 4 additional user selected horizons),
- Automatic risk split into standard/Gauss and extreme/TeRp,
- Automatic choice of appropriate law for multi days horizons (including Cauchy-Lorentz),
- Variation predictions using five (5) different algorithms based on the previous analysis,
- Choice of adjustments (day-of-week, week-of-year, conjecture, square-root(h), etc.),
- Available interface to our ZbBRA-i (Financial Statement Analysis) software,
- Portfolio analysis done by symbol individually as well as global portfolio risk assessment,
- Unique simulation long-horizon methodology including a “break-out” (or divest) feature.

Another of our software applications (ZbBRA-iii/Pro) may also assist portfolio managers in the process of building Risk-Optimized portfolios. Information is available upon request.

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Detailed user guides for these applications are available upon request to: zbbra@zbbra.com
7.2 **Sample distribution fit graphic**

![Sample distribution fit graphic](image)

7.3 **Sample ZbBRA-VaR output reports**

7.3.1 **Portfolio report:**

<table>
<thead>
<tr>
<th>Symbol/Correlation</th>
<th>$price</th>
<th>#Shares</th>
<th>Total Value</th>
<th>VaR%95%</th>
<th>Total VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - ACET</td>
<td>0.98289</td>
<td>18.19$</td>
<td>250</td>
<td>45475.00$</td>
<td>0.608$ 1519.40$</td>
</tr>
<tr>
<td>2 - ADEP</td>
<td>0.95160</td>
<td>16.24$</td>
<td>10000</td>
<td>162450.00$</td>
<td>1.129$ 11292.05$</td>
</tr>
<tr>
<td>3 - CAMP</td>
<td>0.97438</td>
<td>31.44$</td>
<td>2000</td>
<td>62880.00$</td>
<td>1.507$ 3013.10$</td>
</tr>
<tr>
<td>4 - COBN</td>
<td>0.98104</td>
<td>11.63$</td>
<td>1200</td>
<td>13956.00$</td>
<td>0.600$ 720.49$</td>
</tr>
<tr>
<td>5 - HIBB</td>
<td>0.97308</td>
<td>58.58$</td>
<td>900</td>
<td>52717.50$</td>
<td>1.328$ 1195.26$</td>
</tr>
<tr>
<td>6 - HALI</td>
<td>0.97983</td>
<td>8.52$</td>
<td>5500</td>
<td>46660.00$</td>
<td>0.332$ 1824.92$</td>
</tr>
<tr>
<td>7 - FARM</td>
<td>0.97902</td>
<td>20.68$</td>
<td>1200</td>
<td>24816.00$</td>
<td>0.844$ 1012.76$</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td></td>
<td></td>
<td>23300</td>
<td>409154.50$</td>
<td>5.0294% 20577.98$</td>
</tr>
</tbody>
</table>

---

**Consolidation VaR for Portfolio set P007**

<table>
<thead>
<tr>
<th>Symbol/Correlation</th>
<th>$price</th>
<th>#Shares</th>
<th>Total Value</th>
<th>VaR%95%</th>
<th>Total VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A Port 7</strong></td>
<td>0.95624</td>
<td>17.56$</td>
<td>23300</td>
<td>409154.50$</td>
<td>0.412$ 9591.30$</td>
</tr>
</tbody>
</table>

7.3.2 **Symbol VaR analysis report:**

(see next page, ZbBRA-VaR report for HIBB, 2014-03-14)

ZBBRA-ii/VaR Historical Hybrid Gauss+TeRp analysis completed with correlation 97.31%, & annual volatility of 17.69%

[1] Basic (Day +1 horizon) Value at risk as a percentage by tolerance level:

<table>
<thead>
<tr>
<th>Tolerance Level</th>
<th>&gt; ---99%---</th>
<th>---98%---</th>
<th>---97%---</th>
<th>---96%---</th>
<th>---95%---</th>
<th>---94%---</th>
<th>---93%---</th>
<th>---92%---</th>
<th>---91%---</th>
<th>---90%---</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Regular (Gauss) risk:</td>
<td>0.15142</td>
<td>0.51174</td>
<td>0.75386</td>
<td>0.88637</td>
<td>0.95722</td>
<td>1.00664</td>
<td>1.00727</td>
<td>0.99885</td>
<td>0.98433</td>
<td></td>
</tr>
<tr>
<td>2. Extreme (Tail) event:</td>
<td>3.88828</td>
<td>2.66592</td>
<td>1.99135</td>
<td>1.58282</td>
<td>1.31008</td>
<td>1.13979</td>
<td>0.96698</td>
<td>0.85161</td>
<td>0.75849</td>
<td>0.68161</td>
</tr>
</tbody>
</table>
*** Var/threshold totals: | 4.03970 | 3.17766 | 2.74522 | 2.46919 | 2.26730 | 2.01616 | 1.97362 | 1.85888 | 1.75734 | 1.66594 |

[2] Frequency distribution comparative correlations table:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Correlation</th>
<th>Median</th>
<th>-L-Deviation-(R+)-</th>
<th>Description</th>
<th>Res. Deviation</th>
<th>A.A.E</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/02:</td>
<td>0.9554</td>
<td>0.1164</td>
<td>1.0000</td>
<td>Normal, median shift:</td>
<td>3.943</td>
<td>2.056</td>
<td>6.610</td>
</tr>
<tr>
<td>C/06:</td>
<td>0.9612</td>
<td>0.1116</td>
<td>1.3332</td>
<td>Gaussian, asymmetric:</td>
<td>2.719</td>
<td>1.616</td>
<td>4.699</td>
</tr>
<tr>
<td>G/04:</td>
<td>0.9728</td>
<td>0.1256</td>
<td>1.2921</td>
<td>Cauchy, median shift:</td>
<td>2.378</td>
<td>1.275</td>
<td>4.017</td>
</tr>
<tr>
<td>H/08:</td>
<td>0.9731</td>
<td>0.1354</td>
<td>1.2176</td>
<td>Hybrid, Gauss + TeRp:</td>
<td>2.271</td>
<td>1.309</td>
<td>3.934</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal-Gaussian estimates:</td>
<td>$58.642</td>
<td>$58.708</td>
<td>$58.842</td>
<td>$59.110</td>
<td>$59.379</td>
</tr>
<tr>
<td>Historical projection (L):</td>
<td>$58.616</td>
<td>$58.658</td>
<td>$58.740</td>
<td>$58.906</td>
<td>$59.071</td>
</tr>
<tr>
<td>Hybrid, Gauss + TeRp laws:</td>
<td>$58.705</td>
<td>$58.836</td>
<td>$59.098</td>
<td>$59.625</td>
<td>$60.157</td>
</tr>
<tr>
<td>Hybrid, G+T/Markov copula:</td>
<td>$58.548</td>
<td>$58.521</td>
<td>$58.464</td>
<td>$58.345</td>
<td>$58.218</td>
</tr>
<tr>
<td>Hybrid+Fourrier Synthesis:</td>
<td>$58.548</td>
<td>$58.521</td>
<td>$58.464</td>
<td>$58.345</td>
<td>$58.218</td>
</tr>
</tbody>
</table>

6. Predictive downfall risks: (None detected within the 13 upcoming market days context horizon)


<table>
<thead>
<tr>
<th>Market days</th>
<th>--$$/Unit--</th>
<th>=Total%</th>
<th>=Regular</th>
<th>=Extreme</th>
<th>=$$VaR= Risk/Return</th>
<th>=Total%</th>
<th>=Regular</th>
<th>=Extreme</th>
<th>=$$VaR= Risk/Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1/Mar.14</td>
<td>$58.71</td>
<td>2.267%</td>
<td>0.957%</td>
<td>1.310%</td>
<td>$1.33</td>
<td>9.95/1</td>
<td>1.666%</td>
<td>0.894%</td>
<td>0.682%</td>
</tr>
<tr>
<td>+2/Mar.17</td>
<td>$58.84</td>
<td>2.649%</td>
<td>0.672%</td>
<td>1.977%</td>
<td>$1.55</td>
<td>5.81/1</td>
<td>1.850%</td>
<td>0.853%</td>
<td>0.997%</td>
</tr>
<tr>
<td>+4/Mar.19</td>
<td>$59.11</td>
<td>2.686%</td>
<td>0.968%</td>
<td>1.718%</td>
<td>$1.57</td>
<td>2.94/1</td>
<td>1.903%</td>
<td>1.081%</td>
<td>0.823%</td>
</tr>
<tr>
<td>+8/Mar.25</td>
<td>$59.65</td>
<td>2.057%</td>
<td>2.057%</td>
<td>(n/a)</td>
<td>$1.20</td>
<td>1.12/1</td>
<td>1.526%</td>
<td>1.526%</td>
<td>(n/a)</td>
</tr>
<tr>
<td>+12/Mar.31</td>
<td>$60.20</td>
<td>1.955%</td>
<td>1.955%</td>
<td>(n/a)</td>
<td>$1.15</td>
<td>1/1.42</td>
<td>1.413%</td>
<td>1.413%</td>
<td>(n/a)</td>
</tr>
</tbody>
</table>

[5] Important notes:

1. All projected values past day+1 are assuming the previous projections were met;
2. The historical data is kindly provided by the S&P 500 website;
3. Do NOT make use of the projections provided here for investment purposes.

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