Comments on the Consultative document
Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability
by Bank for International Settlements

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Emilio Barucci
Dipartimento di Matematica - Politecnico di Milano
Piazza Leonardo da Vinci, 32 - 20133 Milano (Italy)

Luca Del Viva
ESADE Business School
Av. Pedralbes, 60-62, Barcelona, 08034, (Spain)
luca.delviva@esade.edu
1 Introduction

The financial crisis has shown that the current regulation based on capital requirements is not enough to prevent default of large financial intermediaries. In particular, subordinated debt and Tier-2 capital financial instruments seem to be unable to provide an “absorbing barrier” for losses and to reduce the default probability.

In the post financial crisis debate, hybrid securities have been proposed to render financial intermediaries safer. More specifically, Flannery (2002), Flannery (2009), Duffie (2009), Albul et al. (2010), Pennacchi (2010), Pennacchi et al. (2010), McDonald (2010), Squam Lake Working Group (2009) have proposed several different types of contingent capital: bonds to be converted into shares as the financial condition of the bank deteriorates, on the debate see also Hart and Zingales (2009), Kashyap et al. (2008), Admati and Pfleiderer (2009). This instrument in principle could play an important role in reducing the probability of going bankrupt and bankruptcy costs of a bank, but its effectiveness is seriously affected by the trigger mechanism.

In the Consultative Document by the Bank for International Settlements, contingent capital has been proposed with a trigger mechanism defined by the regulatory authority: the authority has the power to impose the conversion of bonds in shares.

The main features of the proposal are the following:
1. All non-common Tier 1 instruments and Tier 2 instruments at internationally active banks must have a clause in their terms and conditions that requires them to be written-off on the occurrence of the trigger event.
2. Any compensation paid to the instrument holders as a result of the write-off must be paid immediately in the form of common stock (or its equivalent in the case of non-joint stock companies).

3. The issuing bank must maintain at all times all prior authorisation necessary to immediately issue the relevant number of shares specified in the instrument’s terms and conditions should the trigger event occur.

4. The trigger event is the earlier of: (1) the decision to make a public sector injection of capital, or equivalent support, without which the firm would have become non-viable, as determined by the relevant authority; and (2) a decision that a write-off, without which the firm would become non-viable, is necessary, as determined by the relevant authority.

5. The issuance of any new shares as a result of the trigger event must occur prior to any public sector injection of capital so that the capital provided by the public sector is not diluted.

In this note exploiting the analysis contained in [Barucci and Del Viva (2010)] we analyze the impact of the instrument on bankruptcy costs and on the capital structure. In [Barucci and Del Viva (2010)] we have analyzed the capital structure of a company issuing contingent capital and bonds with an option conversion in the hands of shareholders or with a conversion trigger defined by a conversion barrier on the leverage of the company. In what follows, we model the decision of the regulatory authority to impose conversion as a trigger-barrier on the probability of default, i.e., when the probability of default of the company reaches a certain level the authority imposes the
conversion of contingent capital.

We analyze the optimal capital structure in a setting \textit{a la} Goldstein et al. (2001). Our main results are the following:

- it is necessary to fix the spread of the contingent capital coupon over that of bonds, otherwise the optimal capital structure problem doesn’t admit a solution as the net company value is increasing in the coupon of contingent capital;

- contingent capital reduces bankruptcy costs in the following cases: a low enough default probability trigger, a large spread of contingent capital coupon over that of straight debt, the bank is not too risky and the costs of liquidating the assets is not too high;

- a late conversion of contingent capital (high probability trigger) increases the spread of straight debt, decreases that of contingent capital and increases the level of leverage with a transfer of wealth from shareholders to bondholders;

This note is organized as follows. In Section 2 we introduce the setting. In Section 3 we present comparative static results.

2 Capital structure and contingent capital

We consider a company issuing two classes of notes in $t = 0$: straight debt and contingent capital.
Straight debt is a perpetual consol bond paying a constant coupon $CD$ in continuous time as long as the firm remains solvent. Contingent capital pays a constant coupon $CC$ as long as the firm remains solvent and bonds are not converted in equity. If contingent capital is converted then the conversion occurs at the current value of the shares: bondholders receive equities for a value equal to the principal of contingent capital ($K$) at the current price of the stocks. Note the peculiarity: contingent capital bonds are to be converted in common shares, i.e., conversion in shares is mandatory, it is not possible to repay bonds through asset sales or the issuance of other notes as in [Goldstein et al. (2001)]. This feature produces a loss for shareholders after conversion, i.e., they have to sell their shares or issue new shares, bondholders instead are not affected by the conversion. There isn’t a lock up, contingent bondholders can sell equities in the market receiving the principal as cash. Differently from bonds analyzed in [Benston et al. (2000), Chemmanur et al. (2004), Glasserman and Wang (2009)], the notes are perpetuities with no maturity. The coupon cannot be canceled or deferred and therefore these notes are lower Tier-2 financial instruments.

In $t = 0$, before conversion of bonds, we have three claimants (shareholders, straight debt holders, contingent capital holders) on the total value of the payout flow of the company. After conversion (period 1) of callable bonds into shares, we have two claimants (equity holders, straight debt holders).

As in [Leland (1994)], the company faces a static capital structure problem. The company as a whole maximizes in $t = 0$ the total value of the three claims choosing the coupon of straight bonds $CD$ and the coupon of contingent capital bonds $CC$. 


Shareholders determine the time of bankruptcy maximizing the value of equity, the authority imposes the conversion of contingent capital on equity. These two events are triggered by a barrier defined on $V(t)$: conversion into equity of contingent capital bonds occurs when the value of the assets approaches $V_l$ from above, liquidation of the assets occurs when their value approaches $V_b$ from above. Conversion occurs before bankruptcy. Below we will show that the conversion barrier can be recovered from a barrier on the probability of default.

Bankruptcy is costly and in particular the fraction $\alpha$ of the assets represents the cost of liquidating the company, i.e., $\alpha V_b$, and $(1 - \alpha)V_b$ represents the value of the company after default. Instead, conversion of contingent into equity doesn’t induce a cost.

As in [Goldstein et al. (2001)], the tax structure includes personal and corporate taxes: interest payments are taxed at the rate $\tau_i$, corporate profits and dividends are taxed respectively at the rate $\tau_c$ and $\tau_d$. As a consequence, shareholders receive the fraction $(1 - \tau_g) = (1 - \tau_c)(1 - \tau_d)$ of non interest payments and $(1 - \tau_i)$ of interest payments.

The total payout flow value of the company under the risk neutral measure evolves as

$$\frac{dV(t)}{V(t)} = \left( r - \frac{\delta}{V(t)} \right) dt + \sigma dW(t), \quad V(0) = V_0, \tag{1}$$

where $r$ is the pre tax risk-free interest rate and $\delta/V$ is the payout rate of the company that includes taxes, dividends and coupons ($EBIT$). $\sigma$ denotes the riskiness of the asset value and therefore of the company.
Fixed the conversion $V_l$ and the default $V_b$ barriers, we can easily obtain the claim value as a function of the value of the company, $E_0$ $E_1$ represent equity value in period 0 and 1, $D_0$, $D_1$ represent debt value in period 0 and 1, $CB_0$ represents contingent capital value in period 0, $B_0$, $B_1$ represent bankruptcy costs in period 0 and 1, see Barucci and Del Viva (2010):

\[
E_0(V) = \left( V - \frac{CD}{r} - \frac{CC}{r} - \left( V_l - \frac{CD}{r} - \frac{CC}{r} \right) \left( \frac{V}{V_l} \right)^{-x_0} \right) (1 - \tau_g) \\
+ ((1 - \phi)E_1(V_l) - K) \left( \frac{V}{V_l} \right)^{-x_0}
\]

\[
D_0(V) = \frac{CD}{r} \left( 1 - \left( \frac{V}{V_l} \right)^{-x_0} \right) (1 - \tau_i) + D_1(V_l) \left( \frac{V}{V_l} \right)^{-x_0}
\]

\[
CB_0(V) = \frac{CC}{r} \left( 1 - \left( \frac{V}{V_l} \right)^{-x_0} \right) (1 - \tau_i) + (K + \phi E_1(V_l)) \left( \frac{V}{V_l} \right)^{-x_0}
\]

\[
B_0(V) = \left( \frac{V}{V_l} \right)^{-x_0} B_1(V_l)
\]

where:

\[
E_1(V) = \left( V - \frac{CD}{r} - \left( V_b - \frac{CD}{r} \right) \left( \frac{V}{V_b} \right)^{-x_1} \right) (1 - \tau_g)
\]

\[
D_1(V) = \frac{CD(1 - \tau_i)}{r} + \left( (1 - \alpha) V_b (1 - \tau_g) - \frac{CD(1 - \tau_i)}{r} \right) \left( \frac{V}{V_b} \right)^{-x_1}
\]

\[
B_1(V) = \alpha V_b \left( \frac{V}{V_b} \right)^{-x_1}.
\]

The optimal capital structure is then obtained maximizing the net company value with respect to the coupons $CD$ and $CC$:

\[
\max_{CD,CC} (E_0(V) + D_0(V) + CB_0(V))
\]
Unfortunately the net company value turns out to be a linear function of the coupon of contingent capital $CC$ and therefore it is impossible to maximize with respect to $CC$. To overcome this problem we assume that the coupon of contingent capital convertible bond is proportional to the coupon of straight debt:

$$CC = CD \times s, \quad s > 0$$

If $s > 1$ then the coupon of the contingent capital is strictly greater than the coupon of straight debt.

We assume that the authority establishes the conversion of contingent capital as the probability of default of the bank approaches a certain level. As shown in [Leland (1994), Bielecki and Rutkowski (2001)], in this setting the discounted probability of default is given by

$$P_B(V) = \left( \frac{V}{V_b} \right)^{-x_1}. \tag{2}$$

$P_B(V)$ represents the present value of 1 unit of money contingent on future bankruptcy, see [Leland (1994)]. $P_B$ can be rewritten as

$$P_B(V) = \int_0^\infty e^{-rt} dP(t; V, V_b) \, dt$$

where $P(t; V, V_b)$ is the distribution function of the time of the first passage through $V_b$ starting from $V$ when the value of the assets of the company follows (1).

In this setting, bankruptcy can occur only after conversion, therefore conditioning on $V = V_i$ we obtain

$$P_B(V|V_i) = \left( \frac{V_i}{V_b} \right)^{-x_1} \tag{3}$$
Given a value of $P_B(V|V_l)$, we can solve for $V_l$ obtaining the conversion barrier that corresponds to a particular value of the discounted probability of default.

The derivation of (2) is straightforward. The first passage time can be written as

$$\tau = \inf \{ t \in [0, T] : V(t) \leq V_b \}$$

From a well known result we have that the distribution function of the first passage for (1) is

$$\mathbb{P}\{\tau \leq T|\mathcal{F}_s\} = N\left( \frac{\ln \left( \frac{V_b}{V_s} \right) - \nu (T - s)}{\sigma \sqrt{T - s}} \right) + \frac{(V_b/V_s)^2 \nu}{\sigma} N\left( \frac{\ln \left( \frac{V_b}{V_s} \right) + \nu (T - s)}{\sigma \sqrt{T - s}} \right)$$

where:

$$\nu = r - \frac{\delta}{V_0} - \frac{1}{2} \sigma^2$$

Now applying [Bielecki and Rutkowski (2001) Lemma 3.2.1] with $T \to \infty$ and conditioning on $V_l$ we have (3).

From (3), for a particular value of the discounted probability, the level of the conversion boundary is given by:

$$V_l = P_B \left( \frac{-x_1}{\sigma^2} \right) \frac{x_1 C}{(1 + x_1) r}$$

that can be rewritten in more compact form as:

$$V_l = P_B \left( \frac{-x_1}{\sigma^2} \right) V_b$$

where $V_b$ is the optimal bankruptcy level obtained in [Goldstein et al. (2001)].

We have to impose that

$$V_l = P_B \left( \frac{-x_1}{\sigma^2} \right) V_b < V_0$$

Note that $V_l$ goes up when the triggering probability tends to zero.
3 Comparative statics

In this section we focus on three main issues:

- bankruptcy costs
- capital structure
- welfare transfer among claimants.

The comparison is performed with respect to the benchmark model of Goldstein et al. (2001) for a company with equity and debt. Table 1 provides the values of the parameters used in the numerical analysis.

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Table 1: Set of parameters for comparative static analysis.

We analyze the model assuming two different trigger discounted probabilities: \( P_B = 0.55 \) and \( P_B = 0.80 \).

**Trigger fixed at \( P_B = 0.55 \)**

Let us assume that the authority imposes the conversion of contingent capital in eq-
Figure 1: Bankruptcy costs for different combinations of the spread of contingent capital ($s$) and volatility, $\alpha = 0.4$ and $P_B = 55\%$.

Let us assume that the authority impose the conversion of contingent capital in equity
Figure 2: Bankruptcy costs for different combinations of the spread ($s$) and loss rate $\alpha$, $\sigma = 0.15$ and $P_B = 55\%$.

as the probability of default reaches the 80%. In this case bankruptcy costs of a bank financed partially with contingent capital can be greater than those of a company issuing only straight debt. Numerical results are illustrated in Figure 3 and 4

The triggering probability plays a significant role on the magnitude of bankruptcy costs. For a low value of the trigger probability, i.e., a safer company since conversion is triggered at a greater distance from default, contingent capital reduces the size of bankruptcy costs compared to a standard equity-debt company. For instance the average reduction of bankruptcy cost in Figure 2 is 48.19%. Bankruptcy gain associated with contingent capital is decreasing in the riskiness of the company and is increasing in the spread of contingent capital. The increase of the trigger produces an increase of the bankruptcy costs. The rationale is that in the presence of contingent capital, an increase of the triggering probability leads to a higher optimal coupon of the straight
Figure 3: Bankruptcy costs for different combinations of the spread \( (s) \) and volatility, \( \alpha = 0.4 \) and \( P_B = 80\% \).

Figure 4: Bankruptcy costs for different combinations of the spread \( (s) \) and loss rate \( \alpha, \sigma = 0.15 \) and \( P_B = 80\% \).
debt, see Figure 5 and 6. A higher coupon of the straight debt increases the speed of reduction of the value of the company (driven by the drift of our process that depends on the coupon) and the bankruptcy event becomes more likely. Indeed, a large probability trigger reduces the conversion boundary $V_0$, for that value of the conversion barrier the net company value is maximized for a higher straight debt coupon. As a result we observe a transfer of wealth from shareholders to debtholders (contingent and in part straight) as the triggering probability increases, see Figure 9.

In Figure 5 we show the relationship between volatility, spread and optimal coupon of straight debt for $P_B = 55\%$: an increase of the spread size reduces the value of the optimal coupon of straight debt. Volatility instead has a positive or a negative impact on the optimal coupon of the straight debt, depending on the spread. For a low value of the spread an increase in volatility has a positive impact, for a large spread the impact is mixed. The effect on the optimal coupon of straight debt produced by an increase in volatility is always positive considering a high value of the trigger, see Figure 6 for $P_B = 80\%$. In this case company riskiness leads to a higher coupon of straight debt. The behavior of the coupon of contingent capital for different combinations of volatility and spread is similar and is shown in Figures 7 and 8.

As far as the spread is concerned, we have that the spread of contingent capital decreases in average as the triggering probability increases while the spread of straight debt goes up, see Figure 10. The difference is due to the fact that the value of contingent capital (positively) depends on the probability triggering conversion - the period before conversion is longer - and the value of straight bonds (negatively) depends on the
probability triggering conversion and of default. As a consequence as \( P_B \) goes from 55% to 80% - the probability of default at conversion goes up and the period before conversion becomes longer - we have that straight bondholders require a larger spread and contingent capital holders are satisfied with a smaller spread.

The level of leverage defined as \((D_0 + C_0)/(D_0 + C_0 + E_0)\) is increasing in the spread \( s \) and decreasing in volatility. As a consequence of the structure of contingent capital (exogenous trigger and fixed spread over the coupon of straight bonds), both straight debt and contingent capital decrease as the volatility increases. As it can be inferred from Figure 5 an increase on the trigger probability increases the level of leverage.

Our analysis shows that contingent capital could in principle reduce bankruptcy costs. However their role in reducing bankruptcy costs strongly depends on the triggering mechanism: the regulatory authority should act well in advance to convert
Figure 6: Optimal coupon of straight debt for different combinations of spread and volatility, $P_B = 80\%$.

Figure 7: Optimal coupon of contingent capital for different combinations of spread and volatility, $P_B = 55\%$. 
Figure 8: Optimal coupon of contingent capital for different combinations of spread and volatility, $P_B = 80\%$.

Figure 9: Average portion of each claim value on the total net company value when $P_B = 55\%$ (left chart) and $P_B = 80\%$ (right chart). The average is taken for volatility ranging from 0.05 to 0.3 and spreads ranging from 0 to 2, see Figure [1]. The values of the remaining parameters are contained in Table [1].
bonds with respect to the default. Otherwise costs of bankruptcy are high and there is a welfare transfer from shareholders to bondholders.

We can maximize the net company value without a fixed spread over bonds, in this case we obtain that the optimum is reached for a coupon of straight debt equal to zero. Thus basically the capital structure would be composed only by contingent capital (equity will be negative for high values of the coupon of contingent capital). So basically we can have an optimal coupon of straight debt only for low values of the coupon of contingent capital.

References

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