

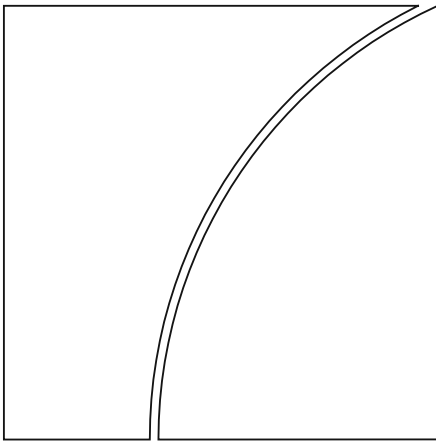
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Consumer Inflation Expectations in Turkey

by Ece Oral

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Consumer Inflation Expectations in Turkey

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Abstract

The expectations obtained from surveys play an important role as leading indicators for the application of the monetary policies. The ability to measure inflation expectations is an integral part of central bank policy especially for central banks that are implementing inflation-targeting regime. A forward-looking perspective is essential to the success of inflation targeting. Therefore, a central bank which has the primary objective of price stability is interested in inflation expectations. Qualitative data on inflation expectations obtained from surveys can be quantified into numerical indicators of the expected rates of price change. This paper presents the results of different quantification methods such as Carlson-Parkin method, balance method, regression method put into action in order to estimate Turkish consumer inflation predictions based on monthly consumer surveys. Carlson-Parkin method quantifies qualitative survey data on expectations assuming aggregate expectations are normally distributed. In order to capture non-normality, stable distributions are also considered. The quantification techniques are compared with each other as well. The regression method is found to be the closest one to realizations. Therefore, expectations via this method is used for all the remaining analyses. Actual inflation and inflation expectations are found to have a cointegration relation. Unbiasedness assumption under REH is rejected within VECM. After rejecting a rational model of the formation of inflation expectations, hybrid model of expectations formation is considered. The “pure” backward and forward looking expectations hypotheses are rejected. As a final result, there exists the strong backward looking nature of expectations in the long run.

¹ The views expressed are those of the author and should not be attributed to the Central Bank of the Republic of Turkey.

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Introduction

A new era began in Turkey in terms of fiscal and monetary policies following the financial and currency crisis in 2001. On the fiscal front, Turkey initiated a set of policies aiming to reduce the public deficit to acceptable levels. On the monetary front, policy switched from a fixed exchange rate to an inflation targeting regime. In this new regime, inflation expectations play a valuable dual role. On the one hand, the new monetary policy emphasizes the need to manage public expectations to curtail inertia in inflation dynamics by switching economic agents' inflation expectations from a backward to a forward-looking perspective, in order to reduce the inflation rate rapidly. By monitoring the credibility gap of the monetary policy, the gap between the inflation target and inflation expectations, the central bank may see how successful it is in anchoring inflation expectations that were damaged by a past history of high inflation rates. On the other hand, agents' inflation expectations may contain some information regarding a future path of inflation that is unknown to the central bank. The success of the inflation-targeting regime crucially depends on a central bank's ability to foresee changes in inflation rates due to long lags in the monetary transmission channel. Thus, inflation expectation surveys may be a valuable source for central banks to collect information regarding the prospect of inflation rates (Oral et al., 2011).

Turkey implemented a disinflation program after the 2001 crisis in 2001. Then, CBRT decided to implement a full fledged inflation targeting regime at the beginning of 2006. When the economy was hit by a series of adverse shocks, drifting inflation away from the target, the regime faced its first stress test in the first half of 2006. Although the central bank reacted determinedly by tightening monetary policy, inflation stayed at relatively high levels due to lagged effects of the accumulated shocks. Inflation deviating the targets by a large margin in the first two years of the new regime increased the role of expectations management in sustaining the disinflation process, as manifested by the persistent gap between expectations and the target (Başkaya et al., 2008).

Inflation expectations play an important part in an inflation targeting regime. Consumers' inflation expectations could potentially impact inflation via their influence on consumption and investment decisions. For a given path of nominal market interest rates, if households expect higher inflation, this implies lower expected real interest rates, making spending more attractive relative to saving.

Survey based data is used to obtain inflation expectations. Expectations obtained from surveys are not directly observable due to qualitative survey data. Direct measure of expectations is needed in order to analyze the behaviour of the inflation expectations. Therefore quantification techniques are needed to quantify survey expectations.

This paper focuses on inflation expectations obtained from Consumer Tendency Survey (CTS) of the CBRT (Central Bank of the Republic of Turkey) and Turkish Statistical Institute (TurkStat). The variables have only been presented in the form of qualitative statistic. Qualitative survey can only provide a direction of change for a given variable instead of an exact figure. Therefore, expectations collected as qualitative survey data are converted into quantitative estimates of the variables under consideration. There are different methods to quantify the qualitative survey results. The main aim of this paper is to quantify the inflation expectations of the consumers. The study is composed of four sections. The aims of the study are presented in the introduction part. The detailed knowledge about the survey and the explanation of the quantification methods are given in the second section. The quantified inflation expectations are given in the third section. The fourth section gives brief knowledge about the formation process of consumer inflation expectations. Finally, the conclusion part gives the final results.

2. Consumer Tendency Survey and Quantification Methods

2.1. Consumer Tendency Survey

Consumer Tendency Survey (CTS), which is the source for the consumer confidence index, was annexed to Household Labour Force Survey in the form of a module. CTS has been conducted since December 2003. CTS has been constructed in order to find out consumer tendencies and expectations for general economic course, job opportunities, personal financial standing and market developments in order to assess their expenditure behavior as well as their expectations, and therefore deciding monthly consumer tendencies in the short-run. CTS covers four fields in measuring consumer tendencies and expectations that are Personal Financial Standing, General Economic Situation, Expenditures and Price Expectations.

The target population used for the survey is the individuals at the age of 15 and above having a job in rural and urban areas of Turkey that provide income. Therefore, the survey covers all individuals at the age of 15 and above who are employed as samples in Household Labour Force Survey meeting these criteria. The survey includes all settlements (rural and urban) in Turkey.

The price question in CTS can be given as follows:

In comparison to the realizations, how do you expect that prices will develop over the next 12 months?

1. Increase more rapidly
2. Increase at the same rate
3. Increase at a slower rate
4. Stay about the same
5. Fall
6. No idea

2.2. Quantification Methods

Inflation expectations have an important role in modern macroeconomic theory. The importance of expectations has been emphasized by the recent inflation experiences of most countries. Direct measurement of expectations can be made through the tendency survey data. The quantitative expectations data are gathered in some surveys. However, the respondents indicate whether prices will fall, rise or remain unchanged for some months ahead in the other surveys. The data gathered from these surveys do not have a mean value because they are qualitative. There are several techniques to quantify the qualitative survey data (Batchelor, 1982). Different quantification methods such as Carlson-Parkin method, balance method, regression method have been considered with the aim of quantifying survey data on inflation expectations.

Carlson-Parkin method used in the pentachotomous case as in this study has two advantages. It does not impose unbiasedness and response thresholds are permitted to vary over time. One disadvantage is that the expectations via this method give different results

due to the choice of probability distribution. On the other hand, regression method doesn't require probability distribution. However, there are two shortcomings of the regression method. One of them is that long time series of survey data is needed in order to implement this method. The other shortcoming is that estimating the regression model of perception with the official current inflation on its left side and survey responses to the question on inflation perception on its right hand side, regression method imposes unbiasedness of inflation perceptions. When balance method is considered, although this method doesn't measure expectations directly, it is not influenced by the assumptions imposed in other two methods.

2.2.1. Probability Method

The probability method which is well known as Carlson-Parkin method (Carlson and Parkin, 1975) was first employed by Theil (1952). The original method has been derived for a trichotomous survey, i.e. the survey participants have three possible answer categories. In this context, the price expectations having three categories such as 'price will increase', 'price will decrease' and 'no change in price' (Batchelor and Orr, 1988) are used. However, in this study, the price question that is used from CTS has five possible answer options. Therefore the pentachotomous case will be considered (Berk, 1999).

The CP method assumes that respondents standing at time t (month) have formed an expectation π_{t+1}^e about inflation in the $t+1$ months when answering the survey. The individual subjective probability distributions can be aggregated to give the joint probability distribution $f(x_{t+1} | \Omega_t)$, where x_{t+1} is the future percentage change of prices at time t for the period $t + 1$ and Ω_t the information set at time t . It is assumed that this distribution has finite first and second order moments and that $E[x_{t+1} | \Omega_t] = \pi_{t+1}^e$, where π_{t+1}^e is the expected value of x at time t for the period $t+1$. Another assumption on the pentachotomous case can be given as follows:

There exists an interval $(-\delta_t^L, \delta_t^U)$ around 0, with $\delta_t^L, \delta_t^U > 0$, such that the participants report 'no change' in prices if the price change expected by them lies within this interval. There exists also an interval $(\pi_t^p - \varepsilon_t^L, \pi_t^p + \varepsilon_t^U)$ around the subjective mean perceived inflation rate π_t^p , with ε_t^L and $\varepsilon_t^U > 0$, such that the individuals report that prices 'increase at the same rate' if the expected price change is covered by this interval. The participants answer therefore in the following manner:

Prices will

fall, if $x_{t+1} \leq -\delta_t^L$

stay about the same, if $-\delta_t^L < x_{t+1} \leq \delta_t^U$

increase at slower rate, if $\delta_t^U < x_{t+1} \leq \pi_t^p - \varepsilon_t^L$

increase at same rate, if $\pi_t^p - \varepsilon_t^L < x_{t+1} < \pi_t^p + \varepsilon_t^U$

increase more rapidly, if $\pi_t^p + \varepsilon_t^U \leq x_{t+1}$.

The proportions of the total response, denoted as ${}_tA_{t+1}$ 'fall', ${}_tB_{t+1}$ 'stay about the same', ${}_tC_{t+1}$ 'increase at slower rate', ${}_tD_{t+1}$ 'increase at same rate' and ${}_tE_{t+1}$ 'increase more rapidly' are written in terms of the aggregated probability distribution as

$$P(x_{t+1} \leq -\delta_t^L) = \int_{-\infty}^{-\delta_t^L} f(x_{t+1}) dx_{t+1} = F(-\delta_t^L) = {}_tA_{t+1}$$

$$P(-\delta_t^L < x_{t+1} \leq \delta_t^U) = \int_{-\delta_t^L}^{\delta_t^U} f(x_{t+1}) dx_{t+1} = F(\delta_t^U) - F(-\delta_t^L) = {}_tB_{t+1}$$

$$P(\delta_t^U < x_{t+1} \leq \pi_t^p - \varepsilon_t^L) = \int_{\delta_t^U}^{\pi_t^p - \varepsilon_t^L} f(x_{t+1}) dx_{t+1} = F(\pi_t^p - \varepsilon_t^L) - F(\delta_t^U) = {}_tC_{t+1}$$

$$P(\pi_t^p - \varepsilon_t^L < x_{t+1} < \pi_t^p + \varepsilon_t^U) = \int_{\pi_t^p - \varepsilon_t^L}^{\pi_t^p + \varepsilon_t^U} f(x_{t+1}) dx_{t+1} = F(\pi_t^p + \varepsilon_t^U) - F(\pi_t^p - \varepsilon_t^L) = {}_tD_{t+1}$$

$$P(\pi_t^p + \varepsilon_t^U \leq x_{t+1}) = \int_{\pi_t^p + \varepsilon_t^U}^{\infty} f(x_{t+1}) dx_{t+1} = 1 - F(\pi_t^p + \varepsilon_t^U) = {}_tE_{t+1}$$

A standardized variable is used with a specified distribution function. It is assumed that the indifference intervals are symmetric, i.e. $\delta_t^L = \delta_t^U = \delta_t$ and $\varepsilon_t^L = \varepsilon_t^U = \varepsilon_t$. However, time-variation is allowed for the intervals. The equations above give solution to the unknown parameters:

$$\pi_{t+1}^e = \pi_t^p ({}_t a_{t+1} + {}_t b_{t+1}) {}_t q_{t+1}$$

$$\sigma_{t+1} = -2 \pi_t^p {}_t q_{t+1}$$

$$\delta_t = \pi_t^p ({}_t a_{t+1} - {}_t b_{t+1}) {}_t q_{t+1}$$

$$\varepsilon_t = \pi_t^p ({}_t c_{t+1} - {}_t d_{t+1}) {}_t q_{t+1}$$

where ${}_t q_{t+1}^{-1} = {}_t a_{t+1} + {}_t b_{t+1} - {}_t c_{t+1} - {}_t d_{t+1}$. The parameters depend on the choice of the distribution and the perceived inflation rate, π_t^p . The distribution function can be chosen as Normal (Carlson and Parkin, 1975). However, the normal distribution may not be appropriate for the price expectations. Some empirical studies, based on financial market data or quantitative data on expectations, recommend that the actual distribution of expectations can be positively skewed in times of high inflation and heavier tails compared to normal. Thus, alternatively other types of distributions are applied in the literature, such as the uniform distribution (Pesaran (1987)), the logistic distribution (Batchelor and Orr (1988), Nielsen (2003)), the central and non-central t distributions (Berk (1999), Nielsen (2003)). To capture the deviation from normality; logistic, uniform, central-t which are more peaked than the normal distribution and chi-square distribution which is positively skewed are employed (Nielsen, 2003). In addition to these well-known distributions, Stable distribution is also applied in order to quantify qualitative data. There are several reasons for using a stable distribution to describe a system. The most important reason is the Generalized Central Limit Theorem which states that the only possible non-trivial limit of normalized sums of independent identically distributed terms is stable. This theorem states that regardless of the existence of the variance, the limiting distribution of a sum of independent and identically

distributed random variables is stable (Borak, Härdle and Weron, 2005). In addition to this intriguing statistical property, stable distributions are preferred with respect to the Student's-t or the GED since they are a rich class of probability distributions that also allow skewness. Therefore, although the lack of closed formulas for most stable densities and distribution functions has been a major drawback to their use, they have been widely employed in modeling non-normal economic & financial data because of their flexibility. Stable distributions, also called "Levy-Pareto distributions", are used to describe complex systems in physics, biology, sociology and economics as well (Zolotarev, 1986). In general, the upper and lower tails of stable distributions decrease like a power function which generates the heavy tails.

Stable distributions are a rich class of probability distributions that allow skewness and heavy tails and have many intriguing mathematical properties (Nolan, 2009; <http://academic2.american.edu/~jpnolan/stable/chap1.pdf>). Since densities and distributions are not known in closed form for most stable distributions (exceptions being the normal, Cauchy and Levy distributions), they are usually defined by their characteristic functions (Mitchell, 2002):

$$\phi_x(t) = E(e^{itx}) = \exp\left(i\delta t - |ct|^\alpha \left[1 + i\beta \operatorname{sgn}(t) w(t, \alpha)\right]\right) \quad (1)$$

where

$$w(t, \alpha) = \begin{cases} -\tan\left(\frac{\pi\alpha}{2}\right) & , \quad \alpha \neq 1 \\ (2/\pi) \ln|t| & , \quad \alpha = 1 \end{cases}$$

$$-\infty < t < \infty, 0 < \alpha \leq 2, |\beta| \leq \min(\alpha, 1 - \alpha), c > 0, -\infty < \delta < \infty.$$

A stable distribution has four parameters; α , β , δ and γ ($\gamma = c^\alpha$). α is called characteristic exponent and interpreted as a shape parameter. The Normal distribution is stable with $\alpha=2$ and is the only stable distribution which second and higher absolute moments exist. When $\alpha < 2$, absolute moments of order equal to and greater than α do not exist while those of order less than α do. The distribution becomes heavy tailed. The tail thickness increases as α decreases. δ and c are the location and scale parameters respectively. When β (skewness parameter) is positive (negative), the distribution is skewed to the right (left). If β is zero, the distribution becomes symmetric about δ (location parameter). As α approaches to 2, the distribution approaches to a Normal distribution regardless of β (Fama and Roll, 1968).

A variety of measures for the scaling parameter, π_t^p , have been used in the literature. As it should reflect the observed inflation rate, the most recent rate available to the survey participants, i.e. π_{t-1} , where π_t is the officially published inflation rate, can be used for the scaling parameter. Due to the delay in publication the lagged inflation is considered rather than π_t . A second possibility is the mean of the actual inflation rate over the whole observed period, but this would imply that the participants base their decisions in part on information that is not available at the time the decision is made. Therefore not the mean over the whole sample can be used, but instead the mean over the period that precedes the time of the decision. This is the running mean of inflation from the beginning of the sample to the point where expectations are surveyed. Another choice can be the linear interpolation between the average value of inflation over the first half of the sample and that over the second half of the sample after those values are assigned to the first and last months in the sample, respectively (Millet, 2006).

In contrast to original Carlson-Parkin approach, where the scaling parameter is estimated by imposing unbiased expectations, an important advantage is that it does not impose

unbiasedness. Second advantage is that the response thresholds are permitted to vary over time (Forsells and Kenny, 2002).

2.2.2. Regression Method

Pesaran (1984, 1987) developed the 'regression approach' that originates in Anderson (1952). The quantified expectations are a function of a specific regression model rather than a function of a specific probability distribution.

This method is based on the estimation of the relationship between current inflation as measured by official statistics and its survey perception by respondents. It is assumed then that the same relationship holds between qualitative opinions of respondents concerning future price changes and expected inflation, so it serves as a yardstick for quantification of respondents' expectations (Lyziak, 2010).

If the percentage change in prices, π_t , is composed of a weighted combination of respondents having experienced increasing (superscript +) or falling (superscript -) prices, then

$$\pi_t = \sum w_{i,t}^+ \pi_{i,t}^+ + \sum w_{i,t}^- \pi_{i,t}^- \quad (2)$$

Where $w_{i,t}^+$ ($w_{i,t}^-$) is the weight on the i^{th} respondent reporting an increase (decrease) during period t .

Assume that all respondents reporting an increase (decrease) give the same increase (decrease) up to a random disturbance, that is, $\pi_{i,t}^+ = \alpha + v_{i,t}^+$ and $\pi_{i,t}^- = -\beta + v_{i,t}^-$. It follows that

$$\pi_t = \alpha R_t - \beta F_t + \varepsilon_t$$

$$\varepsilon_t = \sum w_{i,t}^+ v_{i,t}^+ + \sum w_{i,t}^- v_{i,t}^-$$

where it is assumed that $w_{i,t}^+ = w_{i,t}^- = 1$ for all i and t .

Pesaran (1987) assumes that during inflationary periods, there exists an asymmetrical relationship between the rate of change of individually experienced prices and overall inflation, depending on the direction of change reported:

$$\pi_{i,t}^+ = \alpha + \lambda \pi_t + v_{i,t}^+ \quad \alpha \geq 0, 0 \leq \lambda \leq 1 \quad (3)$$

$$\pi_{i,t}^- = -\beta + v_{i,t}^-, \beta \geq 0 \quad (4)$$

where $v_{i,t}^+$ and $v_{i,t}^-$ are independent white-noise processes. Asymmetric behaviour means that all respondents reporting an increase give additionally more increase up to a random distance whereas these reporting a decrease give the same decrease up to a random distance. After substituting the relations in (3) & (4) into (2) above, we get

$$\pi_t = \frac{\alpha R_t - \beta F_t}{(1 - \lambda R_t)} + \varepsilon_t \quad (5)$$

$$\varepsilon_t = (\sum w_{i,t}^+ v_{i,t}^+ + \sum w_{i,t}^- v_{i,t}^-) / (1 - \lambda R_t)$$

where R_t and F_t denote the percentages of respondents reporting price rises or falls in their answer to the perceptions question, respectively. Once the coefficients from equation (5)

have been recovered, it is possible to apply them to the survey proportions relating to expectations this time, thereby deriving a measure of inflation expectations:

$$\pi_{t+1}^e = \frac{\hat{\alpha}R_t^e - \hat{\beta}F_t^e}{(1 - \hat{\lambda}R_t^e)} \quad (6)$$

The assumption is that the estimated relationship between survey data and inflation holds not only for realizations, but also for expectations. This can be thought to be a strong assumption. Pesaran (1987) points out that a regression like (5) “is not a causal explanation of price changes but simply identifies the relationship between two different sources of information (namely official statistics and survey results), and serves as a ‘yardstick’ by means of which categorical responses concerning the direction of future changes in prices can be converted into quantitative measures”. Pesaran (1984) further recommended correcting for the residual autocorrelation in equation (5) by imposing an AR structure on the error term:

$$\pi_t = \frac{\alpha R_t - \beta F_t + \rho_1((1 - \lambda R_{t-1})\pi_{t-1} - \alpha R_{t-1} - \beta F_{t-1}) + \rho_2((1 - \lambda R_{t-2})\pi_{t-2} - \alpha R_{t-2} - \beta F_{t-2})}{(1 - \lambda R_t)} + \varepsilon_t \quad (7)$$

2.2.3. Balance Method

Balance method is the easiest technique to quantify qualitative data. The calculation of the balance statistic is compiled in accordance with the balance method of European Union (User Guide, 2003). The possible outcomes are -1, -0.5, 0, 0.5 and 1 for a pentachotomous survey. These outcomes are associated with the sample proportions ${}_tA_{t+1}$, ${}_tB_{t+1}$, ${}_tC_{t+1}$, ${}_tD_{t+1}$, ${}_tE_{t+1}$ respectively.

The expected mean of this random variable, denoted as π_{t+1}^b is then for a pentachotomous survey defined as:

$$\begin{aligned} \pi_{t+1}^b &= -1 * {}_tA_{t+1} - 0.5 * {}_tB_{t+1} + 0 * {}_tC_{t+1} + 0.5 * {}_tD_{t+1} + 1 * {}_tE_{t+1} \\ &= {}_tE_{t+1} + 0.5 * {}_tD_{t+1} - 0.5 * {}_tB_{t+1} - {}_tA_{t+1} \end{aligned}$$

3. Quantified Expectations

The expected inflation question of CTS is quantified in order to get quantitative inflation expectations of the consumers. The methods described above are used to obtain the quantified expectations series. The latest officially published inflation rate belongs to March 2011, so the survey period used in the study covers the period from December 2003 to April 2010. Then inflation expectations can be calculated for the period from November 2004 to March 2011 and compared with the realizations.

The probability method is employed to the inflation expectations gathered from CTS by using different distribution functions. Normal distribution is used in many studies since it is easy to handle. However, normal distribution may not be suitable with the empirical findings. Therefore, chi-square distribution, central t-distribution and Stable distribution are also applied in addition to normal, logistic and uniform distributions.

To account for the peakedness of the actual price changes, logistic and central-t distributions are used. To model the asymmetric behaviour of price changes, chi-square distribution is used. To capture not only the asymmetric behaviour but also the heavy tailed pattern, Stable

distribution is applied. A grid search is used in order to derive quantified expectations series via different Stable distribution across different α and β values (α in the range 0.1 to 2 and β in the range -1 to 1 at intervals of 0.05). The accuracy of each estimated series is calculated by comparing against the realization.

Four different choices for the scaling parameter are applied. First one is the most recent officially published inflation rate available to the survey participants. The second choice used is the linear interpolation. The third option is the running mean of inflation. The final choice is the mean of the actual inflation rate over the whole observed period. The forecasting performances of different models can be seen in Table 1. Four measures of scaling parameter are analyzed for each distribution and running mean of inflation has the least error values for all distributions except for chi-square distribution.

When the performances are compared with each other for the probability method, Stable distribution having parameter values 0.30 for α and -0.25 for β with the linear interpolation as threshold parameter shows the best performance due to low mean absolute error, mean square error and their U statistics.

(Table 1)

Figure 1 illustrates the expectations derived via different distributions. It can be concluded that the best fit to actual values of inflation can be attained by the expectations derived from Stable distribution. Table 1 supports this claim.

(Figure 1)

The question of price expectations in CTS has five categories. The proportions are added in order to get three-category options. 'Fall' option is equal to ${}_tA_{t+1}$, 'Same' option is equal to ${}_tB_{t+1}$ and 'Rise' option is equal to ${}_tC_{t+1}+{}_tD_{t+1}+{}_tE_{t+1}$. After having three-option categories, the model given in equation (7) is applied. The question of inflation perceptions does not exist in CTS, so the regression model is constructed for the expected inflation question. The results are given in Table 2 and Figure 2.

(Table 2)

12-month lagged inflation rate is used in order to avoid overlapping of periods for the error term. All the parameters in the model are found to be significant.

(Figure 2)

The balance method can be seen in Figure 3. The figure indicates that there is no similar pattern with realizations in tendency. However, it should be mentioned that this statistic can only give information about tendency instead of quantity of inflation expectations.

(Figure 3)

The comparisons of the models can be seen in Figure 4. The best model for quantified price expectations can be said to be the regression model.

(Figure 4)

The official inflation rate for the period between November 2004 and March 2011 is considered with the aim of investigating the performances of the quantification methods at different time periods. I divide the data into two parts such as the first 20 observations and the remaining ones. I calculate the standard deviations of each portions and take the differences of the standard deviations. Then I roll the sample and keep calculating the differences for the other following samples. After calculating for entire sample, I choose the split date as "April 2008" according to the comparison of the differences. This month has the maximum value for difference. The 1st period to be analyzed now becomes November 2004-April 2008 and the 2nd period is May 2008-March 2011. I compare each method due to these two periods separately.

(Table 3a)

(Table 3b)

The comparison results given in Tables 3a and 3b make us rely on that regression method outperforms the remaining methods for each period. Therefore, it can be concluded that for different periods, we can still choose regression method as best for quantification purposes.

I also calculate 1-step ahead (static) forecasts for the expectations derived by regression method. I compare these forecasts with forecasts of Naive Expectations which are the expected inflation series equal to the current rate of inflation. I measure 1-step ahead forecasts in both ways such as rolling and recursive windows for the regression method. The comparisons of different methods are given in Table 4. In line with the results, recursive window forecasts via regression method performs better than naive expectations for 1-step ahead forecast.

(Table 4)

4. Formation of Turkish Consumer Inflation Expectations

The expectations have been important issue in macroeconomics for many years. Since the way in which expectations are formed has important implications for economic behaviour, many economists have used survey data to test hypotheses about expectation formation (Keane & Runkle, 1990). What is of concern for monetary policy makers are signs that expectations have become de-anchored, which we can interpret as being the case if the public reacts to a short period of higher-than-expected inflation by increasing their long-run expectations. Measuring when inflation expectations have become de-anchored is undoubtedly not easy. One of the problems is that how individuals form expectations is not known. Indeed, it is probably impossible to generalize, as individuals are likely to form their expectations using different information sets, relying on different models. Driver and Windram (2007) found that some households may form their expectations based on a structural relationship, such as the trade-off between inflation and unemployment or demand; others may use an empirical approach, such as their recent memories of inflation data. Furthermore, people may be entirely forward looking or entirely backward looking, or a combination of both.

The expectation formation models in the literature range from simple, purely backward-looking to explicitly modeling learning processes to the hypothesis of perfectly rational expectations (Pesaran, 1989). Backward-looking models assume that agents use only past price developments and earlier forecast errors to form expectations while other influences are disregarded. Muth (1961) assumes that the subjective expectations of economic agents match the predictions of the relevant economic theory in his “rational expectations” hypothesis. Thus, a crucial feature of his definition of rational expectations is that economic agents do not make systematic errors. Many critics have pointed to the importance of information problems and have stressed the need to take into account the costs of making optimal forecasts and also to explicitly model learning processes. However, since its adoption by Lucas (1972, 1975), Sargent (1973), Barro (1977) and others, the rational expectations hypothesis has become one of the broadly accepted paradigms of macroeconomic analysis.

The non-linear regression model is found to be the best model to measure inflation expectations; hence the analyses in this section will be based on the expectations attained via this model.

First of all, the time series have to be tested for a unit root before further analysis. Standard Augmented-Dickey-Fuller (ADF) tests are applied. Lag lengths are chosen according to Schwarz Information criterion (SIC). The results of these tests are presented in Table 5. It

can be concluded that actual year on year inflation and expected inflation have unit root (they are both I(1)).

(Table 5)

Since both actual inflation and consumer inflation expectations are non-stationary, the cointegration between these variables is tested. Before the cointegration test is applied, I obtain the optimal lag-lengths for the VAR process via VAR lag order selection criteria. Lags 1, 2 and 11 can be chosen due to different criteria such as Likelihood-Ratio Test (LR), Final Prediction Error (FPE), Hannan-Quinn (HQ), Akaike (AIC) and Schwarz information (SC). The results of the Johansen test are given in Table 6:

(Table 6)

The null hypothesis that the cointegration rank r is zero can be rejected at lag 11 showing that there is one cointegrating equation at 5% significance level.

In an effort to determine the short run causality among the two variables, Granger Causality/Block Exogeneity Wald Tests based upon VEC model is applied. The results of Table 7 show that short run causality is from the inflation to inflation expectations.

(Table 7)

I also perform Granger causality tests between change in inflation expectations and unobserved future change in the inflation rate (Table 8). The results show that the changes in future inflation rate Granger cause inflation expectations but not the other way. In other words, expectations are actually somewhat forward looking. However, past changes in inflation expectations do not have any significant effect on changes in the inflation rate.

(Table 8)

Additionally, the variables are tested for weak exogeneity in order to test whether the cointegration relation is significant for both endogeneous variables or not. The results can be seen in Table 9.

(Table 9)

The test-statistics point out that the hypothesis of weak exogeneity can be rejected for the expected inflation rate, but not for the actual inflation rate. Consequentially, error correction model can be estimated for the expected inflation rate as dependent variable.

The long-run relationship between consumer inflation expectations and actual inflation (standard errors in parentheses) can be given as:

$$\pi_t^e = -13.093 + 2.464 \pi_t + \varepsilon_t$$

(3.773) (0.429)

Sample: 2006:11-2011:03

Trace test indicates 1 cointegrating equation(s) at 5% level

Max-eigenvalue test indicates 1 cointegrating equation(s) 5% level

The cointegrating vector should have a null constant term and opposite coefficients for expected and actual inflation under the rational expectations hypothesis (unbiasedness assumption). The joint restriction of a unit coefficient and zero constant within VECM is rejected with Chi-Square statistic and probability value equal to 12.11 and 0.0005 respectively. It can be concluded that the expectations are not unbiased. Consequently, the expectations are said to be not rational.

After rejecting a rational model of the formation of inflation expectations, hybrid model of expectations formation is considered for the purpose of measuring the formation, where expectations are comprised of not only forward-looking but also backward-looking portions (Lyziak, 2012).

Following the inflation expectation equation presented in Heinemann and Ullrich (2006), which is an extended version of Carlson and Valev (2002) and Gerberding (2001), the inflation expectations model can be represented in such a way as to contain both forward (rational) and backward-looking elements as in equation below:

$$\pi_t^e = \beta_1 \pi_t + (1 - \beta_1)[\pi_{t-12}^e + \beta_2(\pi_{t-12}^e - \pi_{t-12}) + \beta_3(\pi_{t-12} - \pi_{t-23}) + \beta_4(\pi_{t-1}^e - \pi_{t-12}^e)] + \varepsilon_t \quad (8)$$

In this formulation, expectation formation can be partially characterized as rational, while the backward looking perspective still plays a role. The relative importance of the rational versus adaptive components of expectations is measured by β_1 , where expectations are considered fully rational if $\beta_1 = 1$. This equation is rather flexible in incorporating adaptive nature of expectations such that β_2 measures agents' speed of adjustment to their past forecast errors, while β_3 and β_4 measure the weights of the regressive part of expectations formation.

Equation (8) which was presented in constrained form is re-estimated in unconstrained form in equation below by allowing the sum of the rational and adaptive terms to be different from one.

$$\pi_t^e = \beta'_1 \pi_t + \beta'_2 \pi_{t-12}^e + \beta'_3(\pi_{t-12}^e - \pi_{t-12}) + \beta'_4(\pi_{t-12} - \pi_{t-23}) + \beta'_5(\pi_{t-1}^e - \pi_{t-12}^e) + \varepsilon_t \quad (9)$$

It is seen in Table 10 that the estimated coefficient for the future inflation is 0.11 in the 1st column. The hypothesis of pure backward-looking expectations is therefore rejected. The coefficient of the past inflation expectation however is much bigger (0.89) representing the strong backward looking nature of expectations in the long run. Therefore, the pure forward-looking expectations hypothesis is also rejected. In addition, expectations are found to be highly regressive with the coefficient 0.73. The impact of the change in past inflation on the formation of expectations is, however, unexpectedly small and has a negative sign. The structure of the expectations does not change when we constrain the sum of the coefficients of forward and backward looking elements to be equal to one (Table 11).

(Table 10)

(Table 11)

Conclusions

Surveys are useful because they provide independent (or relatively non-model dependent) measures of inflation expectations, a key variable that a central bank can use in its design of an optimal monetary policy geared toward the achievement of price stability.

This paper has attempted to analyze the qualitative inflation expectations gathered from the survey data. The survey results are examined and the qualitative inflation expectations are quantified by using different methods such as Carlson-Parkin method, balance method, regression method. These methods are compared by using several statistical criteria, like mean square error, mean absolute error and Theil's inequality coefficient.

Carlson-Parkin method is applied for the pentachotomous survey question. In this approach, one advantage is that the scaling parameter is not estimated by imposing unbiased expectations. Another advantage is that the thresholds are permitted to vary over time. Different distributions are applied and expectations obtained by Stable distribution show the best performance due to statistical criteria. The balance method is applied and it is found that consumers are backward-looking. Thirdly, nonlinear regression model is constructed to get the inflation expectations series.

Finally, the expectations derived from three different techniques are analyzed and the nonlinear regression model is found to be the closest one to realizations. The inflation expectations derived from the nonlinear regression model is used in order to analyze the formation of inflation expectations. Actual inflation and inflation expectations are found to have a cointegration relation. Unbiasedness assumption under Rational Expectations Hypothesis is rejected within VECM. Hence, hybrid model of expectations formation is constructed. One important result attained from the model is that the hypothesis of pure backward-looking expectations is rejected. The coefficient of the past inflation expectation however is much bigger that indicates the strong backward looking nature of expectations in the long run. Therefore, the pure forward-looking expectations hypothesis is also rejected. In addition, expectations are found to be highly regressive. As a final remark, the impact of the change in past inflation on the formation of expectations is quite small.

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Tables & Figures

Quantification Methods		MAE*	MSE**	TU1***
Probability Method	Threshold Parameter			
Normal Distribution	Most recent inflation rate	3.2898	15.8759	0.4582
	Linear interpolation	2.4995	8.4202	0.3337
	Running mean of inflation	2.3537	8.3760	0.3328
	Mean of the actual inflation rate	2.7740	10.2919	0.3689
Uniform Distribution	Most recent inflation rate	3.3737	16.7022	0.4700
	Linear interpolation	2.5662	8.8733	0.3426
	Running mean of inflation	2.4242	8.9778	0.3446
	Mean of the actual inflation rate	2.8429	10.7996	0.3779
Logistic Distribution	Most recent inflation rate	3.2250	14.5285	0.4383
	Linear interpolation	2.5735	8.7223	0.3396
	Running mean of inflation	2.2611	7.2071	0.3087
	Mean of the actual inflation rate	2.7933	10.0175	0.3640
Central-t distribution	Most recent inflation rate	3.2126	15.2122	0.4485
	Linear interpolation	2.4310	8.0111	0.3255
	Running mean of inflation	2.2954	7.9299	0.3238
	Mean of the actual inflation rate	2.7054	9.8449	0.3608
Chi-square distribution	Most recent inflation rate	1.6905	5.6456	0.2732
	Linear interpolation	1.8500	5.0477	0.2584
	Running mean of inflation	1.9360	6.3207	0.2891
	Mean of the actual inflation rate	1.5259	3.4798	0.2145
Stable Distribution		1.3933	2.9161	0.0504
<i>Balance Method</i>		17.7834	480.3493	2.5204
<i>Regression Method</i>		1.1807	2.0375	0.1635

$$* MAE = \sum_{i=1}^n |P_t - P_t^e| / n \text{ (mean absolute error of prediction)}$$

$$** MSE = \sum_{i=1}^n (P_t - P_t^e)^2 / n \text{ (mean square error of prediction)}$$

$$*** TU1 = \left[\frac{\sum_{i=1}^n (P_t - P_t^e)^2}{\sum_{i=1}^n (P_t)^2} \right]^{1/2} \text{ (Theil's inequality coefficient)}$$

where P_t and P_t^e denote actual inflation and inflation expectations respectively.

Table 2 Regression Model	
Coefficient	Pesaran AR(12) Correction
Probability Method	Threshold Parameter
α	-0.076*
β	-0.164*
λ	2.033*
ρ	-0.624*
R^2	0.49

* Significant at 5 % level.

Table 3a Comparisons of Quantification Methods for the 1 st Period				
Quantification Methods		MAE*	MSE**	TU1***
Probability Method	Threshold Parameter			
Normal Distribution	Most recent inflation rate	2.4113	9.8457	0.3500
	Linear interpolation	2.4561	9.1789	0.3379
	Running mean of inflation	1.8955	5.4857	0.2612
	Mean of the actual inflation rate	2.1930	7.1621	0.2985
Uniform Distribution	Most recent inflation rate	2.4489	9.9670	0.3521
	Linear interpolation	2.4668	9.2093	0.3385
	Running mean of inflation	1.9175	5.5777	0.2634
	Mean of the actual inflation rate	2.2014	7.1506	0.2983
Logistic Distribution	Most recent inflation rate	2.5331	10.5943	0.3630
	Linear interpolation	2.6447	10.1792	0.3559
	Running mean of inflation	1.9500	5.9357	0.2717
	Mean of the actual inflation rate	2.3623	8.0503	0.3165
Central-t distribution	Most recent inflation rate	2.3653	9.6710	0.3469
	Linear interpolation	2.4238	9.0448	0.3354
	Running mean of inflation	1.8777	5.3933	0.2590
	Mean of the actual inflation rate	2.1674	7.0731	0.2966
Chi-square distribution	Most recent inflation rate	2.3795	9.7245	0.3478
	Linear interpolation	2.4344	9.0885	0.3362
	Running mean of inflation	1.8825	5.4195	0.2597
	Mean of the actual inflation rate	2.1761	7.1031	0.2973
Balance Method		9.6913	153.9728	1.3840
Regression Method		0.6970	0.6779	0.0900
Stable Distribution		1.4002	2.2608	0.0502

Quantification Methods		MAE*	MSE**	TU1***
Probability Method	Threshold Parameter			
Normal Distribution	Most recent inflation rate	4.3438	23.1122	0.5751
	Linear interpolation	3.2794	13.5539	0.4404
	Running mean of inflation	2.9036	11.8443	0.4117
	Mean of the actual inflation rate	2.8750	10.2969	0.3838
Uniform Distribution	Most recent inflation rate	4.4834	24.7845	0.5955
	Linear interpolation	3.4202	14.9645	0.4627
	Running mean of inflation	3.0323	13.0578	0.4322
	Mean of the actual inflation rate	2.9976	11.1971	0.4003
Logistic Distribution	Most recent inflation rate	4.0552	19.2497	0.5248
	Linear interpolation	3.1037	11.8053	0.4110
	Running mean of inflation	2.6344	8.7327	0.3535
	Mean of the actual inflation rate	2.7218	9.6969	0.3725
Central-t distribution	Most recent inflation rate	4.2293	21.8615	0.5593
	Linear interpolation	3.1641	12.4928	0.4228
	Running mean of inflation	2.7966	10.9738	0.3962
	Mean of the actual inflation rate	2.7762	9.5970	0.3706
Chi-square distribution	Most recent inflation rate	4.2616	22.2051	0.5637
	Linear interpolation	3.1966	12.7846	0.4277
	Running mean of inflation	2.8269	11.2093	0.4005
	Mean of the actual inflation rate	2.8042	9.7910	0.3743
Balance Method		27.4939	872.0010	3.5322
Regression Method		1.0074	1.3709	0.1621
Stable Distribution		2.0585	5.7129	0.0738

Methods	MAE*	MSE**	TU1***
Regression Method (rolling window forecast)	3.07	12.19	0.42
Regression Method (recursive window forecasts)	3.03	11.75	0.41
Naive Expectations	3.52	15.59	0.47

$$* MAE = \sum_{i=1}^n |P_t - P_t^e| / n \text{ (mean absolute error of prediction)}$$

$$** MSE = \sum_{i=1}^n (P_t - P_t^e)^2 / n \text{ (mean square error of prediction)}$$

$$*** TU1 = \left[\frac{\sum_{i=1}^n (P_t - P_t^e)^2}{\sum_{i=1}^n (P_t)^2} \right]^{1/2} \text{ (Theil's inequality coefficient)}$$

where P_t and P_t^e denote actual inflation and inflation expectations respectively.

Variable	lags	Test-statistic	Probability*	Result
π_t^e	0	-2.152	0.2257	I(1)
π_t	2	-2.798	0.0679	I(1)
$\Delta\pi_t^e$	1	-4.339	0.0009	I(0)
$\Delta\pi_t$	0	-4.851	0.0002	I(0)

* MacKinnon one-sided p-values.

Lag length	Criterion	Null hypothesis	Trace-statistic	Prob.*
1	SC, HQ	r=0	10.55	0.585
		r=1	3.06	0.569
11	LR, FPE	r=0	30.52	0.001
		r=1	6.51	0.155
12	AIC	r=0	12.32	0.420
		r=1	4.79	0.308

* MacKinnon-Haug-Michelis p-values.

The null hypothesis that the cointegration rank r is zero.

Dependent Variable	Excluded	Chi square statistic	Degrees of freedom	Prob.
$\Delta\pi_t$	$\Delta\pi_t^e$	10.50	11	0.486
$\Delta\pi_t^e$	$\Delta\pi_t$	40.48	11	0.000

Expectation Versus Future Inflation	F-statistic	Probability
Change in inflation expectations formed at time t-11 ($\Delta\pi_t^e$) does not Granger Cause change in the inflation rate at time t ($\Delta\pi_t$)	1.128	0.379
Change in the inflation rate at time t ($\Delta\pi_t$) does not Granger Cause change in inflation expectations formed at time t-11 ($\Delta\pi_t^e$)	8.453	0.000

* Lag length is chosen as 12.

Table 9			
Weak Exogeneity Test			
Analyzed Variable	Lag length	Chi square statistic	Prob.
$\Delta\pi_t$	11	0.85	0.356
$\Delta\pi_t^e$	11	17.49	0.000

Table 10		
Structure of Expectations (Unconstrained Model)		
Coefficients		
π_t	0.11** (1.824)	0.12** (1.944)
π_{t-12}^e	0.89* (16.526)	0.87* (16.078)
$(\pi_{t-12}^e - \pi_{t-12})$	0.07 (0.712)	-
$(\pi_{t-12} - \pi_{t-23})$	-0.11 (-1.423)	-0.12** (-1.837)
$(\pi_{t-1}^e - \pi_{t-12}^e)$	0.73* (8.682)	0.72* (9.337)
R ²	0.82	0.82

* Significant at 5 % level.

** Significant at 10 % level.

Table 11		
Structure of Expectations (Constrained Model)		
Coefficients		
π_t	0.11** (1.984)	0.13* (2.322)
$(\pi_{t-12}^e - \pi_{t-12})$	0.10 (1.074)	-
$(\pi_{t-12} - \pi_{t-23})$	-0.11 (-1.609)	-0.13** (-1.978)
$(\pi_{t-1}^e - \pi_{t-12}^e)$	0.83* (8.285)	0.83* (8.270)
R ²	0.82	0.81

* Significant at 5 % level.

** Significant at 10 % level.

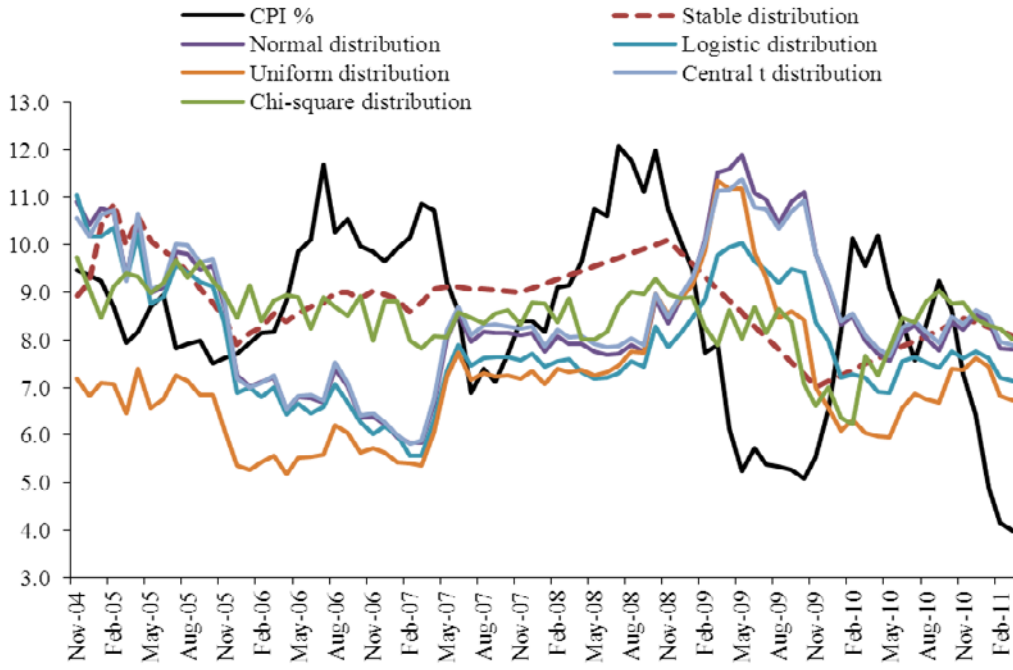


Figure 1. Quantified Expectations by Carlson-Parkin Method

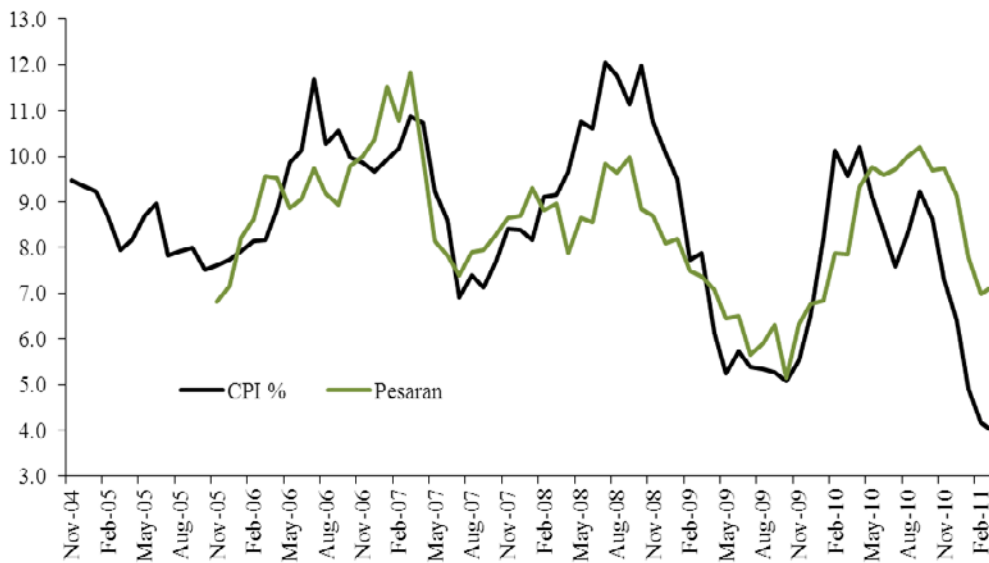


Figure 2. Quantified Expectations by Regression Method

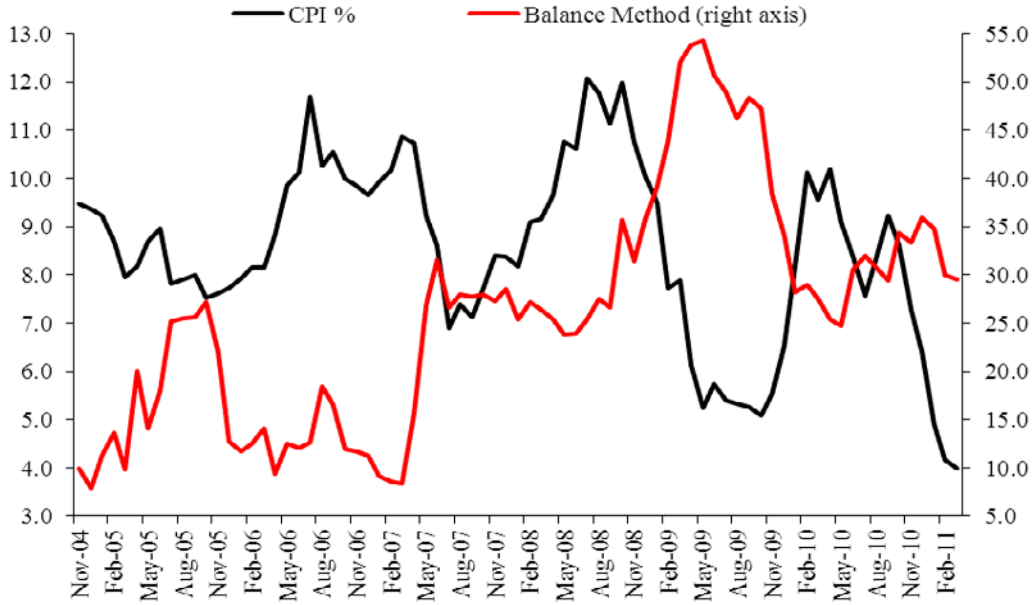


Figure 3. Quantified Expectations by Balance Method

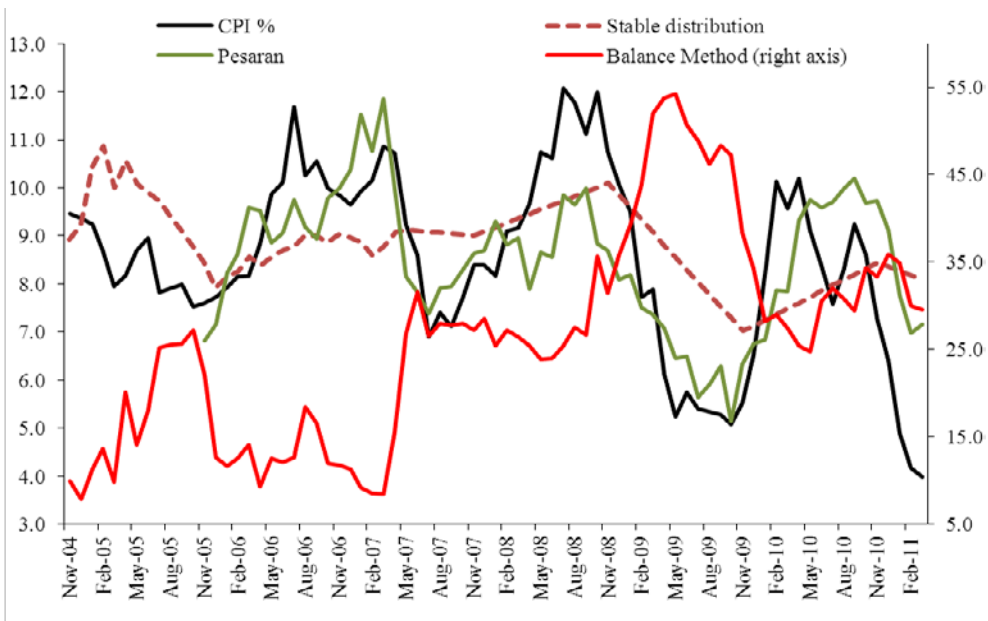


Figure 4. Comparisons of Expectations obtained by Different Methods