Denoised least squares forecasting of GDP changes using indexes of consumer and business sentiment

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1. Introduction

Several studies in the economic literature have examined the usefulness of leading indicators in forecasting GDP changes. The two main sources of information used in this direction are (a) indexes of consumer and business sentiment and (b) monetary and financial data. Consumer and business sentiment indexes are based on sample surveys conducted by both national statistical services and industry groups who survey consumer and business expectations about the economy and individual income. Monetary and financial data usually consist of variables such as the stock market index, money supply, interest rates and yield curves.

The empirical results regarding the predictive ability of indexes of consumer and business sentiment have not been clear. For example, Howrey (2001) reports that indexes of consumer confidence sharpen predictions but provide little gain compared to distributed lag models in forecasting GDP and Bram and Ludvigson (1998) report that although information on consumer confidence improved forecasts, this was by a statistically insignificant amount. Similar results were reported by Fan and Wong (1998) regarding the predictive capacity of consumer sentiment indexes for consumption. Garcia-Ferrer and Bujosa-Brun (2000) find, on the contrary, that qualitative survey data improves forecasting of industrial turning points in OECD countries. And Huth, Eppright and Taube (1994), Easaw and Heravi (2004) and Easaw, Garratt and Heravi (2005) find that consumer sentiment indexes are useful predictors of household consumption.

These mixed results further support the view that different indicators tend to perform differently in different countries and in different times (Emerson and Hendry 1996). And Evans (2003) argues that, when relying on historical data, no method is likely to work without adjustment and that enhanced methods of calculating leading indicators should play a more important role in the future.

In this article I propose one such adjustment through the use of a modern denoising method that can effectively remove measurement errors from survey data. Such errors are inherent in consumer and business sentiment indexes that are based on sample surveys and are therefore subject to sampling and nonsampling errors (Thompson 2002: 5). In a related study, Van Oest and Franses (2008) emphasize that net changes in confidence may be largely driven by the different respondent samples over time and do not always represent real changes in confidence for the entire population. In addition, the consumer sentiment index in a given country generally exhibits several irregular short-term cyclical fluctuations attributed to several factors that are not necessarily related to the economy and that further disturb its predictive capacity (Lemmens, Croux and Dekimpe 2007). Such factors can include, for example, the political climate in the country (see Garner 1981) and events that

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inspire national pride (e.g., performance in international sporting events) or national sorrow (e.g., a natural disaster).

Oller and Tallbom (1996) considered the problem of forecasting with noisy data and proposed a flexible exponential smoothing approach in order to construct entirely new business-cycle indicators. The authors considered both measurement errors and irregular quarter-to-quarter changes in the underlying indicators as being meaningless noise in the data. They also emphasized that the design of business tendency surveys is compromised between getting a fast reply and being highly accurate; therefore such surveys are subject to measurement errors. In this article the problem of noisy consumer and business tendency data is addressed with wavelets. Previous research on wavelets by Yogo (2008) employed wavelet analysis in order to decompose economic time series into trend, cycle and noise, while Clar, Duque and Moreno (2007) applied wavelet filtering in order to seasonally adjust several consumer and business survey indicators.

Measurement errors in the independent variables of linear regression models lead to inconsistent coefficient estimates. One solution for the inconsistency problem was proposed by Cai, Naik and Tsai (2000) in the form of denoised least squares. Denoising of the data with wavelets is performed first, then in a second stage the regression model is estimated with the ordinary least squares method. In this study the denoised least squares method is used in order to provide consistent coefficient estimates and forecasts in models for GDP changes that use consumer and business sentiment indexes as predictors. In addition, I show that wavelet denoising can also provide efficient coefficient estimates in regression models that explain consumer sentiment index variations. Empirical applications are provided with data on the US economy.

2. Wavelet denoising

Wavelets are orthonormal sets of functions whose shape, as the name suggests, is like a little wave. They have compact local support but decay quickly to zero elsewhere. Wavelets can provide approximations of both stationary and nonstationary time series. They are particularly effective for time series characterized by abrupt changes, spikes and periodic cycles. Consumer and business sentiment indexes are characterized by such features. These important properties have inspired several applications of discrete wavelet transforms in economics (see Crowley 2007). The wavelet approximation of an observed time series is similar to the Fourier transform and has the following form:

$$\chi_t = \sum_{k \in Z} c_{j_0,k} \phi_{j_0,k}(t) + \sum_{j \in Z} \sum_{k \in Z} g_{j,k} \psi_{j,k}(t)$$

where Z is the set of integers. This is an orthogonal decomposition that involves J time-scales (where, $j = 1,..., J$) with $k \in Z$ coefficients at each scale. The set of father (φ) and mother (ψ) wavelets that form an orthonormal basis are defined as:

$$\phi_{j_0,k}(t) = 2^{-j_0/2} \phi(2^{-j_0} t - k) \text{ and } \psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j} t - k)$$

and their respective scaling and wavelet coefficients are:

$$c_{j_0,k} = \int_R \chi_t \, \phi_{j_0,k}(t) \; dt \text{ and } g_{j,k} = \int_R \chi_t \, \psi_{j,k}(t) \; dt .$$

For a discrete time series, the discrete wavelet transform is used. In order to obtain the vector of wavelet coefficients w, the $1 \times T$ vector of noisy data χ is multiplied by an appropriate $T \times T$ wavelet matrix W (whose elements depend on a specific wavelet family):

$$w = W\chi$$

The vector of wavelet coefficients consists of different sub-vectors, each of length 2^j ($j = 1,..., J$), which represent different resolution levels of the data. For a dyadic length time series with monthly sampling frequency the first resolution level captures frequency variation with a duration of 2–4 months. Analogously, the second resolution level captures variation of 4–8 months, the level 3 resolution captures variation of 8–16 months and so on, up to level *J*.

Since the data contain measurement errors (noise) this will also be transferred to specific wavelet coefficients. Donoho and Johnstone (1994, 1995) proposed a soft thresholding rule in order to remove the noisy wavelet coefficients associated with the highest frequencies (short-term cyclical fluctuations) and construct noise free estimates of the original data vector χ. In the first stage, the following thresholding rule is applied to the data:

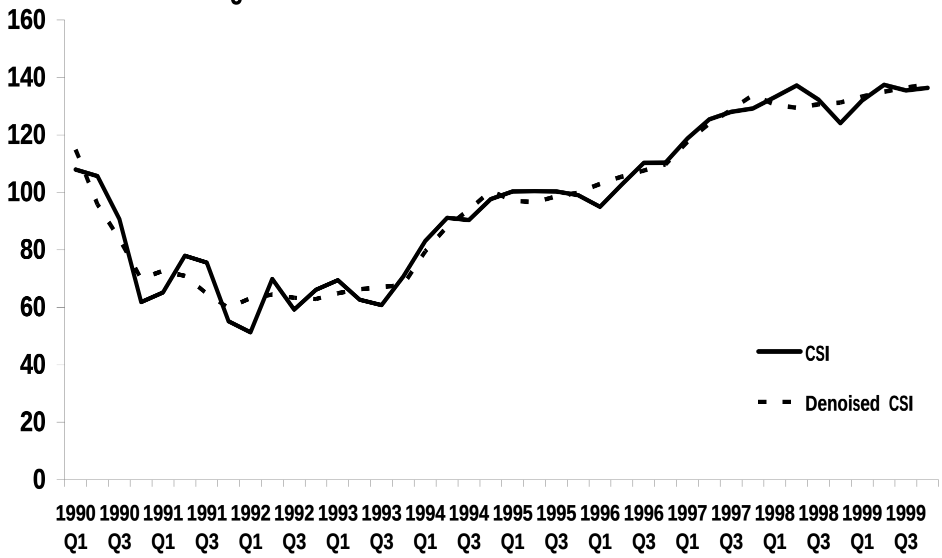
$$\hat{w} = \begin{cases} \operatorname{sgn}(w)\,(|w| - \tau) & if\ |w| \geq \tau \\ 0 & if\ |w| < \tau \end{cases} \quad \text{where } \operatorname{sgn}(w) = \begin{cases} +1 & if\ \ w > 0 \\ 0 & if\ \ w = 0 \\ -1 & if\ \ w < 0 \end{cases} .$$

This rule pushes all coefficients towards zero, but when their magnitude is smaller than the threshold τ, which defines the level of noise in the data, they are set to zero. The resulting wavelet coefficients $\hat{w}$ are free from noise. In the second stage, an inverse wavelet transform is applied to the vector $\hat{w}$ in order to obtain noise free estimates of the original data vector χ as follows:

$$\hat{\chi} = W^{-1}\hat{w}$$

Obviously, the choice of the threshold is critical, and this subject is extensively researched in the statistics literature. In the empirical applications of this article I use the universal threshold, $\tau = \hat{\sigma}_\tau \sqrt{2 \log N}$, proposed by Donoho and Johnstone (1994), where $\hat{\sigma}_\tau$ is the standard deviation of the wavelet coefficients at the finest level of detail. In all the applications in this article the Daubechies least asymmetric family of wavelets was used with a filter of length 10. Figure 1 exhibits the original and denoised values of the US consumer sentiment index published by the Conference Board for the period 1990–1999 after eliminating the two finest levels of wavelet coefficients associated with the highest frequencies in the data. Wavelet denoising produced a smoother signal that preserved the main characteristics of the index but removed noise and short-term irregular variation from the data. This is consistent with the findings of Van Oest and Franses (2008), who report that monthly changes in consumer confidence are not often large. Furthermore, wavelets are particularly effective in handling time series with abrupt changes, spikes and periodic cycles which frequently characterize the consumer sentiment index.

Figure 1 US consumer sentiment index: 1990–1999



Source: Conference Board.

In subsequent sections I use regression models that include consumer and business sentiment indexes as variables. In Section 3 wavelet denoising has been applied only to the dependent variable that represents the consumer sentiment index while in Section 4 wavelet denoising has been applied to three independent variables that concern the consumer sentiment index, the index of homebuilders' sentiment and the index of manufacturing activity in the United States.

3. Determinants of the consumer sentiment index

The economic determinants of the consumer sentiment index are studied in psychological economics (Kantona 1975), where regression models are usually used in order to test several hypotheses and identify possible causal links with the index. Garner (1981) formed several regression models in this direction and confirmed that a small set of objective variables explains most variations in the index. Using similar variables for the US economy for the period 1990–2000, and after careful specification testing, I estimated the following model for the consumer sentiment index:

$$\log CSI_t = a + \beta_1 \log INFLATION_t + \beta_2 \log GDP_t + \beta_3 \log UNEMPLOYMENT_t \qquad (1)$$

where *t* is the time subscript, *CSI* is the consumer sentiment index published by the Conference Board, *INFLATION* is the percentage change in the consumer price index, *GDP* is gross domestic product and *UNEMPLOYMENT* is the harmonized unemployment rate. The CSI data were obtained from the publications of the Conference Board, while data for the other variables were obtained from the OECD statistical database.

When the dependent variable in a linear regression model is measured with error, the ordinary least squares (OLS) estimator provides inefficient estimates. It is possible, however, to improve the efficiency of the coefficient estimates in a model like (1) by first applying wavelet denoising to the dependent variable, then estimating the equation with the OLS method. In order to formally present the efficiency gains associated with wavelet denoising, consider the classical linear regression model satisfying all the Gauss-Markov assumptions:

$$y_i^* = \beta_1 + \sum_{j=2}^{k} \beta_j x_{ij} + \varepsilon_i$$

where y^* is the dependent variable, x_{ij} are the $k-1$ explanatory variables (j = 2,...,k), ε_i is the error term with variance $Var(\varepsilon) = \sigma^2$ and there are N observations available ($i = 1,...,N$). Also assume that the dependent variable is measured with error according to the following model:

$$y_i = y_i^* + r_i$$

where y is the observed dependent variable, y^* is the underlying true signal and r is the additive measurement error with properties, $E(r_i) = 0$, $Var(r_i) = \omega^2$ and $Cov(r_i, r_m) = 0$, $\forall i \neq m$. If the estimated model is based on the observed dependent variable, the OLS estimator of the coefficient vector will be inefficient since the model becomes:

$$y_i = \beta_1 + \sum_{j=2}^{k} \beta_j x_{ij} + u_i$$

with new error term $u_i = \varepsilon_i + r_i$, which results in higher variance for the least squares estimator of the coefficient vector (see Davidson and MacKinnon 2004: 312–313). The error variance-covariance matrix will now be:

$$E(uu') = \Omega = \begin{bmatrix} \sigma^2 + \omega^2 & 0 & ... & 0 \\ 0 & \sigma^2 + \omega^2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \sigma^2 + \omega^2 \end{bmatrix}$$

where $u = (u_1,...,u_N)'$.

In order to estimate the true dependent variable vector y^* and obtain a more efficient estimator, wavelet denoising is applied to the dependent variable as described in the previous section such that:

$$\hat{y} = Wy = y^*$$

and the model becomes:

$$\hat{y}_i = \beta_1 + \sum_{j=2}^{k} \beta_j x_{ij} + \tilde{u}_i$$

with error level $\tilde{u}_i = \varepsilon_i < u = \varepsilon_i + r_i$ and $Var(\tilde{u}_i) < Var(u_i)$.

In order to empirically test the proposed procedure the coefficients of model (1) were estimated with and without wavelet denoising of the dependent variable. The results are included in Table 1. The coefficients estimated for the model with wavelet denoising on the dependent variable are included under column DLS1. Wavelet denoising has significantly reduced the standard errors of all the coefficients and improved the overall model fit. The respective coefficient values are similar in both cases and have the expected signs. Unemployment in particular had a strong negative influence on the consumer sentiment index for the period examined.

Table 1 Coefficient estimates for CSI regressions

No wavelet denoising

Coefficient	OLS	St. errors	t-value
INTERCEPT	3.83	4.36	0.88
INFLATION	-0.04	0.05	-0.76
GDP	0.32	0.46	0.69
UNEMPLOYMENT	-1.34	0.22	-6.20
R-square	0.90		

Wavelet denoising of the dependent variable

Coefficient	DLS1	St. errors	t-value
INTERCEPT	1.33	2.76	0.48
INFLATION	-0.04	0.03	-1.22
GDP	0.57	0.29	1.94
UNEMPLOYMENT	-1.19	0.14	-8.72
R-square	0.95		

4. Forecasting GDP changes using indexes of consumer and business sentiment

In this section I use consumer and business sentiment indexes in order to forecast US GDP changes. The difference when compared to the previous section is that wavelet denoising is now applied to the independent variables of the regression model. Wavelet denoising is necessary in this case because measurement errors in the independent variables lead to inconsistent coefficient estimates (see Davidson and MacKinnon 2004: 313) and, as a consequence, to inaccurate forecasts. In order to demonstrate this, consider the linear regression model $y = X\beta + \varepsilon$ where X is the $(N \times k)$ matrix of k independent variables and $\varepsilon \sim N(0, \sigma^2 I)$. The prediction error in this case is (see Steward and Gill 1998: 78):

$$f = y_f - \hat{y}_f = y_f - X_f \hat{\beta}$$

The subscript f denotes future (out of sample) values of the variables and $\hat{\beta}$ is the OLS estimator of the coefficient vector. It is assumed that the same model holds for both the within sample and the future periods as follows:

$$y_f = X_f \beta + \varepsilon_f \tag{2}$$

and the future values of the independent variables in X_f are known with certainty. If the future error term has the following properties: $E(\varepsilon_f)=0$, $\mathrm{var}(\varepsilon_f)=\sigma^2 I$ and $\mathrm{cov}(\varepsilon,\varepsilon_f)=0$, the prediction error becomes:

$$f = X_f(\beta-\hat{\beta})+\varepsilon_f$$

This expression shows that the prediction error consists of two components: (1) the error due to the difference between the estimate of the coefficient vector and its true value; and (2) the random disturbances in the future period. Consequently, when the coefficients are inconsistently estimated from noisy data, the associated forecasts will also be inconsistent and the prediction error large.

When the future values of the independent variables are not known a priori but have to be estimated, an additional source of uncertainty is introduced into the model and the forecast error will be even higher. In order to demonstrate this let the estimated future values of the independent variables be characterized by the forecast error v_f such that $\hat{X}_f = X_f + v_f$. In addition assume that $E(v_f)=0$, $\mathrm{var}(v_f)=\sigma^2 I$ and $\mathrm{cov}(v_f,\varepsilon_f)=0$. Then the forecast error becomes:

$$f = X_f(\beta-\hat{\beta}) - v_f\hat{\beta}+\varepsilon_f .$$

This expression will also lead to higher forecast error variance for the model (see Tashman, Bakken and Buzas, 2000). Further, it should be expected that the forecast error, v_f, will be even higher when the data based on which the forecasts were generated are subject to measurement errors. By applying wavelet denoising to the independent variables in this case, more accurate forecasts of their future values can be generated and the coefficients of the model can be estimated consistently, which will also lead to more accurate forecasts of the dependent variable and lower forecast error variance for the model. The denoised least squares (DLS) estimator and the exact conditions under which it provides consistency in linear regression models were proposed by Cai, Naik and Tsai (2000) for cases when the independent variables are measured with error.

To estimate a model for GDP changes with consumer and business sentiment indexes as predictors, the DLS estimator proceeds in two stages. In the first stage, wavelet denoising is applied to the indexes in order to remove measurement errors, and in the second stage the model is estimated with OLS. In order to empirically test the forecasting performance of the DLS estimator in comparison to OLS, I estimated the following model for the US economy using quarterly data for the period 1990–1999:

$$DGDP_t = a + \beta_1\, CSI_{t-1} + \beta_2\, NAHB_{t-2} + \beta_3\, NAPM_{t-1} + \beta_4\, NAPM_{t-2} + \beta_5\, LEAD_{t-2}\,. \tag{3}$$

DGDP is the percentage change in the real GDP, *CSI* is the consumer sentiment index published by the Conference Board, *NAHB* is the index of homebuilders' sentiment, *NAPM* is the index of manufacturing activity of the national association of purchasing managers and *LEAD* is the monetary component of the index of leading indicators (stock prices, changes in the real money supply and yield spread). The variables were obtained from Evans (2003: 474–478), who used a similar model specification but incorporated two lags for each independent variable. The specification in (3) was preferred because it provided a better fit to the data and better forecasting performance.

Two versions of the model were estimated. The first version included first differences of all the independent variables, and the coefficients were estimated with OLS. In the second version the coefficients were estimated with DLS (included under the DLS2 column) and wavelet denoising was applied to the variables *CSI*, *NAHB* and *NAPM*. In the case of *CSI*, wavelet denoising was applied to the two finest (resolution) levels of wavelet coefficients, while for the other two variables wavelet denoising was applied to the three finest levels of wavelet coefficients, since the indexes exhibited short-term irregular variation at the respective frequencies. The results from the two estimation procedures are presented in Table 2 (lagged periods are in parentheses). By removing measurement errors from the data and isolating the information content of the indexes that is relevant to GDP, wavelet denoising has significantly improved the R-square of the model and the statistical significance of all the coefficient estimates. Only the coefficients for *NAPM*(1) and *LEAD*(2) are not statistically significant at the 95% significance level; however, their level improved considerably compared to the OLS case.

This improvement is also reflected in the out of sample forecasts generated with each method. The four quarters of 2000 were kept for out of sample evaluation. For the independent variables forecasts were generated with ARIMA models as in Clar, Duque and Moreno (2007), which were then inserted in model (2) in order to forecast GDP changes. The forecasting results are included in Table 3. DLS provided better forecasts and a significantly reduced mean squared forecast error (MSFE) compared to OLS, successfully predicting the reduction in the GDP growth rate in quarter 3.

Table 2 Coefficient estimates for DGDP regressions

No wavelet denoising

Coefficient	OLS	St. errors	t-value
INTERCEPT	0.71	0.12	5.96
CSI(1)	0.01	0.01	0.67
NHBM(2)	0.01	0.02	0.65
NAPM(1)	-0.01	0.03	-0.39
NAPM(2)	-0.02	0.03	-0.47
LEAD(2)	0.27	0.23	1.17
R-square	0.12		

Wavelet denoising of the independent variables

Coefficient	DLS2	St. errors	t-value
INTERCEPT	0.65	0.09	7.57
CSI(1)	0.05	0.02	3.18
NHBM(2)	0.08	0.04	2.14
NAPM(1)	0.10	0.07	1.61
NAPM(2)	-0.20	0.11	-1.83
LEAD(2)	0.16	0.13	1.24
R-square	0.49		

Table 3 Comparison of LS and DLS predictions for GDP changes

Quarter	Actual	OLS	DLS2
2000: Q1	0.58	0.91	1.03
2000: Q2	1.39	0.83	1.12
2000: Q3	0.33	0.80	0.36
2000: Q4	0.47	0.81	0.53
MSFE		0.18	0.04

5. Conclusions

Consumer and business tendency surveys are characterized by sampling and nonsampling errors that result in measurement errors in the corresponding indexes of consumer and business sentiment. This deteriorates their predictive capability for GDP changes since it leads to inconsistent coefficient estimates when they are used as independent variables in linear regression models. It also leads to inefficient estimates when they are used as dependent variables in models that aim to identify the determinants of the consumer sentiment index. The denoised least squares method can improve econometric estimation in both cases by effectively applying wavelet denoising to the indexes and then using the OLS estimation framework as the best linear unbiased predictor. Wavelet denoising is particularly effective when the time series are characterized by abrupt changes, spikes and periodic cycles that frequently characterize consumer and business sentiment indexes.

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