Improving Data Quality and Closing Data Gaps with Machine Learning

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Abstract

The identification and correction of measurement errors often involves labour intensive case-by-case evaluations by statisticians. We show how machine learning can increase the efficiency and effectiveness of these evaluations. Our approach proceeds in two steps. In the first step, a supervised learning algorithm exploits data on decisions to flag data points as erroneous to approximate the results of the human decision making process. In the second step, the algorithm applies the first-step knowledge to predict the probability of measurement errors for newly reported data points. We show that, for data on securities holdings of German banks, the algorithm yields accurate out-of-sample predictions and increases the efficiency of data quality management. While the main focus of our analysis is on an application of machine learning to data quality management, we demonstrate that the potential of machine learning for official statistics is not limited to the prediction of measurement errors. Another important problem that machine learning can help to overcome is missing data. Using simulations, we show that out-of-sample predictions of missing values with machine learning algorithms can help to close data gaps in a wide range of datasets.

Keywords: Data Quality Management, Measurement Errors, Data Gaps, Machine Learning, Supervised Learning

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Further improving and maintaining high data quality is a central goal of official statistics. In the field of data quality management (DQM), the collection of data on human decisions in the DQM process creates an opportunity to increase the efficiency and effectiveness of DQM with machine learning. This paper shows how computers can learn to predict measurement errors on the basis of data on human decisions to flag data points as erroneous. These predicted probabilities of measurement errors facilitate the work of statisticians and form the basis for a novel machine-learning-based approach to automating checks.

While the main focus of our analysis is on an application of machine learning to DQM, we demonstrate that the potential of machine learning for official statistics is not limited to the prediction of measurement errors. Another important problem that machine learning can help to overcome are data gaps. In both applications, we fill in missing information with predictions. To support DQM, the machine learning algorithms predict if a human decision maker would flag data points. In the application to data gaps, the algorithms predict missing values. Using simulations, we establish the algorithms' ability to successfully close data gaps in a wide range of datasets, illustrating their potential beyond an application to DQM.

In this paper, our goal is not to isolate causal effects of data points' features on the probability of measurement errors but, to accurately predict the probability of a measurement error for each data point with machine learning algorithms.[1] Machine learning algorithms fall into two categories: *Supervised* and *unsupervised* learning algorithms. In supervised learning, we provide the algorithm with an outcome and features, in our case, with decisions to flag data points (outcome) and the information that is stored in the data point (features). The task of the algorithm is to learn how to approximate the result of the human decision making process to predict the probability that a data point has to be flagged as erroneous.

In unsupervised learning, we provide the algorithm with unlabelled data, i.e. a dataset without feature and outcome labels. The task of the algorithm is to structure the data, by clustering data points with similar characteristics. Because these clusters are not labelled, they do not necessarily allow us to differentiate between accepted and flagged data points, which is why we focus on supervised learning.[2]

This paper aims to provide an intuitive introduction to the application of machine learning methods to improve data quality. We focus on two types of supervised learning algorithms. The first type of algorithm builds upon assumptions on the functional form of the relationship between features and the outcome. Popular examples for these algorithms are *Linear regressions* and *Logistic regressions*. We demonstrate the application of the structural algorithms with the Logit model. The second type of algorithms is more flexible and does not require structural assumptions. Examples are *decision trees*, *random forests*, and the *K-nearest-neighbors* algorithm. In this paper, we illustrate the premise of these algorithms with the decision tree and the random forest algorithm. Whereas the Logistic regression is applied in econometrics, random forests are "machine learning natives". The comparison of the approaches therefore also hints at advantages and disadvantages of machine learning in comparison to an econometric approach to data analysis.

To illustrate how machine learning can improve data quality, we apply the algorithms to securities holdings data that is reported by German banks to the German central bank (Deutsche Bundesbank). For each reported security, the dataset shows whether a compiler decided to flag the security as erroneous. We begin by splitting the data into a *training dataset* and a *validation dataset*. The application of the algorithms proceeds in two steps. In the first step, the algorithms use the training dataset to learn about the relationship between features of securities and human decisions to flag securities as erroneous. In the second step, the algorithms apply what they have learned in the first step to the validation dataset and make predictions of the probability that securities have to be flagged as erroneous. Importantly, securities in the validation dataset were not used for learning in the first step. Therefore, predictions of measurement errors in the validation dataset are *out-of-sample* and allow us to judge the accuracy of predictions for newly reported securities holdings for which compiler decisions are still unknown.

On the basis of the out-of-sample accuracy of predictions, we compare the performance of the Logit and the random forest algorithm. We find that the random forest algorithm yields a large improvement in accuracy. Building on our findings, we outline possibilities to implement the algorithms and highlight their potential for increasing the efficiency of the DQM process.

The good performance of the algorithms in predicting measurement errors provides the basis for

[1]The machine learning approach to data analysis differs from the econometric approach. In econometrics, causal inference is the name of the game (Angrist and Pischke, 2009). In contrast, the goal in machine learning is to make accurate predictions (Friedman *et al.*, 2001).

[2]Because data on human decisions to flag data points are not always available, supervised learning is not applicable in some contexts. In these cases, unsupervised learning algorithms can provide a viable alternative. In this paper, we do not further discuss unsupervised learning algorithms.

extending the analysis to data gaps. Using simulated data, we show that our findings are not limited to an application of machine learning to DQM. The same methods can be applied to the imputation of missing data with predictions in a wide range of datasets.

Our contributions to the literature are twofold. First, with the application of machine learning to DQM, we introduce a novel approach to the literature on methodologies for the improvement of data quality.[3] Second, our application to data gaps contributes to a literature that discusses the use of machine learning algorithms for the imputation of missing data. Our findings complement earlier work that has mainly focused on closing data gaps in medical databases (see, e.g., Batista and Monard, 2003; Jerez *et al.*, 2010).

The remainder of the paper is organized as follows. Section 1 outlines the background of the Bundesbank's data quality management process for securities holdings data. Section 2 describes the data. Section 3 details the machine learning models. Section 4 presents the results in terms of prediction accuracy and assesses the external validity of the findings. Section 5 describes the implementation of the algorithms in the DQM process. Section 6 discusses the application of machine learning algorithms to close data gaps. Section 7 concludes.

1 Background

German Banks report securities holdings security-by-security to the German central bank (Deutsche Bundesbank) on a monthly basis (Amann *et al.*, 2012).[4] Besides their own securities portfolios, banks report holdings of domestic and foreign depositors, broken down by the depositors' economic sector and country of origin. The reported securities are identified by their International Securities Identification Number (ISIN) and are linked to data about their issuer and market price.

To guarantee high data quality, three types of quality checks are in place. If a security does not pass a check, the reporting bank is asked to explain the reasons for not passing the check and to correct the reported data if necessary. The first type of checks evaluates the consistency of the data and ensures compliance with data format requirements. The second type of checks assesses the plausibility of the data. The third type of checks makes comparisons with other data sources.

In this paper, we focus on a plausibility check of securities that were reported after the expiration of their maturity date. When the maturity date has expired, a security typically has been redeemed and is no longer in the holder's portfolio. In this case, the security should not be reported by the bank. However, if an issuer of a security has financial problems, securities are sometimes redeemed after their maturity date. In this case, the security is still in the holder's portfolio and should be reported. Because of this ambiguity, there is no simple deterministic rule that allows the central bank to automate the check. Instead, compilers make security-by-security evaluations to determine whether a security was erroneously reported after the security's maturity date or not.

2 Data

The algorithms exploit two data sources. The first data source consists of the reported securities' characteristics. The data comprises information on: the number of banks that reported the security (*nreporting*), the number of days that have passed since the maturity date (*days*), the maturity of the security (*maturity*), the type of the security (*stype*), and the nominal value of the security (*nvalue*). The second data source consists of security-by-security compiler decisions. For each security, we use information on a compiler decision to accept the security or to flag the security. There are three types of flags: *"bankrupt"* (12% of flagged securities) if information on a default of the issuer is available, *"erroneous"* (44% of flagged securities) if the maturity date is incorrect, and *"implausible"* (44% of flagged securities) if there is suspicion of an issuer default or an error from the reporting bank. In the following, we differentiate between flagged and accepted securities but do not further distinguish between the three types of flags.

After linking the data on securities' characteristics to the compiler decisions, we end up with a dataset of $N = 4\,495$ securities that were reported after the expiration of their maturity date between February

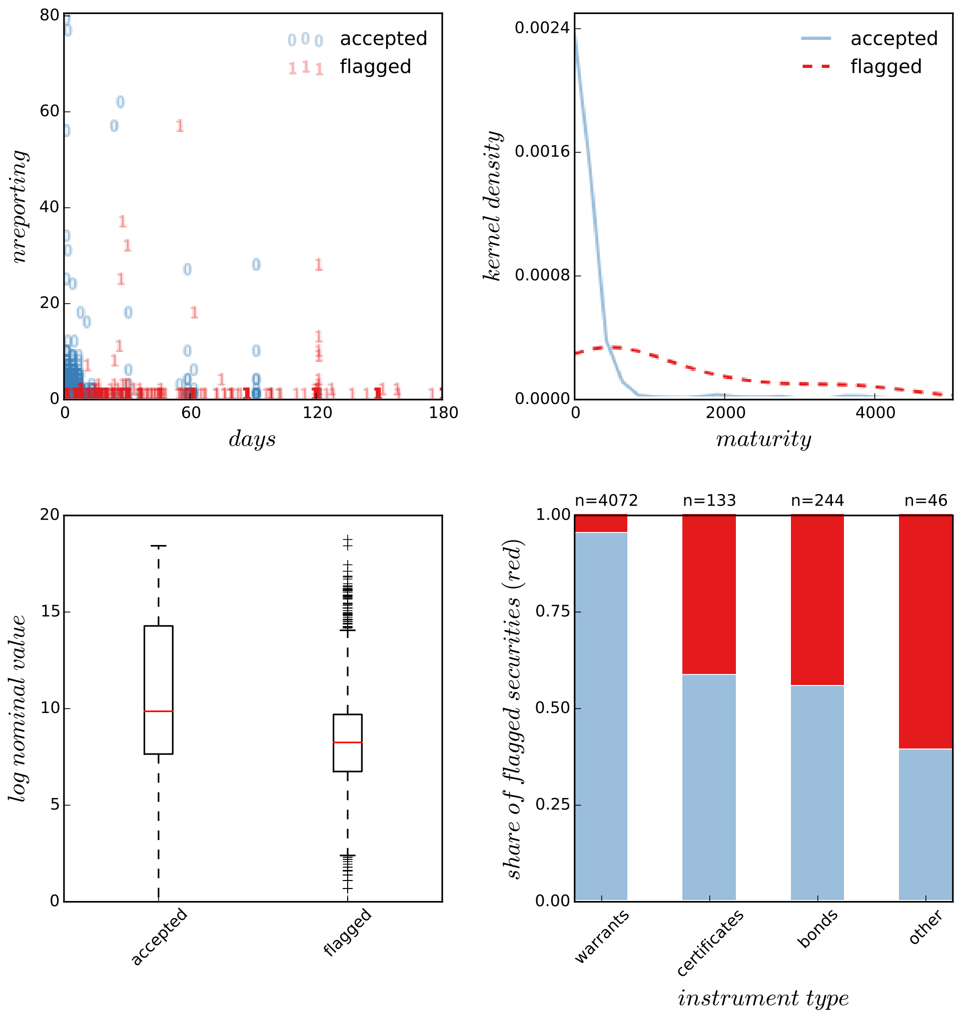
[3] For a literature review, see Batini *et al.* (2009).

[4] The reported securities comprise negotiable bonds and debt securities, negotiable money market papers, shares, participating certificates and investment fund certificates.

and September of 2016. Out of the securities in this dataset, 385 securities (9%) were flagged by compilers.

Figure 1 shows descriptive statistics for securities that were flagged and securities that were accepted. Overall, the figure indicates that characteristics of flagged securities differ in a number of ways from accepted securities. On their own, these descriptive statistics hint at a possibility to predict which securities are flagged on the basis of securities' features. However, the relationships between the characteristics of a security and the probability of a flag is likely complex. Therefore, in the next section, we fit models that can capture complex relationships to the data and make predictions on the basis of these models.

Figure 1: Descriptive Statistics of Features for Accepted and Flagged Securities



Note: The figure shows descriptive statistics of features for accepted and flagged securities. The upper left panel shows a scatter plot of accepted (blue) and flagged (red) securities. The upper right panel shows kernel density plots of the distribution of the securities's maturity dates (in days) for accepted and flagged securities. The lower left panel shows box-plots of the log-normalized nominal values of securities. The lower right panel shows the ratio of accepted and flagged securities by security type. For illustrative purposes, we exclude securities in the top one percentile of days and reporting banks in the upper left panel and the top one percentile of maturity in the upper right panel of this figure.

3 Models

Our goal is to support the compiler by providing the probability that a reported security has to be flagged, conditional on the features of the security:

$$P(y_i = 1|x_i). \tag{1}$$

The outcome y_i is a dummy variable that takes the value one if security $i \in \{1, 2, \ldots, N\}$ is flagged and zero otherwise. The vector x_i consists of the features of security i. We compare two types of models that provide a prediction of the conditional probability in eq. (1): Models that base predictions on structural assumptions and data-driven models without structural assumptions.

3.1 Models with Structural Assumptions

To predict the probability in eq. (1), we start by making assumptions on the relationship between features of the reported security and the probability of compilers flagging the security. A common assumption is that the data-generating process can adequately be described by a transformation function that maps a *single linear index* z_i to the probability $P(y_i = 1) \in [0, 1]$. We construct the index from the security's features under the assumptions of linearity and additive separability. We get:

$$z_i = \alpha + x_i^T \beta, \tag{2}$$

where α is a constant and β is a parameter vector that captures separable effects of the security's features on the index and ultimately the compiler decision.[5] The sigmoid function $\Lambda(\cdot)$ transforms the index z_i in the domain $[-\infty, \infty]$ into a probability in the domain $[0, 1]$:

$$\Lambda(z_i) = (1 + e^{-z_i})^{-1}. \tag{3}$$

Equipped with these assumptions, we can express the conditional probability in eq. (1) as:

$$P\big(y_i = 1|x_i\big) = \Lambda\big(z_i\big) = \Lambda\big(\alpha + x_i^T \beta\big). \tag{4}$$

This is the *Logit* model.[6]

An important difference to an econometric application of the Logit model is that our goal is not to interpret estimated parameters causally but to make a prediction of the probability with high out-of-sample accuracy. Therefore, we do not discuss potential endogeneity problems of the model in our application.

An alternative to the Logit model that follows a similar approach, i.e. starting with assumptions on the structural relationship between features and the outcome, is the *linear regression model*. In contrast to the Logit model, the linear regression model lacks the transformation function (3) and models the conditional probability in eq. (1) as a linear combination of the features: $P(y_i = 1|x_1) = \alpha + x_i^T \beta$. Comparing the Logit and the linear regression model, the linear regression model has the attractive property of providing the best (minimum mean squared error) linear approximation to the data. The Logit model does not have this property but can provide a better fit to binary dependent variables by restricting the domain of $\widehat{P}$ to $[0, 1]$ (Angrist and Pischke, 2009). In the following, we focus on the Logit model.

Fitting the Logit Model to the Data To fit the Logit model to the data, we estimate the sample equivalents of the population parameters in (4) under the assumption that compilers evaluate securities independent from one another.[7] The estimation maximizes the log likelihood function with the Broyden-

[5]For a bivariate model, with the features $days_i$ and $nreporting_i$, we get $z_i = \alpha + \beta_1 days_i + \beta_2 nreporting_i$. In this example, the linearity assumption implies that the index increases by a constant value β_1 for each additional day that has passed since the maturity date and by a constant value β_2 for each additional bank that reports the security. The separability assumption implies that there is no interaction between the features, i.e. the marginal effect of an additional day on the index is independent of the number of reporting banks.

[6]From an econometric perspective, we can derive the Logit model from a decision model, if we interpret the index function (2) as a latent score that the compiler uses to decide whether to flag security i (see Section B of the Appendix). In Machine Learning, we put no emphasis on the decision model underlying the Logit and focus on the ability of the model to provide a good fit to the data.

[7]For human decisions to flag data points, the independence assumption is likely violated. This is because a compiler might flag clusters of data points as erroneous on the basis of information that is not part of the dataset. Then, in terms of our model, the unobserved element in decisions is correlated within these clusters of data points. To evaluate the influence of these correlations on the performance of the algorithms, we simulate datasets with correlated data points. Because we find that in our application, the negative effect of a violation of the independence assumption on prediction accuracy is negligible for a moderate cluster size, we do not further discuss the independence assumption. For detailed simulation results, see section C.5 in the appendix.

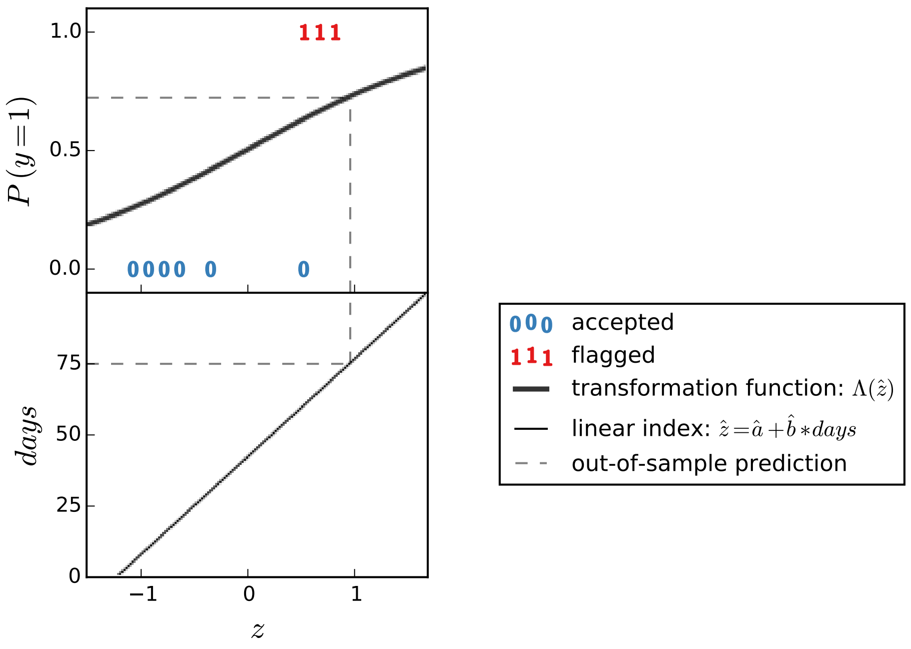
Fletcher-Goldfarb-Shanno algorithm.[8] With the estimated parameters, we can predict the probability of a flag, $\widehat{P}$, for any representation x of the feature vector x_i as:

$$\widehat{P}\big(y_i = 1 | x_i = x\big) = \Lambda\big(\widehat{\alpha} + x^T \widehat{\beta}\big), \tag{5}$$

where $\widehat{\alpha}$ and $\widehat{\beta}$ are the estimates of the intercept and slope parameters.

Figure 2 shows a stylized illustration of a Logit model that includes the number of days that have passed since the maturity date as a single feature. For this univariate model, the estimation with Maximum Likelihood yields the estimates $\widehat{a}$ (intercept) and $\widehat{b}$ (slope). With these parameter estimates, we construct the linear index function $\widehat{z}(x)$ in the lower panel. By plugging the index into the transformation function in the upper panel, we map from the index to a probability. The grey dashed lines illustrate how we make a prediction for an exemplary security that has been reported $x = 75$ days past its maturity date.[9] In the example, the predicted probability of a flag is $\widehat{P}\big(y_i = 1 | days_i = 75\big) = \Lambda\big(-1.22 + 0.03 * 75\big) =$ 72%. The Figure also illustrates the restrictions that our assumptions impose on the model: By assumption, the index function is a linear combination of our features (lower panel) and the transformation function yields a continuous S-shaped curve (upper panel).

Figure 2: Logit Model and Predictions



Note: The figure shows an exemplary illustration of the Logit model (solid lines) and a prediction with the model (grey dashed lines). The blue (red) data points represent accepted (flagged) securities.

[8] The “individual” likelihood function L_i describes the likelihood of observing the outcome y_i for a security i with features x_i, given the parameter α and the parameter vector β. For our model, we get:

$$L_i = \begin{cases} \Lambda\big(\alpha + x_i^T \beta\big) & for \;\; y_i = 1 \\ 1 - \Lambda\big(\alpha + x_i^T \beta\big) & for \;\; y_i = 0 \end{cases} .$$

We can summarize this piecewise function as $L_i = \Lambda\big(\alpha + x_i^T \beta\big)^{y_i} \big(1 - \Lambda\big(\alpha + x_i^T \beta\big)^{y_i - 1}\big)$. If we are willing to assume independence across securities, the likelihood of observing the different feature-outcome combinations in our sample is simply the product of the individual likelihood functions $L = \prod_{i=1}^{N} L_i$. A model provides a good fit to the data if it results in a high value of L, i.e. a high likelihood of observing the feature-outcome combinations in the sample, given the parameter values of α and β. Maximum Likelihood is a method that allows us to choose the optimal values of α and β, i.e. the values that provide the best fit to the data, by maximizing the logarithm of L. The iterative numerical optimization routine that we use to maximize L is the BFGS algorithm.

[9] For the illustrative example, we first construct the index in the lower panel and then use the sigmoid function to map to a probability in the upper panel.

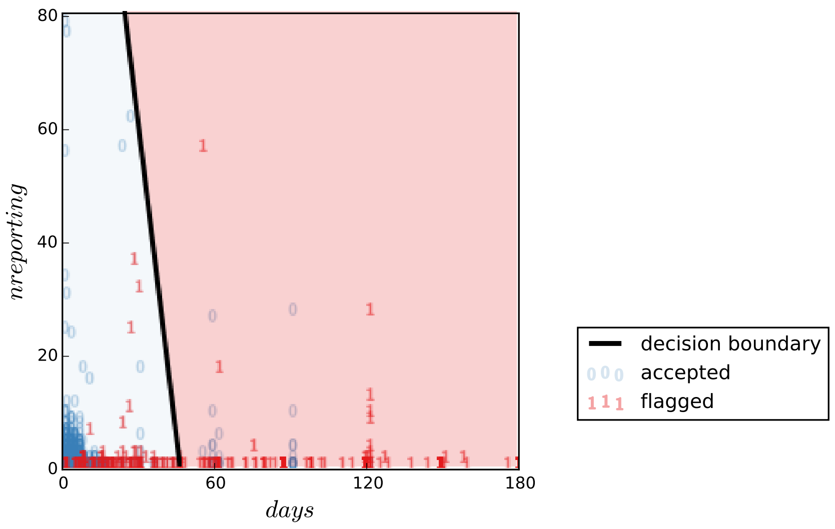
Decision Boundary of the Logit Model So far, we have predicted the probability of a flag. To move from the predicted probabilities to a prediction of the binary outcome, we follow the rule:

$$\widehat{y}_i = \begin{cases} 0 & for \quad \widehat{P}(y_i = 1|x_i = x) < 50\% \\ 1 & for \quad \widehat{P}(y_i = 1|x_i = x) \geq 50\% \end{cases} , \tag{6}$$

predicting a flag for securities with $\widehat{P}$ equal or greater than 50%. From eq. (6), we can derive a decision boundary that separates securities for which we predict a flag from securities for which we predict acceptance.

Figure 3 shows the decision boundary for a bivariate model with days and the number of reporting institutions as features. In the figure, we predict acceptance for securities with nreporting-days feature combinations in the blue area and flags for securities with nreporting-days feature combinations in the red area. Because the index is a linear combination of the features, the decision boundary is linear.[10]

Figure 3: Logit Model and Decision Boundary



Note: The figure shows an illustration of the bivariate Logit model's decision boundary (solid line). The blue (red) data points represent accepted (flagged) securities. For securities with a days-nreporting combination in the light red (blue) area, we predict a flag (acceptance).

Strengths and Weaknesses of Models With Structural Assumptions A strength of the approach are accurate out-of-sample predictions for observations outside the sample's feature-space if our assumptions yield a good representation of the data-generating process. Intuitively, for Figure 2, we use the sample to estimate *a* and *b*. Once we have estimated these parameter values, we can make a prediction for a security that has longer passed the maturity date than the securities in the sample by extrapolating the index curve and the transformation function. Another strength is transparency due to the structural underpinning of the model and the limited number of parameters.

The main weakness of the approach is that the modelling assumptions restrict how flexibly we can fit the model to the data. If the assumptions result in a bad fit of the model to the true data-generating process, the model yields inaccurate predictions. In Figure 3, the linear decision boundary that follows from the assumption of linearity and separability of the features in the index function of the Logit model does not provide a close fit to the data.

One way to increase the flexibility of the Logit model and to allow for non-linearities in the decision boundary is to expand the index function by polynomials of the features and interactions between the

[10]At the decision boundary, the probability of a flag is 50%. With the sigmoid transformation function, we get a predicted probability of 50% for securities with an index function value of zero: $\Lambda(z = 0) = 50\%$. Consequently, to depict the index function in a two dimensional *nreporting-days* diagram, we first set the index function to zero: $0 = a + b_1 \, days + b_2$nreporting. By solving for *nreporting*, we get the boundary: $nreporting = \frac{-a}{b_2} + \frac{b_1}{b_2} days$.

features.[11] However, the method described in section 3.2 provides a more direct approach to flexibly capture non-linearities and might therefore be preferable to the expansion of the index function.

3.2 Models without Structural Assumptions

In the model with structural assumptions, we described the relationship between features and the probability of a flag with the linear index function and the transformation function in eq. (4). In contrast, we now make no assumptions on the functional form of the relationship. We only assume that securities with the same characteristics have the same probability of being flagged. With this assumption, our prediction is a very direct representation of the conditional probability:

$$\widehat{P}\big(y_i = 1|x_i = x\big) = Ave\big(y_i|x_i = x\big). \tag{7}$$

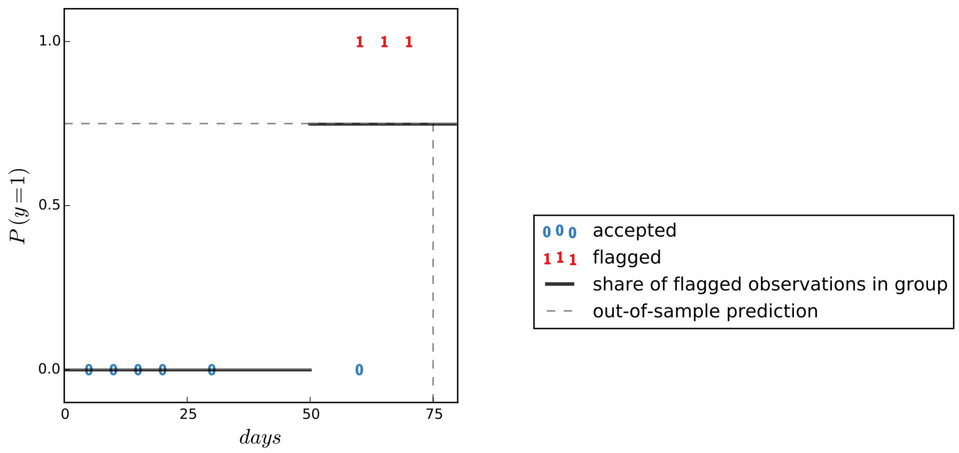
Following eq. (7), to predict the conditional probability of a flag for a security with the feature vector x, we simply take the average over the binary outcome in the group of securities with characteristics $x_i = x$.[12]

There is usually not a large enough number of securities with the exact feature values in x to allow for a precise prediction of the conditional probability with eq. (7). Therefore, we estimate the probability by considering securities with *similar* characteristics. To this end, we define non-overlapping regions $N_r(x)$ with $r \in \{1, 2, \ldots, R\}$ of securities with similar feature values. Formally, we get:

$$\widehat{P}\big(y_i = 1|x_i = x\big) = Ave\big(y_i|x_i \in N_r(x)\big). \tag{8}$$

The estimation of the conditional probability boils down to calculating the average over the outcomes of flagged securities in a reference group with characteristics x_i that are similar to x, i.e. securities that are part of the same region N_r like x. For our binary outcome, this average is simply the share of flagged securities in the region.

Figure 4: Model without Structural Assumptions and Predictions



Note: The figure shows an illustration of a model without structural assumptions(solid lines) and a prediction with the model (grey dashed lines). The blue (red) data points represent accepted (flagged) securities.

Figure 4 illustrates the conceptual idea of the approach for a bivariate model with the features *days* and *nreporting*. We first separate the securities by region, according to the number of days that have passed since the maturity date. In the figure, one region comprises securities for which more than 50 days have passed since the maturity date and the other region comprises securities for which 50 days or less have passed since the maturity date. In a second step, we calculate the share of flagged securities in both regions. For the first region, the share is zero. For the second region, the share is 0.75. Conditional on knowing the number of days that have passed since the maturity date, these shares correspond to our prediction of the probability that a security in the region has to be flagged. The

[11] *Support vector machines* with nonlinear kernels are another popular model that allows us to classify securities as flagged or accepted by fitting a nonlinear decision boundary to the data.

[12] The average over the binary outcome is simply the share of flagged securities with characteristics $x_i = x$.

grey dashed lines provide an exemplary prediction for a security that was reported $x = 75$ days past its maturity date. In the example, the predicted probability of a flag is 75%, the share of flagged securities for which more than 75 days have passed since the maturity date.

The conceptual idea is representative for a larger class of algorithms: decision trees, random forests, and the K-nearest-neighbors algorithm. All of these algorithms follow the premise of eq. (8). They make predictions of the probability of a flag, conditional on the feature values in x in two steps. In the first step, they define a reference group that comprises observations with feature values that are similar to x. In the second step, they make a prediction of the outcome by averaging over the outcomes in this reference group. The algorithms differ in the way in which they define the reference groups. Decision trees and random forests define reference groups by sequentially splitting the feature-space into regions. The reference group of a security with feature values x are the observations that fall into the same region as the security. In the K-nearest-neighbors algorithm, the reference group of a security with the feature values x are the K observations whose feature vector has the smallest Euclidean distance to x. In the following, we focus on random forests, starting with a description of the decision tree algorithm on which the random forest algorithm rests.

Decision Trees Because we do not have data that allows us to construct a separate region for every possible feature combination, the task for our algorithm is to optimally divide the theoretical feature space into regions. Finding optimal splitting rules on the basis of the training data is equivalent to fitting the Logit model to the data.

An optimal region is homogeneous, i.e. securities in the region all are of the same type (flagged or accepted). In such a homogeneous regions, our predictions are perfectly aligned with the actual decisions of the compilers. We either predict a conditional probability of a flag of 100% for all securities in the region and all securities are actually flagged, or we predict a probability of 0% for all securities in the region and none of the securities is actually flagged. Put differently, in homogeneous regions we get all predictions exactly right which is why homogeneity intuitively is an adequate target for our algorithm. To measure the homogeneity within a region, we use the *gini index*:

$$G = 2\mu_r(1-\mu_r) \quad \in \quad [0, 0.5], \tag{9}$$

where μ_r is the share of flagged securities in region r. If all securities in a region are flagged $\mu_r = 1$ and $G = 0$.[13] G increases with the heterogeneity of the region up to a maximum of 0.5.[14]

To divide the feature-space, we sequentially split the dataset into ever smaller, more homogeneous regions. A split allocates each observation to one out of two regions on the basis of a feature k and a cutpoint s, so that securities in one region satisfy $x_k \leq s$ and securities in the other region satisfy $x_k > s$. The approach is *top-down greedy*. We start with the full dataset and compare splits for all features at all potential cutpoints. We then select the feature and cutpoint that maximizes the resulting regions' homogeneity. In the following steps, we repeat the process and successively split the regions, optimizing for homogeneity with each split. By sequentially splitting the dataset, the regions become smaller and increasingly homogeneous. We stop splitting a region either when all observations in the region are of the same type (flagged or accepted), when the region contains a pre-defined minimal number of securities, or after a pre-defined number of steps.[15]

Figure 5 shows the optimal decision tree for the feature set $J = \{days, nreporting\}$, a maximal splitting depth of three and a minimal number of securities in a region of 18. The first split separates securities for which 15 or less days have passed since the maturity date from securities for which more than 15 days have passed since the maturity date. After a second and third split, we end up with eight regions (leafs). To make a prediction, we calculate the share of flagged securities in a leaf which equals the predicted probability that a security is flagged. To make a prediction of the binary outcome, we predict a flag for securities for which the predicted probability of a flag is equal to or greater than 50% (see, eq. (6)). This corresponds to majority voting, where each security casts a vote according to its own type.[16]

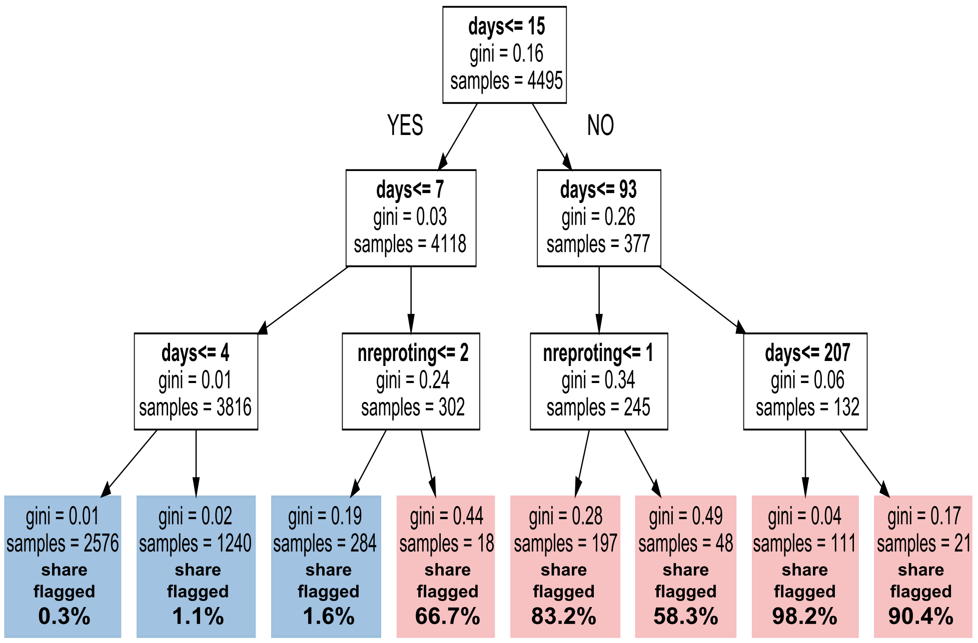
[13]Likewise, we get $G = 0$ if none of the securities in the region is flagged, i.e. if $\mu_r = 0$.

[14]For our binary outcome, we get maximum heterogeneity ($G = 0.5$) if 50% of the securities in the region are flagged.

[15]The depth is the maximal number of successive splits.

[16]Figure A.1 in the appendix shows the corresponding decision boundary.

Figure 5: Decision Tree for Bivariate Model



Note: The figure shows the result of an algorithm that determines the optimal decision tree for the bivariate model with days and nreporting ass features. The algorithm operated under the restrictions of a maximal depth of three and a minimal number of samples in a leaf of 18.

Random Forests A disadvantage of decision trees is that the division into regions strongly depends on the random draw from the population that our sample represents. If we draw different samples from the population and compute decision trees, the trees look very different from one another. Ideally, we would like to have a stable prediction that is not affected by the random selection from the population, the sample represents.[17]

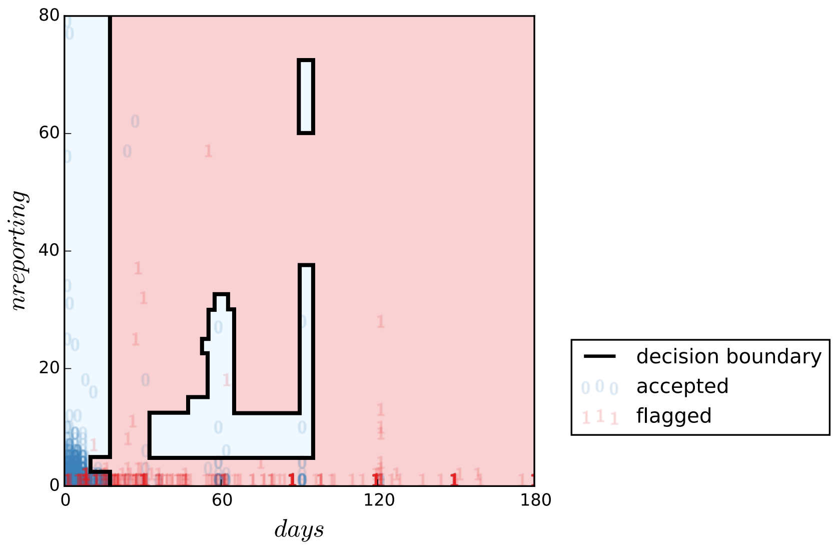
We can overcome this problem by computing a decision tree on the basis of the entire population or by making repeated, independent draws from the population and averaging over the corresponding decision trees. Usually, both of these approaches are not viable because we do not have the population data. However, we can still approximate random draws of samples from the population by using *bootstrapping*. To construct a bootstrap sample, we draw N observations with replacement from the original sample of size N. We then compute a decision tree for the bootstrap sample. We repeat this many times and end up with a large number of decision trees, a forest, which allows us to average over estimated probabilities. In a nutshell, we approximate repeated draws from the population by drawing repeatedly with replacement from the sample.

A caveat of the approach is that often, predictions are correlated across trees. The reason is that if there is a particular feature that has a strong effect on predictions, all trees likely use this feature. The resulting correlations between trees are problematic for the prediction accuracy if they reflect relationships between features and the outcome that are characteristic of the sample but not the population. To overcome this problem, random forests use a different subset of randomly selected features for each bootstrap sample. This is referred to as *decorrelation* of the trees.

Decision Boundary of Random Forests Figure 6 shows the decision boundary for a random forest of 300 trees, a minimum number of securities in a leaf of seven and the feature set $\{nreporting, days\}$. As for the Logit model, we flag securities with a predicted probability equal or greater to 50%. Accordingly, we predict flag for securities with nreporting-days feature combinations in the red area and acceptance for securities with nreporting-days feature combinations in the blue area. In contrast to the Logit model's decision boundary in Figure 3, we did not impose any functional form restrictions or distributional assumptions on the model. This flexibility is reflected in the nonlinear decision boundary.

[17]Another way of framing this is in terms of variance. Varying predictions for different random samples imply that estimates with decision trees have a high variance. In terms of the estimator's properties, this implies that decision trees are inefficient estimators.

Figure 6: Random Forest and Decision Boundary



Note: The figure shows an illustration of the decision boundary (solid line) resulting from bivariate classification with a random forest. The blue (red) data points represent accepted (flagged) securities. For securities with a days-nreporting combination in the light red (blue) area, we predict a flag (acceptance).

Strengths and Weaknesses of Models Without Structural Assumptions Conceptually, the approach is very different from starting with structural assumptions on the relationship between the features and the outcome. Because there are no functional form restrictions that constrain the relationship between features and the probability of a flag, the algorithm can flexibly portray interdependencies and non-linearities in the data. Nonlinear and complex links between features and the outcome do not clash with rigid assumptions on linearity, separability and the shape of the transformation function. Because of this flexibility, the approach is a very direct representation of the conditional probability in eq. (1). The nonlinear decision boundary that results from the random forest in Figure 3 illustrates this flexibility.

The weaknesses of the approach are twofold. First, there is a tradeoff between the gain in flexibility and the accuracy of predictions when we apply models without structural assumptions to high dimensional (large number of features) data. Following the law of large numbers (LLN), sample averages converge to expected values with an increasing number of observations in the sample. Because we get convergence to population parameters only if the number of observations underlying the prediction tends to infinity (LLN), the accuracy of predictions based on the premise of non-structural models (eq. (8)) decreases exponentially with an increasing number of features. This is because in eq. (8), the number of observations underlying a prediction is the number of observations with similar feature values. If the number of features increases, the number of observations with similar feature values decreases exponentially. Intuitively, whereas we might find securities with the same number of days since the maturity date, it is unlikely that we find a security with the same number of days, number of reporting banks, maturity, and security type. This is often called the *curse of dimensionality*. In contrast, in the structural Logit model, our assumptions on the functional form of the relationship between features and the outcome allow us to use the *entire* sample to estimate model parameters. Because we do not rely on a subset with the same feature values, our estimates allow for accurate predictions even if a particular combination of feature values in x is not common in the sample.

Second, with higher flexibility, we run the risk of fitting the model too closely to the data, i.e. overfitting the model. In case of overfitting, our model picks up relationships between features and the outcome that are characteristic of the random sample but not the population. Picking up such sample-specific relationships makes for low accuracy in out-of-sample predictions.[18]

[18]Overfitting can affect both, the Logit model and the random forest. However, due to its higher flexibility, there is more potential for overfitting of the random forest algorithm. For both models, there are ways to prevent overfitting. A first line of defence against the risk of overfitting is to judge the models with respect to the accuracy of out-of-sample predictions. For the Logit, a second line of defence is *regularization*. In a nutshell, with regularization, we add a penalty term to the Likelihood function. The term penalizes large parameter estimates and thereby provides a counter weight to the increase in the likelihood function, we achieve by overfitting (e.g., by adding additional features to the model or overestimating the size of coefficients). To include a penalty term in the individual Likelihood function in Footnote 8, we expand L by adding $-\lambda\beta'\beta$, the sum of squared parameter estimates. By

4 Results and External Validity

To compare the Logit model to the random forest, we evaluate the accuracy of both models' out-of-sample predictions.[19] A simple and straightforward method to assess out-of-sample prediction accuracy is cross-validation with the *validation set approach*. In the first step, we randomly split the securities into a training dataset and a validation dataset and fit the models to the training data. In the second step, we make out-of-sample predictions for securities in the validation dataset on the basis of the first-stage estimates. We then compare the accuracy of both models' predictions in terms of two metrics: *recall* and *precision*. Recall is formally defined as:

$$R = \frac{T_p}{T_p + F_n}, \tag{10}$$

where T_p is the number of true positives (the number of flagged securities for which we correctly predicted flags) and F_n is the number of false negatives (the number of flagged securities for which we incorrectly predicted acceptance). The nominator of eq. (10) equals the number of correctly predicted flags and the denominator equals the actual number of flags in the validation dataset. Taken together, R is the share of flags in the validation dataset that were correctly predicted. Precision is formally defined as:

$$P = \frac{T_p}{T_p + F_p}, \tag{11}$$

where F_p is the number of false positives (the number of accepted securities for which we incorrectly predicted a flag). The nominator of eq. (11) equals the number of correctly predicted flags, whereas the denominator equals the overall number of predicted flags. Intuitively, P measures the share of securities for which we predicted a flag that were actually flagged.

The results of the validation set approach rest on a single random split of the data into a training dataset and a validation dataset. Therefore, the values of our performance metrics depend on the random nature of the split. To overcome this problem, we implement *stratified k-fold* cross-validation. In stratified k-fold cross-validation, we randomly split the data into k equal sized datasets. To make sure that the number of flags is the same in each of these datasets, we stratify the sample by our outcome variable. We then, in turn, use each of the k datasets as training data, computing the recall and precision metric for out-of-sample predictions in the other $k-1$ datasets. This provides us with k values for recall and precision, each representing an estimation with a different randomly selected training dataset. By averaging over these k-values, we get average values for recall and precision that are stable in the sense of not depending on a single random split.

Table 1: Prediction accuracy

	Logit Regression	Random Forest Algorithm
Recall	0.63	0.86
Precision	0.85	0.89

Note: The table shows average recall and precision with stratified five-fold cross-validation for the Logistic regression and the random forest algorithm. Each of the algorithms uses the feature set that minimizes average recall.

Table 1 shows average recall and precision with stratified five-fold cross-validation for the Logistic regression and the random forest algorithm. For each method, we use the feature set that maximizes the model's performance in terms of recall.[20]

choosing λ, we can calibrate the size of the penalty. For the random forest, a strategy that is equivalent to regularization is limiting the maximum number of splits (depth of decision trees). This prevents the algorithm from producing regions with few observations and low out-of-sample prediction accuracy.

[19]We implement both models with the *sklearn* package in *python*.

[20]For the random forest, we use the feature set $\{$*days, nreporting, mvalue, stype*$\}$, where *stype* is a set of dummy variables for the instrument type. The Logistic regression uses the same set of features except for the nominal value. For the random forest algorithm, we compute 300 trees and implement a maximum depth of six, again to maximize recall.

We find that both models have a similar performance in terms of precision. In terms of recall, the random forest algorithm (R = 0.86) yields a 37% percent improvement over the Logistic regression (R = 0.63). Predicting flags with the random forest algorithm, we correctly identify 86% of the securities that were flagged by compilers.

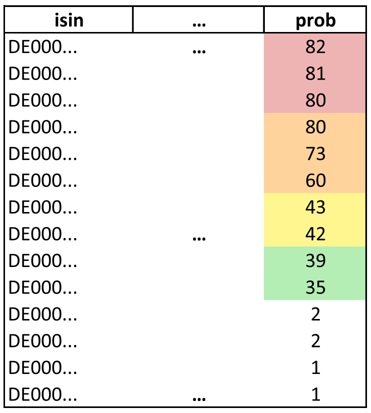
So far, we have treated human decisions as the benchmark for the algorithms. However, there is a margin for decision errors by compilers. If the compilers make random errors in their assessment, an ideal algorithm should produce a non-zero share of false positives and negatives. Following this line of reasoning, the algorithms can yield results that are superior to a human decision maker by filtering out random decision errors.[21]

In order to establish the external validity of our results, we predict two further types of measurement errors using Logit regressions and random forests. The first type of errors concerns erroneous reporting of securities before the emission date. The second type of errors concerns missing reference data. In both cases, we get results that are similar to our findings in Section 4.[22]

5 Implementation

Because of its higher out-of-sample accuracy, we focus on the implementation of the random forest algorithm. To implement the algorithm, we use all 4.495 reported securities and compiler decisions as training data. We then predict the probabilities of flags for newly reported securities and add the predictions to the data table, compilers use to evaluate securities for measurement errors. Figure 7 shows an illustration of the data table with predicted probabilities for securities that were reported in October, 2016. Securities with a high predicted probability of a flag appear on the top of the table. The conditional formatting allows compilers to easily spot potentially problematic securities. By providing compilers with the predicted probability instead of making binary predictions of flags, we take advantage of all the information, the machine learning algorithm provides.

Figure 7: Result of additional out-of-sample predictions



isin	...	prob
DE000...	...	82
DE000...		81
DE000...		80
DE000...		80
DE000...		73
DE000...		60
DE000...		43
DE000...	...	42
DE000...		39
DE000...		35
DE000...		2
DE000...		2
DE000...		1
DE000...	...	1

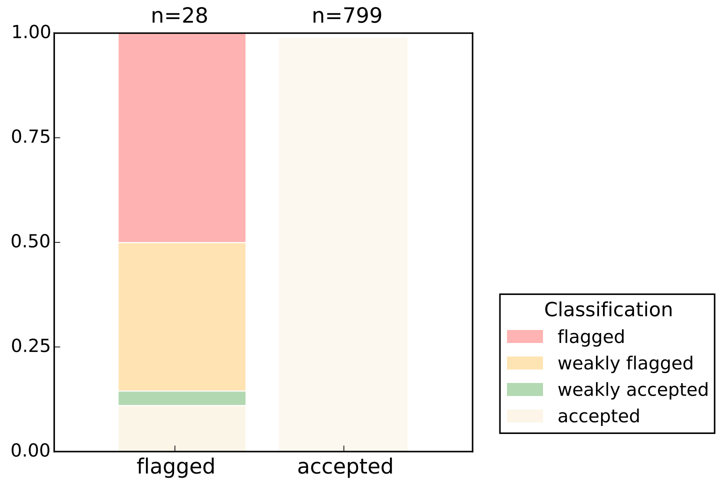
Note: The figure shows the data table after an implementation of the random forest algorithm. The predicted probabilities are shown in the last column. The other columns show the isin that identifies the security and the features of the security (not in the figure).

The sorting of securities by predicted probabilities of measurement errors increases the efficiency of the DQM process. Because compilers know which securities are likely subject to a measurement error, they can allocate their time more efficiently to these securities. The ordering furthermore leads to a higher effectiveness of the evaluation process. Because the attention of compilers and their ability to evaluate a security correctly decreases with each security they review, checking securities with a high probability of a measurement error first increases the success of compilers' efforts to identify measurement errors in the data.

[21] Of course, if decision errors are structural, the algorithm might learn to make the same mistakes as the compiler.

[22] For securities that were reported before their emission date, predictions of flags with the random forest algorithm achieve a recall of 77% and a precision of 88%. Predictions with Logit regression yield a recall of 50% and a precision of 86%. Regarding the attribution of a lack of reference data to a measurement error, predictions with the random forest algorithm have a recall of 96% and a precision of 97%. The Logit regression produces predictions with a recall of 39% and a precision of 97%.

Figure 8: Result of additional out-of-sample predictions



Note: The figure shows the result of additional out-of-sample predictions for securities reported in October, 2016. The left (right) bar shows securities that were flagged (accepted) by compilers. The colours correspond to the predicted probabilities.

Figure 8 shows the results of out-of-sample predictions for the securities that were reported in October, 2016 and were not part of the training dataset.[23] The left (right) bar shows securities that were flagged (accepted) by compilers. We find that the algorithm classifies more than 85% of securities that were flagged by compilers as weakly flagged or flagged. If we had informed compilers about the classification, they would have found all of the 28 flagged securities in the top-35 of the list of 827 securities which they received for evaluation.

6 Using Machine Learning to Close Data Gaps

To illustrate that the potential of machine learning for official statistics is not limited to DQM, we evaluate the possibility of using predictions with machine learning algorithms to close data gaps. Essentially, there are two prerequisites for the algorithms' ability to close data gaps with out-of-sample predictions of the missing values. First, there have to be dependencies between the outcome variable that suffers from data gaps and other features of a data point. Second, we have to observe some data points for which the outcome is not missing. In a nutshell, by learning about the dependencies between features and the outcome from data points where the outcome values are not missing, the algorithm can predict the missing values.

To evaluate the success of the algorithms in closing data gaps, we use simulated data. By simulating data, we can vary the environments in which we evaluate the algorithms' performance in a controlled fashion. For simplicity, we simulate a binary outcome variable $y_i \in \{0, 1\}$ with data gaps. The binary variable allows us to use the same performance metrics as in the DQM application. Intuitively, recall and precision measure the success of the algorithm in predicting the missing values, i.e. the share of observations with data gaps for which the prediction equals the unobserved value. While recall measures our success in predicting the missing outcome $y_i = 1$, precisions shows the performance of the algorithms in predicting the missing outcome $y_i = 0$.

In our simulations, we cover predictions in a wide range of datasets with different: binomial distributions (shares of data points with $y_i = 1$), numbers of data points, data quality, degrees of correlation between data points, and nonlinear feature-outcome relationships. The simulations show that the application of machine learning to close data gaps has a large potential in many environments.[24] If the outcome variable measures occurrences of a rare event, the performance of the models suffers but

[23] If manual checks by compilers are replaced by predictions with a machine learning algorithm, problems can arise when structural changes in the data occur. Because the algorithm draws on historic training data, there might be no possibility to learn about new ways in which a human decision maker would evaluate the data. To investigate whether this is a problem in our implementation, we evaluate whether predictions for October, 2016 on the basis of training data from more recent months are superior to predictions using older training data. We find no systematic difference in performance and conclude that the relationships between the outcome and features in our application are stable over time. Figure A.2 in the Appendix summarizes the results of the analysis.

[24] For a detailed description of the simulation results, see section C of the Appendix.

can be improved by collecting larger datasets. Whereas correlated observations have a minor effect on performance, data quality has a large positive effect on recall and precision. When we introduce non-linear feature-outcome relationships, the random forest algorithm is clearly superior to the Logit model. Taken together, the results show that machine learning algorithms have applications beyond DQM. In particular, they can be tested against other algorithms for the imputation of missing values to close data gaps.

7 Conclusion

There is a large potential to improve Data Quality Management if data on human decisions to flag observations as erroneous is available. With data on decisions, supervised learning algorithms can approximate the result of the human decision making process and predict probabilities of measurement errors. We showed that, for data on German securities holdings, out-of-sample predictions using a random forest algorithm are superior to predictions with the Logit model. The predicted probabilities allow for a prioritization in the DQM process and an increase in the effectiveness and efficiency of the search for measurement errors.

The application of machine learning to close data gaps illustrates that the potential of the methods for officieal statistics is not limited to DQM. Simulations show that the Logit model and the Random Forest accurately predict missing data over a wide range of datasets, with a superior performance of the random forest algorithm in datasets with complex feature-outcome relationships. These findings suggest that machine learning algorithms provide a powerful alternative to established imputation methods.

Future research on the application of machine learning to DQM could discuss unsupervised learning algorithms. Unsupervised learning algorithms can help to identify measurement errors if no information on human classifications of observations as erroneous is available.

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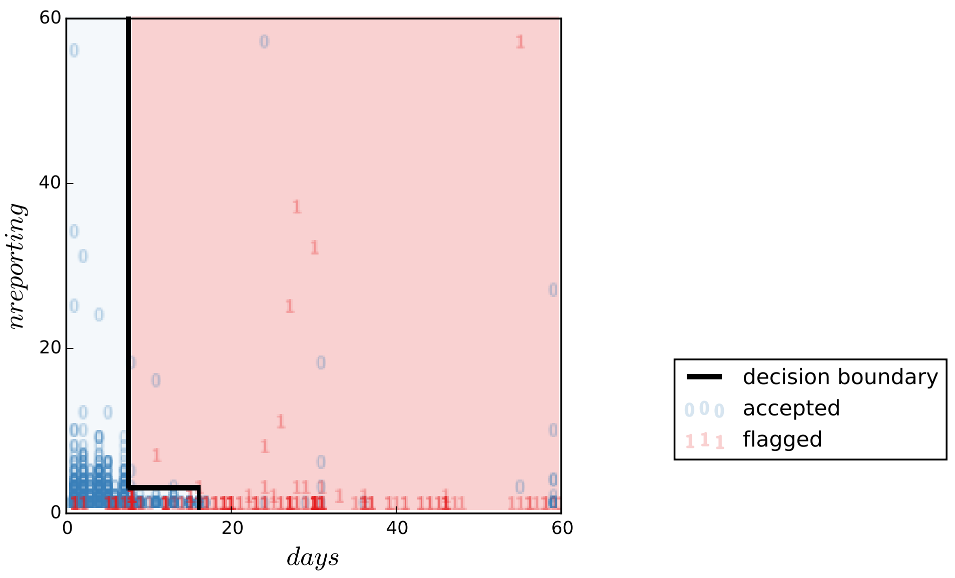
Appendix

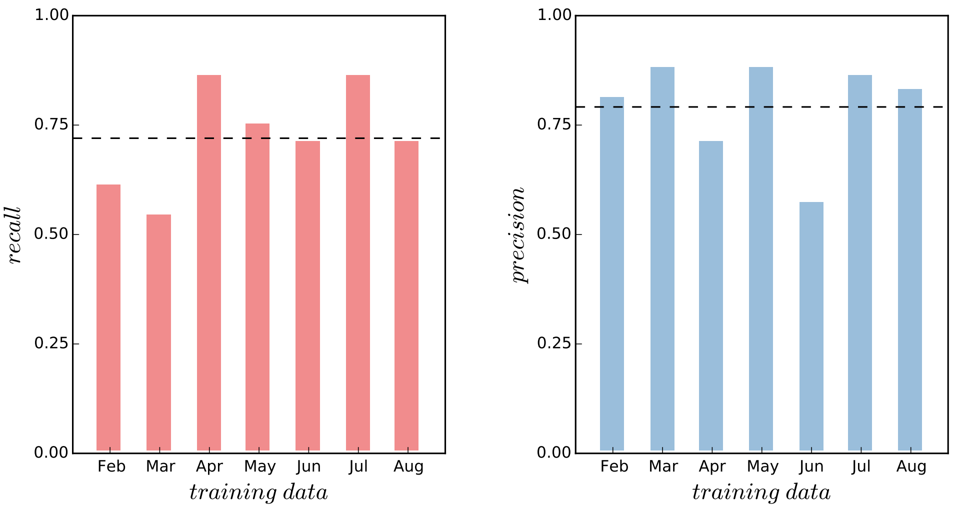
A Figures

Figure A.1: Decision Tree and Decision Boundary



Note: The figure shows an illustration of the bivariate decision tree's decision boundary (solid line). The blue (red) data points represent accepted (flagged) securities. For securities with a days-nreporting combination in the red (blue) area, we predict a flag (acceptance).

Figure A.2: Decision Tree and Decision Boundary



Note: The figure shows the performance of the random forest algorithm in terms of recall (left panel) and precision (right panel) for out-of-sample predictions of measurement errors in securities data, reported in October, 2016. To evaluate a potential influence of the time lag between the collection of the training data and the reporting date of the data for which we make predictions, we use training data from different months. For the first bar of each panel, for example, we only use data from February, 2016 as training data. The figure shows no indication of a systematic negative relationship between the time lag and performance. The dashed lines show the mean recall and precision.

B Derivation of the Logit Model

From an econometric perspective, we can derive the Logit model from a decision model, if we interpret the index function (2) as a latent score that the compiler uses to decide whether to flag security i. We start by assuming that there is an additive, logistically distributed decision error u_i. Then, we get the latent score:

$$z_i^* = \alpha + x_i^T \beta + u_i.$$

If the compiler flags security i for $z_i^* > 0$ and accepts the security otherwise, we get:

$$\begin{aligned} P(y_i = 1|x_i) &= P(z_i^* > 0|x_i) \\ &= P(u_i > -x_i^T \beta | x_i) \\ &= 1 - P(u_i \leq -x_i^T \beta | x_i) \end{aligned}$$

Because we assumed that u_i is logistically distributed, the conditional probability $P(u_i \leq -x_i^T \beta | x_i)$ is the density of the cumulative logistic distribution function $\Lambda(\cdot)$ (sigmoid function) at $-x_i^T \beta$. Hence, we get:

$$1 - P(u_i \leq -x_i^T \beta | x_i) = 1 - \Lambda(-x_i^T \beta).$$

Because of the symmetry of the sigmoid function, this reduces to:

$$\begin{aligned} 1 - \Lambda(-x_i^T \beta) &= \Lambda(x_i^T \beta) \\ &= \Lambda(z_i). \end{aligned}$$

In sum, we have shown that $P(y_i = 1|x_i) = \Lambda(z_i)$ if decisions are based on a latent score with an additive decision error that is drawn from the logistic distribution. In Machine Learning, we put no emphasis on the decision model underlying the Logit but focus on the ability of the model to provide a good fit to the data.

C Using Machine Learning to Close Data Gaps: Simulations

We proceed in two steps to simulate a dataset. First, for each of N data points, we randomly draw a feature value x_i from the normal distribution $\mathcal{N}(\mu, \sigma^2)$ and an unobserved element e_i from the logistic distribution $\mathcal{L}(\mu, s)$. Second, we determine the outcome for each data point with the univariate model:

$$y_i = \begin{cases} 0 & for \quad \alpha + \gamma x_i + e_i < 0 \\ 1 & for \quad \alpha + \gamma x_i + e_i \geq 0 \end{cases} \quad . \tag{1}$$

We end up with a simulated dataset with data points for which $y_i = 1$ and data points for which $y_i = 0$. By constructing the outcome with eq. (1), we implement a structure in the data that the algorithms can learn to predict the outcome.

After constructing a simulated dataset, we evaluate the algorithms' performance in terms of precision and recall using stratified five-fold cross-validation. In the cross-validation procedure, we make out-of-sample predictions for a randomly chosen subset of observations. Essentially, we treat the outcome of the subset as a data gap and fill this gap with the algorithm's predictions. In constructing the performance metrics, we evaluate the success of the algorithms in predicting the missing values, i.e. the share of observations with data gaps for which the prediction equals the unobserved value.

Because the performance metrics also depend on our random draws of x and e, we repeatedly: draw features and unobserved elements, construct a simulated dataset, implement the algorithms, and calculate the performance metrics. This provides us with distributions of the precision and the recall metric.

C.1 Benchmark: Linking the Simulation Results for Data Gaps to the Prediction of Measurement Errors

Simulating a binary variable with data gaps allows us to draw additional conclusions about the performance of machine learning algorithms in the application to measurement errors. This is because we can interpret simulated data points with $y_i = 1$ as "flagged" and simulated data points with $y_i = 0$ as "accepted" observations.

To facilitate drawing conclusions about the performance of machine learning in DQM from the simulation results, we include a simulated dataset with characteristics that are common in the DQM application as a benchmark and show performance in the benchmark (grey dotted lines) in the Figures C.2, C.3, C.4, and C.6.[25] To mirror the fact that measurement errors are a relatively rare event in most datasets, we choose the distributions of x and e and the parameter values in eq. (1) in the benchmark such that the average share of flagged data points is small (15%).[26] Furthermore, human evaluations are usually contingent on a pre-selection of data points from the original dataset. We conjecture that because of this pre-selection, the number of data points is likely in the lower four-digit range and choose $N = 2\,000$ as the number of data points in our benchmark.[27]

Figure C.1 illustrates the recall and precision of predictions for the benchmark simulations. The left panel shows that the median recall is similar for both models. The right panel indicates that the Logit model is slightly superior in terms of precision. Overall, the random forests' performance is more disperse.

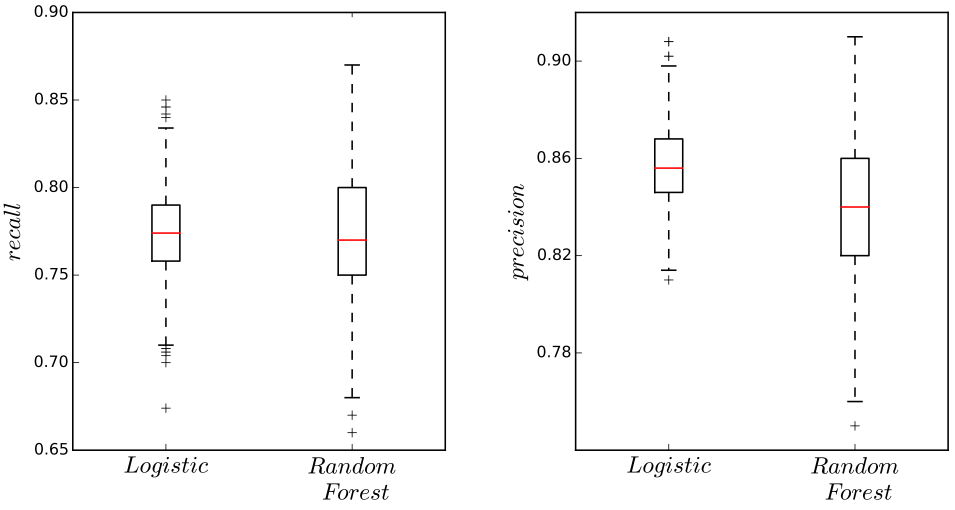
The improvement of the Logit model's performance for the simulated data compared to the application to German securities holdings data is due to the simulated data generating process closely following the Logit's modelling assumptions.

[25] In Figure C.6, the benchmark performance is the performance at the intercept.

[26] In eq. (1), we set alpha to –1.09 and γ to one. For the distributions, we choose $x \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$ and $e \sim \mathcal{L}(\mu = 0, s = 0.17)$. The scale parameter of $s = 0.17$ corresponds to a variance of 0.1. We implement a lower variance for the unobservable element than for the feature in order to give the algorithms a better chance to approximate the result of the simulated decision making process. For a detailed discussion of the performance of the algorithms in environments with a higher variance of the unobservable element, see Section C.4.

[27] We provide an evaluation of the algorithms for different numbers of data points in Section C.3.

Figure C.1: Monte Carlo Simulation – Benchmark



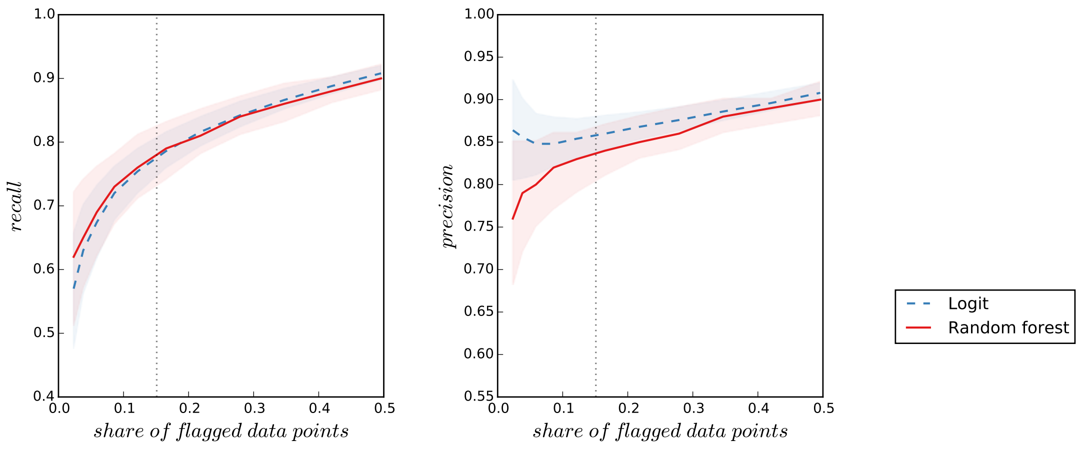
Note: The figure shows the distribution of the recall (left panel) and the precision metric (right panel) for the Logistic regression and the random forest algorithm. Underlying the figure are 1 000 randomly drawn datasets.

C.2 Rare Events (Distribution of Outcome)

To evaluate the performance of the algorithms for different shares of observations with $y_i = 1$, we vary the size of the constant α in eq. (1). Figure C.2 shows the results of the simulations. With an increasing share of observations with $y_i = 1$, i.e. $y_i = 1$ becoming less *rare* as an event, there are gains in recall and precision for both models.

To explain this increase in performance, note that datasets with a small share of observations with $y_i = 1$ contain little information on data points with $y_i = 1$ and lot of information on data points with $y_i = 0$. In terms of the overall information content of a dataset, we can not fully substitute for a lack of information in the first group of data points with additional information on data points in the second group. Therefore, in datasets with a small share of data points with $y_i = 1$, an increase in the share of this group of data points increases the overall information content of the dataset and makes it easier for the algorithm to make predictions. This explains the improvement in the performance metrics illustrated by Figure C.2.

Figure C.2: Monte Carlo Simulations – Share of Flagged Data Points

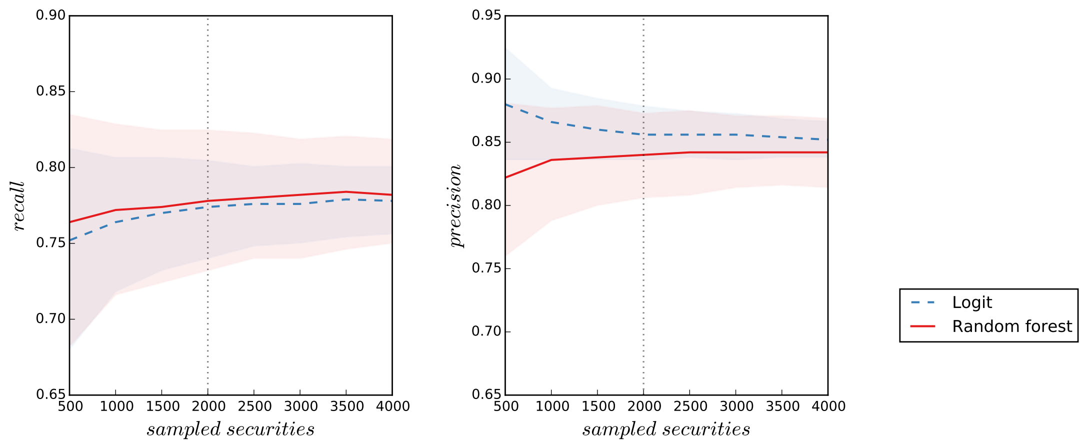


Note: The figure shows the median recall (left panel) and precision (right panel) for the random forest algorithm (red solid lines) and the Logistic regression (blue dashed lines) for 1 000 randomly drawn datasets for each of the shares of data points with $y_i = 1$. The light red and blue areas depict 90% confidence intervals. The grey dotted lines show the benchmark.

C.3 Number of Data Points

One way to increase the accuracy of predictions in datasets with a small share of data points with $y_i = 1$ is to collect more data. Figure C.3 shows the influence of the sample size on the performance metrics. The random forest algorithm shows an improvement in recall and precision over an increase in the number of data points. This is consistent with the idea that the accuracy of estimates increases with a rising number of data points. In contrast, precision decreases with sample size in the Logit model. An explanation for this finding is that in small samples, Logit models underestimate the probability of rare events (Tomz *et al.*, 2003). In our simulated data, this implies a lower probability of false positives in smaller samples and a higher precision at the cost of a lower recall.

Figure C.3: Monte Carlo Simulation – Number of Data Points



Note: The figure shows the median recall (left panel) and precision (right panel) for the random forest algorithm (red solid lines) and the Logistic regression (blue dashed lines) for 1 000 randomly drawn datasets for each of the sample sizes. The light red and blue areas depict 90% confidence intervals based on the normal distribution. The grey dotted lines show the benchmark.

C.4 Data Quality

In the model underlying our simulations, we construct the outcome from a feature x and an unobserved element e. The feature is an observable characteristic of a data point that affects the probability that $y_i = 1$. Our algorithms can use this information to make predictions. The unobserved element, on the other hand, represents characteristics of the data point that affect the propensity of y_i taking the value one but are not observable to us. Because these influences are not observable, the algorithms can not exploit them to make predictions. Consequently, data quality increases with observable feature variation and decreases with variation in the unobservable element. Following this notion, we define:

$$DQ = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2} \tag{2}$$

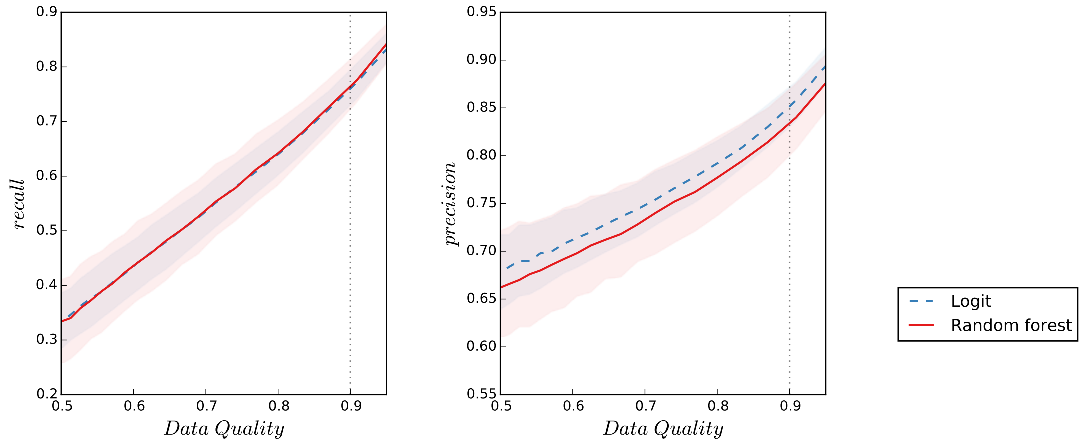
as our measure of data quality, where σ_x^2 is the variance of the observable feature and σ_e^2 is the variance of the unobservable element.[28]

To implement different data qualities in our simulations, we alter the variance of the $\mathcal{L}$-distribution from which we draw the unobserved element.[29] Figure C.4 shows the simulation results. For both, the Logit regression and the random forest algorithm, data quality has a strong positive effect on performance in terms of recall and precision.

[28]The nominator measures observable feature variation and the denominator measures overall variation. Our measure of data quality is similar to the coefficient of determination.

[29]Because tempering with the variance of the unobservable element affects the share of observations with $y_i = 1$, which in turn has an effect on performance (see Figure C.2), we fix the share of observations with $y_i = 1$ at 15% by adjusting the constant parameter α.

Figure C.4: Monte Carlo Simulation – Data Quality



Note: The figure shows the median recall (left panel) and precision (right panel) for the random forest algorithm (red solid lines) and the Logistic regression (blue dashed lines) for 1 000 randomly drawn datasets for each of the different degrees of data quality. The light red and blue areas depict 90% confidence intervals. The grey dotted lines show the benchmark.

The effect of data quality on the accuracy of predictions is smaller if the observable features and the unobserved element are positively correlated. This is because a positive correlation implies that some of the unobserved variation is reflected in the observable feature variation and can be exploited to make accurate predictions.[30]

C.5 Correlated Data Points

One of the assumptions in the derivation of the Logit model is that the data points are uncorrelated. In many applications, this assumptions is likely violated. Therefore, we evaluate the robustness of our results if the unobserved element in eq. (1) is correlated within clusters of data points.

To assess the influence of intra-cluster correlations on the performance of the algorithms, we simulate datasets with correlated data points. To simulate correlated observations, we first set up a block diagonal correlation matrix (C) that captures the correlations within clusters of data points. For clusters comprising two-data points ($n = 2$), for example, we use the block-diagonal correlation matrix:

$$C = \begin{bmatrix} \begin{bmatrix} 1 & \rho_c \\ \rho_c & 1 \end{bmatrix} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \begin{bmatrix} 1 & \rho_c \\ \rho_c & 1 \end{bmatrix} \end{bmatrix},$$

where ρ_c is the intra-cluster correlation. We then use the Cholesky decomposition and get:

$$C = LL^T,$$

where L is a lower triangular matrix. With L, we can finally use a simple transformation to turn the vector of independent unobserved elements $\mathbf{e}$ into a vector of correlated elements $\mathbf{e_c}$ with correlation structure C:

$$\mathbf{e_c} = L^*\mathbf{e}.$$

We find that the effects of the intra-cluster correlations on prediction accuracy are negligible for the Logit regression and the random forest algorithm. From the baseline with independent unobserved elements, a high intra-cluster correlation in the unobserved elements of $\rho_e = 0.9$ decreases recall and precision by less than one percent. The negative impact increases with cluster size but remains below the one percent mark for a moderate cluster size of ten data points.

[30]For inference in an econometric applications of regression models, such correlations with omitted variables are a problem because they bias parameter estimates.

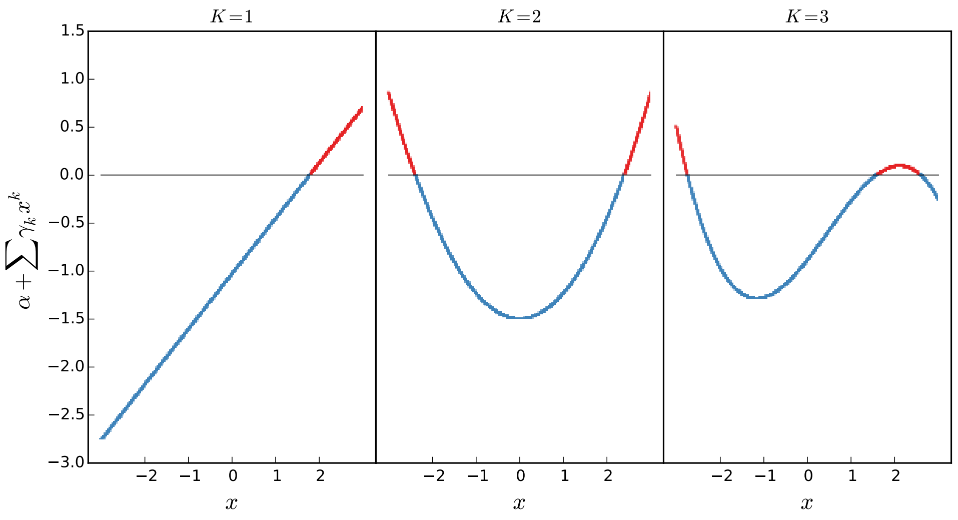
C.6 Nonlinear Feature-Outcome Relationships

So far, the simulations indicated that the Logit model's performance is similar to the performance of the random forest in most applications. This assessment changes, once we introduce datasets with complex structures. To model nonlinear feature-outcome relationships, we use a power-series of order K:

$$y_i = \begin{cases} 0 & \text{for} \quad \alpha + \sum_{k=1}^{K} \gamma_k x_i^k + e_i < 0 \\ 1 & \text{for} \quad \alpha + \sum_{k=1}^{K} \gamma_k x_i^k + e_i \geq 0 \end{cases}, \tag{3}$$

where we draw the γ coefficients from the normal distribution $\mathcal{N}(0,1)$.[31] For a power series of order $K = 1$, we get the benchmark model in eq. (1). With an increase in K, the relationship between feature and outcome becomes nonlinear and increasingly complex.

Figure C.5: Decision Tree and Decision Boundary



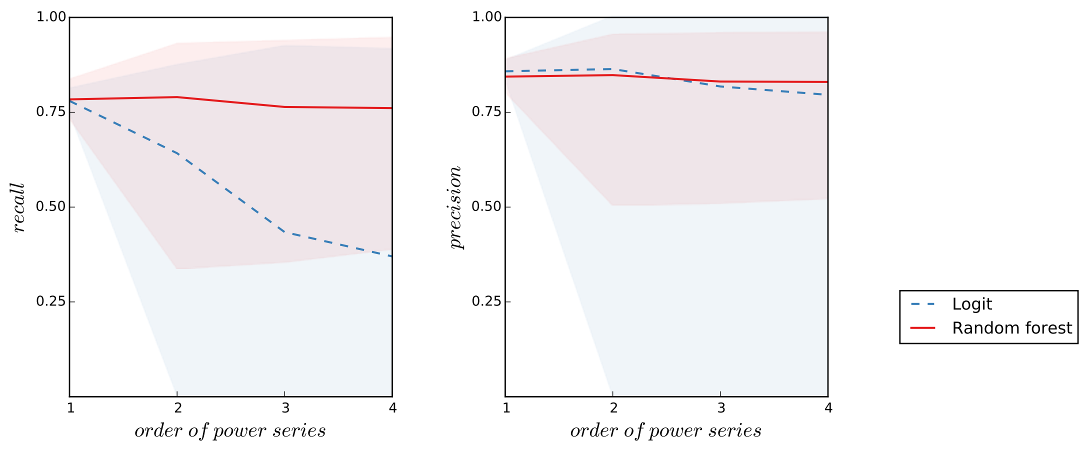
Note: The figure shows an illustration of nonlinear feature-outcome relationships that result from the models with power-series of order K, outlined in eq. (3).

Figure C.5 shows an illustration of nonlinear feature-outcome relationships that result from the models with power-series of order K, outlined in eq. (3). Data points with values below zero (blue) get the outcome zero and data points with values above zero get the outcome one (red). The figure shows that with an increasing order of the power-series, classification into outcome-groups becomes more difficult. Whereas for power-series of order $K = 1$, there is a single critical value of x that separates the groups, for power-series of order $K = 2$ there are two critical values, and for power-series of order $K = 3$ there are three critical values. The higher complexity of the data structure that is reflected in the number of critical values increases the difficulty of classifying data points for the algorithms.

Figure C.6 shows the performance of the Logit model and the random forest in datasets with nonlinear feature-outcome relationships. We find that the good performance of the Logit model deteriorates, once we introduce a nonlinear structure. In contrast, the random forest algorithm seems well equipped to capture the nonlinear relationship. For data generating processes described by high order power series, there is only a small decrease in recall and precision of predictions with the random forest algorithm. At the same time, the accuracy of predictions with the Logit model is very unstable over different random draws of datasets. Taking this instability which is illustrated by the broad confidence bands into account, the random forest algorithm has a clearly superior performance in datasets with nonlinear feature-outcome relationships.

[31] We adjust α so that the share of observations with $y_i = 1$ is not affected by the transformations with the power-series.

Figure C.6: Monte Carlo Simulation – Nonlinear Feature-outcome Relationships



Note: The figure shows the median recall (left panel) and precision (right panel) for the random forest algorithm (red solid lines) and the Logistic regression (blue dashed lines) for 1 000 randomly drawn datasets for each of the orders of the power series. The light red and blue areas depict 90% confidence intervals. The grey dotted lines show the benchmark.