# Fan Chart: The Art and Science of Communicating Uncertainty

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## Abstract

Economic forecasting is an important aspect of policy decision-making. While the baseline forecast is important, a good understanding of the risks, in terms of their bias and uncertainties, surrounding the baseline forecasts is equally important. This paper aims to discuss these risks and uncertainties in two parts. First is on quantifying these risks and uncertainties through the representation of fan charts. In particular, the paper focuses on the technical derivation the balance of risks surrounding the inflation forecasts for Malaysia, incorporating the uncertainty of each of the variables affecting inflation outlook i.e. the risk factors. The paper highlights several methods discussed in the current literature and suggests alternative methodology using financial market volatility indicators and time-varying coefficients to incorporate the uncertainty of the risk factors in a dynamic manner. Second, the paper discusses the assessment of these risks and uncertainties surrounding the forecasts. With the derived probability distributions of the forecasts, vast amount of information can be derived from the fan chart. The paper presents the use of probabilistic assessment to quantify the balance of risks of the forecasts or the likelihood of it falling within certain a range, and discusses the relevance of these analyses for policy making.

Key words: Forecast uncertainty, fan chart, balance of risks, two-piece normal distribution

JEL classifications: C19, C53, C82, E37, E59

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# 1. Introduction

While discussions on uncertainties surrounding economic forecasts have always been an important agenda for central bankers, the debates on the point forecasts usually take the centre stage in policy forums. The assessment and communication of uncertainties surrounding the forecasts, however, gained prominence following the publication of fan charts by the Bank of England in 1996, aimed to improve communication and shift the discussions from the accuracy of the point forecasts to the subjective assessment of risks and uncertainties surrounding the medium-term outlook.

In the past two decades, numerous methodologies have been introduced on the derivation of the fan chart especially on quantifying forecast uncertainty. This paper will build upon the existing literature and contribute by providing variations to existing methodologies in calibrating the degree of uncertainty of the point forecasts. More specifically, this paper focuses on the systematic derivation of fan chart which incorporates the uncertainty of the forecasting model(s) and also the uncertainties and bias of the assumptions used in the forecasting model(s) i.e. the risk factors. The construction of the fan chart is based on the two-piece normal distribution (to take into account the balance of risks surrounding risk factors) and volatility indicators to reflect the uncertainty in forecasting the risk factors, which would in turn affect the uncertainties surrounding the baseline forecasts. Given the dynamic nature of Malaysia's inflation forecasting model which involves lags of both dependent and independent variables, we also propose the use of time-variant coefficients to impute these uncertainties into the inflation fan chart.

While the methodology is fairly statistical in orientation, appropriate subjective judgement is necessary to ensure consistency in the impact of the various risk factors to the forecasts. For example, when constructing the fan charts for both growth and inflation, the assessment on the common risk factors affecting the two variables need to be consistent and in line with theoretical foundations.

The fan chart, depicting the probability distribution of the forecasts, contains vast amount of information on the risk factors affecting the forecasts and the uncertainties surrounding it. Assessment of the fan chart enables policy makers determine the sources of the upside and downside risks, and perform probabilistic assessment of the forecasts being over a range of different outcomes, as well as the likelihood of the forecasts being above, below or within certain threshold levels. This information is very helpful for central banks when deciding monetary policy towards securing stable growth path and maintaining price stability. In particular, the paper also discusses how the different approaches of the probabilistic assessment on the fan chart can promote different discussion on risks and uncertainties surrounding the forecasts and support policy decision-making.

The remainder of the paper is structured as follows. Section 2 provides a brief overview on communication of uncertainty and the methodologies used in constructing fan charts. Section 3 detailed the methodology used to derive the fan charts in Malaysia, including the variations that have been introduced to the existing methodology. This is followed by the assessment of risks and uncertainties surrounding the forecasts using the fan charts in Section 4. Section 5 concludes.

# 2. Literature review

Assessing and communicating uncertainty has often been highlighted by policy makers as a crucial but difficult task. Good understanding of forecast uncertainty is important to provide policy makers with all the information that are available regarding the key economic variables to support forward-looking policies. In addition, well-thought communication of forecast uncertainty is important to help anchor public expectations on the outlook of the economy.[2]

Forecast uncertainty can generally be presented in three ways - 1) quantitative assessment using fan charts where distributions of all possible outcomes are presented; 2) scenario analysis which presents the forecasts for next most probable scenarios using different sets of assumptions; and 3) sensitive analysis where the forecast is estimated based on the changes in one key assumption or shock.

While most central banks may assess uncertainty using all the three methods, fan charts are commonly used as a communication tool for forecast uncertainty. A study by Franta et al (2014) shows that central banks especially the inflation targeters, communicate their inflation forecasts with the help of fan charts. The study also highlighted the different methodologies used by these inflation targeters to produce fan charts, stem mainly from two approaches. The first approach, as implemented by central banks such as the Czech National Bank, is based on past forecast errors assuming a normal distribution of the forecasts. Fan charts constructed in this approach, however, only reflect the uncertainty in the forecasting model(s) and have minimal economic meaning. Therefore, to enrich the economic representation in the fan charts, central banks such as the Bank of England and Risksbank, impute the assessment of the uncertainty of the risk factors affecting the forecasts into the construction of fan charts and allow for asymmetric distribution of the forecasts with the assumption of a two-piece normal distribution.[3] In this approach, the fan chart does not only serve as a communication tool, but also facilitates discussions among policy makers on the impact of the uncertainties of the risks affecting the forecasts.

In the recent two decades, much research has been done in this area, proposing various new methodologies of constructing the fan chart. Franta et al (2014) for instance, proposes that use of Bayesian vector autoregression model (BVAR) to allow for risk asymmetries and to incorporate the expert judgment in the form of a non-negativity constraint on the nominal interest rate. Jordà and Marcellino (2010) proposes the construction of the confidence bands using the joint predictive density to summarise the range of the possible path the predicted variable might follow for a given confidence level.

While calibrated differently, most fan charts represent a density forecasts i.e. a probability distribution of the forecasts at each forecast period. It covers not only the most probable outcome of the variable (baseline forecasts), but also illustrates a range of different possible outcomes at different confidence levels.

## 3. The Science: Methodology in constructing fan chart

The framework on the construction of fan chart discussed in this paper is employing the second approach, built upon the methodologies introduced in Blix and Sellin (1997) and Elekdag and Kannan (2009).

As proposed by the two papers, for the construction of the Malaysia fan charts, the distribution of the forecasts is assumed to be two-piece normal (TPN).[4] This is an important assumption to allow for the

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2 As discussed in Aikman et al (2010).

3 Even though both Bank of England and the Risksbank assume asymmetry in the construction of the fan charts, the methodology employed by both central banks are very different. According to Blix and Sellin (1997), the Bank of England's fan chart is based on a "top-down" approach where the MPC members determine the degree of uncertainty and asymmetry of the fan chart while the Risksbank's fan chart is based on a "bottom-up" approach where the degree of uncertainty and asymmetry of the fan chart is derived from the uncertainty and skew of the risks factors.

4 See John (1982).

asymmetry of balance of risks surrounding the baseline forecasts, which is more realistic and relevant for discussions on the risks and uncertainties surrounding the forecasts in policy forums. Under the TPN, it is assumed that the standard deviation above and below the central projection need not be the same. Therefore, for the construction of fan chart under TPN, the upper and lower standard deviations, $\sigma_1$ and $\sigma_2$, need to be first derived.

For that, three key parameters of the distribution of the forecast at each period need to be computed. First is the central projection i.e. the mode of the variable. Second is the degree of uncertainty of the central projection which will be reflected as the width of the fan chart. And third is the skew of the distributions of the forecasts which will also illustrate the balance of risks of the forecasts.

As commonly used in the literature, the central projection is the baseline forecast which can be obtained from internal forecasting model(s). This is the mode of the distribution and represents the most probable outcome that is anticipated. For Malaysia's inflation fan chart, this is the baseline inflation forecast using all the baseline assumptions in the core forecasting model.

The degree of uncertainty reflects how uncertain we are about the forecast; the more uncertain we are, the wider the width of the fan chart. This, essentially is the forecast error of the baseline. However, as proposed by Blix and Sellin (1997), the degree of uncertainty should also reflect how uncertain we are compared to historical period or previous forecast rounds. This uncertainty can arise from the uncertainty surrounding the key assumptions in the forecasting model(s), identified as risk factors. This, as proposed, can be imputed by adjusting the historical forecast errors with a multiplicative factor which will reflect the relative uncertainties of these risk factors.

The degree of uncertainty of the baseline forecast, $\sigma_y^2$ can therefore be represented as the variance of historical forecast errors[5] of the distribution, $\sigma_e^2$, adjusted for the uncertainties of the risk factors.

$$\boldsymbol{\sigma_y^2 = \sigma_e^2 \; \frac{\sum w_i x_i}{\sum w_i \bar{x}_i}} \qquad \textbf{--- (1)}$$

where $w_i$ is the elasticity[6] of the key assumptions while $x_i$ and $\bar{x}_i$ are the current and historical measure of uncertainty of the risk factors.

In this regard, one of the main contributions of this paper would be on the variation on the measure of how uncertain we are as compared to historical i.e. the derivation of the multiplicative factor. Blix and Sellin (1997) has proposed the use of the variances of the risk factors as the measure of uncertainty. However, in our view this may not necessary reflect the relative uncertainty or difficulties of forecasting the assumptions, but only reflect the historical uncertainty of these factors. We also attempted to replicate the measure suggested by Elekdag and Kannan (2009) using survey-based data or option prices. We found that survey-based data can be too subjective and is highly dependent on the specific analysts' forecasts that are included. While option prices would be a good leading indicator of uncertainty, lack of actively traded options on some of the Malaysian variables becomes a constraint for this method to be implemented for the Malaysian data.

Therefore, this paper proposes a more simplified and practical variation using volatility indicators as a proxy for uncertainties surrounding the assumptions. The volatility indicators used can vary depending on the risk factors. For instance, for inflation forecasts in Malaysia, key assumptions that are subject to high uncertainty, namely the ringgit exchange rate, global oil price and domestic growth, are identified as risk factors to the inflation outlook. The uncertainty of these risk factors are proxied by the respective volatility indicators - 30-day standard deviation for the ringgit exchange rate, CBOE oil volatility index for global oil price and 12-month standard deviation of the leading indicator for domestic growth. The

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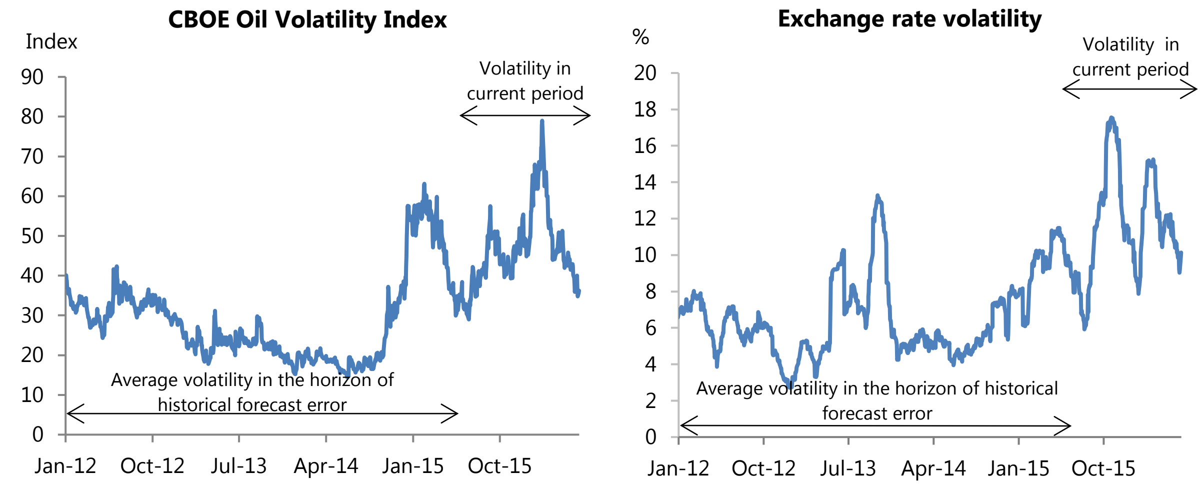
[5] Historical forecast errors used can be the actual forecast errors or the model forecast errors.

[6] Elasticity coefficients of the key assumptions can be obtained from the coefficients of the assumptions in the forecasting model(s) or a separately estimated single equation model.

notion for this is that if these assumptions are more volatile now compared to historical, then the assumptions used are actually more uncertain and hence more difficult to forecast. This will lead to greater uncertainties surrounding the baseline forecasts i.e. a wider fan chart. For instance, in the recent period, the sharp decline in global oil prices and large depreciation in ringgit have led to increased volatility making it extremely difficult to forecast these variables as reflected in the higher relative volatility (Figure 3.1).

Volatility indicators of key assumptions | Figure 3.1




The third parameter which is the skew of the distribution, $\gamma_y$ or the asymmetry surrounding the central projection, is derived as the linear combination of the skew coefficients of each risk factors, $\gamma_i$. [7]

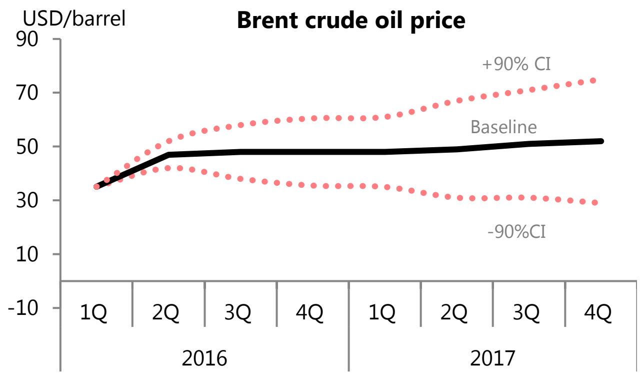
$$\gamma_y = \sum w_i \gamma_i \qquad \text{--- (2)}$$

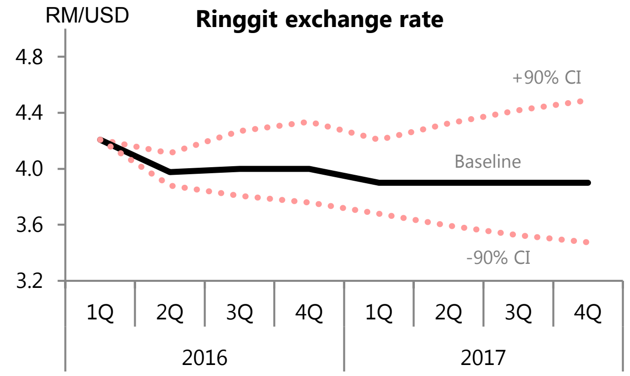
The skew of the risk factors can be obtained from the assessment by individual subject matter specialist on how high and how low each variable can be at each forecasting period. This would be most robust and appropriate as assessment by subject matter specialists would include all readily available information including analysts' survey and option-based data. With the maximum, minimum and mode of the variables, the upper and lower standard deviations as defined in the TPN can be computed.[8] For Malaysia, while the baseline assumptions of the risk factors are imputed in the computation of the baseline inflation forecasts, the maximum and minimum of these risk factors at each forecast period are obtained from the respective teams (Figure 3.2) to derive the skew of the risk factors that should be imputed into the skew of the fan chart based on (2).

[7] As proposed by Blix and Selin (1997), $\gamma = \sqrt{\frac{2}{\pi}}(\sigma_2 - \sigma_1)$ where $\sigma_2$ and $\sigma_1$ are the upper and lower standard error as defined in the TPN distribution.

[8] With the assumption that the maximum is the upper 95% confidence level and the minimum is the lower 95% confidence level, $\sigma_2$ and $\sigma_1$ can be computed using the usual normal distribution formula.

Balance of risks of key assumptions Figure 3.2




Another contribution of the paper is on the type of elasticity used to impute the uncertainties and skew of the risk factors into the fan chart. We propose the use of time-variant coefficients to reflect the dynamic effects of the different variables on the forecast variable. Time-variant coefficients i.e. the impact multiplier which changes over time, can be computed using the autoregressive distributive lag (ARDL) model.[9]

For the case of Malaysia, a simple single-equation forecasting model based on New Keynesian Philip Curve is estimated to derive the coefficients of each risks factor.[10]

$$\pi_t = \alpha_1 \pi_{t-1} + \beta_1(y_t - y_t^p) + \beta_2(\pi_t^* - e_t) + \beta_3(c_t - s_t) + \varepsilon_t \qquad \text{--- (3)}$$

where y refers to GDP, $y^p$ is the potential output, $\pi^*$ is the imported inflation, $\boldsymbol{e}$ is the nominal effective exchange rate, $\boldsymbol{c}$ is the commodity prices and $\boldsymbol{s}$ is the ringgit exchange rate against the US dollar. The time-variant elasticities of these risk factors are estimated using the ARDL(1,0) model.[11] Specifically, at period *t+h,* the elasticity for risk factors *i*, is

$$w_{i,t+h} = \alpha_1^h \beta_i \qquad \text{--- (4)}$$

The coefficient $w_{i,t+h}$ varies depending on the number of forecast period h ahead.

With the properties of the TPN distribution, and the three key parameters, the lower and upper standard deviation, $\sigma_1$ and $\sigma_2$, can be derived (See Appendix for technical details) and the lower and upper values of the confidence intervals, $z_1$ and $z_2$, can be computed:

9 The time-variant coefficients are obtained by expressing the ARDL model in the moving-average representation.

10 The single-equation model illustrated is the simplified version of the internal forecasting model which contains richer lag terms for both endogenous and exogenous variables and also the wage-productivity factor and imported inflation.

11 In this simplified single equation, ARDL (1,0) is used. This will vary depending on the lags of the dependent and independent variables in the forecasting model. A more complex ARDL model would introduce a richer dynamics.

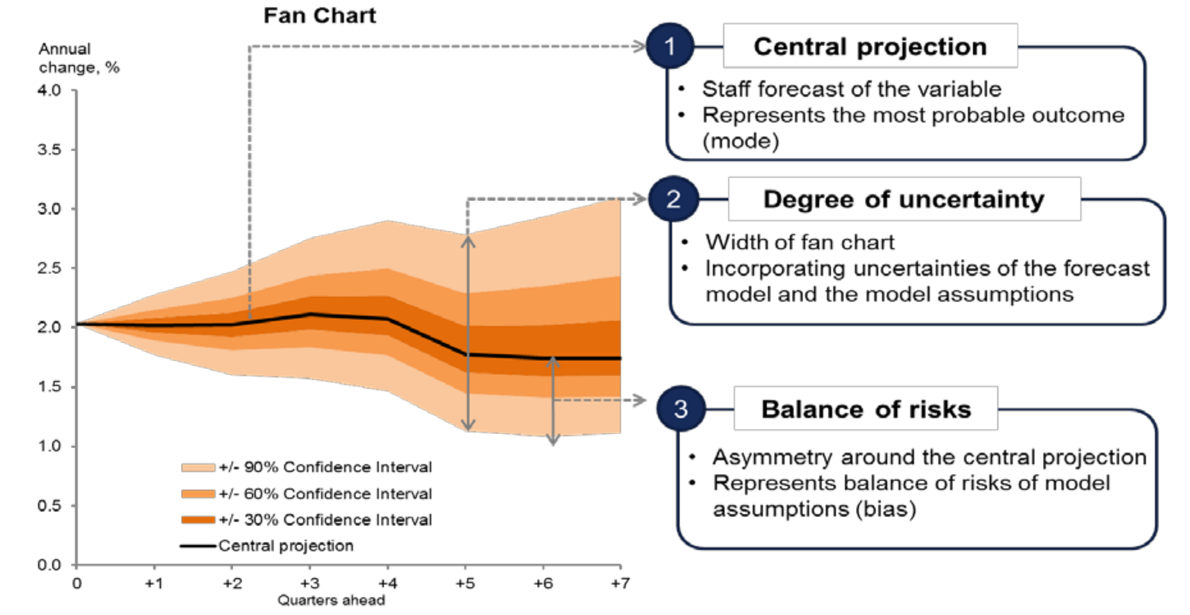
$$z_1 = \mu - \frac{\sigma_1}{\sigma_2}(z_2 - \mu) \qquad \text{--- (5)}$$

$$z_2 = \mu + \sigma_2\, \Phi^{-1}(\tfrac{1+q}{2}) \qquad \text{--- (6)}$$

where $\Phi^{-1}$ is the inverse of the standard normal distribution and q is the level of confidence.

The fan chart is usually plotted using different shades of colour to represent the different confidence intervals. The widest band usually reflects the 90% confidence interval i.e. with 90% probability that the band captures the true value of the forecast variable.

Three key parameters of the fan chart — Figure 3.3




The construction process of the fan chart has thus far been fairly systematic with little subjectivity required. However, necessary judgement is needed to ensure robustness and consistency between the balance of risks of the assumptions i.e. the risk factors and that the economic relationships between the assumptions and the forecast remain intact. Like any forecasting process, it is an art.

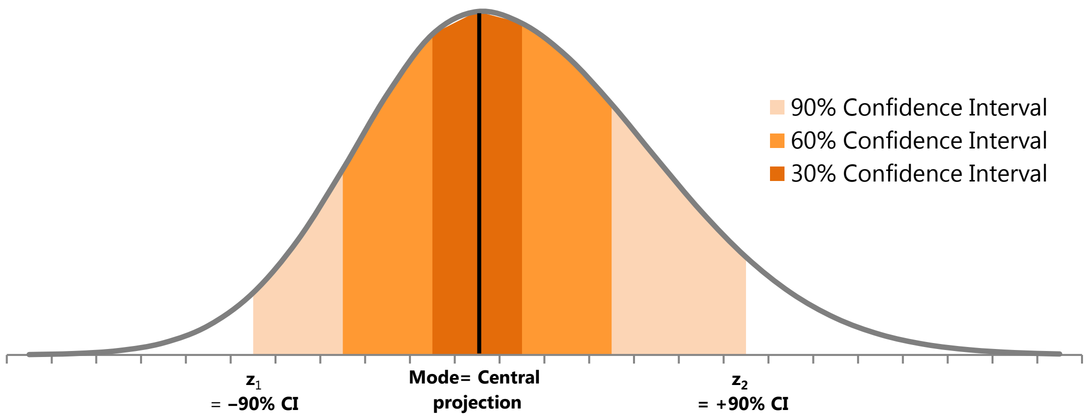
## 4. The Art: Assessment of the fan chart

There are abundant of information that can be derived from the fan chart. With the central projection as the staff's baseline forecasts, the width of the fan chart represents the degree of uncertainty surrounding the baseline forecast at a specific point in time; the wider the fan chart, the greater the uncertainty of the forecast. Therefore, as the forecast horizon increases, the width becomes larger as the baseline forecasts become more uncertain. Based on the method of construction, the width incorporates two types of uncertainties - the uncertainty of the forecasting model(s) and the uncertainties surrounding the assumptions used in the forecasts.

The skew of the fan chart reflects the balance of risks to the central projection. If the distribution is positively skewed i.e. upside bias, the width above the central projection will be larger than the width below. In another word, the probability of the true value to fall above the central projection is higher. Given the methodology above, the skew also reflects the net impact of the balance of risks of the risks factors on the forecasts. Therefore, the balance of risks of the forecasts can be attributed to the relevant key assumptions.

At each forecast period, a cross-section of the fan chart represents the probability distribution of the forecast at t+h, with mode $\mu_{t+h}$, which is the baseline forecast at h periods ahead. As the widest band of the fan chart usually reflects the 90% confidence interval, the remaining 10% that is not captured by the fan chart represents the likelihood of tail-end risks events. These risks are the outlier events that are assessed to occur with very low likelihood.[12]

Probability distribution of forecast at *t+h* — Figure 4.1




As distribution of the forecasts at each period is known, the probabilistic assessment on the uncertainties surrounding the forecast can be performed to assess the balance of risks of the outlook at each period.

[12] Nonetheless, it should be noted that the impact of these events are usually large and may have prolonged impact on the forecast variables and/or macroeconomy.

In Malaysia, the main purpose of constructing the fan chart is to create a framework that assesses the risks and uncertainties to growth and inflation in a more systematic manner. Specifically, it is important to ensure that the assessments on the common risks factors of both growth and inflation are the same to maintain the consistency across the analyses done for policy making discussions.

The assessment of the fan charts can be done in two manners. One is based on the probability of the forecasts falling above or below the baseline projection, which reflects the skew of the distribution. Tracking the changes in the skew across the forecast rounds will provide us with the changes in balance of risks of the risk factors. Second, is based on the probability of the forecast falling above or below a certain range such as the inflation target or official forecast range. For instance, given the inflation forecast range for Malaysia in 2016 is 2-3%, if the probability of inflation falling below 2% for 2016 is larger than the probability of inflation being above 3%, there is larger downside risks for inflation with respect to the forecast range. More importantly, the methodology of deriving the width and the skew of the fan chart also allows us to determine the driving factors behind the changes in balance of risks compared to previous forecast rounds. Linking the uncertainty and skew of the fan chart systematically to the uncertainties and bias of the risk factors enables us to trace and explain the evolution of the risks and uncertainties surrounding the forecast. The assessment in this manner is more meaningful for central banks as it provides the MPC with the likelihood of growth and inflation breaching a certain steady growth path that may warrant policy actions.

It is also important to note that while the confidence levels around the central projection can be computed given the probability distribution, the exact level of the risk factors that lead to the specific confidence level is not able to be determined as it is a permutation of the different levels of the key assumptions. Therefore, the probability of a certain scenario occurring (with specified levels for each assumptions) will not able to be determined using the fan chart. This assessment, however, can be supported using scenario analysis and/or sensitivity analysis.

## 5. Conclusion

The fan chart is an important tool for policy makers especially at times of heightened uncertainty to illustrate the key risks and uncertainties affecting the outlook. Central banks like Bank Negara Malaysia do not only use the information imputed in the baseline projections for policy decision-making but also consider all the other information reflected in the fan chart. Central banks need this information to get a good gauge on the outlook of the economy to ensure policy actions are pre-emptive and timely. In addition, for central banks that publish the fan chart, it is also an important tool to help communicate to public the views of the central banks on the outlook of the economy. Bank of England for instance, illustrated the increased in uncertainty of the domestic economy via the wider fan charts published in its quarterly reports.[13]

Thus, the methodology of computing the fan chart is pinnacle in ensuring the type of information that can be derived from the fan chart. In Malaysia, the central bank recognises the importance of identifying the sources of uncertainty and hence imputed the uncertainties that are identifiable i.e. the risk factors into the construction of the fan charts. While there are many methodologies that have been proposed, most of these methods may be difficult to implement due to data availability and other constraints. Therefore, this paper attempts to provide a more simplified and adaptable variation that can be customised according to the risks specific to different economies such as emerging market economies that typically experience data availability issues.

As the literature in this area continues to grow and the methodology of deriving the fan chart continues to evolve, policy makers need to be cautious of the information that is being understood by the public. Policy makers and/or forecasters need to educate the users of the fan chart to ensure that they understand the information that can be derived from the fan chart so that the main objective of the fan chart can be achieved.

[13] Various Bank of England Inflation reports. http://www.bankofengland.co.uk/publications/Pages/inflationreport/default.aspx

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## Appendix: The Two-piece Normal Distribution

The two-piece normal (TPN) distribution has been widely used in the literature on fan charts due to its asymmetric shape and the relative ease in computing the cumulative density function.

The probability density function of the TPN can be thought as the combination of two halves of two normal distributions with same mode μ but different standard deviations[14], $\sigma_1$ and $\sigma_2$. The probability density function for the TPN is therefore:

$$f(x) = Aexp\left\{-\frac{(x-\mu)^2}{2\sigma_1^2}\right\} \quad for\ x \leq \mu$$

$$Aexp\left\{-\frac{(x-\mu)^2}{2\sigma_2^2}\right\} \quad for\ x < \mu$$

where $A = \sqrt{\frac{2}{\pi}}\,(\sigma_1 + \sigma_2)^{-1}$ is a constant of proportionality introduced to ensure that the distribution is continuous and integrates to one.

The key properties of the distribution as introduced by John (1982) are:

Mean: $E(x) = \mu - k(\sigma_2 - \sigma_1)$..........................................(1)

Variance: $V(x) = \sigma_1\sigma_2 + (1-k^2)(\sigma_2 - \sigma_1)^2$.............................(2)

Skew: $\gamma(x) = k(\sigma_2 - \sigma_1)[(2k^2 - 1)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2]$............(3)

where $k = \sqrt{\frac{2}{\pi}}$.

Therefore, with the mode, variance and skew of the distribution, equation (2) and (3) can be solved for $\sigma_1$ and $\sigma_2$.

From the probability density function, the probability of the being between any two points, $L_1$ and $L_2$ i.e the area under the chart can be computed.

$$P(L_1 < X < L_2) = \int_{L_1}^{L_2} f(x).dx \; = 2(\sigma_1 + \sigma_2)^{-1}[\sigma_2\Phi\left(\frac{L_2 - \mu}{\sigma_2}\right) - \sigma_1\Phi\left(\frac{L_1 - \mu}{\sigma}\right) + (\frac{\sigma_1 - \sigma_2}{2})]$$

where $\Phi(.)$is the cumulative distribution function of the standard normal.

[14] If $\sigma_1 = \sigma_2$, then the distribution collapses to a balanced normal distribution where $\mu = mode = mean$.