Probability-of-default curve calibration and the validation of internal rating systems

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Abstract

The purpose of this article is to present calibration methods which give accurate estimations of default probabilities and validation techniques for evaluating calibration power. Applying both these aspects to real data produces accurate verification and conclusions. The empirical analysis was based on individual data from different sources (from the years 2007 to 2012), i.e. from Prudential Reporting, The National Court Register, AMADEUS, and Notoria OnLine.

This article deals with the issue of rating system calibration, i.e. allocation of rating classes to entities in order to ensure that the calibration power of the division created is as high as possible. The methods presented can be divided into two groups. The first contains methods for approximating conditional score distributions for defaults and entities with a good financial standing to a parametric distribution which can be expressed with the use of a density function and distribution functions. The second group covers a number of variants of regression on binary variables denoting the default status of a given company. The use of k-fold cross-validation and repetition of calculations for different master scales and differing data sets means that the results should be highly robust against distribution of the variable in the training set.

Keywords: probability of default, calibration, rating

JEL classification: C13, G24

Contents

Introduction

Appropriate risk assessment is one of the most important aspects of the activities of financial institutions. In 1999, the Basel Committee on Banking Supervision published several proposal for changes to the current regulations in terms of the capital adequacy structure of financial institutions, which contributed to the preparation of the New Capital Agreement, known as Basel II. The main modification proposed was the reinforcement of the risk management process in the banking sector. One of the key changes in this area was the introduction of the possibility of internal risk management, and therefore the determination of minimal capital requirements. In particular, the bank can select among three approaches. The first of them, which is a continuation of the approach contained in the previous regulation, obliges the bank to maintain the ratio between the minimum capital requirements and the sum of risk-weighted assets at the level of 8%, where the weights are determined by the national regulatory body. As part of the second approach, called IRB (*Internal Rating Based*), the bank is obliged to prepare an internal estimation of the likelihood of the obligation not being fulfilled (*probability of default*). The other risk parameters, such as the loss coefficient arising from the failure to fulfil the commitment (*loss given default*) and the exposure at the time of insolvency (*exposure at default*) are provided by the regulatory body. The third and at the same time the broadest approach, known as Advanced IRB, enables banks to estimate all risk parameters.

Each bank is obliged, using one of the IRB approaches, to estimate the likelihood of insolvency for each loan granted. A popular method of achieving this is credit scoring. Financial institutions can use external scoring or rating assessments (*external rating approach*); however, they are applicable to only a small number of the largest business entities. In the vast majority of cases an internally developed risk assessment method (*internal rating approach*) is used. The use of the bank's own rating boards, called master scales, is a common practice. Entities with low risk levels are grouped together and assigned to one rating class. Each rating class has a top and bottom threshold expressed by the default probability, as well as an average value. The allocation of a given entity to one of the rating classes automatically determines its *default probability*, which is equal to the average value for the given class. The number of classes depends on the bank's individual approach; however, at least seven classes are required for solvent entities. Usually, lower probability values are assigned to the "upper" classes, which are denoted by digits or appropriate abbreviations, such as "AAA". This is, therefore, a process of discretization of the default probability estimations. On the one hand this approach causes a certain loss of accuracy; on the other it has several important benefits. Firstly, it facilitates further aggregate analysis, simplifies the reporting and model monitoring process. Secondly, it allows for expert knowledge to be used by way of relocation of entities to higher or lower rating classes.

The *default* probability determination model and the master scale are known as the rating system. This is used to forecast the default probability of each entity, expressed by a rating class. There are two approaches used to establish a rating system. The first, called PIT (*point in time*), assumes maximum adjustment to changes resulting from the business cycle. The default probability estimation includes individual and macroeconomic components. A high level of migration of units to lower classes is expected in a period of economic growth, and to higher classes at a time of crisis. The second approach, known as TTC (*trough the cycle*), maximally reduces the influence of the macroeconomic component. All changes are only determined by changes in the individual estimation component, while the percentage share of entities should remain relatively unchanged (Heitfeld, 2005). There is also a broad range of intermediate hybrid approaches, which include individual elements of both the above methods.

This article deals with the issue of rating system calibration, i.e. the allocation of rating classes to entities in order to ensure that the calibration power of the division created is as high as possible. At first, the shape of the function depicting the transition of score into default probability is estimated. The methods presented can be divided into two groups. The first contains the methods for approximating the conditional score distributions of defaults and entities with a good financial standing to their parametric distribution, which can be expressed with the use of a density function and distribution functions. Taking into consideration that these distributions are usually skewed either rightward or leftward of the median, only those types of distribution that allow for a description of both density function asymmetry variants with the use of the appropriate parameters (e.g. asymmetric Gauss distribution, asymmetric Laplace distribution, skew normal distribution and scaled beta distribution) are described. On this basis, and with the use of Bayes' formula, it is possible to define PD values. These methods are recommended for the purpose of calibrating a score which is not interpreted as a probability (e.g. the score as a result of discriminant analysis).

The second group covers a number of variants of regression on binary variables denoting the default status of a given company. These are universal methods which facilitate the calibration of a score which can be interpreted as a probability (e.g. the score as a result of logistic regression). Firstly, apart from the most popular transition functions (probit and logit), others have also been suggested: cauchit and the complementary log-log function. Another alternative is the application of Platt scaling and Box-Cox transformations to the explanatory variable. The polygonal curve model can also be used for each regression, and a further option is the quasi-moment-matching method and isotonic regression. Based on the probability values found, the rating is allocated with the use of a master scale with set threshold values for individual classes.

Regarding the fact that the main purpose of using the rating system is risk assessment determined in terms of probability, the verification of calibration power is the main part of validation. As noted by Blöchlinger and Leippold (2006), inappropriately calibrated probabilities result in significant losses, even if differences seem to be small. This is a difficult task, because it is impossible to assess the real probability for every assessed unit; in statistical terms it is a latent variable. To solve this problem, it is necessary to use rating classes that are intended to include units of similar risk. Therefore, calibration validation is a comparison of valued *ex-ante* probabilities and the observed *ex-post* indicators of insolvency for particular classes, as well as the verification of the significance of statistical differences between those indicators.

While validating model calibration, it is worth testing the calibration power of individual classes, as well as the entire rating system. Testing individual classes mainly involves the binomial test, with all its modifications. A crucial aspect here is to take into consideration the *default* correlation between entities. Therefore three additional tests will be carried out: the *one-factor-model,* the *moment matching approach* and *granularity adjustment*. While assessing the calibration power of the rating system on the basis of multiple tests carried out on individual classes, the error of decreasing the value of the established *p-value* level is made. One solution to this problem is to use the Bonferroni or Sidak correction. Another method is to follow the Holm, Hochberg or Hommel procedures. The most popular test of the entire rating system is the Hosmer-Lemeshow test, which involves examination of the differences between observed and the estimated default probability. For the purpose of this research, the Spiegelhalter and Blöchlinger tests were also used; these facilitate verification of the calibration power achieved in a different manner to the Hosmer-Lemeshow test.

The basic purpose of this article is to present calibration methods which provide accurate estimations of default probabilities and validation techniques of calibration power. Using those both aspects on real data provides accurate verification and appropriate conclusions. The subject matter of this article is important and actual, as there is no consensus among practitioners regarding the selection of calibration methods and ways of testing them, so the comparison of methods constitutes a significant added value. According to the author's best knowledge, some methods will be used for the first time with regard to rating systems calibration. This also confirms the significant value of this work.

Two main research questions will be addressed. The first seeks to verify whether there is a calibration method that gives estimations of probabilities of significantly better quality in logistic regression than others. One of the main assumptions of the new regulation was that banks should be free to select a method for insolvency probability estimation. Logistic regression is the method usually used by banks. This method is also recommended by some Analytics[1]. Comparing the precision of estimates obtained by means of different approaches will provide an unambiguous evaluation of the quality of the models used.

The second question concerns rating system structure: does the number of rating classes really impact calibration quality? Breinlinger et al. (2003) proved that by increasing the number of classes, at other fixed parameters, the amount of minimum capital requirements decreases. They also found that with a large number of classes it is impossible to meet assumptions concerning the monotonicity of insolvency probability. This might be caused by a significant worsening of estimation calibration. A detailed determination regarding the number of rating classes used is a rarely mentioned but crucial problem, especially for the rating system structure process.

The conclusions presented in this article are mainly directed to banking sector employees concerned with identifying the best way to calibrate internal credit risk systems. A detailed presentation and comparison of different methods allows for a comparison of particular approaches and the selection of the best one. The part of this work that concerns the testing process may be a valuable source of information for validators of risk models.

Data description

The empirical analysis was based on the individual data from different sources (from the years 2007 to 2012):

– Data on banking defaults are drawn from **Prudential Reporting (NB300)** managed by Narodowy Bank Polski. The Act of the Board of the Narodowy Bank Polski no. 53/2011, dated 22 September 2011 concerning the procedure for and detailed principles of the handing over by banks to the Narodowy Bank Polski data, indispensable for monetary policy, for the periodical evaluation of monetary policy, evaluation of the financial situation of banks and banking sector risks.

– Data on insolvencies/bankruptcies come from a database managed by The **National Court Register (KRS)**, which is the national network of official business register.

– Financial statement data (**AMADEUS (*Bureau van Dijk*); Notoria OnLine**). Amadeus (Bureau van Dijk) is a database of comparable financial and business information on Europe's biggest 510,000 public and private companies by assets.

[1] CRISIL Global Research & Analytics, *Credit Risk Estimation Techniques*.

Amadeus includes standardized annual accounts (consolidated and unconsolidated), financial ratios, sectoral activities and ownership data. A standard Amadeus company report includes 25 balance sheet items; 26 profit-and-loss account items; and 26 ratios. *Notoria OnLine* is the standardized format of financial statements for all companies listed on the Stock Exchange in Warsaw.

The following sectors were taken from the Polish Classification of Activities 2007 sample: section A (Agriculture, forestry and fishing), K (Financial and insurance activities). The following legal forms were analyzed: partnerships (unlimited partnerships, professional partnerships, limited partnerships, joint stock-limited partnerships); capital companies (limited liability companies, joint stock companies); and civil law partnerships, state owned enterprises, foreign enterprises.

For the definition of the total number of obligors the following selection criteria were used:

- the company is existent (operating and not liquidated/in liquidation) throughout the entire respective year;
- the company is not in default at the beginning of the year;
- the total exposure reported to be at least 1.5 Mio EUR for each reporting date.

The dataset, after its initial preparation and while keeping only those observations upon which the model can be based, contained 5091 records. However, the number of observations marked as "*bad*" was 298 (Table 1).

General statistics for 2012

Source: author's own calculation

Table 1

Number of Obligors	Thereof Insolvent	Thereof defaulted	Insolvency Rate	Default rate
5091	28	298	0,55%	5,85%

The preliminary stage was the implementation of the scoring model with the use of the *Nehrebecka approach (2015)*. On the basis of this model, point scores were achieved for each enterprise. The value of the *score* was interpreted by an undefined scoring model. This approach is based on the assumption that this dependence (which is not necessarily linear) is monotonic; that is, that a lesser value correlates to a higher probability of the default state.

The *score* distribution was rescaled so that the values fell between 0 and 1. This is caused mainly by the application of the root in certain calibration methods. With this modified variable *score* and a variable binary defining the *default* state, a validation of the discriminatory power of the model was carried out. AUC, AR, the Pietra index, the *Information Value* index, and the Kolmogorov-Smirnov test were used. A master scale employed by KBC bank (9 classes) in 2011, for corporate clients in Pekao bank (9 classes) in 2013, in Millenium bank (14 classes) in 2011, and in ING bank (19 classes) in 2012 was used. From the research point of view, it is extremely interesting to ascertain whether the quality of the calibration depends on the master scale.

Calibration and verification using a test for the whole rating system

The results presented present the assessment of the function of the transformation of the *score* into a *default* probability, which was used to create a rating for the units. Special attention should be paid to the shape of the function for low probability values, as typically the majority of classes used in master scales cover the first 20% of the probability of the *default* state. The relatively small differences in models for low probabilities will result in relatively large changes in the rating structure. The tests which were carried out allow us to assess the various calibration methods.

1. The quasi-moment-matching method [Tasche, 2009]

In order to use the *quasi-moment-matching* method, the target PD value must be established, assumed as the participation of the units in the *default* state of the total units, and the target value of the AUC (*Cumulative Accuracy Profile*), which has been determined according to the following formula:

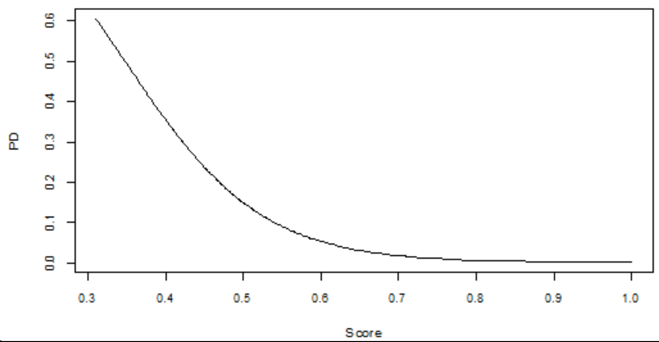
$$AUC = (n_D n_{ND})^{-1} \sum_{i=1}^{n_D} \sum_{j=1}^{n_{ND}} \delta_{y_j}(x_i) \qquad [1]$$

where: n_D - the number of defaulted borrowers, n_{ND} - number of surviving (non-defaulted) borrowers, $\delta_w(z) = \begin{cases} 1 & where \quad z \leq w \\ 0 & \text{where} \quad z > w \end{cases}$.

To numerically estimate the parameters the Broyden, or Newton, was used. The calculated probability of default is shown in Graph 1.

The transition of the score into probability of default using the quasi-moment-matching method

Source: author's own calculation — Graph 1



2. Methods of approximating parametric distribution

2.1. Skew normal distribution

The approximation of conditional *score* distributions in families with symmetric distributions is a fallible method. A better approach is calibration of asymmetric distributions. This approach has not been universally applied in the context of the development of *score* mapping functions for conditional PD distributions. In this paper, three methods of estimating the values of these parameters were used. The first of them (MLM1) used the approach described by Dey (2010), involving the numeric estimation of the parameters μ, σ and λ of the *skew normal distribution*:

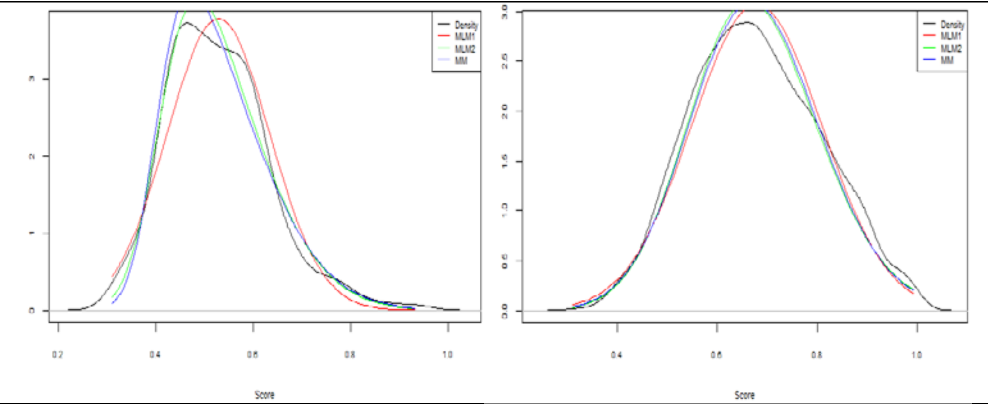
$$f(x) = \frac{2}{\sigma} * \phi\left(\frac{x-\mu}{\sigma}\right) * \Phi\left(\lambda * \frac{x-\mu}{\sigma}\right) \qquad [2]$$

where: $\mu -$ mean, σ – variance, λ – skew, $\phi()$ – density of standard normal distribution, $\Phi(\quad)$ – distribution.

A major problem with the numeric estimation of parameters is the high absolute values of the estimators. This means that the density function is very „steep", which is liable to cause so-called *overfitting*, excessive matching of the model to the data. To avoid this, the starting point should be a low value and a maximum of two steps should be allowed for the iteration of the algorithm. The second method (MLM2) involves the estimation of parameters for which the sum of the density function logarithm is the highest. This is the maximum likelihood method proposed without additional adaptations, as in Dey. Theoretically, the parameters of both methods should be equal. The third method is based on the method of moment (MM).

Empirical density and skew normal density function

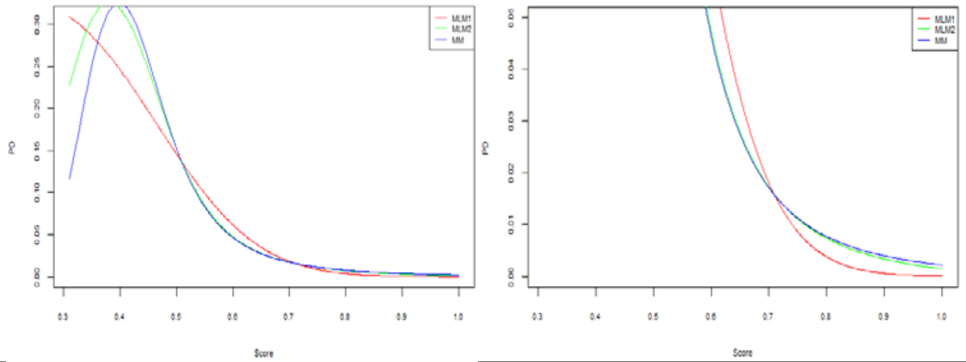
Source: author's own calculation — Graph 2



The distribution parameters for units in the default / non-default obtained by maximum likelihood method proposed proposed by Dey and calculated numerically are not equivalent. An empirical density chart with estimated parameters is presented in Graph 2, whereas a chart of the transformation from score into PD is presented in Graph 3. The bimodality of the distribution for default units causes a significant divergence between the estimated and empirical distribution, caused by the lower values of their averages. In the case of a single mode distribution observed for units not in the default state, this problem does not occur, and the estimated distribution is nearer to the empirical equivalent. When analysing the lowest values of probability for high score values, the greatest probability is observed for the method of moment (MM), and the lowest for the MLM1 method. There is a point at about 0.72 from which the above relation is reversed. When the shape of the graph is known along the whole axis from 0 to 1, it can be seen that another point exists at which the relation is reversed, equal to 0.52. In the analysis of the highest PD values for MLM2 and MM, an increase in probability is observed in the score value below 0.4. This is most likely a result of overfitting.

The transition of score into probability of default using the skew normal distribution method

Source: author's own calculation Graph 3



2.2. Scaled beta distribution

In order to apply the method to the investigation of the distributions (*default / non-default*) using scaled beta distribution, a numeric method was employed for the calculation of the set of non-linear equations (MLM1); the second method (MLM2) was used in a numeric search for the maximum likelihood function, while the third and fourth methods (MM1 and MM2) were applied to the estimation of parameters using the method of moment. These two methods were distinguished by different estimations of the length of the segment of subsets in the distribution $(b-a)$ based on two methods:

$$\widehat{(b-a)} = \sqrt{\widehat{Var(X)}} * \sqrt{6 + 5*(\hat{\alpha}+\hat{\beta}) + \frac{(2+\hat{\alpha}+\hat{\beta})*(3+\hat{\alpha}+\hat{\beta})}{6} * \widehat{\gamma_2}} \quad [3]$$

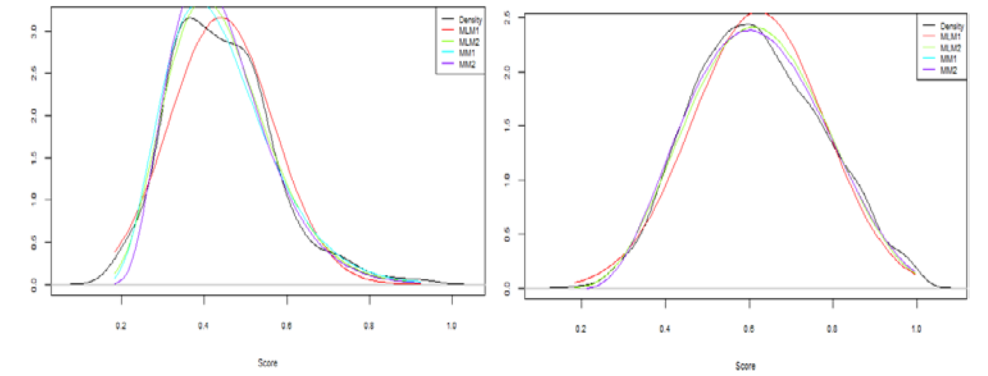
$$\widehat{(b-a)} = \frac{\sqrt{\widehat{Var(X)}}}{2} * \sqrt{(2+\hat{\alpha}+\hat{\beta})^2 * \widehat{\gamma_1}^2 + 16*(1+\hat{\alpha}+\hat{\beta})} \quad [4]$$

where: empirical skew $(\widehat{\gamma_1})$, empirical kurtosis $(\widehat{\gamma_2})$, parametrs $(\hat{\alpha}, \hat{\beta})$.

An empirical density chart with estimated parameters is presented in Graph 4, whereas a chart of the transformation from score into PD is presented in Graph 5.

Empirical density and scaled beta density function

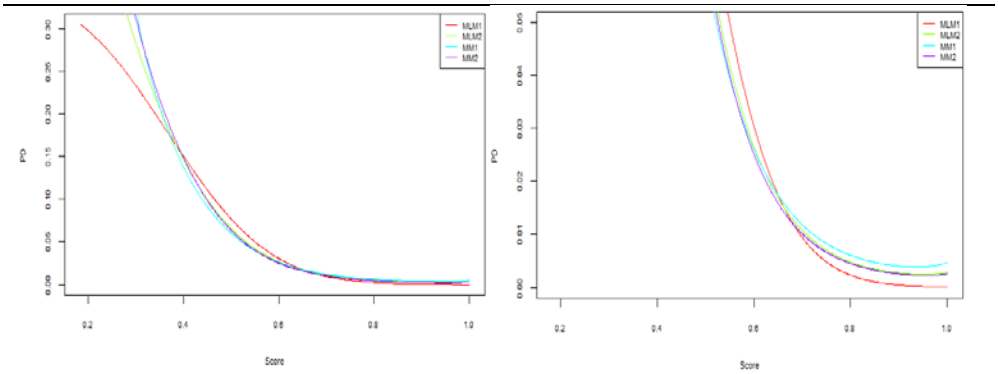
Source: author's own calculation Graph 4



Typical shapes display functions with estimated parameters, as in the maximum likelihood function and the method of moment. Here it is worth noting the difference between the initial values of probability of default. For large *score* values, estimations of probability using the MLM1 method approach 0, whereas the MM1 method gives estimates at an average level of 1%, and these values rise only slightly along with an increase in the *score*.

The transition of score into probability of default using the scaled beta distribution method

Source: author's own calculation Graph 5

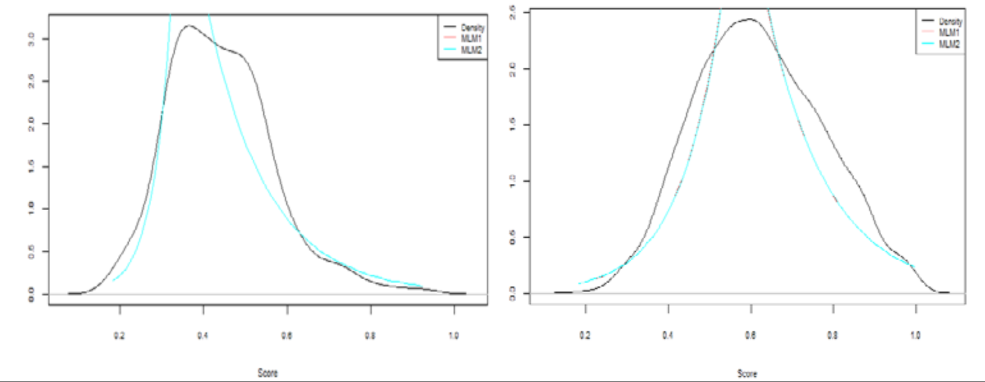


2.3. Asymmetric Laplace distribution

A further approach was proposed by Bennett (2003), who presented a method for approximating the empirical distribution to the asymmetric Laplace distribution and the asymmetric Gauss distribution. The estimate of the parameters achieved using the first method (MLM1) is the result of an algorithm proposed by Bennett, and the second (MLM) is the result of an algorithm searching for the maximum likelihood function. The results of the estimation of distribution parameters achieved by both of these methods is presented in Graphs 6 and 7.

Empirical density and asymmetric Laplace density function Source: author’s own calculation

Graph 6

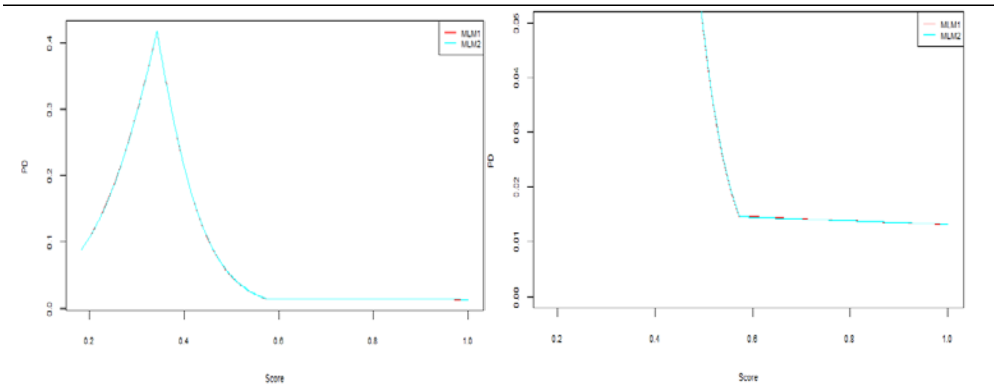


For a sample *score* distribution, significant differences were not noted in the estimates of parameters achieved using the Bennett method and the numeric method of searching for the maximum likelihood function. For a population of units not in the *default* state, parameter estimators have identical values. The adaptation of the Laplace distribution to a greater degree takes into account the mode of the adapted distribution. The bimodality of the distribution for units in the *default* state causes the previously discussed adaptation of the distributions to be shifted towards the lesser mode. An undoubted advantage of using this distribution is the adaptation to the larger mode, meaning that the method is resistant to the bimodality of the distribution and to changes of moment in the distribution due to the observation of outliers. One flaw of the distribution, however, is the increased mass of probability in the central part of the distribution, giving a high kurtosis value. The peaked shape of the density function creates characteristic distortions in the function mapping the *score* into PD.

The transition of score into probability of default using asymmetric Laplace distribution

Source: author’s own calculation

Graph 7

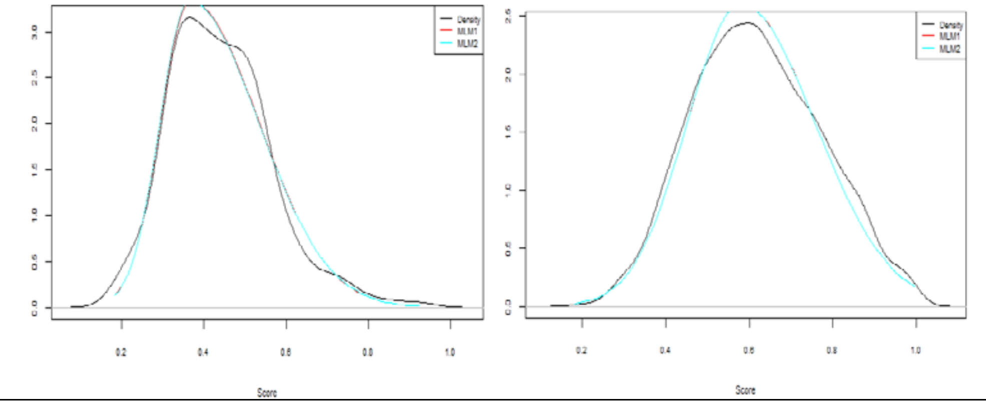


2.4. Asymmetric Gauss distribution

This method assumes the calibration of the empirical distribution to an asymmetric Gauss distribution[2] with three parameters. For the estimate, the method presented by Bennett (MLM1) and the numeric estimate of the maximum likelihood function (MLM2) were used. An empirical density chart with estimated parameters is presented in Graph 8, whereas a chart of the transformation from score into PD is presented in Graph 9.

Empirical density and asymmetric Gauss density function

Source: author’s own calculation | Graph 8

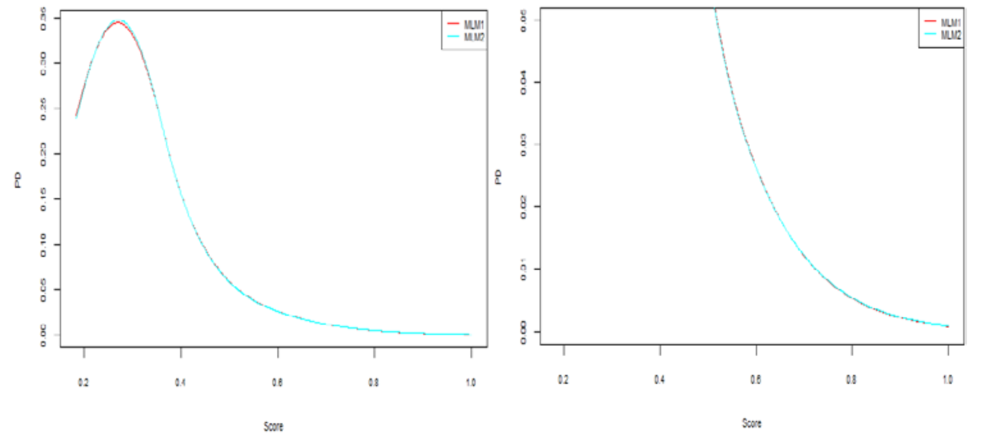


The estimates of parameters achieved using MLM1 and MLM differ, but these differences are minimal. For example, the estimator of the θ parameter for units not in the *default* state for the MLM1 method equals 0.3662, while for the MLM method the same parameter equals 0.364. The relative symmetricity of both empirical distributions creates small differences between the values of the σ_l and σ_r values. For the *score* distribution for units in default, the proportion of standard deviation for *scores* less than θ to standard deviation for arguments greater than θ is 1.15, whereas for the distribution for units not in *default*, this proportion is 0.85. The low value of the bias of the empirical distribution means that this approach does not significantly differ from the probit model, a particular case of the asymmetric Gauss distribution with equal σ_l and σ_r parameters.

[2] $f(x) = \frac{2}{\sqrt{2\pi}(\sigma_l+\sigma_r)} * \begin{cases} \exp\left[-\frac{(x-\theta)^2}{2\sigma_r^2}\right] & \text{gdy} \quad x > \theta \\ \exp\left[-\frac{(x-\theta)^2}{2\sigma_l^2}\right] & \text{gdy} \quad x \le \theta \end{cases}$, where θ, σ_l,σ_r are the model parameters.

The transition of score into probability of default using asymmetric Gauss distribution

Source: author's own calculation Graph 9



3. Regression analysis and other methods

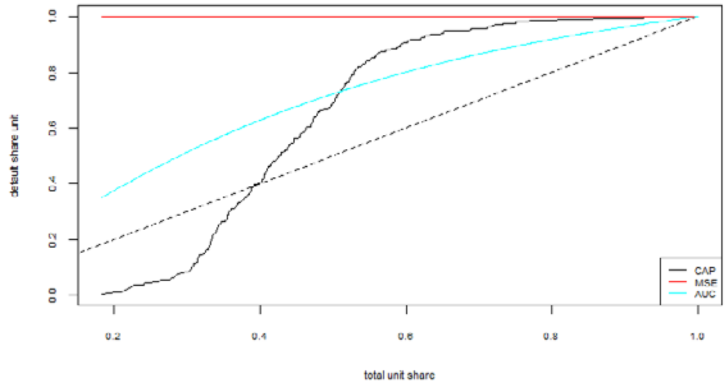
3.1. An approach based on ROC and CAP curves

The method proposed by Van der Burgt involves the estimation of the κ parameter, defining the CAP[3] curve. A derivative of this function allows us to calculate the transformation function of the score into PD. For the estimation of the parameter, two methods were used. The first, MSE (*Mean Square Error*), proposed by Tasche (2009), involves the minimisation of the sum of the squares of the remainders between the empirical CAP curve and its equivalent parameter. The second method (hereinafter referred to AUC – *Area Under Curve*) assumes the use of the dependence between the area under the ROC (AUC) curve and the CAP (*Cumulative Accuracy Profile*) curve, and generates an estimate of the parameter based on this. The results achieved using these two methods are presented in Graphs 10 and 11.

3 An interesting approach was proposed by Van der Burgt (2007), which involved the approximation of the CAP function with one κ parameter indicating the concavity of the function: $C_{\kappa}(u) = \frac{1-\exp(-\kappa * u)}{1-\exp(-\kappa)}$.

Empirical and two parametric CAP curves

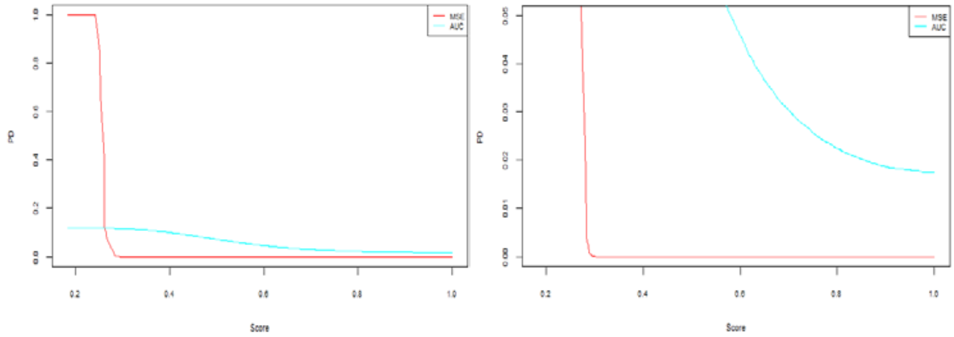
Source: author's own calculation Graph 10



For a given model, the CAP curve has a quite unusual shape - a dynamic growth of the value of the CAP function above the diagonal curve is observed only at 0.4, whereas this normally takes place at the beginning of distribution coordinates. This presents an interesting problem for the algorithm estimating the CAP curve parameter. An estimate using the method of minimising the sum of the squares of remainders defined the parameter as equal to 1000. In turn, the method using AUC gave a result equal to 1.92. The greater the value of the curve parameter, the less flat is its form, which is why the CAP curve obtained using the method of minimising the sum of squares of remainders has a steeper shape and more quickly approaches 1. A chart of the transformation from score into PD is presented in Graph 11.

The transition of score into probability of default using a method based on the calibration curve CAP

Source: author's own calculation Graph 11



For the AUC method, the transformation function of the *score* into PD takes on a sigmoid shape, although the maximum values are significantly lower than those observed in the previously described methods. The maximum size depends on the accepted unconditional probability of default. It should also be noted that the minimum values of probability obtained using the AUC method are greater than analogous values derived from the use of the MSE method. This is an especially relevant property in the context of assigning rating classes.

For the MSE method, the shape of the function is most certainly incorrect, as the vast majority of units will be assigned extreme values of probability, nearing either 0 or 1. An effect of this is the assignment of units to extreme classes, either the best or the worst. In spite of this, it was decided not to disqualify this approach. It is not out of the question that the case under consideration is in some way specific, and that for another data set the transformation function may have a gentler shape.

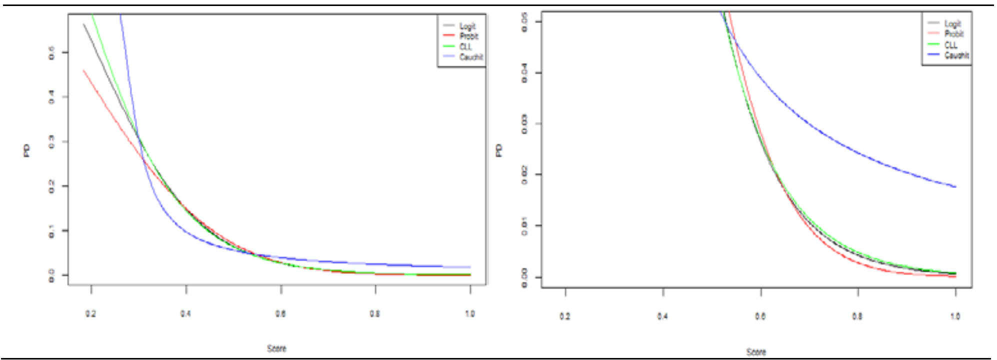
3.2 Logit and probit model, complementary log-log (CLL) function, cauchit function

Another method for the estimation of the function mapping *score* into PD involves the use of linear regression with a binary explanatory variable. One example of a study which addressed the issue of the construction of a transformation function from a synthetic *score* indicator into PD is the work done by Neagu et al. (2009). The authors concentrated on finding an alternative to the basic approaches of the logit and probit models, in which PD value is explained by the *score* constant and variable. As the authors themselves noted with regard to the example analysed, these models overestimate the probability of *default* for units with a low *score*, and underestimate for those with a high *score*. This results mainly from the bias of the *score* distribution. In order to return the distribution to a normal state, the Box-Cox (B-C) and Box-Tidwell (B-T) transformations were used on a variable *score*. The use of these transformations is certainly an interesting operation, which allows us to approximate the asymmetrical distribution to a symmetrical one. It is also worth noting that various distributions „react" differently to this transformation. It is harder to obtain a symmetrical shape when transforming a left-bias distribution, as the change in the transformation parameter results in a greater change in bias with regard to the absolute value and also to the diagnostic statistics of the Anderson-Darling test.

For the article, four transformation functions were used; logit, probit, CLL, and cauchit (Graph 12).

The transition of score into probability of default using the method of linear regression with different transition functions

Source: author's own calculation | Graph 12



The use of the cauchit transformation function gives the least ambiguous classification (the steepest curve). The probabilities obtained with this method are as a rule the largest among the analysed transformation functions. The initial (lowest) probability values obtained using the probit function method are lower than those obtained by the logit function. The use of the CLL function gives similar results to those obtained with the logit function. Concurrent with a decrease in the *score*, the values of probability rapidly rise, reaching their highest values at a certain moment. It is worth noting the *score* values, as the probabilities are relatively low. This is crucial in the context of the use of a master scale; a regression using the cauchit function significantly inflates these probabilities. This may cause a significant difference in the classification of units to higher rating classes, which usually require a very low probability.

3.3 Platt scaling

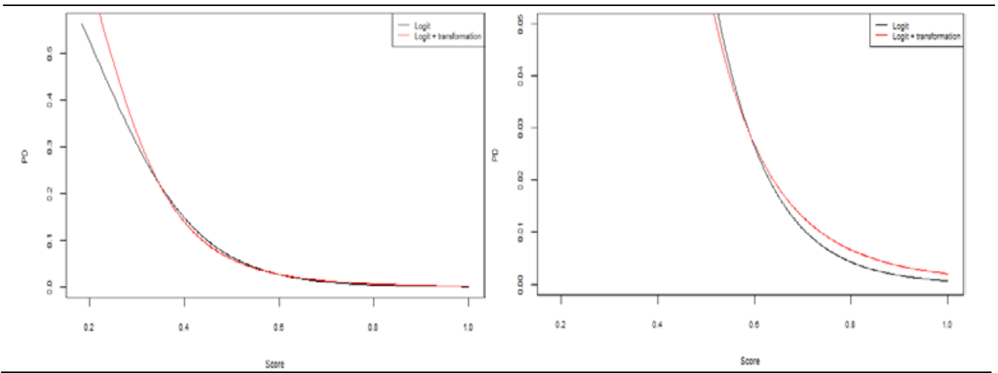
The use of the logit model was also suggested by Platt (1999). Initially, his method was used to transform the results of an SVM (*Support Vector Machines*) study, belonging to the [-∞, +∞] category, for probabilities within a closed range [0, 1]. The use of the Platt correction for the sample analysed does not significantly influence its results. This is caused mainly by the high number of observations (4112) for which the model was estimated. A significant influence will be observable on the basis of only a few samples.

3.4 Transformation function

For each transformation function a separate regression was performed (with a different set of parameter results) on the transformed *score* variable, using the Box-Cox and Box-Tidwell transformations. As was the case with Neagu, the Hosmer-Lemeshow test was used as the optimisation criterion. The differences in the shape of the function in both approaches are meliorated by the regression parameters. The differences between the transformation *score* function into PD for the logistic function, both taking and not taking into account the transformation, are shown in Graph 13.

The transition of score into probability of default using logit regression and logit regression with different transition functions

Source: author's own calculation | Graph 13



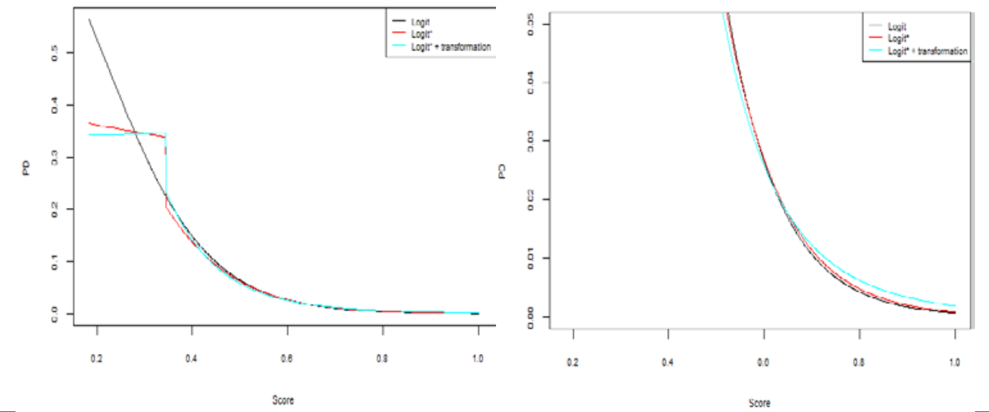
Based on analysis of the graphs of both functions above, it can be seen that the shape of the function has been modified. The function taking into account the transformation begins to grow later, meaning that for a greater number of units, low values of probability will be observed. The growth of the value of this function is also highly dynamic, meaning that from a *score* near 0.38 the probability values are higher than in the model without the transformation. Furthermore, the shape of the function is gradually flattened out and in the segment from 0 to 0.23 probability values are considerably lower.

3.5 Broken curve model

A further modification of linear regression is the application of a broken curve model. The approach proposed by Neagu assumed a search for a point for which the difference between the results of a *„normal"* linear regression and the actual percentage of units in *default* for a given *score* is the highest. Establishing the method used to calculate the last value was a serious problem, as the authors did not give an unambiguous definition of it. For the purposes of this article it was decided that the value of this function for the *score* (s) is equal to the proportion of units in *default* whose *score* fell within the range $\left[s - \frac{sd(score)}{10}; s + \frac{sd(score)}{10}\right]$. This amount was next weighted with the value of the density function at point (s). If the value of the density function were not weighted, this method would most often identify a point equal to the *score* value of units not in default with the highest *score* for that group of units. The application of weighting means that the point identified belongs to the „central" part of the distribution wherever the differences are relatively high, although certainly not the highest. This method is resistant to possible outlying points. Additionally, in the broken curve model (as a separate model), an algorithm for searching for Box-Cox and Box-Tidwell transformation parameters has been taken into account, based on a principle identical to those previously described. In contrast to the approach proposed by Neagu, the algorithm was not applied in several steps until a certain criterion was achieved. For optimisation reasons, this algorithm was applied only once, regardless of the values of the function of probability at the extreme ends of both ranges. The results of the model of a broken curve for the function logit is shown in Graph 14.

The transition of score into probability of default using a broken curve model with logit regression and logit regression with different transition function

Source: author's own calculation. Logit* - Logit (broken curve) Graph 14



This same algorithm for searching for the point with the greatest weighted difference meant that the discontinuity points of the functions obtained using both methods were equal. The use of the broken curve causes the value of probability, concurrent with a decrease in *score* values until the breaking point is reached, to increase more quickly in comparison with the standard approach. From this moment onwards, as the *score* values further decreased, the increase in the probability value was significant. The application of the Box-Cox and Box-Tidwell transformations did not cause significant differences other than an increase in the maximum value of probability. When analysing the dependences for low probability values, it can be seen that in comparison to normal linear regression, the application of the broken curve model means that for a greater number of observations, the estimate of the probability of default is obtained at a very low level. In other words, the normal logistic regression overestimates this probability for units with high *score* values.

3.6 Isotonic regression

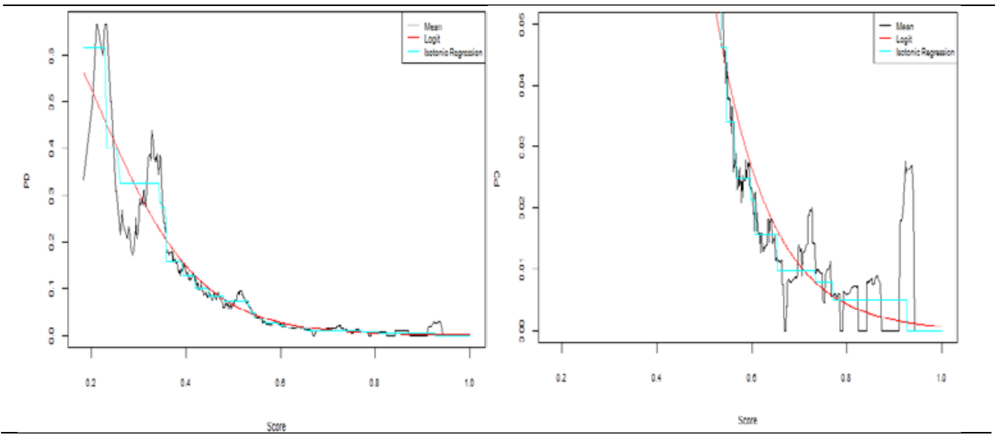
Another approach was presented by Zadrozny and Elkan (2002). The transformation function of the *score* classifier into a probability value may be achieved by the use of isotonic regression. This method can be described as a problem of minimising the weighted sum of error of an estimate (measured as the square of the difference between the estimate and the actual value), with the assumption that the function is non-decreasing.

The shape of the curve obtained by the use of isotonic regression primarily reflects the empirical average of the proportion of units in *default* with a similar *score* value. In accordance with the assumptions of this method, wherever this proportion decreases, the function decreases, and where an increase in proportion is observed, this function remains at the same level. It is worth noting the irregular shape of the average error curve. The stepped shape means that none of the approaches which lead to a sigmoid shape are able to correctly reflect this. An effective solution to this problem is the application of a broken curve or isotonic regression, which theoretically best reflect the actual, irregular shape of the curve. A significant limitation of this approach is the impossibility of expressing the results in the shape of a function. The only viable option is expression as a function, assuming a given value for a defined range. A further flaw is the method's high sensitivity to *default* for units with a high *score* value. The occurrence of such observations means that all observations with a lower *score* value will have a slightly inflated probability of *default*.

The transition of score into probability of default using isotonic regression

Source: author's own calculation

Graph 15



4. Verification using a test for the whole rating system

After estimating the *score* transformation function into the probability of *default*, each unit is assigned a rating, using for this purpose the master scale employed by a given bank. Next, for each rating class an actual default rate is calculated, as the proportion of units in *default* to units in a given rating class. This data is next used to calculate quality tests for calibration of the rating system. There is a risk that the above analysis may in large part be dependent on the *score* distribution for the population of bankruptcies and units in good financial condition. It has also been noted that for one subpopulation, this distribution is bimodal, which is not necessarily the rule for other data sets. This feature may have a substantial impact on the results obtained and the conclusions reached on their basis. To avoid this error, a *k-fold cross-validation* was applied. This involves the division of the original set into *k* subsets. Next, one subset is chosen as a test case, and the rest are treated as a training cases. The estimates obtained for the training cases were then verified with the test case. This step was repeated *k* times for another test case. In the validation set, the shape of the transformation function was estimated, and was next applied to the test case. Subsequently, the *k* obtained for such sets of units, along with the rating assigned to them ,were combined in one set of data based on the average (i.e., the average number of units in a given rating class), on which a calibration test was performed. A value of 4 was assumed for *k*, which on the one hand allows us to obtain relatively resistant results, and on the other does not significantly extend the process of calculation.

Generally, a higher quality of adaptation is observed for master scales with a smaller number of rating classes (Table 3, Table 4, Table 5, Table 6). For a master scale with 9 classes (KBC and Pekao), lower statistical results are observed in a Spiegelhalter test, higher *p-values* for the Hosmer-Lemeshow test, lower values for the shape component of the Blöchlinger test, and as a result lower values for the general statistics and higher *p-values* for this test are observed. This dependence is caused by the fact that with a less granular range, individual rating classes include „wider" ranges of probability of *default*, leading to an increase in the averages of probability in the initial classes. As has been mentioned previously, the lower the values of probability are, the slower the convergence to an χ^2 distribution is. A further consequence is an increased number of this observances, a fact which is crucial in the context of the asymptomatic properties of calibration tests. The more precise the range, the greater the variance in the ranks of the proportion of defaults, as the measurement becomes more precise and more biased away from the average. With a large number of observations in a given class, the influence of outliers on the actual value of probability for a given class lessens. In other words, the variance of deviation of the actual proportion *default*s decreases from the average estimate, meaning that the estimate must be more precise (better calibrated). It must be remembered, however, that the aim of the creation of a rating scale is not to maximise the calibration power of the model it uses. On the assumption that further such dependences can be obtained, and are highly likely to be correct, the most adequate division is one which defines several classes, in extreme cases as few as one. Such an approach is not appropriate. The reason for using master scales is, after all, to maximise business potential, and not to maximise the broadly understood efficiency of the model.

In terms of Spiegelhalter test statistics and their equivalent *p-values* of ratings ranges using all master scales, on average the highest quality calibration was observed for regression with a cauchit transformation function in the case of master scales comprising 9 classes (KBC, Pekao) for the model approximating the asymmetric Laplace distribution (Laplace-MLM1, Laplace-MLM2), the skew normal approximation method (Skewnormal – MLM1), the scaled beta distribution (Beta – MLM1), and the approximation method using the ROC curved (ROC-AUC). The use of Platt's correction brings excellent results. A dependence was observed in which the normal method of moment delivers as a result a lower quality of adapted probabilities. The relatively large differences between both methods in the approximation of the ROC curve should also be stressed. Unambiguously for all master scales, a *p-value* equal to 0.000 is observed for the ROC (ROC-MSE) method of approximating the curve. Additionally, in the case of the Millenium Bank master scale, a *p-value* equal to 0.000 was also noted for the QMM method, and for regression accounting for a broken curve with transformation functions other than *Logit (Broken curve)* (logit, probit, cauchit and CLL).

In performing an analysis of the *p-value* of the Hosmer-Lemeshow test for a range using all master scales, the highest quality is observed for a regression taking into account a broken curve with the *logit* transformation function. In each case, a value equal to 1 is observed. This is especially important for „large" master scales, in which for the majority of models the *p-value* was equal to 0.000. It should also be noted that the simple application and interpretation of the results of this test is problematic, as for values of observed probability near 0 or 1 the values of the test statistics approach infinity. For the model of approximation with an asymmetric beta distribution, both of the method of moment deliver an average statistical test value higher by 2*10^12. This is to be expected, as in this method a relatively small number of units is assigned to a relatively high number of classes. If in a particular case this is one unit which is not in default, the probability of default observed for this class with one unit will equal 0, in turn delivering a high statistical test value. Conclusions on the quality of calibration based on the Hosmer-Lemeshow test may thus be made more difficult. For this reason the application of different, alternative approaches such as the Blöchlinger test is exceptionally valuable.

In terms of the general value of the statistics in the Blöchlinger test and their resultant *p-values*, the worst quality of calibration was obtained when using the two methods of approximating the ROC curve. In turn, the best quality was observed for regression taking into account a broken curve with a logit transformation function, regression with a logit curve, probit, CLL, and cauchit with the Box-Cox transformation. With the exception of regression, for all of these models the quality of calibration rises with the increase in the granularity of the range.

Summing up, the calibration power of the entire rating system was obtained with different calibration methods and different master scales using three tests which took into account k-fold cross-validation. The use of these tests allowed us to study several aspects of high-quality calibration. The Spiegelhalter test verifies the extent to which the estimate of probability for units diverge from the observed proportion of defaults in a given group of units. The shape of the test statistics is regular and is the result of a process of standardisation. A similar zero hypothesis is obtained by the Hosmer-Lemeshow test, though the unit is the rating class, and the statistics have an χ^2 distribution. The Blöchlinger test verifies two basic elements of high-quality calibrations – a size component measured as the average of probability of default with regard to the average proportion of defaults for the entire sample, and a shape component measured as the deviation from the observed ROC curve.

In comparing the quality of calibration to normal logistic regression, a set of methods with a significantly better precision of estimates was identified. There are also methods which always give better calibrated estimates of probability when compared to all of the methods applied in the tests. These are regressions that take into account a broken curve with logit transformation, logit regression, probit, CLL, and cauchit with the Box-Cox transformation.

A further conclusion is the considerable improvement in calibration that is achieved when the Box-Cox transformation is taken into account for the regression analysed. A significant increase in the average *p-values* for the Blöchlinger and Hosmer-Lemeshow tests is observed. The *p-values* of the Spiegelhalter test remain relatively stable.

In performing an analysis of the calibration with regard to the number of classes in the master scale, a significant decrease in the precision of the estimates of probability can be observed with an increase in the number of classes. This dependence is especially visible for the Blöchlinger test.

Conclusion

The basic aim of this paper was to present methods of calibration that allow us to obtain precise estimates of the probability of default, and techniques of validation based on their calibrating power. The use of k-fold cross-validation and repetition of calculations for different master scales and differing data sets means that the results should be highly resistant for the distribution variable in the training set.

The use of several tests allows us to take into account different definitions of high-quality calibration. The results obtained were not unambiguous, but they do allow us to answer the basic research question. First, we can say that there are methods which deliver considerably better calibrated estimates of probability in comparison with logistic regression estimators. The difference observed is relatively large and also concerns calibration of the system as a whole.

The second research question concerned the quality of calibration for different numbers of classes in the master scales. Using four different approaches (master scales including from 9 to 19 classes), whose sources were bank reports on risk, a decrease in the quality of calibration of the whole rating system was noted, in conjunction with an increase in the number of classes. This dependence is dictated first and foremost by the wider ranges of probability for particular classes, and by the properties of the statistical tests applied.

These conclusions definitely do not fully address the issue of the construction of a model that would give the best calibrated estimates. This issue is highly complex, due mainly to the lack of a single, unambiguous method of calibration and a method of estimating the correlation of default between units. A very interesting development of this study would be the use of simulation techniques and an attempt to take into account the cost (expressed in units of time) for the calculation of samples of various sizes.

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Distribution of units on rating classes for different calibration methods and calibration tests for the whole rating system based on a master scale - KBC bank.

Source: author's own calculation

Table 2

Method	Rating 1	2	3	4	5	6	7	8	9	Spiegelhalter	Spiegelhalter p-value	Hosmer-Lemeshow	Hosmer-Lemeshow p-value	Birer Score	Blochlinger Chi2	Blochlinger p-value
Master scale: Upper	0,00%	0,10%	0,20%	0,40%	0,80%	1,60%	3,20%	6,40%	12,80%							
Master scale: Lower	0,10%	0,20%	0,40%	0,80%	1,60%	3,20%	6,40%	12,80%	100%							
QMM	57	137	317	460	582	727	735	633	464	-0,3921	0,6525	9,2857	0,2328	0,0008	0,9929	0,6087
Skewnormal - MLM1	327	194	253	348	441	584	697	729	539	-0,6614	0,7458	36,4415	0,0000	0,0003	0,7943	0,6722
Skewnormal - MLM2	0	57	249	523	734	799	722	517	511	-0,4636	0,6785	8,7482	0,1882	0,0005	0,7990	0,6706
Skewnormal - MM	0	0	214	577	778	818	717	502	506	-0,3718	0,6450	7,3424	0,1964	0,0006	0,6332	0,7286
Beta - MLM1	309	187	253	347	440	592	705	757	522	-0,6189	0,7320	34,8075	0,0000	0,0003	0,6180	0,7342
Beta - MLM2	0	0	419	535	599	713	732	650	464	-0,3421	0,6339	4,3948	0,4941	0,0009	0,7244	0,6961
Beta - MM1	0	0	97	698	704	814	791	593	415	-0,1602	0,5636	5,3872	0,3705	0,0026	0,9845	0,6112
Beta - MM2	0	0	452	542	609	717	725	603	464	-0,4065	0,6578	5,6195	0,3450	0,0022	0,8965	0,6388
Laplace - MLM1	0	0	0	0	2387	416	394	345	570	-0,6560	0,7441	27,3488	0,0000	0,0010	1,4238	0,4907
Laplace - MLM1	0	0	0	0	2378	425	384	351	574	-0,6773	0,7509	26,6065	0,0000	0,0010	1,4856	0,4758
Gauss - MLM1	20	101	270	438	648	853	796	518	468	-0,4127	0,6601	9,9288	0,1926	0,0006	0,6840	0,7103
Gauss - MLM1	18	100	263	442	654	861	795	511	468	-0,3903	0,6518	9,8438	0,1976	0,0006	0,6352	0,7279
ROC – MSE	4071	0	2	6	0	3	3	7	20	163,3894	0,0000	6641797	0,0000	0,0002	123,1925	0,0000
ROC – AUC	0	0	0	0	0	1304	1487	1321	0	-0,6517	0,7427	60,3700	0,0000	0,0001	8,0220	0,0181
Logit	57	143	319	458	576	727	750	618	464	-0,3861	0,6503	8,1855	0,3165	0,0009	0,9083	0,6350
Logit - Platt	57	127	317	456	579	730	749	633	464	-0,4056	0,6575	8,5176	0,2892	0,0008	0,8725	0,6464
Logit - Box-Cox	0	20	205	492	721	861	827	561	425	-0,2960	0,6164	3,6196	0,7280	0,0013	0,9905	0,6094
Probit	231	198	268	384	482	654	712	696	487	-0,4829	0,6854	15,1820	0,0337	0,0006	0,8206	0,6634
Probit – Platt	224	193	268	372	486	646	740	704	479	-0,4965	0,6902	15,8352	0,0267	0,0005	0,8435	0,6559
Probit - Box-Cox	0	0	208	489	729	873	834	567	412	-0,2144	0,5849	3,6169	0,6058	0,0014	0,8675	0,6481
CLL	20	128	293	471	587	767	784	608	454	-0,3632	0,6418	7,5552	0,3734	0,0010	1,0587	0,5890
CLL - Platt	20	123	292	459	597	763	796	608	454	-0,3846	0,6497	7,4704	0,3816	0,0010	1,1238	0,5701
CLL - Box-Cox	0	0	80	414	772	986	906	592	362	-0,1353	0,5538	4,4086	0,4922	0,0015	0,9481	0,6225
Cauchit	0	0	0	0	0	1412	1892	587	221	-1,3317	0,9085	23,3999	0,0000	0,0019	4,7105	0,0949
Cauchit – Platt	0	0	0	0	0	1401	1903	587	221	-1,3336	0,9088	23,3383	0,0000	0,0019	4,8233	0,0897
Cauchit- Box-Cox	0	0	20	421	825	1014	874	546	412	-0,5178	0,6977	3,8313	0,5739	0,0010	0,8677	0,6480
Isotonic	99	0	0	858	1066	411	284	1071	323	-0,3861	0,6503	8,1855	0,3165	0,0009	0,9083	0,6350
Logit (Broken curve)	20	131	293	474	592	770	796	621	415	-0,3937	0,6531	0,0000	1,0000	0,0010	0,8410	0,6567
Logit - Platt (Broken curve)	20	123	286	465	597	775	810	624	412	-0,3965	0,6541	7,6606	0,3635	0,0009	1,0528	0,5907
Logit - Box-Cox (Broken curve)	0	0	214	514	759	866	801	524	434	-0,3010	0,6183	3,4684	0,6282	0,0009	0,6681	0,7160
Probit (Broken curve)	124	182	253	405	521	711	814	738	364	-0,4131	0,6602	16,1469	0,0238	0,0009	1,3129	0,5187
Probit - Platt (Broken curve)	120	166	257	401	528	705	833	740	362	-0,4305	0,6666	16,5331	0,0207	0,0009	1,1963	0,5498
Probit - Box-Cox (Broken curve)	0	20	246	508	717	856	791	540	434	-0,3116	0,6223	3,4907	0,7452	0,0009	0,6617	0,7183
CLL (Broken curve)	20	109	297	468	605	790	798	610	415	-0,3679	0,6435	7,6705	0,3625	0,0009	1,0228	0,5997
CLL - Platt (Broken curve)	19	104	287	460	617	793	804	616	412	-0,3869	0,6506	7,9413	0,3378	0,0009	1,0793	0,5830
CLL - Box-Cox (Broken curve)	0	20	220	516	729	862	796	544	425	-0,2952	0,6161	3,5131	0,7422	0,0009	0,8016	0,6698
Cauchit (Broken curve)	0	0	0	0	444	1858	1118	377	315	-0,3242	0,6271	20,5298	0,0001	0,0006	1,3413	0,5114
Cauchit - Platt (Broken curve)	0	0	0	0	436	1863	1119	379	315	-0,3383	0,6324	20,3797	0,0001	0,0006	1,3899	0,4991
Cauchit- Box-Cox (Broken curve)	0	0	57	454	846	990	840	516	409	-0,3491	0,6365	4,9884	0,4173	0,0009	0,9112	0,6341

Distribution of units on rating classes for different calibration methods and calibration tests for the whole rating system based on a master scale - PEKAO bank

Source: author's own calculation Table 3

Method	Rating 1	2	3	4	5	6	7	8	9	Spiegelhalter	Spiegelhalter p-value	Hosmer-Lemeshow	Hosmer-Lemeshow p-value	Birer Score	Blochlinger Chi2	Blochlinger p-value
Master scale – Upper	0,00%	0,15%	0,27%	0,45%	0,75%	1,27%	2,25%	4,00%	8,50%							
Master scale – Lower	0,15%	0,27%	0,45%	0,75%	1,27%	2,25%	4,00%	8,50%	100,00%							
QMM	120	204	247	350	405	555	647	791	793	-0,5914	0,7229	11,4721	0,1193	0,0014	2,1780	0,3366
Skewnormal - MLM1	435	191	208	258	309	419	527	807	958	-0,7709	0,7796	17,5370	0,0142	0,0006	3,0928	0,2130
Skewnormal - MLM2	0	120	257	392	528	674	649	706	786	-0,6506	0,7424	7,2802	0,2957	0,0010	1,8239	0,4017
Skewnormal - MM	0	57	235	429	572	700	648	695	776	-0,5730	0,7167	5,0380	0,5389	0,0011	1,5082	0,4704
Beta - MLM1	410	194	201	252	311	440	522	824	958	-0,7236	0,7654	20,0760	0,0054	0,0006	3,0029	0,2228
Beta - MLM2	0	125	380	398	423	555	639	784	808	-0,5223	0,6993	5,9251	0,4316	0,0014	2,4325	0,2963
Beta - MM1	0	0	252	484	525	620	698	807	726	-0,4499	0,6736	3,8312	0,5740	0,0033	1,7101	0,4253
Beta - MM2	0	176	361	407	447	572	602	761	786	-0,6658	0,7472	4,7846	0,5717	0,0029	1,8959	0,3875
Laplace - MLM1	0	0	0	0	0	2595	334	414	769	-0,9539	0,8299	27,1495	0,0000	0,0018	3,3986	0,1828
Laplace - MLM1	0	0	0	0	0	2595	334	414	769	-0,9678	0,8334	26,7992	0,0000	0,0018	3,3952	0,1831
Gauss - MLM1	82	132	230	330	459	626	751	762	740	-0,6307	0,7359	7,8222	0,3485	0,0012	1,3339	0,5133
Gauss - MLM1	77	123	236	337	460	635	752	759	733	-0,6080	0,7284	6,0849	0,5299	0,0012	1,6055	0,4481
ROC – MSE	4071	1	2	5	0	2	2	3	26	180	0,0000	6 641 597	0,0000	0,0009	123	0,0000
ROC – AUC	0	0	0	0	0	548	1250	1605	709	-0,6436	0,7401	74,9658	0,0000	0,0001	3,9866	0,1362
Logit	121	205	250	351	407	547	647	791	793	-0,5873	0,7215	11,3454	0,1242	0,0014	2,1635	0,3390
Logit - Platt	118	199	242	359	402	552	648	799	793	-0,6015	0,7263	11,3506	0,1240	0,0014	2,2556	0,3237
Logit - Box-Cox	0	75	195	381	521	671	736	824	709	-0,5233	0,6996	4,8129	0,5680	0,0019	1,3651	0,5053
Probit	344	190	222	283	340	478	565	818	872	-0,6410	0,7392	16,8767	0,0182	0,0010	2,2248	0,3288
Probit – Platt	327	197	212	278	354	475	573	824	872	-0,6523	0,7429	17,1691	0,0163	0,0010	2,2585	0,3233
Probit - Box-Cox	0	85	207	369	517	665	736	824	709	-0,4639	0,6787	4,7023	0,5825	0,0020	1,3534	0,5083
CLL	92	171	248	334	442	572	669	808	776	-0,5734	0,7168	7,4560	0,3830	0,0016	1,8116	0,4042
CLL - Platt	88	161	247	338	438	585	671	808	776	-0,5899	0,7224	5,5149	0,5974	0,0016	1,7104	0,4252
CLL - Box-Cox	57	151	244	353	456	596	674	817	764	-0,5534	0,7100	4,7905	0,6855	0,0017	1,5555	0,4594
Cauchit	0	0	0	0	0	358	1809	1477	468	-1,6106	0,9464	31,4054	0,0000	0,0028	2,8728	0,2378
Cauchit – Platt	0	0	0	0	0	353	1807	1484	468	-1,6083	0,9461	31,3468	0,0000	0,0027	2,9616	0,2275
Cauchit- Box-Cox	0	0	20	251	581	825	888	882	665	-0,8234	0,7949	6,6585	0,2473	0,0017	1,3600	0,5066
Isotonic	99	0	0	605	872	541	493	858	644	-0,5873	0,7215	11,3454	0,1242	0,0014	2,1635	0,3390
Logit (Broken curve)	92	171	248	341	435	581	663	841	740	-0,7253	0,7659	0,0000	1,0000	0,0019	1,3469	0,5099
Logit - Platt (Broken curve)	88	152	256	338	438	585	674	841	740	-0,6461	0,7409	6,2755	0,5080	0,0016	1,6754	0,4327
Logit - Box-Cox (Broken curve)	0	58	219	390	554	687	722	780	702	-0,5486	0,7083	3,5685	0,7348	0,0016	1,5399	0,4630
Probit (Broken curve)	219	189	218	295	358	529	640	900	764	-0,6941	0,7562	8,3422	0,3034	0,0017	1,8322	0,4001
Probit - Platt (Broken curve)	194	197	220	292	363	513	655	914	764	-0,7091	0,7609	9,1188	0,2442	0,0017	1,6645	0,4351
Probit - Box-Cox (Broken curve)	0	98	246	377	528	649	722	783	709	-0,5604	0,7124	4,6394	0,5908	0,0016	1,4342	0,4882
CLL (Broken curve)	85	146	263	335	450	589	683	828	733	-0,6220	0,7330	5,6898	0,5764	0,0016	1,6982	0,4278
CLL - Platt (Broken curve)	81	144	243	351	442	596	684	838	733	-0,6390	0,7386	4,7138	0,6948	0,0016	1,5243	0,4667
CLL - Box-Cox (Broken curve)	57	143	252	358	456	602	697	817	730	-0,6090	0,7287	4,5928	0,7095	0,0016	1,7440	0,4181
Cauchit (Broken curve)	0	0	0	0	80	1260	1444	825	503	-0,6300	0,7357	18,6548	0,0003	0,0014	1,7638	0,4140
Cauchit - Platt (Broken curve)	0	0	0	0	77	1257	1450	825	503	-0,6396	0,7388	18,6037	0,0003	0,0014	1,7865	0,4093
Cauchit- Box-Cox (Broken curve)	0	0	57	326	611	817	859	795	647	-0,5949	0,7240	5,8131	0,3248	0,0015	0,9582	0,6194

Distribution of units on rating classes for different calibration methods and calibration tests for the whole rating system based on a master scale - Millenium bank

Source: own calculation

Table 4

Method / Master scale	Rating 1	2	3	4	5	6	7	8	9	10	11	12	13	14	Spiegelhalter	Spiegelhalter p-value	Hosmer-Lemeshow	Hosmer-Lemeshow p-value	Birer Score	Blochlinger Chi2	Blochlinger p-value
Upper	0,00%	0,05%	0,07%	0,14%	0,28%	0,53%	0,95%	1,73%	2,92%	4,67%	7,00%	9,77%	13,61%	27,21%							
Lower	0,05%	0,07%	0,14%	0,28%	0,53%	0,95%	1,73%	2,92%	4,67%	7,00%	9,77%	13,61%	27,21%	100,00%							
QMM	4099	1	0	5	0	0	0	0	0	0	1	1	1	4	350,9	0,0000	311161324	0,0000	0,0000	139	0,0000
Skewnormal - MLM1	173	67	175	231	275	300	386	453	450	419	379	336	450	18	-0,6103	0,7292	44,5856	0,0000	0,0002	0,2987	0,8613
Skewnormal - MLM2	0	0	0	124	366	507	651	618	518	374	269	214	345	126	-0,3160	0,6240	7,2546	0,6106	0,0001	0,2918	0,8643
Skewnormal - MM	0	0	0	75	340	570	675	629	514	373	271	200	332	133	-0,2094	0,5829	3,4684	0,9428	0,0001	0,2257	0,8933
Beta - MLM1	158	66	167	226	271	302	403	465	444	427	390	332	445	16	-0,5556	0,7108	46,2002	0,0000	0,0002	0,3189	0,8526
Beta - MLM2	0	0	0	159	467	466	529	556	512	424	314	260	354	71	-0,1301	0,5517	12,1923	0,2027	0,0004	0,4912	0,7822
Beta - MM1	0	0	0	0	419	558	616	624	547	433	304	247	269	95	0,3747	0,3540	10,0139	0,2641	0,0012	0,3327	0,8467
Beta - MM2	0	0	0	214	447	469	550	551	502	405	309	231	339	95	0,0315	0,4874	12,8902	0,1676	0,0011	0,3382	0,8444
Laplace - MLM1	0	0	0	0	0	0	2434	326	250	235	176	148	316	227	-0,3268	0,6281	42,9160	0,0000	0,0002	0,9029	0,6367
Laplace - MLM1	0	0	0	0	0	0	2422	331	257	230	181	148	316	227	-0,3590	0,6402	40,4916	0,0000	0,0002	0,9488	0,6223
Gauss - MLM1	0	0	68	157	312	445	571	664	574	406	285	191	295	144	-0,2276	0,5900	5,5288	0,8532	0,0001	0,2063	0,9020
Gauss - MLM1	0	0	57	164	310	446	586	657	578	411	273	194	290	146	-0,1967	0,5780	5,9365	0,8206	0,0001	0,1989	0,9053
ROC – MSE	4065	1	5	1	7	0	2	1	2	1	2	5	0	20	163,6	0,0000	21795091	0,0000	0,0002	123,73	0,0000
ROC – AUC	0	0	0	0	0	0	0	1107	1010	869	711	415	0	0	-0,6269	0,7346	97,3184	0,0000	0,0000	1,1393	0,5657
Logit	0	18	93	229	337	425	519	556	525	431	314	231	339	95	-0,1583	0,5629	17,2348	0,1011	0,0003	0,2953	0,8627
Logit - Platt	0	1	103	223	334	431	515	566	523	430	321	240	332	93	-0,1818	0,5721	16,5813	0,1209	0,0003	0,3379	0,8445
Logit - Box-Cox	0	0	0	88	300	490	640	671	585	442	289	235	273	99	0,0487	0,4806	6,8660	0,6511	0,0004	0,2568	0,8795
Probit	111	46	169	222	292	343	438	502	472	435	356	287	379	60	-0,3460	0,6353	48,2051	0,0000	0,0002	0,4302	0,8064
Probit – Platt	104	44	164	222	295	343	431	502	482	443	356	290	378	58	-0,3658	0,6427	50,5933	0,0000	0,0002	0,3601	0,8352
Probit - Box-Cox	0	0	81	187	339	417	559	590	535	446	314	238	311	95	-0,0767	0,5306	6,6947	0,7539	0,0003	0,2260	0,8931
CLL	0	0	85	186	340	428	544	590	535	435	325	235	314	95	-0,0929	0,5370	7,9892	0,6299	0,0003	0,2462	0,8842
CLL - Platt	0	0	82	184	338	415	557	591	535	441	325	235	314	95	-0,1208	0,5481	7,2503	0,7016	0,0003	0,2253	0,8934
CLL - Box-Cox	0	0	82	189	340	413	559	590	535	446	314	235	314	95	-0,0914	0,5364	6,5256	0,7693	0,0003	0,2271	0,8927
Cauchit	0	0	0	0	0	0	0	1107	1513	807	327	155	129	74	-0,9210	0,8215	67,2232	0,0000	0,0007	0,8303	0,6602
Cauchit – Platt	0	0	0	0	0	0	0	1107	1503	817	327	158	126	74	-0,9280	0,8233	71,2522	0,0000	0,0007	0,6136	0,7358
Cauchit- Box-Cox	0	0	0	0	92	459	769	803	668	460	284	218	249	110	-0,2761	0,6088	8,7527	0,3636	0,0003	0,3117	0,8557
Isotonic	99	0	0	0	605	253	1066	377	318	46	704	321	138	185	-0,1583	0,5629	17,2348	0,1011	0,0003	0,2953	0,8627
Logit (Broken curve)	0	0	84	192	340	422	554	580	545	450	324	258	205	158	-0,1322	0,5526	0,0000	1,0000	0,0003	0,2920	0,8642
Logit - Platt (Broken curve)	0	0	81	184	332	416	562	591	551	450	324	261	202	158	4,2061	0,0000	1037,7944	0,0000	0,0000	0,2135	0,8987
Logit - Box-Cox (Broken curve)	0	0	0	91	327	525	639	648	572	432	270	221	229	158	4,3686	0,0000	751,6186	0,0000	0,0001	0,2276	0,8924
Probit (Broken curve)	46	41	101	228	286	368	482	542	535	503	359	297	166	158	4,1234	0,0000	1725,5089	0,0000	0,0000	0,2236	0,8942
Probit - Platt (Broken curve)	45	39	90	228	289	361	483	549	545	503	363	301	158	158	4,0675	0,0000	1682,5030	0,0000	0,0000	0,2182	0,8966
Probit - Box-Cox (Broken curve)	0	0	46	174	337	445	580	606	564	434	309	252	207	158	4,2950	0,0000	920,8078	0,0000	0,0000	0,2211	0,8953
CLL (Broken curve)	0	0	76	183	332	427	564	594	570	429	320	252	207	158	4,2776	0,0000	1045,2887	0,0000	0,0000	0,2183	0,8966
CLL - Platt (Broken curve)	0	0	57	173	330	442	566	604	560	443	320	254	205	158	4,2197	0,0000	929,5544	0,0000	0,0000	0,2166	0,8973
CLL - Box-Cox (Broken curve)	0	0	56	174	330	446	576	606	564	431	312	252	207	158	4,2892	0,0000	921,1887	0,0000	0,0000	0,2226	0,8947
Cauchit (Broken curve)	0	0	0	0	0	0	645	1439	960	447	186	142	116	177	3,8386	0,0001	224,7046	0,0000	0,0001	0,2516	0,8818
Cauchit - Platt (Broken curve)	0	0	0	0	0	0	630	1444	955	462	185	143	116	177	3,8003	0,0001	222,3489	0,0000	0,0001	0,2557	0,8800
Cauchit- Box-Cox (Broken curve)	0	0	0	0	91	500	809	819	639	449	254	191	202	158	4,1822	0,0000	454,0612	0,0000	0,0001	0,2439	0,8852

Distribution of units on rating classes for different calibration methods and calibration tests for the whole rating system based on a master scale - ING bank

Source: author's own calculation

Table 5

Method	Rating 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Spiegelhalter	Spiegelhalter p-value	Hosmer-Lemeshow	Hosmer-Lemeshow p-value	Birer Score	Blochlinger Chi2	Blochlinger p-value
Master scale – Upper	0,00%	0,01%	0,02%	0,04%	0,05%	0,06%	0,08%	0,11%	0,17%	0,29%	0,51%	0,89%	1,54%	2,67%	4,62%	8,01%	13,88%	20,00%	30,00%							
Master scale – Lower	0,01%	0,02%	0,04%	0,05%	0,06%	0,08%	0,11%	0,17%	0,29%	0,51%	0,89%	1,54%	2,67%	4,62%	8,01%	13,88%	20,00%	30,00%	100,00							
QMM	0	0	0	0	0	20	60	68	210	296	385	471	575	604	570	441	209	140	63	-0,1322	0,5526	22,4454	0,0328	0,0002	0,2005	0,9046
Skewnormal - MLM1	30	58	60	25	41	63	81	105	191	249	280	347	458	514	585	566	300	155	4	-0,5891	0,7221	84,8333	0,0000	0,0001	0,2728	0,8725
Skewnormal - MLM2	0	0	0	0	0	0	0	20	124	304	496	574	655	601	497	380	195	197	69	-0,2972	0,6168	10,4497	0,4020	0,0001	0,2080	0,9012
Skewnormal - MM	0	0	0	0	0	0	0	0	85	303	533	597	671	602	492	371	194	183	81	-0,1897	0,5752	8,5445	0,4803	0,0001	0,2414	0,8863
Beta - MLM1	20	58	46	34	26	76	67	117	187	239	286	343	451	535	602	586	292	144	3	-0,5537	0,7101	108	0,0000	0,0001	0,3310	0,8475
Beta - MLM2	0	0	0	0	0	0	0	0	184	414	426	486	575	593	567	458	232	130	47	-0,0889	0,5354	9,3763	0,4033	0,0003	0,2010	0,9044
Beta - MM1	0	0	0	0	0	0	0	0	0	391	527	554	645	636	578	423	188	95	75	0,4464	0,3277	24,3487	0,0020	0,0010	0,2922	0,8641
Beta - MM2	0	0	0	0	0	0	0	0	231	409	441	488	566	583	547	435	209	135	68	0,1112	0,4557	25,4699	0,0025	0,0009	0,2592	0,8784
Laplace - MLM1	0	0	0	0	0	0	0	0	0	0	0	2372	330	308	309	253	172	182	186	-0,2934	0,6154	44,3942	0,0000	0,0001	0,9417	0,6245
Laplace - MLM1	0	0	0	0	0	0	0	0	0	0	0	2362	340	296	321	256	163	188	186	-0,3121	0,6225	40,4449	0,0000	0,0001	0,9879	0,6102
Gauss - MLM1	0	0	0	0	0	0	20	72	146	283	406	499	679	679	542	351	171	168	96	-0,2089	0,5827	7,9660	0,7163	0,0001	0,2202	0,8957
Gauss - MLM1	0	0	0	0	0	0	20	72	137	282	410	517	667	686	535	352	170	161	103	-0,1779	0,5706	7,9472	0,7180	0,0001	0,2128	0,8991
ROC – MSE	4058	3	4	0	1	5	0	0	1	2	5	0	2	3	2	6	0	0	20	163,6131	0,0000	143246075	0,0000	0,0002	120,7016	0,0000
ROC – AUC	0	0	0	0	0	0	0	0	0	0	0	0	918	1177	1176	841	0	0	0	-0,6358	0,7376	68,1846	0,0000	0,0001	3,1924	0,2027
Logit	0	0	0	0	0	20	61	70	208	296	392	463	585	601	563	441	209	135	68	-0,1258	0,5500	23,3483	0,0249	0,0002	0,2018	0,9040
Logit - Platt	0	0	0	0	0	20	55	70	199	302	378	475	576	614	562	449	212	139	61	-0,1497	0,5595	21,4656	0,0440	0,0002	0,1984	0,9056
Logit - Box-Cox	0	0	0	0	0	0	0	20	128	299	456	535	647	662	579	414	172	122	78	0,0217	0,4914	17,7018	0,0602	0,0003	0,2399	0,8870
Probit	0	30	62	19	18	47	87	120	176	264	321	386	507	550	597	494	264	139	31	-0,3158	0,6239	132,6172	0,0000	0,0001	0,1735	0,9169
Probit – Platt	0	20	68	16	19	46	71	133	175	257	322	391	505	556	597	511	255	139	31	-0,3369	0,6319	155,0342	0,0000	0,0001	0,1751	0,9162
Probit - Box-Cox	0	0	0	0	0	0	0	0	82	249	443	594	707	689	584	412	161	110	81	0,1954	0,4225	19,6716	0,0201	0,0004	0,2643	0,8762
CLL	0	0	0	0	0	0	45	66	175	290	409	492	591	634	572	452	188	130	68	-0,0581	0,5232	30,1901	0,0015	0,0002	0,2600	0,8781
CLL - Platt	0	0	0	0	0	0	42	62	178	279	416	488	595	636	578	452	189	129	68	-0,0869	0,5346	31,6973	0,0009	0,0002	0,2503	0,8824
CLL - Box-Cox	0	0	0	0	0	0	0	0	98	271	422	566	691	699	596	417	175	103	74	0,1245	0,4504	18,1938	0,0330	0,0004	0,2389	0,8874
Cauchit	0	0	0	0	0	0	0	0	0	0	0	0	823	1764	999	329	86	50	61	-0,8665	0,8069	75,8417	0,0000	0,0006	0,5387	0,7639
Cauchit – Platt	0	0	0	0	0	0	0	0	0	0	0	0	819	1760	1007	329	86	51	60	-0,8704	0,8080	74,7127	0,0000	0,0006	0,5816	0,7477
Cauchit- Box-Cox	0	0	0	0	0	0	0	0	0	0	47	575	1020	1066	749	381	114	82	78	-0,5498	0,7088	29,6169	0,0001	0,0003	0,2567	0,8795
Isotonic	99	0	0	0	0	0	0	0	0	605	253	619	824	210	606	573	126	41	156	-0,1258	0,5500	23,3483	0,0249	0,0002	0,2018	0,9040
Logit (Broken curve)	0	0	0	0	0	0	46	65	181	300	396	489	598	633	600	452	189	5	158	-0,1230	0,5489	0,0000	1,0000	0,0003	0,3179	0,8530
Logit - Platt (Broken curve)	0	0	0	0	0	0	42	62	173	284	416	488	603	634	606	462	181	3	158	-0,0612	0,5244	16,0861	0,1380	0,0001	0,2550	0,8803
Logit - Box-Cox (Broken curve)	0	0	0	0	0	0	18	70	150	299	434	506	618	638	593	427	190	11	158	-0,0232	0,5092	6,4494	0,8418	0,0001	0,2384	0,8876
Probit (Broken curve)	0	0	20	27	30	21	47	115	169	256	334	439	535	632	649	538	141	1	158	-0,0805	0,5321	26,9415	0,0292	0,0001	0,2626	0,8770
Probit - Platt (Broken curve)	0	0	20	26	12	34	37	102	184	247	341	423	562	632	654	546	133	1	158	-0,1025	0,5408	34,4180	0,0030	0,0001	0,2582	0,8789
Probit - Box-Cox (Broken curve)	0	0	0	0	0	0	0	0	20	272	503	655	717	666	546	357	149	69	158	0,0575	0,4771	3,5298	0,9396	0,0001	0,2880	0,8659
CLL (Broken curve)	0	0	0	0	0	0	20	79	171	288	419	495	613	633	601	435	191	9	158	-0,0324	0,5129	6,7619	0,8180	0,0001	0,2290	0,8918
CLL - Platt (Broken curve)	0	0	0	0	0	0	20	72	169	282	421	494	610	650	601	441	186	8	158	-0,0568	0,5226	6,9662	0,8018	0,0001	0,2304	0,8912
CLL - Box-Cox (Broken curve)	0	0	0	0	0	0	0	0	20	243	493	656	748	684	538	358	145	68	159	0,0708	0,4718	4,1293	0,9027	0,0001	0,4228	0,8095
Cauchit (Broken curve)	0	0	0	0	0	0	0	0	0	0	0	373	1456	1186	558	248	64	58	169	-0,1263	0,5503	16,5599	0,0110	0,0001	1,2777	0,5279
Cauchit - Platt (Broken curve)	0	0	0	0	0	0	0	0	0	0	0	363	1457	1190	563	248	64	60	167	-0,1433	0,5570	16,3799	0,0119	0,0001	1,2273	0,5414
Cauchit- Box-Cox (Broken curve)	0	0	0	0	0	0	0	0	0	82	419	728	846	776	569	340	137	56	159	-0,0485	0,5193	6,8013	0,5582	0,0001	0,3755	0,8288