A Systematic Approach to Systemic Risk
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Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England or to state Bank of England policy.
Motivation

• emergence of structure as factor in economic analysis
  – financialization at macro, meso, and micro levels of economy
    (Van der Zwan, 2014)
  – propagation of influence and the challenges to causal analysis

• need for a comprehensive exploration of systemic risk
  (Schwarcz, 2011)

  system-of-systems model with macro, meso, and micro sub-systems
  (Haldane, 2015); (Sergueieva, 2016; Sergueieva & Bholat, 2016, 2017)
  – single-layer network vs multiplex at macro level
    (De Domenico et al., 2013); (Buldyrev et al., 2010);
    (Sergueieva, 2013b, 2016; Sergueieva & Bholat, 2016, 2017)
  – extending the scope of analysis beyond the banking system:
    systemic risk is not contained within the banking system
    (Sergueieva, 2016; Sergueieva & Bholat, 2016, 2017)

• requirement for new approaches to modelling interconnectedness
  – network links vs tensors balancing behavioural tendencies of
    nodes/institutions
    (Sergueieva, 2016)
Motivation

- evaluation of *multichannel* contagion dynamics and systemic impact
  - *multichannel* vs single-channel contagion
  - systemic impact in a single market vs *systemic impact within multiple interconnected markets*

  (Markose, 2012); (Serguieva, 2016; Serguieva & Bholat, 2016, 2017)
Conclusions

• we formulate and empirically evaluate the minimum-cost strategy for multiple-market stabilisation: the strategy is capable of containing contagion and involves all UK-incorporated deposit takers and significant investment firms within the strongly connected component of the structure of inter-institutional impacts

• the stabilisation strategy involves carefully estimated and minimally invasive rebalancing of how the system copes with the emerged structure of exposures

• this work formulates and empirically evaluates structural measures of systemic-risk and systemic-resilience for the banking system as a whole, as well as relative systemic-risk indexes for each institution and the absolute contribution of each institution to the overall systemic risk

• systemic-stability conditions of banks’ interactions in multiple interconnected markets are evaluated as a structural characteristic and does not depend on the trigger or type of crisis

• the multiple-market (multiplex) systemic-risk measures and indexes are a more realistic basis for regulation

• the formulation of the tensor-multiplex facilitates the simulation of multichannel contagion dynamics and the design of stabilisation strategies
Conclusions

• different institutions have the higher systemic impacts in multiple markets vs single markets, and as institutions realistically interact in multiple markets then stabilisation strategies should be based on institutions' multiple-market impact

• different institutions have the higher systemic impacts under EAD and NAC scenarios, and as both scenarios are plausible though in different parts of the system at different stages of the contagion process, then the rebalancing of impact should be based on both scenarios

• evaluating empirically multiple-market centralities of the wider system of inter-institutional exposures - involving banks, non-bank financial institutions, non-financial corporates, governments, central banks, and supranational organisations – allowed us to identify concentration of vulnerabilities within the wider economic system

• tensors are more realistic models of complex structures, as they capture not only magnitudes of interlinkages but also tensions within the structure due to behavioural tendencies of nodes with potential to change the magnitudes, and this can be used to extend the scope of stress-testing scenarios
Findings and empirical results

- **formulation and empirical simulation of:**
  - stability conditions
  - the dynamics of multi-channel vs single-channel contagion
  - multiple-market stabilisation processes vs single-market processes

- different institutions have the higher systemic impacts in multiple interconnected markets vs single markets
**Multiplex**: Four-dimensional decomposition of inter- and intra-link magnitudes of impact $S$

It captures the impact magnitudes among institutions within each financial market and between any pair of markets.
<table>
<thead>
<tr>
<th>multi-channel systemic impact, through the fixed-income, SFT and derivatives markets</th>
<th>single-channel systemic impact, through the derivatives-market</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of reporting banks</td>
<td>$n = 22$</td>
</tr>
<tr>
<td>number of banks in the strongly connected subtensor</td>
<td>$m_{total_impact} = 19$ (18 overlapping banks)</td>
</tr>
<tr>
<td>$p_{total_impact_connected}$</td>
<td>0.14573</td>
</tr>
<tr>
<td>stability (insolvency) condition</td>
<td>$\lambda_{total_impact_connected}^{max} &lt; 0.14573$</td>
</tr>
<tr>
<td>for $\gamma_{total_impact} = 0$ (no stabilisation step implemented)</td>
<td>0.47440</td>
</tr>
<tr>
<td>$\lambda_{total_impact_connected}^{max}$ for $\gamma_{total_impact} = 0.02850$ (a stabilisation step implemented)</td>
<td>0.14568</td>
</tr>
</tbody>
</table>
Findings and empirical results

- *formulation and empirical simulation of stability conditions, and the dynamics of multi-channel vs single-channel contagion*
  - different institutions have the higher systemic impacts in multiple interconnected markets vs single markets

<table>
<thead>
<tr>
<th>Institution $i$</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>rank</strong> at $\gamma_{total_impact} = 0$, (multiple-market contagion dynamics)</td>
<td>2</td>
<td>10</td>
<td>0  (not participating in the multiplex strongly-connected component)</td>
</tr>
<tr>
<td><strong>rank</strong> at $\gamma_{derivatives} = 0$, (single-market contagion dynamics)</td>
<td>17</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td><strong>SRI</strong>$<em>{index_total_impact}(i)$ at $\gamma</em>{total_impact} = 0$ (multiple-market systemic-risk index)</td>
<td>16.34%</td>
<td>0.33%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>SRI</strong>$<em>{derivatives_EAD_index}(i)$ at $\gamma</em>{derivatives} = 0$ (single-market systemic-risk index)</td>
<td>0.28%</td>
<td>4.05%</td>
<td>0.23%</td>
</tr>
</tbody>
</table>
Findings and empirical results

• **formulation and empirical simulation of single-market stabilisation processes**

\[
S_{\text{EAD connected}}^{\text{rebalanced}} = \left[ s(ij)_{\text{EAD connected}}^{\text{rebalanced}} \right] = \\
\begin{bmatrix}
S(ij)_{\text{EAD connected}} \\
1 + \sum_{i=1}^{m_{\text{EAD}}} \gamma_{\text{EAD}} SRI(i)_{\text{EAD index connected}} \left( \frac{C(i)_{\text{EAD modified connected}}}{p_{\text{min connected}}} \right) \left( \frac{C(j)_{\text{EAD modified connected}}}{m_{\text{EAD}}} \right) \sum_{q=1}^{i-1} S(iq)_{\text{EAD connected}}
\end{bmatrix}
\]

\[
s(ij)_{\text{EAD connected}} = \begin{cases} 
\frac{EAD(iij)_{\text{EAD connected}}}{\text{EAD}} C(j)_{\text{modified connected}} & \geq 0, i \neq j \\
0, i = j
\end{cases} ; \\
p_i = \frac{A(i)_{\text{EAD connected}}}{C(i)_{\text{EAD connected}}}
\]
Findings and empirical results

- **formulation and empirical simulation of stabilisation processes in multiple interconnected markets**

\[
\begin{align*}
\text{for } 0 & \leq \gamma_{\text{total impact}} < 1; \\
1 & \leq i, j \leq m_{\text{total impact}}; 1 \leq k, \ell \leq 3; \\
S_{\text{total impact connected}}^{\text{rebalanced}} &= \left[ s(ijk\ell)_{\text{total impact connected}}^{\text{rebalanced}} \right] = \\
&= \left[ \begin{array}{c}
S(ijk\ell)_{\text{total impact connected}} \\
\sum_{i=1}^{m_{\text{total impact}}} \sum_{\text{index connected}} SRI(i)_{\text{total impact}} \\
1 + \sum_{i=1}^{m_{\text{total impact}}} \gamma_{\text{total impact}} \\
\end{array} \right] \\
&= s(ijk\ell)_{\text{total impact connected}}^{\text{modified connected}} \begin{array}{c}
\sum_{z=1}^{3} \sum_{y=1}^{3} \left( s(ijk\ell)_{\text{total impact connected}}^{\text{rebalanced}} \right) \\
\sum_{z,y=1}^{3} \sum_{q=1}^{m_{\text{total impact}}} \left( s(iqyz)_{\text{total impact connected}} \right) \\
\end{array}
\end{align*}
\]

\[
s(ijk\ell)_{\text{total impact connected}}^{\text{modified connected}} = \begin{cases} 
\frac{\sum_{v=1}^{n} s(vikk)}{C(j)_{\text{modified connected}}} & , i = j, k \neq \ell \\
C(i)_{\text{modified connected}} & , i \neq j, k \neq \ell \\
s(ij) & , i \neq j, k = \ell 
\end{cases}
\]

\[
C(i)_{\text{modified connected}} = \min_{1 \leq i \leq m_{\text{total impact}}} \frac{p_i C(i)_{\text{connected}}}{(p_i)}
\]
Findings and empirical results

• formulation and empirical evaluation of absolute vs relative systemic-risk measures

- relative systemic-risk indexes

\[
SRI(i)_{\text{total\_impact \ index connected}} = \left\{
\begin{array}{l}
= SRI(i)_{\text{total\_impact \ connected}} = \frac{r(i)_{S_{\text{total\_impact \ connected}}}}{\sum_{i=1}^{m_{\text{total\_impact \ connected}}}} (r(i)_{S_{\text{total\_impact \ connected}}}) \\
\text{for } r_{S_{\text{total\_impact \ connected}}} = U_{S_{\text{total\_impact \ connected}}}[1\, 1\, 1]' \text{ and } 1 \leq i \leq m_{\text{total\_impact \ connected}} \\
= SRI(i)_{\text{total\_impact \ index unconnected}} = 0 \text{ for } m_{\text{total\_impact \ connected}} \leq i \leq n
\end{array}
\right.
\]

- absolute systemic risk measure

\[
SR_{R_{\text{risk}}}^{\text{total\_impact}} = \left\{
\begin{array}{l}
\lambda_{\text{total\_impact \ connected}}^{\max} - p_{\text{min \ connected}}^{\text{total\_impact}} > 0 \\
0, \text{ if } \lambda_{\text{total\_impact \ connected}}^{\max} - p_{\text{min \ connected}}^{\text{total\_impact}} < 0
\end{array}
\right.
\]
Findings and empirical results

- formulation and empirical simulation of scenarios in the derivatives market

<table>
<thead>
<tr>
<th>going-concern scenario</th>
<th>at-default scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of reporting banks</td>
<td>$n = 22$</td>
</tr>
<tr>
<td>number of banks in the strongly connected subtensor</td>
<td>$m_{NAC} = 16$</td>
</tr>
<tr>
<td>$p_{NAC}^{\text{connected}}$</td>
<td>0.26843</td>
</tr>
<tr>
<td>stability condition</td>
<td>( \lambda_{NAC}^{\text{connected}} \leq 0.26843 )</td>
</tr>
<tr>
<td>( \lambda_{NAC}^{\text{connected}} ) for ( \gamma_{NAC} = 0 ) (no regulatory surcharges implemented)</td>
<td>0.00715</td>
</tr>
</tbody>
</table>
Findings and empirical results

- formulation and empirical simulation of scenarios in the derivatives market

<table>
<thead>
<tr>
<th>institution $i$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>rank at $\gamma_{NAC} = 0$, (going-concern systemic dynamics)</td>
<td>10</td>
<td>8</td>
<td>0 (not participating in strongly-connected subtensor)</td>
<td>7</td>
</tr>
<tr>
<td>rank at $\gamma_{EAD} = 0$, (at-default systemic dynamics)</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>$SRI_{\text{index connected}}^{NAC} (i)$ at $\gamma_{NAC} = 0$</td>
<td>2.34%</td>
<td>3.01%</td>
<td>0%</td>
<td>4.00%</td>
</tr>
<tr>
<td>$SRI_{\text{index connected}}^{EAD} (i)$ at $\gamma_{EAD} = 0$</td>
<td>13.15%</td>
<td>5.09%</td>
<td>4.03%</td>
<td>0.81%</td>
</tr>
</tbody>
</table>

- systemic stability and quality can be achieved through targeting different institutions to a different extent under the two scenarios: one surcharge:

$$SRS(i)_{\text{surcharge}}^{\text{derivatives}} = f \left( \gamma_{EAD} SRI(i)_{\text{index connected}}^{EAD}, \gamma_{NAC} SRI(i)_{\text{index connected}}^{NAC} \right)$$

- systemic-resilience measures
Findings and empirical results

- **theoretical formulation, domain interpretation, and empirical evaluation of the tensor-multiplex and tensor-monoplexes, and the corresponding multiple-market vs single-market centralities of institutions in the wider system** incorporating banks, non-bank financial institutions, non-financial corporations, central banks, governments, and supranational organisation (over 1,500 institutions)
Findings and empirical results

empirical:

wider system
prior to introducing
the multiplex structure
Findings and empirical results

exposures due to the fixed-income market

exposures due to the securities-financing market

exposures due to the derivatives market

empirical: wider economic system as multiplex structure
Multiplex: Four-dimensional decomposition of inter- and intra-market magnitudes of exposure $W$

It captures the magnitudes of exposures among institutions within each financial market and between any pair of markets.

derivatives market ($D$): a three-dimensional decomposition of size $n \times n \times 3$ of exposures, where the exposed institutions are in market $D$

securities-financing market ($SFT$): a three-dimensional decomposition of size $n \times n \times 3$ of exposures, where the exposed institutions are in market $SFT$

fixed-income market ($FI$): a three-dimensional decomposition of size $n \times n \times 3$ of exposures, where the exposed institutions are in market $FI$
Findings and empirical results

- empirical evaluation *for the wider economic system*

## Strength Centralities – in, out, total

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SFT rank</td>
<td>SFT rank</td>
<td>SFT rank</td>
<td>SFT rank</td>
</tr>
<tr>
<td></td>
<td>(in-strength)</td>
<td>(out-strength)</td>
<td>(total-strength)</td>
<td>(in-strength)</td>
</tr>
<tr>
<td>XXI</td>
<td>6 (4)</td>
<td>4 (-2)</td>
<td>4 (-2)</td>
<td>8 (+2)</td>
</tr>
<tr>
<td>XXII</td>
<td>9 (22 (+13))</td>
<td>24 (+15)</td>
<td>11 (+2)</td>
<td>19 (+10)</td>
</tr>
<tr>
<td>XXIII</td>
<td>12 (0)</td>
<td>18 (+6)</td>
<td>12 (0)</td>
<td>14 (+2)</td>
</tr>
<tr>
<td>XXIV</td>
<td>13 (22 (+9))</td>
<td>28 (+15)</td>
<td>173 (+160)</td>
<td>19 (+6)</td>
</tr>
<tr>
<td>XXV</td>
<td>14 (22 (+8))</td>
<td>29 (+15)</td>
<td>10 (-4)</td>
<td>19 (+5)</td>
</tr>
<tr>
<td>XXVI</td>
<td>19 (3 (-16))</td>
<td>3 (-16)</td>
<td>75 (+56)</td>
<td>2 (-17)</td>
</tr>
<tr>
<td>XXVII</td>
<td>22 (12 (-10))</td>
<td>12 (-10)</td>
<td>42 (+20)</td>
<td>9 (-13)</td>
</tr>
<tr>
<td>XXVIII</td>
<td>23 (22 (-1))</td>
<td>22 (-1)</td>
<td>21 (-2)</td>
<td>12 (-11)</td>
</tr>
<tr>
<td>XXIX</td>
<td>29 (10 (19))</td>
<td>10 (19)</td>
<td>36 (+7)</td>
<td>6 (-23)</td>
</tr>
<tr>
<td>XXX</td>
<td>39 (7 (-32))</td>
<td>7 (-32)</td>
<td>32 (-7)</td>
<td>5 (-34)</td>
</tr>
</tbody>
</table>
Findings and empirical results

- empirical evaluation *for the wider economic system*

**Katz-Bonacich Centrality**

March 2015

<table>
<thead>
<tr>
<th>institutions</th>
<th>fixed-income rank (Katz-Bonacich)</th>
<th>fixed-income rank (total-strength)</th>
<th>fixed-income rank gap</th>
<th>SFT rank (Katz-Bonacich)</th>
<th>SFT rank (total-strength)</th>
<th>SFT rank gap</th>
<th>derivatives rank (Katz-Bonacich)</th>
<th>derivatives rank (total-strength)</th>
<th>derivatives rank gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>XXXI</td>
<td>2</td>
<td>8</td>
<td>+6</td>
<td>1</td>
<td>11</td>
<td>+10</td>
<td>5</td>
<td>24</td>
<td>+19</td>
</tr>
<tr>
<td>XXXII</td>
<td>4</td>
<td>13</td>
<td>+9</td>
<td>19</td>
<td>211</td>
<td>+192</td>
<td>13</td>
<td>46</td>
<td>+33</td>
</tr>
<tr>
<td>XXXIII</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>18</td>
<td>168</td>
<td>+150</td>
<td>1</td>
<td>5</td>
<td>+4</td>
</tr>
<tr>
<td>XXXIV</td>
<td>8</td>
<td>33</td>
<td>+25</td>
<td>9</td>
<td>90</td>
<td>+81</td>
<td>15</td>
<td>90</td>
<td>+75</td>
</tr>
<tr>
<td>XXXV</td>
<td>10</td>
<td>16</td>
<td>+6</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>12</td>
<td>21</td>
<td>+9</td>
</tr>
<tr>
<td>XXXVI</td>
<td>12</td>
<td>14</td>
<td>+2</td>
<td>13</td>
<td>23</td>
<td>+10</td>
<td>2</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>XXXVII</td>
<td>14</td>
<td>25</td>
<td>+11</td>
<td>6</td>
<td>2</td>
<td>-4</td>
<td>8</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>XXXVIII</td>
<td>16</td>
<td>22</td>
<td>+6</td>
<td>12</td>
<td>87</td>
<td>+75</td>
<td>17</td>
<td>145</td>
<td>+128</td>
</tr>
<tr>
<td>XXXIX</td>
<td>18</td>
<td>30</td>
<td>+12</td>
<td>14</td>
<td>12</td>
<td>-2</td>
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<td>+432</td>
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<tr>
<td>XL</td>
<td>20</td>
<td>36</td>
<td>+16</td>
<td>19</td>
<td>55</td>
<td>+36</td>
<td>21</td>
<td>20</td>
<td>-1</td>
</tr>
</tbody>
</table>
Findings and empirical results

• *theoretical formulation and domain interpretation of the tensor-multiplex model*

  - a link between two nodes may be represented through a binary number, a scalar, a vector, a tensor of rank 2, or a tensor of rank 4 - each of these mathematical constructs, in that order, progressively captures and communicates more complex and realistic information

  - the structure is not just the magnitudes of the exposures themselves but also the tension within the system due to the behaviour of institutions with potential to change exposures – behavioural tendencies of institutions are part of the tensorial structure through the basis and dual-basis vectors
Findings and empirical results

• formulation and domain interpretation of the tensor-multiplex model for the structure of inter-institutional exposures within the wider economic system (1,500 institutions of different type)

\[
W = \sum_{k=1}^{m} \sum_{\ell=1}^{m} \sum_{j=1}^{n} \sum_{i=1}^{n} \left( W_{ij}^{\ell} \right) \Gamma(x_i, x_j, f_\ell, f_k) = \\
= \sum_{k=1}^{m} \sum_{\ell=1}^{m} \sum_{j=1}^{n} \sum_{i=1}^{n} \left( W_{ij}^{\ell} \right) \tilde{\varepsilon}_i \otimes \tilde{\omega}^j \otimes \tilde{\varepsilon}_\ell \otimes \tilde{\omega}^k'
\]

for \( W_{ij}^{\ell} \) \( = 0 \) if \( i = j \land \ell = k \lor i \neq j \land \ell \neq k \)
\( \neq 0 \) if \( i \neq j \land \ell = k \lor i = j \land \ell \neq k \)
Findings and empirical results

• empirical evaluation for the wider economic system

  – multiplex centrality

<table>
<thead>
<tr>
<th>institution</th>
<th>average rank (Katz vector)</th>
<th>weighted rank (Katz vector)</th>
<th>tensor rank (Katz tensor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XLI</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>XLII</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>XLIII</td>
<td>9</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>XLIV</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>XLV</td>
<td>14</td>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

  – Katz-tensor centrality is mathematical extension of the original definition – the aggregation is performed within the calculation and does not require any heuristic choice, as it is already accounting for the whole interconnected structure.
Findings and empirical results

• formulation and empirical evaluation of influence-tensor among financial stability factors in the orthonormal reference frame of their principal components (the selected factors are of regulatory significance for detecting early warning signals of system-wide vulnerabilities)

  – factors as vectors in the reference frame of factors’ principal components

  \[ \vec{f}_p = \begin{bmatrix} f_{p,1} & \cdots & f_{p,m} \\ \cdots & \cdots & \cdots \\ f_{p,1} & \cdots & f_{p,m} \end{bmatrix} = \sum_{\ell=1}^{m} f_{p,\ell} (pc \, \vec{E}_\ell) \]

  – tensor product as a link (influence) between two factors

  \[ pc \, F = \sum_{q=1}^{v} \sum_{p=1}^{v} \sum_{k=1}^{m} \sum_{\ell=1}^{m} f_{p,\ell} f_{q,k} \left( pc \, \vec{E}_\ell \otimes pc \, \vec{E}_k' \right) = \sum_{k} \sum_{\ell} pc \, F_{k}^{\ell} \left( pc \, \vec{E}_\ell \otimes pc \, \vec{E}_k' \right) \]
Conclusions
(slides 4-5)

Questions