Simultaneous-Equations Model for Global-Flow-Funds Analysis

Nan Zhang

Faculty of Economic Sciences

Hiroshima Shudo University

Hiroshima, 731-3159 Japan

zhang@shudo-u.ac.jp

Global-Flow-of-Funds (abbreviated to GFOF) are flow of funds that relates to domestic flows and international capital flows. The analysis of GFOF is an analysis which shows the characteristics and the structure in the flows of fund, includes the flows of all domestic funds with investment-savings, links current balance, and connects international capital flows.

1. The Mechanism of Global-Flow-of-Funds

The financial markets indicate the debts and credits of funds as a whole plus the total process of financial liquidity. Investigated more carefully, items of financial markets include inflows of domestic funds, overseas funds by domestic savings and credit loans of banks on the side of fund-sources (funds inflow). On the other hand, funds split into supply of funds to the domestic economy and funds outflow overseas in fund uses (funds outflow). The figure 1 shows the mechanism of GFOF.

Figure 1 shows the mechanism of GFOF between three countries of A, B and C, an international financial market, and an international organization. The economy of three countries consists of the balance of savings-investment which shows real economy activity, and the financial market which indicates the financial circulation of funds. As spread between Domestic and Overseas balance of the economy in each country, real economy (savings-investment balance) serves as a current balance, and the fund loan balance in the financial market serves as the capital balance. And the current transaction and capital transaction of each country are connected mutually internationally, and a part of capital transaction link to international financial market, IMF, the World Bank, etc., that is formed with the ring of the whole of GFOF.

On the Fig.1, the excess savings corresponding to the black figures of the current balance occur in a capital supplier (in A country), and it becomes a relation which financial assets carry out net increase in financial sector. Financial market receives inflow of the funds from the sectors of Domestic and Overseas, and supplies funds to the two sectors simultaneously. In the fund loan balance of A country financial market, it becomes the net inflow of funds corresponds with excess savings balance in domestic sector, and the net outflow of funds (include the change in foreign exchange reserves) correspond to the current account surplus in the overseas sector. That is, the net increase of the financial assets which offsets the excess of domestic savings is balanced with the net inflow of funds from the domestic sector in financial market. And the external claimable assets formed corresponding to current balance surplus becomes the net outflow of the funds from financial market as supply of overseas lending.
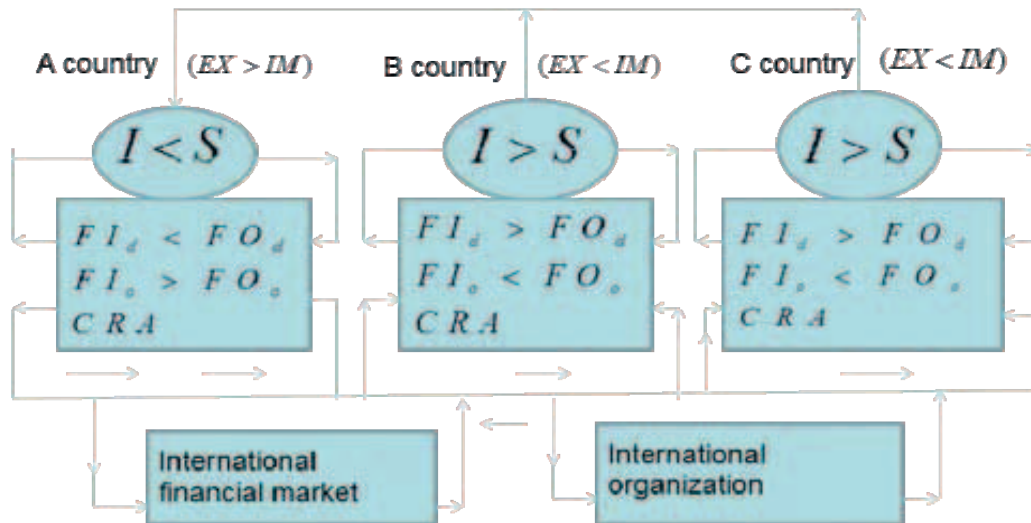
Fig. 1 the Mechanism of Global-Flow-of-Funds

Notes: FI_d : domestic inflow of funds, FI_o : overseas inflow of funds
FO_d : domestic outflow of funds, FO_o : overseas outflow of funds
CRA : Changes in Reserve Assets

On the other hand, in a capital importer as a country of B or C, current balance deficit become a relation that is with the domestic excess of investment (savings deficit) and the net increase of the financial liability in financial sector. In the financial market, it becomes an excess of credit with the domestic excess of investment, and current account deficit is financed by the net inflow of funds (capital balance surplus) from overseas. Therefore, on the flow of funds balance with the sectors of the domestic-and-oversea in the financial market of B and C country, it becomes a net outflow of funds with the domestic sector, and the net inflow of funds with the overseas sector. The net inflow of funds of the overseas sector becomes a source of funds for domestic sector that try to keep a balance of credit. And the net outflow of funds into domestic sector in the financial market will serve as over-borrowing about the domestic sector, that is, become the net increase on the financial liability.

2. Analytical Framework

When the flow of funds in financial markets is tied up with the international balance of payments, the overseas sector will become fund outflow excess (net capital outflows) if the current account is in surplus. Conversely, the domestic sector will become fund inflow excess. Therefore, when the real economy side of the domestic economy and overseas is analyzed under an open economic system, the balance of savings-investment of the domestic economy corresponds to the current account balance. According to the dynamic process of external flow of funds and the definitional equation of a System of National Account, from the accounting identity as follows,

$$Y = C + I + G + EX - IM$$

We obtain the equilibrium condition below with through arranging the above formula.

$$S - I = EM - IM$$

The right side of equilibrium condition shows Current Account, and the left side of equilibrium condition shows the balance of savings-investment, or Net Financial Investment. So we obtain the relationship of equilibrium condition equations as follows.

$$S - I = EX - IM = NFI$$

However, domestic net funds outflow correspond with the capital account balance, when the relationship between domestic and overseas on the financial side is examined. Therefore, relations between the domestic savings-investment balance, the financial surplus or deficit, the current account, and the overseas net fund outflow will be expressed in the following structural formulae.

Savings-Investment and Current Account Balance

$$S - I = \Delta FA - \Delta FL = EX - IM \qquad (1)$$

Overseas Income and Expenditures Balance

$$EX - IM = (FO - FI) + CRA \qquad (2)$$

Changing the above formula to:

$$(EX - IM) + (FI + FO) = CRA$$

Financial-Markets Balance

$$FO_d + FO_o + CRA = FI_d + FI_o \qquad (3)$$

External Flow of Funds Balance as follows:

$$FO_o - FI_o + CRA = FI_d - FO_d \qquad (4)$$

We can get the constitution of net overseas flow of funds

$$(FO_o - FI_o) = DI + PI + OI + CaA \qquad (5)$$

Where S: gross savings, I: gross investments, ΔFA: change in financial assets, ΔFL: change in financial liabilities, EX: exports, IM: imports, FO: fund outflow, FI: funds inflow, DI: Direct investment, PI: Portfolio investment, OI: Other investment, CaA: Capital account.

Formula (1) shows a relationship between the savings-investment and domestic fund flows and current account. When $S > I$, $\Delta FA > \Delta FL$ is capital surplus, it will become $CA > 0$, that is mean current account is a surplus. On the other side, when $S < I$ and $\Delta FA < \Delta FL$, there exists financial deficit in the flow of funds, it will become $CA < 0$, that is mean current account is deficit. By the second formula, we can know that global flow of funds and international flow of goods is like the two side of a coin.

When current account is in surplus, capital account will be in deficit ($FO > FI$, only outflow of funds), or an increase in foreign reserves can make a country's external claim to increase. Otherwise, when domestic investment is larger than domestic savings, current account is in deficit, external debt can only increase through inflow of capital ($FO < FI$, capital balance is in surplus), or decreasing foreign reserves as a compensation for the deficit. From changing formula (2), we can get the constitution of foreign reserves, which shows that the simultaneous existence of current balance and capital balance surplus will increase foreign reserves. In other words, when current balance is in surplus while overseas capital flows in, account system shows capital balance become surplus. As a result, foreign reserves increases rapidly, which leads to a systematic problem in external flow of funds. From formula (2), main factors that affect the change in foreign reserves are current balance and capital balance. Since the change in current balance is due to the balance of savings-investment, change in foreign reserves actually depends on the change in the structure of flow of funds. Also exchange rate, which often affects the current balance, international

market rates which affects capital expenditure, as well as the benchmark interest rate of Central Bank are basic factors affecting foreign reserves.

Formula (3) shows the balance in general financial market and reflects the relationship between domestic capital flows and international capital flows. This process is the so called change in GFOF. And the right side of (4) is net increase of domestic financial assets, and is the surplus or deficit items in the flow of funds account; same as the current balance in Balance of Payments (see formula (1)). Left hand side shows that net external flow of funds plus the change in Reserve Assets, where net external flow of funds corresponds to the Capital & Financial Account in Balance of Payments. When $S > I$ and current balance is surplus, net external flow of funds becomes a path for a country to accumulate wealth[1]. Otherwise, when current balance is deficit, we can compensate for the deficit only through decreasing external assets or increase external debt. The decrease in external assets shows that this process is an operation of capital expenditure. Besides, when current balance is zero, however frequent the operation for capital expenditure is, there is no influence on net foreign assets. Therefore, we know that a country's change in Reserve Assets come from the accumulation of the current balance surplus, and not directly related to frequent financial trade.

It is apparent that the net overseas flow of funds $(FO_o - FI_o)$ corresponds with the Capital & Financial Account in Balance of Payment by (4) and Capital & Financial Account is constituted of Financial Account and Capital Account, the net overseas flow of funds as formula (5). It shows the balance relationship among savings investment gaps, funds surplus or deficit, and balance of payments, as well as the structure of net external flows of funds. Consequently formula (1) to formula (5) are used to create the analytical framework of GFOF. Based on this analytical framework, we will perform descriptive statistical analysis on the characteristics and mirror image relationship of the external flow of funds between the US and China.

3. The Empirical Model

Three techniques are generally used for joint estimation of the entire system of equations: Three-Stage Least Squares (3SLS), Generalized Method of Moments (GMM), and Maximum Likelihood. The model of GFOF was built by 3SLS method. When Two-Stage Least Squares (2SLS) was used for joint estimation of the entire system of equation, 2SLS assumed no correlation between error terms ε_1 and ε_2 in simultaneous equations. On the other hand, 3SLS is presuming that exist correlation between error terms ε_1 and ε_2 of simultaneous equations.

As the presumed method, 2SLS is how to solve the equation of each of structural equations separately. But compared with this method, 3SLS is the methods for presuming simultaneous equations that take into consideration all directions of a simultaneous-equations system using the variance-covariance matrix of the error term between equations. Intuition would surely suggest that systems method, 3SLS is to be preferred to single-equation methods 2SLS. The estimator of presumption that was by 3SLS is a consistent estimator, and when the disturbance terms of each structural equation have correlation, it becomes the estimator of effective presumption more asymptotically than 2SLS

We begin with a slightly abstract linear model

$$Y = \alpha + X\beta + \varepsilon_i \qquad (6)$$

Where Y is an observable $n \times 1$ random vector, X is an observable $n \times p$ random matrix, and β is an

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[1] By the formula (1), we can know that *S=I + overseas net assets*. So accumulation of national wealth of a country contains domestic fixed capital formation and overseas net assets.

unobservable $p \times 1$ parameter vector. The ε_i are IID with mean 0 and finite variance σ^2; they are unobservable random errors. This is the standard regression model, except that X is endogenous, i.e., X and α are dependent. Conditional on X, the OLS estimates are biased by $(X^{'}X)^{-1}X^{'}E(\alpha \mid X)$. This is simultaneity bias.

The model of GFOF is based on *IS-LM* theory and general equilibrium theory, and it was created by the form of simultaneous-equation. Creation of this model has two purposes. One of the purposes is related to the immanent factors of many economic variables. It estimates the quantity relation between change of the savings and investment, export and import, external flow of funds and economic growth in certain country, and reflects the structural changes of the flow of funds in a medium-to-long period of time.

There are 11 endogenous variables and 17 exogenous variables in the model of GFOF. This model referred to the principle of Applied General Equilibrium model. This model takes the change in the formation of expectation and risk, it with a lag structure or immanent relevant factors of many economic variables, and explains the state of the funds flow in the continuous adjustment process to the balance from imbalance, and in a medium-to-long period of time. It also belongs to a kind of dynamic model. A system of simultaneous equations is a model of financial market equilibrium, consisting of the follows.

Through those simultaneous-equations models, we want to observe systematically the structural factor and cyclic factor in flow-of-funds; how the profit factor and the risk factor affect international capital flows, and how the change of the pattern of funds flow affect domestic economy growth. In order to perform prediction of a future flow-of-funds trend, and the simulation of the policy effect at the end of the model, three definitional equations called Net Flow of Funds, Current Account Balance, National Income identical equation are formed.

Structural Equations

(1) Savings
$$S_t = b_{11} + b_{12}DI_t + b_{13}C_{t-1} + b_{14}R_t + \varepsilon_{t1}$$

(2) Investment
$$I_t = b_{21} + b_{22}\Delta Y_{t-1} + b_{23}G_t + b_{24}R_t + \varepsilon_{t2}$$

(3) Import
$$IM_t = b_{31} + b_{32}IPI_t + b_{33}Y_t + \varepsilon_{t3}$$

(4) Export
$$EX_t = b_{41} + b_{42}REER_t + b_{43}WGDP_t + b_{44}CEUS_t + \varepsilon_{t4}$$

(5) Capital inflow
$$FI_t = b_{51} + b_{52}YR_{t-1} + b_{53}PER_t + b_{54}FDI_t + b_{55}DR_t + b_{56}D_t + \varepsilon_{t5}$$

(6) Capital outflow
$$FO_t = b_{61} + b_{62}CRA_t + b_{63}PI_t + b_{64}RCB_t + b_{65}FFR_t + b_{66}D_t + \varepsilon_{t6}$$

(7) Reserve asset
$$CRA_t = b_{71} + b_{72}CA_t + b_{73}FI_t + b_{74}RCB_t + b_{75}GBYUS_t + b_{76}REX_t + \varepsilon_{t7}$$

(8) Exchange rate
$$REX_t = b_{81} + b_{82}DR + b_{83}PER + b_{84}YR_{-1} + b_{85}REX_{-1} + \varepsilon_{t8}$$

Identities Equation

(9) Net Fund flow definition
$$NFI_t = FO_t - FI_t$$

(10) Current balance definition
$$CA_t = NFI_t + CRA_t$$

(11) GDP identical equation
$$Y_t = C_t + I_t + NFI_t$$

4. Conclusions

The estimating method of the simultaneous equation can use the way with 2SLS and 3SLS, and we found that presumed method of 3SLS is more exact than 2SLS. We also explained the mechanism of the model of GFOF using the flowchart, and explained the causal relationship of each endogenous variable and exogenous variables in the model funds from three different viewpoints. In the end, the model was built by simultaneous equation. In order to create this model, a huge set of statistical data is required. This model can be more realistic by uniting descriptive analysis and inferential analysis, and the better result will be obtained when change of GFOF is seen in dynamic state. If the time series of the statistical data used in the model are too long, we need to examine whether there exists a problem of spurious regression in the time series. How to take in the time-series-analysis technique to simultaneous-equation model is our next research project.

Table 1 The list of Variables and Definition

Variable	Variable name	Classification	Variable	Variable name	Classification
Y	GDP	Endogenous	FO	Fund outflows	Endogenous
Y2	Dif2 of lag of GDP	Exogenous	FI	Fund inflows	Endogenous
S	Gross Savings	Endogenous	NFI	Net financial investment	Endogenous
DI	Disposable Income	Exogenous	YR	Economic growth rate	Exogenous
I	Gross Investment	Endogenous	FDI	Foreign direct investment	Exogenous
C	Final Consumption	Exogenous	RCB	Interest rates of central bank	Exogenous
EX	Export	Endogenous	G	Government expenditure	Exogenous
IM	Import	Endogenous	PI	Profit from Investment	Exogenous
CA	Current account	Endogenous	FFR	Federal Funds Rate of the US	Exogenous
REER	Real Effective Exchange Rate	Exogenous	WGDP	Total GDP of Japan ,USA and EU	Exogenous
REX	Exchange rates	Endogenous	IPI	Import Price Index	Exogenous
R	One-year loans interest	Exogenous	DR	Difference of Interest Rate	Exogenous
PER	Price earnings ratio	Exogenous	D	Dummy Variables	Exogenous
CRA	Changes in Reserve Assets	Exdogenous	GBYUS	Government Bonds Yields of the US	Exogenous
			CEUS	Consumption Expenditure of the US	Exogenous

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